1. **Problem**
In a deck of strange cards, there are 764 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	gray	orange	red	violet
cat	45	97	48	19	17
dog	82	32	54	27	91
wheel	43	47	64	72	26

(a) What is the probability a random card is a wheel given it is red?

(b) What is the probability a random card is either a wheel or gray?

(c) What is the probability a random card is violet given it is a wheel?

(d) What is the probability a random card is orange?

(e) What is the probability a random card is a cat?

(f) What is the probability a random card is both a cat and orange?

Solution

(a) $P(\text{wheel given red}) = \frac{72}{19+27+72} = 0.61$

(b) $P(\text{wheel or gray}) = \frac{43+47+64+72+26+97+32+47-47}{764} = 0.499$

(c) $P(\text{violet given wheel}) = \frac{26}{43+47+64+72+26} = 0.103$

(d) $P(\text{orange}) = \frac{48+54+64}{764} = 0.217$

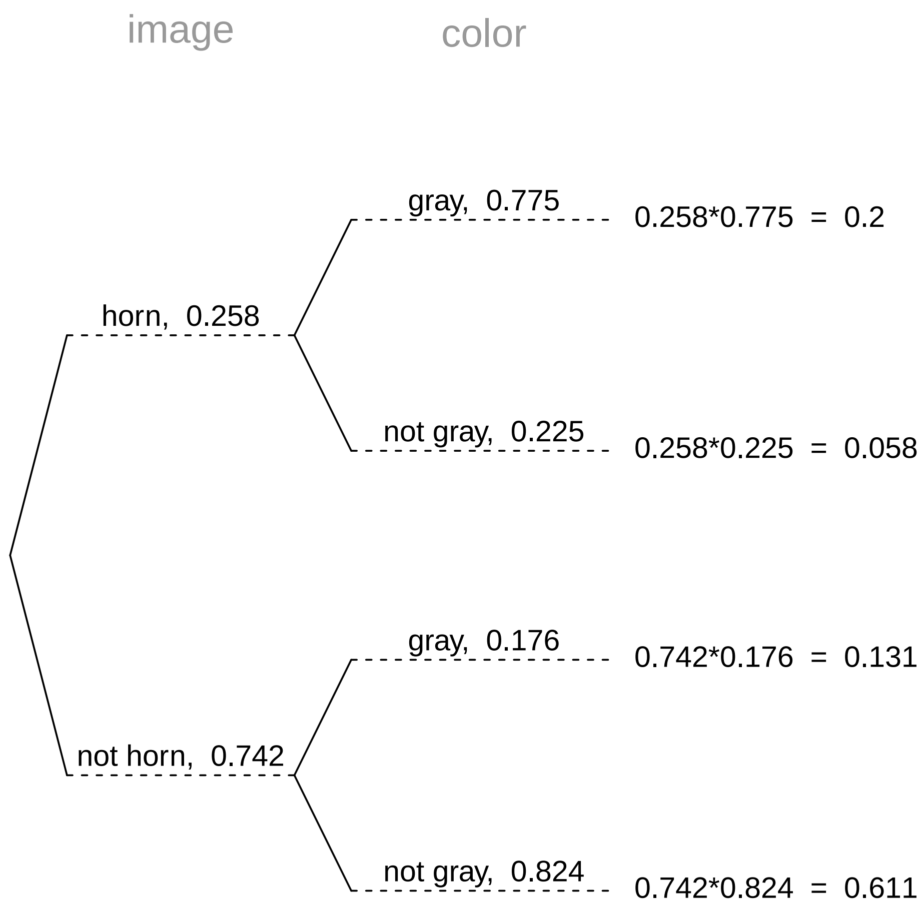
(e) $P(\text{cat}) = \frac{45+97+48+19+17}{764} = 0.296$

(f) $P(\text{cat and orange}) = \frac{48}{764} = 0.0628$

2. **Problem**
   In a deck of strange cards, the chance of drawing a horn is 25.8%. If a horn is drawn, there is a 77.5% chance that it is gray. If a card that is not a horn is drawn, there is a 17.6% chance that it is gray.

   Now, someone draws a random card and reveals it is gray. What is the chance the card is not a horn?

Solution
I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not horn" given "gray"}) = \frac{0.058}{0.058 + 0.611} = 0.0867$$

3. **Problem**
   Let each trial have a chance of success $p = 0.12$. If 81 trials occur, what is the probability of getting exactly 14 successes?

   In other words, let $X \sim \text{Bin}(n = 81,\ p = 0.12)$ and find $P(X = 14)$.

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1 - p)^{n-k}$$

$$P(X = 14) = \binom{81}{14}(0.12)^{14}(1 - 0.12)^{81-14}$$

$$P(X = 14) = \binom{81}{14}(0.12)^{14}(0.88)^{67}$$

$$P(X = 14) = 0.0446$$

4. **Problem**
   Let each trial have a chance of success $p = 0.4$. If 109 trials occur, what is the probability of getting more than 45 but at most 56 successes?

   In other words, let $X \sim \text{Bin}(n = 109,\ p = 0.4)$ and find $P(45 < X \leq 56)$.

   Use a normal approximation along with the continuity correction.

