Key ID: 011

Name: ______________________________

1. **Problem**
An experiment has n_1 = 8 plants in the treatment group and n_2 = 7 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	9.1	11.4	9.7	8.9	11.1	8.1	9.2	13.3
sample 2:	12.4	12.4	18.6	14.5	13.9	12.1	10.6	

(a) Determine degrees of freedom.

(b) Determine $t^{\star}$ for a 95% confidence interval.

(c) Determine SE.

(d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.

(e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.

(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.

(g) Determine a lower bound of the two-tail p-value.

(h) Determine an upper bound of two-tail p-value.

(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) **6 . 0 0 0**

(b) **2 . 4 5 0**

(c) **1 . 1 4 5**

(d) **0 . 5 9 5**

(e) **6 . 2 0 5**

(f) **2 . 9 6 8**

(g) **0 . 0 2 0**

(h) **0 . 0 4 0**

(i) **yes**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 10.1$$
$$\overline{x_2} = 13.5$$
$$s_1 = 1.7$$
$$s_2 = 2.58$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\, n_2) - 1 = \min(8, 7) - 1 = 6$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.95$

$$t^\star = 2.45$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(1.7)^2}{8} + \frac{(2.58)^2}{7}} = 1.145$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (0.595,\, 6.205)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(13.5 - 10.1) - 0}{1.145} = 2.97$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 2.97$$

We use the table to determine bounds on p-value. Remember, $df = 6$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.02 < p\text{-value} < 0.04$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 6
(b) 2.45
(c) 1.145
(d) 0.595
(e) 6.205
(f) 2.968
(g) 0.02
(h) 0.04
(i) yes