1. **Problem**
An experiment has $n_1 = 4$ plants in the treatment group and $n_2 = 6$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	16.4	14.2	19.4	17.3		
sample 2:	10.3	9.9	9.4	11	10.4	10.7

(a) Determine degrees of freedom.
(b) Determine $t^\star$ for a 98% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.02$? (yes or no)

Solution
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 16.8$$
$$\overline{x_2} = 10.3$$
$$s_1 = 2.15$$
$$s_2 = 0.571$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(4, 6) - 1 \;=\; 3$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 4.54$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(2.15)^2}{4} + \frac{(0.571)^2}{6}} = 1.1$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI \;=\; (-11.494,\, -1.506)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(10.3 - 16.8) - 0}{1.1} = -5.91$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 5.91$$

We use the table to determine bounds on p-value. Remember, $df = 3$ and $p\text{-value} = P(|T| > |t_{\text{obs}}|)$.

$$0.005 \ < \ p\text{-value} \ < \ 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^{\star}$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 3

(b) 4.54

(c) 1.1

(d) -11.494

(e) -1.506

(f) 5.909

(g) 0.005

(h) 0.01

(i) yes