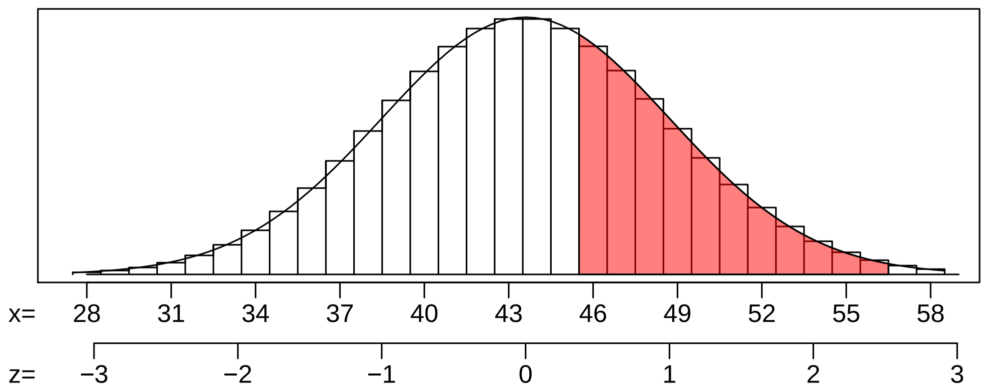
Solution
Find the mean.

$$\mu = np = (109)(0.4) = 43.6$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(109)(0.4)(1-0.4)} = 5.1147$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{45.5 - 43.6}{5.1147} = 0.37$$

$$z_2 = \frac{56.5 - 43.6}{5.1147} = 2.52$$

Calculate the probability.

$$P(45 < X \leq 56) = \Phi(2.52) - \Phi(0.37) = 0.35$$

5. **Problem**
A roughly symmetric population has a mean $\mu = 160$ and standard deviation $\sigma = 31$. What is the probability that a sample of size $n = 150$ has a mean less than 164.43?

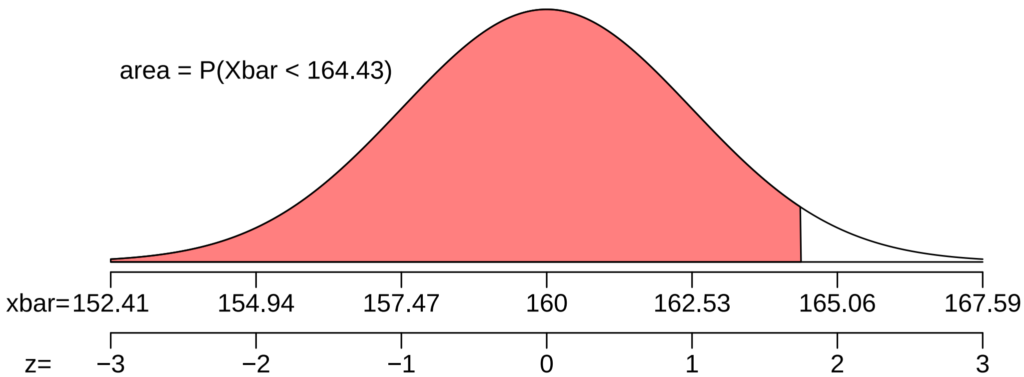
Solution
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{31}{\sqrt{150}} = 2.53$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(160,\ 2.53)$$

Draw a sketch.



Calculate a z score.

$$z = \frac{164.43 - 160}{2.53} = 1.75$$

Determine the probability.

$$P(X < 164.43) = 0.96$$

6. **Problem**
An experiment is run with a treatment group of size 99 and a control group of size 65. The results are summarized in the table below.

	treatment	control
angry	55	50
not angry	44	15

Using a significance level of 0.01, determine whether the treatment causes an effect on the proportion of cases that are angry.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution
State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.01$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.58$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{55}{99} = 0.556$$
$$\hat{p}_2 = \frac{50}{65} = 0.769$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.769 - 0.556 = 0.213$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{55 + 50}{99 + 65} = 0.64$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.64)(0.36)}{99} + \frac{(0.64)(0.36)}{65}} \\ &= 0.0766 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.0766)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.769 - 0.556) - 0}{0.0766} \\ &= 2.78 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.78) \\ &= 0.0054 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0 : \quad p_2 - p_1 = 0$
(b) $H_A : \quad p_2 - p_1 \neq 0$
(c) $z^\star = 2.58$
(d) $SE = 0.0766$
(e) $|z_{\text{obs}}| = 2.78$
(f) p-value = 0.0054
(g) reject the null

1. (a) P(**wheel given red**) = 0.61

   (b) P(**wheel or gray**) = 0.499

   (c) P(**violet given wheel**) = 0.103

   (d) P(**orange**) = 0.217

   (e) P(**cat**) = 0.296

   (f) P(**cat and orange**) = 0.0628

2. P(**"not horn" given "gray"**) = 0.0867

3. $P(X = 14) = 0.0446$

4. $P(45 < X \leq 56) = 0.35$

5. $P(X < 164.43) = 0.96$

6. (a) $H_0 : \quad p_2 - p_1 = 0$

   (b) $H_{\mathbf{A}} : \quad p_2 - p_1 \neq 0$

   (c) $z^{\star} = 2.58$

   (d) $SE = 0.077$

   (e) $\mid z_{\mathbf{obs}} \mid = 2.78$

   (f) p-**value** = 0.0054

   (g) **reject**