1. (a) P(**wheel given red**) = 0.61

   (b) P(**wheel or gray**) = 0.499

   (c) P(**violet given wheel**) = 0.103

   (d) P(**orange**) = 0.217

   (e) P(**cat**) = 0.296

   (f) P(**cat and orange**) = 0.0628

2. P(**"not kite" given "gray"**) = 0.881

3. $P(68.39 < X < 68.91) = 0.7856$

4. (a) $P(X = 19) = 0.1032$

   (b) $P(13 \leq X \leq 25) = 0.8758$

5. $\mathbf{(17.1, 20.4)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.72$

   (d) $SE = 8.005$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.5$

   (f) $0.01 <$ p**-value** < 0.02

   (g) **retain**

7. (a) **LB of p CI = 0.709 or** 70.9%

   (b) **UB of p CI = 0.763 or** 76.3%

8. (a) $\boxed{H_0 : \quad p_2 - p_1 = 0}$

(b) $\boxed{H_{\mathbf{A}} : \quad p_2 - p_1 \neq 0}$

(c) $\boxed{z^\star = 2.58}$

(d) $\boxed{SE = 0.029}$

(e) $\boxed{\mid z_{\mathbf{obs}} \mid = 2.76}$

(f) $\boxed{p\textbf{-value} = 0.0058}$

(g) $\boxed{\textbf{reject}}$

1. In a deck of strange cards, there are 764 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	gray	orange	red	violet
cat	45	97	48	19	17
dog	82	32	54	27	91
wheel	43	47	64	72	26

(a) What is the probability a random card is a wheel given it is red?

(b) What is the probability a random card is either a wheel or gray (or both)?

(c) What is the probability a random card is violet given it is a wheel?

(d) What is the probability a random card is orange?

(e) What is the probability a random card is a cat?

(f) What is the probability a random card is both a cat and orange?

Solution

(a) P(wheel given red) = $\frac{72}{19+27+72}$ = 0.61

(b) P(wheel or gray) = $\frac{43+47+64+72+26+97+32+47-47}{764}$ = 0.499

(c) P(violet given wheel) = $\frac{26}{43+47+64+72+26}$ = 0.103

(d) P(orange) = $\frac{48+54+64}{764}$ = 0.217

(e) P(cat) = $\frac{45+97+48+19+17}{764}$ = 0.296

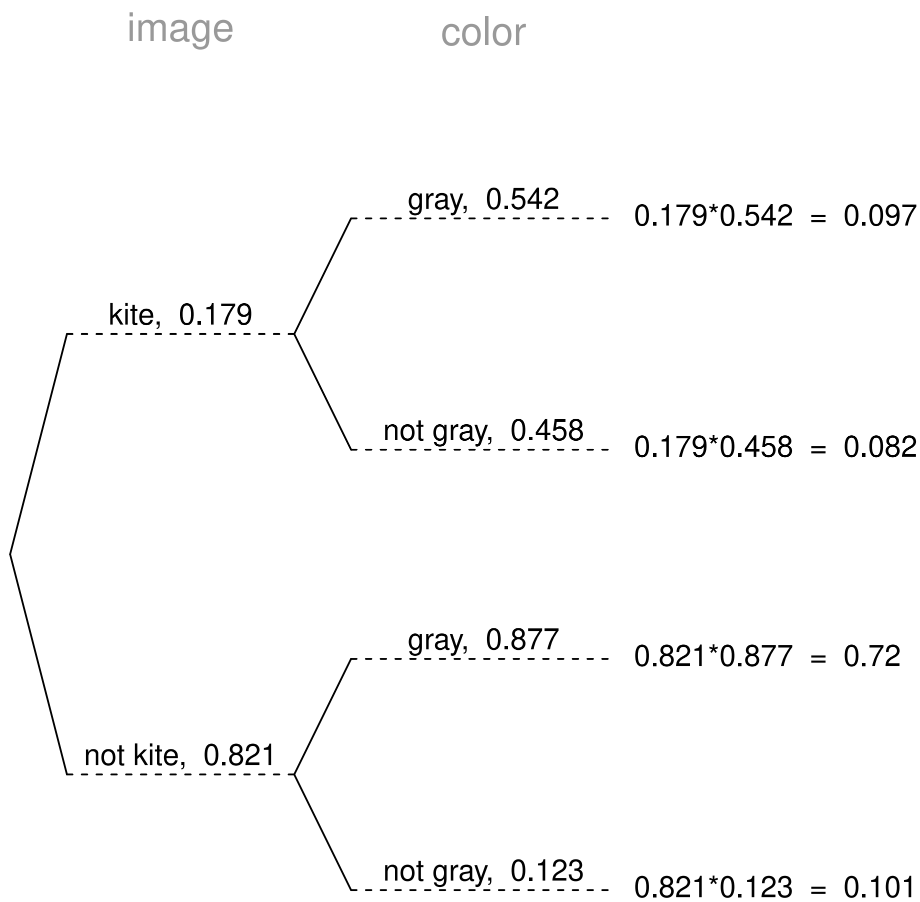
(f) P(cat and orange) = $\frac{48}{764}$ = 0.0628

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a kite is 17.9%. If a kite is drawn, there is a 54.2% chance that it is gray. If a card that is not a kite is drawn, there is a 87.7% chance that it is gray.

   Now, someone draws a random card and reveals it is gray. What is the chance the card is not a kite?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not kite" given "gray"}) = \frac{0.72}{0.72 + 0.097} = 0.881$$

3. In a very large pile of toothpicks, the mean length is 68.61 millimeters and the standard deviation is 2.92 millimeters. If you randomly sample 200 toothpicks, what is the chance the sample mean is between 68.39 and 68.91 millimeters?

Solution

Label the given information.

$$\mu = 68.61$$
$$\sigma = 2.92$$
$$n = 200$$
$$\bar{x}_{\text{lower}} = 68.39$$
$$\bar{x}_{\text{upper}} = 68.91$$

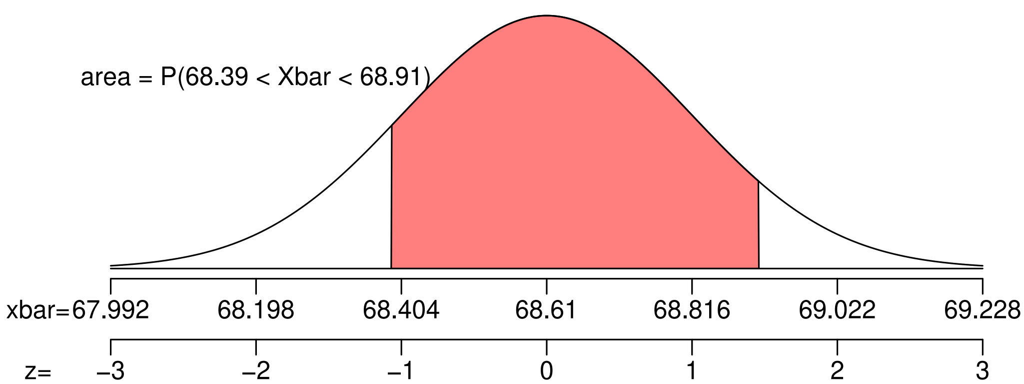
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2.92}{\sqrt{200}} = 0.206$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(68.61,\ 0.206)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{68.39 - 68.61}{0.206} = -1.07$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{68.91 - 68.61}{0.206} = 1.46$$

Determine the probability.

$$\begin{aligned} P(68.39 < X < 68.91) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.46) - \Phi(-1.07) \\ &= 0.7856 \end{aligned}$$

4. In a game, there is a 31% chance to win a round. You will play 65 rounds.

   (a) What is the probability of winning exactly 19 rounds?

   (b) What is the probability of winning at least 13 but at most 25 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 19) = \binom{65}{19}(0.31)^{19}(1-0.31)^{65-19}$$

$$P(X = 19) = \binom{65}{19}(0.31)^{19}(0.69)^{46}$$

$$P(X = 19) = 0.1032$$

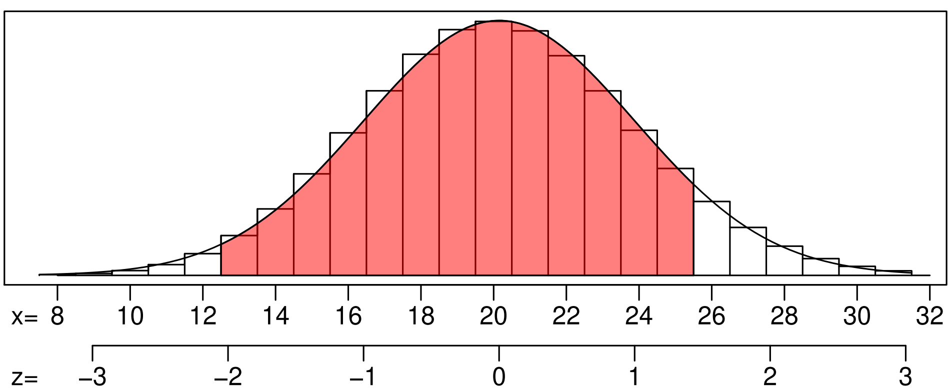
Find the mean.

$$\mu = np = (65)(0.31) = 20.15$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(65)(0.31)(1-0.31)} = 3.7287$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{12.5 - 20.15}{3.7287} = -1.92$$

$$z_2 = \frac{25.5 - 20.15}{3.7287} = 1.3$$

Calculate the probability.

$$P(13 \leq X \leq 25) = \Phi(1.3) - \Phi(-1.92) = 0.8758$$

(a) $P(X = 19) = 0.1032$

(b) $P(13 \leq X \leq 25) = 0.8758$

5. As an ornithologist, you wish to determine the average body mass of *Vireo olivaceus*. You randomly sample 20 adults of *Vireo olivaceus*, resulting in a sample mean of 18.75 grams and a sample standard deviation of 2.6 grams. Determine a 99% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$
\begin{aligned}
n &= 20 \\
\bar{x} &= 18.75 \\
s &= 2.6 \\
CL &= 0.99
\end{aligned}
$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 19$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.99$.

$$t^\star = 2.86$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.6}{\sqrt{20}} = 0.581$$

We want to make an inference about the population mean.

$$\mu \;\approx\; \bar{x} \pm t^\star SE$$

Determine the bounds.

$$
\begin{aligned}
CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\
&= (18.75 - 2.86 \times 0.581,\ 18.75 + 2.86 \times 0.581) \\
&= (17.1,\ 20.4)
\end{aligned}
$$

We are 99% confident that the population mean is between 17.1 and 20.4.

6. A treatment group of size 15 has a mean of 123 and standard deviation of 22.6. A control group of size 32 has a mean of 103 and standard deviation of 31. If you decided to use a signficance level of 0.01, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 36.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned}
n_1 &= 15\\
\bar{x}_1 &= 123\\
s_1 &= 22.6\\
n_2 &= 32\\
\bar{x}_2 &= 103\\
s_2 &= 31\\
\alpha &= 0.01\\
df &= 36
\end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.01$ by using a t table.

$$t^\star = 2.72$$

Calculate the standard error.

$$\begin{aligned}
SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}\\
&= \sqrt{\frac{(22.6)^2}{15} + \frac{(31)^2}{32}}\\
&= 8.005
\end{aligned}$$

Determine the test statistic.

$$\begin{aligned}
t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE}\\
&= \frac{(103 - 123) - (0)}{8.005}\\
&= -2.5
\end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_{\mathrm{A}}: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^{\star} = 2.72$

(d) $SE = 8.005$

(e) $|t_{\mathrm{obs}}| = 2.5$

(f) $0.01 < p\text{-value} < 0.02$

(g) retain the null

7. From a very large population, a random sample of 1100 individuals was taken. In that sample, 73.6% were special. Determine a 96% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.96$.

$$z^\star = 2.05$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.736)(1-0.736)}{1100}} = 0.0133$$

Calculate the margin of error.

$$ME = z^\star SE = (2.05)(0.0133) = 0.0273$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.709\,,\ 0.763)$$

We are 96% confident that the true population proportion is between 70.9% and 76.3%.

(a) The lower bound = 0.709, which can also be expressed as 70.9%.

(b) The upper bound = 0.763, which can also be expressed as 76.3%.

8. An experiment is run with a treatment group of size 147 and a control group of size 170. The results are summarized in the table below.

	treatment	control
green	130	164
not green	17	6

Using a significance level of 0.01, determine whether the treatment causes an effect on the proportion of cases that are green.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.01$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.58$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{130}{147} = 0.884$$
$$\hat{p}_2 = \frac{164}{170} = 0.965$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.965 - 0.884 = 0.081$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{130 + 164}{147 + 170} = 0.927$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.927)(0.073)}{147} + \frac{(0.927)(0.073)}{170}} \\ &= 0.0293 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0293)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.965 - 0.884) - 0}{0.0293} \\ &= 2.76 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.76) \\ &= 0.0058 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \ p_2 - p_1 = 0$

(b) $H_A: \ p_2 - p_1 \neq 0$

(c) $z^\star = 2.58$

(d) $SE = 0.0293$

(e) $|z_{obs}| = 2.76$

(f) p-value = 0.0058

(g) reject the null

1. (a) P(**wheel given red**) = 0.61

   (b) P(**wheel or gray**) = 0.499

   (c) P(**violet given wheel**) = 0.103

   (d) P(**orange**) = 0.217

   (e) P(**cat**) = 0.296

   (f) P(**cat and orange**) = 0.0628

2. P(**"not kite" given "gray"**) = 0.881

3. $P(68.39 < X < 68.91) = 0.7856$

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   (b) $P(13 \leq X \leq 25) = 0.8758$

5. **(17.1, 20.4)**

6. (a) $H_0 : \ \mu_2 - \mu_1 = 0$

   (b) $H_0 : \ \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.72$

   (d) $SE = 8.005$

   (e) $\mid t_{\textbf{obs}} \mid = 2.5$

   (f) $0.01 < p$-**value** < 0.02

   (g) **retain**

7. (a) **LB of p CI = 0.709 or** 70.9%

   (b) **UB of p CI = 0.763 or** 76.3%

8. (a) $H_0: \quad p_2 - p_1 = 0$

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(d) $SE = 0.029$

(e) $|\, z_{\mathbf{obs}} \,| = 2.76$

(f) p-**value** $= 0.0058$

(g) **reject**

1. In a deck of strange cards, there are 764 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	gray	orange	red	violet
cat	45	97	48	19	17
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(a) What is the probability a random card is a wheel given it is red?

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(f) What is the probability a random card is both a cat and orange?

Solution

(a) $P(\text{wheel given red}) = \frac{72}{19+27+72} = 0.61$

(b) $P(\text{wheel or gray}) = \frac{43+47+64+72+26+97+32+47-47}{764} = 0.499$

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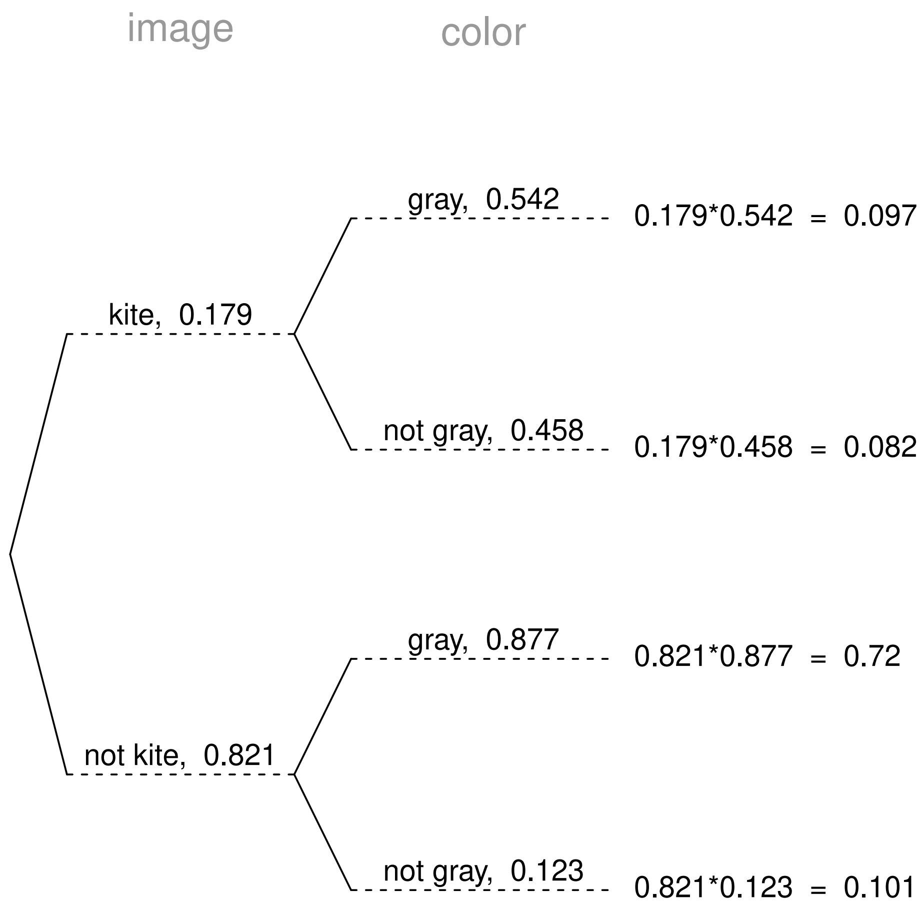
2. In a deck of strange cards, each card has an image and a color. The chance of drawing a kite is 17.9%. If a kite is drawn, there is a 54.2% chance that it is gray. If a card that is not a kite is drawn, there is a 87.7% chance that it is gray.

   Now, someone draws a random card and reveals it is gray. What is the chance the card is not a kite?

Solution
I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



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3. In a very large pile of toothpicks, the mean length is 68.61 millimeters and the standard deviation is 2.92 millimeters. If you randomly sample 200 toothpicks, what is the chance the sample mean is between 68.39 and 68.91 millimeters?

Solution

Label the given information.

$$\mu = 68.61$$
$$\sigma = 2.92$$
$$n = 200$$
$$\bar{x}_{\text{lower}} = 68.39$$
$$\bar{x}_{\text{upper}} = 68.91$$

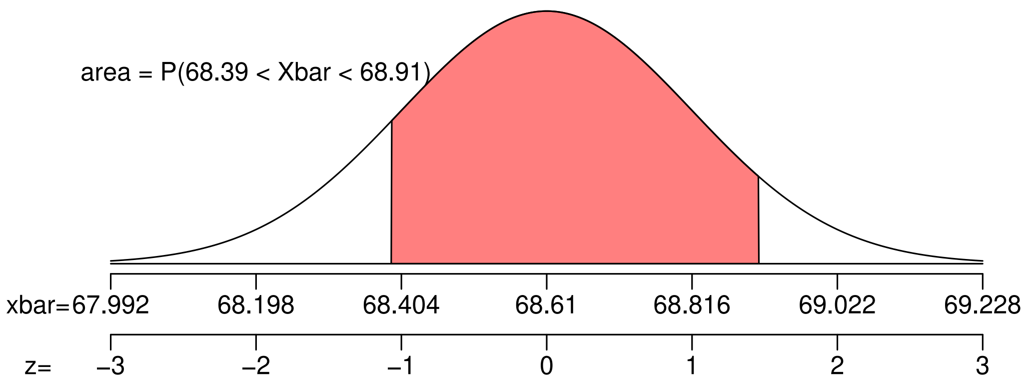
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2.92}{\sqrt{200}} = 0.206$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(68.61,\ 0.206)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{68.39 - 68.61}{0.206} = -1.07$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{68.91 - 68.61}{0.206} = 1.46$$

Determine the probability.

$$\begin{aligned} P(68.39 < X < 68.91) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.46) - \Phi(-1.07) \\ &= 0.7856 \end{aligned}$$

4. In a game, there is a 31% chance to win a round. You will play 65 rounds.
   (a) What is the probability of winning exactly 19 rounds?
   (b) What is the probability of winning at least 13 but at most 25 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 19) = \binom{65}{19}(0.31)^{19}(1-0.31)^{65-19}$$

$$P(X = 19) = \binom{65}{19}(0.31)^{19}(0.69)^{46}$$

$$P(X = 19) = 0.1032$$

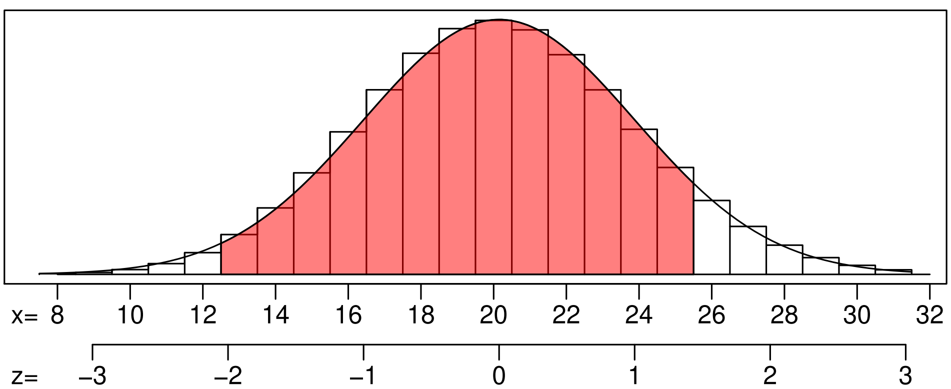
Find the mean.

$$\mu = np = (65)(0.31) = 20.15$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(65)(0.31)(1-0.31)} = 3.7287$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{12.5 - 20.15}{3.7287} = -1.92$$

$$z_2 = \frac{25.5 - 20.15}{3.7287} = 1.3$$

Calculate the probability.

$$P(13 \leq X \leq 25) = \Phi(1.3) - \Phi(-1.92) = 0.8758$$

(a) $P(X = 19) = 0.1032$

(b) $P(13 \leq X \leq 25) = 0.8758$

5. As an ornithologist, you wish to determine the average body mass of *Vireo olivaceus*. You randomly sample 20 adults of *Vireo olivaceus*, resulting in a sample mean of 18.75 grams and a sample standard deviation of 2.6 grams. Determine a 99% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 20 \\ \bar{x} &= 18.75 \\ s &= 2.6 \\ CL &= 0.99 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 19$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.99$.

$$t^\star = 2.86$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.6}{\sqrt{20}} = 0.581$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (18.75 - 2.86 \times 0.581,\ 18.75 + 2.86 \times 0.581) \\ &= (17.1,\ 20.4) \end{aligned}$$

We are 99% confident that the population mean is between 17.1 and 20.4.

6. A treatment group of size 15 has a mean of 123 and standard deviation of 22.6. A control group of size 32 has a mean of 103 and standard deviation of 31. If you decided to use a signficance level of 0.01, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 36.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 15 \\ \bar{x}_1 &= 123 \\ s_1 &= 22.6 \\ n_2 &= 32 \\ \bar{x}_2 &= 103 \\ s_2 &= 31 \\ \alpha &= 0.01 \\ df &= 36 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.01$ by using a t table.

$$t^\star = 2.72$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(22.6)^2}{15} + \frac{(31)^2}{32}} \\ &= 8.005 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(103 - 123) - (0)}{8.005} \\ &= -2.5 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.72$

(d) $SE = 8.005$

(e) $|t_{obs}| = 2.5$

(f) $0.01 < p\text{-value} < 0.02$

(g) retain the null

7. From a very large population, a random sample of 1100 individuals was taken. In that sample, 73.6% were special. Determine a 96% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.96$.

$$z^{\star} = 2.05$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.736)(1-0.736)}{1100}} = 0.0133$$

Calculate the margin of error.

$$ME = z^{\star}SE = (2.05)(0.0133) = 0.0273$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.709\,,\ 0.763)$$

We are 96% confident that the true population proportion is between 70.9% and 76.3%.

(a) The lower bound = 0.709, which can also be expressed as 70.9%.

(b) The upper bound = 0.763, which can also be expressed as 76.3%.

8. An experiment is run with a treatment group of size 147 and a control group of size 170. The results are summarized in the table below.

	treatment	control
green	130	164
not green	17	6

Using a significance level of 0.01, determine whether the treatment causes an effect on the proportion of cases that are green.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.01$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.58$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{130}{147} = 0.884$$
$$\hat{p}_2 = \frac{164}{170} = 0.965$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.965 - 0.884 = 0.081$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{130 + 164}{147 + 170} = 0.927$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.927)(0.073)}{147} + \frac{(0.927)(0.073)}{170}} \\ &= 0.0293 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0293)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.965 - 0.884) - 0}{0.0293} \\ &= 2.76 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.76) \\ &= 0.0058 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.58$

(d) $SE = 0.0293$

(e) $|z_{\text{obs}}| = 2.76$

(f) p-value = 0.0058

(g) reject the null

1. (a) $P(\textbf{pink given bike}) = 0.321$

   (b) $P(\textbf{tree given pink}) = 0.21$

   (c) $P(\textbf{red}) = 0.283$

   (d) $P(\textbf{bike and red}) = 0.112$

   (e) $P(\textbf{bike or red}) = 0.502$

   (f) $P(\textbf{pig}) = 0.208$

2. $P(\textbf{"not tree" given "not yellow"}) = 0.511$

3. $P(61.55 < X < 62.3) = 0.8251$

4. (a) $P(X = 30) = 0.0884$

   (b) $P(34 \leq X \leq 39) = 0.2244$

5. $\mathbf{(54.2, 57)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 1.99$

   (d) $SE = 20.518$

   (e) $\mid t_{\textbf{obs}} \mid = 1.95$

   (f) $0.05 < p\textbf{-value} < 0.1$

   (g) **retain**

7. (a) **LB of p CI = 0.0182 or** 1.82%

   (b) **UB of p CI = 0.0198 or** 1.98%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.05$

(d) $SE = 0.166$

(e) $|\, z_{\mathbf{obs}} \,| = 2.14$

(f) p-**value** $= 0.0324$

(g) **reject**

1. In a deck of strange cards, there are 827 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	pink	red	white
bike	24	88	93	69
pig	87	27	42	16
tree	21	38	47	58
wheel	51	28	52	86

(a) What is the probability a random card is pink given it is a bike?

(b) What is the probability a random card is a tree given it is pink?

(c) What is the probability a random card is red?

(d) What is the probability a random card is both a bike and red?

(e) What is the probability a random card is either a bike or red (or both)?

(f) What is the probability a random card is a pig?

Solution

(a) P(pink given bike) = $\frac{88}{24+88+93+69}$ = 0.321

(b) P(tree given pink) = $\frac{38}{88+27+38+28}$ = 0.21

(c) P(red) = $\frac{93+42+47+52}{827}$ = 0.283

(d) P(bike and red) = $\frac{93}{827}$ = 0.112

(e) P(bike or red) = $\frac{24+88+93+69+93+42+47+52-93}{827}$ = 0.502

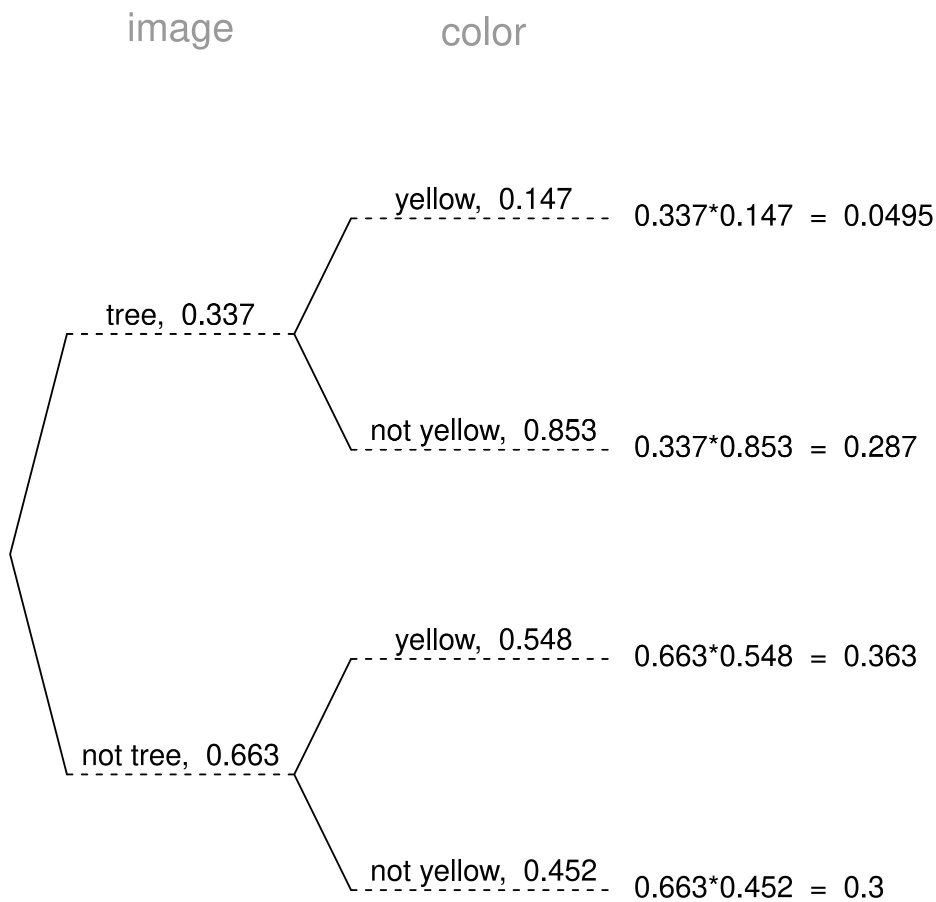
(f) P(pig) = $\frac{87+27+42+16}{827}$ = 0.208

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a tree is 33.7%. If a tree is drawn, there is a 14.7% chance that it is yellow. If a card that is not a tree is drawn, there is a 54.8% chance that it is yellow.

   Now, someone draws a random card and reveals it is not yellow. What is the chance the card is not a tree?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not tree" given "not yellow"}) = \frac{0.3}{0.3 + 0.287} = 0.511$$

3. In a very large pile of toothpicks, the mean length is 61.98 millimeters and the standard deviation is 3.52 millimeters. If you randomly sample 169 toothpicks, what is the chance the sample mean is between 61.55 and 62.3 millimeters?

Solution

Label the given information.

$$\mu = 61.98$$
$$\sigma = 3.52$$
$$n = 169$$
$$\bar{x}_{\text{lower}} = 61.55$$
$$\bar{x}_{\text{upper}} = 62.3$$

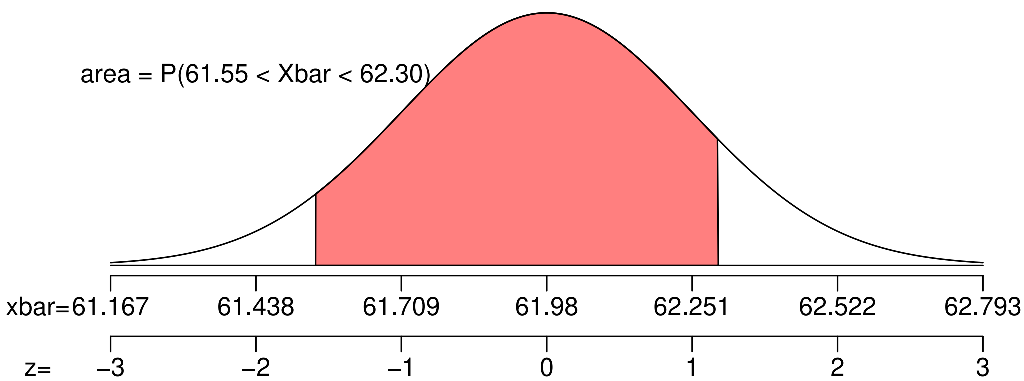
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.52}{\sqrt{169}} = 0.271$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(61.98,\ 0.271)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{61.55 - 61.98}{0.271} = -1.59$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{62.3 - 61.98}{0.271} = 1.18$$

Determine the probability.

$$\begin{aligned} P(61.55 < X < 62.3) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.18) - \Phi(-1.59) \\ &= 0.8251 \end{aligned}$$

4. In a game, there is a 39% chance to win a round. You will play 80 rounds.
   (a) What is the probability of winning exactly 30 rounds?
   (b) What is the probability of winning at least 34 but at most 39 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 30) = \binom{80}{30}(0.39)^{30}(1-0.39)^{80-30}$$

$$P(X = 30) = \binom{80}{30}(0.39)^{30}(0.61)^{50}$$

$$P(X = 30) = 0.0884$$

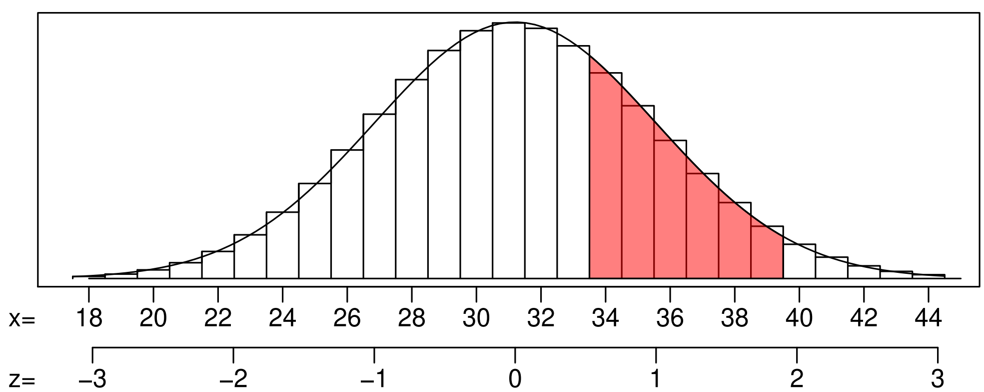
Find the mean.

$$\mu = np = (80)(0.39) = 31.2$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(80)(0.39)(1-0.39)} = 4.3626$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{33.5 - 31.2}{4.3626} = 0.64$$

$$z_2 = \frac{39.5 - 31.2}{4.3626} = 1.79$$

Calculate the probability.

$$P(34 \leq X \leq 39) = \Phi(1.79) - \Phi(0.64) = 0.2244$$

(a) $P(X = 30) = 0.0884$

(b) $P(34 \leq X \leq 39) = 0.2244$

5. As an ornithologist, you wish to determine the average body mass of *Hylocichla mustelina*. You randomly sample 29 adults of *Hylocichla mustelina*, resulting in a sample mean of 55.61 grams and a sample standard deviation of 5.63 grams. Determine a 80% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 29 \\ \bar{x} &= 55.61 \\ s &= 5.63 \\ CL &= 0.8 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 28$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.8$.

$$t^\star = 1.31$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{5.63}{\sqrt{29}} = 1.05$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (55.61 - 1.31 \times 1.05,\ 55.61 + 1.31 \times 1.05) \\ &= (54.2,\ 57) \end{aligned}$$

We are 80% confident that the population mean is between 54.2 and 57.

6. A treatment group of size 32 has a mean of 1060 and standard deviation of 77.5. A control group of size 40 has a mean of 1020 and standard deviation of 96.6. If you decided to use a signficance level of 0.05, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 69.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 32$$
$$\bar{x}_1 = 1060$$
$$s_1 = 77.5$$
$$n_2 = 40$$
$$\bar{x}_2 = 1020$$
$$s_2 = 96.6$$
$$\alpha = 0.05$$
$$df = 69$$

State the hypotheses.

$$H_0 : \quad \mu_2 - \mu_1 = 0$$
$$H_A : \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.05$ by using a t table.

$$t^\star = 1.99$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(77.5)^2}{32} + \frac{(96.6)^2}{40}} \\ &= 20.518 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1020 - 1060) - (0)}{20.518} \\ &= -1.95 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 1.99$

(d) $SE = 20.518$

(e) $|t_{obs}| = 1.95$

(f) $0.05 < p\text{-value} < 0.1$

(g) retain the null

7. From a very large population, a random sample of 52000 individuals was taken. In that sample, 1.9% were angry. Determine a 80% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.8$.

$$z^{\star} = 1.28$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.019)(1-0.019)}{52000}} = 0.000599$$

Calculate the margin of error.

$$ME = z^{\star}SE = (1.28)(0.000599) = 0.000767$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.0182\,,\ 0.0198)$$

We are 80% confident that the true population proportion is between 1.82% and 1.98%.

(a) The lower bound = 0.0182, which can also be expressed as 1.82%.

(b) The upper bound = 0.0198, which can also be expressed as 1.98%.

8. An experiment is run with a treatment group of size 16 and a control group of size 21. The results are summarized in the table below.

	treatment	control
abysmal	5	14
not abysmal	11	7

Using a significance level of 0.04, determine whether the treatment causes an effect on the proportion of cases that are abysmal.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.04$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.05$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{5}{16} = 0.312$$
$$\hat{p}_2 = \frac{14}{21} = 0.667$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.667 - 0.312 = 0.355$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{5 + 14}{16 + 21} = 0.514$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.514)(0.486)}{16} + \frac{(0.514)(0.486)}{21}} \\ &= 0.166 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.166)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.667 - 0.312) - 0}{0.166} \\ &= 2.14 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.14) \\ &= 0.0324 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.05$

(d) $SE = 0.166$

(e) $|z_{\text{obs}}| = 2.14$

(f) p-value = 0.0324

(g) reject the null

1. (a) P(**wheel and orange**) = 0.0495

   (b) P(**blue**) = 0.18

   (c) P(**tree**) = 0.229

   (d) P(**pig given red**) = 0.208

   (e) P(**indigo given tree**) = 0.307

   (f) P(**wheel or orange**) = 0.408

2. P(**"cat" given "not green"**) = 0.22

3. $P(66.76 < X < 66.99) = 0.6043$

4. (a) $P(X = 37) = 0.0722$

   (b) $P(38 \leq X \leq 48) = 0.3432$

5. **(63.1, 70.9)**

6. (a) $H_0 : \ \mu_2 - \mu_1 = 0$

   (b) $H_0 : \ \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.13$

   (d) $SE = 6.063$

   (e) $|\ t_{\mathbf{obs}}\ | = 1.85$

   (f) $0.05 <$ p**-value** < 0.1

   (g) **retain**

7. (a) **LB of p CI = 0.0981 or** 9.81%

   (b) **UB of p CI = 0.104 or** 10.4%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.64$

(d) $SE = 0.021$

(e) $\mid z_{\mathbf{obs}} \mid = 1.88$

(f) p-**value** $= 0.0602$

(g) **reject**

1. In a deck of strange cards, there are 1253 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	indigo	orange	red	violet
flower	14	67	47	40	10
pig	52	98	48	51	20
shovel	53	41	90	55	27
tree	70	88	73	21	35
wheel	36	61	62	78	16

(a) What is the probability a random card is both a wheel and orange?

(b) What is the probability a random card is blue?

(c) What is the probability a random card is a tree?

(d) What is the probability a random card is a pig given it is red?

(e) What is the probability a random card is indigo given it is a tree?

(f) What is the probability a random card is either a wheel or orange (or both)?

Solution

(a) $P(\text{wheel and orange}) = \frac{62}{1253} = 0.0495$

(b) $P(\text{blue}) = \frac{14+52+53+70+36}{1253} = 0.18$

(c) $P(\text{tree}) = \frac{70+88+73+21+35}{1253} = 0.229$

(d) $P(\text{pig given red}) = \frac{51}{40+51+55+21+78} = 0.208$

(e) $P(\text{indigo given tree}) = \frac{88}{70+88+73+21+35} = 0.307$

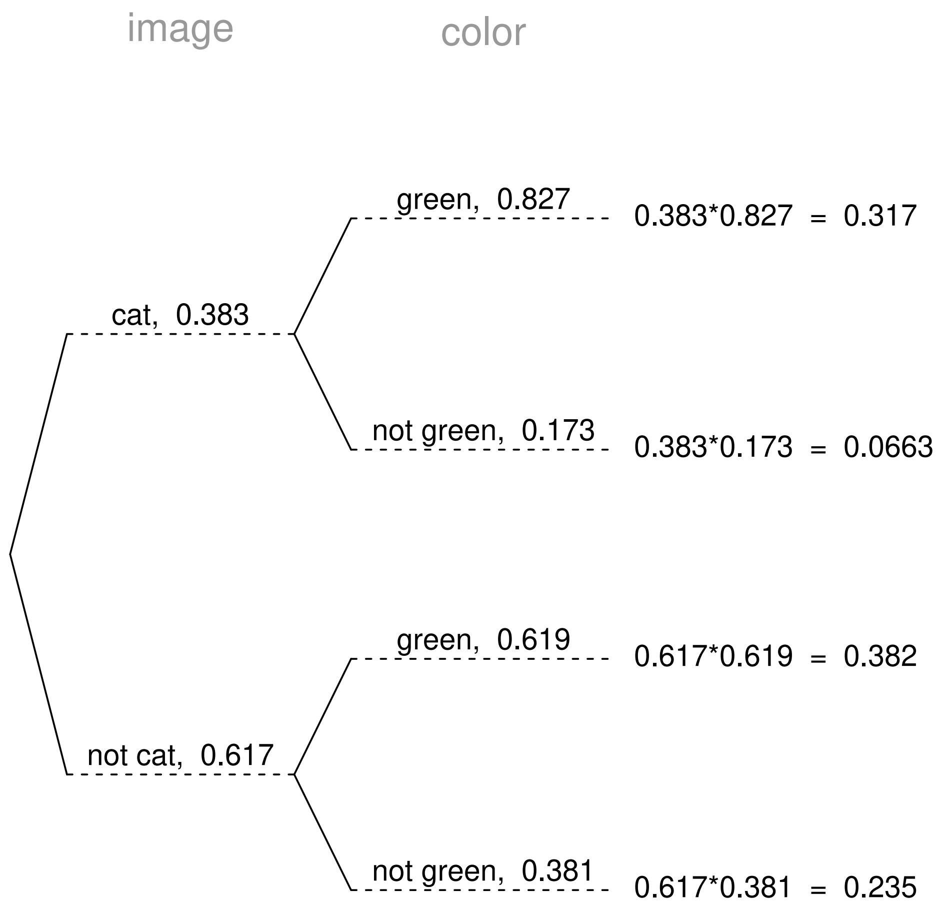
(f) $P(\text{wheel or orange}) = \frac{36+61+62+78+16+47+48+90+73+62-62}{1253} = 0.408$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a cat is 38.3%. If a cat is drawn, there is a 82.7% chance that it is green. If a card that is not a cat is drawn, there is a 61.9% chance that it is green.

   Now, someone draws a random card and reveals it is not green. What is the chance the card is a cat?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"cat" given "not green"}) = \frac{0.0663}{0.0663 + 0.235} = 0.22$$

3. In a very large pile of toothpicks, the mean length is 66.87 millimeters and the standard deviation is 1.79 millimeters. If you randomly sample 175 toothpicks, what is the chance the sample mean is between 66.76 and 66.99 millimeters?

Solution
Label the given information.

$$\mu = 66.87$$
$$\sigma = 1.79$$
$$n = 175$$
$$\bar{x}_{\text{lower}} = 66.76$$
$$\bar{x}_{\text{upper}} = 66.99$$

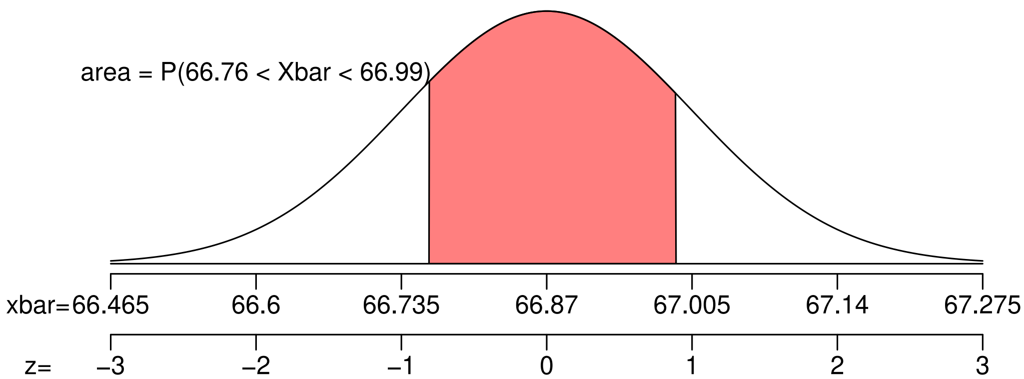
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.79}{\sqrt{175}} = 0.135$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(66.87,\ 0.135)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{66.76 - 66.87}{0.135} = -0.81$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{66.99 - 66.87}{0.135} = 0.89$$

Determine the probability.

$$\begin{aligned} P(66.76 < X < 66.99) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.89) - \Phi(-0.81) \\ &= 0.6043 \end{aligned}$$

4. In a game, there is a 20% chance to win a round. You will play 180 rounds.
   (a) What is the probability of winning exactly 37 rounds?
   (b) What is the probability of winning at least 38 but at most 48 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 37) = \binom{180}{37}(0.2)^{37}(1-0.2)^{180-37}$$

$$P(X = 37) = \binom{180}{37}(0.2)^{37}(0.8)^{143}$$

$$P(X = 37) = 0.0722$$

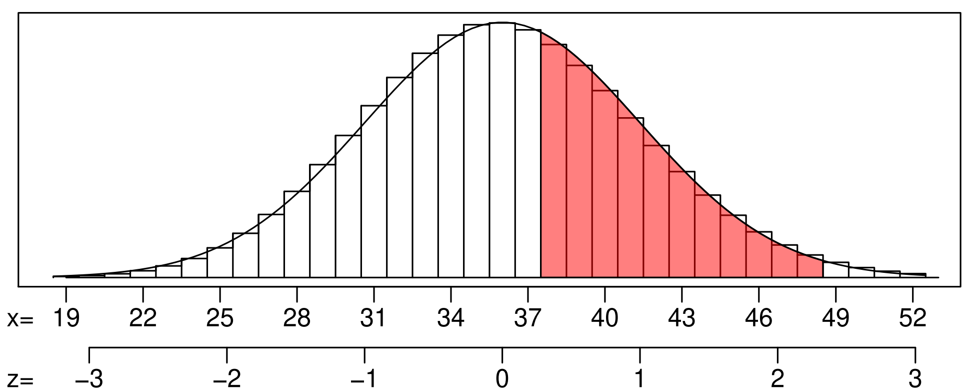
Find the mean.

$$\mu = np = (180)(0.2) = 36$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(180)(0.2)(1-0.2)} = 5.3666$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{37.5 - 36}{5.3666} = 0.37$$

$$z_2 = \frac{48.5 - 36}{5.3666} = 2.24$$

Calculate the probability.

$$P(38 \leq X \leq 48) = \Phi(2.24) - \Phi(0.37) = 0.3432$$

(a) $P(X = 37) = 0.0722$

(b) $P(38 \leq X \leq 48) = 0.3432$

5. As an ornithologist, you wish to determine the average body mass of *Porzana carolina*. You randomly sample 27 adults of *Porzana carolina*, resulting in a sample mean of 66.97 grams and a sample standard deviation of 9.8 grams. Determine a 95% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 27 \\ \bar{x} &= 66.97 \\ s &= 9.8 \\ CL &= 0.95 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 26$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.95$.

$$t^\star = 2.06$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{9.8}{\sqrt{27}} = 1.89$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (66.97 - 2.06 \times 1.89,\ 66.97 + 2.06 \times 1.89) \\ &= (63.1,\ 70.9) \end{aligned}$$

We are 95% confident that the population mean is between 63.1 and 70.9.

6. A treatment group of size 21 has a mean of 108 and standard deviation of 18.3. A control group of size 20 has a mean of 96.8 and standard deviation of 20.4. If you decided to use a signficance level of 0.04, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 38.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 21$$
$$\bar{x}_1 = 108$$
$$s_1 = 18.3$$
$$n_2 = 20$$
$$\bar{x}_2 = 96.8$$
$$s_2 = 20.4$$
$$\alpha = 0.04$$
$$df = 38$$

State the hypotheses.

$$H_0 : \quad \mu_2 - \mu_1 = 0$$
$$H_A : \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.04$ by using a t table.

$$t^\star = 2.13$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(18.3)^2}{21} + \frac{(20.4)^2}{20}} \\ &= 6.063 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(96.8 - 108) - (0)}{6.063} \\ &= -1.85 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.13$

(d) $SE = 6.063$

(e) $|t_{\text{obs}}| = 1.85$

(f) $0.05 < p\text{-value} < 0.1$

(g) retain the null

7. From a very large population, a random sample of 59000 individuals was taken. In that sample, 10.1% were asleep. Determine a 98% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.98$.

$$z^{\star} = 2.33$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.101)(1-0.101)}{59000}} = 0.00124$$

Calculate the margin of error.

$$ME = z^{\star} SE = (2.33)(0.00124) = 0.00289$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.0981\ ,\ 0.104)$$

We are 98% confident that the true population proportion is between 9.81% and 10.4%.

