Key ID: 023

Name:

1. Problem

An experiment has $n_1 = 3$ plants in the treatment group and $n_2 = 6$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	1.07	0.98	1			
sample 2:	1.86	2.77	2.8	1.91	2.37	2.21

- (a) Determine degrees of freedom.
- (b) Determine t^* for a 99% confidence interval.
- (c) Determine SE.
- (d) Determine a lower bound of the 99% confidence interval of $\mu_2 \mu_1$.
- (e) Determine an upper bound of the 99% confidence interval of $\mu_2 \mu_1$.
- (f) Determine $|t_{\rm obs}|$ under the null hypothesis $\mu_2-\mu_1=0$.
- (g) Determine a lower bound of the two-tail *p*-value.
- (h) Determine an upper bound of two-tail *p*-value.
- (i) Do you reject the null hypothesis with a two-tail test using a significance level α = 0.01? (yes or no)

1.	(a)	2	. 0	0	0	
	(b)	9	. 9	2	0	
	(c)	0	. 1	6	8	
	(d)	- 0	. 3	6	7	
	(e)	2	. 9	6	7	
	(f)	7	. 7	2	0	
	(g)	0	. 0	1	0	
	(h)	0	. 0	2	0	

(i) no

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1. Solution

These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{X_1} = 1.02$$

$$\overline{X_2} = 2.32$$

$$s_1 = 0.0473$$

$$s_2 = 0.407$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(3, 6) - 1 = 2$$

We use the t table to find t^* such that $P(|T| < t^*) = 0.99$

$$t^* = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.0473)^2}{3} + \frac{(0.407)^2}{6}} = 0.168$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^* SE$$

$$CI = (-0.367, 2.967)$$

We find t_{obs} .

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(2.32 - 1.02) - 0}{0.168} = 7.72$$

We find $|t_{obs}|$.

$$|t_{\rm obs}| = 7.72$$

We use the table to determine bounds on *p*-value. Remember, df = 2 and *p*-value = $P(|T| > |t_{\text{obs}}|)$.

$$0.01 < p$$
-value < 0.02

We should consider both comparisons to make our decision.

$$|t_{\sf obs}| < t^{\star}$$

$$p$$
-value $> \alpha$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

- (a) 2
- (b) 9.92
- (c) 0.168
- (d) -0.367
- (e) 2.967
- (f) 7.72
- (g) 0.01
- (h) 0.02
- (i) no