Bunker Hill Community College

Name: _____

This take-home exam is due **Wednesday, May 8**, at the beginning of class.

You may use any notes, textbook, or online tools; however, you may not request help from any other human.

You will show your work on the pages with questions. When you are sure of your answers, you will **put those answers in the boxes** on the first few pages.

Unless you have an objection to doing so, please **copy the honor-code text below and sign**.

I understand that outside help is NOT allowed on this exam. On my honor, the work herein is my own.

_____

_____

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Signature: _____

1. (a) $P(\textbf{pink}) = 0.232$

   (b) $P(\textbf{flower or black}) = 0.514$

   (c) $P(\textbf{bike}) = 0.26$

   (d) $P(\textbf{gem given pink}) = 0.0491$

   (e) $P(\textbf{wheel and red}) = 0.0248$

   (f) $P(\textbf{pink given gem}) = 0.147$

2. $P(\textbf{"wheel" given "orange"}) = 0.243$

3. $P(61.92 < X < 62.35) = 0.7054$

4. (a) $P(X = 17) = 0.0905$

   (b) $P(13 \leq X \leq 25) = 0.8846$

5. $(14.4, 17.1)$

6. (a) $H_0 : \quad \mu_2 - \mu_1 = 0$

   (b) $H_0 : \quad \mu_2 - \mu_1 \neq 0$

   (c) $t^\star = 2.51$

   (d) $SE = 6.522$

   (e) $\mid t_{\textbf{obs}} \mid = 2.45$

   (f) $0.02 < p\textbf{-value} < 0.04$

   (g) **retain**

7. (a) **LB of p CI = 0.31 or** 31%

   (b) **UB of p CI = 0.33 or** 33%

8. (a) $H_0 : \quad p_2 - p_1 = 0$

    (b) $H_{\mathbf{A}} : \quad p_2 - p_1 \neq 0$

    (c) $z^\star = 1.64$

    (d) $SE = 0.076$

    (e) $\mid z_{\mathbf{obs}} \mid = 1.92$

    (f) $p\text{-}\mathbf{value} = 0.0548$

    (g) **reject**

1. In a deck of strange cards, there are 967 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	pink	red	yellow
bike	76	71	55	49
flower	84	74	68	22
gem	33	11	21	10
pig	73	56	48	79
wheel	67	12	24	34

(a) What is the probability a random card is pink?

(b) What is the probability a random card is either a flower or black (or both)?

(c) What is the probability a random card is a bike?

(d) What is the probability a random card is a gem given it is pink?

(e) What is the probability a random card is both a wheel and red?

(f) What is the probability a random card is pink given it is a gem?

Solution

(a) $P(\text{pink}) = \frac{71+74+11+56+12}{967} = 0.232$

(b) $P(\text{flower or black}) = \frac{84+74+68+22+76+84+33+73+67-84}{967} = 0.514$