(a) The lower bound = 0.0981, which can also be expressed as 9.81%.

(b) The upper bound = 0.104, which can also be expressed as 10.4%.

8. An experiment is run with a treatment group of size 235 and a control group of size 268. The results are summarized in the table below.

	treatment	control
cold	19	11
not cold	216	257

Using a significance level of 0.1, determine whether the treatment causes an effect on the proportion of cases that are cold.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.1$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.64$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{19}{235} = 0.0809$$
$$\hat{p}_2 = \frac{11}{268} = 0.041$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.041 - 0.0809 = -0.0399$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{19 + 11}{235 + 268} = 0.0596$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.0596)(0.9404)}{235} + \frac{(0.0596)(0.9404)}{268}} \\ &= 0.0212 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0212)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.041 - 0.0809) - 0}{0.0212} \\ &= -1.88 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-1.88) \\ &= 0.0602 \end{aligned}$$

Compare the *p*-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \ p_2 - p_1 = 0$

(b) $H_A: \ p_2 - p_1 \neq 0$

(c) $z^\star = 1.64$

(d) $SE = 0.0212$

(e) $|z_{\text{obs}}| = 1.88$

(f) p-value = 0.0602

(g) reject the null

1. (a) P(**horn given black**) = 0.4

   (b) P(**pig or teal**) = 0.614

   (c) P(**teal given pig**) = 0.358

   (d) P(**horn and indigo**) = 0.0707

   (e) P(**black**) = 0.269

   (f) P(**pig**) = 0.367

2. P(**"not horn" given "black"**) = 0.821

3. $P(71.36 < X < 71.65) = 0.493$

4. (a) $P(X = 41) = 0.0718$

   (b) $P(33 \leq X \leq 51) = 0.8808$

5. **(41.3, 43.5)**

6. (a) $H_0 : \ \mu_2 - \mu_1 = 0$

   (b) $H_0 : \ \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.45$

   (d) $SE = 4.179$

   (e) $| \ t_{\mathbf{obs}} \ | = 2.39$

   (f) $0.02 <$ p**-value** < 0.04

   (g) **retain**

7. (a) **LB of p CI = 0.0569 or** 5.69%

   (b) **UB of p CI = 0.0731 or** 7.31%

8. (a) $H_0: \; p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \; p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.05$

(d) $SE = 0.056$

(e) $\mid z_{\mathbf{obs}} \mid = 2.31$

(f) p-**value** $= 0.0208$

(g) **reject**

1. In a deck of strange cards, there are 594 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	gray	indigo	teal
flower	51	44	17	91
horn	64	11	42	56
pig	45	32	63	78

(a) What is the probability a random card is a horn given it is black?

(b) What is the probability a random card is either a pig or teal (or both)?

(c) What is the probability a random card is teal given it is a pig?

(d) What is the probability a random card is both a horn and indigo?

(e) What is the probability a random card is black?

(f) What is the probability a random card is a pig?

Solution

(a) $P(\text{horn given black}) = \frac{64}{51+64+45} = 0.4$

(b) $P(\text{pig or teal}) = \frac{45+32+63+78+91+56+78-78}{594} = 0.614$

(c) $P(\text{teal given pig}) = \frac{78}{45+32+63+78} = 0.358$

(d) $P(\text{horn and indigo}) = \frac{42}{594} = 0.0707$

(e) $P(\text{black}) = \frac{51+64+45}{594} = 0.269$

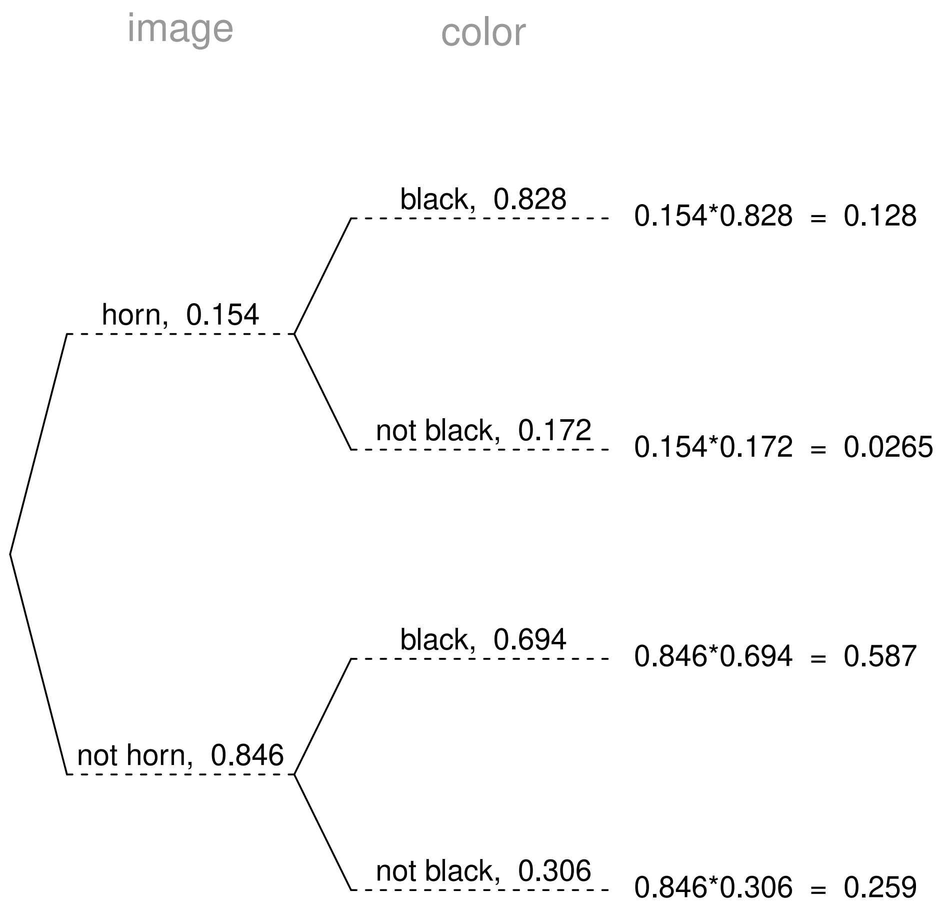
(f) $P(\text{pig}) = \frac{45+32+63+78}{594} = 0.367$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a horn is 15.4%. If a horn is drawn, there is a 82.8% chance that it is black. If a card that is not a horn is drawn, there is a 69.4% chance that it is black.

   Now, someone draws a random card and reveals it is black. What is the chance the card is not a horn?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not horn" given "black"}) = \frac{0.587}{0.587 + 0.128} = 0.821$$

3. In a very large pile of toothpicks, the mean length is 71.49 millimeters and the standard deviation is 2.84 millimeters. If you randomly sample 169 toothpicks, what is the chance the sample mean is between 71.36 and 71.65 millimeters?

Solution

Label the given information.

$$\mu = 71.49$$
$$\sigma = 2.84$$
$$n = 169$$
$$\bar{x}_{\text{lower}} = 71.36$$
$$\bar{x}_{\text{upper}} = 71.65$$

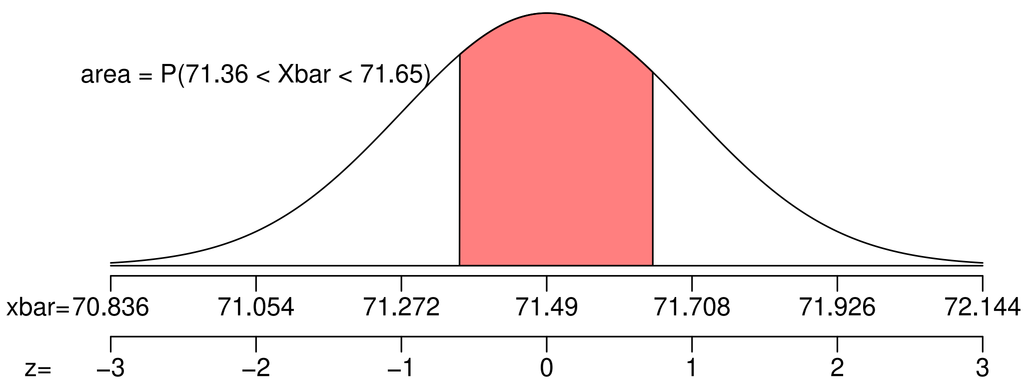
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2.84}{\sqrt{169}} = 0.218$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(71.49,\ 0.218)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{71.36 - 71.49}{0.218} = -0.6$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{71.65 - 71.49}{0.218} = 0.73$$

Determine the probability.

$$\begin{aligned} P(71.36 < X < 71.65) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.73) - \Phi(-0.6) \\ &= 0.493 \end{aligned}$$

4. In a game, there is a 27% chance to win a round. You will play 148 rounds.
   (a) What is the probability of winning exactly 41 rounds?
   (b) What is the probability of winning at least 33 but at most 51 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 41) = \binom{148}{41}(0.27)^{41}(1-0.27)^{148-41}$$

$$P(X = 41) = \binom{148}{41}(0.27)^{41}(0.73)^{107}$$

$$P(X = 41) = 0.0718$$

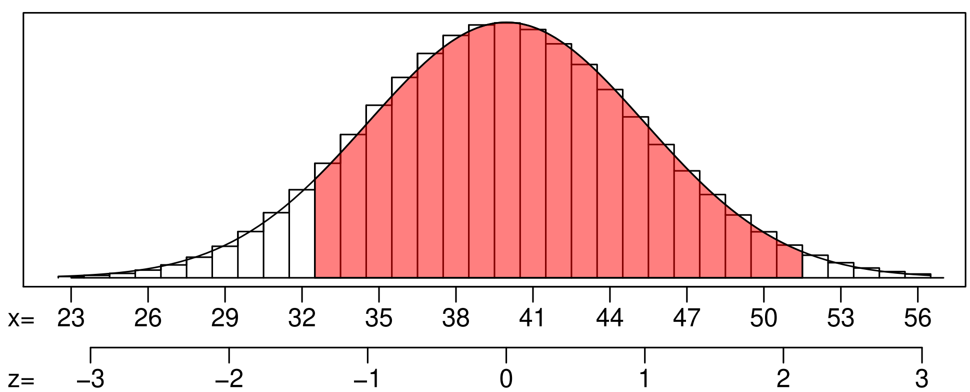
Find the mean.

$$\mu = np = (148)(0.27) = 39.96$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(148)(0.27)(1-0.27)} = 5.401$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{32.5 - 39.96}{5.401} = -1.29$$

$$z_2 = \frac{51.5 - 39.96}{5.401} = 2.04$$

Calculate the probability.

$$P(33 \le X \le 51) = \Phi(2.04) - \Phi(-1.29) = 0.8808$$

(a) $P(X = 41) = 0.0718$

(b) $P(33 \le X \le 51) = 0.8808$

5. As an ornithologist, you wish to determine the average body mass of *Catharus fuscescens*. You randomly sample 26 adults of *Catharus fuscescens*, resulting in a sample mean of 42.41 grams and a sample standard deviation of 4.23 grams. Determine a 80% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 26 \\ \bar{x} &= 42.41 \\ s &= 4.23 \\ CL &= 0.8 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 25$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.8$.

$$t^\star = 1.32$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{4.23}{\sqrt{26}} = 0.83$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (42.41 - 1.32 \times 0.83,\ 42.41 + 1.32 \times 0.83) \\ &= (41.3,\ 43.5) \end{aligned}$$

We are 80% confident that the population mean is between 41.3 and 43.5.

6. A treatment group of size 18 has a mean of 110 and standard deviation of 14.7. A control group of size 30 has a mean of 100 and standard deviation of 12.8. If you decided to use a signficance level of 0.02, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 32.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 18 \\ \bar{x}_1 &= 110 \\ s_1 &= 14.7 \\ n_2 &= 30 \\ \bar{x}_2 &= 100 \\ s_2 &= 12.8 \\ \alpha &= 0.02 \\ df &= 32 \end{aligned}$$

State the hypotheses.

$$H_0 : \quad \mu_2 - \mu_1 = 0$$
$$H_A : \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^{\star}$ such that $P(|T| > t^{\star}) = 0.02$ by using a t table.

$$t^{\star} = 2.45$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(14.7)^2}{18} + \frac{(12.8)^2}{30}} \\ &= 4.179 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(100 - 110) - (0)}{4.179} \\ &= -2.39 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^{\star}$.

$$|t_{\text{obs}}| < t^{\star}$$

We can determine an interval for the p-value using the t table.

$$0.02 < p\text{-value} < 0.04$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.45$

(d) $SE = 4.179$

(e) $|t_{\text{obs}}| = 2.39$

(f) $0.02 < p\text{-value} < 0.04$

(g) retain the null

7. From a very large population, a random sample of 6200 individuals was taken. In that sample, 6.5% were purple. Determine a 99% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.99$.

$$z^\star = 2.58$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.065)(1-0.065)}{6200}} = 0.00313$$

Calculate the margin of error.

$$ME = z^\star SE = (2.58)(0.00313) = 0.00808$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.0569\,,\ 0.0731)$$

We are 99% confident that the true population proportion is between 5.69% and 7.31%.

(a) The lower bound = 0.0569, which can also be expressed as 5.69%.

(b) The upper bound = 0.0731, which can also be expressed as 7.31%.

8. An experiment is run with a treatment group of size 144 and a control group of size 186. The results are summarized in the table below.

	treatment	control
abysmal	59	100
not abysmal	85	86

Using a significance level of 0.04, determine whether the treatment causes an effect on the proportion of cases that are abysmal.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.04$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.05$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{59}{144} = 0.41$$
$$\hat{p}_2 = \frac{100}{186} = 0.538$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.538 - 0.41 = 0.128$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{59 + 100}{144 + 186} = 0.482$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.482)(0.518)}{144} + \frac{(0.482)(0.518)}{186}} \\ &= 0.0555 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0555)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.538 - 0.41) - 0}{0.0555} \\ &= 2.31 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.31) \\ &= 0.0208 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.05$

(d) $SE = 0.0555$

(e) $|z_{\text{obs}}| = 2.31$

(f) p-value = 0.0208

(g) reject the null

1. (a) P(**flower**) = 0.434

   (b) P(**wheel or yellow**) = 0.596

   (c) P(**red**) = 0.281

   (d) P(**pig given white**) = 0.0855

   (e) P(**white given flower**) = 0.0901

   (f) P(**flower and white**) = 0.0391

2. P(**"not shovel" given "not white"**) = 0.533

3. $P(65.28 < X < 65.85) = 0.839$

4. (a) $P(X = 14) = 0.1225$

   (b) $P(8 \leq X \leq 21) = 0.9508$

5. $\mathbf{(24.5, 26.5)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.02$

   (d) $SE = 0.01$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.3$

   (f) $0.02 < p$**-value** < 0.04

   (g) **reject**

7. (a) **LB of p CI = 0.852 or** 85.2%

   (b) **UB of p CI = 0.874 or** 87.4%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.33$

(d) $SE = 0.043$

(e) $\mid z_{\mathbf{obs}} \mid = 2.5$

(f) p-**value** = 0.0124

(g) **reject**

1. In a deck of strange cards, there are 512 cards. Each card has an image and a color. The amounts are shown in the table below.

	indigo	red	white	yellow
flower	30	91	20	81
pig	27	29	10	48
wheel	44	24	87	21

(a) What is the probability a random card is a flower?

(b) What is the probability a random card is either a wheel or yellow (or both)?

(c) What is the probability a random card is red?

(d) What is the probability a random card is a pig given it is white?

(e) What is the probability a random card is white given it is a flower?

(f) What is the probability a random card is both a flower and white?

Solution

(a) $P(\text{flower}) = \frac{30+91+20+81}{512} = 0.434$

(b) $P(\text{wheel or yellow}) = \frac{44+24+87+21+81+48+21-21}{512} = 0.596$

(c) $P(\text{red}) = \frac{91+29+24}{512} = 0.281$

(d) $P(\text{pig given white}) = \frac{10}{20+10+87} = 0.0855$

(e) $P(\text{white given flower}) = \frac{20}{30+91+20+81} = 0.0901$

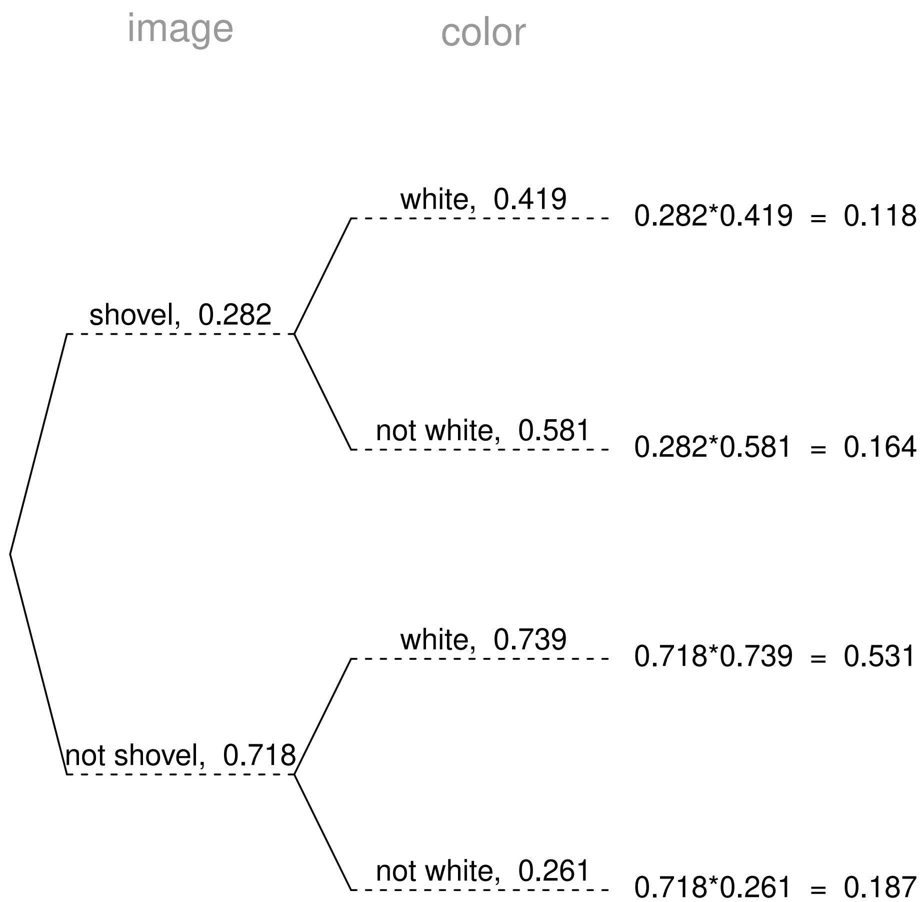
(f) $P(\text{flower and white}) = \frac{20}{512} = 0.0391$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a shovel is 28.2%. If a shovel is drawn, there is a 41.9% chance that it is white. If a card that is not a shovel is drawn, there is a 73.9% chance that it is white.

   Now, someone draws a random card and reveals it is not white. What is the chance the card is not a shovel?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not shovel" given "not white"}) = \frac{0.187}{0.187 + 0.164} = 0.533$$

3. In a very large pile of toothpicks, the mean length is 65.69 millimeters and the standard deviation is 1.94 millimeters. If you randomly sample 150 toothpicks, what is the chance the sample mean is between 65.28 and 65.85 millimeters?

Solution

Label the given information.

$$\mu = 65.69$$
$$\sigma = 1.94$$
$$n = 150$$
$$\bar{x}_{\text{lower}} = 65.28$$
$$\bar{x}_{\text{upper}} = 65.85$$

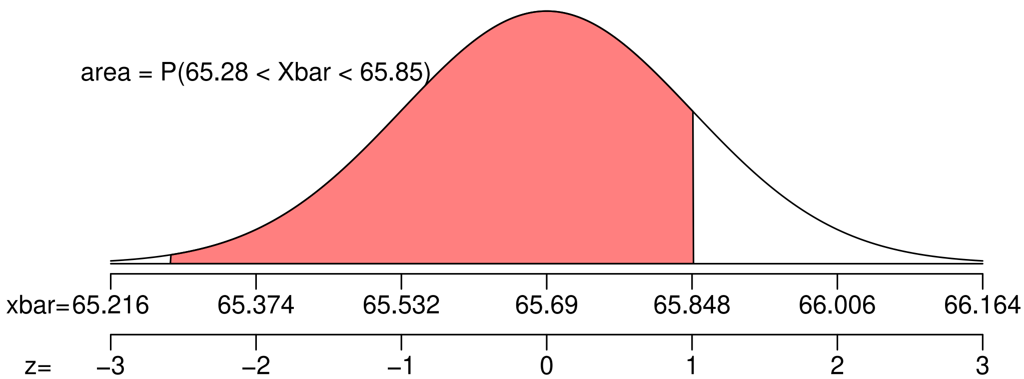
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.94}{\sqrt{150}} = 0.158$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(65.69,\ 0.158)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{65.28 - 65.69}{0.158} = -2.59$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{65.85 - 65.69}{0.158} = 1.01$$

Determine the probability.

$$\begin{aligned} P(65.28 < X < 65.85) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.01) - \Phi(-2.59) \\ &= 0.839 \end{aligned}$$

4. In a game, there is a 26% chance to win a round. You will play 52 rounds.
   (a) What is the probability of winning exactly 14 rounds?
   (b) What is the probability of winning at least 8 but at most 21 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 14) = \binom{52}{14}(0.26)^{14}(1-0.26)^{52-14}$$

$$P(X = 14) = \binom{52}{14}(0.26)^{14}(0.74)^{38}$$

$$P(X = 14) = 0.1225$$

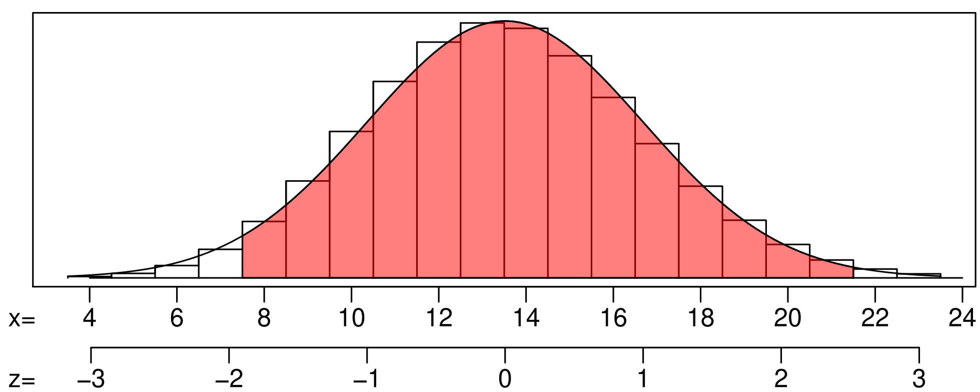
Find the mean.

$$\mu = np = (52)(0.26) = 13.52$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(52)(0.26)(1-0.26)} = 3.163$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{7.5 - 13.52}{3.163} = -1.75$$

$$z_2 = \frac{21.5 - 13.52}{3.163} = 2.36$$

Calculate the probability.

$$P(8 \leq X \leq 21) = \Phi(2.36) - \Phi(-1.75) = 0.9508$$

(a) $P(X = 14) = 0.1225$

(b) $P(8 \leq X \leq 21) = 0.9508$

5. As an ornithologist, you wish to determine the average body mass of *Passer domesticus*. You randomly sample 25 adults of *Passer domesticus*, resulting in a sample mean of 25.5 grams and a sample standard deviation of 2.47 grams. Determine a 95% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 25 \\ \bar{x} &= 25.5 \\ s &= 2.47 \\ CL &= 0.95 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 24$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.95$.

$$t^{\star} = 2.06$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.47}{\sqrt{25}} = 0.494$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^{\star} SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^{\star} SE,\ \bar{x} + t^{\star} SE) \\ &= (25.5 - 2.06 \times 0.494,\ 25.5 + 2.06 \times 0.494) \\ &= (24.5,\ 26.5) \end{aligned}$$

We are 95% confident that the population mean is between 24.5 and 26.5.

6. A treatment group of size 20 has a mean of 1.02 and standard deviation of 0.0297. A control group of size 25 has a mean of 0.998 and standard deviation of 0.0345. If you decided to use a signficance level of 0.05, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 42.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 20 \\ \bar{x}_1 &= 1.02 \\ s_1 &= 0.0297 \\ n_2 &= 25 \\ \bar{x}_2 &= 0.998 \\ s_2 &= 0.0345 \\ \alpha &= 0.05 \\ df &= 42 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.05$ by using a t table.

$$t^\star = 2.02$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.0297)^2}{20} + \frac{(0.0345)^2}{25}} \\ &= 0.01 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(0.998 - 1.02) - (0)}{0.01} \\ &= -2.3 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.02 < p\text{-value} < 0.04$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.02$

(d) $SE = 0.01$

(e) $|t_{obs}| = 2.3$

(f) $0.02 < p\text{-value} < 0.04$

(g) reject the null

7. From a very large population, a random sample of 5200 individuals was taken. In that sample, 86.3% were glowing. Determine a 98% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.863)(1-0.863)}{5200}} = 0.00477$$

Calculate the margin of error.

$$ME = z^\star SE = (2.33)(0.00477) = 0.0111$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.852\,,\ 0.874)$$

We are 98% confident that the true population proportion is between 85.2% and 87.4%.

(a) The lower bound = 0.852, which can also be expressed as 85.2%.

(b) The upper bound = 0.874, which can also be expressed as 87.4%.

8. An experiment is run with a treatment group of size 259 and a control group of size 227. The results are summarized in the table below.

	treatment	control
special	183	136
not special	76	91

Using a significance level of 0.02, determine whether the treatment causes an effect on the proportion of cases that are special.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.02$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.33$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{183}{259} = 0.707$$
$$\hat{p}_2 = \frac{136}{227} = 0.599$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.599 - 0.707 = -0.108$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{183 + 136}{259 + 227} = 0.656$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.656)(0.344)}{259} + \frac{(0.656)(0.344)}{227}} \\ &= 0.0432 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0432)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.599 - 0.707) - 0}{0.0432} \\ &= -2.5 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.5) \\ &= 0.0124 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \ p_2 - p_1 = 0$

(b) $H_A: \ p_2 - p_1 \neq 0$

(c) $z^\star = 2.33$

(d) $SE = 0.0432$

(e) $|z_{\text{obs}}| = 2.5$

(f) p-value = 0.0124

(g) reject the null

1. (a) P(**shovel**) = 0.247

   (b) P(**gray given bike**) = 0.311

   (c) P(**gem and orange**) = 0.0291

   (d) P(**bike given yellow**) = 0.239

   (e) P(**gem or violet**) = 0.444

   (f) P(**violet**) = 0.258

2. P(**"not shovel" given "not blue"**) = 0.777

3. $P(69.04 < X < 69.24) = 0.6538$

4. (a) $P(X = 59) = 0.0411$

   (b) $P(70 \leq X \leq 76) = 0.1751$

5. **(12.5, 13.2)**

6. (a) $H_0 : \quad \mu_2 - \mu_1 = 0$

   (b) $H_0 : \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 1.72$

   (d) $SE = 0.489$

   (e) $\mid t_{\textbf{obs}} \mid = 1.43$

   (f) $0.1 <$ p**-value** < 0.2

   (g) **retain**

7. (a) **LB of p CI = 0.715 or** 71.5%

   (b) **UB of p CI = 0.743 or** 74.3%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.28$

(d) $SE = 0.038$

(e) $\mid z_{\mathbf{obs}} \mid = 1.39$

(f) p-**value** = 0.1646

(g) **reject**

1. In a deck of strange cards, there are 928 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	orange	violet	yellow
bike	79	86	37	52
cat	50	53	69	54
gem	56	27	46	90
shovel	61	59	87	22

(a) What is the probability a random card is a shovel?

(b) What is the probability a random card is gray given it is a bike?

(c) What is the probability a random card is both a gem and orange?

(d) What is the probability a random card is a bike given it is yellow?

(e) What is the probability a random card is either a gem or violet (or both)?

(f) What is the probability a random card is violet?

Solution

(a) $P(\text{shovel}) = \frac{61+59+87+22}{928} = 0.247$

(b) $P(\text{gray given bike}) = \frac{79}{79+86+37+52} = 0.311$

(c) $P(\text{gem and orange}) = \frac{27}{928} = 0.0291$

(d) $P(\text{bike given yellow}) = \frac{52}{52+54+90+22} = 0.239$

(e) $P(\text{gem or violet}) = \frac{56+27+46+90+37+69+46+87-46}{928} = 0.444$

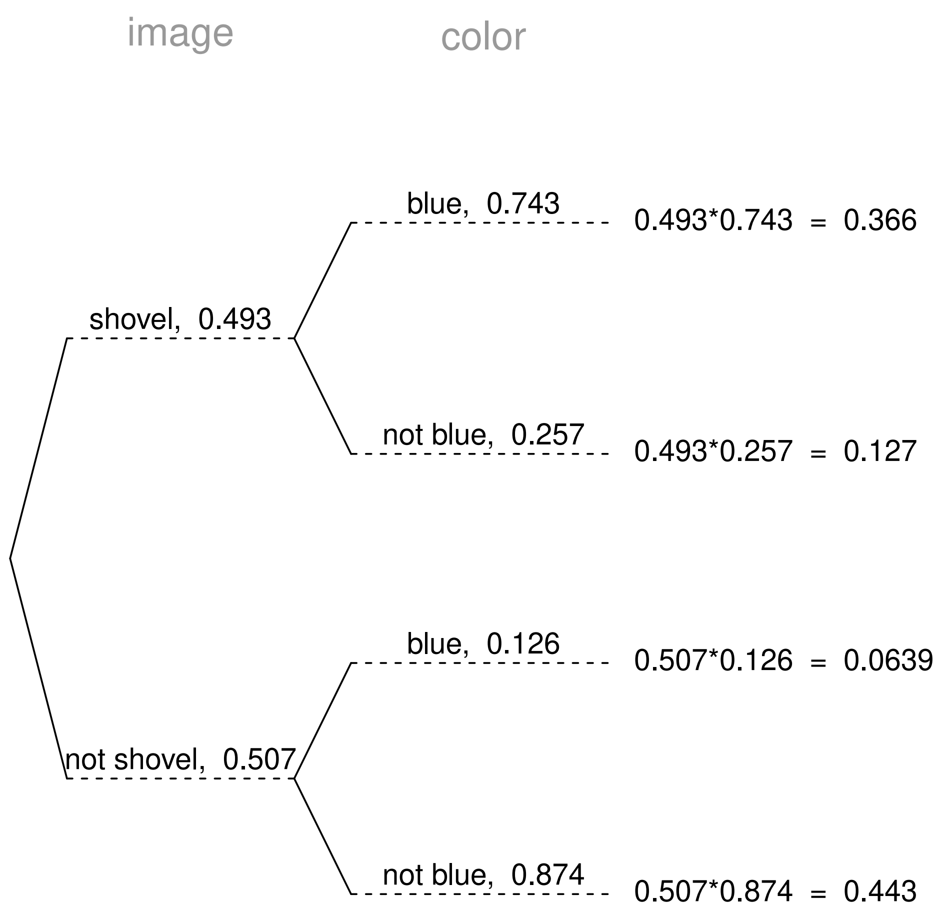
(f) $P(\text{violet}) = \frac{37+69+46+87}{928} = 0.258$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a shovel is 49.3%. If a shovel is drawn, there is a 74.3% chance that it is blue. If a card that is not a shovel is drawn, there is a 12.6% chance that it is blue.

   Now, someone draws a random card and reveals it is not blue. What is the chance the card is not a shovel?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not shovel" given "not blue"}) = \frac{0.443}{0.443 + 0.127} = 0.777$$

3. In a very large pile of toothpicks, the mean length is 69.19 millimeters and the standard deviation is 1.01 millimeters. If you randomly sample 120 toothpicks, what is the chance the sample mean is between 69.04 and 69.24 millimeters?

Solution

Label the given information.

$$\mu = 69.19$$
$$\sigma = 1.01$$
$$n = 120$$
$$\bar{x}_{\text{lower}} = 69.04$$
$$\bar{x}_{\text{upper}} = 69.24$$

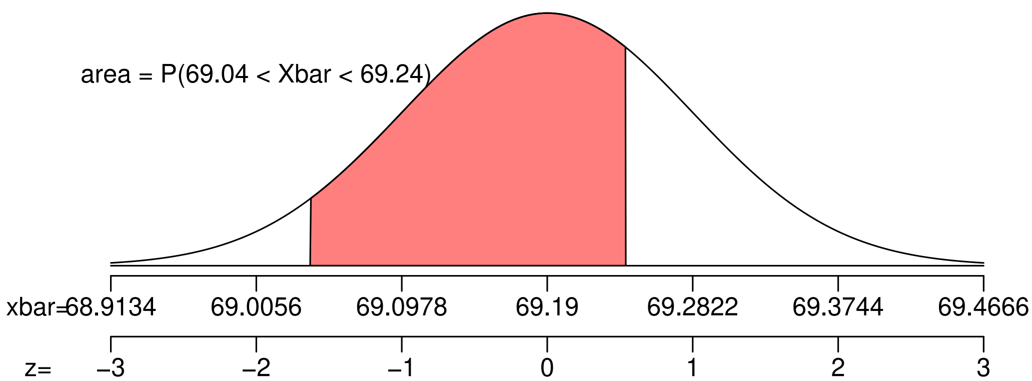
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.01}{\sqrt{120}} = 0.0922$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(69.19,\ 0.0922)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{69.04 - 69.19}{0.0922} = -1.63$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{69.24 - 69.19}{0.0922} = 0.54$$

Determine the probability.

$$\begin{aligned} P(69.04 < X < 69.24) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.54) - \Phi(-1.63) \\ &= 0.6538 \end{aligned}$$

4. In a game, there is a 34% chance to win a round. You will play 191 rounds.

   (a) What is the probability of winning exactly 59 rounds?

   (b) What is the probability of winning at least 70 but at most 76 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 59) = \binom{191}{59}(0.34)^{59}(1-0.34)^{191-59}$$

$$P(X = 59) = \binom{191}{59}(0.34)^{59}(0.66)^{132}$$

$$P(X = 59) = 0.0411$$

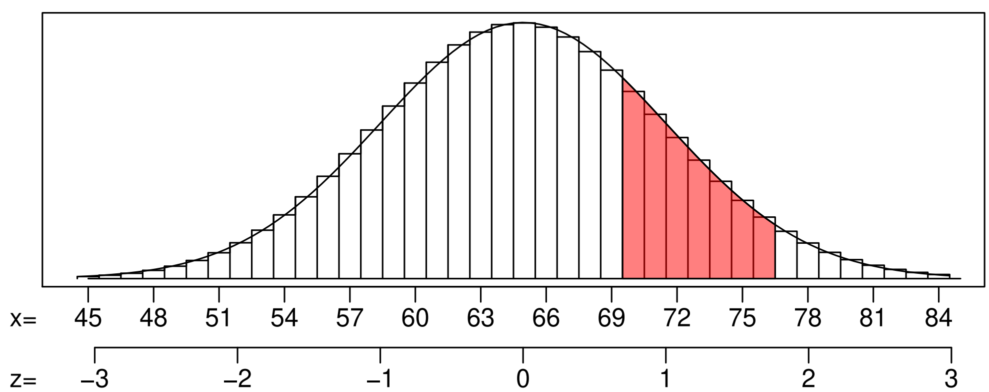
Find the mean.

$$\mu = np = (191)(0.34) = 64.94$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(191)(0.34)(1-0.34)} = 6.5468$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{69.5 - 64.94}{6.5468} = 0.77$$

$$z_2 = \frac{76.5 - 64.94}{6.5468} = 1.69$$

Calculate the probability.

$$P(70 \leq X \leq 76) = \Phi(1.69) - \Phi(0.77) = 0.1751$$

(a) $P(X = 59) = 0.0411$

(b) $P(70 \leq X \leq 76) = 0.1751$

5. As an ornithologist, you wish to determine the average body mass of *Vermivora peregrina*. You randomly sample 35 adults of *Vermivora peregrina*, resulting in a sample mean of 12.82 grams and a sample standard deviation of 1.26 grams. Determine a 90% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 35$$
$$\bar{x} = 12.82$$
$$s = 1.26$$
$$CL = 0.9$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 34$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.9$.

$$t^\star = 1.69$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.26}{\sqrt{35}} = 0.213$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (12.82 - 1.69 \times 0.213,\ 12.82 + 1.69 \times 0.213) \\ &= (12.5,\ 13.2) \end{aligned}$$

We are 90% confident that the population mean is between 12.5 and 13.2.

6. A treatment group of size 17 has a mean of 10.5 and standard deviation of 1.04. A control group of size 13 has a mean of 11.2 and standard deviation of 1.51. If you decided to use a signficance level of 0.1, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 20.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 17$$
$$\bar{x}_1 = 10.5$$
$$s_1 = 1.04$$
$$n_2 = 13$$
$$\bar{x}_2 = 11.2$$
$$s_2 = 1.51$$
$$\alpha = 0.1$$
$$df = 20$$

State the hypotheses.

$$H_0 : \quad \mu_2 - \mu_1 = 0$$
$$H_A : \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.1$ by using a t table.

$$t^\star = 1.72$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(1.04)^2}{17} + \frac{(1.51)^2}{13}} \\ &= 0.489 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(11.2 - 10.5) - (0)}{0.489} \\ &= 1.43 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.1 < p\text{-value} < 0.2$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^{\star} = 1.72$

(d) $SE = 0.489$

(e) $|t_{\text{obs}}| = 1.43$

(f) $0.1 < p\text{-value} < 0.2$

(g) retain the null

7. From a very large population, a random sample of 3800 individuals was taken. In that sample, 72.9% were super. Determine a 95% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.95$.

$$z^\star = 1.96$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.729)(1-0.729)}{3800}} = 0.00721$$

Calculate the margin of error.

$$ME = z^\star SE = (1.96)(0.00721) = 0.0141$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.715\,,\ 0.743)$$

We are 95% confident that the true population proportion is between 71.5% and 74.3%.

(a) The lower bound = 0.715, which can also be expressed as 71.5%.

(b) The upper bound = 0.743, which can also be expressed as 74.3%.

8. An experiment is run with a treatment group of size 167 and a control group of size 135. The results are summarized in the table below.

	treatment	control
omnivorous	16	20
not omnivorous	151	115

Using a significance level of 0.2, determine whether the treatment causes an effect on the proportion of cases that are omnivorous.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.2$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.28$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{16}{167} = 0.0958$$
$$\hat{p}_2 = \frac{20}{135} = 0.148$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.148 - 0.0958 = 0.0522$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{16 + 20}{167 + 135} = 0.119$$

Determine the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}$$
$$= \sqrt{\frac{(0.119)(0.881)}{167} + \frac{(0.119)(0.881)}{135}}$$
$$= 0.0375$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0375)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.148 - 0.0958) - 0}{0.0375} \\ &= 1.39 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-1.39) \\ &= 0.1646 \end{aligned}$$

Compare the *p*-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \ p_2 - p_1 = 0$

(b) $H_A: \ p_2 - p_1 \neq 0$

(c) $z^{\star} = 1.28$

(d) $SE = 0.0375$

(e) $|z_{\text{obs}}| = 1.39$

(f) p-value = 0.1646

(g) reject the null

1. (a) P(**white**) = 0.0828

   (b) P(**gem given blue**) = 0.18

   (c) P(**yellow given horn**) = 0.269

   (d) P(**gem or yellow**) = 0.505

   (e) P(**gem and blue**) = 0.0366

   (f) P(**flower**) = 0.389

2. P(**"not horn" given "blue"**) = 0.691

3. $P(65.96 < X < 66.74) = 0.7792$

4. (a) $P(X = 51) = 0.1022$

   (b) $P(49 \leq X \leq 59) = 0.749$

5. $\mathbf{(42.1, 49.1)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 3.05$

   (d) $SE = 0.389$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.88$

   (f) $0.01 <$ p-**value** < 0.02

   (g) **retain**

7. (a) **LB of p CI = 0.0893 or** 8.93%

   (b) **UB of p CI = 0.101 or** 10.1%

8. (a) $H_0 : \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}} : \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.81$

(d) $SE = 0$

(e) $\mid z_{\mathbf{obs}} \mid = NaN$

(f) p**-value** $= NaN$

(g) **reject**

1. In a deck of strange cards, there are 628 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	indigo	white	yellow
flower	61	85	10	88
gem	23	40	28	81
horn	44	97	14	57

(a) What is the probability a random card is white?

(b) What is the probability a random card is a gem given it is blue?

(c) What is the probability a random card is yellow given it is a horn?

(d) What is the probability a random card is either a gem or yellow (or both)?

(e) What is the probability a random card is both a gem and blue?

(f) What is the probability a random card is a flower?

Solution

(a) $P(\text{white}) = \frac{10+28+14}{628} = 0.0828$

(b) $P(\text{gem given blue}) = \frac{23}{61+23+44} = 0.18$

(c) $P(\text{yellow given horn}) = \frac{57}{44+97+14+57} = 0.269$

(d) $P(\text{gem or yellow}) = \frac{23+40+28+81+88+81+57-81}{628} = 0.505$

(e) $P(\text{gem and blue}) = \frac{23}{628} = 0.0366$

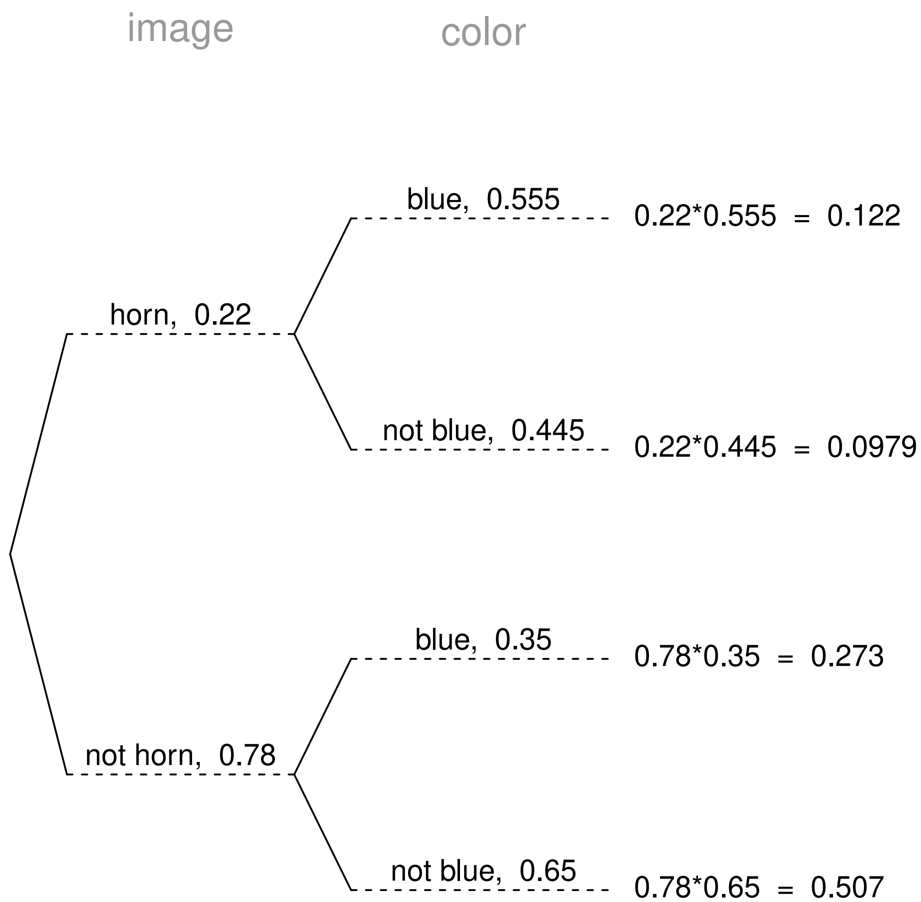
(f) $P(\text{flower}) = \frac{61+85+10+88}{628} = 0.389$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a horn is 22%. If a horn is drawn, there is a 55.5% chance that it is blue. If a card that is not a horn is drawn, there is a 35% chance that it is blue.

   Now, someone draws a random card and reveals it is blue. What is the chance the card is not a horn?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not horn" given "blue"}) = \frac{0.273}{0.273 + 0.122} = 0.691$$

3. In a very large pile of toothpicks, the mean length is 66.22 millimeters and the standard deviation is 3.19 millimeters. If you randomly sample 121 toothpicks, what is the chance the sample mean is between 65.96 and 66.74 millimeters?

Solution

Label the given information.

$$\mu = 66.22$$
$$\sigma = 3.19$$
$$n = 121$$
$$\bar{x}_{\text{lower}} = 65.96$$
$$\bar{x}_{\text{upper}} = 66.74$$

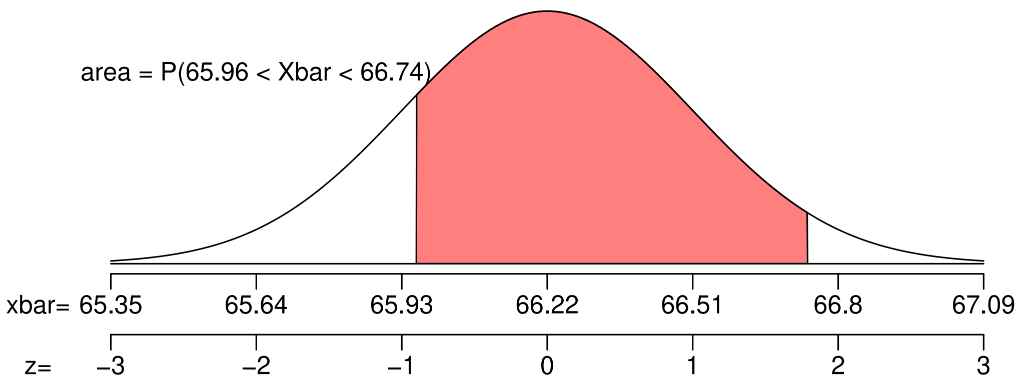
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.19}{\sqrt{121}} = 0.29$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(66.22,\ 0.29)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{65.96 - 66.22}{0.29} = -0.9$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{66.74 - 66.22}{0.29} = 1.79$$

Determine the probability.

$$\begin{aligned} P(65.96 < X < 66.74) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.79) - \Phi(-0.9) \\ &= 0.7792 \end{aligned}$$

4. In a game, there is a 73% chance to win a round. You will play 71 rounds.

   (a) What is the probability of winning exactly 51 rounds?

   (b) What is the probability of winning at least 49 but at most 59 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 51) = \binom{71}{51}(0.73)^{51}(1-0.73)^{71-51}$$

$$P(X = 51) = \binom{71}{51}(0.73)^{51}(0.27)^{20}$$

$$P(X = 51) = 0.1022$$

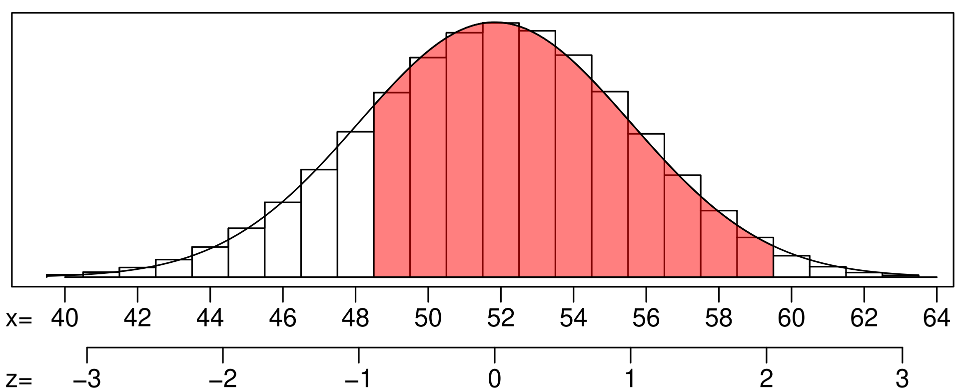
Find the mean.

$$\mu = np = (71)(0.73) = 51.83$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(71)(0.73)(1-0.73)} = 3.7409$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{48.5 - 51.83}{3.7409} = -0.76$$

$$z_2 = \frac{59.5 - 51.83}{3.7409} = 1.92$$

Calculate the probability.

$$P(49 \le X \le 59) = \Phi(1.92) - \Phi(-0.76) = 0.749$$

(a) $P(X = 51) = 0.1022$

(b) $P(49 \le X \le 59) = 0.749$

5. As an ornithologist, you wish to determine the average body mass of *Agelaius Phoeniceus*. You randomly sample 25 adults of *Agelaius Phoeniceus*, resulting in a sample mean of 45.6 grams and a sample standard deviation of 5.6 grams. Determine a 99.5% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 25 \\ \bar{x} &= 45.6 \\ s &= 5.6 \\ CL &= 0.995 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 24$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.995$.

$$t^\star = 3.09$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{5.6}{\sqrt{25}} = 1.12$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (45.6 - 3.09 \times 1.12,\ 45.6 + 3.09 \times 1.12) \\ &= (42.1,\ 49.1) \end{aligned}$$

We are 99.5% confident that the population mean is between 42.1 and 49.1.

