Bunker Hill Community College

Final Statistics Exam 2019-05-02 Exam ID 030

Name: ____________________

This take-home exam is due **Wednesday, May 8**, at the beginning of class.

You may use any notes, textbook, or online tools; however, you may not request help from any other human.

You will show your work on the pages with questions. When you are sure of your answers, you will **put those answers in the boxes** on the first few pages.

Unless you have an objection to doing so, please **copy the honor-code text below and sign**.

I understand that outside help is NOT allowed on this exam. On my honor, the work herein is my own.

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Signature: ____________________

1. (a) P(**green given wheel**) = 0.179

   (b) P(**flower or teal**) = 0.43

   (c) P(**pink**) = 0.23

   (d) P(**tree**) = 0.289

   (e) P(**shovel given pink**) = 0.351

   (f) P(**tree and yellow**) = 0.091

2. P(**"tree" given "not pink"**) = 0.427

3. $P(74.6 < X < 75) = 0.8912$

4. (a) $P(X = 44) = 0.0581$

   (b) $P(33 \leq X \leq 38) = 0.2492$

5. **(13.8, 15.9)**

6. (a) $H_0 : \quad \mu_2 - \mu_1 = 0$

   (b) $H_0 : \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.45$

   (d) $SE = 56.41$

   (e) $\mid t_{\textbf{obs}} \mid = 2.73$

   (f) $0.01 <$ p-**value** < 0.02

   (g) **reject**

7. (a) **LB of p CI = 0.624 or** 62.4%

   (b) **UB of p CI = 0.656 or** 65.6%

8. (a) $H_0 : \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}} : \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 1.64$

(d) $SE = 0.038$

(e) $\mid z_{\mathbf{obs}} \mid = 1.85$

(f) p-**value** = 0.0644

(g) **reject**

1. In a deck of strange cards, there are 758 cards. Each card has an image and a color. The amounts are shown in the table below.

	green	pink	teal	yellow
flower	50	21	57	28
shovel	24	61	27	47
tree	22	49	79	69
wheel	40	43	64	77

(a) What is the probability a random card is green given it is a wheel?

(b) What is the probability a random card is either a flower or teal (or both)?

(c) What is the probability a random card is pink?

(d) What is the probability a random card is a tree?

(e) What is the probability a random card is a shovel given it is pink?

(f) What is the probability a random card is both a tree and yellow?

Solution

(a) $P(\text{green given wheel}) = \frac{40}{40+43+64+77} = 0.179$

(b) $P(\text{flower or teal}) = \frac{50+21+57+28+57+27+79+64-57}{758} = 0.43$

(c) $P(\text{pink}) = \frac{21+61+49+43}{758} = 0.23$

(d) $P(\text{tree}) = \frac{22+49+79+69}{758} = 0.289$

(e) $P(\text{shovel given pink}) = \frac{61}{21+61+49+43} = 0.351$

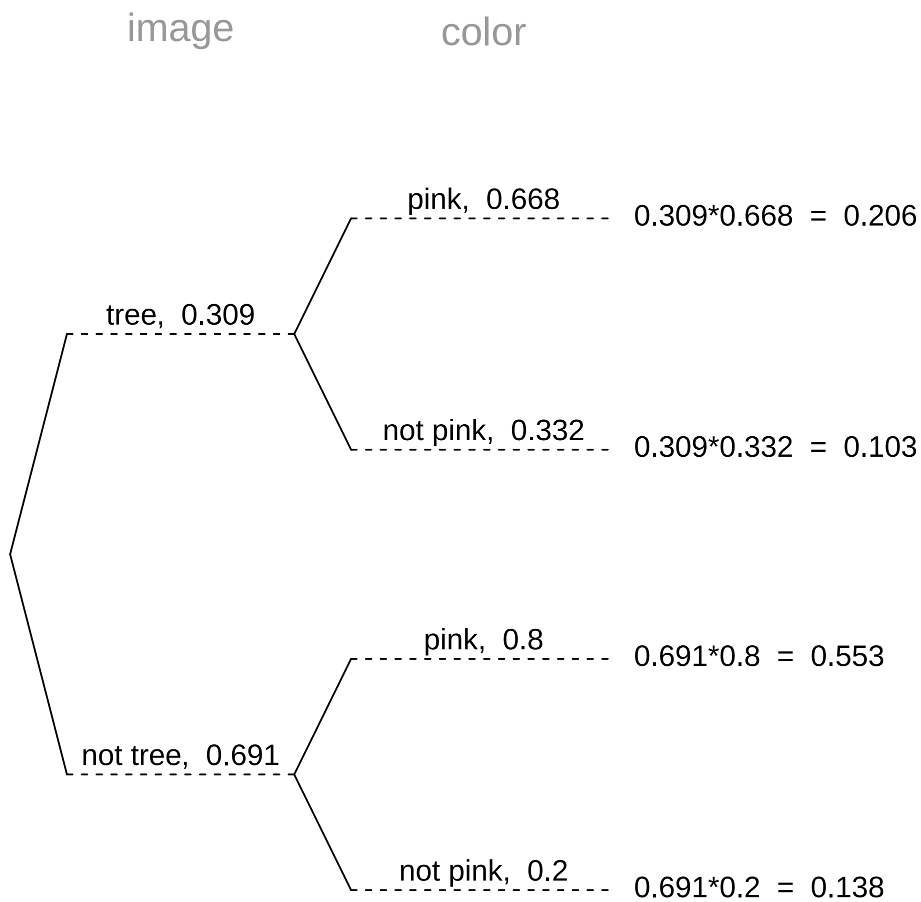
(f) $P(\text{tree and yellow}) = \frac{69}{758} = 0.091$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a tree is 30.9%. If a tree is drawn, there is a 66.8% chance that it is pink. If a card that is not a tree is drawn, there is a 80% chance that it is pink.

