Name: ____________________

1. **Problem**
   An experiment has n_1 = 5 plants in the treatment group and n_2 = 3 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5
sample 1:	9	6.6	5.2	10.8	11.6
sample 2:	21.2	18.9	18.4		

   (a) Determine degrees of freedom.
   (b) Determine t^* for a 95% confidence interval.
   (c) Determine SE.
   (d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 2.000

   (b) 4.300

   (c) 1.490

   (d) 4.453

   (e) 17.267

   (f) 7.289

   (g) 0.010

   (h) 0.020

   (i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 8.64$$
$$\overline{x_2} = 19.5$$
$$s_1 = 2.72$$
$$s_2 = 1.49$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(5, 3) - 1 \;=\; 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.95$

$$t^\star = 4.3$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(2.72)^2}{5} + \frac{(1.49)^2}{3}} = 1.49$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI \;=\; (4.453,\ 17.267)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(19.5 - 8.64) - 0}{1.49} = 7.29$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 7.29$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.01 \;<\; p\text{-value} \;<\; 0.02$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 2
(b) 4.3
(c) 1.49
(d) 4.453
(e) 17.267
(f) 7.289
(g) 0.01
(h) 0.02
(i) yes