Name: ______________________________

1. **Problem**
   An experiment has $n_1 = 6$ plants in the treatment group and $n_2 = 6$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	1.22	1.37	1.33	1.14	1.22	1.29
sample 2:	1.21	1.63	1.1	1.01	1.07	0.81

   (a) Determine degrees of freedom.
   (b) Determine t^* for a 95% confidence interval.
   (c) Determine SE.
   (d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 5.000
   (b) 2.570
   (c) 0.117
   (d) -0.421
   (e) 0.181
   (f) 1.022
   (g) 0.200
   (h) 1.000
   (i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1.26$$
$$\overline{x_2} = 1.14$$
$$s_1 = 0.0842$$
$$s_2 = 0.275$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(6, 6) - 1 = 5$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.95$

$$t^\star = 2.57$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.0842)^2}{6} + \frac{(0.275)^2}{6}} = 0.117$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-0.421, 0.181)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.14 - 1.26) - 0}{0.117} = -1.02$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 1.02$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.2 < p\text{-value} < 1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 5
(b) 2.57
(c) 0.117
(d) -0.421
(e) 0.181
(f) 1.022
(g) 0.2
(h) 1
(i) no