

1. Problem

An experiment has $n_1 = 6$ plants in the treatment group and $n_2 = 8$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	0.81	0.98	1.39	1.34	0.78	1.11		
sample 2:	1.31	1.3	1.45	1.42	1.22	1.37	1.34	1.31

- Determine degrees of freedom.
- Determine t^* for a 98% confidence interval.
- Determine SE .
- Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
- Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
- Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
- Determine a lower bound of the two-tail p -value.
- Determine an upper bound of two-tail p -value.
- Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.02$? (yes or no)

Solution

These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\bar{x}_1 = 1.07$$

$$\bar{x}_2 = 1.34$$

$$s_1 = 0.259$$

$$s_2 = 0.0729$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(6, 8) - 1 = 5$$

We use the t table to find t^* such that $P(|T| < t^*) = 0.98$

$$t^* = 3.36$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.259)^2}{6} + \frac{(0.0729)^2}{8}} = 0.109$$

We find the bounds of the confidence interval.

$$CI = (\bar{x}_2 - \bar{x}_1) \pm t^* SE$$

$$CI = (-0.096, 0.636)$$

We find t_{obs} .

$$t_{\text{obs}} = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.34 - 1.07) - 0}{0.109} = 2.48$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 2.48$$

We use the table to determine bounds on p -value. Remember, $df = 5$ and $p\text{-value} = P(|T| > |t_{\text{obs}}|)$.

$$0.05 < p\text{-value} < 0.1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^*$$

$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

- (a) 5
- (b) 3.36
- (c) 0.109
- (d) -0.096
- (e) 0.636
- (f) 2.481
- (g) 0.05
- (h) 0.1
- (i) no