6. A treatment group of size 31 has a mean of 11 and standard deviation of 0.98. A control group of size 9 has a mean of 9.88 and standard deviation of 1.04. If you decided to use a signficance level of 0.01, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 12.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 31$$
$$\bar{x}_1 = 11$$
$$s_1 = 0.98$$
$$n_2 = 9$$
$$\bar{x}_2 = 9.88$$
$$s_2 = 1.04$$
$$\alpha = 0.01$$
$$df = 12$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.01$ by using a t table.

$$t^\star = 3.05$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.98)^2}{31} + \frac{(1.04)^2}{9}} \\ &= 0.389 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(9.88 - 11) - (0)}{0.389} \\ &= -2.88 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 3.05$

(d) $SE = 0.389$

(e) $|t_{obs}| = 2.88$

(f) $0.01 < p\text{-value} < 0.02$

(g) retain the null

7. From a very large population, a random sample of 4400 individuals was taken. In that sample, 9.5% were salty. Determine a 80% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.8$.

$$z^\star = 1.28$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.095)(1-0.095)}{4400}} = 0.00442$$

Calculate the margin of error.

$$ME = z^\star SE = (1.28)(0.00442) = 0.00566$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.0893\,,\ 0.101)$$

We are 80% confident that the true population proportion is between 8.93% and 10.1%.

(a) The lower bound = 0.0893, which can also be expressed as 8.93%.

(b) The upper bound = 0.101, which can also be expressed as 10.1%.

8. An experiment is run with a treatment group of size 16 and a control group of size 27. The results are summarized in the table below.

	treatment	control
sick	16	27
not sick	0	0

Using a significance level of 0.005, determine whether the treatment causes an effect on the proportion of cases that are sick.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.005$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.81$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{16}{16} = 1$$
$$\hat{p}_2 = \frac{27}{27} = 1$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 1 - 1 = 0$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{16 + 27}{16 + 27} = 1$$

Determine the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}$$
$$= \sqrt{\frac{(1)(0)}{16} + \frac{(1)(0)}{27}}$$
$$= 0$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\, 0)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(1-1) - 0}{0} \\ &= NaN \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-NaN) \\ &= NaN \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \ p_2 - p_1 = 0$
(b) $H_A: \ p_2 - p_1 \neq 0$
(c) $z^{\star} = 2.81$
(d) $SE = 0$
(e) $|z_{\text{obs}}| = NaN$
(f) p-value = NaN
(g) reject the null

1. (a) P(**pig given blue**) = 0.228

   (b) P(**teal**) = 0.269

   (c) P(**pig or gray**) = 0.407

   (d) P(**horn**) = 0.185

   (e) P(**red given wheel**) = 0.212

   (f) P(**flower and red**) = 0.0663

2. P(**"not pig" given "yellow"**) = 0.316

3. $P(69.67 < X < 70.23) = 0.785$

4. (a) $P(X = 37) = 0.0742$

   (b) $P(35 \leq X \leq 48) = 0.5848$

5. $\mathbf{(19, 22.1)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 1.99$

   (d) $SE = 3.921$

   (e) $\mid t_{\textbf{obs}} \mid = 1.79$

   (f) $0.05 <$ p-**value** < 0.1

   (g) **retain**

7. (a) **LB of p CI = 0.934 or** 93.4%

   (b) **UB of p CI = 0.938 or** 93.8%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.96$

(d) $SE = 0.024$

(e) $|\ z_{\mathbf{obs}}\ | = 2.03$

(f) p-**value** = 0.0424

(g) **reject**

1. In a deck of strange cards, there are 966 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	gray	red	teal
flower	98	87	64	89
horn	27	48	16	88
pig	55	44	71	10
wheel	61	78	57	73

(a) What is the probability a random card is a pig given it is blue?

(b) What is the probability a random card is teal?

(c) What is the probability a random card is either a pig or gray (or both)?

(d) What is the probability a random card is a horn?

(e) What is the probability a random card is red given it is a wheel?

(f) What is the probability a random card is both a flower and red?

Solution

(a) $P(\text{pig given blue}) = \frac{55}{98+27+55+61} = 0.228$

(b) $P(\text{teal}) = \frac{89+88+10+73}{966} = 0.269$

(c) $P(\text{pig or gray}) = \frac{55+44+71+10+87+48+44+78-44}{966} = 0.407$

(d) $P(\text{horn}) = \frac{27+48+16+88}{966} = 0.185$

(e) $P(\text{red given wheel}) = \frac{57}{61+78+57+73} = 0.212$

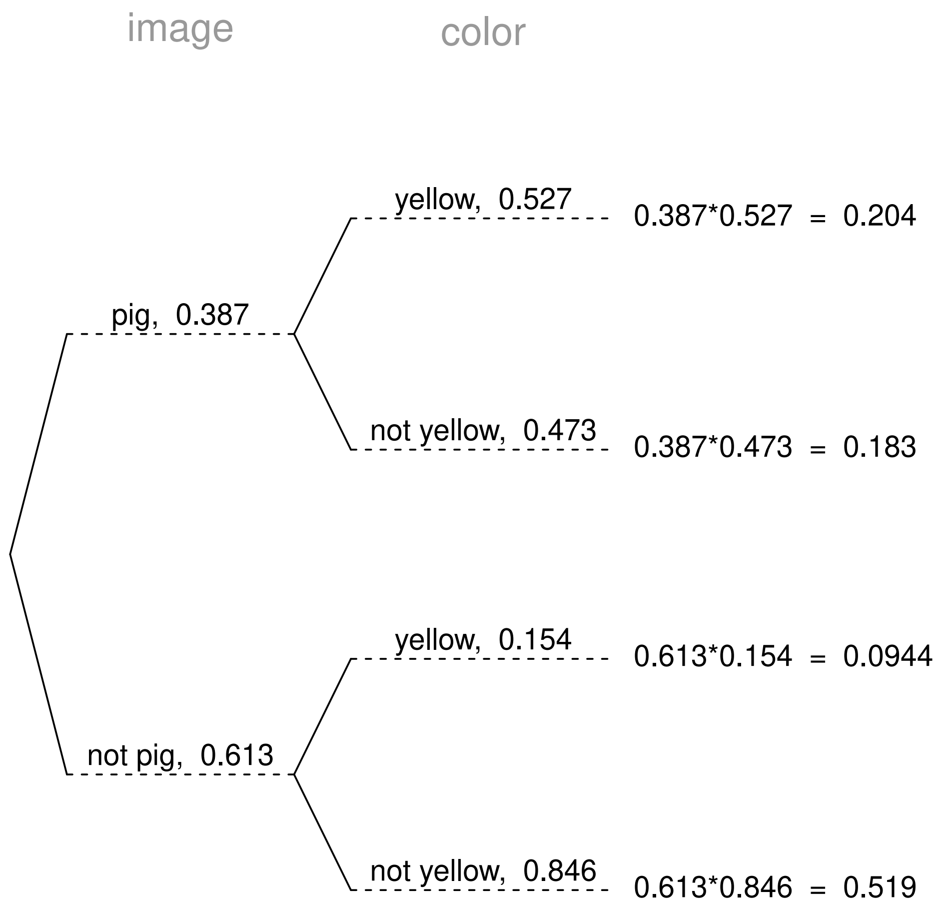
(f) $P(\text{flower and red}) = \frac{64}{966} = 0.0663$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a pig is 38.7%. If a pig is drawn, there is a 52.7% chance that it is yellow. If a card that is not a pig is drawn, there is a 15.4% chance that it is yellow.

   Now, someone draws a random card and reveals it is yellow. What is the chance the card is not a pig?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not pig" given "yellow"}) = \frac{0.0944}{0.0944 + 0.204} = 0.316$$

3. In a very large pile of toothpicks, the mean length is 69.91 millimeters and the standard deviation is 2.89 millimeters. If you randomly sample 169 toothpicks, what is the chance the sample mean is between 69.67 and 70.23 millimeters?

Solution

Label the given information.

$$\mu = 69.91$$
$$\sigma = 2.89$$
$$n = 169$$
$$\bar{x}_{\text{lower}} = 69.67$$
$$\bar{x}_{\text{upper}} = 70.23$$

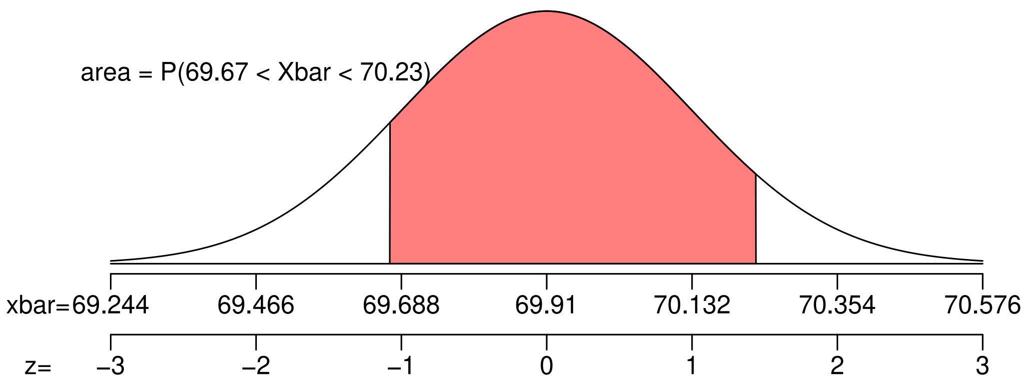
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2.89}{\sqrt{169}} = 0.222$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(69.91,\ 0.222)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{69.67 - 69.91}{0.222} = -1.08$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{70.23 - 69.91}{0.222} = 1.44$$

Determine the probability.

$$\begin{aligned} P(69.67 < X < 70.23) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.44) - \Phi(-1.08) \\ &= 0.785 \end{aligned}$$

4. In a game, there is a 23% chance to win a round. You will play 158 rounds.
   (a) What is the probability of winning exactly 37 rounds?
   (b) What is the probability of winning at least 35 but at most 48 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 37) = \binom{158}{37}(0.23)^{37}(1-0.23)^{158-37}$$

$$P(X = 37) = \binom{158}{37}(0.23)^{37}(0.77)^{121}$$

$$P(X = 37) = 0.0742$$

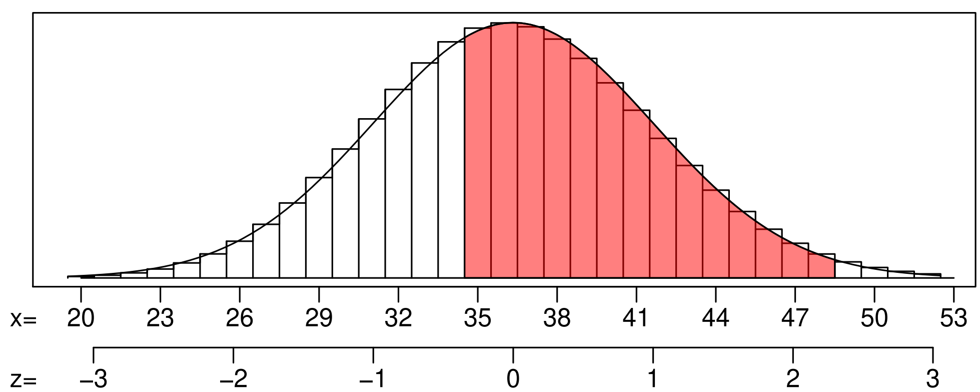
Find the mean.

$$\mu = np = (158)(0.23) = 36.34$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(158)(0.23)(1-0.23)} = 5.2898$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{34.5 - 36.34}{5.2898} = -0.25$$

$$z_2 = \frac{48.5 - 36.34}{5.2898} = 2.2$$

Calculate the probability.

$$P(35 \leq X \leq 48) = \Phi(2.2) - \Phi(-0.25) = 0.5848$$

(a) $P(X = 37) = 0.0742$

(b) $P(35 \leq X \leq 48) = 0.5848$

5. As an ornithologist, you wish to determine the average body mass of *Seiurus noveboracensis*. You randomly sample 33 adults of *Seiurus noveboracensis*, resulting in a sample mean of 20.51 grams and a sample standard deviation of 3.64 grams. Determine a 98% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 33 \\ \bar{x} &= 20.51 \\ s &= 3.64 \\ CL &= 0.98 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 32$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.98$.

$$t^\star = 2.45$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{3.64}{\sqrt{33}} = 0.634$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (20.51 - 2.45 \times 0.634,\ 20.51 + 2.45 \times 0.634) \\ &= (19,\ 22.1) \end{aligned}$$

We are 98% confident that the population mean is between 19 and 22.1.

6. A treatment group of size 33 has a mean of 102 and standard deviation of 15.4. A control group of size 40 has a mean of 109 and standard deviation of 18.1. If you decided to use a signficance level of 0.05, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 70.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$
\begin{aligned}
n_1 &= 33\\
\bar{x}_1 &= 102\\
s_1 &= 15.4\\
n_2 &= 40\\
\bar{x}_2 &= 109\\
s_2 &= 18.1\\
\alpha &= 0.05\\
df &= 70
\end{aligned}
$$

State the hypotheses.

$$
\begin{aligned}
H_0 &: \quad \mu_2 - \mu_1 = 0\\
H_A &: \quad \mu_2 - \mu_1 \neq 0
\end{aligned}
$$

We are using a two-tail test. Find $t^{\star}$ such that $P(|T| > t^{\star}) = 0.05$ by using a t table.

$$t^{\star} = 1.99$$

Calculate the standard error.

$$
\begin{aligned}
SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}\\
&= \sqrt{\frac{(15.4)^2}{33} + \frac{(18.1)^2}{40}}\\
&= 3.921
\end{aligned}
$$

Determine the test statistic.

$$
\begin{aligned}
t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE}\\
&= \frac{(109 - 102) - (0)}{3.921}\\
&= 1.79
\end{aligned}
$$

Compare $|t_{\text{obs}}|$ and $t^{\star}$.

$$|t_{\text{obs}}| < t^{\star}$$

We can determine an interval for the p-value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) H_A : $\mu_2 - \mu_1 \neq 0$

(c) $t^\star = 1.99$

(d) $SE = 3.921$

(e) $|t_{obs}| = 1.79$

(f) $0.05 < p\text{-value} < 0.1$

(g) retain the null

7. From a very large population, a random sample of 49000 individuals was taken. In that sample, 93.6% were asleep. Determine a 96% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.96$.

$$z^{\star} = 2.05$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.936)(1-0.936)}{49000}} = 0.00111$$

Calculate the margin of error.

$$ME = z^{\star}SE = (2.05)(0.00111) = 0.00228$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.934\,,\ 0.938)$$

We are 96% confident that the true population proportion is between 93.4% and 93.8%.

(a) The lower bound = 0.934, which can also be expressed as 93.4%.

(b) The upper bound = 0.938, which can also be expressed as 93.8%.

8. An experiment is run with a treatment group of size 204 and a control group of size 243. The results are summarized in the table below.

	treatment	control
glossy	185	232
not glossy	19	11

Using a significance level of 0.05, determine whether the treatment causes an effect on the proportion of cases that are glossy.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution
State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.05$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.96$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{185}{204} = 0.907$$
$$\hat{p}_2 = \frac{232}{243} = 0.955$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.955 - 0.907 = 0.048$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{185 + 232}{204 + 243} = 0.933$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.933)(0.067)}{204} + \frac{(0.933)(0.067)}{243}} \\ &= 0.0237 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0237)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.955 - 0.907) - 0}{0.0237} \\ &= 2.03 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.03) \\ &= 0.0424 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.96$

(d) $SE = 0.0237$

(e) $|z_{\text{obs}}| = 2.03$

(f) p-value = 0.0424

(g) reject the null

1. (a) P(**tree given orange**) = 0.163

   (b) P(**wheel and gray**) = 0.0172

   (c) P(**wheel or orange**) = 0.439

   (d) P(**flower**) = 0.202

   (e) P(**orange**) = 0.362

   (f) P(**gray given gem**) = 0.18

2. P(**"gem" given "pink"**) = 0.0891

3. $P(70.39 < X < 70.68) = 0.8111$

4. (a) $P(X = 97) = 0.0554$

   (b) $P(78 \leq X \leq 96) = 0.5916$

5. $\mathbf{(12, 13.1)}$

6. (a) $H_0 : \quad \mu_2 - \mu_1 = 0$

   (b) $H_0 : \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.51$

   (d) $SE = 5.215$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.21$

   (f) $0.02 <$ p**-value** < 0.04

   (g) **retain**

7. (a) **LB of p CI = 0.663 or** 66.3%

   (b) **UB of p CI = 0.701 or** 70.1%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.33$

(d) $SE = 0.025$

(e) $\mid z_{\mathbf{obs}} \mid = 2.49$

(f) p-**value** $= 0.0128$

(g) **reject**

1. In a deck of strange cards, there are 816 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	orange	red
bike	56	99	55
flower	77	64	24
gem	27	40	83
tree	62	48	74
wheel	14	44	49

(a) What is the probability a random card is a tree given it is orange?

(b) What is the probability a random card is both a wheel and gray?

(c) What is the probability a random card is either a wheel or orange (or both)?

(d) What is the probability a random card is a flower?

(e) What is the probability a random card is orange?

(f) What is the probability a random card is gray given it is a gem?

Solution

(a) $P(\text{tree given orange}) = \frac{48}{99+64+40+48+44} = 0.163$

(b) $P(\text{wheel and gray}) = \frac{14}{816} = 0.0172$

(c) $P(\text{wheel or orange}) = \frac{14+44+49+99+64+40+48+44-44}{816} = 0.439$

(d) $P(\text{flower}) = \frac{77+64+24}{816} = 0.202$

(e) $P(\text{orange}) = \frac{99+64+40+48+44}{816} = 0.362$

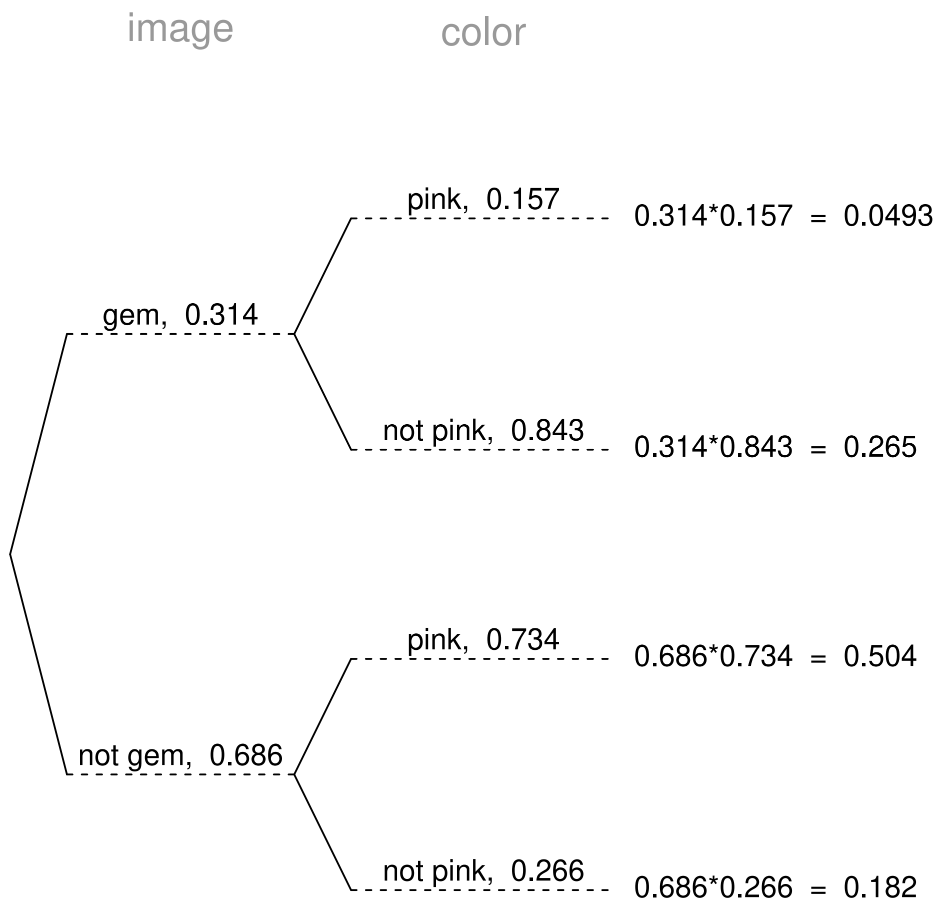
(f) $P(\text{gray given gem}) = \frac{27}{27+40+83} = 0.18$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a gem is 31.4%. If a gem is drawn, there is a 15.7% chance that it is pink. If a card that is not a gem is drawn, there is a 73.4% chance that it is pink.

   Now, someone draws a random card and reveals it is pink. What is the chance the card is a gem?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"gem" given "pink"}) = \frac{0.0493}{0.0493 + 0.504} = 0.0891$$

3. In a very large pile of toothpicks, the mean length is 70.54 millimeters and the standard deviation is 1.35 millimeters. If you randomly sample 150 toothpicks, what is the chance the sample mean is between 70.39 and 70.68 millimeters?

Solution

Label the given information.

$$\mu = 70.54$$
$$\sigma = 1.35$$
$$n = 150$$
$$\bar{x}_{\text{lower}} = 70.39$$
$$\bar{x}_{\text{upper}} = 70.68$$

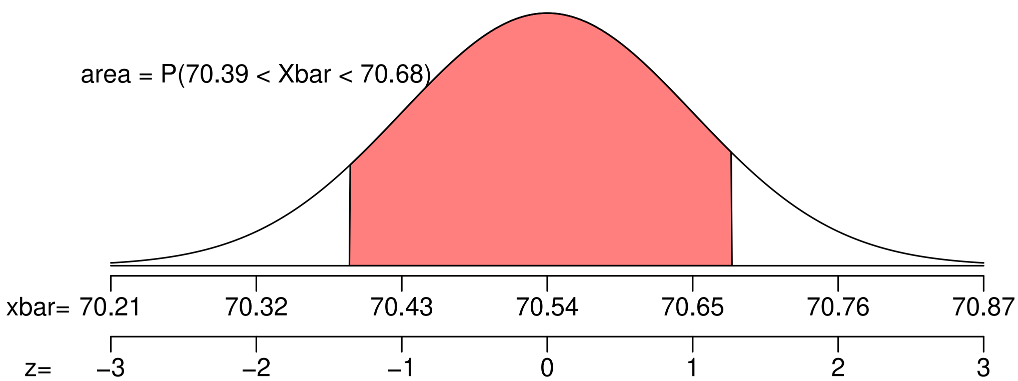
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.35}{\sqrt{150}} = 0.11$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(70.54,\ 0.11)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{70.39 - 70.54}{0.11} = -1.36$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{70.68 - 70.54}{0.11} = 1.27$$

Determine the probability.

$$\begin{aligned} P(70.39 < X < 70.68) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.27) - \Phi(-1.36) \\ &= 0.8111 \end{aligned}$$

4. In a game, there is a 53% chance to win a round. You will play 178 rounds.
   (a) What is the probability of winning exactly 97 rounds?
   (b) What is the probability of winning at least 78 but at most 96 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X=k)=\binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X=97)=\binom{178}{97}(0.53)^{97}(1-0.53)^{178-97}$$

$$P(X=97)=\binom{178}{97}(0.53)^{97}(0.47)^{81}$$

$$P(X=97)=0.0554$$

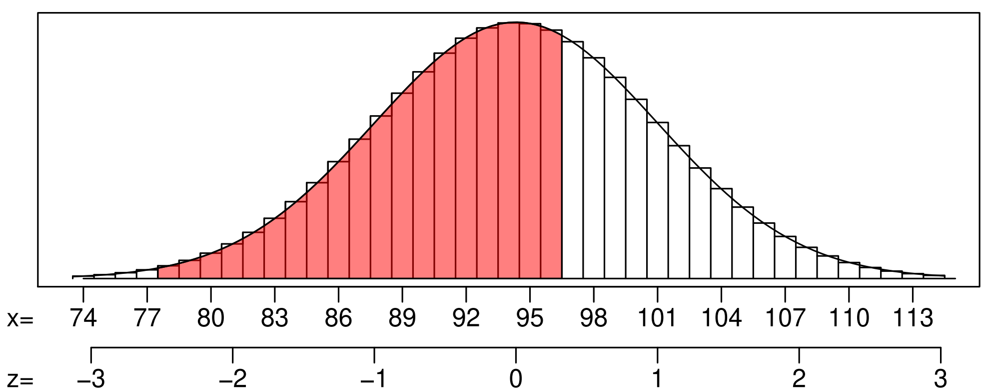
Find the mean.

$$\mu=np=(178)(0.53)=94.34$$

Find the standard deviation.

$$\sigma=\sqrt{np(1-p)}=\sqrt{(178)(0.53)(1-0.53)}=6.6588$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1=\frac{77.5-94.34}{6.6588}=-2.45$$

$$z_2=\frac{96.5-94.34}{6.6588}=0.25$$

Calculate the probability.

$$P(78\le X\le 96)=\Phi(0.25)-\Phi(-2.45)=0.5916$$

(a) $P(X=97)=0.0554$

(b) $P(78\le X\le 96)=0.5916$

5. As an ornithologist, you wish to determine the average body mass of *Vermivora peregrina*. You randomly sample 23 adults of *Vermivora peregrina*, resulting in a sample mean of 12.52 grams and a sample standard deviation of 1.24 grams. Determine a 95% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 23 \\ \bar{x} &= 12.52 \\ s &= 1.24 \\ CL &= 0.95 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 22$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.95$.

$$t^\star = 2.07$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.24}{\sqrt{23}} = 0.259$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (12.52 - 2.07 \times 0.259,\ 12.52 + 2.07 \times 0.259) \\ &= (12,\ 13.1) \end{aligned}$$

We are 95% confident that the population mean is between 12 and 13.1.

6. A treatment group of size 14 has a mean of 98.5 and standard deviation of 16.4. A control group of size 16 has a mean of 110 and standard deviation of 11.3. If you decided to use a signficance level of 0.02, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 22.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 14$$
$$\bar{x}_1 = 98.5$$
$$s_1 = 16.4$$
$$n_2 = 16$$
$$\bar{x}_2 = 110$$
$$s_2 = 11.3$$
$$\alpha = 0.02$$
$$df = 22$$

State the hypotheses.

$$H_0 : \quad \mu_2 - \mu_1 = 0$$
$$H_A : \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^{\star}$ such that $P(|T| > t^{\star}) = 0.02$ by using a t table.

$$t^{\star} = 2.51$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(16.4)^2}{14} + \frac{(11.3)^2}{16}} \\ &= 5.215 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(110 - 98.5) - (0)}{5.215} \\ &= 2.21 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^{\star}$.

$$|t_{\text{obs}}| < t^{\star}$$

We can determine an interval for the p-value using the t table.

$$0.02 < p\text{-value} < 0.04$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.51$

(d) $SE = 5.215$

(e) $|t_{\text{obs}}| = 2.21$

(f) $0.02 < p\text{-value} < 0.04$

(g) retain the null

7. From a very large population, a random sample of 5000 individuals was taken. In that sample, 68.2% were blue. Determine a 99.5% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.995$.

$$z^{\star} = 2.81$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.682)(1-0.682)}{5000}} = 0.00659$$

Calculate the margin of error.

$$ME = z^{\star}SE = (2.81)(0.00659) = 0.0185$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.663\,,\ 0.701)$$

We are 99.5% confident that the true population proportion is between 66.3% and 70.1%.

(a) The lower bound = 0.663, which can also be expressed as 66.3%.

(b) The upper bound = 0.701, which can also be expressed as 70.1%.

8. An experiment is run with a treatment group of size 273 and a control group of size 275. The results are summarized in the table below.

	treatment	control
pink	238	257
not pink	35	18

Using a significance level of 0.02, determine whether the treatment causes an effect on the proportion of cases that are pink.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.02$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.33$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{238}{273} = 0.872$$
$$\hat{p}_2 = \frac{257}{275} = 0.935$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.935 - 0.872 = 0.063$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{238 + 257}{273 + 275} = 0.903$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.903)(0.097)}{273} + \frac{(0.903)(0.097)}{275}} \\ &= 0.0253 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0253)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.935 - 0.872) - 0}{0.0253} \\ &= 2.49 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.49) \\ &= 0.0128 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.33$

(d) $SE = 0.0253$

(e) $|z_{\text{obs}}| = 2.49$

(f) p-value = 0.0128

(g) reject the null

1. (a) P(**green given wheel**) = 0.341

   (b) P(**wheel or pink**) = 0.431

   (c) P(**bike**) = 0.304

   (d) P(**wheel and gray**) = 0.0169

   (e) P(**indigo**) = 0.18

   (f) P(**shovel given pink**) = 0.27

2. P(**"kite" given "not indigo"**) = 0.45

3. $P(71.51 < X < 71.95) = 0.5183$

4. (a) $P(X = 83) = 0.0583$

   (b) $P(70 \leq X \leq 77) = 0.0986$

5. $\mathbf{(64.3, 75.1)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.2$

   (d) $SE = 28.699$

   (e) $\mid t_{\textbf{obs}} \mid = 2.09$

   (f) $0.05 <$ p**-value** < 0.1

   (g) **retain**

7. (a) **LB of p CI = 0.671 or** 67.1%

   (b) **UB of p CI = 0.705 or** 70.5%

8. (a) $H_0: \; p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \; p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.33$

(d) $SE = 0.028$

(e) $|\, z_{\mathbf{obs}} \,| = 2.61$

(f) p**-value** $= 0.009$

(g) **reject**

1. In a deck of strange cards, there are 947 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	green	indigo	pink
bike	81	87	32	88
horn	53	80	44	25
shovel	39	58	65	78
wheel	16	74	29	98

(a) What is the probability a random card is green given it is a wheel?

(b) What is the probability a random card is either a wheel or pink (or both)?

(c) What is the probability a random card is a bike?

(d) What is the probability a random card is both a wheel and gray?

(e) What is the probability a random card is indigo?

(f) What is the probability a random card is a shovel given it is pink?

Solution

(a) $P(\text{green given wheel}) = \frac{74}{16+74+29+98} = 0.341$

(b) $P(\text{wheel or pink}) = \frac{16+74+29+98+88+25+78+98-98}{947} = 0.431$

(c) $P(\text{bike}) = \frac{81+87+32+88}{947} = 0.304$

(d) $P(\text{wheel and gray}) = \frac{16}{947} = 0.0169$

(e) $P(\text{indigo}) = \frac{32+44+65+29}{947} = 0.18$

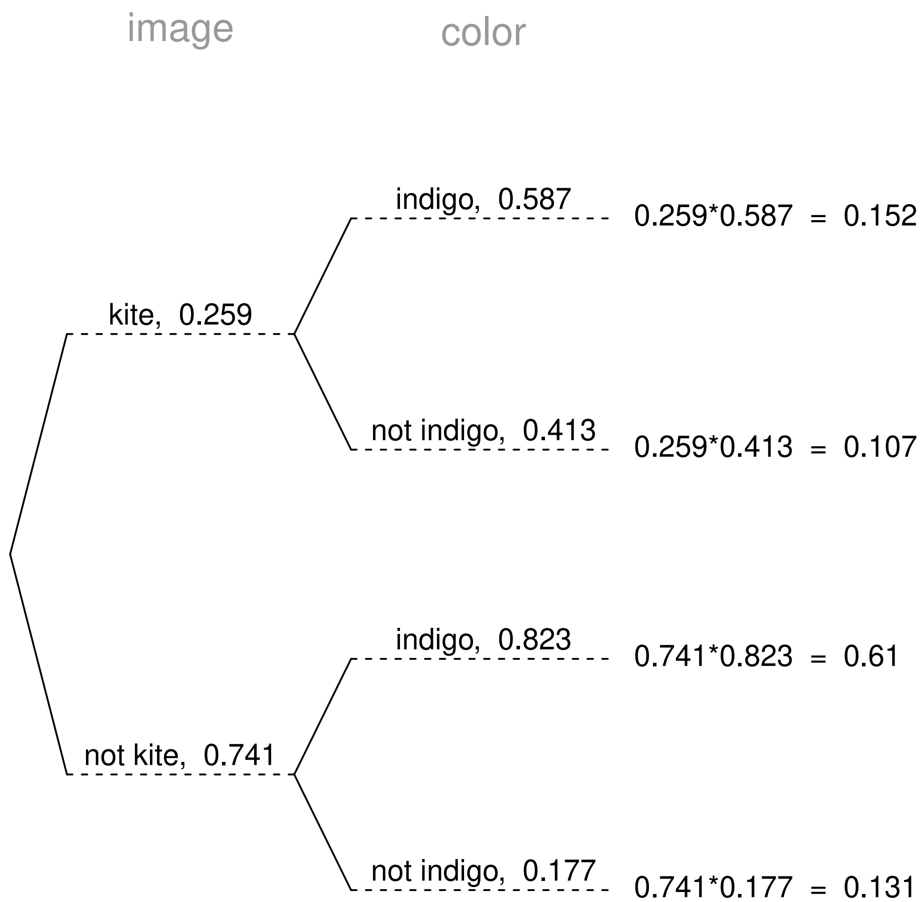
(f) $P(\text{shovel given pink}) = \frac{78}{88+25+78+98} = 0.27$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a kite is 25.9%. If a kite is drawn, there is a 58.7% chance that it is indigo. If a card that is not a kite is drawn, there is a 82.3% chance that it is indigo.

   Now, someone draws a random card and reveals it is not indigo. What is the chance the card is a kite?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"kite" given "not indigo"}) = \frac{0.107}{0.107 + 0.131} = 0.45$$

3. In a very large pile of toothpicks, the mean length is 71.75 millimeters and the standard deviation is 3.42 millimeters. If you randomly sample 120 toothpicks, what is the chance the sample mean is between 71.51 and 71.95 millimeters?

Solution

Label the given information.

$$\mu = 71.75$$
$$\sigma = 3.42$$
$$n = 120$$
$$\bar{x}_{\text{lower}} = 71.51$$
$$\bar{x}_{\text{upper}} = 71.95$$

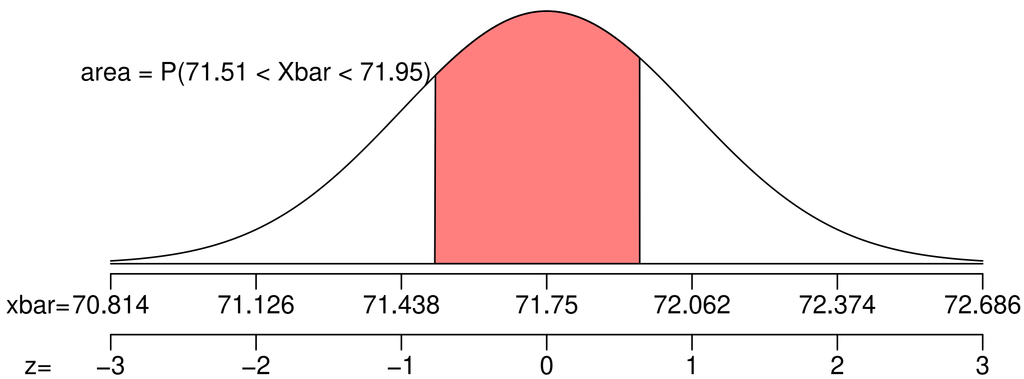
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.42}{\sqrt{120}} = 0.312$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(71.75, 0.312)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{71.51 - 71.75}{0.312} = -0.77$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{71.95 - 71.75}{0.312} = 0.64$$

Determine the probability.

$$\begin{aligned} P(71.51 < X < 71.95) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.64) - \Phi(-0.77) \\ &= 0.5183 \end{aligned}$$

4. In a game, there is a 50% chance to win a round. You will play 170 rounds.
   (a) What is the probability of winning exactly 83 rounds?
   (b) What is the probability of winning at least 70 but at most 77 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 83) = \binom{170}{83}(0.5)^{83}(1-0.5)^{170-83}$$

$$P(X = 83) = \binom{170}{83}(0.5)^{83}(0.5)^{87}$$

$$P(X = 83) = 0.0583$$

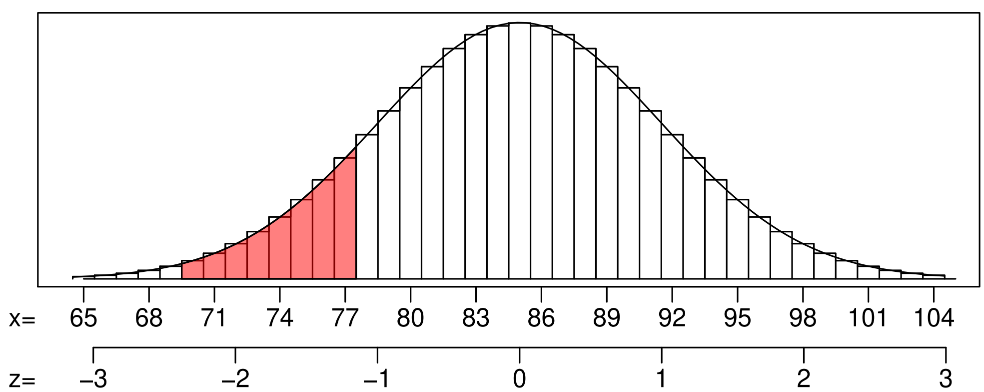
Find the mean.

$$\mu = np = (170)(0.5) = 85$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(170)(0.5)(1-0.5)} = 6.5192$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{69.5 - 85}{6.5192} = -2.3$$

$$z_2 = \frac{77.5 - 85}{6.5192} = -1.23$$

Calculate the probability.

$$P(70 \leq X \leq 77) = \Phi(-1.23) - \Phi(-2.3) = 0.0986$$

(a) $P(X = 83) = 0.0583$

(b) $P(70 \leq X \leq 77) = 0.0986$

5. As an ornithologist, you wish to determine the average body mass of *Porzana carolina*. You randomly sample 27 adults of *Porzana carolina*, resulting in a sample mean of 69.7 grams and a sample standard deviation of 11.3 grams. Determine a 98% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 27 \\ \bar{x} &= 69.7 \\ s &= 11.3 \\ CL &= 0.98 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 26$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.98$.

$$t^\star = 2.48$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{11.3}{\sqrt{27}} = 2.17$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (69.7 - 2.48 \times 2.17,\ 69.7 + 2.48 \times 2.17) \\ &= (64.3,\ 75.1) \end{aligned}$$

We are 98% confident that the population mean is between 64.3 and 75.1.

6. A treatment group of size 12 has a mean of 1090 and standard deviation of 63.8. A control group of size 11 has a mean of 1030 and standard deviation of 73. If you decided to use a signficance level of 0.04, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 19.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$
\begin{aligned}
n_1 &= 12 \\
\bar{x}_1 &= 1090 \\
s_1 &= 63.8 \\
n_2 &= 11 \\
\bar{x}_2 &= 1030 \\
s_2 &= 73 \\
\alpha &= 0.04 \\
df &= 19
\end{aligned}
$$

State the hypotheses.

$$
\begin{aligned}
H_0 &: \quad \mu_2 - \mu_1 = 0 \\
H_A &: \quad \mu_2 - \mu_1 \neq 0
\end{aligned}
$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.04$ by using a t table.

$$t^\star = 2.2$$

Calculate the standard error.

$$
\begin{aligned}
SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\
&= \sqrt{\frac{(63.8)^2}{12} + \frac{(73)^2}{11}} \\
&= 28.699
\end{aligned}
$$

Determine the test statistic.

$$
\begin{aligned}
t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\
&= \frac{(1030 - 1090) - (0)}{28.699} \\
&= -2.09
\end{aligned}
$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^{\star} = 2.2$

(d) $SE = 28.699$

(e) $|t_{\text{obs}}| = 2.09$

(f) $0.05 < p\text{-value} < 0.1$

(g) retain the null

7. From a very large population, a random sample of 1900 individuals was taken. In that sample, 68.8% were cold. Determine a 90% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.9$.

$$z^\star = 1.64$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.688)(1-0.688)}{1900}} = 0.0106$$

Calculate the margin of error.

$$ME = z^\star SE = (1.64)(0.0106) = 0.0174$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.671\ ,\ 0.705)$$

We are 90% confident that the true population proportion is between 67.1% and 70.5%.

(a) The lower bound = 0.671, which can also be expressed as 67.1%.

(b) The upper bound = 0.705, which can also be expressed as 70.5%.

8. An experiment is run with a treatment group of size 275 and a control group of size 226. The results are summarized in the table below.

	treatment	control
cold	237	211
not cold	38	15

Using a significance level of 0.02, determine whether the treatment causes an effect on the proportion of cases that are cold.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.02$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.33$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{237}{275} = 0.862$$
$$\hat{p}_2 = \frac{211}{226} = 0.934$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.934 - 0.862 = 0.072$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{237 + 211}{275 + 226} = 0.894$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.894)(0.106)}{275} + \frac{(0.894)(0.106)}{226}} \\ &= 0.0276 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0276)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.934 - 0.862) - 0}{0.0276} \\ &= 2.61 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.61) \\ &= 0.009 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \; p_2 - p_1 = 0$
(b) $H_A: \; p_2 - p_1 \neq 0$
(c) $z^\star = 2.33$
(d) $SE = 0.0276$
(e) $|z_{\text{obs}}| = 2.61$
(f) p-value = 0.009
(g) reject the null

1. (a) P(**tree**) = 0.26

   (b) P(**shovel or indigo**) = 0.327

   (c) P(**shovel and red**) = 0.0816

   (d) P(**yellow given dog**) = 0.0676

   (e) P(**pig given pink**) = 0.2

   (f) P(**pink**) = 0.201

2. P(**"not wheel" given "green"**) = 0.311

3. $P(68.15 < X < 68.51) = 0.7666$

4. (a) $P(X = 25) = 0.0815$

   (b) $P(23 \leq X \leq 34) = 0.8863$

5. **(9.89, 10.2)**

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.66$

   (d) $SE = 3.192$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.51$

   (f) $0.01 <$ p-**value** < 0.02

   (g) **retain**

7. (a) **LB of p CI = 0.872 or** 87.2%

   (b) **UB of p CI = 0.884 or** 88.4%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.05$

(d) $SE = 0.053$

(e) $\mid z_{\mathbf{obs}} \mid = 2.33$

(f) p-**value** = 0.0198

(g) **reject**

1. In a deck of strange cards, there are 895 cards. Each card has an image and a color. The amounts are shown in the table below.

	green	indigo	pink	red	yellow
dog	16	26	86	65	14
pig	34	18	36	68	90
shovel	31	19	45	73	41
tree	84	40	13	20	76

(a) What is the probability a random card is a tree?

(b) What is the probability a random card is either a shovel or indigo (or both)?

(c) What is the probability a random card is both a shovel and red?

(d) What is the probability a random card is yellow given it is a dog?

(e) What is the probability a random card is a pig given it is pink?

(f) What is the probability a random card is pink?

Solution

(a) $P(\text{tree}) = \frac{84+40+13+20+76}{895} = 0.26$

(b) $P(\text{shovel or indigo}) = \frac{31+19+45+73+41+26+18+19+40-19}{895} = 0.327$

(c) $P(\text{shovel and red}) = \frac{73}{895} = 0.0816$

(d) $P(\text{yellow given dog}) = \frac{14}{16+26+86+65+14} = 0.0676$

(e) $P(\text{pig given pink}) = \frac{36}{86+36+45+13} = 0.2$

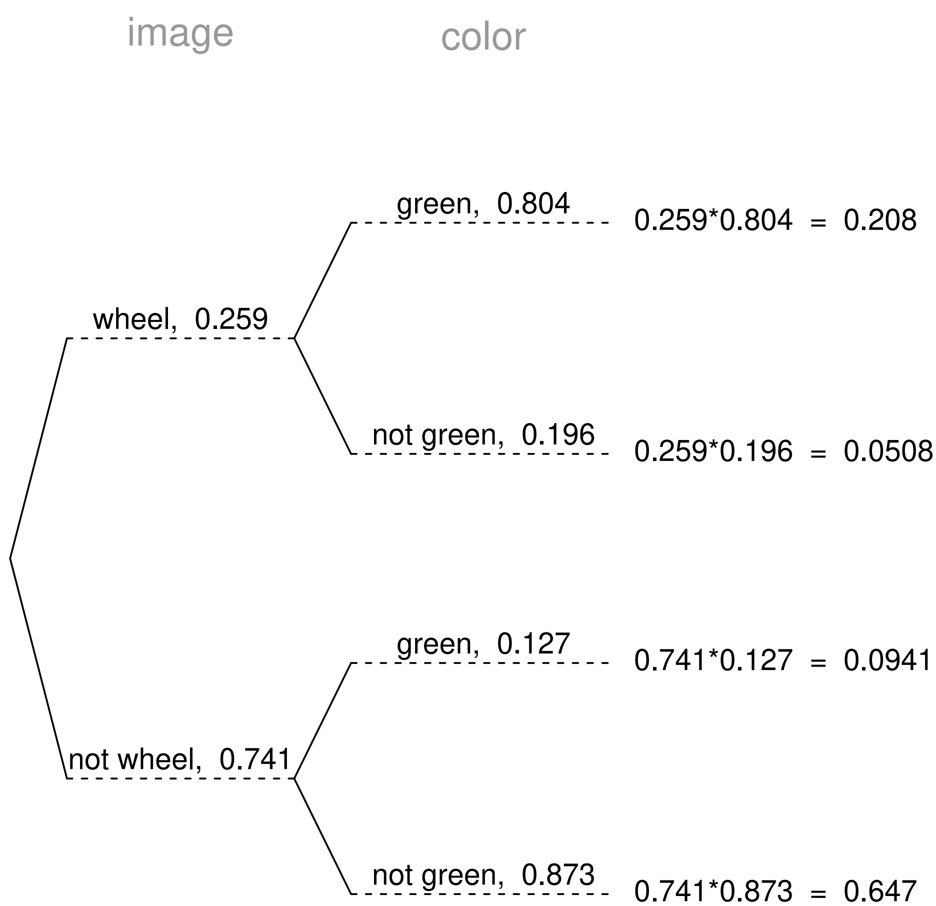
(f) $P(\text{pink}) = \frac{86+36+45+13}{895} = 0.201$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a wheel is 25.9%. If a wheel is drawn, there is a 80.4% chance that it is green. If a card that is not a wheel is drawn, there is a 12.7% chance that it is green.

   Now, someone draws a random card and reveals it is green. What is the chance the card is not a wheel?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not wheel" given "green"}) = \frac{0.0941}{0.0941 + 0.208} = 0.311$$

3. In a very large pile of toothpicks, the mean length is 68.4 millimeters and the standard deviation is 1.99 millimeters. If you randomly sample 225 toothpicks, what is the chance the sample mean is between 68.15 and 68.51 millimeters?