(c) $P(\text{bike}) = \frac{76+71+55+49}{967} = 0.26$

(d) $P(\text{gem given pink}) = \frac{11}{71+74+11+56+12} = 0.0491$

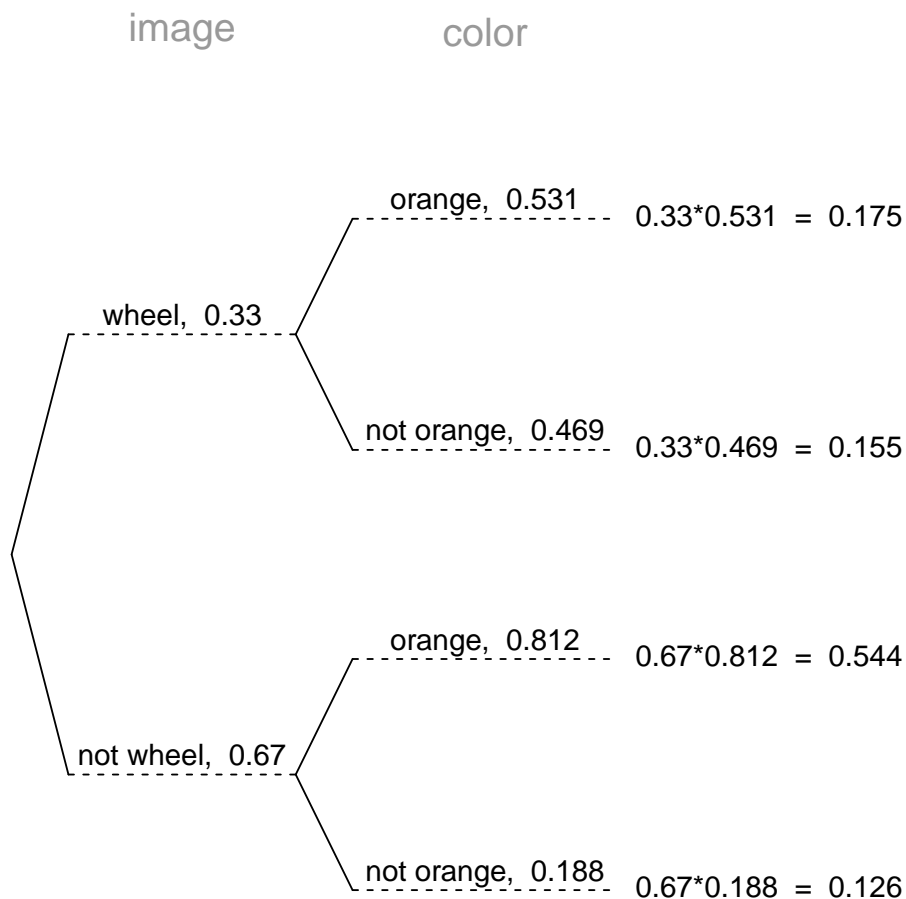(e) $P(\text{wheel and red}) = \frac{24}{967} = 0.0248$

(f) $P(\text{pink given gem}) = \frac{11}{33+11+21+10} = 0.147$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a wheel is 33%. If a wheel is drawn, there is a 53.1% chance that it is orange. If a card that is not a wheel is drawn, there is a 81.2% chance that it is orange.

    Now, someone draws a random card and reveals it is orange. What is the chance the card is a wheel?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilites. The results (products) are joint probabilities.

image   color

orange, 0.531   0.33*0.531 = 0.175

wheel, 0.33

not orange, 0.469   0.33*0.469 = 0.155

orange, 0.812   0.67*0.812 = 0.544

not wheel, 0.67

not orange, 0.188   0.67*0.188 = 0.126

Determine the appropriate conditional probability.

$$P(\text{"wheel" given "orange"}) = \frac{0.175}{0.175 + 0.544} = 0.243$$

3. In a very large pile of toothpicks, the mean length is 62.01 millimeters and the standard deviation is 1.69 millimeters. If you randomly sample 120 toothpicks, what is the chance the sample mean is between 61.92 and 62.35 millimeters?

Solution

Label the given information.

$$\mu = 62.01$$
$$\sigma = 1.69$$
$$n = 120$$
$$\bar{x}_{lower} = 61.92$$
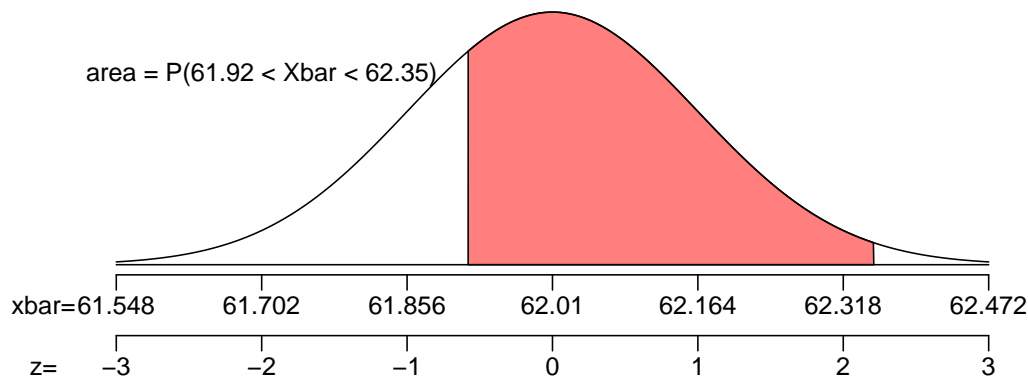$$\bar{x}_{upper} = 62.35$$

Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.69}{\sqrt{120}} = 0.154$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(62.01, \ 0.154)$$

Draw a sketch.

