Name: ______________________________

1. **Problem**
An experiment has $n_1 = 6$ plants in the treatment group and $n_2 = 7$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	91	118	144	104	118	141	
sample 2:	97	120	81	87	97	91	112

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 90% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 90% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 90% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.1$? (yes or no)

1. (a) 5.000

(b) 2.020

(c) 9.896

(d) -41.090

(e) -1.110

(f) 2.132

(g) 0.050

(h) 0.100

(i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 119$$
$$\overline{x_2} = 97.9$$
$$s_1 = 20.6$$
$$s_2 = 13.8$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(6, 7) - 1 = 5$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.9$

$$t^\star = 2.02$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(20.6)^2}{6} + \frac{(13.8)^2}{7}} = 9.896$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-41.09, -1.11)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(97.9 - 119) - 0}{9.896} = -2.13$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 2.13$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.05 < p\text{-value} < 0.1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 5
(b) 2.02
(c) 9.896
(d) -41.09
(e) -1.11
(f) 2.132
(g) 0.05
(h) 0.1
(i) yes