Solution

Label the given information.

$$\mu = 68.4$$
$$\sigma = 1.99$$
$$n = 225$$
$$\bar{x}_{\text{lower}} = 68.15$$
$$\bar{x}_{\text{upper}} = 68.51$$

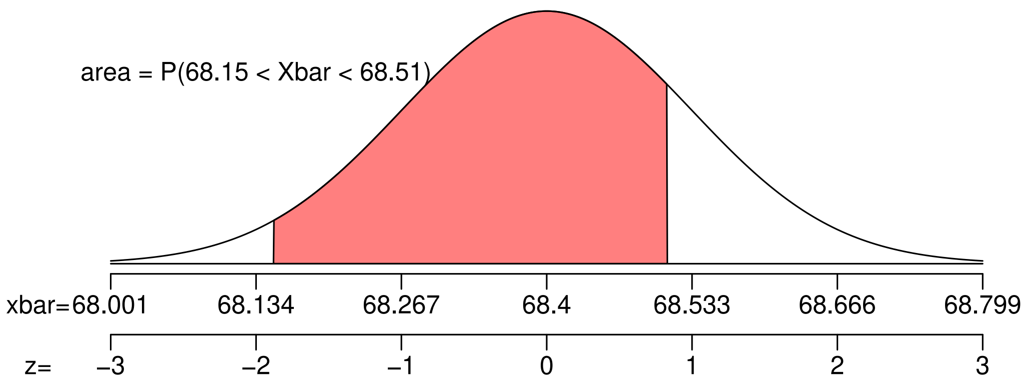
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.99}{\sqrt{225}} = 0.133$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(68.4,\ 0.133)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{68.15 - 68.4}{0.133} = -1.88$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{68.51 - 68.4}{0.133} = 0.83$$

Determine the probability.

$$\begin{aligned} P(68.15 < X < 68.51) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.83) - \Phi(-1.88) \\ &= 0.7666 \end{aligned}$$

4. In a game, there is a 58% chance to win a round. You will play 48 rounds.
   (a) What is the probability of winning exactly 25 rounds?
   (b) What is the probability of winning at least 23 but at most 34 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 25) = \binom{48}{25}(0.58)^{25}(1-0.58)^{48-25}$$

$$P(X = 25) = \binom{48}{25}(0.58)^{25}(0.42)^{23}$$

$$P(X = 25) = 0.0815$$

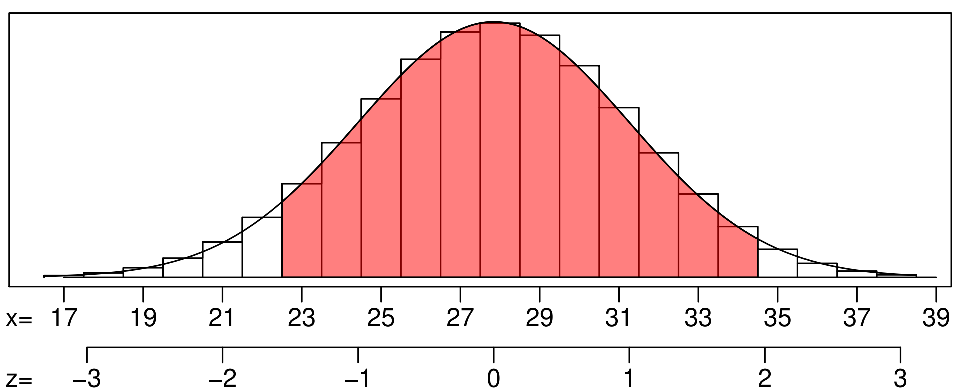
Find the mean.

$$\mu = np = (48)(0.58) = 27.84$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(48)(0.58)(1-0.58)} = 3.4195$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{22.5 - 27.84}{3.4195} = -1.42$$

$$z_2 = \frac{34.5 - 27.84}{3.4195} = 1.8$$

Calculate the probability.

$$P(23 \le X \le 34) = \Phi(1.8) - \Phi(-1.42) = 0.8863$$

(a) $P(X = 25) = 0.0815$

(b) $P(23 \le X \le 34) = 0.8863$

5. As an ornithologist, you wish to determine the average body mass of *Vireo griseus*. You randomly sample 33 adults of *Vireo griseus*, resulting in a sample mean of 10.04 grams and a sample standard deviation of 0.653 grams. Determine a 80% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 33 \\ \bar{x} &= 10.04 \\ s &= 0.653 \\ CL &= 0.8 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 32$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.8$.

$$t^\star = 1.31$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{0.653}{\sqrt{33}} = 0.114$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (10.04 - 1.31 \times 0.114,\ 10.04 + 1.31 \times 0.114) \\ &= (9.89,\ 10.2) \end{aligned}$$

We are 80% confident that the population mean is between 9.89 and 10.2.

6. A treatment group of size 33 has a mean of 109 and standard deviation of 14. A control group of size 29 has a mean of 101 and standard deviation of 11.1. If you decided to use a signficance level of 0.01, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 59.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 33 \\ \bar{x}_1 &= 109 \\ s_1 &= 14 \\ n_2 &= 29 \\ \bar{x}_2 &= 101 \\ s_2 &= 11.1 \\ \alpha &= 0.01 \\ df &= 59 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.01$ by using a t table.

$$t^\star = 2.66$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(14)^2}{33} + \frac{(11.1)^2}{29}} \\ &= 3.192 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(101 - 109) - (0)}{3.192} \\ &= -2.51 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^{\star} = 2.66$

(d) $SE = 3.192$

(e) $|t_{\text{obs}}| = 2.51$

(f) $0.01 < p\text{-value} < 0.02$

(g) retain the null

7. From a very large population, a random sample of 5400 individuals was taken. In that sample, 87.8% were tasty. Determine a 80% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.8$.

$$z^\star = 1.28$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.878)(1-0.878)}{5400}} = 0.00445$$

Calculate the margin of error.

$$ME = z^\star SE = (1.28)(0.00445) = 0.0057$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.872\,,\ 0.884)$$

We are 80% confident that the true population proportion is between 87.2% and 88.4%.

(a) The lower bound = 0.872, which can also be expressed as 87.2%.

(b) The upper bound = 0.884, which can also be expressed as 88.4%.

8. An experiment is run with a treatment group of size 165 and a control group of size 173. The results are summarized in the table below.

	treatment	control
folksy	110	94
not folksy	55	79

Using a significance level of 0.04, determine whether the treatment causes an effect on the proportion of cases that are folksy.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.04$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.05$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{110}{165} = 0.667$$
$$\hat{p}_2 = \frac{94}{173} = 0.543$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.543 - 0.667 = -0.124$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{110 + 94}{165 + 173} = 0.604$$

Determine the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}$$
$$= \sqrt{\frac{(0.604)(0.396)}{165} + \frac{(0.604)(0.396)}{173}}$$
$$= 0.0532$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0532)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$z = \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE}$$
$$= \frac{(0.543 - 0.667) - 0}{0.0532}$$
$$= -2.33$$

Determine the p-value.

$$p\text{-value} = 2 \cdot \Phi(-|z|)$$
$$= 2 \cdot \Phi(-2.33)$$
$$= 0.0198$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.05$

(d) $SE = 0.0532$

(e) $|z_{\text{obs}}| = 2.33$

(f) p-value = 0.0198

(g) reject the null

1. (a) P(**pig given black**) = 0.117

   (b) P(**pig and green**) = 0.0868

   (c) P(**green**) = 0.299

   (d) P(**horn or green**) = 0.483

   (e) P(**green given bike**) = 0.156

   (f) P(**wheel**) = 0.166

2. P(**"not shovel" given "not yellow"**) = 0.577

3. $P(60.8 < X < 61.26) = 0.7409$

4. (a) $P(X = 18) = 0.0924$

   (b) $P(22 \leq X \leq 29) = 0.2799$

5. $\mathbf{(13.8, 16)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.46$

   (d) $SE = 0.756$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.51$

   (f) $0.01 < p$**-value** < 0.02

   (g) **reject**

7. (a) **LB of p CI = 0.274 or** 27.4%

   (b) **UB of p CI = 0.31 or** 31%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 1.96$

(d) $SE = 0.066$

(e) $\mid z_{\mathbf{obs}} \mid = 2.08$

(f) p-**value** $= 0.0376$

(g) **reject**

1. In a deck of strange cards, there are 1002 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	green	orange
bike	41	20	67
flower	88	85	79
horn	91	71	93
pig	38	87	76
wheel	66	37	63

(a) What is the probability a random card is a pig given it is black?

(b) What is the probability a random card is both a pig and green?

(c) What is the probability a random card is green?

(d) What is the probability a random card is either a horn or green (or both)?

(e) What is the probability a random card is green given it is a bike?

(f) What is the probability a random card is a wheel?

Solution

(a) $P(\text{pig given black}) = \frac{38}{41+88+91+38+66} = 0.117$

(b) $P(\text{pig and green}) = \frac{87}{1002} = 0.0868$

(c) $P(\text{green}) = \frac{20+85+71+87+37}{1002} = 0.299$

(d) $P(\text{horn or green}) = \frac{91+71+93+20+85+71+87+37-71}{1002} = 0.483$

(e) $P(\text{green given bike}) = \frac{20}{41+20+67} = 0.156$

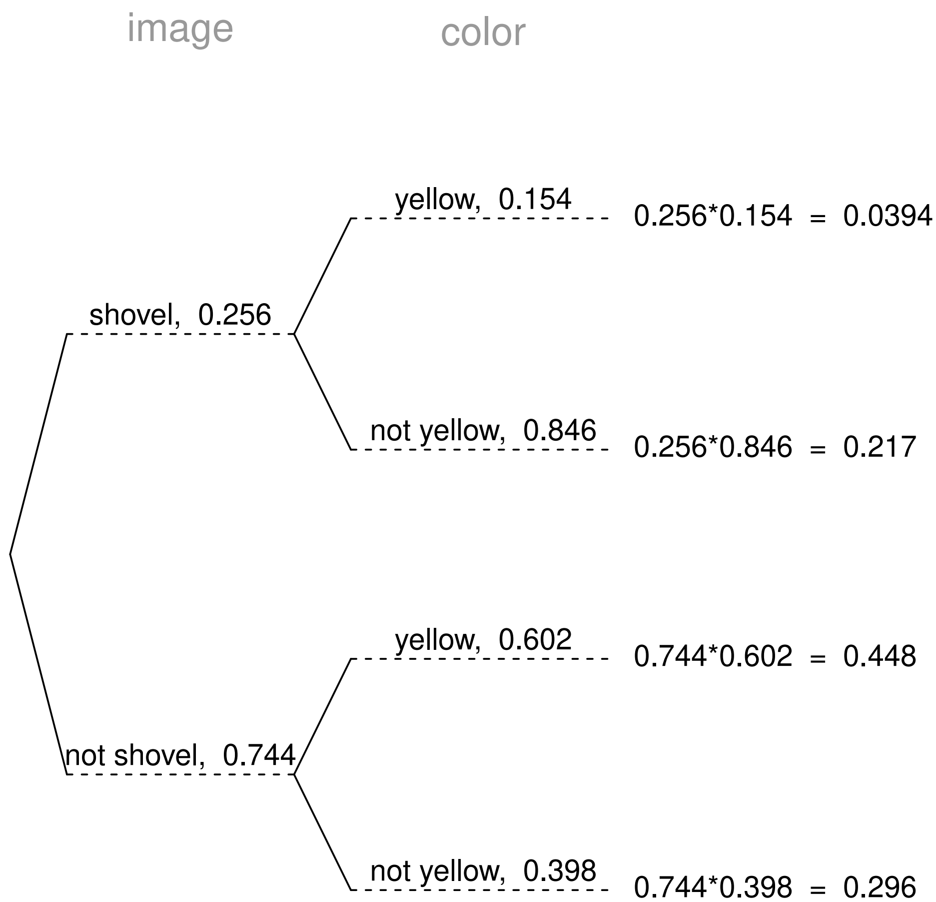
(f) $P(\text{wheel}) = \frac{66+37+63}{1002} = 0.166$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a shovel is 25.6%. If a shovel is drawn, there is a 15.4% chance that it is yellow. If a card that is not a shovel is drawn, there is a 60.2% chance that it is yellow.

   Now, someone draws a random card and reveals it is not yellow. What is the chance the card is not a shovel?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not shovel" given "not yellow"}) = \frac{0.296}{0.296 + 0.217} = 0.577$$

3. In a very large pile of toothpicks, the mean length is 61.04 millimeters and the standard deviation is 2.45 millimeters. If you randomly sample 144 toothpicks, what is the chance the sample mean is between 60.8 and 61.26 millimeters?

Solution

Label the given information.

$$\mu = 61.04$$
$$\sigma = 2.45$$
$$n = 144$$
$$\bar{x}_{\text{lower}} = 60.8$$
$$\bar{x}_{\text{upper}} = 61.26$$

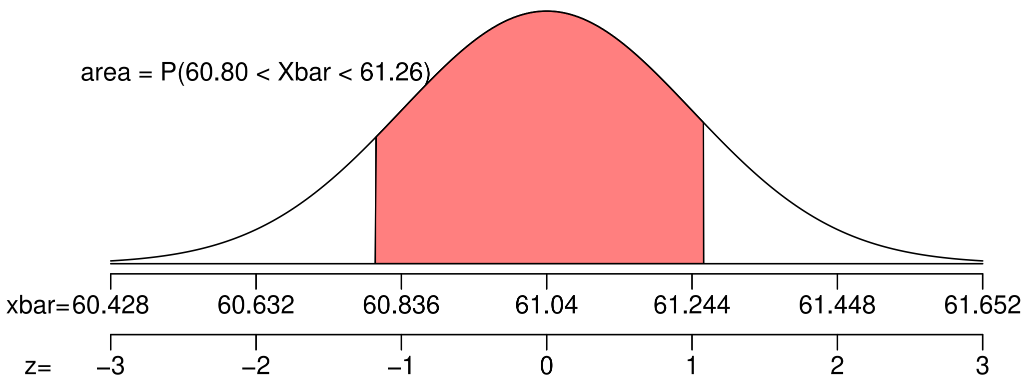
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2.45}{\sqrt{144}} = 0.204$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(61.04,\ 0.204)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{60.8 - 61.04}{0.204} = -1.18$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{61.26 - 61.04}{0.204} = 1.08$$

Determine the probability.

$$\begin{aligned} P(60.8 < X < 61.26) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.08) - \Phi(-1.18) \\ &= 0.7409 \end{aligned}$$

4. In a game, there is a 18% chance to win a round. You will play 110 rounds.
   (a) What is the probability of winning exactly 18 rounds?
   (b) What is the probability of winning at least 22 but at most 29 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 18) = \binom{110}{18}(0.18)^{18}(1-0.18)^{110-18}$$

$$P(X = 18) = \binom{110}{18}(0.18)^{18}(0.82)^{92}$$

$$P(X = 18) = 0.0924$$

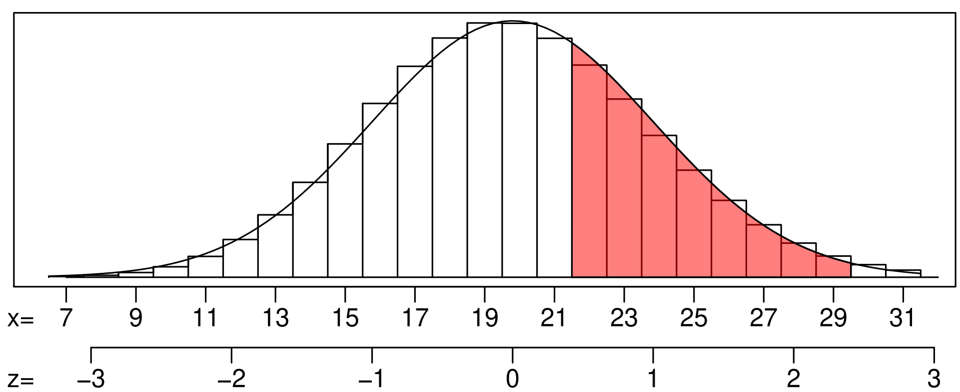
Find the mean.

$$\mu = np = (110)(0.18) = 19.8$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(110)(0.18)(1-0.18)} = 4.0294$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{21.5 - 19.8}{4.0294} = 0.55$$

$$z_2 = \frac{29.5 - 19.8}{4.0294} = 2.28$$

Calculate the probability.

$$P(22 \leq X \leq 29) = \Phi(2.28) - \Phi(0.55) = 0.2799$$

(a) $P(X = 18) = 0.0924$

(b) $P(22 \leq X \leq 29) = 0.2799$

5. As an ornithologist, you wish to determine the average body mass of *Passerina cyanea*. You randomly sample 24 adults of *Passerina cyanea*, resulting in a sample mean of 14.92 grams and a sample standard deviation of 2.16 grams. Determine a 98% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 24 \\ \bar{x} &= 14.92 \\ s &= 2.16 \\ CL &= 0.98 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 23$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.98$.

$$t^{\star} = 2.5$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.16}{\sqrt{24}} = 0.441$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^{\star} SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^{\star} SE,\ \bar{x} + t^{\star} SE) \\ &= (14.92 - 2.5 \times 0.441,\ 14.92 + 2.5 \times 0.441) \\ &= (13.8,\ 16) \end{aligned}$$

We are 98% confident that the population mean is between 13.8 and 16.

6. A treatment group of size 17 has a mean of 12.1 and standard deviation of 1.9. A control group of size 17 has a mean of 10.2 and standard deviation of 2.47. If you decided to use a signficance level of 0.02, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 30.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 17 \\ \bar{x}_1 &= 12.1 \\ s_1 &= 1.9 \\ n_2 &= 17 \\ \bar{x}_2 &= 10.2 \\ s_2 &= 2.47 \\ \alpha &= 0.02 \\ df &= 30 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.02$ by using a t table.

$$t^\star = 2.46$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(1.9)^2}{17} + \frac{(2.47)^2}{17}} \\ &= 0.756 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(10.2 - 12.1) - (0)}{0.756} \\ &= -2.51 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.46$

(d) $SE = 0.756$

(e) $|t_{obs}| = 2.51$

(f) $0.01 < p\text{-value} < 0.02$

(g) reject the null

7. From a very large population, a random sample of 5300 individuals was taken. In that sample, 29.2% were super. Determine a 99.5% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.995$.

$$z^\star = 2.81$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.292)(1-0.292)}{5300}} = 0.00625$$

Calculate the margin of error.

$$ME = z^\star SE = (2.81)(0.00625) = 0.0176$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.274\,,\ 0.31)$$

We are 99.5% confident that the true population proportion is between 27.4% and 31%.

(a) The lower bound = 0.274, which can also be expressed as 27.4%.

(b) The upper bound = 0.31, which can also be expressed as 31%.

8. An experiment is run with a treatment group of size 117 and a control group of size 71. The results are summarized in the table below.

	treatment	control
sick	92	46
not sick	25	25

Using a significance level of 0.05, determine whether the treatment causes an effect on the proportion of cases that are sick.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution
State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.05$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.96$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{92}{117} = 0.786$$
$$\hat{p}_2 = \frac{46}{71} = 0.648$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.648 - 0.786 = -0.138$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{92 + 46}{117 + 71} = 0.734$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.734)(0.266)}{117} + \frac{(0.734)(0.266)}{71}} \\ &= 0.0665 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0665)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.648 - 0.786) - 0}{0.0665} \\ &= -2.08 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.08) \\ &= 0.0376 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 1.96$

(d) $SE = 0.0665$

(e) $|z_{\text{obs}}| = 2.08$

(f) p-value = 0.0376

(g) reject the null

1. (a) P(**orange given dog**) = 0.335

   (b) P(**horn and gray**) = 0.0253

   (c) P(**horn given gray**) = 0.128

   (d) P(**dog**) = 0.322

   (e) P(**gray**) = 0.197

   (f) P(**horn or red**) = 0.402

2. P(**"cat" given "orange"**) = 0.432

3. $P(70.05 < X < 70.54) = 0.6763$

4. (a) $P(X = 28) = 0.0814$

   (b) $P(33 \leq X \leq 40) = 0.1555$

5. **(9.3, 9.94)**

6. (a) $H_0 : \ \mu_2 - \mu_1 = 0$

   (b) $H_0 : \ \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 1.68$

   (d) $SE = 0.021$

   (e) $\mid t_{\textbf{obs}} \mid = 1.46$

   (f) $0.1 <$ p-**value** < 0.2

   (g) **retain**

7. (a) **LB of p CI = 0.412 or** 41.2%

   (b) **UB of p CI = 0.506 or** 50.6%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.81$

(d) $SE = 0.118$

(e) $\mid z_{\mathbf{obs}} \mid = 2.68$

(f) p-**value** = 0.0074

(g) **retain**

1. In a deck of strange cards, there are 752 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	green	orange	red
dog	61	76	81	24
horn	19	63	34	85
wheel	68	90	74	77

   (a) What is the probability a random card is orange given it is a dog?

   (b) What is the probability a random card is both a horn and gray?

   (c) What is the probability a random card is a horn given it is gray?

   (d) What is the probability a random card is a dog?

   (e) What is the probability a random card is gray?

   (f) What is the probability a random card is either a horn or red (or both)?

Solution

(a) $P(\text{orange given dog}) = \frac{81}{61+76+81+24} = 0.335$

(b) $P(\text{horn and gray}) = \frac{19}{752} = 0.0253$

(c) $P(\text{horn given gray}) = \frac{19}{61+19+68} = 0.128$

(d) $P(\text{dog}) = \frac{61+76+81+24}{752} = 0.322$

(e) $P(\text{gray}) = \frac{61+19+68}{752} = 0.197$

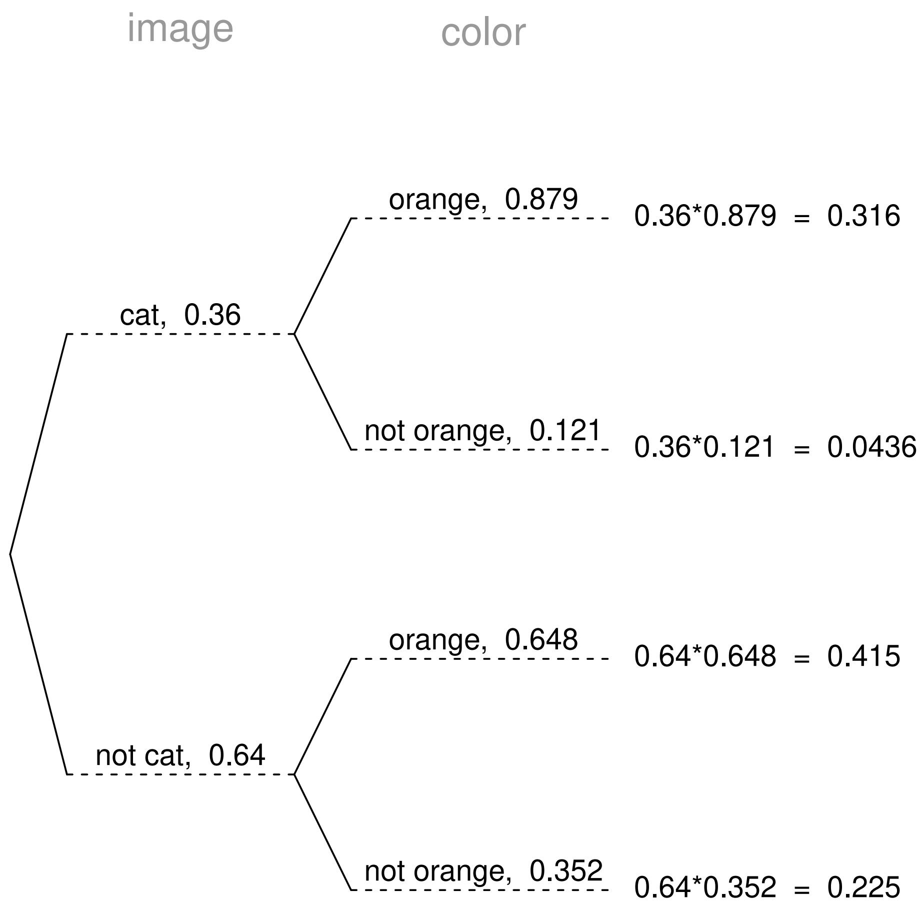
(f) $P(\text{horn or red}) = \frac{19+63+34+85+24+85+77-85}{752} = 0.402$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a cat is 36%. If a cat is drawn, there is a 87.9% chance that it is orange. If a card that is not a cat is drawn, there is a 64.8% chance that it is orange.

   Now, someone draws a random card and reveals it is orange. What is the chance the card is a cat?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"cat" given "orange"}) = \frac{0.316}{0.316 + 0.415} = 0.432$$

3. In a very large pile of toothpicks, the mean length is 70.4 millimeters and the standard deviation is 3.36 millimeters. If you randomly sample 225 toothpicks, what is the chance the sample mean is between 70.05 and 70.54 millimeters?

Solution

Label the given information.

$$\mu = 70.4$$
$$\sigma = 3.36$$
$$n = 225$$
$$\bar{x}_{\text{lower}} = 70.05$$
$$\bar{x}_{\text{upper}} = 70.54$$

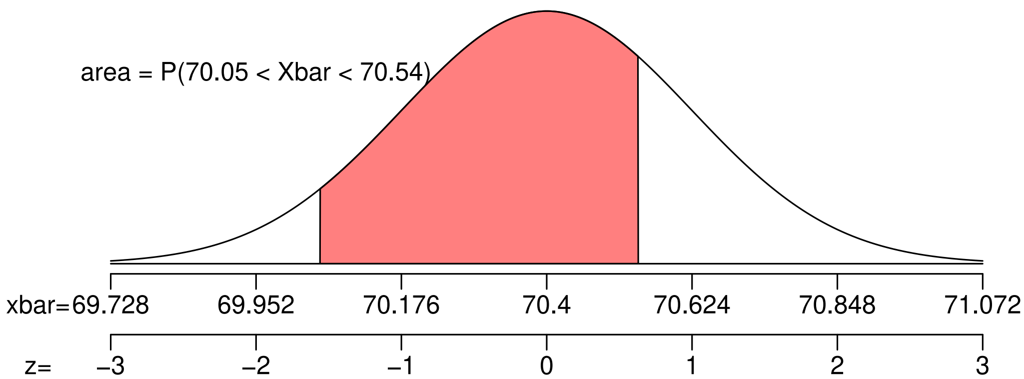
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.36}{\sqrt{225}} = 0.224$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(70.4,\ 0.224)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{70.05 - 70.4}{0.224} = -1.56$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{70.54 - 70.4}{0.224} = 0.63$$

Determine the probability.

$$\begin{aligned} P(70.05 < X < 70.54) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.63) - \Phi(-1.56) \\ &= 0.6763 \end{aligned}$$

4. In a game, there is a 15% chance to win a round. You will play 188 rounds.
   (a) What is the probability of winning exactly 28 rounds?
   (b) What is the probability of winning at least 33 but at most 40 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 28) = \binom{188}{28}(0.15)^{28}(1-0.15)^{188-28}$$

$$P(X = 28) = \binom{188}{28}(0.15)^{28}(0.85)^{160}$$

$$P(X = 28) = 0.0814$$

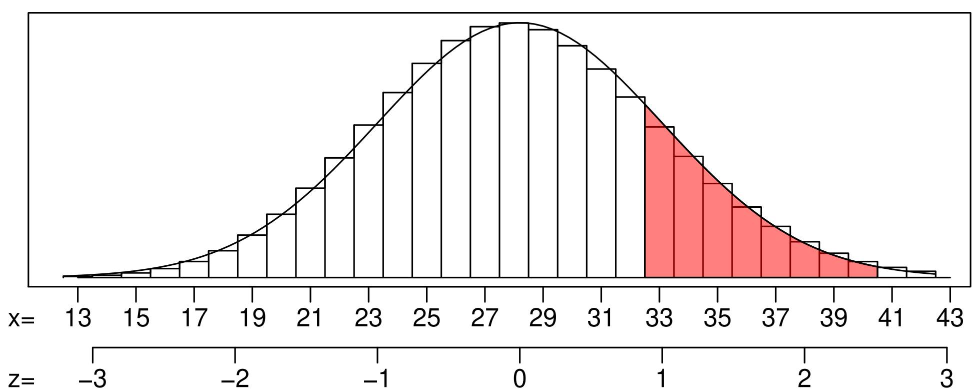
Find the mean.

$$\mu = np = (188)(0.15) = 28.2$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(188)(0.15)(1-0.15)} = 4.8959$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{32.5 - 28.2}{4.8959} = 0.98$$

$$z_2 = \frac{40.5 - 28.2}{4.8959} = 2.41$$

Calculate the probability.

$$P(33 \leq X \leq 40) = \Phi(2.41) - \Phi(0.98) = 0.1555$$

(a) $P(X = 28) = 0.0814$

(b) $P(33 \leq X \leq 40) = 0.1555$

5. As an ornithologist, you wish to determine the average body mass of *Denrdoica magnolia*. You randomly sample 24 adults of *Denrdoica magnolia*, resulting in a sample mean of 9.62 grams and a sample standard deviation of 1.17 grams. Determine a 80% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 24 \\ \bar{x} &= 9.62 \\ s &= 1.17 \\ CL &= 0.8 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 23$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.8$.

$$t^\star = 1.32$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.17}{\sqrt{24}} = 0.239$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (9.62 - 1.32 \times 0.239,\ 9.62 + 1.32 \times 0.239) \\ &= (9.3,\ 9.94) \end{aligned}$$

We are 80% confident that the population mean is between 9.3 and 9.94.

6. A treatment group of size 27 has a mean of 1.02 and standard deviation of 0.066. A control group of size 23 has a mean of 1.05 and standard deviation of 0.0778. If you decided to use a signficance level of 0.1, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 43.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 27 \\ \bar{x}_1 &= 1.02 \\ s_1 &= 0.066 \\ n_2 &= 23 \\ \bar{x}_2 &= 1.05 \\ s_2 &= 0.0778 \\ \alpha &= 0.1 \\ df &= 43 \end{aligned}$$

State the hypotheses.

$$\begin{aligned} H_0 :&\quad \mu_2 - \mu_1 = 0 \\ H_A :&\quad \mu_2 - \mu_1 \neq 0 \end{aligned}$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.1$ by using a t table.

$$t^\star = 1.68$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.066)^2}{27} + \frac{(0.0778)^2}{23}} \\ &= 0.021 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1.05 - 1.02) - (0)}{0.021} \\ &= 1.46 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.1 < p\text{-value} < 0.2$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^{\star} = 1.68$

(d) $SE = 0.021$

(e) $|t_{obs}| = 1.46$

(f) $0.1 < p\text{-value} < 0.2$

(g) retain the null

7. From a very large population, a random sample of 440 individuals was taken. In that sample, 45.9% were tasty. Determine a 95% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.95$.

$$z^{\star} = 1.96$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.459)(1-0.459)}{440}} = 0.0238$$

Calculate the margin of error.

$$ME = z^{\star}SE = (1.96)(0.0238) = 0.0466$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.412\,,\ 0.506)$$

We are 95% confident that the true population proportion is between 41.2% and 50.6%.

(a) The lower bound = 0.412, which can also be expressed as 41.2%.

(b) The upper bound = 0.506, which can also be expressed as 50.6%.

8. An experiment is run with a treatment group of size 49 and a control group of size 20. The results are summarized in the table below.

	treatment	control
green	9	10
not green	40	10

Using a significance level of 0.005, determine whether the treatment causes an effect on the proportion of cases that are green.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.005$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.81$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{9}{49} = 0.184$$

$$\hat{p}_2 = \frac{10}{20} = 0.5$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.5 - 0.184 = 0.316$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{9 + 10}{49 + 20} = 0.275$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.275)(0.725)}{49} + \frac{(0.275)(0.725)}{20}} \\ &= 0.118 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.118)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.5 - 0.184) - 0}{0.118} \\ &= 2.68 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.68) \\ &= 0.0074 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} > \alpha$$

So, we retain the null hypothesis. Thus the difference in proportions is not significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.81$

(d) $SE = 0.118$

(e) $|z_{\text{obs}}| = 2.68$

(f) p-value = 0.0074

(g) retain the null

1. (a) P(**horn or red**) = 0.449

   (b) P(**horn**) = 0.242

   (c) P(**wheel and red**) = 0.0746

   (d) P(**orange**) = 0.357

   (e) P(**shovel given orange**) = 0.0514

   (f) P(**teal given dog**) = 0.453

2. P(**"tree" given "green"**) = 0.0532

3. $P(61.96 < X < 62.41) = 0.6884$

4. (a) $P(X = 32) = 0.1093$

   (b) $P(29 \leq X \leq 35) = 0.6159$

5. **(11.5, 12.6)**

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.13$

   (d) $SE = 0.049$

   (e) $| \; t_{\mathbf{obs}} \; | = 1.83$

   (f) $0.05 <$ p**-value** < 0.1

   (g) **retain**

7. (a) **LB of p CI = 0.496 or** 49.6%

   (b) **UB of p CI = 0.516 or** 51.6%

8. (a) $\boxed{H_0 : \quad p_2 - p_1 = 0}$

(b) $\boxed{H_{\mathbf{A}} : \quad p_2 - p_1 \neq 0}$

(c) $\boxed{z^\star = 2.81}$

(d) $\boxed{SE = 0.141}$

(e) $\boxed{\mid z_{\mathbf{obs}} \mid = 2.75}$

(f) $\boxed{p\textbf{-value} = 0.006}$

(g) **retain**

1. In a deck of strange cards, there are 818 cards. Each card has an image and a color. The amounts are shown in the table below.

	orange	red	teal
dog	75	41	96
gem	63	25	52
horn	94	21	83
shovel	15	42	72
wheel	45	61	33

(a) What is the probability a random card is either a horn or red (or both)?

(b) What is the probability a random card is a horn?

(c) What is the probability a random card is both a wheel and red?

(d) What is the probability a random card is orange?

(e) What is the probability a random card is a shovel given it is orange?

(f) What is the probability a random card is teal given it is a dog?

Solution

(a) $P(\text{horn or red}) = \frac{94+21+83+41+25+21+42+61-21}{818} = 0.449$

(b) $P(\text{horn}) = \frac{94+21+83}{818} = 0.242$

(c) $P(\text{wheel and red}) = \frac{61}{818} = 0.0746$

(d) $P(\text{orange}) = \frac{75+63+94+15+45}{818} = 0.357$

(e) $P(\text{shovel given orange}) = \frac{15}{75+63+94+15+45} = 0.0514$

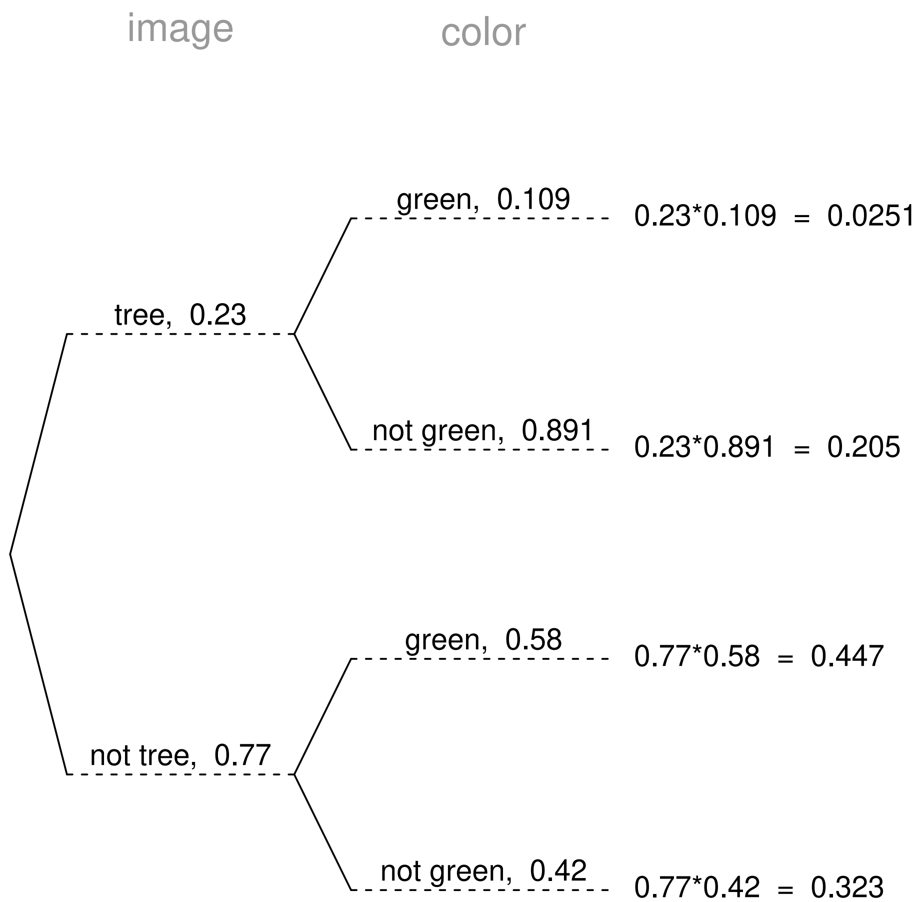
(f) $P(\text{teal given dog}) = \frac{96}{75+41+96} = 0.453$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a tree is 23%. If a tree is drawn, there is a 10.9% chance that it is green. If a card that is not a tree is drawn, there is a 58% chance that it is green.

   Now, someone draws a random card and reveals it is green. What is the chance the card is a tree?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"tree" given "green"}) = \frac{0.0251}{0.0251 + 0.447} = 0.0532$$

3. In a very large pile of toothpicks, the mean length is 62.13 millimeters and the standard deviation is 3.02 millimeters. If you randomly sample 196 toothpicks, what is the chance the sample mean is between 61.96 and 62.41 millimeters?

Solution

Label the given information.

$$\mu = 62.13$$
$$\sigma = 3.02$$
$$n = 196$$
$$\bar{x}_{\text{lower}} = 61.96$$
$$\bar{x}_{\text{upper}} = 62.41$$

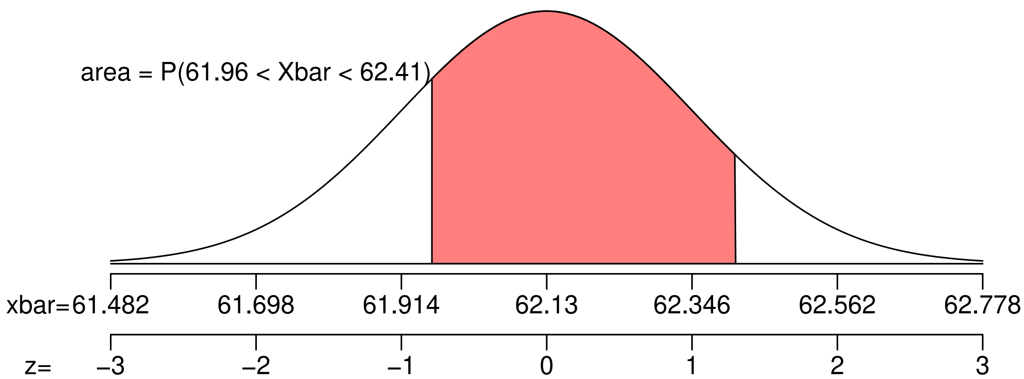
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.02}{\sqrt{196}} = 0.216$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(62.13,\ 0.216)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{61.96 - 62.13}{0.216} = -0.79$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{62.41 - 62.13}{0.216} = 1.3$$

Determine the probability.

$$\begin{aligned} P(61.96 < X < 62.41) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.3) - \Phi(-0.79) \\ &= 0.6884 \end{aligned}$$

4. In a game, there is a 75% chance to win a round. You will play 45 rounds.

   (a) What is the probability of winning exactly 32 rounds?

   (b) What is the probability of winning at least 29 but at most 35 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 32) = \binom{45}{32}(0.75)^{32}(1-0.75)^{45-32}$$

$$P(X = 32) = \binom{45}{32}(0.75)^{32}(0.25)^{13}$$

$$P(X = 32) = 0.1093$$

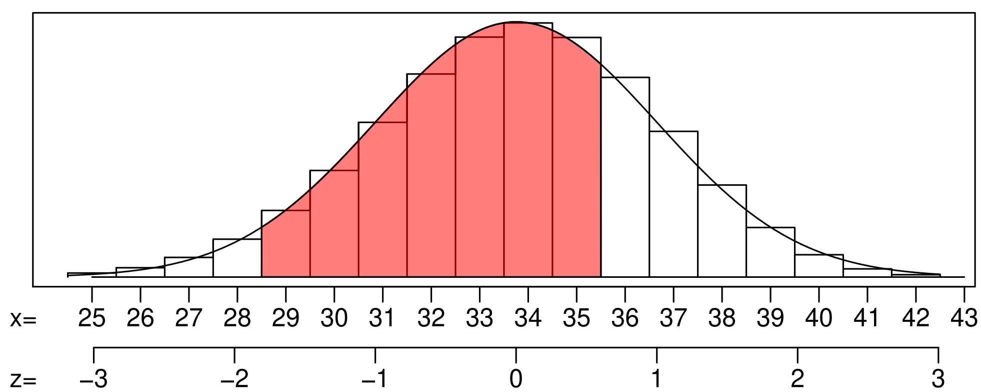
Find the mean.

$$\mu = np = (45)(0.75) = 33.75$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(45)(0.75)(1-0.75)} = 2.9047$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{28.5 - 33.75}{2.9047} = -1.64$$

$$z_2 = \frac{35.5 - 33.75}{2.9047} = 0.43$$

Calculate the probability.

$$P(29 \leq X \leq 35) = \Phi(0.43) - \Phi(-1.64) = 0.6159$$

(a) $P(X = 32) = 0.1093$

(b) $P(29 \leq X \leq 35) = 0.6159$

5. As an ornithologist, you wish to determine the average body mass of *Dendroica coronata*. You randomly sample 14 adults of *Dendroica coronata*, resulting in a sample mean of 12.09 grams and a sample standard deviation of 1.54 grams. Determine a 80% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 14$$
$$\bar{x} = 12.09$$
$$s = 1.54$$
$$CL = 0.8$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 13$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.8$.

$$t^\star = 1.35$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.54}{\sqrt{14}} = 0.412$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (12.09 - 1.35 \times 0.412,\ 12.09 + 1.35 \times 0.412) \\ &= (11.5,\ 12.6) \end{aligned}$$

We are 80% confident that the population mean is between 11.5 and 12.6.

6. A treatment group of size 19 has a mean of 1.12 and standard deviation of 0.166. A control group of size 25 has a mean of 1.03 and standard deviation of 0.155. If you decided to use a signficance level of 0.04, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 37.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 19$$
$$\bar{x}_1 = 1.12$$
$$s_1 = 0.166$$
$$n_2 = 25$$
$$\bar{x}_2 = 1.03$$
$$s_2 = 0.155$$
$$\alpha = 0.04$$
$$df = 37$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.04$ by using a t table.

$$t^\star = 2.13$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.166)^2}{19} + \frac{(0.155)^2}{25}} \\ &= 0.049 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1.03 - 1.12) - (0)}{0.049} \\ &= -1.83 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.13$

(d) $SE = 0.049$

(e) $|t_{obs}| = 1.83$

(f) $0.05 < p\text{-value} < 0.1$

(g) retain the null

7. From a very large population, a random sample of 6300 individuals was taken. In that sample, 50.6% were angry. Determine a 90% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.9$.

$$z^\star = 1.64$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.506)(1-0.506)}{6300}} = 0.0063$$

Calculate the margin of error.

$$ME = z^\star SE = (1.64)(0.0063) = 0.0103$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \;\approx\; \hat{p} \pm ME$$

Determine the interval.

$$(0.496\,,\;0.516)$$

We are 90% confident that the true population proportion is between 49.6% and 51.6%.

(a) The lower bound = 0.496, which can also be expressed as 49.6%.

(b) The upper bound = 0.516, which can also be expressed as 51.6%.

8. An experiment is run with a treatment group of size 55 and a control group of size 14. The results are summarized in the table below.

	treatment	control
happy	14	9
not happy	41	5

Using a significance level of 0.005, determine whether the treatment causes an effect on the proportion of cases that are happy.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.005$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.81$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{14}{55} = 0.255$$
$$\hat{p}_2 = \frac{9}{14} = 0.643$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.643 - 0.255 = 0.388$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{14 + 9}{55 + 14} = 0.333$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.333)(0.667)}{55} + \frac{(0.333)(0.667)}{14}} \\ &= 0.141 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.141)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.643 - 0.255) - 0}{0.141} \\ &= 2.75 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.75) \\ &= 0.006 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} > \alpha$$

So, we retain the null hypothesis. Thus the difference in proportions is not significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.81$

(d) $SE = 0.141$

(e) $|z_{\text{obs}}| = 2.75$

(f) p-value = 0.006

(g) retain the null

1. (a) P(**flower or white**) = 0.426

   (b) P(**pink given bike**) = 0.109

   (c) P(**orange**) = 0.218

   (d) P(**wheel and blue**) = 0.0372

   (e) P(**bike given blue**) = 0.128

   (f) P(**cat**) = 0.32

2. P(**"not llama" given "yellow"**) = 0.886

3. $P(60.5 < X < 60.79) = 0.8051$

4. (a) $P(X = 140) = 0.0519$

   (b) $P(144 \leq X \leq 150) = 0.307$

5. $\mathbf{(9.36, 11.3)}$

6. (a) $H_0 : \quad \mu_2 - \mu_1 = 0$

   (b) $H_0 : \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 1.69$

   (d) $SE = 8.867$

   (e) $\mid t_{\mathbf{obs}} \mid = 1.76$

   (f) $0.05 <$ p**-value** < 0.1

   (g) **reject**

7. (a) **LB of p CI = 0.0974 or** 9.74%

   (b) **UB of p CI = 0.113 or** 11.3%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.05$

(d) $SE = 0.068$

(e) $\mid z_{\mathbf{obs}} \mid = 2.21$

(f) p**-value** = 0.0272

(g) **reject**

1. In a deck of strange cards, there are 1128 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	orange	pink	teal	white
bike	32	28	18	68	19
cat	85	95	52	74	55
flower	91	79	50	71	90
wheel	42	44	94	16	25

(a) What is the probability a random card is either a flower or white (or both)?

(b) What is the probability a random card is pink given it is a bike?

(c) What is the probability a random card is orange?

(d) What is the probability a random card is both a wheel and blue?

(e) What is the probability a random card is a bike given it is blue?

(f) What is the probability a random card is a cat?

Solution

(a) $P(\text{flower or white}) = \frac{91+79+50+71+90+19+55+90+25-90}{1128} = 0.426$

(b) $P(\text{pink given bike}) = \frac{18}{32+28+18+68+19} = 0.109$

(c) $P(\text{orange}) = \frac{28+95+79+44}{1128} = 0.218$

(d) $P(\text{wheel and blue}) = \frac{42}{1128} = 0.0372$

(e) $P(\text{bike given blue}) = \frac{32}{32+85+91+42} = 0.128$

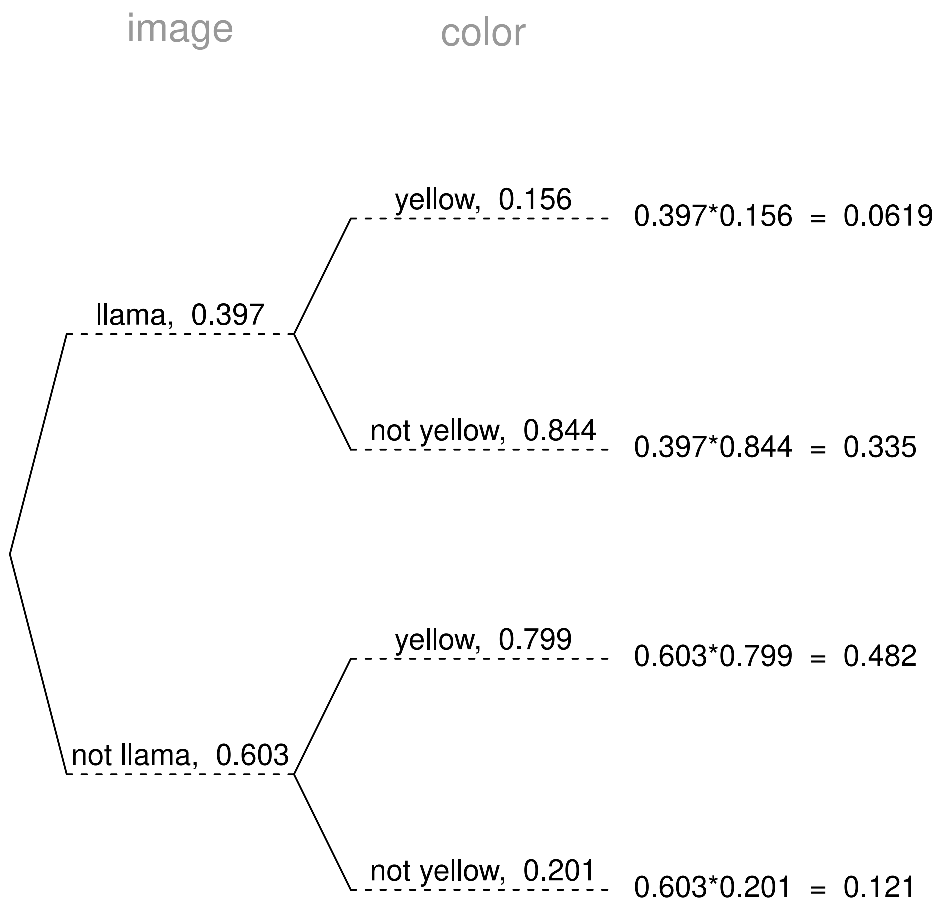
(f) $P(\text{cat}) = \frac{85+95+52+74+55}{1128} = 0.32$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a llama is 39.7%. If a llama is drawn, there is a 15.6% chance that it is yellow. If a card that is not a llama is drawn, there is a 79.9% chance that it is yellow.

   Now, someone draws a random card and reveals it is yellow. What is the chance the card is not a llama?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not llama" given "yellow"}) = \frac{0.482}{0.482 + 0.0619} = 0.886$$

3. In a very large pile of toothpicks, the mean length is 60.6 millimeters and the standard deviation is 1.35 millimeters. If you randomly sample 175 toothpicks, what is the chance the sample mean is between 60.5 and 60.79 millimeters?

