Name: ____________________

1. **Problem**
   An experiment has n_1 = 6 plants in the treatment group and n_2 = 6 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	14.1	11.2	12.8	15.1	13.8	14.1
sample 2:	9.8	8.1	9.8	11.2	11	10.8

   (a) Determine degrees of freedom.
   (b) Determine t^* for a 99% confidence interval.
   (c) Determine *SE*.
   (d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 5.000
   (b) 4.030
   (c) 0.727
   (d) -6.330
   (e) -0.470
   (f) 4.679
   (g) 0.005
   (h) 0.010
   (i) **yes**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 13.5$$
$$\overline{x_2} = 10.1$$
$$s_1 = 1.35$$
$$s_2 = 1.16$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(6, 6) - 1 = 5$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 4.03$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(1.35)^2}{6} + \frac{(1.16)^2}{6}} = 0.727$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-6.33, -0.47)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(10.1 - 13.5) - 0}{0.727} = -4.68$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 4.68$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.005 < p\text{-value} < 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 5
(b) 4.03
(c) 0.727
(d) -6.33
(e) -0.47
(f) 4.679
(g) 0.005
(h) 0.01
(i) yes