Name: ____________________

1. **Problem**
An experiment has n_1 = 4 plants in the treatment group and n_2 = 8 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	143	134	145	151				
sample 2:	108	109	101	110	94	81	96	96

(a) Determine degrees of freedom.
(b) Determine t^* for a 99% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 3.000

(b) 5.840

(c) 4.939

(d) -72.444

(e) -14.756

(f) 8.827

(g) 0.002

(h) 0.004

(i) **yes**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 143$$

$$\overline{x_2} = 99.4$$

$$s_1 = 7.04$$

$$s_2 = 9.8$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(4, 8) - 1 = 3$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 5.84$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(7.04)^2}{4} + \frac{(9.8)^2}{8}} = 4.939$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$

$$CI = (-72.444, -14.756)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(99.4 - 143) - 0}{4.939} = -8.83$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 8.83$$

We use the table to determine bounds on p-value. Remember, $df = 3$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.002 < p\text{-value} < 0.004$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 3
(b) 5.84
(c) 4.939
(d) -72.444
(e) -14.756
(f) 8.827
(g) 0.002
(h) 0.004
(i) yes