   Now, someone draws a random card and reveals it is not pink. What is the chance the card is a tree?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"tree" given "not pink"}) = \frac{0.103}{0.103 + 0.138} = 0.427$$

3. In a very large pile of toothpicks, the mean length is 74.82 millimeters and the standard deviation is 1.37 millimeters. If you randomly sample 125 toothpicks, what is the chance the sample mean is between 74.6 and 75 millimeters?

Solution

Label the given information.

$$\mu = 74.82$$
$$\sigma = 1.37$$
$$n = 125$$
$$\bar{x}_{\text{lower}} = 74.6$$
$$\bar{x}_{\text{upper}} = 75$$

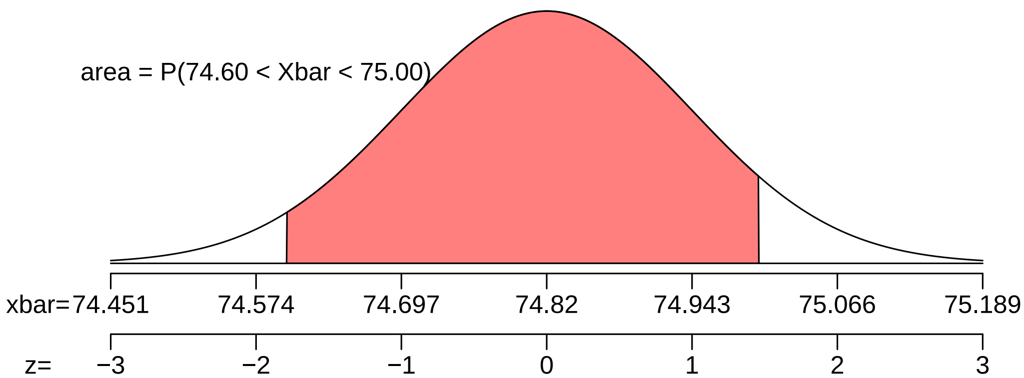
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.37}{\sqrt{125}} = 0.123$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(74.82, 0.123)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{74.6 - 74.82}{0.123} = -1.79$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{75 - 74.82}{0.123} = 1.46$$

Determine the probability.

$$\begin{aligned} P(74.6 < X < 75) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.46) - \Phi(-1.79) \\ &= 0.8912 \end{aligned}$$

4. In a game, there is a 33% chance to win a round. You will play 122 rounds.

   (a) What is the probability of winning exactly 44 rounds?

   (b) What is the probability of winning at least 33 but at most 38 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 44) = \binom{122}{44}(0.33)^{44}(1-0.33)^{122-44}$$

