Bunker Hill Community College

Final Statistics Exam 2019-05-02 Exam ID 007

Name: ____________________

This take-home exam is due **Wednesday, May 8**, at the beginning of class.

You may use any notes, textbook, or online tools; however, you may not request help from any other human.

You will show your work on the pages with questions. When you are sure of your answers, you will **put those answers in the boxes** on the first few pages.

Unless you have an objection to doing so, please **copy the honor-code text below and sign**.

I understand that outside help is NOT allowed on this exam. On my honor, the work herein is my own.

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Signature: ____________________

1. (a) P(**indigo given gem**) = 0.414

   (b) P(**bike or teal**) = 0.574

   (c) P(**shovel given red**) = 0.176

   (d) P(**shovel**) = 0.167

   (e) P(**indigo**) = 0.312

   (f) P(**dog and indigo**) = 0.0713

2. P(**"flower" given "white"**) = 0.175

3. $P(72.57 < X < 72.81) = 0.6332$

4. (a) $P(X = 82) = 0.0498$

   (b) $P(65 \leq X \leq 78) = 0.5509$

5. $\mathbf{(18.6, 20.8)}$

6. (a) $H_0: \mu_2 - \mu_1 = 0$

   (b) $H_0: \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.12$

   (d) $SE = 0.531$

   (e) $\mid t_{\textbf{obs}} \mid = 2.26$

   (f) $0.02 <$ p**-value** < 0.04

   (g) **reject**

7. (a) **LB of p CI = 0.947 or** 94.7%

   (b) **UB of p CI = 0.957 or** 95.7%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.81$

(d) $SE = 0.167$

(e) $\mid z_{\mathbf{obs}} \mid = 2.67$

(f) p-**value** = 0.0076

(g) **retain**

1. In a deck of strange cards, there are 813 cards. Each card has an image and a color. The amounts are shown in the table below.

	indigo	red	teal
bike	89	97	87
dog	58	31	16
gem	53	19	56
shovel	10	49	77
wheel	44	82	45

(a) What is the probability a random card is indigo given it is a gem?

(b) What is the probability a random card is either a bike or teal (or both)?

(c) What is the probability a random card is a shovel given it is red?

(d) What is the probability a random card is a shovel?

(e) What is the probability a random card is indigo?

(f) What is the probability a random card is both a dog and indigo?

Solution

(a) $P(\text{indigo given gem}) = \frac{53}{53+19+56} = 0.414$

(b) $P(\text{bike or teal}) = \frac{89+97+87+87+16+56+77+45-87}{813} = 0.574$

(c) $P(\text{shovel given red}) = \frac{49}{97+31+19+49+82} = 0.176$

(d) $P(\text{shovel}) = \frac{10+49+77}{813} = 0.167$

(e) $P(\text{indigo}) = \frac{89+58+53+10+44}{813} = 0.312$

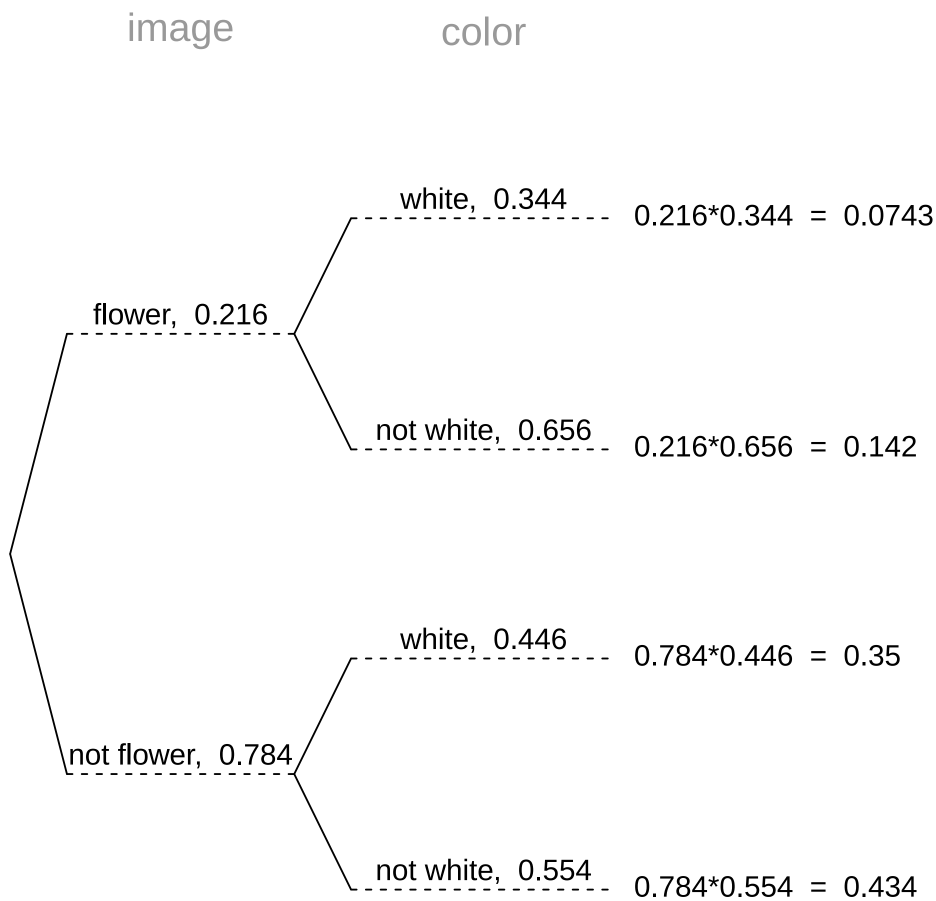
(f) $P(\text{dog and indigo}) = \frac{58}{813} = 0.0713$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a flower is 21.6%. If a flower is drawn, there is a 34.4% chance that it is white. If a card that is not a flower is drawn, there is a 44.6% chance that it is white.

