Name: ____________________

1. **Problem**
An experiment has n_1 = 3 plants in the treatment group and n_2 = 7 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	21.3	21.8	27.2				
sample 2:	8.8	10.9	9.9	10.3	11.4	10.4	9

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 99% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 2.000
(b) 9.920
(c) 1.922
(d) -32.366
(e) 5.766
(f) 6.921
(g) 0.020
(h) 0.040
(i) no

1. **Solution**

   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 23.4$$

$$\overline{x_2} = 10.1$$

$$s_1 = 3.27$$

$$s_2 = 0.949$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(3, 7) - 1 \;=\; 2$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.99$

$$t^{\star} = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(3.27)^2}{3} + \frac{(0.949)^2}{7}} = 1.922$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$

$$CI \;=\; (-32.366,\, 5.766)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(10.1 - 23.4) - 0}{1.922} = -6.92$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 6.92$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and $p\text{-value} = P(|T| > |t_{\text{obs}}|)$.

$$0.02 \;<\; p\text{-value} \;<\; 0.04$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^{\star}$$

$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

   (a) 2
   (b) 9.92
   (c) 1.922
   (d) -32.366
   (e) 5.766
   (f) 6.921
   (g) 0.02
   (h) 0.04
   (i) no