Problem 1

1. There is a normal population X.

$$X \sim \mathcal{N}(\mu = 20, \ \sigma = 5)$$

   Another population Y is determined by X.

$$Y \sim \frac{X}{7} + \frac{X}{7} + \frac{X}{7} + \frac{X}{7} + \frac{X}{7} + \frac{X}{7} + \frac{X}{7}$$

Problem 2

2. You have two populations (random variables): V and W.

$$V \sim \mathcal{N}(\mu = 99\,,\ \sigma = 31)$$

$$W \sim \mathcal{N}(\mu = 77\,,\ \sigma = 11)$$

A (normal) population X is determined by V and W.

$$X \sim \left(\frac{V}{3} + \frac{V}{3} + \frac{V}{3}\right) - \left(\frac{W}{6} + \frac{W}{6} + \frac{W}{6} + \frac{W}{6} + \frac{W}{6} + \frac{W}{6}\right)$$

2.1 Evaluate $\mathrm{E}(X)$.
2.2 Evaluate $\mathrm{Var}(X)$.
2.3 Evaluate $P(X > 25)$.
2.4 Determine x such that $P(X < x) = 0.888$.

Solution 2

2. 2.1 Expected value follows basic rules.
   $E(aA + bB) = aE(A) + bE(B)$

   $$E(X) \ = \ 3\left(\frac{E(V)}{3}\right) - 6\left(\frac{E(W)}{6}\right) \ = \ E(V) - E(W) \ = \ 99 - 77 \ = \ 22$$

   2.2 Variance has a more complicated rule.
   $Var(aA + bB) = a^2 Var(A) + b^2 Var(B)$

   $$\begin{aligned} Var(X) &= 3\left(\frac{Var(V)}{9}\right) + 6\left(\frac{Var(W)}{36}\right) \\ &= \frac{Var(V)}{3} + \frac{Var(W)}{6} \\ &= \frac{31^2}{3} + \frac{11^2}{6} \\ &= 340.5 \end{aligned}$$

   2.3

Problem 3

3. You have two populations (random variables): V and W.

$$V \sim \mathcal{N}(\mu = 99\,,\ \sigma = 31)$$

$$W \sim \mathcal{N}(\mu = 77\,,\ \sigma = 11)$$

A (normal) population Y is determined by V and W.

$$Y \sim \frac{(V - W) + (V - W) + (V - W) + (V - W)}{4}$$

3.1 Evaluate $\mathrm{E}(Y)$.
3.2 Evaluate $\mathrm{Var}(Y)$.
3.3 Evaluate $P(Y > 25)$.
3.4 Determine y such that $P(Y < y) = 0.888$.