

# Bunker Hill Community College

Final Statistics Exam 2019-05-02

Exam ID 005

**Name:** \_\_\_\_\_

This take-home exam is due **Wednesday, May 8**, at the beginning of class.

You may use any notes, textbook, or online tools; however, you may not request help from any other human.

You will show your work on the pages with questions. When you are sure of your answers, you will **put those answers in the boxes** on the first few pages.

Unless you have an objection to doing so, please **copy the honor-code text below and sign**.

*I understand that outside help is NOT allowed on this exam. On my honor, the work herein is my own.*

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**Signature:** \_\_\_\_\_

1. (a)  $P(\text{flower}) = 0.434$
- (b)  $P(\text{wheel or yellow}) = 0.596$
- (c)  $P(\text{red}) = 0.281$
- (d)  $P(\text{pig given white}) = 0.0855$
- (e)  $P(\text{white given flower}) = 0.0901$
- (f)  $P(\text{flower and white}) = 0.0391$
2.  $P(\text{"not shovel" given "not white"}) = 0.533$
3.  $P(65.28 < X < 65.85) = 0.839$
4. (a)  $P(X = 14) = 0.1225$
- (b)  $P(8 \leq X \leq 21) = 0.9508$
5. **(24.5, 26.5)**
6. (a)  $H_0 : \mu_2 - \mu_1 = 0$
- (b)  $H_0 : \mu_2 - \mu_1 \neq 0$
- (c)  $t^* = 2.02$
- (d)  $SE = 0.01$
- (e)  $|t_{\text{obs}}| = 2.3$
- (f)  $0.02 < p\text{-value} < 0.04$
- (g) **reject**
7. (a) **LB of p CI = 0.852 or 85.2%**
- (b) **UB of p CI = 0.874 or 87.4%**

8. (a)  $H_0 : p_2 - p_1 = 0$

(b)  $H_A : p_2 - p_1 \neq 0$

(c)  $z^* = 2.33$

(d)  $SE = 0.043$

(e)  $|z_{\text{obs}}| = 2.5$

(f)  $p\text{-value} = 0.0124$

(g) **reject**

1. In a deck of strange cards, there are 512 cards. Each card has an image and a color. The amounts are shown in the table below.

	indigo	red	white	yellow
flower	30	91	20	81
pig	27	29	10	48
wheel	44	24	87	21

- (a) What is the probability a random card is a flower?
- (b) What is the probability a random card is either a wheel or yellow (or both)?
- (c) What is the probability a random card is red?
- (d) What is the probability a random card is a pig given it is white?
- (e) What is the probability a random card is white given it is a flower?
- (f) What is the probability a random card is both a flower and white?

**Solution**

$$(a) P(\text{flower}) = \frac{30+91+20+81}{512} = 0.434$$

$$(b) P(\text{wheel or yellow}) = \frac{44+24+87+21+81+48+21-21}{512} = 0.596$$

$$(c) P(\text{red}) = \frac{91+29+24}{512} = 0.281$$

$$(d) P(\text{pig given white}) = \frac{10}{20+10+87} = 0.0855$$

$$(e) P(\text{white given flower}) = \frac{20}{30+91+20+81} = 0.0901$$

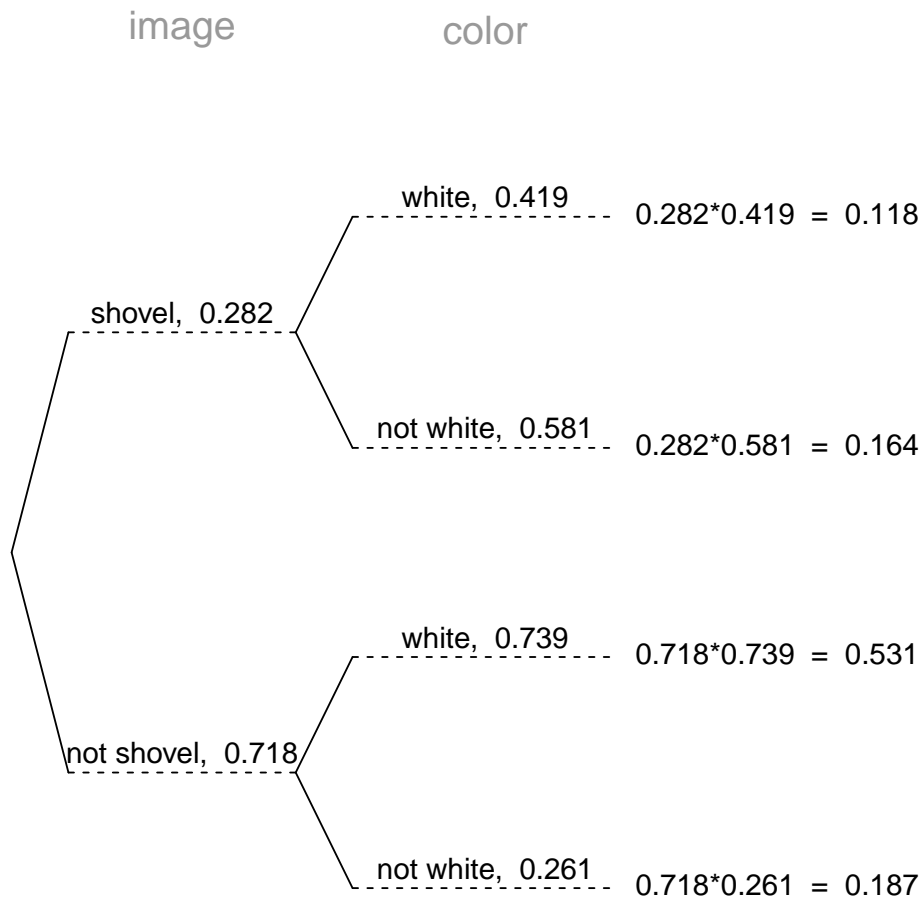
$$(f) P(\text{flower and white}) = \frac{20}{512} = 0.0391$$

