Name: ______________________________

1. **Problem**
An experiment has n_1 = 4 plants in the treatment group and n_2 = 3 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4
sample 1:	10.6	12.7	10.6	10.8
sample 2:	12.4	13	11.9	

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 95% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 2 . 0 0 0
(b) 4 . 3 0 0
(c) 0 . 6 0 1
(d) - 1 . 3 8 4
(e) 3 . 7 8 4
(f) 1 . 9 9 6
(g) 0 . 1 0 0
(h) 0 . 2 0 0
(i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 11.2$$
$$\overline{x_2} = 12.4$$
$$s_1 = 1.02$$
$$s_2 = 0.551$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(4, 3) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.95$

$$t^\star = 4.3$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(1.02)^2}{4} + \frac{(0.551)^2}{3}} = 0.601$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-1.384,\ 3.784)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(12.4 - 11.2) - 0}{0.601} = 2$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 2$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.1 < p\text{-value} < 0.2$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 2
(b) 4.3
(c) 0.601
(d) -1.384
(e) 3.784
(f) 1.996
(g) 0.1
(h) 0.2
(i) no