

# Bunker Hill Community College

Final Statistics Exam 2019-05-02

Exam ID 023

**Name:** \_\_\_\_\_

This take-home exam is due **Wednesday, May 8**, at the beginning of class.

You may use any notes, textbook, or online tools; however, you may not request help from any other human.

You will show your work on the pages with questions. When you are sure of your answers, you will **put those answers in the boxes** on the first few pages.

Unless you have an objection to doing so, please **copy the honor-code text below and sign**.

*I understand that outside help is NOT allowed on this exam. On my honor, the work herein is my own.*

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**Signature:** \_\_\_\_\_

1. (a)  $P(\text{pink given bike}) = 0.388$
- (b)  $P(\text{pig given yellow}) = 0.247$
- (c)  $P(\text{pig and pink}) = 0.0703$
- (d)  $P(\text{wheel}) = 0.179$
- (e)  $P(\text{bike or red}) = 0.526$
- (f)  $P(\text{pink}) = 0.291$
2.  $P(\text{"not wheel" given "not pink"}) = 0.647$
3.  $P(71.02 < X < 71.32) = 0.893$
4. (a)  $P(X = 12) = 0.1113$
- (b)  $P(7 \leq X \leq 14) = 0.786$
5. **(32.2, 39.9)**
6. (a)  $H_0 : \mu_2 - \mu_1 = 0$
- (b)  $H_0 : \mu_2 - \mu_1 \neq 0$
- (c)  $t^* = 2.03$
- (d)  $SE = 6.807$
- (e)  $|t_{\text{obs}}| = 1.91$
- (f)  $0.05 < p\text{-value} < 0.1$
- (g) **retain**
7. (a) **LB of p CI = 0.238 or 23.8%**
- (b) **UB of p CI = 0.242 or 24.2%**

8. (a)  $H_0 : p_2 - p_1 = 0$

(b)  $H_A : p_2 - p_1 \neq 0$

(c)  $z^* = 2.58$

(d)  $SE = 0.088$

(e)  $|z_{\text{obs}}| = 2.7$

(f)  $p\text{-value} = 0.007$

(g) **reject**

1. In a deck of strange cards, there are 725 cards. Each card has an image and a color. The amounts are shown in the table below.

	pink	red	yellow
bike	95	86	64
cat	11	22	69
pig	51	55	72
shovel	17	29	24
wheel	37	30	63

- (a) What is the probability a random card is pink given it is a bike?
- (b) What is the probability a random card is a pig given it is yellow?
- (c) What is the probability a random card is both a pig and pink?
- (d) What is the probability a random card is a wheel?
- (e) What is the probability a random card is either a bike or red (or both)?
- (f) What is the probability a random card is pink?