Solution

Label the given information.

$$\mu = 60.6$$
$$\sigma = 1.35$$
$$n = 175$$
$$\bar{x}_{\text{lower}} = 60.5$$
$$\bar{x}_{\text{upper}} = 60.79$$

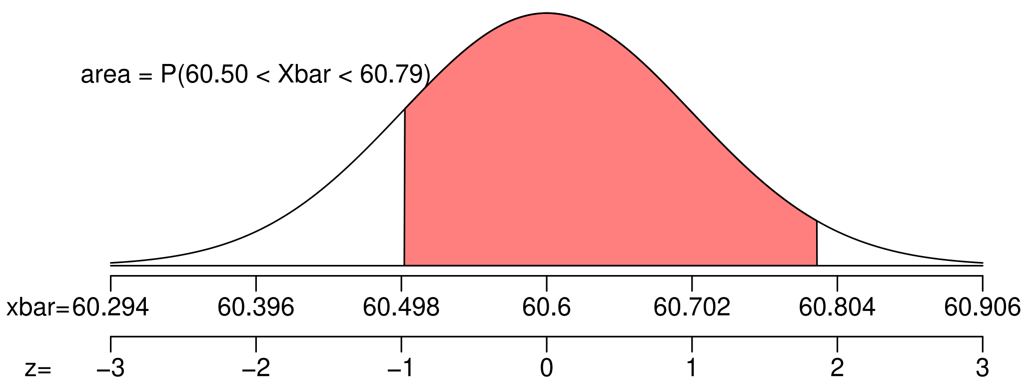
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.35}{\sqrt{175}} = 0.102$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(60.6,\ 0.102)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{60.5 - 60.6}{0.102} = -0.98$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{60.79 - 60.6}{0.102} = 1.86$$

Determine the probability.

$$\begin{aligned} P(60.5 < X < 60.79) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.86) - \Phi(-0.98) \\ &= 0.8051 \end{aligned}$$

4. In a game, there is a 70% chance to win a round. You will play 205 rounds.
   (a) What is the probability of winning exactly 140 rounds?
   (b) What is the probability of winning at least 144 but at most 150 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 140) = \binom{205}{140}(0.7)^{140}(1-0.7)^{205-140}$$

$$P(X = 140) = \binom{205}{140}(0.7)^{140}(0.3)^{65}$$

$$P(X = 140) = 0.0519$$

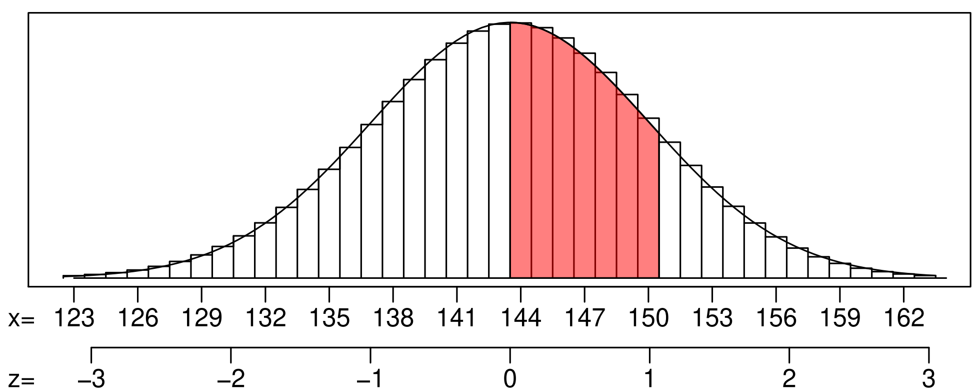
Find the mean.

$$\mu = np = (205)(0.7) = 143.5$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(205)(0.7)(1-0.7)} = 6.5612$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{143.5 - 143.5}{6.5612} = 0.08$$

$$z_2 = \frac{150.5 - 143.5}{6.5612} = 0.99$$

Calculate the probability.

$$P(144 \leq X \leq 150) = \Phi(0.99) - \Phi(0.08) = 0.307$$

(a) $P(X = 140) = 0.0519$

(b) $P(144 \leq X \leq 150) = 0.307$

5. As an ornithologist, you wish to determine the average body mass of *Dendroica dominica*. You randomly sample 25 adults of *Dendroica dominica*, resulting in a sample mean of 10.35 grams and a sample standard deviation of 1.99 grams. Determine a 98% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 25 \\ \bar{x} &= 10.35 \\ s &= 1.99 \\ CL &= 0.98 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 24$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.98$.

$$t^\star = 2.49$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.99}{\sqrt{25}} = 0.398$$

We want to make an inference about the population mean.

$$\mu \;\approx\; \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (10.35 - 2.49 \times 0.398,\ 10.35 + 2.49 \times 0.398) \\ &= (9.36,\ 11.3) \end{aligned}$$

We are 98% confident that the population mean is between 9.36 and 11.3.

6. A treatment group of size 20 has a mean of 94.4 and standard deviation of 25. A control group of size 19 has a mean of 110 and standard deviation of 30. If you decided to use a signficance level of 0.1, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 35.

   (a) State the null hypothesis.
   (b) State the alternative hypothesis.
   (c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)
   (d) Determine the standard error of the relevant sampling distribution.
   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)
   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.
   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 20$$
$$\bar{x}_1 = 94.4$$
$$s_1 = 25$$
$$n_2 = 19$$
$$\bar{x}_2 = 110$$
$$s_2 = 30$$
$$\alpha = 0.1$$
$$df = 35$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.1$ by using a t table.

$$t^\star = 1.69$$

Calculate the standard error.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(25)^2}{20} + \frac{(30)^2}{19}} = 8.867$$

Determine the test statistic.

$$t_{\text{obs}} = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} = \frac{(110 - 94.4) - (0)}{8.867} = 1.76$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 1.69$

(d) $SE = 8.867$

(e) $|t_{\text{obs}}| = 1.76$

(f) $0.05 < p\text{-value} < 0.1$

(g) reject the null

7. From a very large population, a random sample of 8900 individuals was taken. In that sample, 10.5% were purple. Determine a 98% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.105)(1-0.105)}{8900}} = 0.00325$$

Calculate the margin of error.

$$ME = z^\star SE = (2.33)(0.00325) = 0.00757$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.0974\,,\ 0.113)$$

We are 98% confident that the true population proportion is between 9.74% and 11.3%.

(a) The lower bound = 0.0974, which can also be expressed as 9.74%.

(b) The upper bound = 0.113, which can also be expressed as 11.3%.

8. An experiment is run with a treatment group of size 99 and a control group of size 118. The results are summarized in the table below.

	treatment	control
green	61	55
not green	38	63

Using a significance level of 0.04, determine whether the treatment causes an effect on the proportion of cases that are green.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.04$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.05$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{61}{99} = 0.616$$
$$\hat{p}_2 = \frac{55}{118} = 0.466$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.466 - 0.616 = -0.15$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{61 + 55}{99 + 118} = 0.535$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.535)(0.465)}{99} + \frac{(0.535)(0.465)}{118}} \\ &= 0.068 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.068)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.466 - 0.616) - 0}{0.068} \\ &= -2.21 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.21) \\ &= 0.0272 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.05$

(d) $SE = 0.068$

(e) $|z_{\text{obs}}| = 2.21$

(f) p-value = 0.0272

(g) reject the null

1. (a) $P(\textbf{blue}) = 0.389$

   (b) $P(\textbf{red given gem}) = 0.294$

   (c) $P(\textbf{wheel and red}) = 0.0574$

   (d) $P(\textbf{gem or blue}) = 0.531$

   (e) $P(\textbf{gem given red}) = 0.234$

   (f) $P(\textbf{cat}) = 0.193$

2. $P(\textbf{"not cat" given "violet"}) = 0.433$

3. $P(60.84 < X < 61.37) = 0.8953$

4. (a) $P(X = 49) = 0.0444$

   (b) $P(46 \leq X \leq 54) = 0.3119$

5. $\mathbf{(8.95, 11.1)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.7$

   (d) $SE = 48.57$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.49$

   (f) $0.01 < p$**-value** < 0.02

   (g) **retain**

7. (a) **LB of p CI = 0.429 or** 42.9%

   (b) **UB of p CI = 0.451 or** 45.1%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 1.96$

(d) $SE = 0.042$

(e) $\mid z_{\mathbf{obs}} \mid = 2.14$

(f) p**-value** = 0.0324

(g) **reject**

1. In a deck of strange cards, there are 1027 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	green	red
bike	61	28	32
cat	95	57	46
dog	78	84	85
gem	86	77	68
wheel	80	91	59

   (a) What is the probability a random card is blue?

   (b) What is the probability a random card is red given it is a gem?

   (c) What is the probability a random card is both a wheel and red?

   (d) What is the probability a random card is either a gem or blue (or both)?

   (e) What is the probability a random card is a gem given it is red?

   (f) What is the probability a random card is a cat?

Solution

(a) $P(\text{blue}) = \frac{61+95+78+86+80}{1027} = 0.389$

(b) $P(\text{red given gem}) = \frac{68}{86+77+68} = 0.294$

(c) $P(\text{wheel and red}) = \frac{59}{1027} = 0.0574$

(d) $P(\text{gem or blue}) = \frac{86+77+68+61+95+78+86+80-86}{1027} = 0.531$

(e) $P(\text{gem given red}) = \frac{68}{32+46+85+68+59} = 0.234$

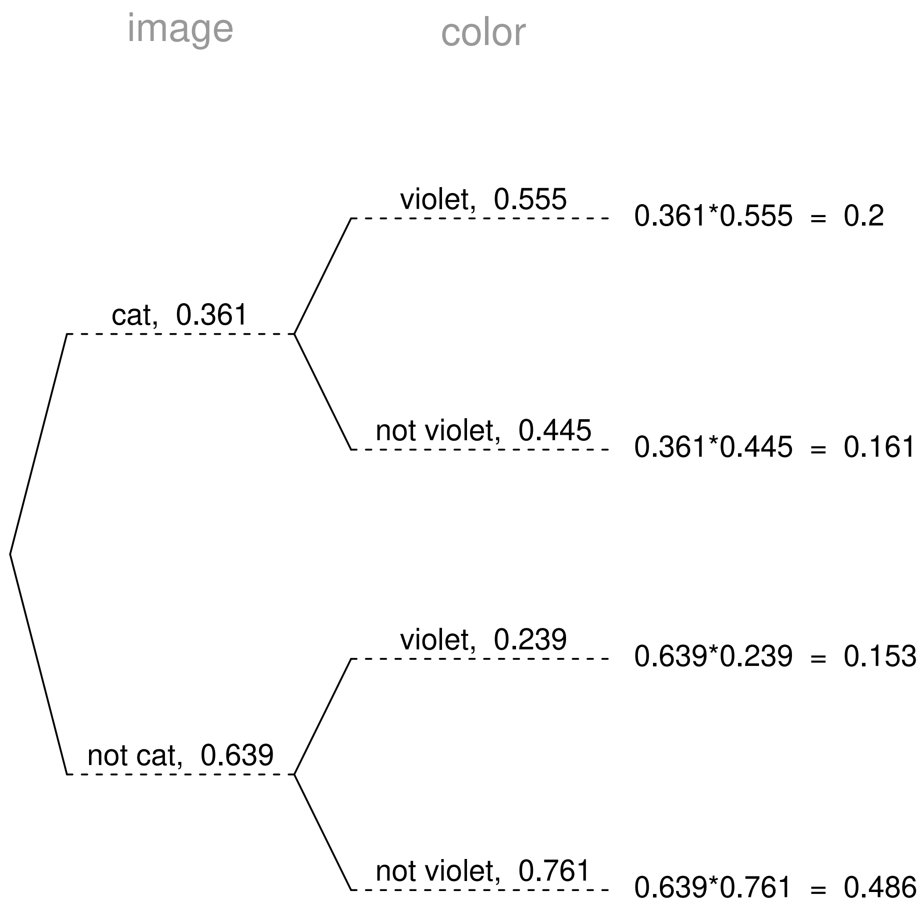
(f) $P(\text{cat}) = \frac{95+57+46}{1027} = 0.193$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a cat is 36.1%. If a cat is drawn, there is a 55.5% chance that it is violet. If a card that is not a cat is drawn, there is a 23.9% chance that it is violet.

   Now, someone draws a random card and reveals it is violet. What is the chance the card is not a cat?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not cat" given "violet"}) = \frac{0.153}{0.153 + 0.2} = 0.433$$

3. In a very large pile of toothpicks, the mean length is 61.02 millimeters and the standard deviation is 1.96 millimeters. If you randomly sample 196 toothpicks, what is the chance the sample mean is between 60.84 and 61.37 millimeters?

Solution

Label the given information.

$$\mu = 61.02$$
$$\sigma = 1.96$$
$$n = 196$$
$$\bar{x}_{\text{lower}} = 60.84$$
$$\bar{x}_{\text{upper}} = 61.37$$

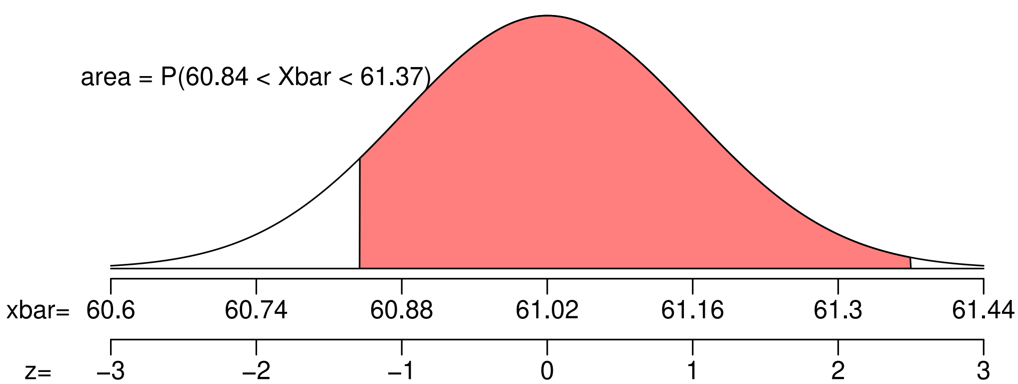
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.96}{\sqrt{196}} = 0.14$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(61.02, 0.14)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{60.84 - 61.02}{0.14} = -1.29$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{61.37 - 61.02}{0.14} = 2.5$$

Determine the probability.

$$\begin{aligned} P(60.84 < X < 61.37) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(2.5) - \Phi(-1.29) \\ &= 0.8953 \end{aligned}$$

4. In a game, there is a 22% chance to win a round. You will play 199 rounds.
   (a) What is the probability of winning exactly 49 rounds?
   (b) What is the probability of winning at least 46 but at most 54 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 49) = \binom{199}{49}(0.22)^{49}(1-0.22)^{199-49}$$

$$P(X = 49) = \binom{199}{49}(0.22)^{49}(0.78)^{150}$$

$$P(X = 49) = 0.0444$$

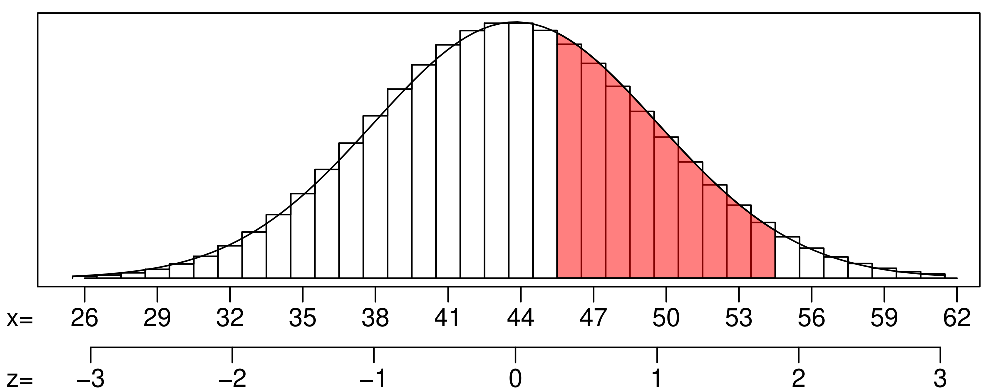
Find the mean.

$$\mu = np = (199)(0.22) = 43.78$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(199)(0.22)(1-0.22)} = 5.8437$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{45.5 - 43.78}{5.8437} = 0.38$$

$$z_2 = \frac{54.5 - 43.78}{5.8437} = 1.75$$

Calculate the probability.

$$P(46 \leq X \leq 54) = \Phi(1.75) - \Phi(0.38) = 0.3119$$

(a) $P(X = 49) = 0.0444$

(b) $P(46 \leq X \leq 54) = 0.3119$

5. As an ornithologist, you wish to determine the average body mass of *Dendroica palmarum*. You randomly sample 13 adults of *Dendroica palmarum*, resulting in a sample mean of 10.01 grams and a sample standard deviation of 1.26 grams. Determine a 99% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 13$$
$$\bar{x} = 10.01$$
$$s = 1.26$$
$$CL = 0.99$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 12$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.99$.

$$t^\star = 3.05$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.26}{\sqrt{13}} = 0.349$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (10.01 - 3.05 \times 0.349,\ 10.01 + 3.05 \times 0.349) \\ &= (8.95,\ 11.1) \end{aligned}$$

We are 99% confident that the population mean is between 8.95 and 11.1.

6. A treatment group of size 19 has a mean of 1120 and standard deviation of 158. A control group of size 31 has a mean of 999 and standard deviation of 180. If you decided to use a signficance level of 0.01, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 42.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 19$$
$$\bar{x}_1 = 1120$$
$$s_1 = 158$$
$$n_2 = 31$$
$$\bar{x}_2 = 999$$
$$s_2 = 180$$
$$\alpha = 0.01$$
$$df = 42$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.01$ by using a t table.

$$t^\star = 2.7$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(158)^2}{19} + \frac{(180)^2}{31}} \\ &= 48.57 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(999 - 1120) - (0)}{48.57} \\ &= -2.49 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^{\star} = 2.7$

(d) $SE = 48.57$

(e) $|t_{obs}| = 2.49$

(f) $0.01 < p\text{-value} < 0.02$

(g) retain the null

7. From a very large population, a random sample of 12000 individuals was taken. In that sample, 44% were glowing. Determine a 98% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.44)(1-0.44)}{12000}} = 0.00453$$

Calculate the margin of error.

$$ME = z^\star SE = (2.33)(0.00453) = 0.0106$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.429\,,\ 0.451)$$

We are 98% confident that the true population proportion is between 42.9% and 45.1%.

(a) The lower bound = 0.429, which can also be expressed as 42.9%.

(b) The upper bound = 0.451, which can also be expressed as 45.1%.

8. An experiment is run with a treatment group of size 185 and a control group of size 220. The results are summarized in the table below.

	treatment	control
pink	52	42
not pink	133	178

Using a significance level of 0.05, determine whether the treatment causes an effect on the proportion of cases that are pink.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.05$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.96$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{52}{185} = 0.281$$
$$\hat{p}_2 = \frac{42}{220} = 0.191$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.191 - 0.281 = -0.09$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{52 + 42}{185 + 220} = 0.232$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.232)(0.768)}{185} + \frac{(0.232)(0.768)}{220}} \\ &= 0.0421 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0421)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.191 - 0.281) - 0}{0.0421} \\ &= -2.14 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.14) \\ &= 0.0324 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.96$

(d) $SE = 0.0421$

(e) $|z_{\text{obs}}| = 2.14$

(f) p-value = 0.0324

(g) reject the null

1. (a) P(**dog given white**) = 0.211

   (b) P(**pig and violet**) = 0.0423

   (c) P(**violet given wheel**) = 0.208

   (d) P(**white**) = 0.341

   (e) P(**dog or white**) = 0.528

   (f) P(**wheel**) = 0.289

2. P(**"llama" given "not white"**) = 0.0695

3. $P(62.52 < X < 63.24) = 0.8035$

4. (a) $P(X = 18) = 0.094$

   (b) $P(14 \leq X \leq 27) = 0.897$

5. $\mathbf{(9.58, 10.8)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 1.69$

   (d) $SE = 2.667$

   (e) $\mid t_{\mathbf{obs}} \mid = 1.95$

   (f) $0.05 < p$**-value** < 0.1

   (g) **reject**

7. (a) **LB of p CI = 0.449 or** 44.9%

   (b) **UB of p CI = 0.535 or** 53.5%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.58$

(d) $SE = 0.063$

(e) $| \; z_{\mathbf{obs}} \; | = 2.71$

(f) p-**value** $= 0.0068$

(g) **reject**

1. In a deck of strange cards, there are 781 cards. Each card has an image and a color. The amounts are shown in the table below.

	indigo	red	violet	white
bike	21	54	17	38
dog	55	34	57	56
pig	72	31	33	87
wheel	65	29	47	85

(a) What is the probability a random card is a dog given it is white?

(b) What is the probability a random card is both a pig and violet?

(c) What is the probability a random card is violet given it is a wheel?

(d) What is the probability a random card is white?

(e) What is the probability a random card is either a dog or white (or both)?

(f) What is the probability a random card is a wheel?

Solution

(a) $P(\text{dog given white}) = \frac{56}{38+56+87+85} = 0.211$

(b) $P(\text{pig and violet}) = \frac{33}{781} = 0.0423$

(c) $P(\text{violet given wheel}) = \frac{47}{65+29+47+85} = 0.208$

(d) $P(\text{white}) = \frac{38+56+87+85}{781} = 0.341$

(e) $P(\text{dog or white}) = \frac{55+34+57+56+38+56+87+85-56}{781} = 0.528$

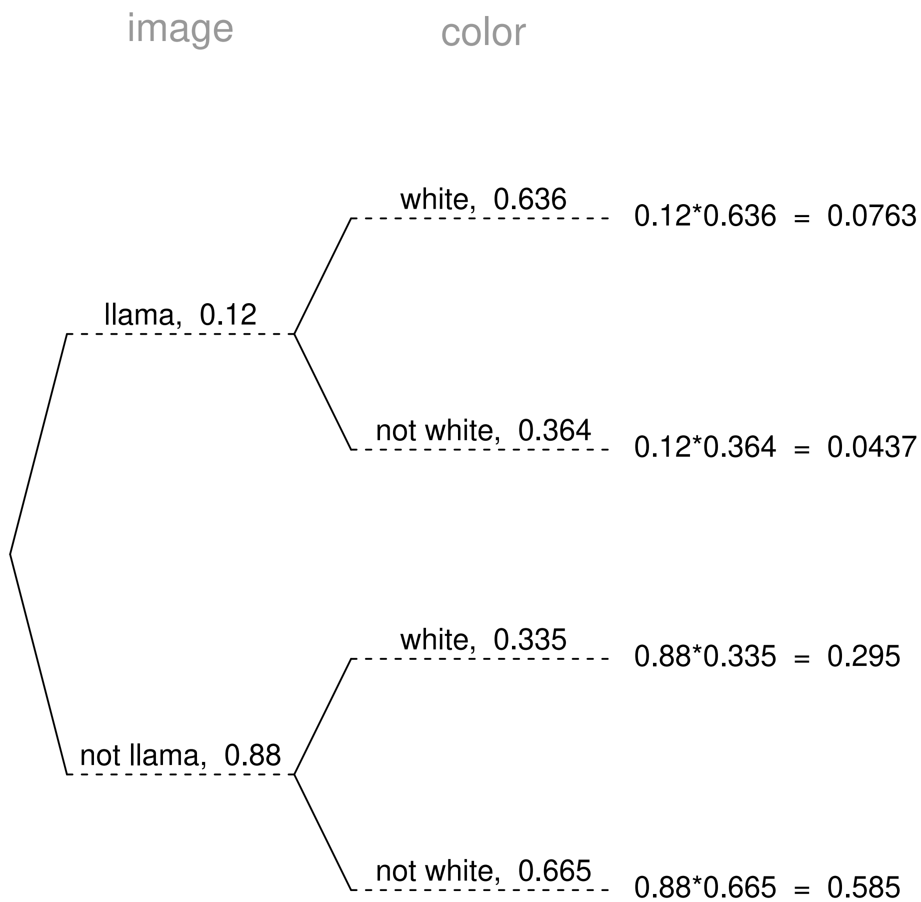
(f) $P(\text{wheel}) = \frac{65+29+47+85}{781} = 0.289$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a llama is 12%. If a llama is drawn, there is a 63.6% chance that it is white. If a card that is not a llama is drawn, there is a 33.5% chance that it is white.

   Now, someone draws a random card and reveals it is not white. What is the chance the card is a llama?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"llama" given "not white"}) = \frac{0.0437}{0.0437 + 0.585} = 0.0695$$

3. In a very large pile of toothpicks, the mean length is 62.84 millimeters and the standard deviation is 3.3 millimeters. If you randomly sample 144 toothpicks, what is the chance the sample mean is between 62.52 and 63.24 millimeters?

Solution

Label the given information.

$$\mu = 62.84$$
$$\sigma = 3.3$$
$$n = 144$$
$$\bar{x}_{\text{lower}} = 62.52$$
$$\bar{x}_{\text{upper}} = 63.24$$

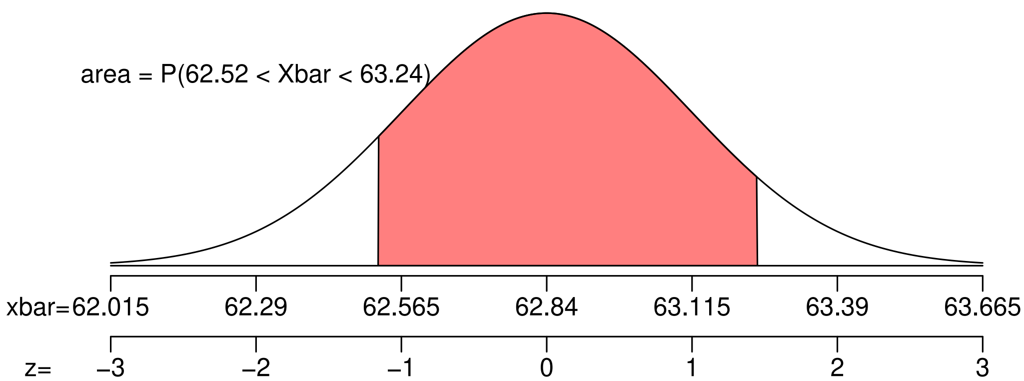
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.3}{\sqrt{144}} = 0.275$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(62.84,\ 0.275)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{62.52 - 62.84}{0.275} = -1.16$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{63.24 - 62.84}{0.275} = 1.45$$

Determine the probability.

$$\begin{aligned} P(62.52 < X < 63.24) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.45) - \Phi(-1.16) \\ &= 0.8035 \end{aligned}$$

4. In a game, there is a 22% chance to win a round. You will play 90 rounds.
   (a) What is the probability of winning exactly 18 rounds?
   (b) What is the probability of winning at least 14 but at most 27 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 18) = \binom{90}{18}(0.22)^{18}(1-0.22)^{90-18}$$

$$P(X = 18) = \binom{90}{18}(0.22)^{18}(0.78)^{72}$$

$$P(X = 18) = 0.094$$

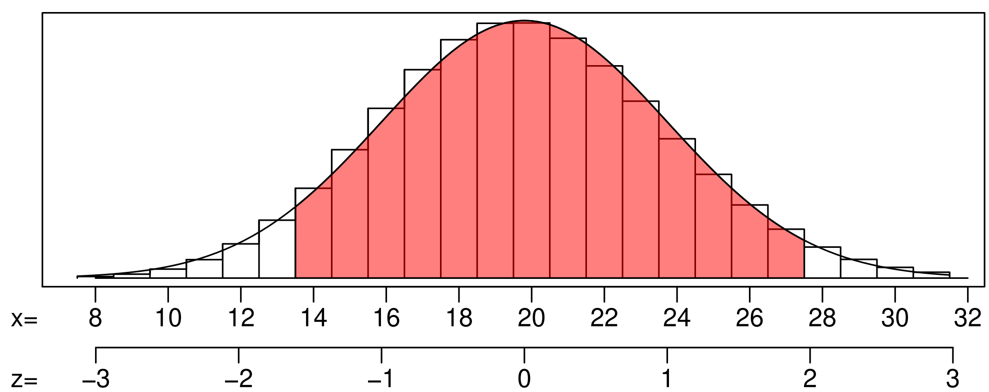
Find the mean.

$$\mu = np = (90)(0.22) = 19.8$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(90)(0.22)(1-0.22)} = 3.9299$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{13.5 - 19.8}{3.9299} = -1.48$$

$$z_2 = \frac{27.5 - 19.8}{3.9299} = 1.83$$

Calculate the probability.

$$P(14 \leq X \leq 27) = \Phi(1.83) - \Phi(-1.48) = 0.897$$

(a) $P(X = 18) = 0.094$

(b) $P(14 \leq X \leq 27) = 0.897$

5. As an ornithologist, you wish to determine the average body mass of *Vireo griseus*. You randomly sample 16 adults of *Vireo griseus*, resulting in a sample mean of 10.18 grams and a sample standard deviation of 0.729 grams. Determine a 99.5% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 16$$
$$\bar{x} = 10.18$$
$$s = 0.729$$
$$CL = 0.995$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 15$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.995$.

$$t^\star = 3.29$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{0.729}{\sqrt{16}} = 0.182$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (10.18 - 3.29 \times 0.182,\ 10.18 + 3.29 \times 0.182) \\ &= (9.58,\ 10.8) \end{aligned}$$

We are 99.5% confident that the population mean is between 9.58 and 10.8.

6. A treatment group of size 16 has a mean of 99.8 and standard deviation of 7.01. A control group of size 21 has a mean of 105 and standard deviation of 9.21. If you decided to use a signficance level of 0.1, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 34.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 16 \\ \bar{x}_1 &= 99.8 \\ s_1 &= 7.01 \\ n_2 &= 21 \\ \bar{x}_2 &= 105 \\ s_2 &= 9.21 \\ \alpha &= 0.1 \\ df &= 34 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$

$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.1$ by using a t table.

$$t^\star = 1.69$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(7.01)^2}{16} + \frac{(9.21)^2}{21}} \\ &= 2.667 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(105 - 99.8) - (0)}{2.667} \\ &= 1.95 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 1.69$

(d) $SE = 2.667$

(e) $|t_{obs}| = 1.95$

(f) $0.05 < p\text{-value} < 0.1$

(g) reject the null

7. From a very large population, a random sample of 360 individuals was taken. In that sample, 49.2% were sweet. Determine a 90% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.9$.

$$z^\star = 1.64$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.492)(1-0.492)}{360}} = 0.0263$$

Calculate the margin of error.

$$ME = z^\star SE = (1.64)(0.0263) = 0.0431$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.449\,,\ 0.535)$$

We are 90% confident that the true population proportion is between 44.9% and 53.5%.

(a) The lower bound = 0.449, which can also be expressed as 44.9%.

(b) The upper bound = 0.535, which can also be expressed as 53.5%.

8. An experiment is run with a treatment group of size 115 and a control group of size 92. The results are summarized in the table below.

	treatment	control
pink	91	57
not pink	24	35

Using a significance level of 0.01, determine whether the treatment causes an effect on the proportion of cases that are pink.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.01$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.58$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{91}{115} = 0.791$$

$$\hat{p}_2 = \frac{57}{92} = 0.62$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.62 - 0.791 = -0.171$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{91 + 57}{115 + 92} = 0.715$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.715)(0.285)}{115} + \frac{(0.715)(0.285)}{92}} \\ &= 0.0631 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0631)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.62 - 0.791) - 0}{0.0631} \\ &= -2.71 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.71) \\ &= 0.0068 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.58$

(d) $SE = 0.0631$

(e) $|z_{\text{obs}}| = 2.71$

(f) p-value = 0.0068

(g) reject the null

1. (a) $P($**tree given white**$) = 0.129$

   (b) $P($**white**$) = 0.26$

   (c) $P($**white given tree**$) = 0.157$

   (d) $P($**shovel or teal**$) = 0.487$

   (e) $P($**tree**$) = 0.213$

   (f) $P($**tree and white**$) = 0.0334$

2. $P($**"not cat" given "teal"**$) = 0.73$

3. $P(74 < X < 74.7) = 0.7567$

4. (a) $P(X = 28) = 0.0905$

   (b) $P(21 \leq X \leq 32) = 0.7656$

5. $\mathbf{(11.7, 12.3)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.62$

   (d) $SE = 8.061$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.79$

   (f) $0.01 <$ p-**value** < 0.02

   (g) **reject**

7. (a) **LB of p CI = 0.917 or** 91.7%

   (b) **UB of p CI = 0.921 or** 92.1%

8. (a) $H_0: \ p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \ p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.33$

(d) $SE = 0.097$

(e) $|\ z_{\mathbf{obs}}\ | = 2.46$

(f) p-**value** = 0.0138

(g) **reject**

1. In a deck of strange cards, there are 809 cards. Each card has an image and a color. The amounts are shown in the table below.

	green	teal	violet	white
bike	32	48	62	29
cat	76	59	58	83
shovel	15	18	86	71
tree	28	97	20	27

(a) What is the probability a random card is a tree given it is white?

(b) What is the probability a random card is white?

(c) What is the probability a random card is white given it is a tree?

(d) What is the probability a random card is either a shovel or teal (or both)?

(e) What is the probability a random card is a tree?

(f) What is the probability a random card is both a tree and white?

Solution

(a) $P(\text{tree given white}) = \frac{27}{29+83+71+27} = 0.129$

(b) $P(\text{white}) = \frac{29+83+71+27}{809} = 0.26$

(c) $P(\text{white given tree}) = \frac{27}{28+97+20+27} = 0.157$

(d) $P(\text{shovel or teal}) = \frac{15+18+86+71+48+59+18+97-18}{809} = 0.487$

(e) $P(\text{tree}) = \frac{28+97+20+27}{809} = 0.213$

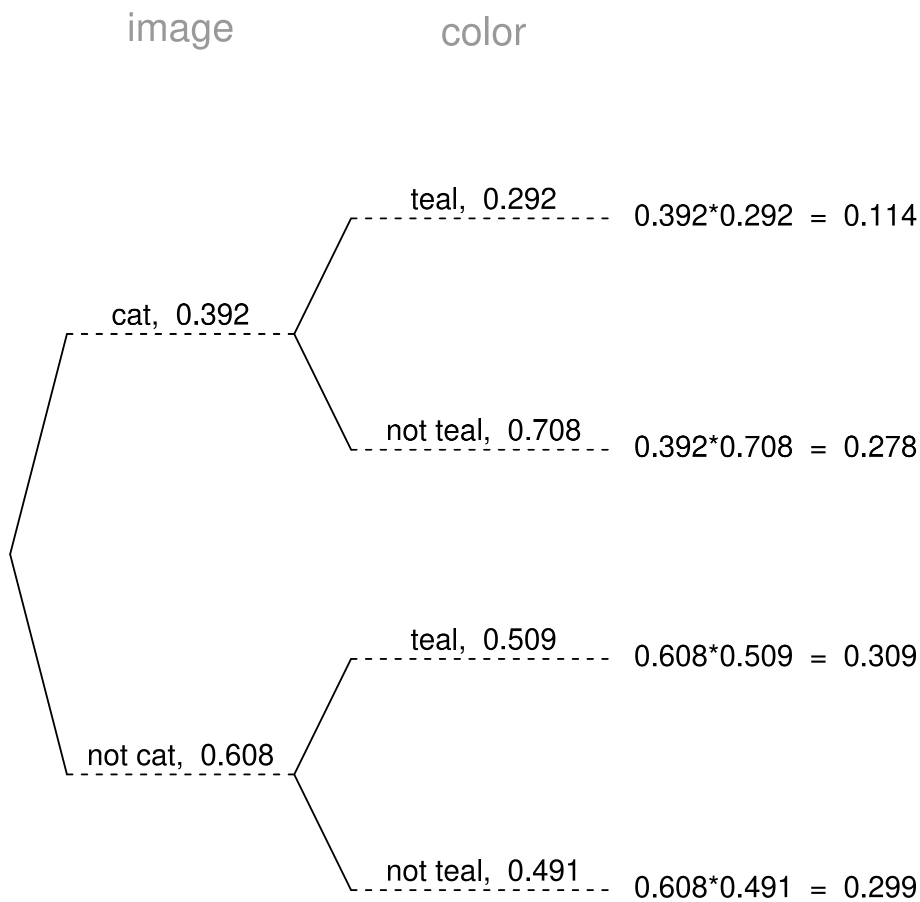
(f) $P(\text{tree and white}) = \frac{27}{809} = 0.0334$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a cat is 39.2%. If a cat is drawn, there is a 29.2% chance that it is teal. If a card that is not a cat is drawn, there is a 50.9% chance that it is teal.

   Now, someone draws a random card and reveals it is teal. What is the chance the card is not a cat?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not cat" given "teal"}) = \frac{0.309}{0.309 + 0.114} = 0.73$$

3. In a very large pile of toothpicks, the mean length is 74.2 millimeters and the standard deviation is 3.33 millimeters. If you randomly sample 169 toothpicks, what is the chance the sample mean is between 74 and 74.7 millimeters?

Solution

Label the given information.

$$\mu = 74.2$$
$$\sigma = 3.33$$
$$n = 169$$
$$\bar{x}_{\text{lower}} = 74$$
$$\bar{x}_{\text{upper}} = 74.7$$

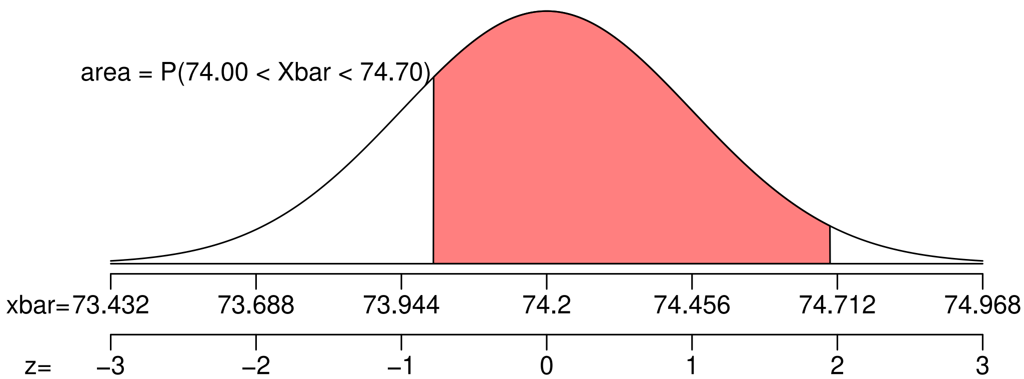
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.33}{\sqrt{169}} = 0.256$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(74.2,\ 0.256)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{74 - 74.2}{0.256} = -0.78$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{74.7 - 74.2}{0.256} = 1.95$$

Determine the probability.

$$\begin{aligned} P(74 < X < 74.7) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.95) - \Phi(-0.78) \\ &= 0.7567 \end{aligned}$$

4. In a game, there is a 31% chance to win a round. You will play 90 rounds.
   (a) What is the probability of winning exactly 28 rounds?
   (b) What is the probability of winning at least 21 but at most 32 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 28) = \binom{90}{28}(0.31)^{28}(1-0.31)^{90-28}$$

$$P(X = 28) = \binom{90}{28}(0.31)^{28}(0.69)^{62}$$

$$P(X = 28) = 0.0905$$

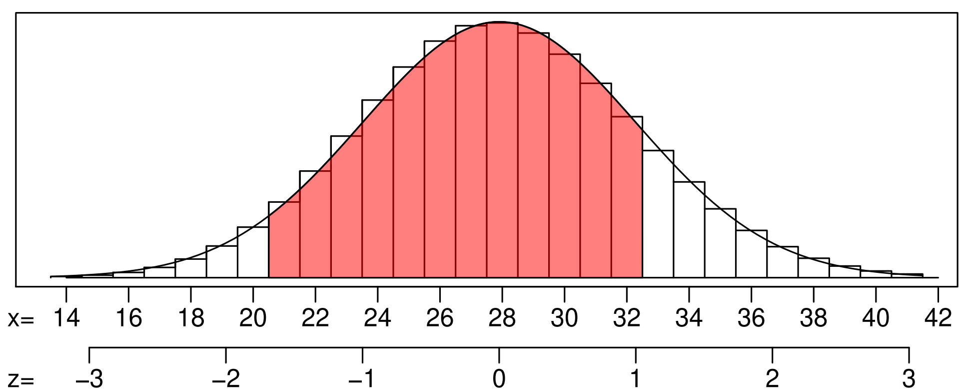
Find the mean.

$$\mu = np = (90)(0.31) = 27.9$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(90)(0.31)(1-0.31)} = 4.3876$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{20.5 - 27.9}{4.3876} = -1.57$$

$$z_2 = \frac{32.5 - 27.9}{4.3876} = 0.93$$

Calculate the probability.

$$P(21 \leq X \leq 32) = \Phi(0.93) - \Phi(-1.57) = 0.7656$$

(a) $P(X = 28) = 0.0905$

(b) $P(21 \leq X \leq 32) = 0.7656$

5. As an ornithologist, you wish to determine the average body mass of *Dendroica coronata*. You randomly sample 34 adults of *Dendroica coronata*, resulting in a sample mean of 11.97 grams and a sample standard deviation of 1.28 grams. Determine a 80% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 34 \\ \bar{x} &= 11.97 \\ s &= 1.28 \\ CL &= 0.8 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 33$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.8$.

$$t^{\star} = 1.31$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.28}{\sqrt{34}} = 0.22$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^{\star} SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^{\star} SE,\ \bar{x} + t^{\star} SE) \\ &= (11.97 - 1.31 \times 0.22,\ 11.97 + 1.31 \times 0.22) \\ &= (11.7,\ 12.3) \end{aligned}$$

We are 80% confident that the population mean is between 11.7 and 12.3.

6. A treatment group of size 10 has a mean of 93.5 and standard deviation of 22.3. A control group of size 21 has a mean of 116 and standard deviation of 17.9. If you decided to use a signficance level of 0.02, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 14.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 10$$
$$\bar{x}_1 = 93.5$$
$$s_1 = 22.3$$
$$n_2 = 21$$
$$\bar{x}_2 = 116$$
$$s_2 = 17.9$$
$$\alpha = 0.02$$
$$df = 14$$

State the hypotheses.

$$H_0 : \quad \mu_2 - \mu_1 = 0$$
$$H_A : \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.02$ by using a t table.

$$t^\star = 2.62$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(22.3)^2}{10} + \frac{(17.9)^2}{21}} \\ &= 8.061 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(116 - 93.5) - (0)}{8.061} \\ &= 2.79 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.62$

(d) $SE = 8.061$

(e) $|t_{obs}| = 2.79$

(f) $0.01 < p\text{-value} < 0.02$

(g) reject the null

7. From a very large population, a random sample of 90000 individuals was taken. In that sample, 91.9% were messy. Determine a 95% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.95$.

$$z^{\star} = 1.96$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.919)(1-0.919)}{90000}} = 0.000909$$

Calculate the margin of error.

$$ME = z^{\star}SE = (1.96)(0.000909) = 0.00178$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.917\,,\ 0.921)$$

We are 95% confident that the true population proportion is between 91.7% and 92.1%.

(a) The lower bound = 0.917, which can also be expressed as 91.7%.

(b) The upper bound = 0.921, which can also be expressed as 92.1%.

8. An experiment is run with a treatment group of size 62 and a control group of size 46. The results are summarized in the table below.

	treatment	control
happy	27	31
not happy	35	15

Using a significance level of 0.02, determine whether the treatment causes an effect on the proportion of cases that are happy.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.02$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.33$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{27}{62} = 0.435$$

$$\hat{p}_2 = \frac{31}{46} = 0.674$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.674 - 0.435 = 0.239$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{27 + 31}{62 + 46} = 0.537$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.537)(0.463)}{62} + \frac{(0.537)(0.463)}{46}} \\ &= 0.097 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\, 0.097)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.674 - 0.435) - 0}{0.097} \\ &= 2.46 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.46) \\ &= 0.0138 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.33$

(d) $SE = 0.097$

(e) $|z_{\text{obs}}| = 2.46$

(f) p-value = 0.0138

(g) reject the null

1. (a) P(**cat or white**) = 0.456

   (b) P(**white**) = 0.418

   (c) P(**tree given white**) = 0.168

   (d) P(**red given dog**) = 0.216

   (e) P(**dog and white**) = 0.105

   (f) P(**bike**) = 0.272

2. P(**"ring" given "violet"**) = 0.706

3. $P(67.06 < X < 67.88) = 0.8396$

4. (a) $P(X = 37) = 0.0718$

   (b) $P(37 \leq X \leq 50) = 0.4721$

5. **(11.2, 12.8)**

6. (a) $H_0 : \ \mu_2 - \mu_1 = 0$

   (b) $H_0 : \ \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 1.67$

   (d) $SE = 0.526$

   (e) $\mid t_{\textbf{obs}} \mid = 1.79$

   (f) $0.05 <$ p**-value** < 0.1

   (g) **reject**

7. (a) **LB of p CI = 0.0795 or** 7.95%

   (b) **UB of p CI = 0.0927 or** 9.27%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.81$

(d) $SE = 0.115$

(e) $\mid z_{\mathbf{obs}} \mid = 2.74$

(f) p-**value** = 0.0062

(g) **retain**

1. In a deck of strange cards, there are 895 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	red	white
bike	87	71	85
cat	22	12	56
dog	58	42	94
flower	68	70	76
tree	81	10	63

(a) What is the probability a random card is either a cat or white (or both)?

(b) What is the probability a random card is white?

(c) What is the probability a random card is a tree given it is white?

(d) What is the probability a random card is red given it is a dog?

(e) What is the probability a random card is both a dog and white?

(f) What is the probability a random card is a bike?

Solution

(a) $P(\text{cat or white}) = \frac{22+12+56+85+56+94+76+63-56}{895} = 0.456$

(b) $P(\text{white}) = \frac{85+56+94+76+63}{895} = 0.418$

(c) $P(\text{tree given white}) = \frac{63}{85+56+94+76+63} = 0.168$

(d) $P(\text{red given dog}) = \frac{42}{58+42+94} = 0.216$

(e) $P(\text{dog and white}) = \frac{94}{895} = 0.105$

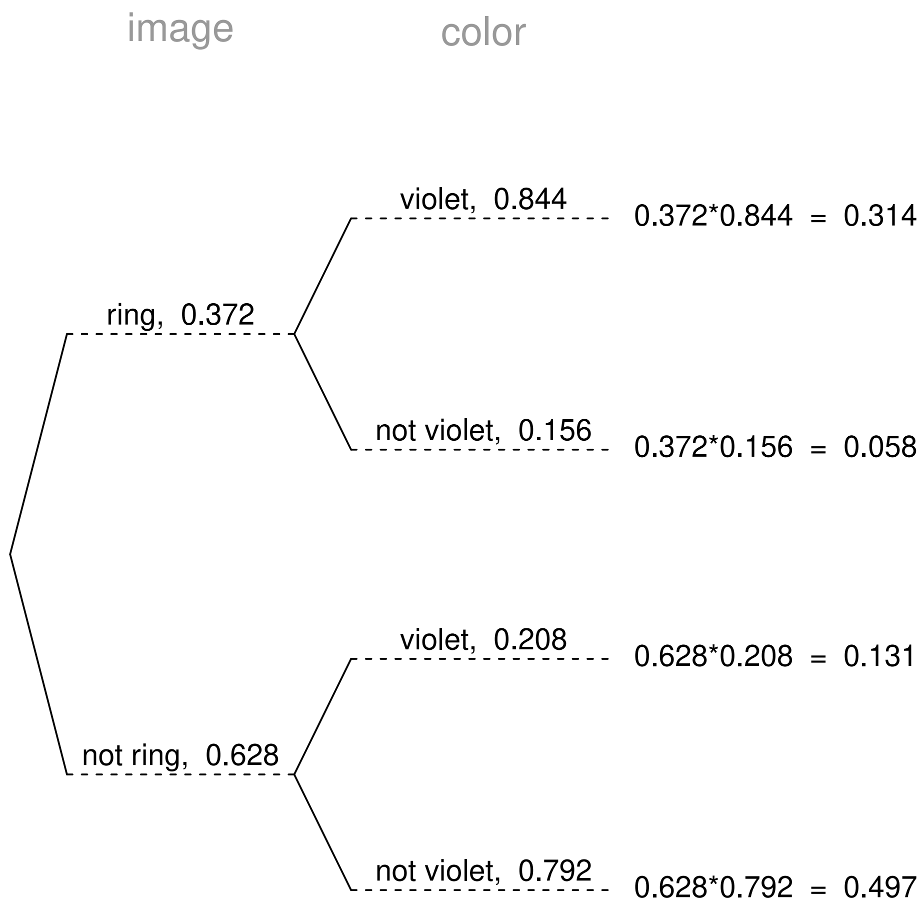
(f) $P(\text{bike}) = \frac{87+71+85}{895} = 0.272$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a ring is 37.2%. If a ring is drawn, there is a 84.4% chance that it is violet. If a card that is not a ring is drawn, there is a 20.8% chance that it is violet.

