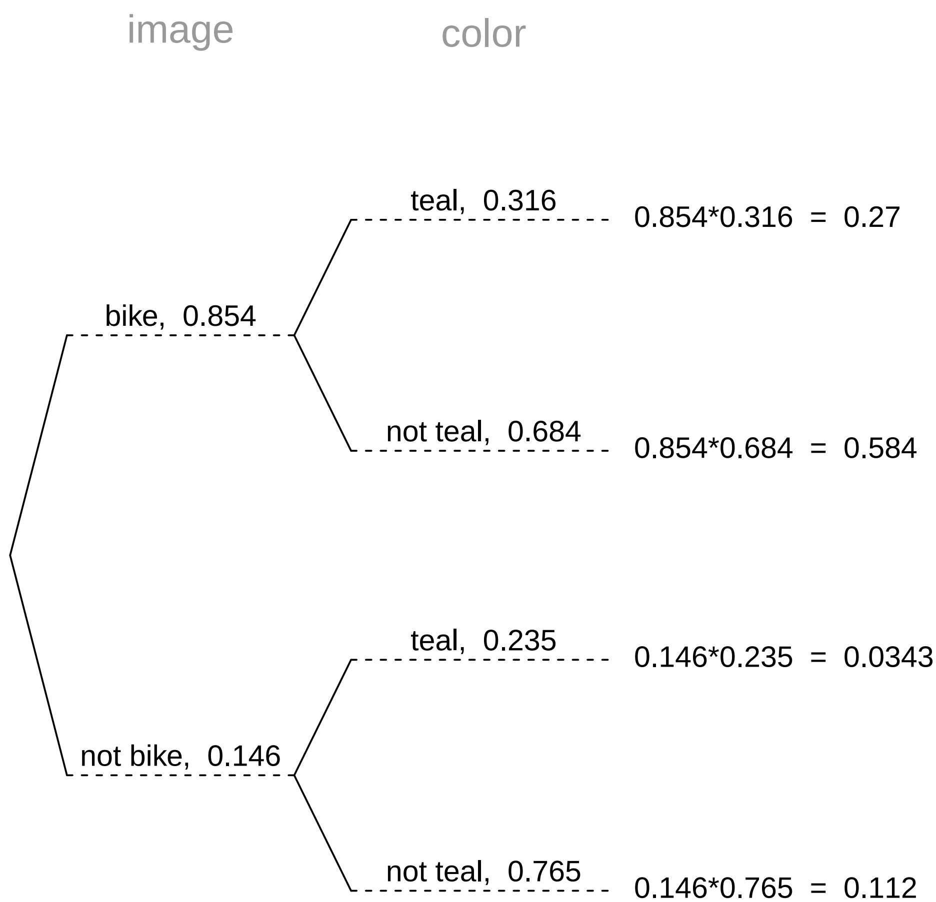
1. **Problem**

   In a deck of strange cards, each card has an image and a color. The chance of drawing a bike is 85.4%. If a bike is drawn, there is a 31.6% chance that it is teal. If a card that is not a bike is drawn, there is a 23.5% chance that it is teal.

   Now, someone draws a random card and reveals it is teal. What is the chance the card is a bike?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"bike" given "teal"}) = \frac{0.27}{0.27 + 0.0343} = 0.887$$

2. **Problem**
A treatment group of size 29 has a mean of 1.11 and standard deviation of 0.239. A control group of size 13 has a mean of 0.953 and standard deviation of 0.254. If you decided to use a signficance level of 0.05, is there sufficient evidence to conclude the treatment causes an effect?

By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 21.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^{\star}$ or $t^{\star}$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution
We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned} n_1 &= 29 \\ \bar{x}_1 &= 1.11 \\ s_1 &= 0.239 \\ n_2 &= 13 \\ \bar{x}_2 &= 0.953 \\ s_2 &= 0.254 \\ \alpha &= 0.05 \\ df &= 21 \end{aligned}$$

State the hypotheses.

$$\begin{aligned} H_0 &: \quad \mu_2 - \mu_1 = 0 \\ H_A &: \quad \mu_2 - \mu_1 \neq 0 \end{aligned}$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star)$ by using a t table.

$$t^\star = 2.08$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.239)^2}{29} + \frac{(0.254)^2}{13}} \\ &= 0.083 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(0.953 - 1.11) - (0)}{0.083} \\ &= -1.89 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and $t^\star$.

$$|t_{\text{obs}}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

3. **Problem**
A roughly symmetric population has a mean $\mu = 220$ and standard deviation $\sigma = 63$. What is the probability that a sample of size $n = 75$ has a mean more than 225.6?