   Now, someone draws a random card and reveals it is white. What is the chance the card is a flower?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"flower" given "white"}) = \frac{0.0743}{0.0743 + 0.35} = 0.175$$

3. In a very large pile of toothpicks, the mean length is 72.68 millimeters and the standard deviation is 1.85 millimeters. If you randomly sample 196 toothpicks, what is the chance the sample mean is between 72.57 and 72.81 millimeters?

Solution

Label the given information.

$$\mu = 72.68$$
$$\sigma = 1.85$$
$$n = 196$$
$$\bar{x}_{\text{lower}} = 72.57$$
$$\bar{x}_{\text{upper}} = 72.81$$

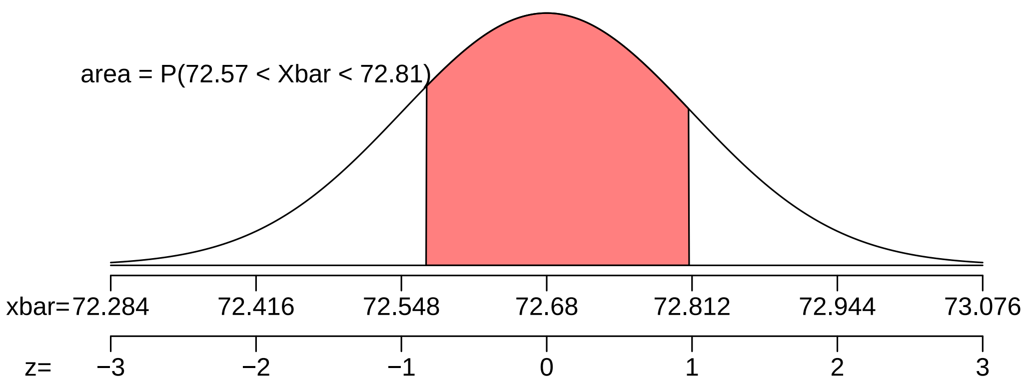
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.85}{\sqrt{196}} = 0.132$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(72.68,\ 0.132)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{72.57 - 72.68}{0.132} = -0.83$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{72.81 - 72.68}{0.132} = 0.98$$

Determine the probability.

$$\begin{aligned} P(72.57 < X < 72.81) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.98) - \Phi(-0.83) \\ &= 0.6332 \end{aligned}$$

4. In a game, there is a 52% chance to win a round. You will play 149 rounds.
   (a) What is the probability of winning exactly 82 rounds?
   (b) What is the probability of winning at least 65 but at most 78 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k} (p)^k (1 - p)^{n-k}$$

$$P(X = 82) = \binom{149}{82} (0.52)^{82} (1 - 0.52)^{149-82}$$

$$P(X = 82) = \binom{149}{82} (0.52)^{82} (0.48)^{67}$$

$$P(X = 82) = 0.0498$$

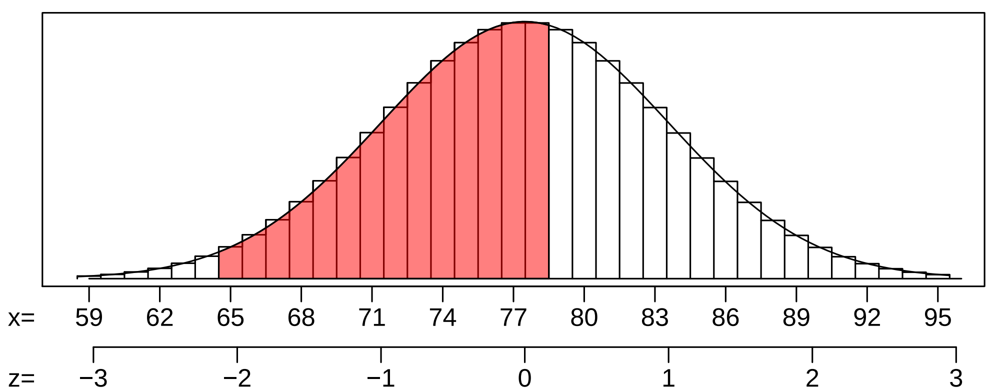
Find the mean.

