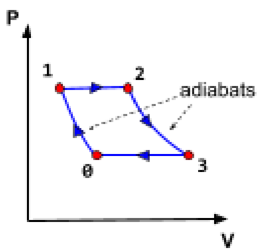
The Thermodynamics of a Ramjet Engine

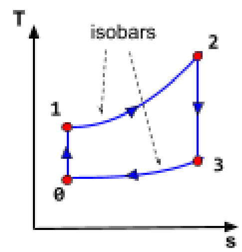
The thermodynamics of a ramjet can be broken down to mirror aspects of a reversible Brayton cycle. Under the Brayton cycle, work is extracted through the application of two isobaric and two adiabatic processes. The working substance for this type of engine is the incoming air, which we treat as an ideal diatomic gas. For a ramjet engine, the cycle begins with an adiabatic compression (*Steps 0-1*) of the incoming air, which reduces the incoming velocity to approximately zero with respect to the aircraft. The engine is increasingly efficient with higher incoming velocities, and is infinitely inefficient (i.e. $\eta = 0$) without an incoming velocity. Fuel is then combusted to supply heat to the stagnant air at constant pressure (*Steps 1-2)*, increasing their temperature and speed of flow. This increase in internal energy results in an adiabatic expansion of the gas after combustion in the nozzle (*Steps 2-3*) and the air is ejected into the open atmosphere to produce thrust (*Steps 3-0)*. This final step, where the ejected gas cools to atmospheric temperature, is considered an isobaric process since open air is considered to be at a constant pressure.

State Variable Analysis

Taking a look at the pressure, temperature and velocity at each step results in a comprehensive overview of the engine cycle. We will assume that the air goes through a



PV and TS Diagrams of the Ramjet



continuous flow process, which means that enthalpy is conserved, to find a relation between velocity and temperature at each step. The equation to find enthalpy is $H = U + PV$where we defined $U = \frac{5}{2}RT$for a diatomic ideal gas. Using the ideal gas law $PV = nRT$, for 1 mole of air, we find that $H = \frac{7}{2}RT$and specific enthalpy is $h(T) = \frac{H}{M} = \frac{7}{2}\frac{RT}{M}$. Using conservation of enthalpy and energy, we derived the equation:

$$h_i + \frac{E_i}{M} = h_f + \frac{E_f}{M}, \ with\, h_i = \frac{7}{2}\frac{RT_i}{M} \,\&\, E_i = \frac{1}{2}Mv_i^{\,2} + Mgy_i$$

since we are assuming we are at a constant height (*y)* this expression becomes:

$$\frac{1}{2}v_i^{\,2} + \frac{7}{2}\frac{RT_i}{M} + q = \frac{1}{2}v_f^{\,2} + \frac{7}{2}\frac{RT_f}{M}$$

where we added a factor of $\frac{Q}{M} = q$ to represent the specific heat added to the system during the combustion process. Rearranging this equation we can now solve for the velocity or temperature at each stage.

$$v_f = \sqrt{v_0^{\,2} + \frac{7R(T_o - T_f)}{M} + 2q} \tag{1}$$

In the initial stage, incoming air is compressed and the velocity is reduced to approximately zero through an adiabatic compression ($v_1 = 0$). Using Equation 1, we confirm the expectation that temperature increases (note: $\frac{Mv_0^2}{7R}$ is always positive).

$$T_1 = T_0 + \frac{Mv_0^2}{7R}$$

Since temperature is increasing during this volume decrease we can infer, using the ideal gas law, that pressure also increases during this process. In the second stage, the gas remains stationary ($v_2 = v_1 = 0$) and is heated at a constant pressure.

$$0 = \sqrt{\frac{7R}{M}(T_1 - T_2) + 2q} \rightarrow T_2 = T_1 + \frac{2Mq}{7R} = T_0 + \frac{Mv_0^2}{7R} + \frac{2Mq}{7R}$$

The final step is executed adiabatically, accelerating the gas molecules until they reach the outside atmospheric pressure. This change in pressure and volume results in a temperature decrease, as some of the internal energy of the system provides thrust to the aircraft. Using the property that $TP^{\zeta} = const$ for adiabatic processes we can derive the relation that $\frac{T_f}{T_0} = \frac{T_2}{T_1}$, or:

$$T_f = \frac{T_0 T_2}{T_1} = T_0\left(1 + \frac{2Mq}{7RT_0 + Mv_0^2}\right)$$

since T_f, T_0 and T_2,T_1 are at the same pressure due to these being isobaric processes. Plugging T_f into the equation we find that the final velocity is given by:

$$v_f = v_0\sqrt{1 + \frac{2qM}{7RT_1 + Mv_0^2}} \qquad T_f = T_1\left(1 + \frac{2q}{7RT_1 + Mv_0^2}\right)$$

Efficiency

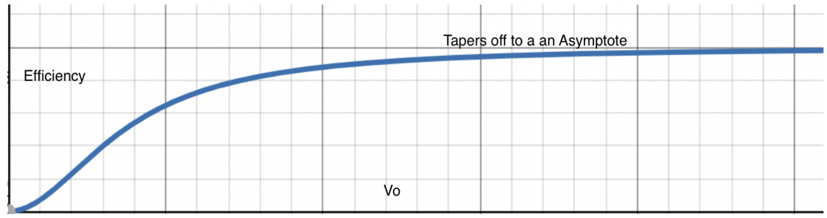
Using the equations above, we can simplify our efficiency down to two variables; T_0 and v_0. With W as the work done by the engine, we assume that $\frac{dW}{dt} = \tau v_0$, defining $\tau \equiv \frac{d(\Delta P)}{dt} = \frac{dm}{dt}(v_f - v_0)$ where ΔP is the change in momentum of the air inside the ramjet. Then, assuming $\frac{dQ}{dt} = \rho A v_0 q$, where we set up the equation for efficiency:

$\dot{m} = A\rho v_0$
$\rho =$ average density of air
$A =$ inlet cross sectional area

$$\eta = \frac{\dot{W}}{\dot{Q}} = \frac{\dot{m}v_0^2}{Av_0\rho_0 q}\left[\sqrt{1 + \frac{2qM}{7RT_0 + 2Mv_0^2}} - 1\right],$$

Which simplifies to:

$$\eta = \frac{v_0^2}{q}\left[\sqrt{1 + \frac{2qM}{7RT_0 + 2Mv_0^2}} - 1\right]$$



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