   Now, someone draws a random card and reveals it is violet. What is the chance the card is a ring?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"ring" given "violet"}) = \frac{0.314}{0.314 + 0.131} = 0.706$$

3. In a very large pile of toothpicks, the mean length is 67.52 millimeters and the standard deviation is 3.17 millimeters. If you randomly sample 121 toothpicks, what is the chance the sample mean is between 67.06 and 67.88 millimeters?

Solution

Label the given information.

$$\mu = 67.52$$
$$\sigma = 3.17$$
$$n = 121$$
$$\bar{x}_{\text{lower}} = 67.06$$
$$\bar{x}_{\text{upper}} = 67.88$$

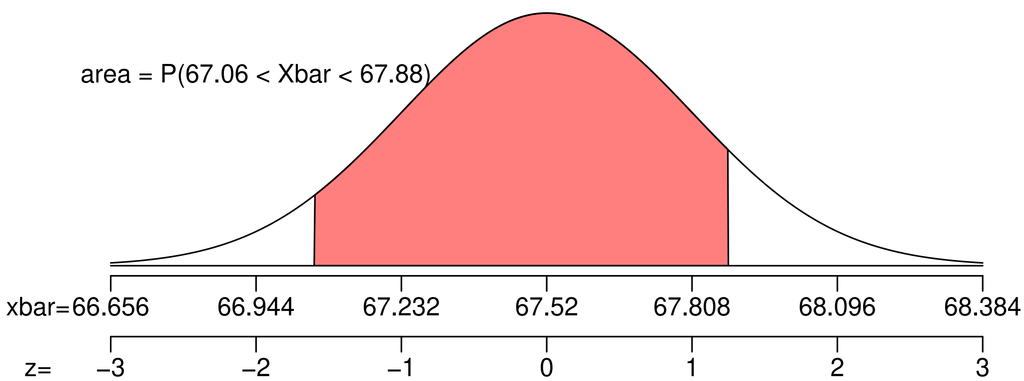
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.17}{\sqrt{121}} = 0.288$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(67.52,\ 0.288)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{67.06 - 67.52}{0.288} = -1.6$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{67.88 - 67.52}{0.288} = 1.25$$

Determine the probability.

$$\begin{aligned} P(67.06 < X < 67.88) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.25) - \Phi(-1.6) \\ &= 0.8396 \end{aligned}$$

4. In a game, there is a 17% chance to win a round. You will play 216 rounds.
   (a) What is the probability of winning exactly 37 rounds?
   (b) What is the probability of winning at least 37 but at most 50 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 37) = \binom{216}{37}(0.17)^{37}(1-0.17)^{216-37}$$

$$P(X = 37) = \binom{216}{37}(0.17)^{37}(0.83)^{179}$$

$$P(X = 37) = 0.0718$$

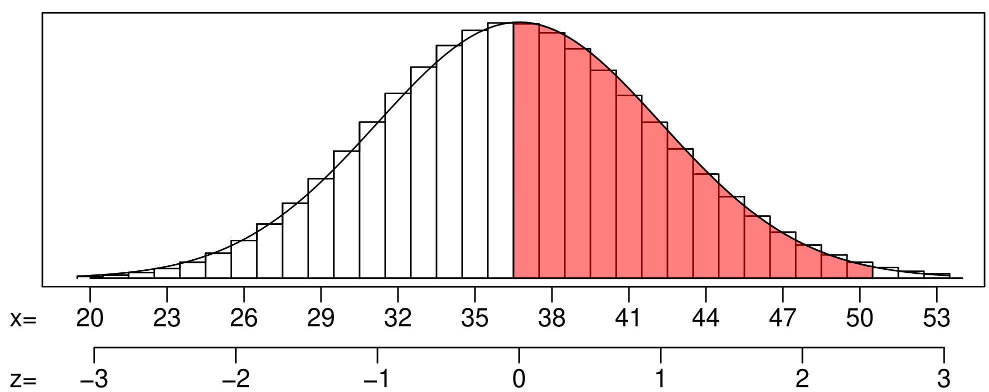
Find the mean.

$$\mu = np = (216)(0.17) = 36.72$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(216)(0.17)(1-0.17)} = 5.5207$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{36.5 - 36.72}{5.5207} = 0.05$$

$$z_2 = \frac{50.5 - 36.72}{5.5207} = 2.41$$

Calculate the probability.

$$P(37 \leq X \leq 50) = \Phi(2.41) - \Phi(0.05) = 0.4721$$

(a) $P(X = 37) = 0.0718$

(b) $P(37 \leq X \leq 50) = 0.4721$

5. As an ornithologist, you wish to determine the average body mass of *Dendroica coronata*. You randomly sample 24 adults of *Dendroica coronata*, resulting in a sample mean of 11.99 grams and a sample standard deviation of 1.25 grams. Determine a 99.5% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 24 \\ \bar{x} &= 11.99 \\ s &= 1.25 \\ CL &= 0.995 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 23$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.995$.

$$t^\star = 3.1$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.25}{\sqrt{24}} = 0.255$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE, \ \bar{x} + t^\star SE) \\ &= (11.99 - 3.1 \times 0.255, \ 11.99 + 3.1 \times 0.255) \\ &= (11.2, \ 12.8) \end{aligned}$$

We are 99.5% confident that the population mean is between 11.2 and 12.8.

6. A treatment group of size 37 has a mean of 9.96 and standard deviation of 2.22. A control group of size 26 has a mean of 10.9 and standard deviation of 1.93. If you decided to use a signficance level of 0.1, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 58.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 37$$
$$\bar{x}_1 = 9.96$$
$$s_1 = 2.22$$
$$n_2 = 26$$
$$\bar{x}_2 = 10.9$$
$$s_2 = 1.93$$
$$\alpha = 0.1$$
$$df = 58$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.1$ by using a t table.

$$t^\star = 1.67$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(2.22)^2}{37} + \frac{(1.93)^2}{26}} \\ &= 0.526 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(10.9 - 9.96) - (0)}{0.526} \\ &= 1.79 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 1.67$

(d) $SE = 0.526$

(e) $|t_{\text{obs}}| = 1.79$

(f) $0.05 < p\text{-value} < 0.1$

(g) reject the null

7. From a very large population, a random sample of 7600 individuals was taken. In that sample, 8.61% were glowing. Determine a 96% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.96$.

$$z^\star = 2.05$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.0861)(1-0.0861)}{7600}} = 0.00322$$

Calculate the margin of error.

$$ME = z^\star SE = (2.05)(0.00322) = 0.0066$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.0795\,,\ 0.0927)$$

We are 96% confident that the true population proportion is between 7.95% and 9.27%.

(a) The lower bound = 0.0795, which can also be expressed as 7.95%.

(b) The upper bound = 0.0927, which can also be expressed as 9.27%.

8. An experiment is run with a treatment group of size 23 and a control group of size 43. The results are summarized in the table below.

	treatment	control
special	11	7
not special	12	36

Using a significance level of 0.005, determine whether the treatment causes an effect on the proportion of cases that are special.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.005$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.81$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{11}{23} = 0.478$$
$$\hat{p}_2 = \frac{7}{43} = 0.163$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.163 - 0.478 = -0.315$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{11 + 7}{23 + 43} = 0.273$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.273)(0.727)}{23} + \frac{(0.273)(0.727)}{43}} \\ &= 0.115 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.115)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.163 - 0.478) - 0}{0.115} \\ &= -2.74 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.74) \\ &= 0.0062 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} > \alpha$$

So, we retain the null hypothesis. Thus the difference in proportions is not significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.81$

(d) $SE = 0.115$

(e) $|z_{\text{obs}}| = 2.74$

(f) p-value = 0.0062

(g) retain the null

1. (a) $P(\textbf{teal}) = 0.205$

   (b) $P(\textbf{horn given red}) = 0.241$

   (c) $P(\textbf{bike or black}) = 0.363$

   (d) $P(\textbf{teal given wheel}) = 0.165$

   (e) $P(\textbf{bike}) = 0.24$

   (f) $P(\textbf{bike and teal}) = 0.0685$

2. $P(\textbf{"gem" given "black"}) = 0.265$

3. $P(72.47 < X < 72.81) = 0.7557$

4. (a) $P(X = 20) = 0.1113$

   (b) $P(24 \leq X \leq 29) = 0.2023$

5. $\mathbf{(9.35, 10.4)}$

6. (a) $H_0: \ \mu_2 - \mu_1 = 0$

   (b) $H_0: \ \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.11$

   (d) $SE = 0.009$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.21$

   (f) $0.02 < p\textbf{-value} < 0.04$

   (g) **reject**

7. (a) **LB of p CI = 0.522 or** 52.2%

   (b) **UB of p CI = 0.548 or** 54.8%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.64$

(d) $SE = 0.042$

(e) $\mid z_{\mathbf{obs}} \mid = 1.76$

(f) p-**value** = 0.0784

(g) **reject**

1. In a deck of strange cards, there are 1271 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	green	pink	red	teal
bike	34	85	68	31	87
dog	71	24	47	19	32
flower	20	69	55	88	35
horn	14	26	65	72	63
wheel	51	40	42	89	44

(a) What is the probability a random card is teal?

(b) What is the probability a random card is a horn given it is red?

(c) What is the probability a random card is either a bike or black (or both)?

(d) What is the probability a random card is teal given it is a wheel?

(e) What is the probability a random card is a bike?

(f) What is the probability a random card is both a bike and teal?

Solution

(a) $P(\text{teal}) = \frac{87+32+35+63+44}{1271} = 0.205$

(b) $P(\text{horn given red}) = \frac{72}{31+19+88+72+89} = 0.241$

(c) $P(\text{bike or black}) = \frac{34+85+68+31+87+34+71+20+14+51-34}{1271} = 0.363$

(d) $P(\text{teal given wheel}) = \frac{44}{51+40+42+89+44} = 0.165$

(e) $P(\text{bike}) = \frac{34+85+68+31+87}{1271} = 0.24$

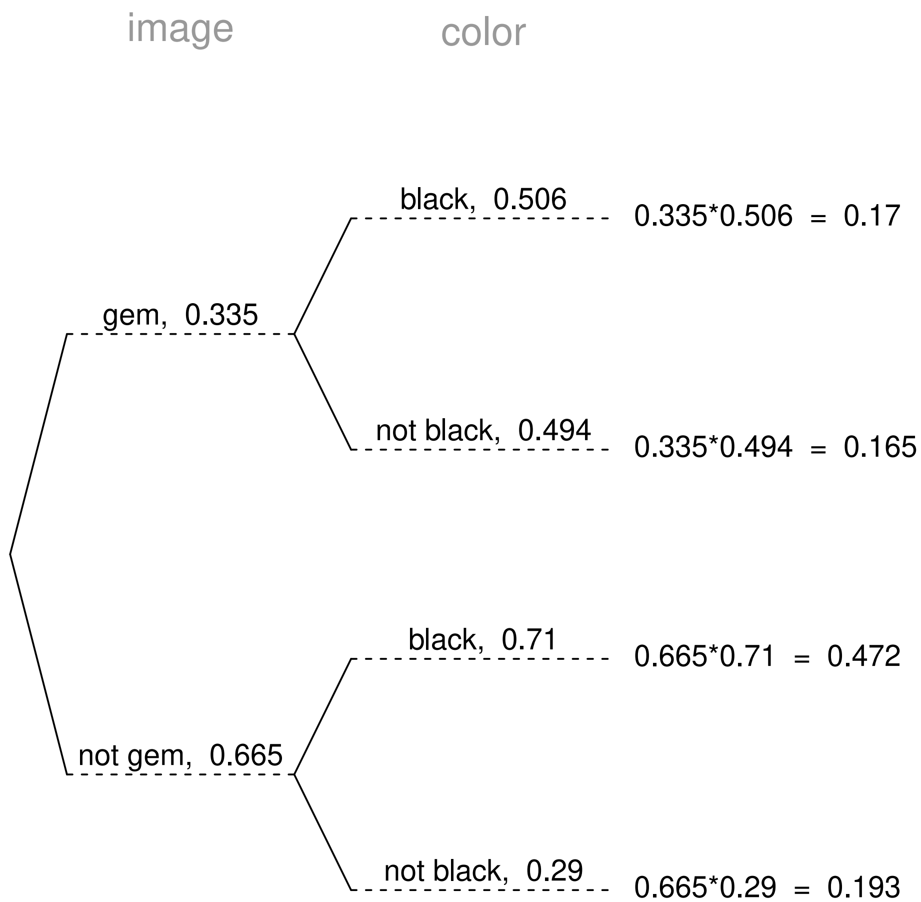
(f) $P(\text{bike and teal}) = \frac{87}{1271} = 0.0685$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a gem is 33.5%. If a gem is drawn, there is a 50.6% chance that it is black. If a card that is not a gem is drawn, there is a 71% chance that it is black.

   Now, someone draws a random card and reveals it is black. What is the chance the card is a gem?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"gem" given "black"}) = \frac{0.17}{0.17 + 0.472} = 0.265$$

3. In a very large pile of toothpicks, the mean length is 72.61 millimeters and the standard deviation is 2.14 millimeters. If you randomly sample 225 toothpicks, what is the chance the sample mean is between 72.47 and 72.81 millimeters?

Solution

Label the given information.

$$\mu = 72.61$$
$$\sigma = 2.14$$
$$n = 225$$
$$\bar{x}_{\text{lower}} = 72.47$$
$$\bar{x}_{\text{upper}} = 72.81$$

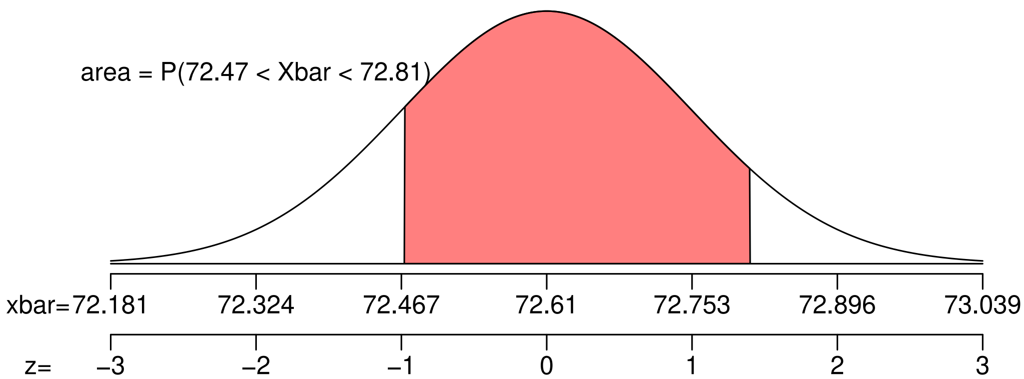
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2.14}{\sqrt{225}} = 0.143$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(72.61,\ 0.143)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{72.47 - 72.61}{0.143} = -0.98$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{72.81 - 72.61}{0.143} = 1.4$$

Determine the probability.

$$\begin{aligned} P(72.47 < X < 72.81) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.4) - \Phi(-0.98) \\ &= 0.7557 \end{aligned}$$

4. In a game, there is a 51% chance to win a round. You will play 42 rounds.
   (a) What is the probability of winning exactly 20 rounds?
   (b) What is the probability of winning at least 24 but at most 29 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 20) = \binom{42}{20}(0.51)^{20}(1-0.51)^{42-20}$$

$$P(X = 20) = \binom{42}{20}(0.51)^{20}(0.49)^{22}$$

$$P(X = 20) = 0.1113$$

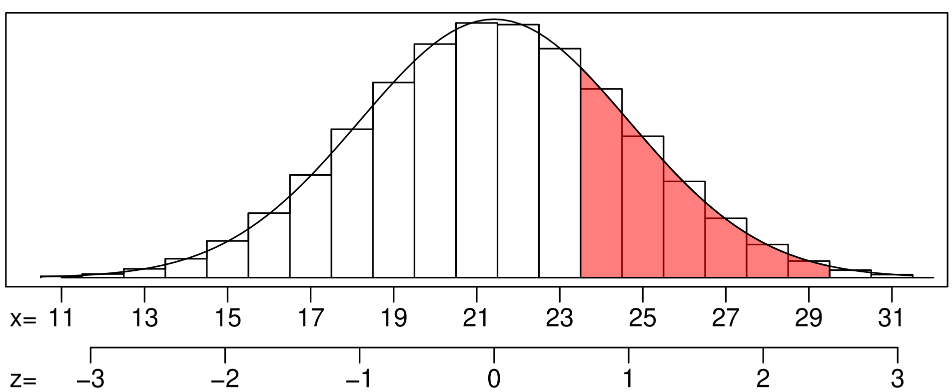
Find the mean.

$$\mu = np = (42)(0.51) = 21.42$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(42)(0.51)(1-0.51)} = 3.2397$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{23.5 - 21.42}{3.2397} = 0.8$$

$$z_2 = \frac{29.5 - 21.42}{3.2397} = 2.34$$

Calculate the probability.

$$P(24 \le X \le 29) = \Phi(2.34) - \Phi(0.8) = 0.2023$$

(a) $P(X = 20) = 0.1113$

(b) $P(24 \le X \le 29) = 0.2023$

5. As an ornithologist, you wish to determine the average body mass of *Dendroica dominica*. You randomly sample 27 adults of *Dendroica dominica*, resulting in a sample mean of 9.87 grams and a sample standard deviation of 1.59 grams. Determine a 90% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 27 \\ \bar{x} &= 9.87 \\ s &= 1.59 \\ CL &= 0.9 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 26$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.9$.

$$t^{\star} = 1.71$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.59}{\sqrt{27}} = 0.306$$

We want to make an inference about the population mean.

$$\mu \;\approx\; \bar{x} \pm t^{\star}SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^{\star}SE,\ \bar{x} + t^{\star}SE) \\ &= (9.87 - 1.71 \times 0.306,\ 9.87 + 1.71 \times 0.306) \\ &= (9.35,\ 10.4) \end{aligned}$$

We are 90% confident that the population mean is between 9.35 and 10.4.

6. A treatment group of size 26 has a mean of 1 and standard deviation of 0.0315. A control group of size 29 has a mean of 1.02 and standard deviation of 0.0356. If you decided to use a signficance level of 0.04, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 52.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 26 \\ \bar{x}_1 &= 1 \\ s_1 &= 0.0315 \\ n_2 &= 29 \\ \bar{x}_2 &= 1.02 \\ s_2 &= 0.0356 \\ \alpha &= 0.04 \\ df &= 52 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.04$ by using a t table.

$$t^\star = 2.11$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.0315)^2}{26} + \frac{(0.0356)^2}{29}} \\ &= 0.009 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1.02 - 1) - (0)}{0.009} \\ &= 2.21 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.02 < p\text{-value} < 0.04$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.11$

(d) $SE = 0.009$

(e) $|t_{obs}| = 2.21$

(f) $0.02 < p\text{-value} < 0.04$

(g) reject the null

7. From a very large population, a random sample of 4000 individuals was taken. In that sample, 53.5% were broken. Determine a 90% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.9$.

$$z^\star = 1.64$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.535)(1-0.535)}{4000}} = 0.00789$$

Calculate the margin of error.

$$ME = z^\star SE = (1.64)(0.00789) = 0.0129$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.522\,,\ 0.548)$$

We are 90% confident that the true population proportion is between 52.2% and 54.8%.

(a) The lower bound = 0.522, which can also be expressed as 52.2%.

(b) The upper bound = 0.548, which can also be expressed as 54.8%.

8. An experiment is run with a treatment group of size 296 and a control group of size 284. The results are summarized in the table below.

	treatment	control
fluorescent	151	124
not fluorescent	145	160

Using a significance level of 0.1, determine whether the treatment causes an effect on the proportion of cases that are fluorescent.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.1$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.64$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{151}{296} = 0.51$$
$$\hat{p}_2 = \frac{124}{284} = 0.437$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.437 - 0.51 = -0.073$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{151 + 124}{296 + 284} = 0.474$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.474)(0.526)}{296} + \frac{(0.474)(0.526)}{284}} \\ &= 0.0415 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0415)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.437 - 0.51) - 0}{0.0415} \\ &= -1.76 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-1.76) \\ &= 0.0784 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.64$

(d) $SE = 0.0415$

(e) $|z_{\text{obs}}| = 1.76$

(f) p-value = 0.0784

(g) reject the null

1. (a) $P($**yellow given cat**$) = 0.439$

   (b) $P($**wheel**$) = 0.281$

   (c) $P($**cat given blue**$) = 0.408$

   (d) $P($**wheel or yellow**$) = 0.603$

   (e) $P($**cat and teal**$) = 0.209$

   (f) $P($**teal**$) = 0.495$

2. $P($**"not tree" given "green"**$) = 0.802$

3. $P(63.31 < X < 63.96) = 0.878$

4. (a) $P(X = 17) = 0.1012$

   (b) $P(9 \leq X \leq 16) = 0.6524$

5. **(52.1, 59.2)**

6. (a) $H_0 : \ \mu_2 - \mu_1 = 0$

   (b) $H_0 : \ \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.01$

   (d) $SE = 0.023$

   (e) $| \ t_{\mathbf{obs}} \ | = 1.93$

   (f) $0.05 <$ p**-value** < 0.1

   (g) **retain**

7. (a) **LB of p CI = 0.852 or** 85.2%

   (b) **UB of p CI = 0.86 or** 86%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.96$

(d) $SE = 0.07$

(e) $\mid z_{\mathbf{obs}} \mid = 2$

(f) p-**value** = 0.0456

(g) **reject**

1. In a deck of strange cards, there are 459 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	teal	yellow
cat	29	96	98
dog	16	41	50
wheel	26	90	13

(a) What is the probability a random card is yellow given it is a cat?

(b) What is the probability a random card is a wheel?

(c) What is the probability a random card is a cat given it is blue?

(d) What is the probability a random card is either a wheel or yellow (or both)?

(e) What is the probability a random card is both a cat and teal?

(f) What is the probability a random card is teal?

Solution

(a) $P(\text{yellow given cat}) = \frac{98}{29+96+98} = 0.439$

(b) $P(\text{wheel}) = \frac{26+90+13}{459} = 0.281$

(c) $P(\text{cat given blue}) = \frac{29}{29+16+26} = 0.408$

(d) $P(\text{wheel or yellow}) = \frac{26+90+13+98+50+13-13}{459} = 0.603$

(e) $P(\text{cat and teal}) = \frac{96}{459} = 0.209$

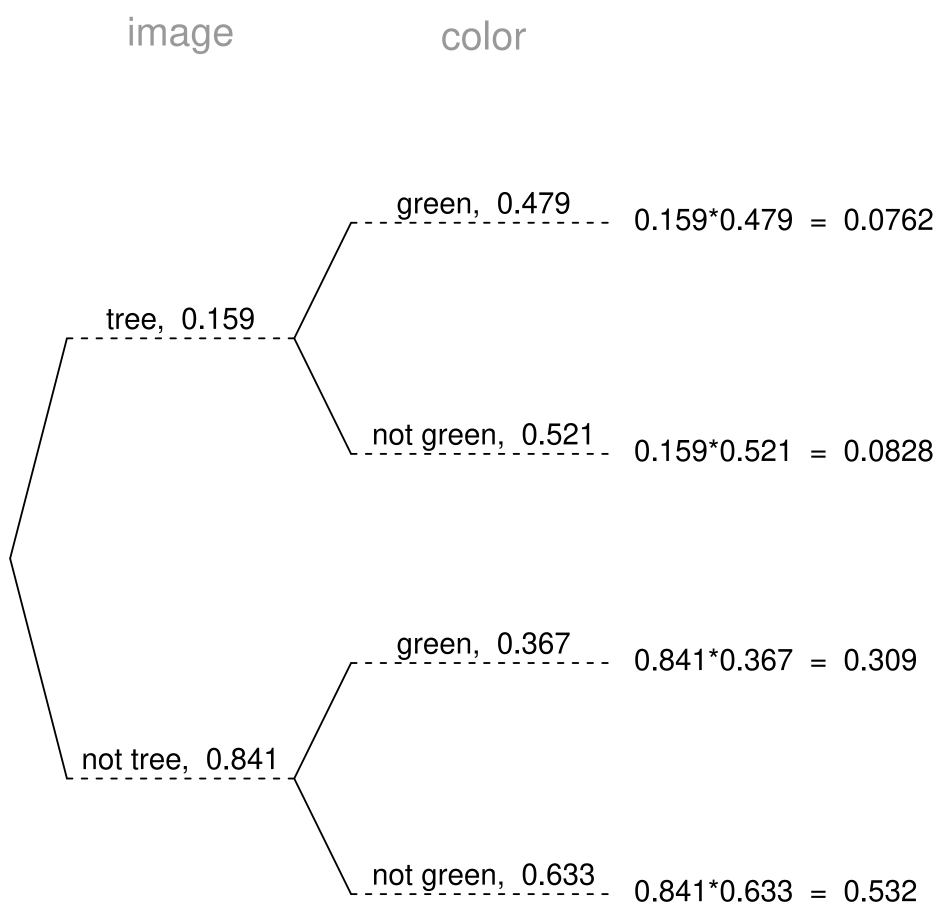
(f) $P(\text{teal}) = \frac{96+41+90}{459} = 0.495$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a tree is 15.9%. If a tree is drawn, there is a 47.9% chance that it is green. If a card that is not a tree is drawn, there is a 36.7% chance that it is green.

   Now, someone draws a random card and reveals it is green. What is the chance the card is not a tree?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not tree" given "green"}) = \frac{0.309}{0.309 + 0.0762} = 0.802$$

3. In a very large pile of toothpicks, the mean length is 63.65 millimeters and the standard deviation is 2.3 millimeters. If you randomly sample 120 toothpicks, what is the chance the sample mean is between 63.31 and 63.96 millimeters?

Solution

Label the given information.

$$\mu = 63.65$$
$$\sigma = 2.3$$
$$n = 120$$
$$\bar{x}_{\text{lower}} = 63.31$$
$$\bar{x}_{\text{upper}} = 63.96$$

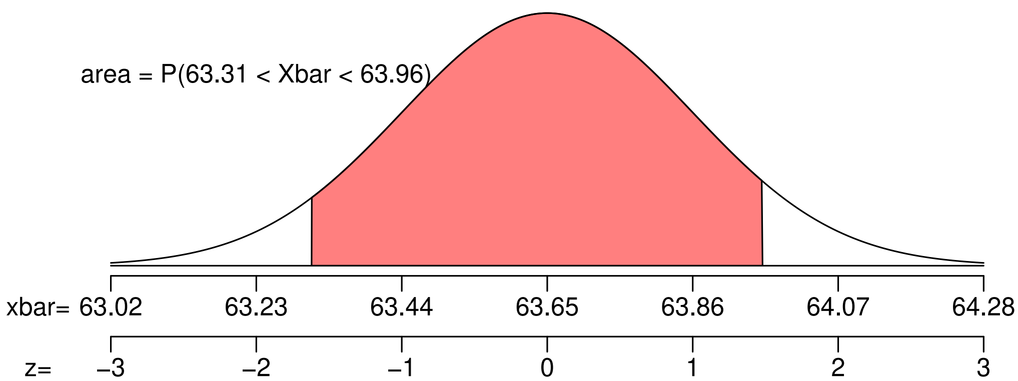
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2.3}{\sqrt{120}} = 0.21$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(63.65,\ 0.21)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{63.31 - 63.65}{0.21} = -1.62$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{63.96 - 63.65}{0.21} = 1.48$$

Determine the probability.

$$\begin{aligned} P(63.31 < X < 63.96) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.48) - \Phi(-1.62) \\ &= 0.878 \end{aligned}$$

4. In a game, there is a 46% chance to win a round. You will play 32 rounds.

   (a) What is the probability of winning exactly 17 rounds?

   (b) What is the probability of winning at least 9 but at most 16 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X=k)=\binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X=17)=\binom{32}{17}(0.46)^{17}(1-0.46)^{32-17}$$

$$P(X=17)=\binom{32}{17}(0.46)^{17}(0.54)^{15}$$

$$P(X=17)=0.1012$$

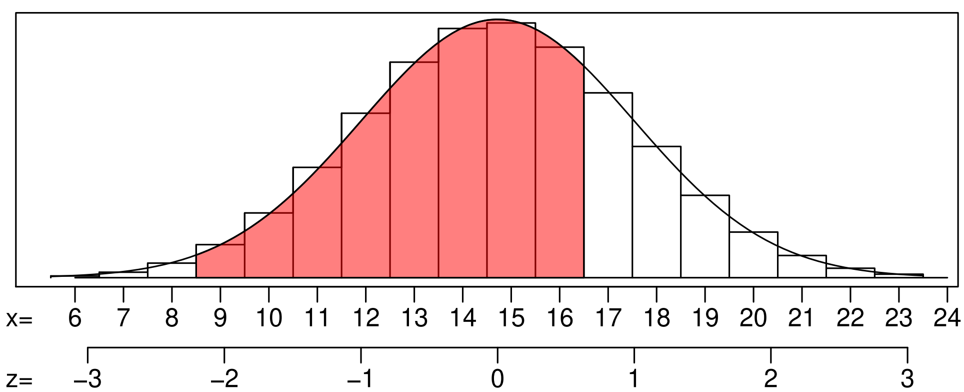
Find the mean.

$$\mu=np=(32)(0.46)=14.72$$

Find the standard deviation.

$$\sigma=\sqrt{np(1-p)}=\sqrt{(32)(0.46)(1-0.46)}=2.8194$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1=\frac{8.5-14.72}{2.8194}=-2.03$$

$$z_2=\frac{16.5-14.72}{2.8194}=0.45$$

Calculate the probability.

$$P(9\le X\le 16)=\Phi(0.45)-\Phi(-2.03)=0.6524$$

(a) $P(X=17)=0.1012$

(b) $P(9\le X\le 16)=0.6524$

5. As an ornithologist, you wish to determine the average body mass of *Hylocichla mustelina*. You randomly sample 35 adults of *Hylocichla mustelina*, resulting in a sample mean of 55.64 grams and a sample standard deviation of 7.77 grams. Determine a 99% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 35 \\ \bar{x} &= 55.64 \\ s &= 7.77 \\ CL &= 0.99 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 34$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.99$.

$$t^{\star} = 2.73$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{7.77}{\sqrt{35}} = 1.31$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^{\star} SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^{\star} SE,\ \bar{x} + t^{\star} SE) \\ &= (55.64 - 2.73 \times 1.31,\ 55.64 + 2.73 \times 1.31) \\ &= (52.1,\ 59.2) \end{aligned}$$

We are 99% confident that the population mean is between 52.1 and 59.2.

6. A treatment group of size 35 has a mean of 0.996 and standard deviation of 0.0738. A control group of size 28 has a mean of 1.04 and standard deviation of 0.101. If you decided to use a signficance level of 0.05, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 48.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 35 \\ \bar{x}_1 &= 0.996 \\ s_1 &= 0.0738 \\ n_2 &= 28 \\ \bar{x}_2 &= 1.04 \\ s_2 &= 0.101 \\ \alpha &= 0.05 \\ df &= 48 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$

$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.05$ by using a t table.

$$t^\star = 2.01$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.0738)^2}{35} + \frac{(0.101)^2}{28}} \\ &= 0.023 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1.04 - 0.996) - (0)}{0.023} \\ &= 1.93 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.01$

(d) $SE = 0.023$

(e) $|t_{\text{obs}}| = 1.93$

(f) $0.05 < p\text{-value} < 0.1$

(g) retain the null

7. From a very large population, a random sample of 49000 individuals was taken. In that sample, 85.6% were glowing. Determine a 99.5% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.995$.

$$z^\star = 2.81$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.856)(1-0.856)}{49000}} = 0.00159$$

Calculate the margin of error.

$$ME = z^\star SE = (2.81)(0.00159) = 0.00447$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.852\,,\ 0.86)$$

We are 99.5% confident that the true population proportion is between 85.2% and 86%.

(a) The lower bound = 0.852, which can also be expressed as 85.2%.

(b) The upper bound = 0.86, which can also be expressed as 86%.

8. An experiment is run with a treatment group of size 73 and a control group of size 109. The results are summarized in the table below.

	treatment	control
angry	29	28
not angry	44	81

Using a significance level of 0.05, determine whether the treatment causes an effect on the proportion of cases that are angry.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.05$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.96$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{29}{73} = 0.397$$
$$\hat{p}_2 = \frac{28}{109} = 0.257$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.257 - 0.397 = -0.14$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{29 + 28}{73 + 109} = 0.313$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.313)(0.687)}{73} + \frac{(0.313)(0.687)}{109}} \\ &= 0.0701 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0701)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.257 - 0.397) - 0}{0.0701} \\ &= -2 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2) \\ &= 0.0456 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.96$

(d) $SE = 0.0701$

(e) $|z_{\text{obs}}| = 2$

(f) p-value = 0.0456

(g) reject the null

1. (a) P(**gem or orange**) = 0.405

   (b) P(**gem**) = 0.262

   (c) P(**teal**) = 0.378

   (d) P(**wheel and gray**) = 0.219

   (e) P(**gem given gray**) = 0.133

   (f) P(**teal given wheel**) = 0.184

2. P(**"not dog" given "not white"**) = 0.31

3. $P(60.24 < X < 60.69) = 0.895$

4. (a) $P(X = 73) = 0.0575$

   (b) $P(66 \leq X \leq 81) = 0.7044$

5. **(20.6, 23.1)**

6. (a) $H_0 : \ \mu_2 - \mu_1 = 0$

   (b) $H_0 : \ \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.38$

   (d) $SE = 0.05$

   (e) $|\ t_{\mathbf{obs}}\ | = 2.42$

   (f) $0.01 <$ p-**value** < 0.02

   (g) **reject**

7. (a) **LB of p CI = 0.44 or** 44%

   (b) **UB of p CI = 0.514 or** 51.4%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.96$

(d) $SE = 0.042$

(e) $\mid z_{\mathbf{obs}} \mid = 2.02$

(f) p-**value** $= 0.0434$

(g) **reject**

1. In a deck of strange cards, there are 447 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	orange	teal
gem	22	49	46
horn	45	16	90
wheel	98	48	33

(a) What is the probability a random card is either a gem or orange (or both)?

(b) What is the probability a random card is a gem?

(c) What is the probability a random card is teal?

(d) What is the probability a random card is both a wheel and gray?

(e) What is the probability a random card is a gem given it is gray?

(f) What is the probability a random card is teal given it is a wheel?

Solution

(a) $P(\text{gem or orange}) = \frac{22+49+46+49+16+48-49}{447} = 0.405$

(b) $P(\text{gem}) = \frac{22+49+46}{447} = 0.262$

(c) $P(\text{teal}) = \frac{46+90+33}{447} = 0.378$

(d) $P(\text{wheel and gray}) = \frac{98}{447} = 0.219$

(e) $P(\text{gem given gray}) = \frac{22}{22+45+98} = 0.133$

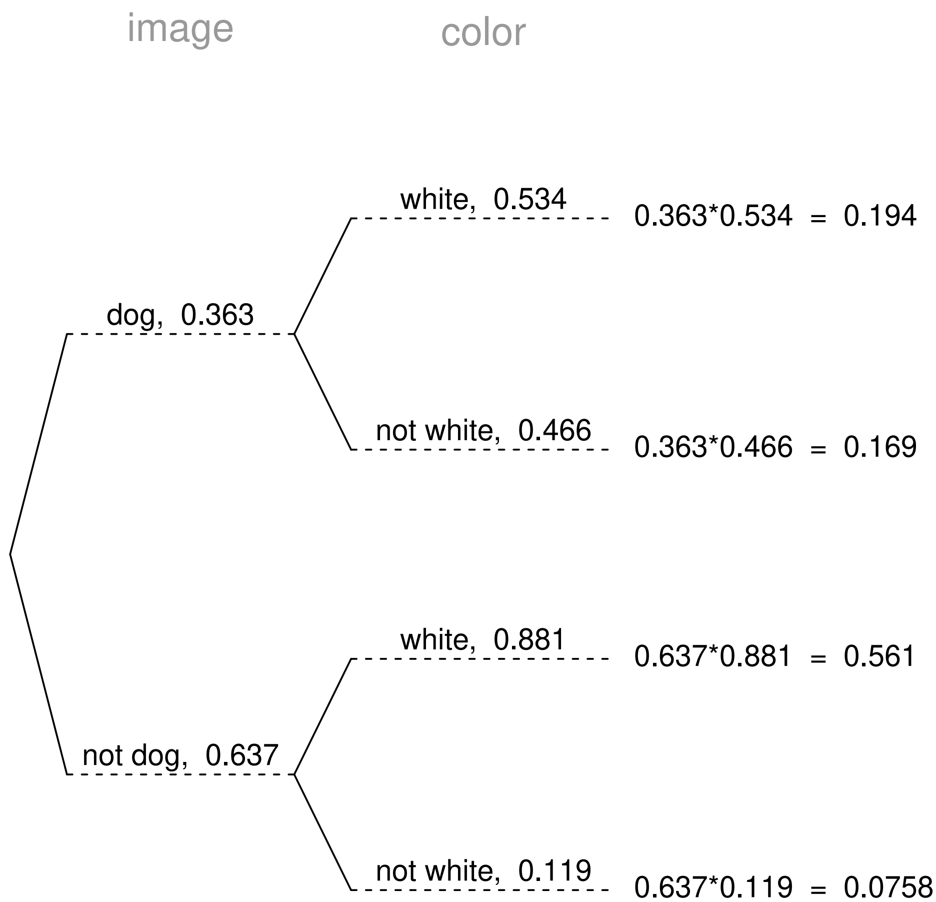
(f) $P(\text{teal given wheel}) = \frac{33}{98+48+33} = 0.184$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a dog is 36.3%. If a dog is drawn, there is a 53.4% chance that it is white. If a card that is not a dog is drawn, there is a 88.1% chance that it is white.

   Now, someone draws a random card and reveals it is not white. What is the chance the card is not a dog?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not dog" given "not white"}) = \frac{0.0758}{0.0758 + 0.169} = 0.31$$

3. In a very large pile of toothpicks, the mean length is 60.51 millimeters and the standard deviation is 1.44 millimeters. If you randomly sample 121 toothpicks, what is the chance the sample mean is between 60.24 and 60.69 millimeters?

Solution

Label the given information.

$$\mu = 60.51$$
$$\sigma = 1.44$$
$$n = 121$$
$$\bar{x}_{\text{lower}} = 60.24$$
$$\bar{x}_{\text{upper}} = 60.69$$

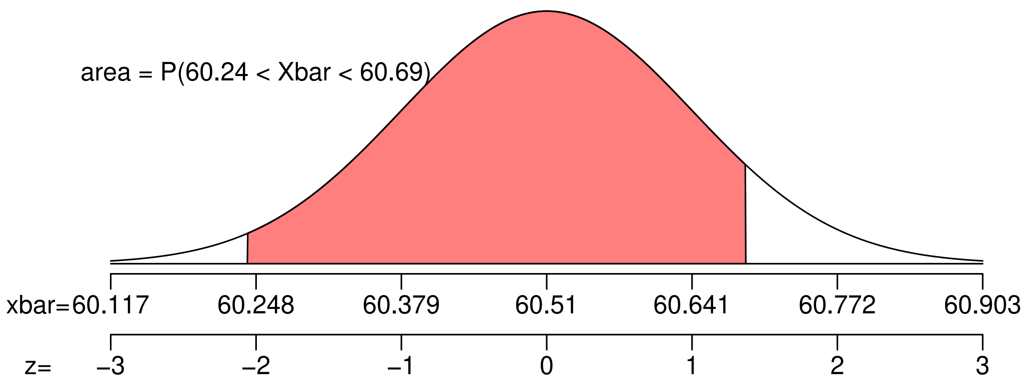
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.44}{\sqrt{121}} = 0.131$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(60.51,\ 0.131)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{60.24 - 60.51}{0.131} = -2.06$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{60.69 - 60.51}{0.131} = 1.37$$

Determine the probability.

$$\begin{aligned} P(60.24 < X < 60.69) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.37) - \Phi(-2.06) \\ &= 0.895 \end{aligned}$$

4. In a game, there is a 60% chance to win a round. You will play 115 rounds.
   (a) What is the probability of winning exactly 73 rounds?
   (b) What is the probability of winning at least 66 but at most 81 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 73) = \binom{115}{73}(0.6)^{73}(1-0.6)^{115-73}$$

$$P(X = 73) = \binom{115}{73}(0.6)^{73}(0.4)^{42}$$

$$P(X = 73) = 0.0575$$

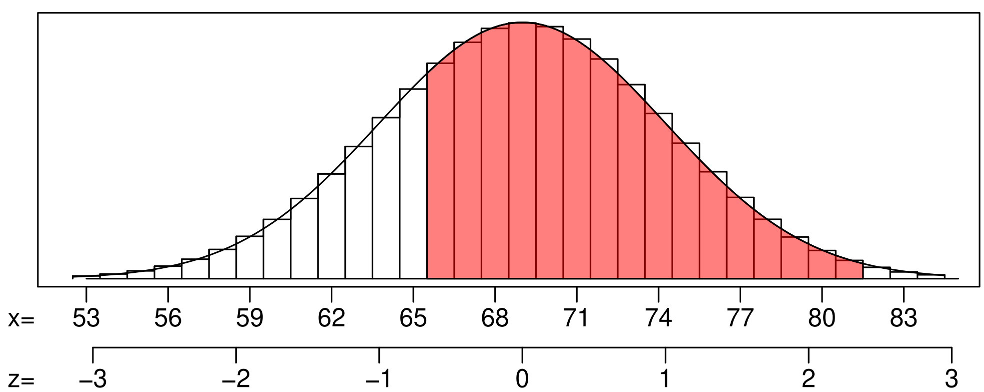
Find the mean.

$$\mu = np = (115)(0.6) = 69$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(115)(0.6)(1-0.6)} = 5.2536$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{65.5 - 69}{5.2536} = -0.57$$

$$z_2 = \frac{81.5 - 69}{5.2536} = 2.28$$

Calculate the probability.

$$P(66 \leq X \leq 81) = \Phi(2.28) - \Phi(-0.57) = 0.7044$$

(a) $P(X = 73) = 0.0575$

(b) $P(66 \leq X \leq 81) = 0.7044$

5. As an ornithologist, you wish to determine the average body mass of *Icterus spurius*. You randomly sample 13 adults of *Icterus spurius*, resulting in a sample mean of 21.81 grams and a sample standard deviation of 2.07 grams. Determine a 95% confidence interval of the true population mean.

Solution
We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 13$$
$$\bar{x} = 21.81$$
$$s = 2.07$$
$$CL = 0.95$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 12$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.95$.

$$t^\star = 2.18$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.07}{\sqrt{13}} = 0.574$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (21.81 - 2.18 \times 0.574,\ 21.81 + 2.18 \times 0.574) \\ &= (20.6,\ 23.1) \end{aligned}$$

We are 95% confident that the population mean is between 20.6 and 23.1.

6. A treatment group of size 35 has a mean of 1.03 and standard deviation of 0.188. A control group of size 38 has a mean of 1.15 and standard deviation of 0.235. If you decided to use a signficance level of 0.02, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 69.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 35 \\ \bar{x}_1 &= 1.03 \\ s_1 &= 0.188 \\ n_2 &= 38 \\ \bar{x}_2 &= 1.15 \\ s_2 &= 0.235 \\ \alpha &= 0.02 \\ df &= 69 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.02$ by using a t table.

$$t^\star = 2.38$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.188)^2}{35} + \frac{(0.235)^2}{38}} \\ &= 0.05 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1.15 - 1.03) - (0)}{0.05} \\ &= 2.42 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.38$

(d) $SE = 0.05$

(e) $|t_{obs}| = 2.42$

(f) $0.01 < p\text{-value} < 0.02$

(g) reject the null

7. From a very large population, a random sample of 770 individuals was taken. In that sample, 47.7% were green. Determine a 96% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.96$.

$$z^\star = 2.05$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.477)(1-0.477)}{770}} = 0.018$$

Calculate the margin of error.

$$ME = z^\star SE = (2.05)(0.018) = 0.0369$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.44\,,\ 0.514)$$

We are 96% confident that the true population proportion is between 44% and 51.4%.

(a) The lower bound = 0.44, which can also be expressed as 44%.

(b) The upper bound = 0.514, which can also be expressed as 51.4%.

8. An experiment is run with a treatment group of size 198 and a control group of size 175. The results are summarized in the table below.

	treatment	control
fluorescent	32	43
not fluorescent	166	132

Using a significance level of 0.05, determine whether the treatment causes an effect on the proportion of cases that are fluorescent.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.05$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.96$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{32}{198} = 0.162$$
$$\hat{p}_2 = \frac{43}{175} = 0.246$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.246 - 0.162 = 0.084$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{32 + 43}{198 + 175} = 0.201$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.201)(0.799)}{198} + \frac{(0.201)(0.799)}{175}} \\ &= 0.0416 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0416)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.246 - 0.162) - 0}{0.0416} \\ &= 2.02 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.02) \\ &= 0.0434 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0 : \quad p_2 - p_1 = 0$

(b) $H_A : \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.96$

(d) $SE = 0.0416$

(e) $|z_{\text{obs}}| = 2.02$

(f) p-value = 0.0434

(g) reject the null

1. (a) $P(\textbf{cat}) = 0.296$

   (b) $P(\textbf{bike or green}) = 0.427$

   (c) $P(\textbf{cat and green}) = 0.0193$

   (d) $P(\textbf{green}) = 0.326$

   (e) $P(\textbf{green given bike}) = 0.451$

   (f) $P(\textbf{shovel given orange}) = 0.245$

2. $P(\textbf{"not wheel" given "blue"}) = 0.943$

3. $P(65.93 < X < 66.24) = 0.7574$

4. (a) $P(X = 85) = 0.1117$

   (b) $P(84 \leq X \leq 92) = 0.7435$

5. $\mathbf{(10.1, 11.1)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.66$

   (d) $SE = 61.965$

   (e) $\mid t_{\textbf{obs}} \mid = 2.9$

   (f) $0.005 < p\textbf{-value} < 0.01$

   (g) **reject**

7. (a) **LB of p CI = 0.806 or** 80.6%

   (b) **UB of p CI = 0.864 or** 86.4%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.33$

(d) $SE = 0.047$

(e) $\mid z_{\mathbf{obs}} \mid = 2.46$

(f) p-**value** = 0.0138

(g) **reject**

1. In a deck of strange cards, there are 724 cards. Each card has an image and a color. The amounts are shown in the table below.

	green	orange	teal	white
bike	60	50	10	13
cat	14	52	71	77
flower	73	18	63	16
shovel	89	39	54	25

(a) What is the probability a random card is a cat?

(b) What is the probability a random card is either a bike or green (or both)?

(c) What is the probability a random card is both a cat and green?

(d) What is the probability a random card is green?

(e) What is the probability a random card is green given it is a bike?

(f) What is the probability a random card is a shovel given it is orange?

Solution

(a) $P(\text{cat}) = \frac{14+52+71+77}{724} = 0.296$

(b) $P(\text{bike or green}) = \frac{60+50+10+13+60+14+73+89-60}{724} = 0.427$

(c) $P(\text{cat and green}) = \frac{14}{724} = 0.0193$

(d) $P(\text{green}) = \frac{60+14+73+89}{724} = 0.326$

(e) $P(\text{green given bike}) = \frac{60}{60+50+10+13} = 0.451$

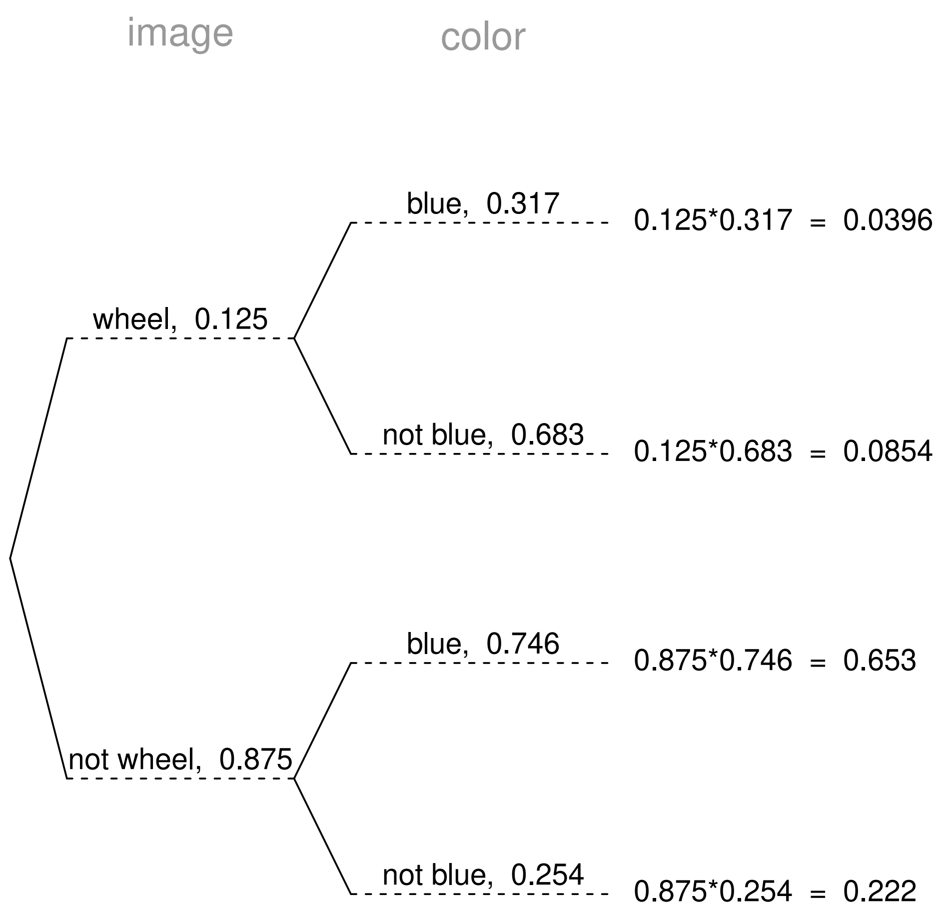
(f) $P(\text{shovel given orange}) = \frac{39}{50+52+18+39} = 0.245$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a wheel is 12.5%. If a wheel is drawn, there is a 31.7% chance that it is blue. If a card that is not a wheel is drawn, there is a 74.6% chance that it is blue.

