Bunker Hill Community College

Final Statistics Exam 2019-05-02 Exam ID 023

Name: ____________________

This take-home exam is due **Wednesday, May 8**, at the beginning of class.

You may use any notes, textbook, or online tools; however, you may not request help from any other human.

You will show your work on the pages with questions. When you are sure of your answers, you will **put those answers in the boxes** on the first few pages.

Unless you have an objection to doing so, please **copy the honor-code text below and sign**.

I understand that outside help is NOT allowed on this exam. On my honor, the work herein is my own.

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Signature: ____________________

1. (a) $P(\textbf{cat}) = 0.296$

   (b) $P(\textbf{bike or green}) = 0.427$

   (c) $P(\textbf{cat and green}) = 0.0193$

   (d) $P(\textbf{green}) = 0.326$

   (e) $P(\textbf{green given bike}) = 0.451$

   (f) $P(\textbf{shovel given orange}) = 0.245$

2. $P(\textbf{"not wheel" given "blue"}) = 0.943$

3. $P(65.93 < X < 66.24) = 0.7574$

4. (a) $P(X = 85) = 0.1117$

   (b) $P(84 \leq X \leq 92) = 0.7435$

5. $\mathbf{(10.1, 11.1)}$

6. (a) $H_0 : \ \mu_2 - \mu_1 = 0$

   (b) $H_0 : \ \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.66$

   (d) $SE = 61.965$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.9$

   (f) $0.005 < p\textbf{-value} < 0.01$

   (g) **reject**

7. (a) **LB of p CI = 0.806 or** 80.6%

   (b) **UB of p CI = 0.864 or** 86.4%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.33$

(d) $SE = 0.047$

(e) $|\ z_{\mathbf{obs}}\ | = 2.46$

(f) p-**value** = 0.0138

(g) **reject**

1. In a deck of strange cards, there are 724 cards. Each card has an image and a color. The amounts are shown in the table below.

	green	orange	teal	white
bike	60	50	10	13
cat	14	52	71	77
flower	73	18	63	16
shovel	89	39	54	25

   (a) What is the probability a random card is a cat?

   (b) What is the probability a random card is either a bike or green (or both)?

   (c) What is the probability a random card is both a cat and green?

   (d) What is the probability a random card is green?

   (e) What is the probability a random card is green given it is a bike?

   (f) What is the probability a random card is a shovel given it is orange?

Solution

(a) $P(\text{cat}) = \frac{14+52+71+77}{724} = 0.296$

(b) $P(\text{bike or green}) = \frac{60+50+10+13+60+14+73+89-60}{724} = 0.427$

(c) $P(\text{cat and green}) = \frac{14}{724} = 0.0193$

(d) $P(\text{green}) = \frac{60+14+73+89}{724} = 0.326$

(e) $P(\text{green given bike}) = \frac{60}{60+50+10+13} = 0.451$

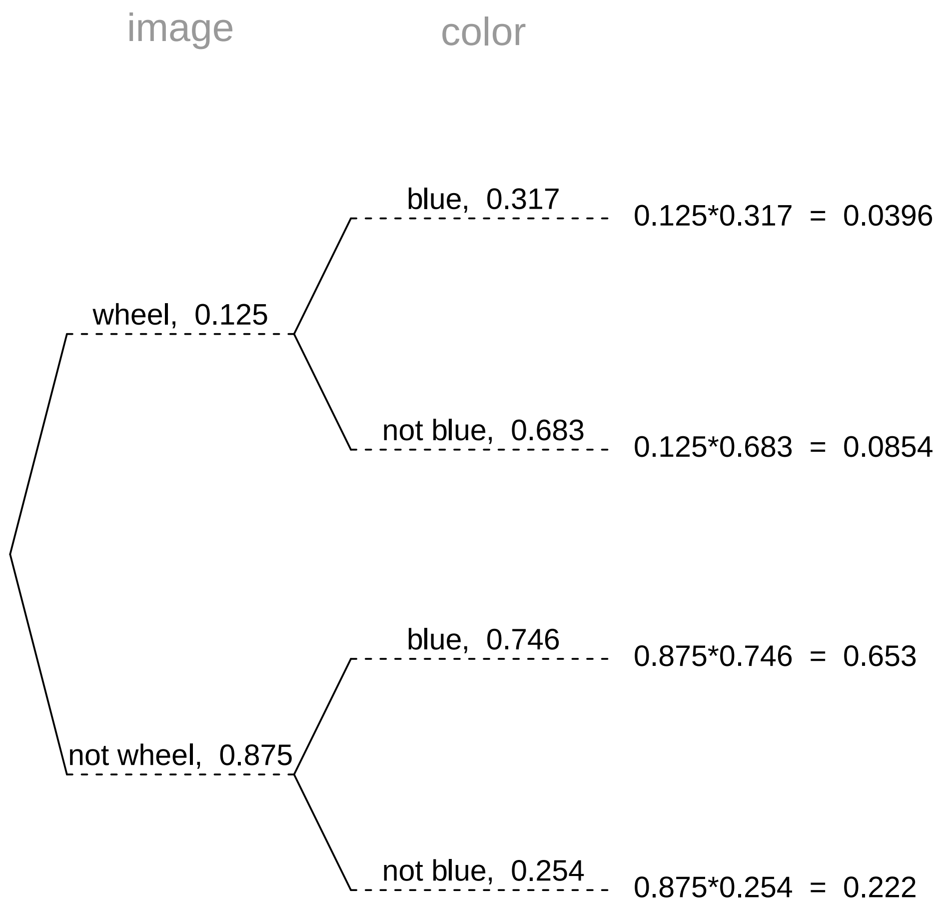
(f) $P(\text{shovel given orange}) = \frac{39}{50+52+18+39} = 0.245$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a wheel is 12.5%. If a wheel is drawn, there is a 31.7% chance that it is blue. If a card that is not a wheel is drawn, there is a 74.6% chance that it is blue.