**Solution**

$$(a) P(\text{pink given bike}) = \frac{95}{95+86+64} = 0.388$$

$$(b) P(\text{pig given yellow}) = \frac{72}{64+69+72+24+63} = 0.247$$

$$(c) P(\text{pig and pink}) = \frac{51}{725} = 0.0703$$

$$(d) P(\text{wheel}) = \frac{37+30+63}{725} = 0.179$$

$$(e) P(\text{bike or red}) = \frac{95+86+64+86+22+55+29+30-86}{725} = 0.526$$

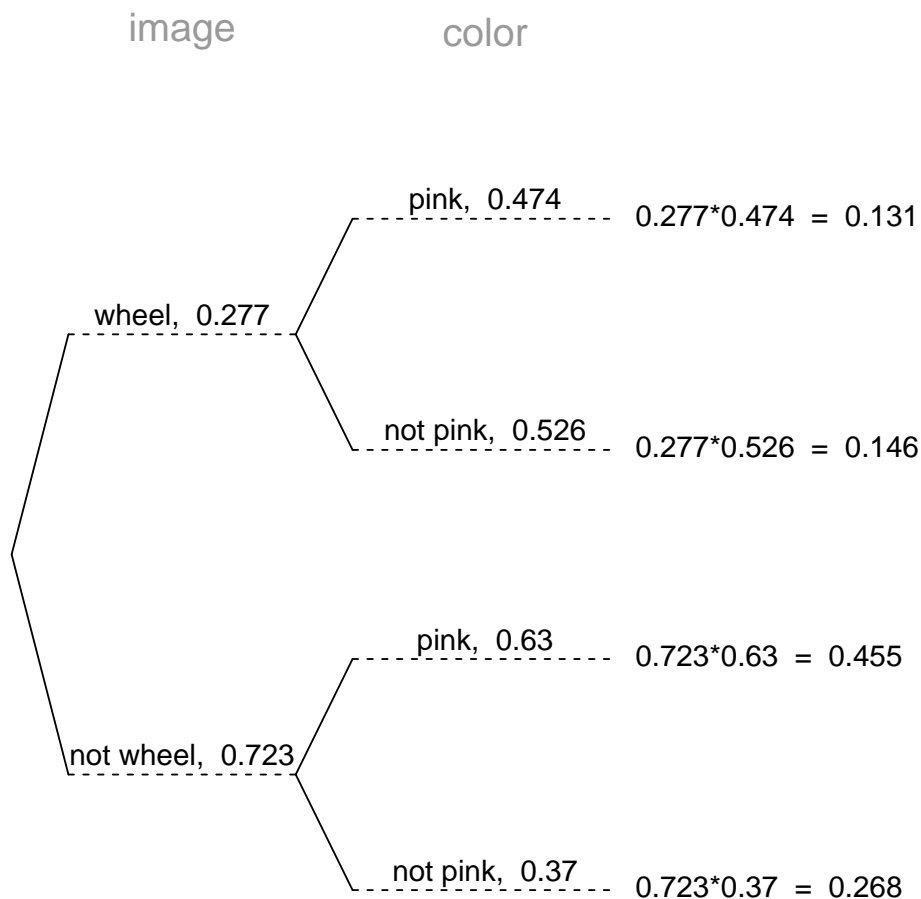
$$(f) P(\text{pink}) = \frac{95+11+51+17+37}{725} = 0.291$$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a wheel is 27.7%. If a wheel is drawn, there is a 47.4% chance that it is pink. If a card that is not a wheel is drawn, there is a 63% chance that it is pink.

Now, someone draws a random card and reveals it is not pink. What is the chance the card is not a wheel?

**Solution**

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilities. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not wheel" given "not pink"}) = \frac{0.268}{0.268 + 0.146} = 0.647$$

3. In a very large pile of toothpicks, the mean length is 71.13 millimeters and the standard deviation is 1.26 millimeters. If you randomly sample 225 toothpicks, what is the chance the sample mean is between 71.02 and 71.32 millimeters?



**Solution**

Label the given information.

$$\mu = 71.13$$

$$\sigma = 1.26$$

$$n = 225$$

$$\bar{x}_{\text{lower}} = 71.02$$

$$\bar{x}_{\text{upper}} = 71.32$$

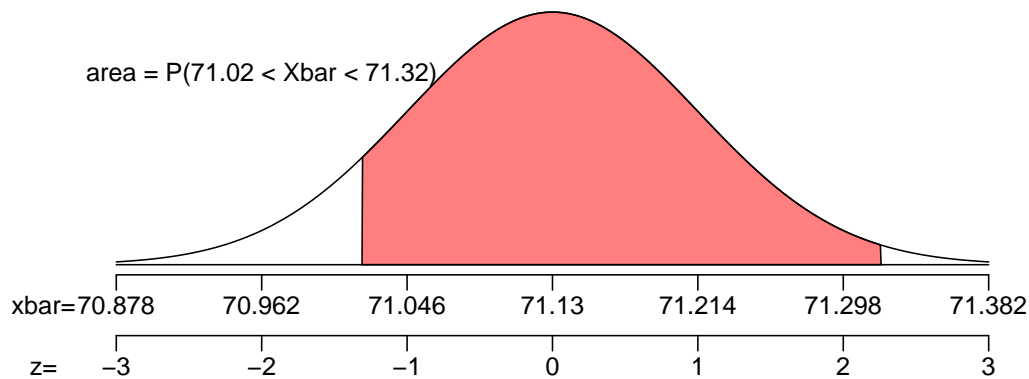
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.26}{\sqrt{225}} = 0.084$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(71.13, 0.084)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{\bar{x}_{\text{lower}} - \mu}{SE} = \frac{71.02 - 71.13}{0.084} = -1.31$$

$$z_{\text{upper}} = \frac{\bar{x}_{\text{upper}} - \mu}{SE} = \frac{71.32 - 71.13}{0.084} = 2.26$$

Determine the probability.

$$\begin{aligned} P(71.02 < \bar{X} < 71.32) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(2.26) - \Phi(-1.31) \\ &= 0.893 \end{aligned}$$

4. In a game, there is a 6% chance to win a round. You will play 181 rounds.
- (a) What is the probability of winning exactly 12 rounds?
  - (b) What is the probability of winning at least 7 but at most 14 rounds?