$$P(X = 44) = \binom{122}{44}(0.33)^{44}(0.67)^{78}$$

$$P(X = 44) = 0.0581$$

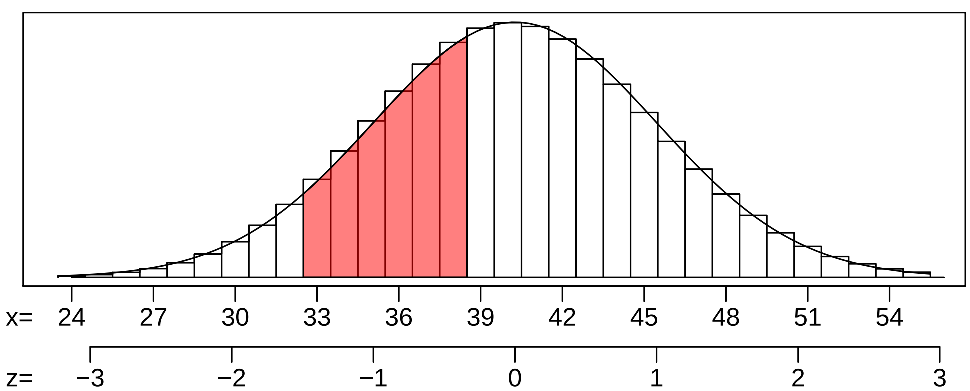
Find the mean.

$$\mu = np = (122)(0.33) = 40.26$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(122)(0.33)(1-0.33)} = 5.1937$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{32.5 - 40.26}{5.1937} = -1.4$$

$$z_2 = \frac{38.5 - 40.26}{5.1937} = -0.44$$

Calculate the probability.

$$P(33 \leq X \leq 38) = \Phi(-0.44) - \Phi(-1.4) = 0.2492$$

(a) $P(X = 44) = 0.0581$

(b) $P(33 \leq X \leq 38) = 0.2492$

5. As an ornithologist, you wish to determine the average body mass of *Passerina cyanea*. You randomly sample 31 adults of *Passerina cyanea*, resulting in a sample mean of 14.86 grams and a sample standard deviation of 2.37 grams. Determine a 98% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 31$$
$$\bar{x} = 14.86$$
$$s = 2.37$$
$$CL = 0.98$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 30$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.98$.

$$t^\star = 2.46$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.37}{\sqrt{31}} = 0.426$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (14.86 - 2.46 \times 0.426,\ 14.86 + 2.46 \times 0.426) \\ &= (13.8,\ 15.9) \end{aligned}$$

We are 98% confident that the population mean is between 13.8 and 15.9.

6. A treatment group of size 32 has a mean of 966 and standard deviation of 204. A control group of size 15 has a mean of 1120 and standard deviation of 168. If you decided to use a signficance level of 0.02, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 32.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 32$$
$$\bar{x}_1 = 966$$
$$s_1 = 204$$
$$n_2 = 15$$
$$\bar{x}_2 = 1120$$
$$s_2 = 168$$
$$\alpha = 0.02$$
$$df = 32$$

State the hypotheses.

$$H_0 : \quad \mu_2 - \mu_1 = 0$$
$$H_A : \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.02$ by using a t table.

$$t^\star = 2.45$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(204)^2}{32} + \frac{(168)^2}{15}} \\ &= 56.41 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1120 - 966) - (0)}{56.41} \\ &= 2.73 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.45$

(d) $SE = 56.41$

(e) $|t_{\text{obs}}| = 2.73$

(f) $0.01 < p\text{-value} < 0.02$

(g) reject the null

7. From a very large population, a random sample of 3600 individuals was taken. In that sample, 64% were glowing. Determine a 95% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.95$.

$$z^{\star} = 1.96$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.64)(1-0.64)}{3600}} = 0.008$$

Calculate the margin of error.

$$ME = z^{\star}SE = (1.96)(0.008) = 0.0157$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.624\,,\ 0.656)$$

We are 95% confident that the true population proportion is between 62.4% and 65.6%.

(a) The lower bound = 0.624, which can also be expressed as 62.4%.

(b) The upper bound = 0.656, which can also be expressed as 65.6%.

8. An experiment is run with a treatment group of size 195 and a control group of size 165. The results are summarized in the table below.

	treatment	control
omnivorous	172	134
not omnivorous	23	31

Using a significance level of 0.1, determine whether the treatment causes an effect on the proportion of cases that are omnivorous.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.1$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.64$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{172}{195} = 0.882$$
$$\hat{p}_2 = \frac{134}{165} = 0.812$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.812 - 0.882 = -0.07$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{172 + 134}{195 + 165} = 0.85$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.85)(0.15)}{195} + \frac{(0.85)(0.15)}{165}} \\ &= 0.0378 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.0378)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.812 - 0.882) - 0}{0.0378} \\ &= -1.85 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-1.85) \\ &= 0.0644 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0 : \quad p_2 - p_1 = 0$

(b) $H_A : \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.64$

(d) $SE = 0.0378$

(e) $|z_{\text{obs}}| = 1.85$

(f) p-value = 0.0644

(g) reject the null