Calculate a z scores.

$$z_{lower} = \frac{x_{lower} - \mu}{SE} = \frac{61.92 - 62.01}{0.154} = -0.58$$

$$z_{upper} = \frac{x_{upper} - \mu}{SE} = \frac{62.35 - 62.01}{0.154} = 2.21$$

Determine the probability.

$$P(61.92 < X < 62.35) = \Phi(z_{upper}) - \Phi(z_{lower})$$
$$= \Phi(2.21) - \Phi(-0.58)$$
$$= 0.7054$$

4. In a game, there is a 10% chance to win a round. You will play 189 rounds.

    (a) What is the probability of winning exactly 17 rounds?

    (b) What is the probability of winning at least 13 but at most 25 rounds?

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k}(p)^k(1 - p)^{n-k}$$

$$P(X = 17) = \binom{189}{17}(0.1)^{17}(1 - 0.1)^{189-17}$$

$$P(X = 17) = \binom{189}{17}(0.1)^{17}(0.9)^{172}$$
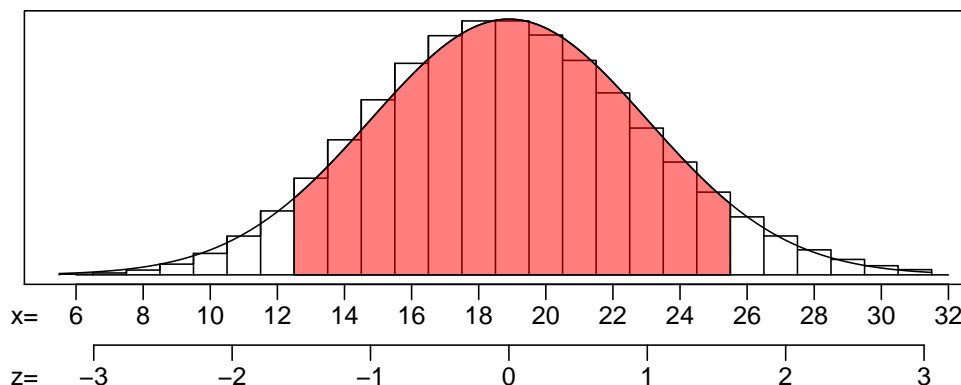
$$P(X = 17) = 0.0905$$

Find the mean.

$$\mu = np = (189)(0.1) = 18.9$$

Find the standard deviation.

$$\sigma = \sqrt{np(1 - p)} = \sqrt{(189)(0.1)(1 - 0.1)} = 4.1243$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.

Find the z scores.

$$z_1 = \frac{12.5 - 18.9}{4.1243} = -1.55$$

$$z_2 = \frac{25.5 - 18.9}{4.1243} = 1.6$$

Calculate the probability.

$$P(13 \leq X \leq 25) = \Phi(1.6) - \Phi(-1.55) = 0.8846$$

(a) $P(X = 17) = 0.0905$

(b) $P(13 \leq X \leq 25) = 0.8846$

5. As an ornithologist, you wish to determine the average body mass of *Oporornis formosus*. You randomly sample 36 adults of *Oporornis formosus*, resulting in a sample mean of 15.75 grams and a sample standard deviation of 2.9 grams. Determine a 99% confidence interval of the true population mean.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 36$$
$$\bar{x} = 15.75$$
$$s = 2.9$$
$$CL = 0.99$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 35$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.99$.

$$t^\star = 2.72$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.9}{\sqrt{36}} = 0.483$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$
\begin{aligned}
CI &= (\bar{x} - t^\star SE, \ \bar{x} + t^\star SE) \\
&= (15.75 - 2.72 \times 0.483, \ 15.75 + 2.72 \times 0.483) \\
&= (14.4, \ 17.1)
\end{aligned}
$$

We are 99% confident that the population mean is between 14.4 and 17.1.

6. A treatment group of size 10 has a mean of 119 and standard deviation of 15.2. A control group of size 18 has a mean of 103 and standard deviation of 18.7. If you decided to use a signficance level of 0.02, is there sufficient evidence to conclude the treatment causes an effect?

    By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 22.

    (a) State the null hypothesis.

    (b) State the alternative hypothesis.

