Key ID: 025

Name: ______________________________

1. **Problem**
An experiment has $n_1 = 7$ plants in the treatment group and $n_2 = 4$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	1.07	1.03	1.39	0.76	0.82	0.83	0.74
sample 2:	1.05	0.84	1.62	1.19			

(a) Determine degrees of freedom.

(b) Determine $t^{\star}$ for a 90% confidence interval.

(c) Determine *SE*.

(d) Determine a lower bound of the 90% confidence interval of $\mu_2 - \mu_1$.

(e) Determine an upper bound of the 90% confidence interval of $\mu_2 - \mu_1$.

(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.

(g) Determine a lower bound of the two-tail p-value.

(h) Determine an upper bound of two-tail p-value.

(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.1$? (yes or no)

1. (a) 3.000

(b) 2.350

(c) 0.187

(d) -0.208

(e) 0.670

(f) 1.235

(g) 0.200

(h) 1.000

(i) no

1. **Solution**
   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 0.949$$
$$\overline{x_2} = 1.18$$
$$s_1 = 0.233$$
$$s_2 = 0.33$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(7, 4) - 1 = 3$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.9$

$$t^\star = 2.35$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.233)^2}{7} + \frac{(0.33)^2}{4}} = 0.187$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-0.208, 0.67)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.18 - 0.949) - 0}{0.187} = 1.24$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 1.24$$

We use the table to determine bounds on p-value. Remember, $df = 3$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.2 < p\text{-value} < 1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

   (a) 3
   (b) 2.35
   (c) 0.187
   (d) -0.208
   (e) 0.67
   (f) 1.235
   (g) 0.2
   (h) 1
   (i) no