   Now, someone draws a random card and reveals it is blue. What is the chance the card is not a wheel?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not wheel" given "blue"}) = \frac{0.653}{0.653 + 0.0396} = 0.943$$

3. In a very large pile of toothpicks, the mean length is 66.17 millimeters and the standard deviation is 1.07 millimeters. If you randomly sample 121 toothpicks, what is the chance the sample mean is between 65.93 and 66.24 millimeters?

Solution

Label the given information.

$$\mu = 66.17$$
$$\sigma = 1.07$$
$$n = 121$$
$$\bar{x}_{\text{lower}} = 65.93$$
$$\bar{x}_{\text{upper}} = 66.24$$

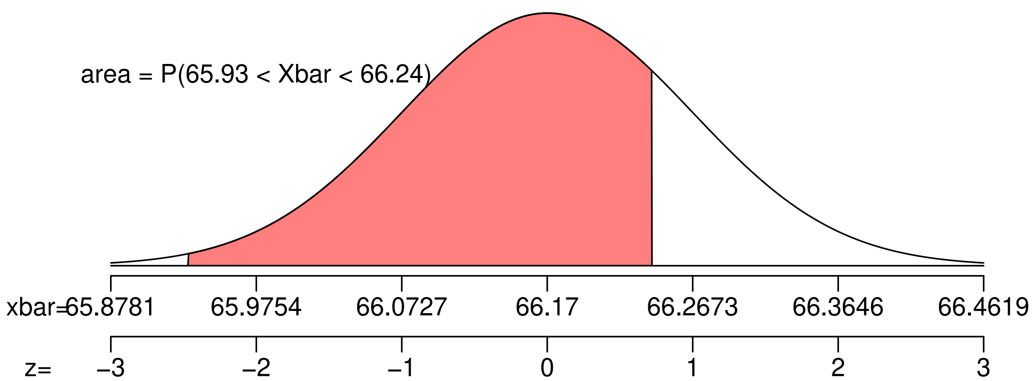
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.07}{\sqrt{121}} = 0.0973$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(66.17,\ 0.0973)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{65.93 - 66.17}{0.0973} = -2.47$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{66.24 - 66.17}{0.0973} = 0.72$$

Determine the probability.

$$\begin{aligned} P(65.93 < X < 66.24) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.72) - \Phi(-2.47) \\ &= 0.7574 \end{aligned}$$

4. In a game, there is a 89% chance to win a round. You will play 97 rounds.

   (a) What is the probability of winning exactly 85 rounds?

   (b) What is the probability of winning at least 84 but at most 92 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 85) = \binom{97}{85}(0.89)^{85}(1-0.89)^{97-85}$$

$$P(X = 85) = \binom{97}{85}(0.89)^{85}(0.11)^{12}$$

$$P(X = 85) = 0.1117$$

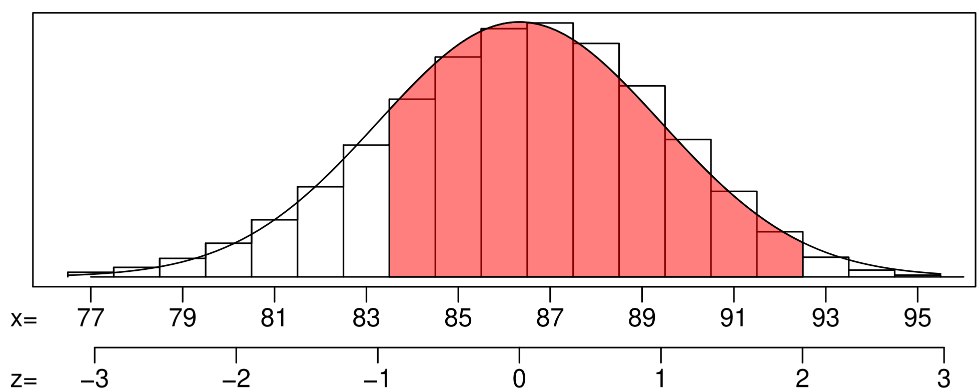
Find the mean.

$$\mu = np = (97)(0.89) = 86.33$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(97)(0.89)(1-0.89)} = 3.0816$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{83.5 - 86.33}{3.0816} = -0.76$$

$$z_2 = \frac{92.5 - 86.33}{3.0816} = 1.84$$

Calculate the probability.

$$P(84 \leq X \leq 92) = \Phi(1.84) - \Phi(-0.76) = 0.7435$$

(a) $P(X = 85) = 0.1117$

(b) $P(84 \leq X \leq 92) = 0.7435$

5. As an ornithologist, you wish to determine the average body mass of *Geothlypis trichas*. You randomly sample 32 adults of *Geothlypis trichas*, resulting in a sample mean of 10.6 grams and a sample standard deviation of 1.09 grams. Determine a 99% confidence interval of the true population mean.

Solution
We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 32 \\ \bar{x} &= 10.6 \\ s &= 1.09 \\ CL &= 0.99 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 31$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.99$.

$$t^{\star} = 2.74$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.09}{\sqrt{32}} = 0.193$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^{\star} SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^{\star} SE,\ \bar{x} + t^{\star} SE) \\ &= (10.6 - 2.74 \times 0.193,\ 10.6 + 2.74 \times 0.193) \\ &= (10.1,\ 11.1) \end{aligned}$$

We are 99% confident that the population mean is between 10.1 and 11.1.

6. A treatment group of size 28 has a mean of 1210 and standard deviation of 204. A control group of size 35 has a mean of 1030 and standard deviation of 287. If you decided to use a signficance level of 0.01, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 60.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 28 \\ \bar{x}_1 &= 1210 \\ s_1 &= 204 \\ n_2 &= 35 \\ \bar{x}_2 &= 1030 \\ s_2 &= 287 \\ \alpha &= 0.01 \\ df &= 60 \end{aligned}$$

State the hypotheses.

$$\begin{aligned} H_0 &: \quad \mu_2 - \mu_1 = 0 \\ H_A &: \quad \mu_2 - \mu_1 \neq 0 \end{aligned}$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.01$ by using a t table.

$$t^\star = 2.66$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(204)^2}{28} + \frac{(287)^2}{35}} \\ &= 61.965 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1030 - 1210) - (0)}{61.965} \\ &= -2.9 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.005 < p\text{-value} < 0.01$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^{\star} = 2.66$

(d) $SE = 61.965$

(e) $|t_{\text{obs}}| = 2.9$

(f) $0.005 < p\text{-value} < 0.01$

(g) reject the null

7. From a very large population, a random sample of 650 individuals was taken. In that sample, 83.5% were super. Determine a 95% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.95$.

$$z^\star = 1.96$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.835)(1-0.835)}{650}} = 0.0146$$

Calculate the margin of error.

$$ME = z^\star SE = (1.96)(0.0146) = 0.0286$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.806\,,\ 0.864)$$

We are 95% confident that the true population proportion is between 80.6% and 86.4%.

(a) The lower bound = 0.806, which can also be expressed as 80.6%.

(b) The upper bound = 0.864, which can also be expressed as 86.4%.

8. An experiment is run with a treatment group of size 221 and a control group of size 234. The results are summarized in the table below.

	treatment	control
cold	103	136
not cold	118	98

Using a significance level of 0.02, determine whether the treatment causes an effect on the proportion of cases that are cold.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution
State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^{\star}$ such that $P(|Z| > z^{\star}) = 0.02$.

$$z^{\star} = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.33$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{103}{221} = 0.466$$
$$\hat{p}_2 = \frac{136}{234} = 0.581$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.581 - 0.466 = 0.115$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{103 + 136}{221 + 234} = 0.525$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.525)(0.475)}{221} + \frac{(0.525)(0.475)}{234}} \\ &= 0.0468 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0468)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.581 - 0.466) - 0}{0.0468} \\ &= 2.46 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.46) \\ &= 0.0138 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.33$

(d) $SE = 0.0468$

(e) $|z_{\text{obs}}| = 2.46$

(f) p-value = 0.0138

(g) reject the null

1. (a) $P(\textbf{yellow given gem}) = 0.209$

   (b) $P(\textbf{dog and red}) = 0.0757$

   (c) $P(\textbf{orange}) = 0.297$

   (d) $P(\textbf{dog or red}) = 0.487$

   (e) $P(\textbf{dog given orange}) = 0.396$

   (f) $P(\textbf{gem}) = 0.383$

2. $P(\textbf{"cat" given "red"}) = 0.708$

3. $P(72.13 < X < 72.47) = 0.8444$

4. (a) $P(X = 12) = 0.083$

   (b) $P(7 \leq X \leq 24) = 0.9775$

5. $\mathbf{(12.7, 14.8)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 1.72$

   (d) $SE = 3.284$

   (e) $\mid t_{\textbf{obs}} \mid = 1.52$

   (f) $0.1 < p\textbf{-value} < 0.2$

   (g) **retain**

7. (a) **LB of p CI = 0.31 or** 31%

   (b) **UB of p CI = 0.318 or** 31.8%

8. (a) $\boxed{H_0 : \quad p_2 - p_1 = 0}$

(b) $\boxed{H_{\mathbf{A}} : \quad p_2 - p_1 \neq 0}$

(c) $\boxed{z^\star = 1.28}$

(d) $\boxed{SE = 0.058}$

(e) $\boxed{\mid z_{\mathbf{obs}} \mid = 1.54}$

(f) $\boxed{p\textbf{-value} = 0.1236}$

(g) **reject**

1. In a deck of strange cards, there are 713 cards. Each card has an image and a color. The amounts are shown in the table below.

	green	orange	red	yellow
dog	11	84	54	26
gem	99	34	83	57
tree	38	94	89	44

   (a) What is the probability a random card is yellow given it is a gem?

   (b) What is the probability a random card is both a dog and red?

   (c) What is the probability a random card is orange?

   (d) What is the probability a random card is either a dog or red (or both)?

   (e) What is the probability a random card is a dog given it is orange?

   (f) What is the probability a random card is a gem?

Solution

(a) $P(\text{yellow given gem}) = \frac{57}{99+34+83+57} = 0.209$

(b) $P(\text{dog and red}) = \frac{54}{713} = 0.0757$

(c) $P(\text{orange}) = \frac{84+34+94}{713} = 0.297$

(d) $P(\text{dog or red}) = \frac{11+84+54+26+54+83+89-54}{713} = 0.487$

(e) $P(\text{dog given orange}) = \frac{84}{84+34+94} = 0.396$

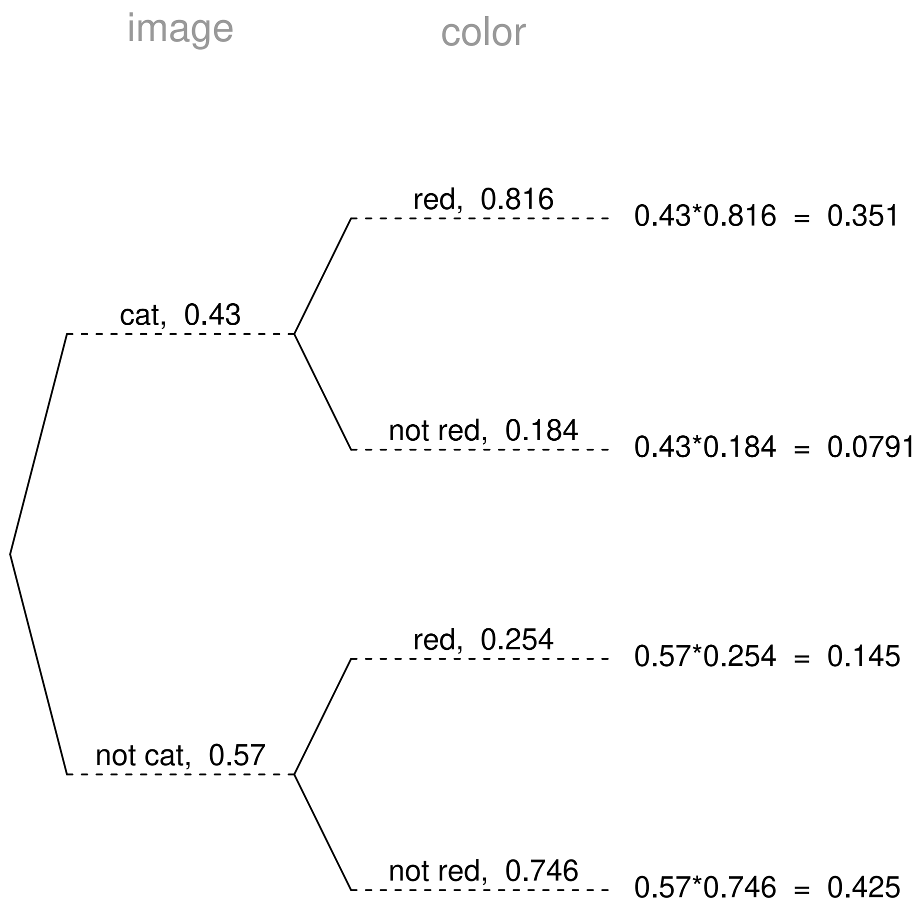
(f) $P(\text{gem}) = \frac{99+34+83+57}{713} = 0.383$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a cat is 43%. If a cat is drawn, there is a 81.6% chance that it is red. If a card that is not a cat is drawn, there is a 25.4% chance that it is red.

   Now, someone draws a random card and reveals it is red. What is the chance the card is a cat?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"cat" given "red"}) = \frac{0.351}{0.351 + 0.145} = 0.708$$

3. In a very large pile of toothpicks, the mean length is 72.37 millimeters and the standard deviation is 1.36 millimeters. If you randomly sample 200 toothpicks, what is the chance the sample mean is between 72.13 and 72.47 millimeters?

Solution

Label the given information.

$$\mu = 72.37$$
$$\sigma = 1.36$$
$$n = 200$$
$$\bar{x}_{\text{lower}} = 72.13$$
$$\bar{x}_{\text{upper}} = 72.47$$

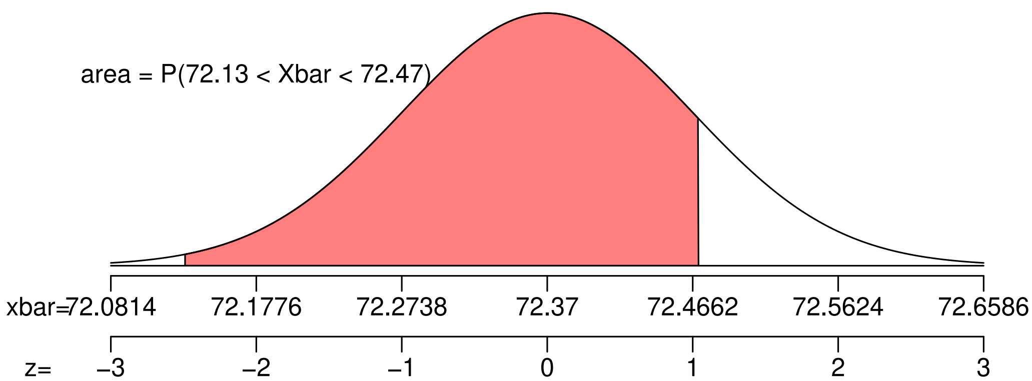
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.36}{\sqrt{200}} = 0.0962$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(72.37,\ 0.0962)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{72.13 - 72.37}{0.0962} = -2.49$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{72.47 - 72.37}{0.0962} = 1.04$$

Determine the probability.

$$\begin{aligned} P(72.13 < X < 72.47) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.04) - \Phi(-2.49) \\ &= 0.8444 \end{aligned}$$

4. In a game, there is a 9% chance to win a round. You will play 167 rounds.

   (a) What is the probability of winning exactly 12 rounds?

   (b) What is the probability of winning at least 7 but at most 24 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 12) = \binom{167}{12}(0.09)^{12}(1-0.09)^{167-12}$$

$$P(X = 12) = \binom{167}{12}(0.09)^{12}(0.91)^{155}$$

$$P(X = 12) = 0.083$$

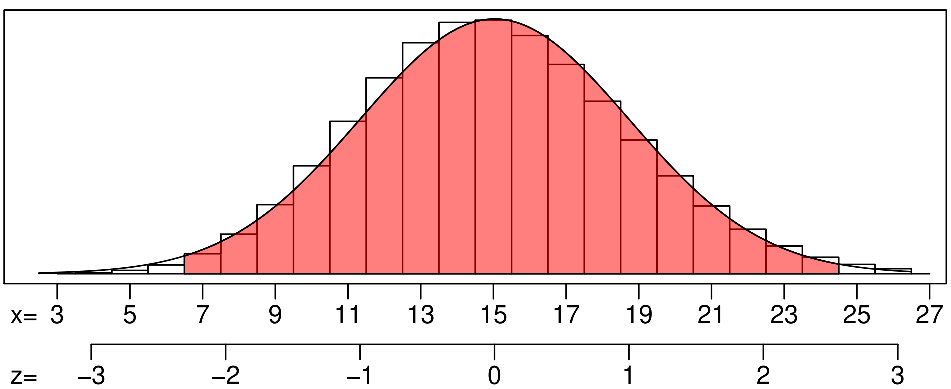
Find the mean.

$$\mu = np = (167)(0.09) = 15.03$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(167)(0.09)(1-0.09)} = 3.6983$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{6.5 - 15.03}{3.6983} = -2.17$$

$$z_2 = \frac{24.5 - 15.03}{3.6983} = 2.43$$

Calculate the probability.

$$P(7 \le X \le 24) = \Phi(2.43) - \Phi(-2.17) = 0.9775$$

(a) $P(X = 12) = 0.083$

(b) $P(7 \le X \le 24) = 0.9775$

5. As an ornithologist, you wish to determine the average body mass of *Protonotaria citrea*. You randomly sample 21 adults of *Protonotaria citrea*, resulting in a sample mean of 13.75 grams and a sample standard deviation of 1.64 grams. Determine a 99% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 21 \\ \bar{x} &= 13.75 \\ s &= 1.64 \\ CL &= 0.99 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 20$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.99$.

$$t^{\star} = 2.85$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.64}{\sqrt{21}} = 0.358$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^{\star} SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^{\star} SE,\ \bar{x} + t^{\star} SE) \\ &= (13.75 - 2.85 \times 0.358,\ 13.75 + 2.85 \times 0.358) \\ &= (12.7,\ 14.8) \end{aligned}$$

We are 99% confident that the population mean is between 12.7 and 14.8.

6. A treatment group of size 13 has a mean of 106 and standard deviation of 10.3. A control group of size 38 has a mean of 101 and standard deviation of 9.99. If you decided to use a signficance level of 0.1, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 20.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 13$$
$$\bar{x}_1 = 106$$
$$s_1 = 10.3$$
$$n_2 = 38$$
$$\bar{x}_2 = 101$$
$$s_2 = 9.99$$
$$\alpha = 0.1$$
$$df = 20$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.1$ by using a t table.

$$t^\star = 1.72$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(10.3)^2}{13} + \frac{(9.99)^2}{38}} \\ &= 3.284 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(101 - 106) - (0)}{3.284} \\ &= -1.52 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.1 < p\text{-value} < 0.2$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 1.72$

(d) $SE = 3.284$

(e) $|t_{obs}| = 1.52$

(f) $0.1 < p\text{-value} < 0.2$

(g) retain the null

7. From a very large population, a random sample of 71000 individuals was taken. In that sample, 31.4% were floating. Determine a 98% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.314)(1-0.314)}{71000}} = 0.00174$$

Calculate the margin of error.

$$ME = z^\star SE = (2.33)(0.00174) = 0.00405$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.31\ ,\ 0.318)$$

We are 98% confident that the true population proportion is between 31% and 31.8%.

(a) The lower bound = 0.31, which can also be expressed as 31%.

(b) The upper bound = 0.318, which can also be expressed as 31.8%.

8. An experiment is run with a treatment group of size 104 and a control group of size 149. The results are summarized in the table below.

	treatment	control
sorry	35	37
not sorry	69	112

Using a significance level of 0.2, determine whether the treatment causes an effect on the proportion of cases that are sorry.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.2$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.28$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{35}{104} = 0.337$$
$$\hat{p}_2 = \frac{37}{149} = 0.248$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.248 - 0.337 = -0.089$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{35 + 37}{104 + 149} = 0.285$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.285)(0.715)}{104} + \frac{(0.285)(0.715)}{149}} \\ &= 0.0577 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0577)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.248 - 0.337) - 0}{0.0577} \\ &= -1.54 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-1.54) \\ &= 0.1236 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.28$

(d) $SE = 0.0577$

(e) $|z_{\text{obs}}| = 1.54$

(f) p-value = 0.1236

(g) reject the null

1. (a) P(**wheel and teal**) = 0.123

   (b) P(**violet given pig**) = 0.366

   (c) P(**pig given black**) = 0.149

   (d) P(**wheel**) = 0.371

   (e) P(**black**) = 0.179

   (f) P(**wheel or teal**) = 0.576

2. P(**"not ring" given "green"**) = 0.83

3. $P(65.97 < X < 66.22) = 0.7038$

4. (a) $P(X = 33) = 0.1018$

   (b) $P(29 \leq X \leq 40) = 0.8364$

5. **(15.2, 18.8)**

6. (a) $H_0 : \quad \mu_2 - \mu_1 = 0$

   (b) $H_0 : \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.68$

   (d) $SE = 22.027$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.81$

   (f) $0.01 <$ p**-value** < 0.02

   (g) **reject**

7. (a) **LB of p CI = 0.294 or** 29.4%

   (b) **UB of p CI = 0.3 or** 30%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.96$

(d) $SE = 0.066$

(e) $\mid z_{\mathbf{obs}} \mid = 2.02$

(f) p**-value** = 0.0434

(g) **reject**

1. In a deck of strange cards, there are 375 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	teal	violet
pig	10	35	26
shovel	32	42	91
wheel	25	46	68

(a) What is the probability a random card is both a wheel and teal?

(b) What is the probability a random card is violet given it is a pig?

(c) What is the probability a random card is a pig given it is black?

(d) What is the probability a random card is a wheel?

(e) What is the probability a random card is black?

(f) What is the probability a random card is either a wheel or teal (or both)?

Solution

(a) $P(\text{wheel and teal}) = \frac{46}{375} = 0.123$

(b) $P(\text{violet given pig}) = \frac{26}{10+35+26} = 0.366$

(c) $P(\text{pig given black}) = \frac{10}{10+32+25} = 0.149$

(d) $P(\text{wheel}) = \frac{25+46+68}{375} = 0.371$

(e) $P(\text{black}) = \frac{10+32+25}{375} = 0.179$

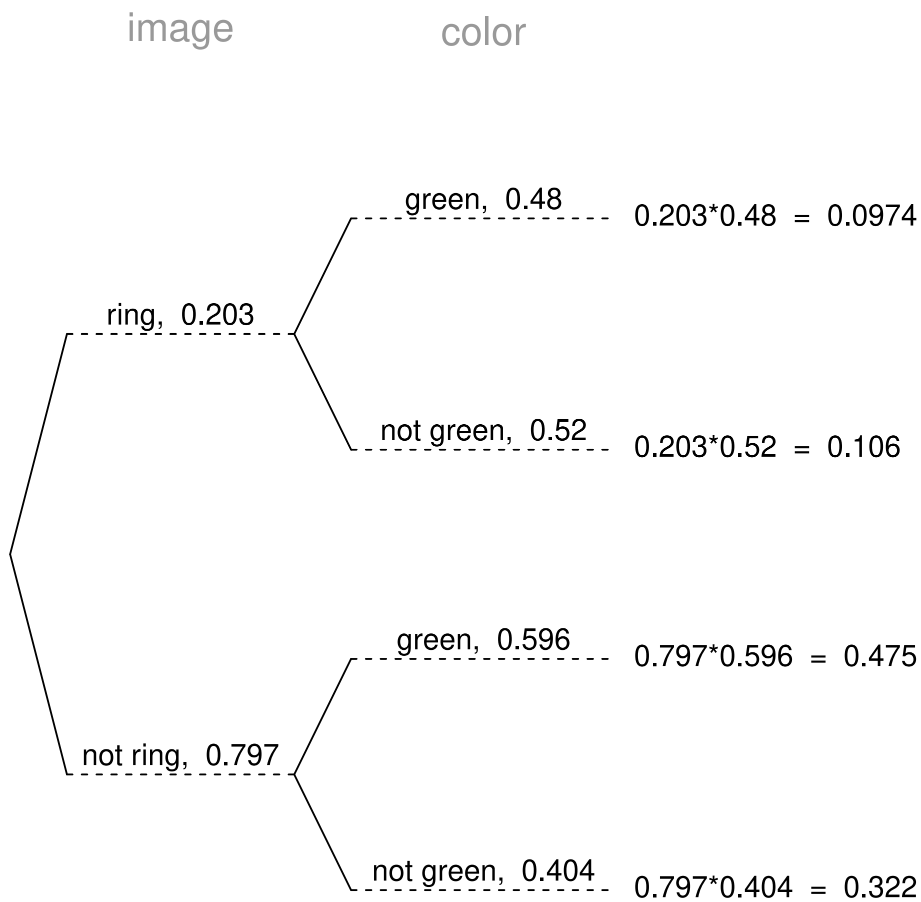
(f) $P(\text{wheel or teal}) = \frac{25+46+68+35+42+46-46}{375} = 0.576$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a ring is 20.3%. If a ring is drawn, there is a 48% chance that it is green. If a card that is not a ring is drawn, there is a 59.6% chance that it is green.

   Now, someone draws a random card and reveals it is green. What is the chance the card is not a ring?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not ring" given "green"}) = \frac{0.475}{0.475 + 0.0974} = 0.83$$

3. In a very large pile of toothpicks, the mean length is 66.17 millimeters and the standard deviation is 1.17 millimeters. If you randomly sample 175 toothpicks, what is the chance the sample mean is between 65.97 and 66.22 millimeters?

Solution

Label the given information.

$$\mu = 66.17$$
$$\sigma = 1.17$$
$$n = 175$$
$$\bar{x}_{\text{lower}} = 65.97$$
$$\bar{x}_{\text{upper}} = 66.22$$

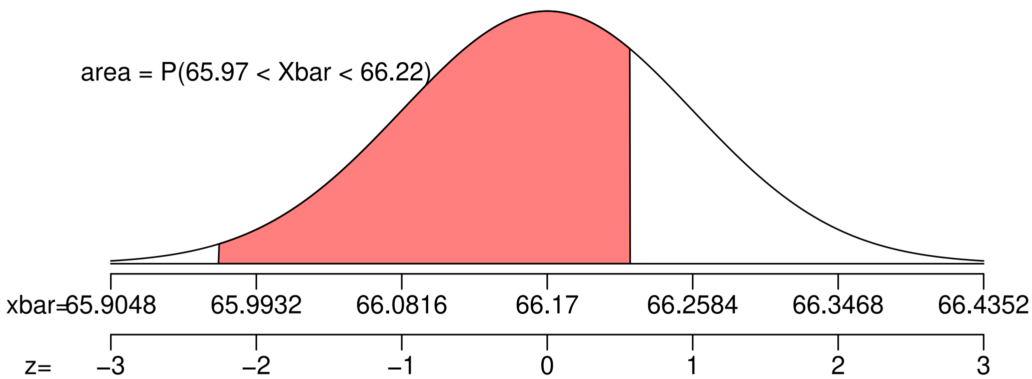
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.17}{\sqrt{175}} = 0.0884$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(66.17,\ 0.0884)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{65.97 - 66.17}{0.0884} = -2.26$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{66.22 - 66.17}{0.0884} = 0.57$$

Determine the probability.

$$\begin{aligned} P(65.97 < X < 66.22) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.57) - \Phi(-2.26) \\ &= 0.7038 \end{aligned}$$

4. In a game, there is a 56% chance to win a round. You will play 60 rounds.

   (a) What is the probability of winning exactly 33 rounds?

   (b) What is the probability of winning at least 29 but at most 40 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 33) = \binom{60}{33}(0.56)^{33}(1-0.56)^{60-33}$$

$$P(X = 33) = \binom{60}{33}(0.56)^{33}(0.44)^{27}$$

$$P(X = 33) = 0.1018$$

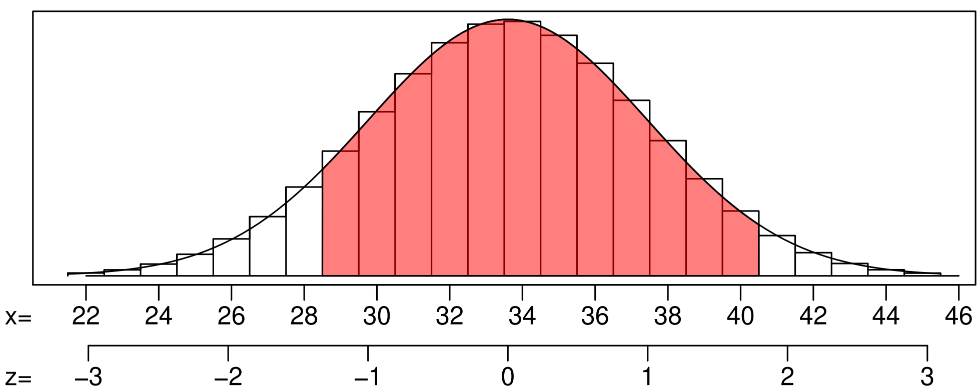
Find the mean.

$$\mu = np = (60)(0.56) = 33.6$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(60)(0.56)(1-0.56)} = 3.845$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{28.5 - 33.6}{3.845} = -1.2$$

$$z_2 = \frac{40.5 - 33.6}{3.845} = 1.66$$

Calculate the probability.

$$P(29 \leq X \leq 40) = \Phi(1.66) - \Phi(-1.2) = 0.8364$$

(a) $P(X = 33) = 0.1018$

(b) $P(29 \leq X \leq 40) = 0.8364$

5. As an ornithologist, you wish to determine the average body mass of *Oporornis formosus*. You randomly sample 13 adults of *Oporornis formosus*, resulting in a sample mean of 17 grams and a sample standard deviation of 1.92 grams. Determine a 99.5% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 13 \\ \bar{x} &= 17 \\ s &= 1.92 \\ CL &= 0.995 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 12$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.995$.

$$t^\star = 3.43$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.92}{\sqrt{13}} = 0.533$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (17 - 3.43 \times 0.533,\ 17 + 3.43 \times 0.533) \\ &= (15.2,\ 18.8) \end{aligned}$$

We are 99.5% confident that the population mean is between 15.2 and 18.8.

6. A treatment group of size 9 has a mean of 1040 and standard deviation of 59.5. A control group of size 40 has a mean of 978 and standard deviation of 60.6. If you decided to use a signficance level of 0.02, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 12.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 9 \\ \bar{x}_1 &= 1040 \\ s_1 &= 59.5 \\ n_2 &= 40 \\ \bar{x}_2 &= 978 \\ s_2 &= 60.6 \\ \alpha &= 0.02 \\ df &= 12 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$

$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.02$ by using a t table.

$$t^\star = 2.68$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(59.5)^2}{9} + \frac{(60.6)^2}{40}} \\ &= 22.027 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(978 - 1040) - (0)}{22.027} \\ &= -2.81 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.68$

(d) $SE = 22.027$

(e) $|t_{obs}| = 2.81$

(f) $0.01 < p\text{-value} < 0.02$

(g) reject the null

7. From a very large population, a random sample of 76000 individuals was taken. In that sample, 29.7% were cold. Determine a 96% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.96$.

$$z^\star = 2.05$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.297)(1-0.297)}{76000}} = 0.00166$$

Calculate the margin of error.

$$ME = z^\star SE = (2.05)(0.00166) = 0.0034$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.294\,,\ 0.3)$$

We are 96% confident that the true population proportion is between 29.4% and 30%.

(a) The lower bound = 0.294, which can also be expressed as 29.4%.

(b) The upper bound = 0.3, which can also be expressed as 30%.

8. An experiment is run with a treatment group of size 78 and a control group of size 96. The results are summarized in the table below.

	treatment	control
glossy	25	18
not glossy	53	78

Using a significance level of 0.05, determine whether the treatment causes an effect on the proportion of cases that are glossy.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.05$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.96$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{25}{78} = 0.321$$
$$\hat{p}_2 = \frac{18}{96} = 0.188$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.188 - 0.321 = -0.133$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{25 + 18}{78 + 96} = 0.247$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.247)(0.753)}{78} + \frac{(0.247)(0.753)}{96}} \\ &= 0.0657 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0657)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.188 - 0.321) - 0}{0.0657} \\ &= -2.02 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.02) \\ &= 0.0434 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 1.96$

(d) $SE = 0.0657$

(e) $|z_{\text{obs}}| = 2.02$

(f) p-value = 0.0434

(g) reject the null

1. (a) P(**flower or indigo**) = 0.44

   (b) P(**indigo given flower**) = 0.278

   (c) P(**flower**) = 0.303

   (d) P(**flower and gray**) = 0.0266

   (e) P(**pig given green**) = 0.165

   (f) P(**green**) = 0.147

2. P(**"not horn" given "pink"**) = 0.83

3. $P(62.06 < X < 62.47) = 0.8183$

4. (a) $P(X = 68) = 0.0508$

   (b) $P(61 \leq X \leq 82) = 0.8702$

5. **(9.44, 10.4)**

6. (a) $H_0: \ \mu_2 - \mu_1 = 0$

   (b) $H_0: \ \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.74$

   (d) $SE = 0.054$

   (e) $\mid t_{\textbf{obs}} \mid = 2.84$

   (f) $0.005 <$ p**-value** < 0.01

   (g) **reject**

7. (a) **LB of p CI = 0.677 or** 67.7%

   (b) **UB of p CI = 0.699 or** 69.9%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.33$

(d) $SE = 0.083$

(e) $\mid z_{\mathbf{obs}} \mid = 2.47$

(f) p-**value** = 0.0136

(g) **reject**

1. In a deck of strange cards, there are 902 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	gray	green	indigo	teal
flower	65	24	48	76	60
pig	44	54	22	37	92
shovel	90	98	63	87	42

(a) What is the probability a random card is either a flower or indigo (or both)?

(b) What is the probability a random card is indigo given it is a flower?

(c) What is the probability a random card is a flower?

(d) What is the probability a random card is both a flower and gray?

(e) What is the probability a random card is a pig given it is green?

(f) What is the probability a random card is green?

Solution

(a) $P(\text{flower or indigo}) = \frac{65+24+48+76+60+76+37+87-76}{902} = 0.44$

(b) $P(\text{indigo given flower}) = \frac{76}{65+24+48+76+60} = 0.278$

(c) $P(\text{flower}) = \frac{65+24+48+76+60}{902} = 0.303$

(d) $P(\text{flower and gray}) = \frac{24}{902} = 0.0266$

(e) $P(\text{pig given green}) = \frac{22}{48+22+63} = 0.165$

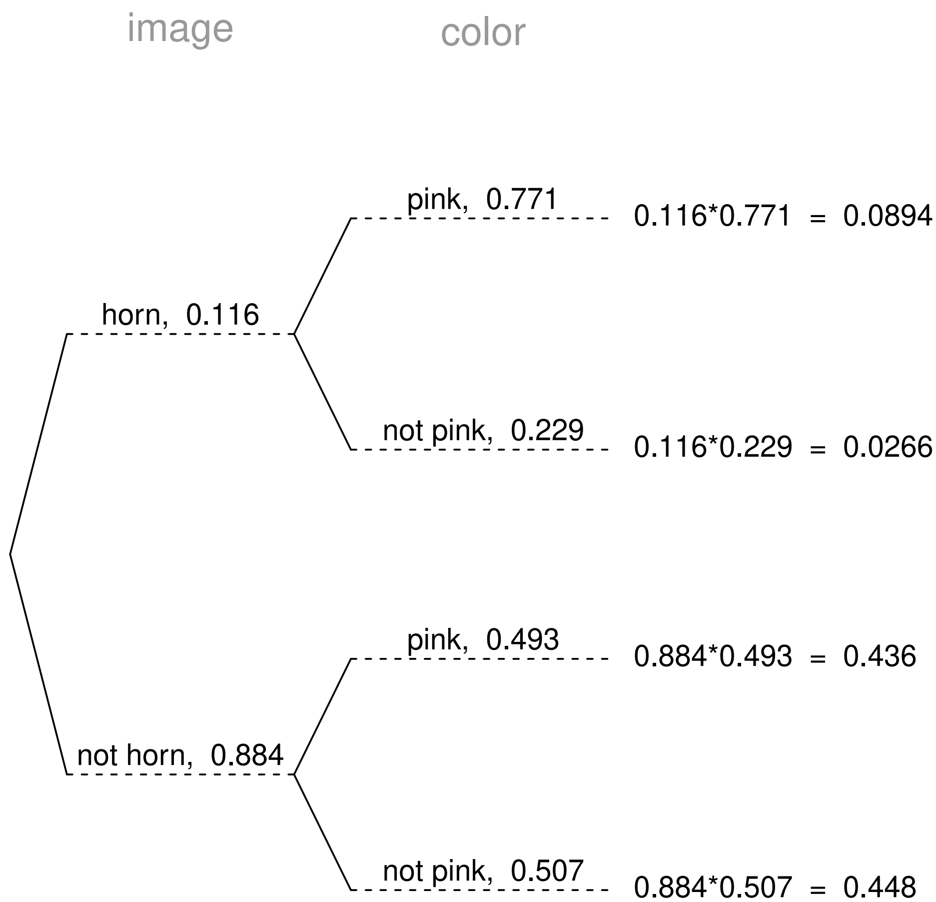
(f) $P(\text{green}) = \frac{48+22+63}{902} = 0.147$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a horn is 11.6%. If a horn is drawn, there is a 77.1% chance that it is pink. If a card that is not a horn is drawn, there is a 49.3% chance that it is pink.

   Now, someone draws a random card and reveals it is pink. What is the chance the card is not a horn?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not horn" given "pink"}) = \frac{0.436}{0.436 + 0.0894} = 0.83$$

3. In a very large pile of toothpicks, the mean length is 62.17 millimeters and the standard deviation is 1.32 millimeters. If you randomly sample 125 toothpicks, what is the chance the sample mean is between 62.06 and 62.47 millimeters?

Solution

Label the given information.

$$\mu = 62.17$$
$$\sigma = 1.32$$
$$n = 125$$
$$\bar{x}_{\text{lower}} = 62.06$$
$$\bar{x}_{\text{upper}} = 62.47$$

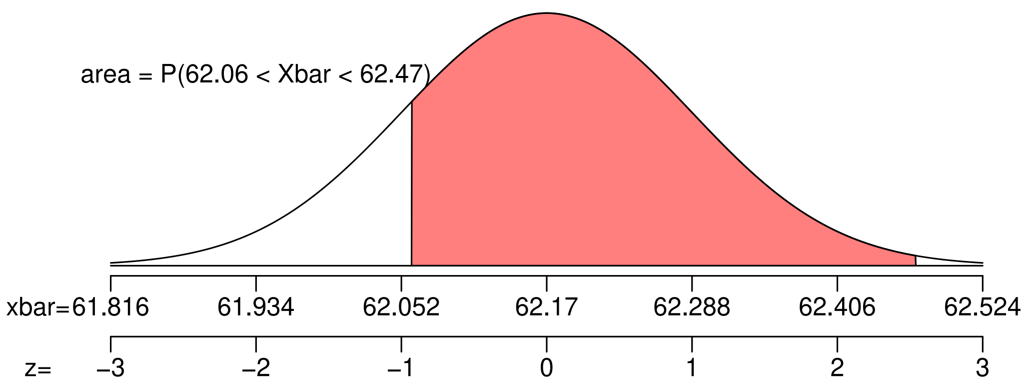
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.32}{\sqrt{125}} = 0.118$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(62.17, 0.118)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{62.06 - 62.17}{0.118} = -0.93$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{62.47 - 62.17}{0.118} = 2.54$$

Determine the probability.

$$\begin{aligned} P(62.06 < X < 62.47) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(2.54) - \Phi(-0.93) \\ &= 0.8183 \end{aligned}$$

4. In a game, there is a 33% chance to win a round. You will play 217 rounds.
   (a) What is the probability of winning exactly 68 rounds?
   (b) What is the probability of winning at least 61 but at most 82 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 68) = \binom{217}{68}(0.33)^{68}(1-0.33)^{217-68}$$

$$P(X = 68) = \binom{217}{68}(0.33)^{68}(0.67)^{149}$$

$$P(X = 68) = 0.0508$$

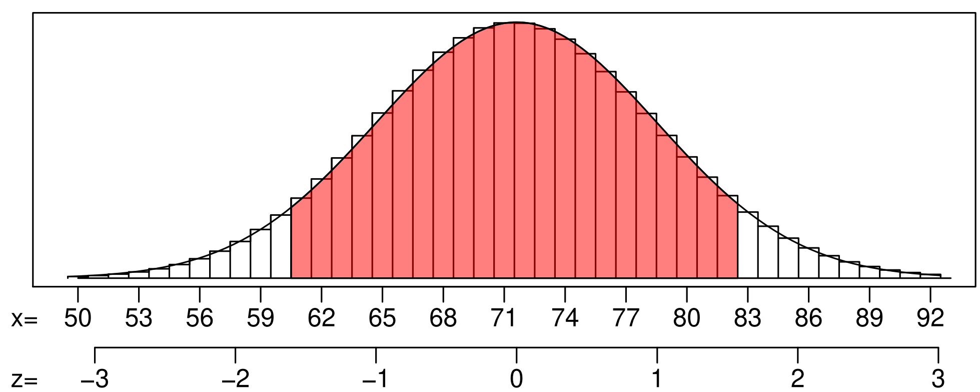
Find the mean.

$$\mu = np = (217)(0.33) = 71.61$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(217)(0.33)(1-0.33)} = 6.9267$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{60.5 - 71.61}{6.9267} = -1.53$$

$$z_2 = \frac{82.5 - 71.61}{6.9267} = 1.5$$

Calculate the probability.

$$P(61 \le X \le 82) = \Phi(1.5) - \Phi(-1.53) = 0.8702$$

(a) $P(X = 68) = 0.0508$

(b) $P(61 \le X \le 82) = 0.8702$

5. As an ornithologist, you wish to determine the average body mass of *Cistothorus palustris*. You randomly sample 26 adults of *Cistothorus palustris*, resulting in a sample mean of 9.9 grams and a sample standard deviation of 1.14 grams. Determine a 95% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 26 \\ \bar{x} &= 9.9 \\ s &= 1.14 \\ CL &= 0.95 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 25$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.95$.

$$t^\star = 2.06$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.14}{\sqrt{26}} = 0.224$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (9.9 - 2.06 \times 0.224,\ 9.9 + 2.06 \times 0.224) \\ &= (9.44,\ 10.4) \end{aligned}$$

We are 95% confident that the population mean is between 9.44 and 10.4.

6. A treatment group of size 19 has a mean of 1.1 and standard deviation of 0.141. A control group of size 18 has a mean of 0.947 and standard deviation of 0.183. If you decided to use a signficance level of 0.01, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 31.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 19 \\ \bar{x}_1 &= 1.1 \\ s_1 &= 0.141 \\ n_2 &= 18 \\ \bar{x}_2 &= 0.947 \\ s_2 &= 0.183 \\ \alpha &= 0.01 \\ df &= 31 \end{aligned}$$

State the hypotheses.

$$\begin{aligned} H_0 &: \quad \mu_2 - \mu_1 = 0 \\ H_A &: \quad \mu_2 - \mu_1 \neq 0 \end{aligned}$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.01$ by using a t table.

$$t^\star = 2.74$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.141)^2}{19} + \frac{(0.183)^2}{18}} \\ &= 0.054 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(0.947 - 1.1) - (0)}{0.054} \\ &= -2.84 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.005 < p\text{-value} < 0.01$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.74$

(d) $SE = 0.054$

(e) $|t_{obs}| = 2.84$

(f) $0.005 < p\text{-value} < 0.01$

(g) reject the null

7. From a very large population, a random sample of 6700 individuals was taken. In that sample, 68.8% were blue. Determine a 95% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.95$.

$$z^{\star} = 1.96$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.688)(1-0.688)}{6700}} = 0.00566$$

Calculate the margin of error.

$$ME = z^{\star} SE = (1.96)(0.00566) = 0.0111$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.677\,,\ 0.699)$$

We are 95% confident that the true population proportion is between 67.7% and 69.9%.

(a) The lower bound = 0.677, which can also be expressed as 67.7%.

(b) The upper bound = 0.699, which can also be expressed as 69.9%.

8. An experiment is run with a treatment group of size 24 and a control group of size 68. The results are summarized in the table below.

	treatment	control
organic	7	6
not organic	17	62

Using a significance level of 0.02, determine whether the treatment causes an effect on the proportion of cases that are organic.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.02$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.33$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{7}{24} = 0.292$$

$$\hat{p}_2 = \frac{6}{68} = 0.0882$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.0882 - 0.292 = -0.2038$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{7+6}{24+68} = 0.141$$

Determine the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}$$

$$= \sqrt{\frac{(0.141)(0.859)}{24} + \frac{(0.141)(0.859)}{68}}$$

$$= 0.0826$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0826)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.0882 - 0.292) - 0}{0.0826} \\ &= -2.47 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.47) \\ &= 0.0136 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.33$

(d) $SE = 0.0826$

(e) $|z_{obs}| = 2.47$

(f) p-value = 0.0136

(g) reject the null

1. (a) P(**flower**) = 0.318

   (b) P(**teal given shovel**) = 0.329

   (c) P(**teal**) = 0.373

   (d) P(**dog or red**) = 0.408

   (e) P(**shovel given red**) = 0.517

   (f) P(**shovel and teal**) = 0.147

2. P(**"not kite" given "orange"**) = 0.577

3. $P(67.5 < X < 67.67) = 0.7121$

4. (a) $P(X = 73) = 0.0704$

   (b) $P(73 \leq X \leq 81) = 0.2949$

5. **(14.3, 17.7)**

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.02$

   (d) $SE = 0.029$

   (e) $|\ t_{\mathbf{obs}}\ | = 2.08$

   (f) $0.04 <$ p**-value** < 0.05

   (g) **reject**

7. (a) **LB of p CI = 0.629 or** 62.9%

   (b) **UB of p CI = 0.653 or** 65.3%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 2.05$

(d) $SE = 0.095$

(e) $| \, z_{\mathbf{obs}} \, | = 2.33$

(f) p-**value** = 0.0198

(g) **reject**

1. In a deck of strange cards, there are 566 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	red	teal
dog	44	54	36
flower	69	19	92
shovel	91	78	83

(a) What is the probability a random card is a flower?

(b) What is the probability a random card is teal given it is a shovel?

(c) What is the probability a random card is teal?

(d) What is the probability a random card is either a dog or red (or both)?

(e) What is the probability a random card is a shovel given it is red?

(f) What is the probability a random card is both a shovel and teal?