2. In a deck of strange cards, each card has an image and a color. The chance of drawing a shovel is 28.2%. If a shovel is drawn, there is a 41.9% chance that it is white. If a card that is not a shovel is drawn, there is a 73.9% chance that it is white.

Now, someone draws a random card and reveals it is not white. What is the chance the card is not a shovel?

**Solution**

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilities. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"not shovel" given "not white"}) = \frac{0.187}{0.187 + 0.164} = 0.533$$

3. In a very large pile of toothpicks, the mean length is 65.69 millimeters and the standard deviation is 1.94 millimeters. If you randomly sample 150 toothpicks, what is the chance the sample mean is between 65.28 and 65.85 millimeters?



**Solution**

Label the given information.

$$\mu = 65.69$$

$$\sigma = 1.94$$

$$n = 150$$

$$\bar{x}_{\text{lower}} = 65.28$$

$$\bar{x}_{\text{upper}} = 65.85$$

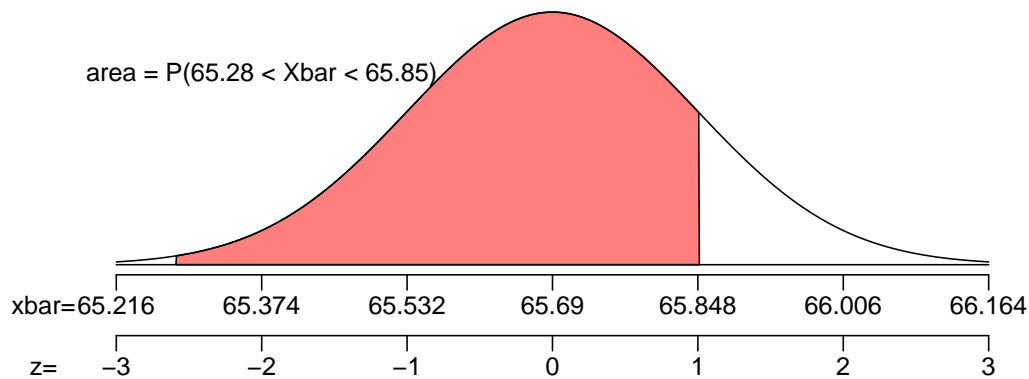
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{1.94}{\sqrt{150}} = 0.158$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(65.69, 0.158)$$

Draw a sketch.



Calculate a z scores.

$$z_{\text{lower}} = \frac{\bar{x}_{\text{lower}} - \mu}{SE} = \frac{65.28 - 65.69}{0.158} = -2.59$$

$$z_{\text{upper}} = \frac{\bar{x}_{\text{upper}} - \mu}{SE} = \frac{65.85 - 65.69}{0.158} = 1.01$$

Determine the probability.

$$\begin{aligned} P(65.28 < \bar{X} < 65.85) &= \Phi(z_{\text{upper}}) - \Phi(z_{\text{lower}}) \\ &= \Phi(1.01) - \Phi(-2.59) \\ &= 0.839 \end{aligned}$$

4. In a game, there is a 26% chance to win a round. You will play 52 rounds.
- (a) What is the probability of winning exactly 14 rounds?
  - (b) What is the probability of winning at least 8 but at most 21 rounds?