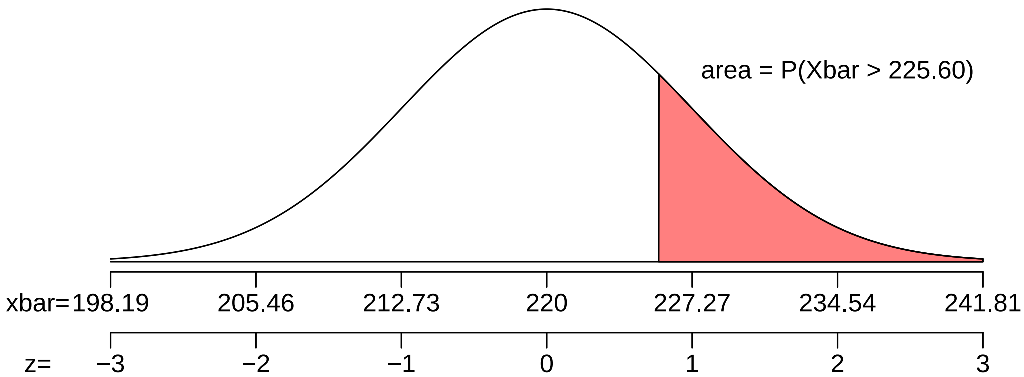
Solution
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{63}{\sqrt{75}} = 7.27$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(220, 7.27)$$

Draw a sketch.



Calculate a z score.

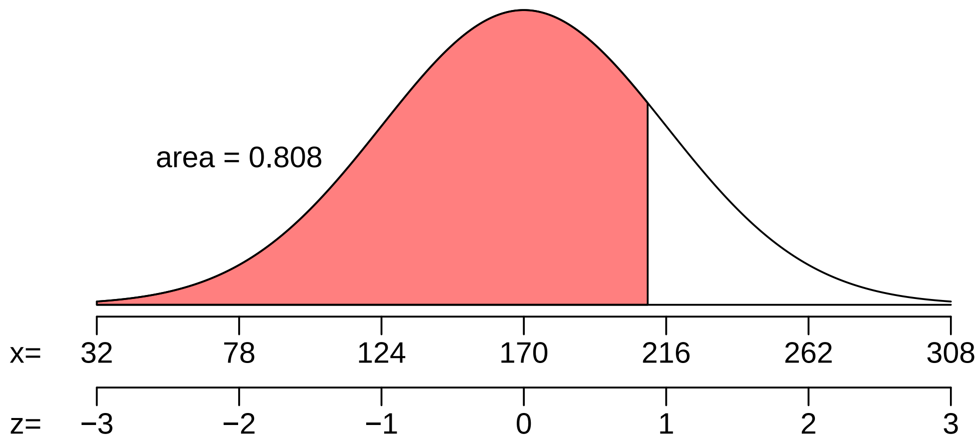
$$z = \frac{225.6 - 220}{7.27} = 0.77$$

Determine the probability.

$$P(X > 225.6) = 0.221$$

4. **Problem**
   Let $X \sim \mathcal{N}(170, 46)$. Determine x such that $P(X < x) = 0.808$.

Solution
We draw a sketch.



We find the z score.

$$z = \Phi^{-1}(0.808) = 0.87$$

We find x.

$$x = \mu + z\sigma = 170 + (0.87)(46) = 210.02$$

5. **Problem**

Let each trial have a chance of success $p = 0.14$. If 157 trials occur, what is the probability of getting at least 15 but less than 26 successes?

In other words, let $X \sim \text{Bin}(n = 157,\ p = 0.14)$ and find $P(15 \leq X < 26)$.