Solution

(a) $P(\text{flower}) = \frac{69+19+92}{566} = 0.318$

(b) $P(\text{teal given shovel}) = \frac{83}{91+78+83} = 0.329$

(c) $P(\text{teal}) = \frac{36+92+83}{566} = 0.373$

(d) $P(\text{dog or red}) = \frac{44+54+36+54+19+78-54}{566} = 0.408$

(e) $P(\text{shovel given red}) = \frac{78}{54+19+78} = 0.517$

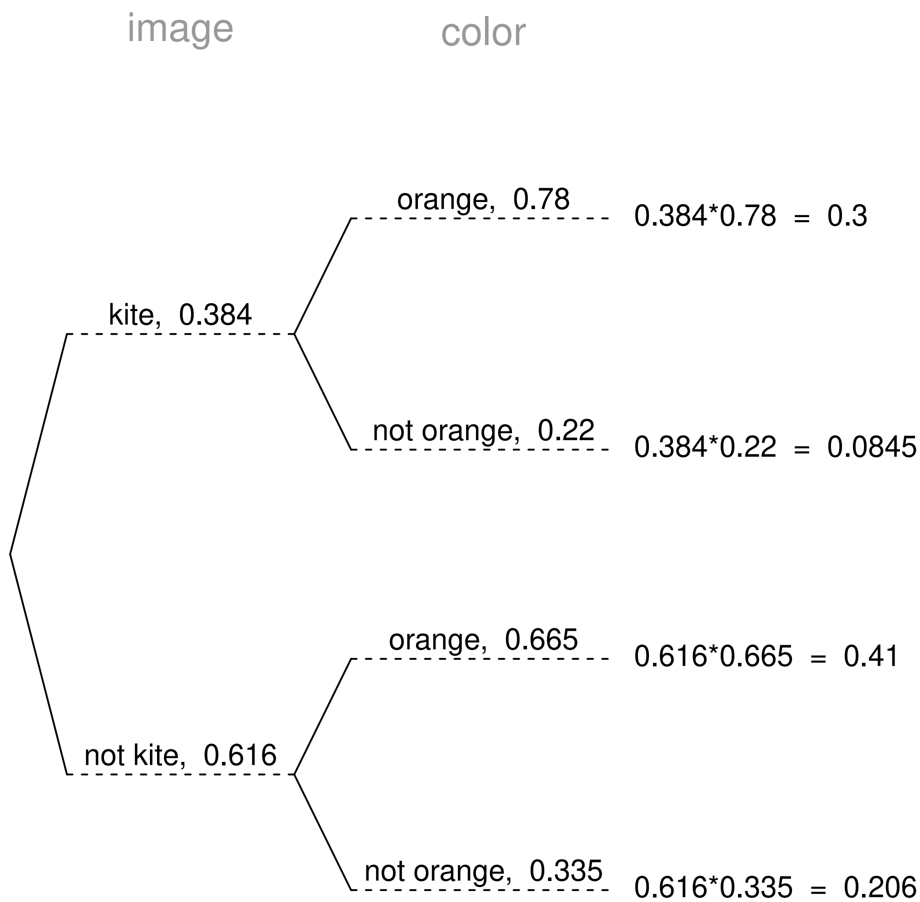
(f) $P(\text{shovel and teal}) = \frac{83}{566} = 0.147$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a kite is 38.4%. If a kite is drawn, there is a 78% chance that it is orange. If a card that is not a kite is drawn, there is a 66.5% chance that it is orange.

   Now, someone draws a random card and reveals it is orange. What is the chance the card is not a kite?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not kite" given "orange"}) = \frac{0.41}{0.41 + 0.3} = 0.577$$

3. In a very large pile of toothpicks, the mean length is 67.58 millimeters and the standard deviation is 1.2 millimeters. If you randomly sample 225 toothpicks, what is the chance the sample mean is between 67.5 and 67.67 millimeters?

Solution

Label the given information.

$$\mu = 67.58$$
$$\sigma = 1.2$$
$$n = 225$$
$$\bar{x}_{\text{lower}} = 67.5$$
$$\bar{x}_{\text{upper}} = 67.67$$

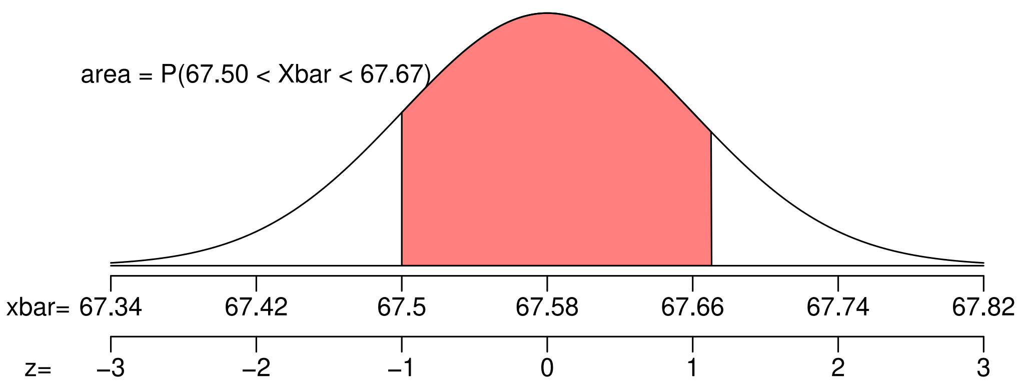
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.2}{\sqrt{225}} = 0.08$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(67.58,\ 0.08)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{67.5 - 67.58}{0.08} = -1$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{67.67 - 67.58}{0.08} = 1.13$$

Determine the probability.

$$\begin{aligned} P(67.5 < X < 67.67) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.13) - \Phi(-1) \\ &= 0.7121 \end{aligned}$$

4. In a game, there is a 63% chance to win a round. You will play 112 rounds.

   (a) What is the probability of winning exactly 73 rounds?

   (b) What is the probability of winning at least 73 but at most 81 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 73) = \binom{112}{73}(0.63)^{73}(1-0.63)^{112-73}$$

$$P(X = 73) = \binom{112}{73}(0.63)^{73}(0.37)^{39}$$

$$P(X = 73) = 0.0704$$

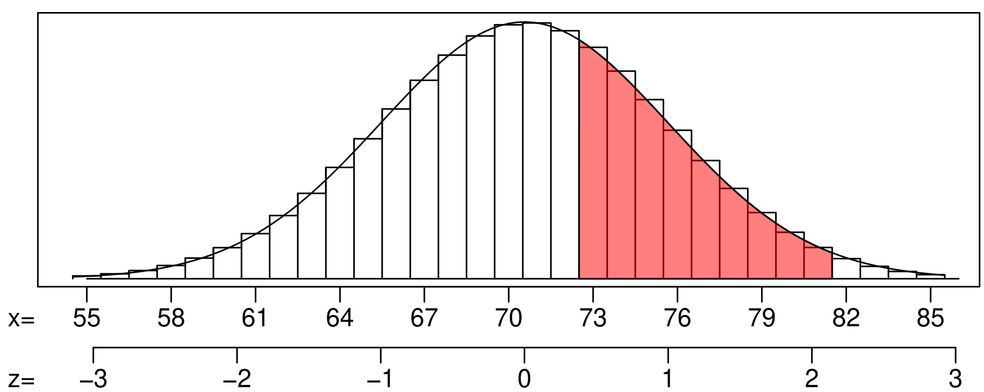
Find the mean.

$$\mu = np = (112)(0.63) = 70.56$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(112)(0.63)(1-0.63)} = 5.1095$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{72.5 - 70.56}{5.1095} = 0.48$$

$$z_2 = \frac{81.5 - 70.56}{5.1095} = 2.04$$

Calculate the probability.

$$P(73 \leq X \leq 81) = \Phi(2.04) - \Phi(0.48) = 0.2949$$

(a) $P(X = 73) = 0.0704$

(b) $P(73 \leq X \leq 81) = 0.2949$

5. As an ornithologist, you wish to determine the average body mass of *Helmitheros vermivorus*. You randomly sample 23 adults of *Helmitheros vermivorus*, resulting in a sample mean of 16.01 grams and a sample standard deviation of 2.66 grams. Determine a 99.5% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 23 \\ \bar{x} &= 16.01 \\ s &= 2.66 \\ CL &= 0.995 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 22$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.995$.

$$t^\star = 3.12$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.66}{\sqrt{23}} = 0.555$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (16.01 - 3.12 \times 0.555,\ 16.01 + 3.12 \times 0.555) \\ &= (14.3,\ 17.7) \end{aligned}$$

We are 99.5% confident that the population mean is between 14.3 and 17.7.

6. A treatment group of size 28 has a mean of 1.06 and standard deviation of 0.108. A control group of size 18 has a mean of 1 and standard deviation of 0.0868. If you decided to use a signficance level of 0.05, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 41.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 28 \\ \bar{x}_1 &= 1.06 \\ s_1 &= 0.108 \\ n_2 &= 18 \\ \bar{x}_2 &= 1 \\ s_2 &= 0.0868 \\ \alpha &= 0.05 \\ df &= 41 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$

$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.05$ by using a t table.

$$t^\star = 2.02$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.108)^2}{28} + \frac{(0.0868)^2}{18}} \\ &= 0.029 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1 - 1.06) - (0)}{0.029} \\ &= -2.08 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.04 < p\text{-value} < 0.05$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.02$

(d) $SE = 0.029$

(e) $|t_{\text{obs}}| = 2.08$

(f) $0.04 < p\text{-value} < 0.05$

(g) reject the null

7. From a very large population, a random sample of 8100 individuals was taken. In that sample, 64.1% were angry. Determine a 98% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.641)(1-0.641)}{8100}} = 0.00533$$

Calculate the margin of error.

$$ME = z^\star SE = (2.33)(0.00533) = 0.0124$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.629\,,\ 0.653)$$

We are 98% confident that the true population proportion is between 62.9% and 65.3%.

(a) The lower bound = 0.629, which can also be expressed as 62.9%.

(b) The upper bound = 0.653, which can also be expressed as 65.3%.

8. An experiment is run with a treatment group of size 72 and a control group of size 32. The results are summarized in the table below.

	treatment	control
omnivorous	47	28
not omnivorous	25	4

Using a significance level of 0.04, determine whether the treatment causes an effect on the proportion of cases that are omnivorous.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.04$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 2.05$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{47}{72} = 0.653$$
$$\hat{p}_2 = \frac{28}{32} = 0.875$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.875 - 0.653 = 0.222$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{47 + 28}{72 + 32} = 0.721$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.721)(0.279)}{72} + \frac{(0.721)(0.279)}{32}} \\ &= 0.0953 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0953)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.875 - 0.653) - 0}{0.0953} \\ &= 2.33 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.33) \\ &= 0.0198 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 2.05$

(d) $SE = 0.0953$

(e) $|z_{\text{obs}}| = 2.33$

(f) p-value = 0.0198

(g) reject the null

1. (a) P(**gem and blue**) = 0.0854

   (b) P(**orange given dog**) = 0.114

   (c) P(**dog**) = 0.183

   (d) P(**yellow**) = 0.154

   (e) P(**gem given red**) = 0.366

   (f) P(**flower or orange**) = 0.316

2. P(**"not shovel" given "pink"**) = 0.75

3. $P(64.77 < X < 65.05) = 0.728$

4. (a) $P(X = 53) = 0.0613$

   (b) $P(45 \leq X \leq 66) = 0.8542$

5. $\mathbf{(15.4, 17)}$

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.65$

   (d) $SE = 0.426$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.54$

   (f) $0.01 <$ p-**value** < 0.02

   (g) **retain**

7. (a) **LB of p CI = 0.27 or** 27%

   (b) **UB of p CI = 0.284 or** 28.4%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.28$

(d) $SE = 0.027$

(e) $\mid z_{\mathbf{obs}} \mid = 1.36$

(f) p-**value** = 0.1738

(g) **reject**

1. In a deck of strange cards, there are 1054 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	blue	orange	red	yellow
dog	46	70	22	23	32
flower	49	36	55	65	26
gem	62	90	67	96	31
tree	80	40	13	78	73

   (a) What is the probability a random card is both a gem and blue?

   (b) What is the probability a random card is orange given it is a dog?

   (c) What is the probability a random card is a dog?

   (d) What is the probability a random card is yellow?

   (e) What is the probability a random card is a gem given it is red?

   (f) What is the probability a random card is either a flower or orange (or both)?

Solution

(a) $P(\text{gem and blue}) = \frac{90}{1054} = 0.0854$

(b) $P(\text{orange given dog}) = \frac{22}{46+70+22+23+32} = 0.114$

(c) $P(\text{dog}) = \frac{46+70+22+23+32}{1054} = 0.183$

(d) $P(\text{yellow}) = \frac{32+26+31+73}{1054} = 0.154$

(e) $P(\text{gem given red}) = \frac{96}{23+65+96+78} = 0.366$

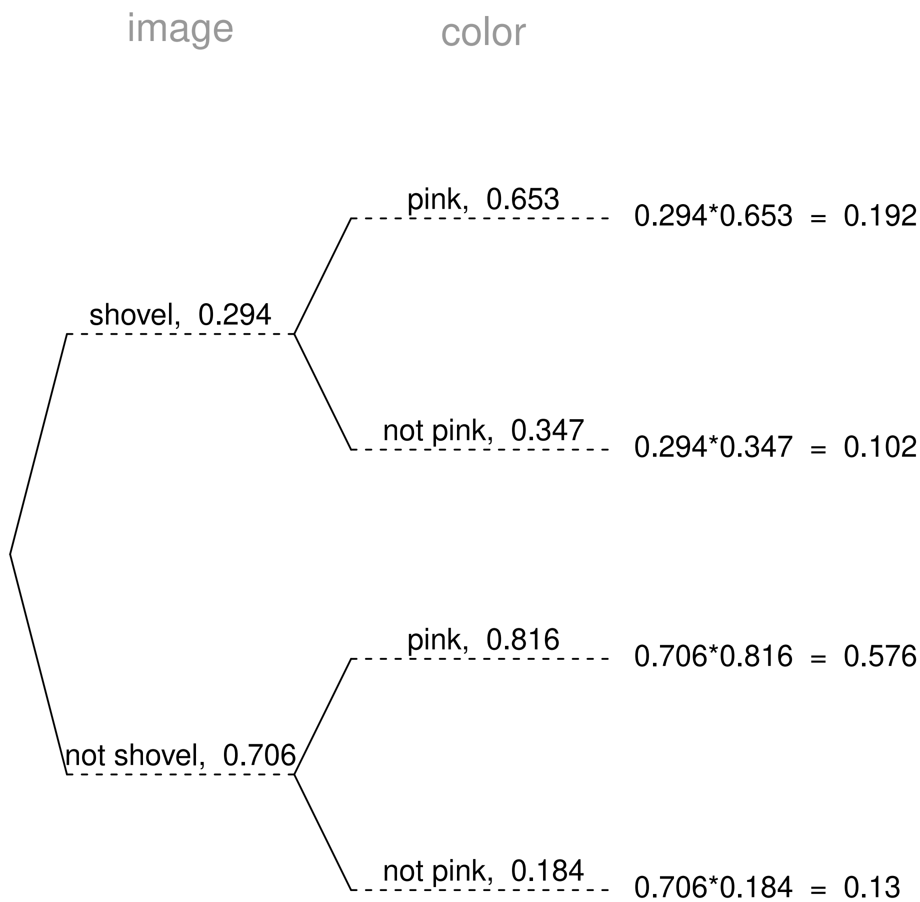
(f) $P(\text{flower or orange}) = \frac{49+36+55+65+26+22+55+67+13-55}{1054} = 0.316$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a shovel is 29.4%. If a shovel is drawn, there is a 65.3% chance that it is pink. If a card that is not a shovel is drawn, there is a 81.6% chance that it is pink.

   Now, someone draws a random card and reveals it is pink. What is the chance the card is not a shovel?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not shovel" given "pink"}) = \frac{0.576}{0.576 + 0.192} = 0.75$$

3. In a very large pile of toothpicks, the mean length is 64.94 millimeters and the standard deviation is 1.74 millimeters. If you randomly sample 196 toothpicks, what is the chance the sample mean is between 64.77 and 65.05 millimeters?

Solution

Label the given information.

$$\mu = 64.94$$
$$\sigma = 1.74$$
$$n = 196$$
$$\bar{x}_{\text{lower}} = 64.77$$
$$\bar{x}_{\text{upper}} = 65.05$$

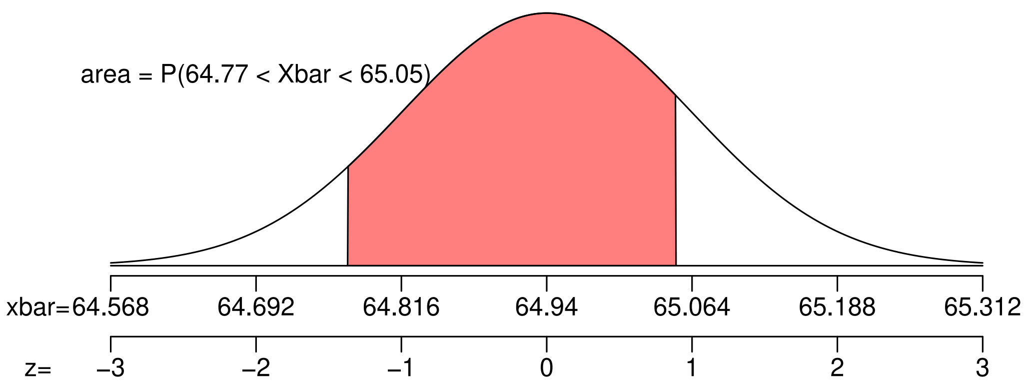
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.74}{\sqrt{196}} = 0.124$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(64.94,\ 0.124)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{64.77 - 64.94}{0.124} = -1.37$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{65.05 - 64.94}{0.124} = 0.89$$

Determine the probability.

$$\begin{aligned} P(64.77 < X < 65.05) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(0.89) - \Phi(-1.37) \\ &= 0.728 \end{aligned}$$

4. In a game, there is a 21% chance to win a round. You will play 249 rounds.
   (a) What is the probability of winning exactly 53 rounds?
   (b) What is the probability of winning at least 45 but at most 66 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 53) = \binom{249}{53}(0.21)^{53}(1-0.21)^{249-53}$$

$$P(X = 53) = \binom{249}{53}(0.21)^{53}(0.79)^{196}$$

$$P(X = 53) = 0.0613$$

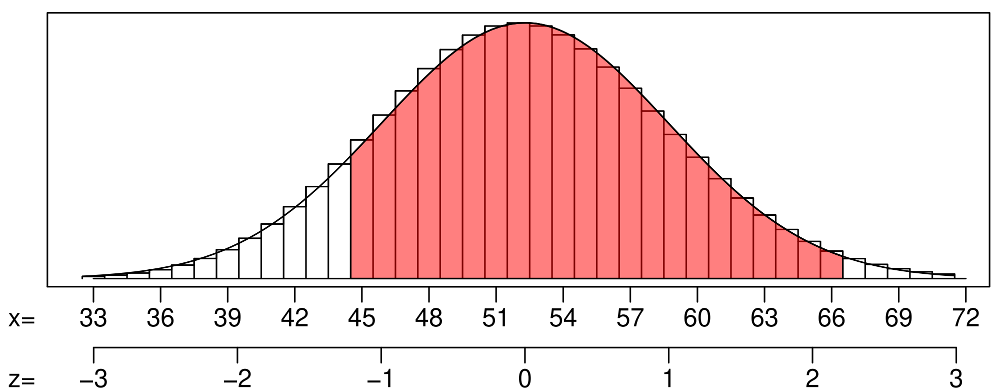
Find the mean.

$$\mu = np = (249)(0.21) = 52.29$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(249)(0.21)(1-0.21)} = 6.4272$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{44.5 - 52.29}{6.4272} = -1.13$$

$$z_2 = \frac{66.5 - 52.29}{6.4272} = 2.13$$

Calculate the probability.

$$P(45 \leq X \leq 66) = \Phi(2.13) - \Phi(-1.13) = 0.8542$$

(a) $P(X = 53) = 0.0613$

(b) $P(45 \leq X \leq 66) = 0.8542$

5. As an ornithologist, you wish to determine the average body mass of *Helmitheros vermivorus*. You randomly sample 18 adults of *Helmitheros vermivorus*, resulting in a sample mean of 16.21 grams and a sample standard deviation of 2.5 grams. Determine a 80% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 18 \\ \bar{x} &= 16.21 \\ s &= 2.5 \\ CL &= 0.8 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 17$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.8$.

$$t^{\star} = 1.33$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.5}{\sqrt{18}} = 0.589$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^{\star} SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^{\star} SE,\ \bar{x} + t^{\star} SE) \\ &= (16.21 - 1.33 \times 0.589,\ 16.21 + 1.33 \times 0.589) \\ &= (15.4,\ 17) \end{aligned}$$

We are 80% confident that the population mean is between 15.4 and 17.

6. A treatment group of size 40 has a mean of 11 and standard deviation of 1.89. A control group of size 30 has a mean of 9.92 and standard deviation of 1.66. If you decided to use a signficance level of 0.01, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 66.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 40$$
$$\bar{x}_1 = 11$$
$$s_1 = 1.89$$
$$n_2 = 30$$
$$\bar{x}_2 = 9.92$$
$$s_2 = 1.66$$
$$\alpha = 0.01$$
$$df = 66$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.01$ by using a t table.

$$t^\star = 2.65$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(1.89)^2}{40} + \frac{(1.66)^2}{30}} \\ &= 0.426 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(9.92 - 11) - (0)}{0.426} \\ &= -2.54 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^{\star} = 2.65$

(d) $SE = 0.426$

(e) $|t_{obs}| = 2.54$

(f) $0.01 < p\text{-value} < 0.02$

(g) retain the null

7. From a very large population, a random sample of 6400 individuals was taken. In that sample, 27.7% were tasty. Determine a 80% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.8$.

$$z^{\star} = 1.28$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.277)(1-0.277)}{6400}} = 0.00559$$

Calculate the margin of error.

$$ME = z^{\star}SE = (1.28)(0.00559) = 0.00716$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.27\,,\ 0.284)$$

We are 80% confident that the true population proportion is between 27% and 28.4%.

(a) The lower bound = 0.27, which can also be expressed as 27%.

(b) The upper bound = 0.284, which can also be expressed as 28.4%.

8. An experiment is run with a treatment group of size 172 and a control group of size 157. The results are summarized in the table below.

	treatment	control
sick	14	7
not sick	158	150

Using a significance level of 0.2, determine whether the treatment causes an effect on the proportion of cases that are sick.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.2$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.28$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{14}{172} = 0.0814$$
$$\hat{p}_2 = \frac{7}{157} = 0.0446$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.0446 - 0.0814 = -0.0368$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{14 + 7}{172 + 157} = 0.0638$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.0638)(0.9362)}{172} + \frac{(0.0638)(0.9362)}{157}} \\ &= 0.027 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\, 0.027)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.0446 - 0.0814) - 0}{0.027} \\ &= -1.36 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-1.36) \\ &= 0.1738 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \ \ p_2 - p_1 = 0$

(b) $H_A: \ \ p_2 - p_1 \neq 0$

(c) $z^\star = 1.28$

(d) $SE = 0.027$

(e) $|z_{obs}| = 1.36$

(f) p-value = 0.1738

(g) reject the null

1. (a) P(**bike or yellow**) = 0.376

   (b) P(**blue**) = 0.21

   (c) P(**pig**) = 0.201

   (d) P(**wheel and blue**) = 0.0604

   (e) P(**teal given gem**) = 0.599

   (f) P(**wheel given blue**) = 0.288

2. P(**"kite" given "green"**) = 0.0572

3. $P(61.1 < X < 61.83) = 0.7739$

4. (a) $P(X = 30) = 0.0558$

   (b) $P(24 \leq X \leq 41) = 0.9097$

5. $\mathbf{(29.6, 41)}$

6. (a) $H_0 : \quad \mu_2 - \mu_1 = 0$

   (b) $H_0 : \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.4$

   (d) $SE = 0.062$

   (e) $\mid t_{\mathbf{obs}} \mid = 2.1$

   (f) $0.04 < p$-**value** < 0.05

   (g) **retain**

7. (a) **LB of p CI = 0.138 or** 13.8%

   (b) **UB of p CI = 0.252 or** 25.2%

8. (a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.64$

(d) $SE = 0.09$

(e) $\mid z_{\mathbf{obs}} \mid = 1.92$

(f) p-**value** = 0.0548

(g) **reject**

1. In a deck of strange cards, there are 1043 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	orange	teal	yellow
bike	16	22	92	34
gem	28	17	82	10
pig	51	52	24	83
shovel	61	38	60	39
wheel	63	86	89	96

(a) What is the probability a random card is either a bike or yellow (or both)?

(b) What is the probability a random card is blue?

(c) What is the probability a random card is a pig?

(d) What is the probability a random card is both a wheel and blue?

(e) What is the probability a random card is teal given it is a gem?

(f) What is the probability a random card is a wheel given it is blue?

Solution

(a) P(bike or yellow) = $\frac{16+22+92+34+34+10+83+39+96-34}{1043}$ = 0.376

(b) P(blue) = $\frac{16+28+51+61+63}{1043}$ = 0.21

(c) P(pig) = $\frac{51+52+24+83}{1043}$ = 0.201

(d) P(wheel and blue) = $\frac{63}{1043}$ = 0.0604

(e) P(teal given gem) = $\frac{82}{28+17+82+10}$ = 0.599

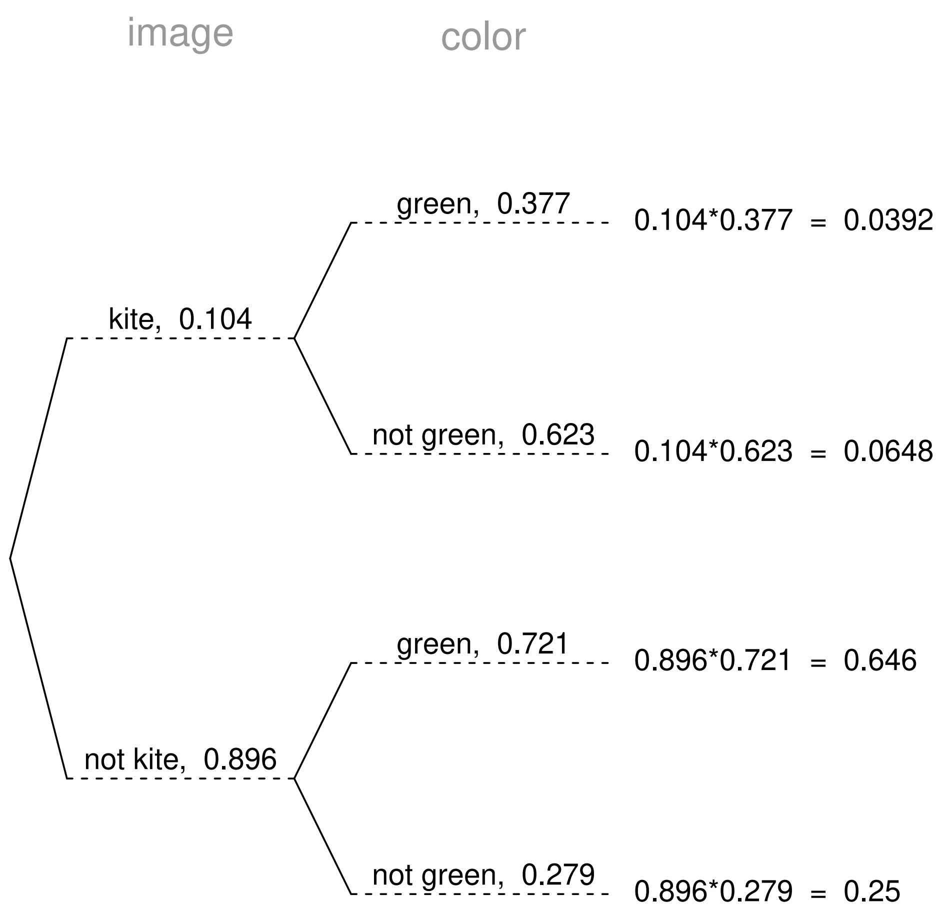
(f) P(wheel given blue) = $\frac{63}{16+28+51+61+63}$ = 0.288

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a kite is 10.4%. If a kite is drawn, there is a 37.7% chance that it is green. If a card that is not a kite is drawn, there is a 72.1% chance that it is green.

   Now, someone draws a random card and reveals it is green. What is the chance the card is a kite?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"kite" given "green"}) = \frac{0.0392}{0.0392 + 0.646} = 0.0572$$

3. In a very large pile of toothpicks, the mean length is 61.42 millimeters and the standard deviation is 3.94 millimeters. If you randomly sample 175 toothpicks, what is the chance the sample mean is between 61.1 and 61.83 millimeters?

Solution

Label the given information.

$$\mu = 61.42$$
$$\sigma = 3.94$$
$$n = 175$$
$$\bar{x}_{\text{lower}} = 61.1$$
$$\bar{x}_{\text{upper}} = 61.83$$

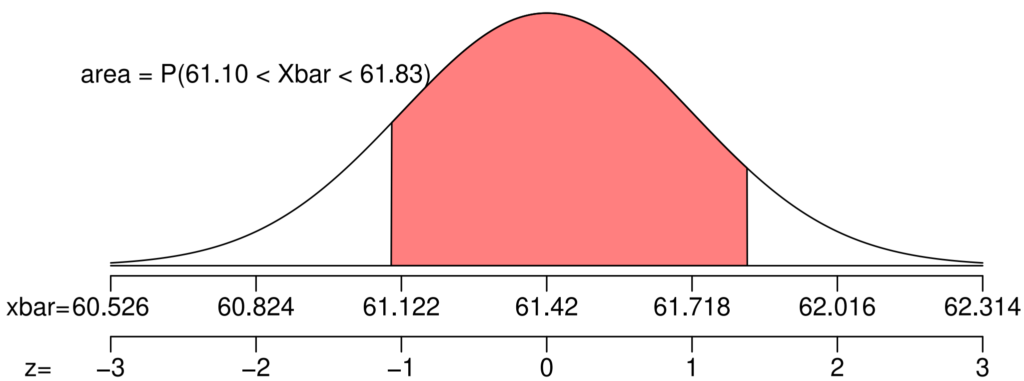
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.94}{\sqrt{175}} = 0.298$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(61.42, 0.298)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{61.1 - 61.42}{0.298} = -1.07$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{61.83 - 61.42}{0.298} = 1.38$$

Determine the probability.

$$\begin{aligned} P(61.1 < X < 61.83) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.38) - \Phi(-1.07) \\ &= 0.7739 \end{aligned}$$

4. In a game, there is a 37% chance to win a round. You will play 93 rounds.

   (a) What is the probability of winning exactly 30 rounds?

   (b) What is the probability of winning at least 24 but at most 41 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 30) = \binom{93}{30}(0.37)^{30}(1-0.37)^{93-30}$$

$$P(X = 30) = \binom{93}{30}(0.37)^{30}(0.63)^{63}$$

$$P(X = 30) = 0.0558$$

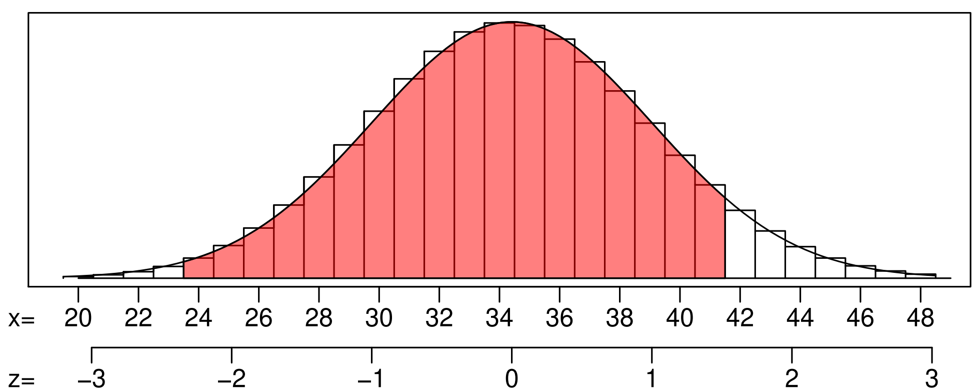
Find the mean.

$$\mu = np = (93)(0.37) = 34.41$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(93)(0.37)(1-0.37)} = 4.656$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{23.5 - 34.41}{4.656} = -2.24$$

$$z_2 = \frac{41.5 - 34.41}{4.656} = 1.42$$

Calculate the probability.

$$P(24 \leq X \leq 41) = \Phi(1.42) - \Phi(-2.24) = 0.9097$$

(a) $P(X = 30) = 0.0558$
(b) $P(24 \leq X \leq 41) = 0.9097$

5. As an ornithologist, you wish to determine the average body mass of *Dolichonyx orizivorus*. You randomly sample 20 adults of *Dolichonyx orizivorus*, resulting in a sample mean of 35.29 grams and a sample standard deviation of 8.9 grams. Determine a 99% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 20 \\ \bar{x} &= 35.29 \\ s &= 8.9 \\ CL &= 0.99 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 19$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.99$.

$$t^\star = 2.86$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{8.9}{\sqrt{20}} = 1.99$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (35.29 - 2.86 \times 1.99,\ 35.29 + 2.86 \times 1.99) \\ &= (29.6,\ 41) \end{aligned}$$

We are 99% confident that the population mean is between 29.6 and 41.

6. A treatment group of size 23 has a mean of 1.14 and standard deviation of 0.209. A control group of size 31 has a mean of 1.01 and standard deviation of 0.245. If you decided to use a signficance level of 0.02, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 50.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 23$$
$$\bar{x}_1 = 1.14$$
$$s_1 = 0.209$$
$$n_2 = 31$$
$$\bar{x}_2 = 1.01$$
$$s_2 = 0.245$$
$$\alpha = 0.02$$
$$df = 50$$

State the hypotheses.

$$H_0 : \quad \mu_2 - \mu_1 = 0$$
$$H_A : \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.02$ by using a t table.

$$t^\star = 2.4$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.209)^2}{23} + \frac{(0.245)^2}{31}} \\ &= 0.062 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1.01 - 1.14) - (0)}{0.062} \\ &= -2.1 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.04 < p\text{-value} < 0.05$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0 : \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \quad \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.4$

(d) $SE = 0.062$

(e) $|t_{obs}| = 2.1$

(f) $0.04 < p\text{-value} < 0.05$

(g) retain the null

7. From a very large population, a random sample of 380 individuals was taken. In that sample, 19.5% were purple. Determine a 99.5% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.995$.

$$z^{\star} = 2.81$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.195)(1-0.195)}{380}} = 0.0203$$

Calculate the margin of error.

$$ME = z^{\star}SE = (2.81)(0.0203) = 0.057$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.138\,,\ 0.252)$$

We are 99.5% confident that the true population proportion is between 13.8% and 25.2%.

(a) The lower bound = 0.138, which can also be expressed as 13.8%.

(b) The upper bound = 0.252, which can also be expressed as 25.2%.

8. An experiment is run with a treatment group of size 72 and a control group of size 52. The results are summarized in the table below.

	treatment	control
organic	36	35
not organic	36	17

Using a significance level of 0.1, determine whether the treatment causes an effect on the proportion of cases that are organic.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution
State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.1$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.64$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{36}{72} = 0.5$$
$$\hat{p}_2 = \frac{35}{52} = 0.673$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.673 - 0.5 = 0.173$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{36 + 35}{72 + 52} = 0.573$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.573)(0.427)}{72} + \frac{(0.573)(0.427)}{52}} \\ &= 0.09 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\, 0.09)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.673 - 0.5) - 0}{0.09} \\ &= 1.92 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-1.92) \\ &= 0.0548 \end{aligned}$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 1.64$

(d) $SE = 0.09$

(e) $|z_{\text{obs}}| = 1.92$

(f) p-value = 0.0548

(g) reject the null

1. (a) P(**green given wheel**) = 0.179

   (b) P(**flower or teal**) = 0.43

   (c) P(**pink**) = 0.23

   (d) P(**tree**) = 0.289

   (e) P(**shovel given pink**) = 0.351

   (f) P(**tree and yellow**) = 0.091

2. P(**"tree" given "not pink"**) = 0.427

3. $P(74.6 < X < 75) = 0.8912$

4. (a) $P(X = 44) = 0.0581$

   (b) $P(33 \leq X \leq 38) = 0.2492$

5. **(13.8, 15.9)**

6. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.45$

   (d) $SE = 56.41$

   (e) $\mid t_{\textbf{obs}} \mid = 2.73$

   (f) $0.01 < p$**-value** < 0.02

   (g) **reject**

7. (a) **LB of p CI = 0.624 or** 62.4%

   (b) **UB of p CI = 0.656 or** 65.6%

8. (a) $H_0: \ p_2 - p_1 = 0$

(b) $H_{\mathbf{A}}: \ p_2 - p_1 \neq 0$

(c) $z^\star = 1.64$

(d) $SE = 0.038$

(e) $|\ z_{\mathbf{obs}}\ | = 1.85$

(f) p-**value** $= 0.0644$

(g) **reject**

1. In a deck of strange cards, there are 758 cards. Each card has an image and a color. The amounts are shown in the table below.

	green	pink	teal	yellow
flower	50	21	57	28
shovel	24	61	27	47
tree	22	49	79	69
wheel	40	43	64	77

(a) What is the probability a random card is green given it is a wheel?

(b) What is the probability a random card is either a flower or teal (or both)?

(c) What is the probability a random card is pink?

(d) What is the probability a random card is a tree?

(e) What is the probability a random card is a shovel given it is pink?

(f) What is the probability a random card is both a tree and yellow?

Solution

(a) $P(\text{green given wheel}) = \frac{40}{40+43+64+77} = 0.179$

(b) $P(\text{flower or teal}) = \frac{50+21+57+28+57+27+79+64-57}{758} = 0.43$

(c) $P(\text{pink}) = \frac{21+61+49+43}{758} = 0.23$

(d) $P(\text{tree}) = \frac{22+49+79+69}{758} = 0.289$

(e) $P(\text{shovel given pink}) = \frac{61}{21+61+49+43} = 0.351$

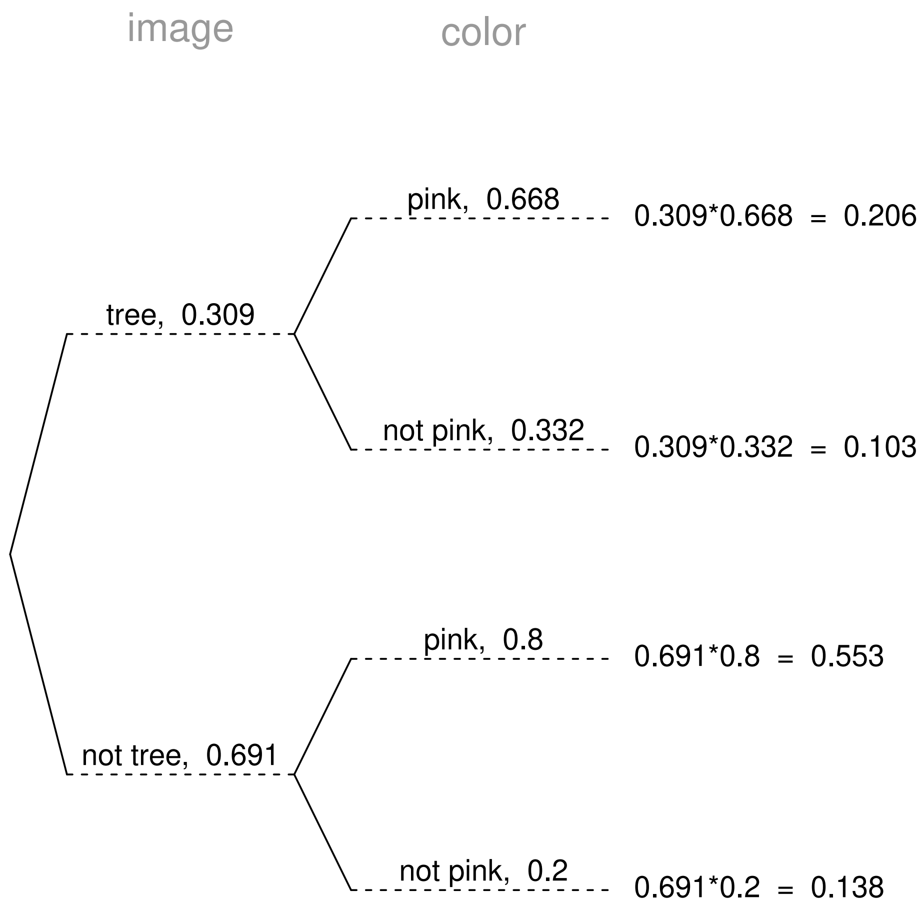
(f) $P(\text{tree and yellow}) = \frac{69}{758} = 0.091$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a tree is 30.9%. If a tree is drawn, there is a 66.8% chance that it is pink. If a card that is not a tree is drawn, there is a 80% chance that it is pink.

   Now, someone draws a random card and reveals it is not pink. What is the chance the card is a tree?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"tree" given "not pink"}) = \frac{0.103}{0.103 + 0.138} = 0.427$$

3. In a very large pile of toothpicks, the mean length is 74.82 millimeters and the standard deviation is 1.37 millimeters. If you randomly sample 125 toothpicks, what is the chance the sample mean is between 74.6 and 75 millimeters?

Solution
Label the given information.

$$\mu = 74.82$$
$$\sigma = 1.37$$
$$n = 125$$
$$\bar{x}_{\text{lower}} = 74.6$$
$$\bar{x}_{\text{upper}} = 75$$

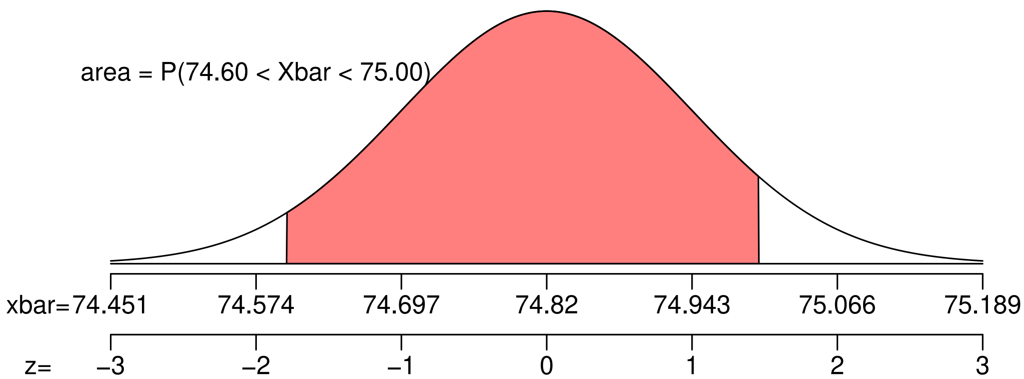
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.37}{\sqrt{125}} = 0.123$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(74.82,\ 0.123)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{x_{\text{lower}} - \mu}{SE} = \frac{74.6 - 74.82}{0.123} = -1.79$$

$$z_{\text{upper}} = \frac{x_{\text{upper}} - \mu}{SE} = \frac{75 - 74.82}{0.123} = 1.46$$

Determine the probability.

$$\begin{aligned} P(74.6 < X < 75) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.46) - \Phi(-1.79) \\ &= 0.8912 \end{aligned}$$

4. In a game, there is a 33% chance to win a round. You will play 122 rounds.
   (a) What is the probability of winning exactly 44 rounds?
   (b) What is the probability of winning at least 33 but at most 38 rounds?

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 44) = \binom{122}{44}(0.33)^{44}(1-0.33)^{122-44}$$

$$P(X = 44) = \binom{122}{44}(0.33)^{44}(0.67)^{78}$$

$$P(X = 44) = 0.0581$$

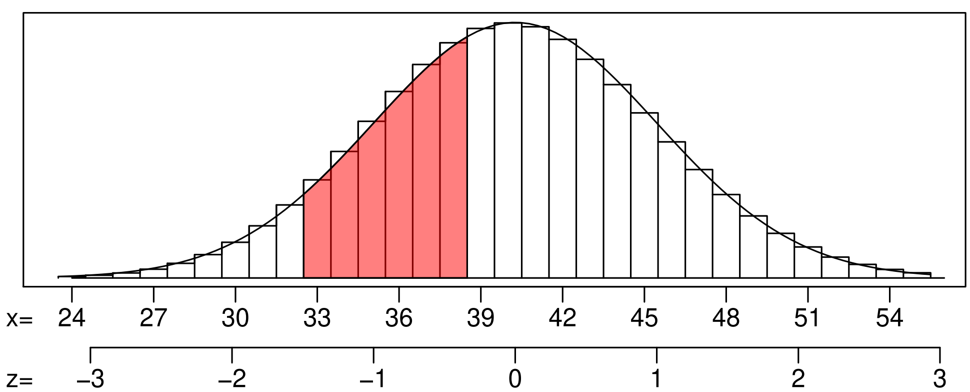
Find the mean.

$$\mu = np = (122)(0.33) = 40.26$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(122)(0.33)(1-0.33)} = 5.1937$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{32.5 - 40.26}{5.1937} = -1.4$$

$$z_2 = \frac{38.5 - 40.26}{5.1937} = -0.44$$

Calculate the probability.

$$P(33 \leq X \leq 38) = \Phi(-0.44) - \Phi(-1.4) = 0.2492$$

(a) $P(X = 44) = 0.0581$

(b) $P(33 \leq X \leq 38) = 0.2492$

5. As an ornithologist, you wish to determine the average body mass of *Passerina cyanea*. You randomly sample 31 adults of *Passerina cyanea*, resulting in a sample mean of 14.86 grams and a sample standard deviation of 2.37 grams. Determine a 98% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 31 \\ \bar{x} &= 14.86 \\ s &= 2.37 \\ CL &= 0.98 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 30$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.98$.

$$t^\star = 2.46$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.37}{\sqrt{31}} = 0.426$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (14.86 - 2.46 \times 0.426,\ 14.86 + 2.46 \times 0.426) \\ &= (13.8,\ 15.9) \end{aligned}$$

We are 98% confident that the population mean is between 13.8 and 15.9.

6. A treatment group of size 32 has a mean of 966 and standard deviation of 204. A control group of size 15 has a mean of 1120 and standard deviation of 168. If you decided to use a signficance level of 0.02, is there sufficient evidence to conclude the treatment causes an effect?

   By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 32.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

   (d) Determine the standard error of the relevant sampling distribution.

   (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

   (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

   (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 32 \\ \bar{x}_1 &= 966 \\ s_1 &= 204 \\ n_2 &= 15 \\ \bar{x}_2 &= 1120 \\ s_2 &= 168 \\ \alpha &= 0.02 \\ df &= 32 \end{aligned}$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$

$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.02$ by using a t table.

$$t^\star = 2.45$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(204)^2}{32} + \frac{(168)^2}{15}} \\ &= 56.41 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(1120 - 966) - (0)}{56.41} \\ &= 2.73 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| > t^\star$$

We can determine an interval for the p-value using the t table.

$$0.01 < p\text{-value} < 0.02$$

Compare p-value and α.

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A: \ \mu_2 - \mu_1 \neq 0$

(c) $t^\star = 2.45$

(d) $SE = 56.41$

(e) $|t_{obs}| = 2.73$

(f) $0.01 < p\text{-value} < 0.02$

(g) reject the null

7. From a very large population, a random sample of 3600 individuals was taken. In that sample, 64% were glowing. Determine a 95% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.95$.

$$z^\star = 1.96$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.64)(1-0.64)}{3600}} = 0.008$$

Calculate the margin of error.

$$ME = z^\star SE = (1.96)(0.008) = 0.0157$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.624\,,\ 0.656)$$

We are 95% confident that the true population proportion is between 62.4% and 65.6%.

(a) The lower bound = 0.624, which can also be expressed as 62.4%.

(b) The upper bound = 0.656, which can also be expressed as 65.6%.

8. An experiment is run with a treatment group of size 195 and a control group of size 165. The results are summarized in the table below.

	treatment	control
omnivorous	172	134
not omnivorous	23	31

Using a significance level of 0.1, determine whether the treatment causes an effect on the proportion of cases that are omnivorous.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0: \quad p_2 - p_1 = 0$$
$$H_A: \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.1$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.64$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{172}{195} = 0.882$$
$$\hat{p}_2 = \frac{134}{165} = 0.812$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.812 - 0.882 = -0.07$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{172 + 134}{195 + 165} = 0.85$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.85)(0.15)}{195} + \frac{(0.85)(0.15)}{165}} \\ &= 0.0378 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0: \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0,\ 0.0378)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.812 - 0.882) - 0}{0.0378} \\ &= -1.85 \end{aligned}$$

Determine the p-value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-1.85) \\ &= 0.0644 \end{aligned}$$

Compare the *p*-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^{\star} = 1.64$

(d) $SE = 0.0378$

(e) $|z_{\text{obs}}| = 1.85$

(f) p-value = 0.0644

(g) reject the null