   Now, someone draws a random card and reveals it is blue. What is the chance the card is not a wheel?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not wheel" given "blue"}) = \frac{0.653}{0.653 + 0.0396} = 0.943$$

3. In a very large pile of toothpicks, the mean length is 66.17 millimeters and the standard deviation is 1.07 millimeters. If you randomly sample 121 toothpicks, what is the chance the sample mean is between 65.93 and 66.24 millimeters?

Solution

Label the given information.

$$\mu = 66.17$$
$$\sigma = 1.07$$
$$n = 121$$
$$\bar{x}_{\text{lower}} = 65.93$$
$$\bar{x}_{\text{upper}} = 66.24$$

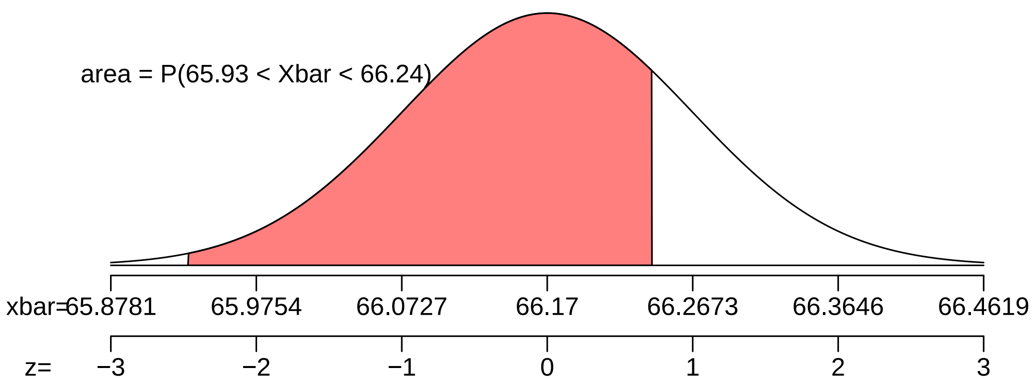
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.07}{\sqrt{121}} = 0.0973$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(66.17,\ 0.0973)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{65.93 - 66.17}{0.0973} = -2.47$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{66.24 - 66.17}{0.0973} = 0.72$$

Determine the probability.

$$\begin{aligned} P(65.93 < X < 66.24) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.72) - \Phi(-2.47) \\ &= 0.7574 \end{aligned}$$

4. In a game, there is a 89% chance to win a round. You will play 97 rounds.
   (a) What is the probability of winning exactly 85 rounds?
   (b) What is the probability of winning at least 84 but at most 92 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 85) = \binom{97}{85}(0.89)^{85}(1-0.89)^{97-85}$$