Use a normal approximation along with the continuity correction.

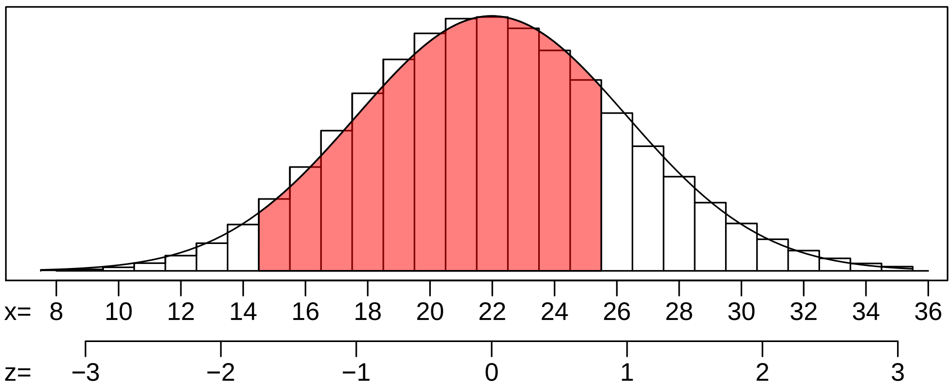
Solution
Find the mean.

$$\mu = np = (157)(0.14) = 21.98$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(157)(0.14)(1-0.14)} = 4.3477$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{14.5 - 21.98}{4.3477} = -1.72$$

$$z_2 = \frac{25.5 - 21.98}{4.3477} = 0.81$$

Calculate the probability.

$$P(15 \leq X < 26) = \Phi(0.81) - \Phi(-1.72) = 0.748$$

6. **Problem**
As an ornithologist, you wish to determine the average body mass of *Catharus guttatus*. You randomly capture 17 adults of *Catharus guttatus*, resulting in a sample mean of 29.52 grams and a sample standard deviation of 1.6 grams. You decide to report a 90% confidence interval.

   (a) Determine the lower bound of the confidence interval.

   (b) Determine the upper bound of the confidence interval.

Solution
We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 17 \\ \bar{x} &= 29.52 \\ s &= 1.6 \\ CL &= 0.9 \end{aligned}$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 16$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.9$.

$$t^\star = 1.75$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.6}{\sqrt{17}} = 0.388$$

We want to make an inference about the population mean.

$$\mu \;\approx\; \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (29.52 - 1.75 \times 0.388,\ 29.52 + 1.75 \times 0.388) \\ &= (28.8,\ 30.2) \end{aligned}$$

We are 90% confident that the population mean is between 28.8 and 30.2.

(a) Lower bound = 28.8

(b) Upper bound = 30.2

7. **Problem**
   Let each trial have a chance of success $p = 0.62$. If 150 trials occur, what is the probability of getting exactly 106 successes?

   In other words, let $X \sim \text{Bin}(n = 150,\ p = 0.62)$ and find $P(X = 106)$.

Solution
We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(X = 106) = \binom{150}{106}(0.62)^{106}(1-0.62)^{150-106}$$
$$P(X = 106) = \binom{150}{106}(0.62)^{106}(0.38)^{44}$$

In a TI calculator, the expression will look like:

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(150 nCr 106)*0.62^106*(1-0.62)^(150-106)
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In Geogebra:

`nCr(150 , 106)`$\cdot 0.62^{106}(1-0.62)^{150-106}$

$$P(X = 106) = 0.006$$

1. $P($**"bike" given "teal"**$) = 0.887$

2. (a) $H_0: \quad \mu_2 - \mu_1 = 0$

   (b) $H_0: \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^{\star} = 2.08$

   (d) $SE = 0.083$

   (e) $\mid t_{\textbf{obs}} \mid = 1.89$

   (f) $0.05 <$ ***p*-value** < 0.1

   (g) **retain**

3. $P(X > 225.6) = 0.221$

4. $x = 210.02$

5. $P(15 \leq X < 26) = 0.748$

6. (a) **LB = 28.8**

   (b) **UB = 30.2**

7. $P(X = 106) = 0.006$