    (c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

    (d) Determine the standard error of the relevant sampling distribution.

    (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

    (f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

    (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 10$$
$$\bar{x}_1 = 119$$
$$s_1 = 15.2$$
$$n_2 = 18$$
$$\bar{x}_2 = 103$$
$$s_2 = 18.7$$
$$\alpha = 0.02$$
$$df = 22$$

State the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find $t^\star$ such that $P(|T| > t^\star) = 0.02$ by using a t table.

$$t^\star = 2.51$$

Calculate the standard error.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}$$
$$= \sqrt{\frac{(15.2)^2}{10} + \frac{(18.7)^2}{18}}$$
$$= 6.522$$

Determine the test statistic.

$$t_{obs} = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE}$$
$$= \frac{(103 - 119) - (0)}{6.522}$$
$$= -2.45$$

Compare $|t_{obs}|$ and $t^\star$.

$$|t_{obs}| < t^\star$$

We can determine an interval for the p-value using the t table.

$$0.02 < p\text{-value} < 0.04$$

Compare p-value and α.

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

(a) $H_0: \quad \mu_2 - \mu_1 = 0$

(b) $H_A : \quad \mu_2 - \mu_1 \neq 0$

(c) $t^* = 2.51$

(d) $SE = 6.522$

(e) $|t_{obs}| = 2.45$

(f) $0.02 < p\text{-value} < 0.04$

(g) retain the null

7. From a very large population, a random sample of 9200 individuals was taken. In that sample, 32% were frigid. Determine a 95% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.95$.

$$z^\star = 1.96$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \sqrt{\frac{(0.32)(1 - 0.32)}{9200}} = 0.00486$$

Calculate the margin of error.

$$ME = z^\star SE = (1.96)(0.00486) = 0.00953$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.31 \,,\; 0.33)$$

We are 95% confident that the true population proportion is between 31% and 33%.

(a) The lower bound = 0.31, which can also be expressed as 31%.

(b) The upper bound = 0.33, which can also be expressed as 33%.

8. An experiment is run with a treatment group of size 68 and a control group of size 95. The results are summarized in the table below.

	treatment	control
omnivorous	38	67
not omnivorous	30	28

Using a significance level of 0.1, determine whether the treatment causes an effect on the proportion of cases that are omnivorous.

(a) State the null hypothesis.

(b) State the alternative hypothesis.

(c) Evaluate the critical value. (The critical value is either $z^\star$ or $t^\star$. Determine its value.)

(d) Determine the standard error of the relevant sampling distribution.

(e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs}. Determine its absolute value.)

(f) If possible, evaluate the p-value. Otherwise, describe an interval containing the p-value.

(g) Do we reject or retain the null?

Solution

State the hypotheses.

$$H_0 : \quad p_2 - p_1 = 0$$
$$H_A : \quad p_2 - p_1 \neq 0$$

Find $z^\star$ such that $P(|Z| > z^\star) = 0.1$.

$$z^\star = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.64$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{38}{68} = 0.559$$

$$\hat{p}_2 = \frac{67}{95} = 0.705$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.705 - 0.559 = 0.146$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{38 + 67}{68 + 95} = 0.644$$

Determine the standard error.

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n_1} + \frac{\hat{p}(1 - \hat{p})}{n_2}}$$

$$= \sqrt{\frac{(0.644)(0.356)}{68} + \frac{(0.644)(0.356)}{95}}$$

$$= 0.0761$$

We can be more specific about what the null hypothesis claims.

$$H_0 : \quad \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.0761)$$

We want to describe how unusual our observation is under the null by finding the p-value. To do so, first find the z score.

$$z = \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE}$$

$$= \frac{(0.705 - 0.559) - 0}{0.0761}$$

$$= 1.92$$

Determine the p-value.

$$p\text{-value} = 2 \cdot \Phi(-|z|)$$
$$= 2 \cdot \Phi(-1.92)$$
$$= 0.0548$$

Compare the p-value to the signficance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a) $H_0: \quad p_2 - p_1 = 0$

(b) $H_A: \quad p_2 - p_1 \neq 0$

(c) $z^\star = 1.64$

(d) $SE = 0.0761$

(e) $|z_{\text{obs}}| = 1.92$

(f) $p\text{-value} = 0.0548$

(g) reject the null