Name: _____

1. **Problem**
An experiment has n_1 = 3 plants in the treatment group and n_2 = 4 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4
sample 1:	112	114	100	
sample 2:	211	204	233	223

(a) Determine degrees of freedom.
(b) Determine $t^\star$ for a 98% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail *p*-value.
(h) Determine an upper bound of two-tail *p*-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level α = 0.02? (yes or no)

1. (a) **2.000**
(b) **6.960**
(c) **7.750**
(d) **55.060**
(e) **162.940**
(f) **14.065**
(g) **0.005**
(h) **0.010**
(i) **yes**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 109$$

$$\overline{x_2} = 218$$

$$s_1 = 7.57$$

$$s_2 = 12.8$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(3, 4) - 1 \;=\; 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 6.96$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(7.57)^2}{3} + \frac{(12.8)^2}{4}} = 7.75$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$

$$CI \;=\; (55.06,\, 162.94)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(218 - 109) - 0}{7.75} = 14.06$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 14.06$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.005 \;<\; p\text{-value} \;<\; 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 2
(b) 6.96
(c) 7.75
(d) 55.06
(e) 162.94
(f) 14.065
(g) 0.005
(h) 0.01
(i) yes