Name: ______________________

1. **Problem**
An experiment has n_1 = 6 plants in the treatment group and n_2 = 3 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	215	232	210	204	217	215
sample 2:	76	104	92			

(a) Determine degrees of freedom.
(b) Determine t^* for a 98% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.02$? (yes or no)

1. (a) 2.000
(b) 6.960
(c) 8.939
(d) -187.515
(e) -63.085
(f) 14.017
(g) 0.005
(h) 0.010
(i) yes

1. **Solution**

These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 216$$
$$\overline{x_2} = 90.7$$
$$s_1 = 9.35$$
$$s_2 = 14$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\ n_2) - 1 = \min(6, 3) - 1 = 2$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.98$

$$t^{\star} = 6.96$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(9.35)^2}{6} + \frac{(14)^2}{3}} = 8.939$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$
$$CI = (-187.515,\ -63.085)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(90.7 - 216) - 0}{8.939} = -14.02$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 14.02$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.005 < p\text{-value} < 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^{\star}$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 2
(b) 6.96
(c) 8.939
(d) -187.515
(e) -63.085
(f) 14.017
(g) 0.005
(h) 0.01
(i) yes