**Solution**

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k} (p)^k (1 - p)^{n-k}$$

$$P(X = 12) = \binom{181}{12} (0.06)^{12} (1 - 0.06)^{181-12}$$

$$P(X = 12) = \binom{181}{12} (0.06)^{12} (0.94)^{169}$$

$$P(X = 12) = 0.1113$$

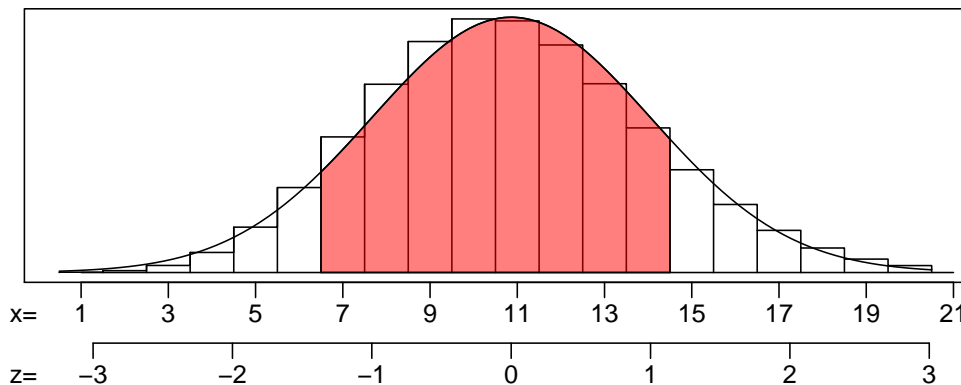
Find the mean.

$$\mu = np = (181)(0.06) = 10.86$$

Find the standard deviation.

$$\sigma = \sqrt{np(1 - p)} = \sqrt{(181)(0.06)(1 - 0.06)} = 3.1951$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{6.5 - 10.86}{3.1951} = -1.36$$

$$z_2 = \frac{14.5 - 10.86}{3.1951} = 1.14$$

Calculate the probability.

$$P(7 \leq X \leq 14) = \Phi(1.14) - \Phi(-1.36) = 0.786$$

(a)  $P(X = 12) = 0.1113$

(b)  $P(7 \leq X \leq 14) = 0.786$

5. As an ornithologist, you wish to determine the average body mass of *Catharus ustulatus*. You randomly sample 19 adults of *Catharus ustulatus*, resulting in a sample mean of 36.09 grams and a sample standard deviation of 5.24 grams. Determine a 99.5% confidence interval of the true population mean.

**Solution**

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 19$$

$$\bar{x} = 36.09$$

$$s = 5.24$$

$$CL = 0.995$$

Determine the degrees of freedom (because we don't know  $\sigma$  and we are doing inference so we need to use the  $t$  distribution).

$$df = n - 1 = 18$$

Determine the critical  $t$  value,  $t^*$ , such that  $P(|T| < t^*) = 0.995$ .

$$t^* = 3.2$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{5.24}{\sqrt{19}} = 1.2$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^* SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^* SE, \bar{x} + t^* SE) \\ &= (36.09 - 3.2 \times 1.2, 36.09 + 3.2 \times 1.2) \\ &= (32.2, 39.9) \end{aligned}$$

We are 99.5% confident that the population mean is between 32.2 and 39.9.

6. A treatment group of size 20 has a mean of 114 and standard deviation of 21.4. A control group of size 19 has a mean of 101 and standard deviation of 21.1. If you decided to use a significance level of 0.05, is there sufficient evidence to conclude the treatment causes an effect?

By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 36.

- (a) State the null hypothesis.
- (b) State the alternative hypothesis.
- (c) Evaluate the critical value. (The critical value is either  $z^*$  or  $t^*$ . Determine its value.)
- (d) Determine the standard error of the relevant sampling distribution.
- (e) Evaluate the absolute value of the test statistic. (The test statistic is either  $z_{\text{obs}}$  or  $t_{\text{obs}}$ . Determine its absolute value.)
- (f) If possible, evaluate the  $p$ -value. Otherwise, describe an interval containing the  $p$ -value.
- (g) Do we reject or retain the null?

