Name: _____

1. **Problem**
An experiment has n_1 = 7 plants in the treatment group and n_2 = 5 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	1.1	1.05	1.08	1.05	0.86	1.29	0.6
sample 2:	1.62	1.4	1.51	1.17	1.46		

(a) Determine degrees of freedom.
(b) Determine $t^\star$ for a 98% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail *p*-value.
(h) Determine an upper bound of two-tail *p*-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level α = 0.02? (yes or no)

1. (a) **4.000**
(b) **3.750**
(c) **0.111**
(d) **0.014**
(e) **0.846**
(f) **3.867**
(g) **0.010**
(h) **0.020**
(i) **yes**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1$$

$$\overline{x_2} = 1.43$$

$$s_1 = 0.218$$

$$s_2 = 0.167$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(7, 5) - 1 \;=\; 4$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 3.75$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.218)^2}{7} + \frac{(0.167)^2}{5}} = 0.111$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$

$$CI \;=\; (0.014,\, 0.846)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.43 - 1) - 0}{0.111} = 3.87$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 3.87$$

We use the table to determine bounds on p-value. Remember, $df = 4$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.01 \;<\; p\text{-value} \;<\; 0.02$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 4
(b) 3.75
(c) 0.111
(d) 0.014
(e) 0.846
(f) 3.867
(g) 0.01
(h) 0.02
(i) yes