Name: ____________________

1. **Problem**
   An experiment has n_1 = 8 plants in the treatment group and n_2 = 8 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	9.4	9.7	10.6	10.9	12.3	9.9	9.6	12
sample 2:	13.1	11.4	11.1	9.6	12.8	10.6	10.3	14.4

   (a) Determine degrees of freedom.
   (b) Determine $t^{\star}$ for a 95% confidence interval.
   (c) Determine SE.
   (d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 7.000

(b) 2.360

(c) 0.694

(d) -0.538

(e) 2.738

(f) 1.584

(g) 0.100

(h) 0.200

(i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 10.6$$
$$\overline{x_2} = 11.7$$
$$s_1 = 1.11$$
$$s_2 = 1.62$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(8, 8) - 1 \;=\; 7$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.95$

$$t^{\star} = 2.36$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(1.11)^2}{8} + \frac{(1.62)^2}{8}} = 0.694$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$
$$CI \;=\; (-0.538,\, 2.738)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(11.7 - 10.6) - 0}{0.694} = 1.58$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 1.58$$

We use the table to determine bounds on p-value. Remember, $df = 7$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.1 \;<\; p\text{-value} \;<\; 0.2$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^{\star}$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 7
(b) 2.36
(c) 0.694
(d) -0.538
(e) 2.738
(f) 1.584
(g) 0.1
(h) 0.2
(i) no