**Solution**

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned}n_1 &= 20 \\ \bar{x}_1 &= 114 \\ s_1 &= 21.4 \\ n_2 &= 19 \\ \bar{x}_2 &= 101 \\ s_2 &= 21.1 \\ \alpha &= 0.05 \\ df &= 36\end{aligned}$$

State the hypotheses.

$$\begin{aligned}H_0 : \mu_2 - \mu_1 &= 0 \\ H_A : \mu_2 - \mu_1 &\neq 0\end{aligned}$$

We are using a two-tail test. Find  $t^*$  such that  $P(|T| > t^*) = 0.05$  by using a  $t$  table.

$$t^* = 2.03$$

Calculate the standard error.

$$\begin{aligned}SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(21.4)^2}{20} + \frac{(21.1)^2}{19}} \\ &= 6.807\end{aligned}$$

Determine the test statistic.

$$\begin{aligned}t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(101 - 114) - (0)}{6.807} \\ &= -1.91\end{aligned}$$

Compare  $|t_{\text{obs}}|$  and  $t^*$ .

$$|t_{\text{obs}}| < t^*$$

We can determine an interval for the  $p$ -value using the  $t$  table.

$$0.05 < p\text{-value} < 0.1$$

Compare  $p$ -value and  $\alpha$ .

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

$$(a) H_0 : \mu_2 - \mu_1 = 0$$

(b)  $H_A : \mu_2 - \mu_1 \neq 0$

(c)  $t^* = 2.03$

(d)  $SE = 6.807$

(e)  $|t_{\text{obs}}| = 1.91$

(f)  $0.05 < p\text{-value} < 0.1$

(g) retain the null



7. From a very large population, a random sample of 80000 individuals was taken. In that sample, 24% were salty. Determine a 80% confidence interval of the population proportion.
- (a) Find the lower bound of the confidence interval.
  - (b) Find the upper bound of the confidence interval.

**Solution**

Determine  $z^*$  such that  $P(|Z| < z^*) = 0.8$ .

$$z^* = 1.28$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \sqrt{\frac{(0.24)(1 - 0.24)}{80000}} = 0.00151$$

Calculate the margin of error.

$$ME = z^* SE = (1.28)(0.00151) = 0.00193$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.238, 0.242)$$

We are 80% confident that the true population proportion is between 23.8% and 24.2%.

- (a) The lower bound = 0.238, which can also be expressed as 23.8%.
- (b) The upper bound = 0.242, which can also be expressed as 24.2%.

8. An experiment is run with a treatment group of size 62 and a control group of size 69. The results are summarized in the table below.

	treatment	control
cold	38	26
not cold	24	43

Using a significance level of 0.01, determine whether the treatment causes an effect on the proportion of cases that are cold.

- (a) State the null hypothesis.
- (b) State the alternative hypothesis.
- (c) Evaluate the critical value. (The critical value is either  $z^*$  or  $t^*$ . Determine its value.)
- (d) Determine the standard error of the relevant sampling distribution.
- (e) Evaluate the absolute value of the test statistic. (The test statistic is either  $z_{\text{obs}}$  or  $t_{\text{obs}}$ . Determine its absolute value.)
- (f) If possible, evaluate the  $p$ -value. Otherwise, describe an interval containing the  $p$ -value.
- (g) Do we reject or retain the null?

**Solution**

State the hypotheses.

$$H_0 : p_2 - p_1 = 0$$

$$H_A : p_2 - p_1 \neq 0$$

Find  $z^*$  such that  $P(|Z| > z^*) = 0.01$ .

$$z^* = \Phi^{-1} \left( 1 - \frac{\alpha}{2} \right) = 2.58$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{38}{62} = 0.613$$

$$\hat{p}_2 = \frac{26}{69} = 0.377$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.377 - 0.613 = -0.236$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{38 + 26}{62 + 69} = 0.489$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1 - \hat{p})}{n_1} + \frac{\hat{p}(1 - \hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.489)(0.511)}{62} + \frac{(0.489)(0.511)}{69}} \\ &= 0.0875 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0 : \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.0875)$$

We want to describe how unusual our observation is under the null by finding the  $p$ -value. To do so, first find the  $z$  score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.377 - 0.613) - 0}{0.0875} \\ &= -2.7 \end{aligned}$$

Determine the  $p$ -value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.7) \\ &= 0.007 \end{aligned}$$

Compare the  $p$ -value to the significance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

- (a)  $H_0 : p_2 - p_1 = 0$
- (b)  $H_A : p_2 - p_1 \neq 0$
- (c)  $z^* = 2.58$
- (d)  $SE = 0.0875$
- (e)  $|z_{\text{obs}}| = 2.7$
- (f)  $p\text{-value} = 0.007$
- (g) reject the null