$$P(X = 85) = \binom{97}{85}(0.89)^{85}(0.11)^{12}$$

$$P(X = 85) = 0.1117$$

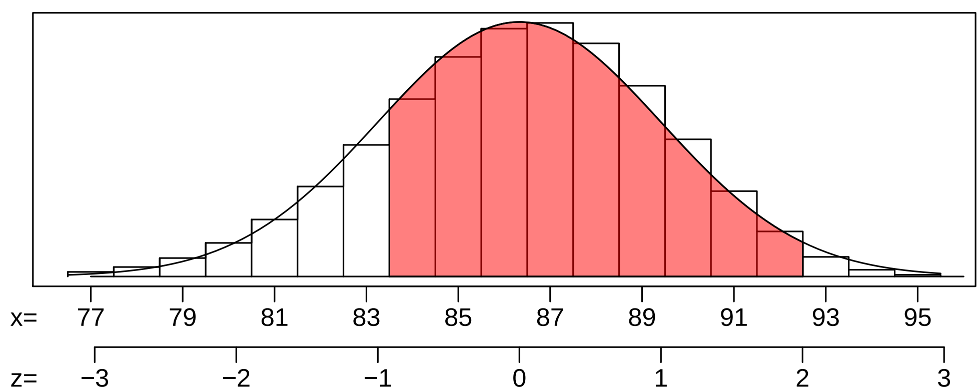
Find the mean.

$$\mu = np = (97)(0.89) = 86.33$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(97)(0.89)(1-0.89)} = 3.0816$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{83.5 - 86.33}{3.0816} = -0.76$$

$$z_2 = \frac{92.5 - 86.33}{3.0816} = 1.84$$

Calculate the probability.

$$P(84 \leq X \leq 92) = \Phi(1.84) - \Phi(-0.76) = 0.7435$$

(a) $P(X = 85) = 0.1117$

(b) $P(84 \leq X \leq 92) = 0.7435$

5. As an ornithologist, you wish to determine the average body mass of *Geothlypis trichas*. You randomly sample 32 adults of *Geothlypis trichas*, resulting in a sample mean of 10.6 grams and a sample standard deviation of 1.09 grams. Determine a 99% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 32 \\ \bar{x} &= 10.6 \\ s &= 1.09 \\ CL &= 0.99 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 31$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.99$.

$$t^{\star} = 2.74$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.09}{\sqrt{32}} = 0.193$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^{\star} SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^{\star} SE,\ \bar{x} + t^{\star} SE) \\ &= (10.6 - 2.74 \times 0.193,\ 10.6 + 2.74 \times 0.193) \\ &= (10.1,\ 11.1) \end{aligned}$$

We are 99% confident that the population mean is between 10.1 and 11.1.

6. A treatment group of size 28 has a mean of 1210 and standard deviation of 204. A control group of size 35 has a mean of 1030 and standard deviation of 287. If you decided to use a signficance level of 0.01, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 60.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 28$$
$$\bar{x}_1 = 1210$$
$$s_1 = 204$$
$$n_2 = 35$$
$$\bar{x}_2 = 1030$$
$$s_2 = 287$$
$$\alpha = 0.01$$
$$df = 60$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.01$ by using a t table.

$$t^\star = 2.66$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(204)^2}{28} + \frac{(287)^2}{35}} \\ &= 61.965 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1030 - 1210) - (0)}{61.965} \\ &= -2.9 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.005 < p\text{-value} < 0.01$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.66$

(d) $SE = 61.965$

(e) $|t_{obs}| = 2.9$

(f) $0.005 < p\text{-value} < 0.01$

(g) reject the null

7. From a very large population, a random sample of 650 individuals was taken. In that sample, 83.5% were super. Determine a 95% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.95$.

$$z^{\star} = 1.96$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.835)(1-0.835)}{650}} = 0.0146$$

Calculate the margin of error.

$$ME = z^{\star} SE = (1.96)(0.0146) = 0.0286$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.806\,,\ 0.864)$$

We are 95% confident that the true population proportion is between 80.6% and 86.4%.

(a) The lower bound = 0.806, which can also be expressed as 80.6%.

(b) The upper bound = 0.864, which can also be expressed as 86.4%.

8. An experiment is run with a treatment group of size 221 and a control group of size 234. The results are summarized in the table below.

	treatment	control
cold	103	136
not cold	118	98

Using a significance level of 0.02, determine whether the treatment causes an effect on the proportion of cases that are cold.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.02$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.33$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{103}{221} = 0.466$$
$$\hat{p}_2 = \frac{136}{234} = 0.581$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.581 - 0.466 = 0.115$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{103 + 136}{221 + 234} = 0.525$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.525)(0.475)}{221} + \frac{(0.525)(0.475)}{234}} \\ &= 0.0468 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0468)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.581 - 0.466) - 0}{0.0468} \\ &= 2.46 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.46) \\ &= 0.0138 \end{aligned}$$

Compare the *p*-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.33$

(d) $SE = 0.0468$

(e) $|z_{obs}| = 2.46$

(f) p-value = 0.0138

(g) reject the null