$$\mu = np = (149)(0.52) = 77.48$$

Find the standard deviation.

$$\sigma = \sqrt{np(1 - p)} = \sqrt{(149)(0.52)(1 - 0.52)} = 6.0984$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{64.5 - 77.48}{6.0984} = -2.13$$

$$z_2 = \frac{78.5 - 77.48}{6.0984} = 0.17$$

Calculate the probability.

$$P(65 \leq X \leq 78) = \Phi(0.17) - \Phi(-2.13) = 0.5509$$

(a) $P(X = 82) = 0.0498$

(b) $P(65 \leq X \leq 78) = 0.5509$

5. As an ornithologist, you wish to determine the average body mass of *Vireo olivaceus*. You randomly sample 28 adults of *Vireo olivaceus*, resulting in a sample mean of 19.67 grams and a sample standard deviation of 2.38 grams. Determine a 98% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 28 \\ \bar{x} &= 19.67 \\ s &= 2.38 \\ CL &= 0.98 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 27$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.98$.

$$t^\star = 2.47$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.38}{\sqrt{28}} = 0.45$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (19.67 - 2.47 \times 0.45,\ 19.67 + 2.47 \times 0.45) \\ &= (18.6,\ 20.8) \end{aligned}$$

We are 98% confident that the population mean is between 18.6 and 20.8.

6. A treatment group of size 21 has a mean of 10 and standard deviation of 1.98. A control group of size 35 has a mean of 11.2 and standard deviation of 1.83. If you decided to use a signficance level of 0.04, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 39.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 21 \\ \bar{x}_1 &= 10 \\ s_1 &= 1.98 \\ n_2 &= 35 \\ \bar{x}_2 &= 11.2 \\ s_2 &= 1.83 \\ \alpha &= 0.04 \\ df &= 39 \end{aligned}$$

State the hypotheses.

$$\begin{aligned} H_0 &: \quad \mu_2 - \mu_1 = 0 \\ H_A &: \quad \mu_2 - \mu_1 \neq 0 \end{aligned}$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.04$ by using a t table.

$$t^\star = 2.12$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(1.98)^2}{21} + \frac{(1.83)^2}{35}} \\ &= 0.531 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(11.2 - 10) - (0)}{0.531} \\ &= 2.26 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.02 < p\text{-value} < 0.04$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.12$

(d) $SE = 0.531$

(e) $|t_{obs}| = 2.26$

(f) $0.02 < p\text{-value} < 0.04$

(g) reject the null

7. From a very large population, a random sample of 9500 individuals was taken. In that sample, 95.2% were floating. Determine a 98% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.952)(1-0.952)}{9500}} = 0.00219$$

Calculate the margin of error.

$$ME = z^\star SE = (2.33)(0.00219) = 0.0051$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.947\,,\ 0.957)$$

We are 98% confident that the true population proportion is between 94.7% and 95.7%.

(a) The lower bound = 0.947, which can also be expressed as 94.7%.

(b) The upper bound = 0.957, which can also be expressed as 95.7%.

8. An experiment is run with a treatment group of size 26 and a control group of size 10. The results are summarized in the table below.

	treatment	control
pink	22	4
not pink	4	6

Using a significance level of 0.005, determine whether the treatment causes an effect on the proportion of cases that are pink.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution
State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.005$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.81$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{22}{26} = 0.846$$

$$\hat{p}_2 = \frac{4}{10} = 0.4$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.4 - 0.846 = -0.446$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{22 + 4}{26 + 10} = 0.722$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.722)(0.278)}{26} + \frac{(0.722)(0.278)}{10}} \\ &= 0.167 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.167)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.4 - 0.846) - 0}{0.167} \\ &= -2.67 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.67) \\ &= 0.0076 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} > \alpha$$

So, we retain the null hypothesis. Thus the difference in proportions is not significant.

(a) $H_0: \quad p_2 - p_1 = 0$
(b) $H_A: \quad p_2 - p_1 \neq 0$
(c) $z^\star = 2.81$
(d) $SE = 0.167$
(e) $|z_{\text{obs}}| = 2.67$
(f) p-value = 0.0076
(g) retain the null