

Name: \_\_\_\_\_

1. Problem

An experiment has  $n_1 = 6$  plants in the treatment group and  $n_2 = 5$  plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	10.6	9.4	9.9	9.4	11.6	9.8
sample 2:	19.4	14.2	14	15.9	15.7	

- (a) Determine degrees of freedom.
- (b) Determine  $t^*$  for a 99% confidence interval.
- (c) Determine  $SE$ .
- (d) Determine a lower bound of the 99% confidence interval of  $\mu_2 - \mu_1$ .
- (e) Determine an upper bound of the 99% confidence interval of  $\mu_2 - \mu_1$ .
- (f) Determine  $|t_{\text{obs}}|$  under the null hypothesis  $\mu_2 - \mu_1 = 0$ .
- (g) Determine a lower bound of the two-tail  $p$ -value.
- (h) Determine an upper bound of two-tail  $p$ -value.
- (i) Do you reject the null hypothesis with a two-tail test using a significance level  $\alpha = 0.01$ ? (yes or no)

1. (a) 

					4
--	--	--	--	--	---

 . 

0	0	0
---	---	---

(b) 

					4
--	--	--	--	--	---

 . 

6	0	0
---	---	---

(c) 

					1
--	--	--	--	--	---

 . 

0	3	1
---	---	---

(d) 

					0
--	--	--	--	--	---

 . 

9	5	7
---	---	---

(e) 

				1	0
--	--	--	--	---	---

 . 

4	4	3
---	---	---

(f) 

					5
--	--	--	--	--	---

 . 

5	3	1
---	---	---

(g) 

					0
--	--	--	--	--	---

 . 

0	0	5
---	---	---

(h) 

					0
--	--	--	--	--	---

 . 

0	1	0
---	---	---

(i) 

yes
-----

## 1. Solution

These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\bar{x}_1 = 10.1$$

$$\bar{x}_2 = 15.8$$

$$s_1 = 0.85$$

$$s_2 = 2.17$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(6, 5) - 1 = 4$$

We use the  $t$  table to find  $t^*$  such that  $P(|T| < t^*) = 0.99$

$$t^* = 4.6$$

We use the  $SE$  formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.85)^2}{6} + \frac{(2.17)^2}{5}} = 1.031$$

We find the bounds of the confidence interval.

$$CI = (\bar{x}_2 - \bar{x}_1) \pm t^* SE$$

$$CI = (0.957, 10.443)$$

We find  $t_{\text{obs}}$ .

$$t_{\text{obs}} = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} = \frac{(15.8 - 10.1) - 0}{1.031} = 5.53$$

We find  $|t_{\text{obs}}|$ .

$$|t_{\text{obs}}| = 5.53$$

We use the table to determine bounds on  $p$ -value. Remember,  $df = 4$  and  $p\text{-value} = P(|T| > |t_{\text{obs}}|)$ .

$$0.005 < p\text{-value} < 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^*$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

- (a) 4
- (b) 4.6
- (c) 1.031
- (d) 0.957
- (e) 10.443
- (f) 5.531
- (g) 0.005
- (h) 0.01
- (i) yes