Name: ____________________

1. **Problem**
   An experiment has n_1 = 6 plants in the treatment group and n_2 = 6 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	139	127	120	142	119	142
sample 2:	111	98	94	81	67	125

   (a) Determine degrees of freedom.
   (b) Determine $t^{\star}$ for a 95% confidence interval.
   (c) Determine *SE*.
   (d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail *p*-value.
   (h) Determine an upper bound of two-tail *p*-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 5.000

(b) 2.570

(c) 9.532

(d) -60.497

(e) -11.503

(f) 3.777

(g) 0.010

(h) 0.020

(i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 132$$

$$\overline{x_2} = 96$$

$$s_1 = 10.8$$

$$s_2 = 20.7$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(6, 6) - 1 = 5$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.95$

$$t^\star = 2.57$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(10.8)^2}{6} + \frac{(20.7)^2}{6}} = 9.532$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$

$$CI = (-60.497, -11.503)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(96 - 132) - 0}{9.532} = -3.78$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 3.78$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.01 < p\text{-value} < 0.02$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 5
(b) 2.57
(c) 9.532
(d) -60.497
(e) -11.503
(f) 3.777
(g) 0.01
(h) 0.02
(i) yes