**Solution**

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k} (p)^k (1 - p)^{n-k}$$

$$P(X = 14) = \binom{52}{14} (0.26)^{14} (1 - 0.26)^{52-14}$$

$$P(X = 14) = \binom{52}{14} (0.26)^{14} (0.74)^{38}$$

$$P(X = 14) = 0.1225$$

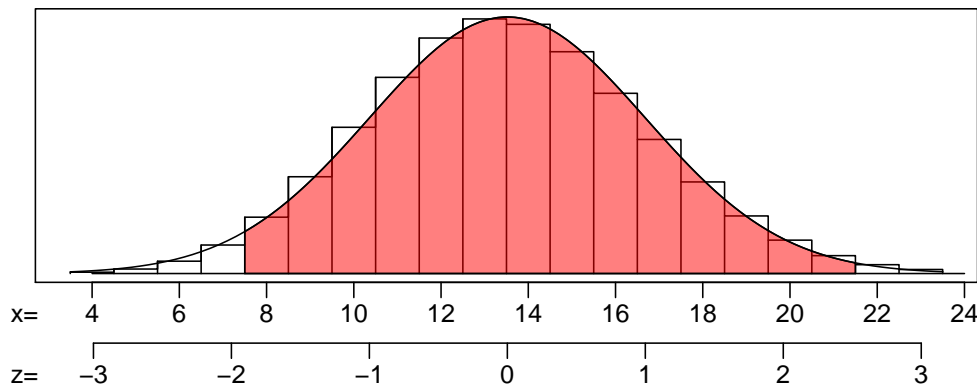
Find the mean.

$$\mu = np = (52)(0.26) = 13.52$$

Find the standard deviation.

$$\sigma = \sqrt{np(1 - p)} = \sqrt{(52)(0.26)(1 - 0.26)} = 3.163$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{7.5 - 13.52}{3.163} = -1.75$$

$$z_2 = \frac{21.5 - 13.52}{3.163} = 2.36$$

Calculate the probability.

$$P(8 \leq X \leq 21) = \Phi(2.36) - \Phi(-1.75) = 0.9508$$

(a)  $P(X = 14) = 0.1225$

(b)  $P(8 \leq X \leq 21) = 0.9508$

5. As an ornithologist, you wish to determine the average body mass of *Passer domesticus*. You randomly sample 25 adults of *Passer domesticus*, resulting in a sample mean of 25.5 grams and a sample standard deviation of 2.47 grams. Determine a 95% confidence interval of the true population mean.

**Solution**

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 25$$

$$\bar{x} = 25.5$$

$$s = 2.47$$

$$CL = 0.95$$

Determine the degrees of freedom (because we don't know  $\sigma$  and we are doing inference so we need to use the  $t$  distribution).

$$df = n - 1 = 24$$

Determine the critical  $t$  value,  $t^*$ , such that  $P(|T| < t^*) = 0.95$ .

$$t^* = 2.06$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.47}{\sqrt{25}} = 0.494$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^* SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^* SE, \bar{x} + t^* SE) \\ &= (25.5 - 2.06 \times 0.494, 25.5 + 2.06 \times 0.494) \\ &= (24.5, 26.5) \end{aligned}$$

We are 95% confident that the population mean is between 24.5 and 26.5.

6. A treatment group of size 20 has a mean of 1.02 and standard deviation of 0.0297. A control group of size 25 has a mean of 0.998 and standard deviation of 0.0345. If you decided to use a significance level of 0.05, is there sufficient evidence to conclude the treatment causes an effect?

By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 42.

- (a) State the null hypothesis.
- (b) State the alternative hypothesis.
- (c) Evaluate the critical value. (The critical value is either  $z^*$  or  $t^*$ . Determine its value.)
- (d) Determine the standard error of the relevant sampling distribution.
- (e) Evaluate the absolute value of the test statistic. (The test statistic is either  $z_{\text{obs}}$  or  $t_{\text{obs}}$ . Determine its absolute value.)
- (f) If possible, evaluate the  $p$ -value. Otherwise, describe an interval containing the  $p$ -value.
- (g) Do we reject or retain the null?

**Solution**

We are given unpaired data. We are considering a difference of means. Label the given information.

$$\begin{aligned}n_1 &= 20 \\ \bar{x}_1 &= 1.02 \\ s_1 &= 0.0297 \\ n_2 &= 25 \\ \bar{x}_2 &= 0.998 \\ s_2 &= 0.0345 \\ \alpha &= 0.05 \\ df &= 42\end{aligned}$$

State the hypotheses.

$$\begin{aligned}H_0 : \mu_2 - \mu_1 &= 0 \\ H_A : \mu_2 - \mu_1 &\neq 0\end{aligned}$$

We are using a two-tail test. Find  $t^*$  such that  $P(|T| > t^*) = 0.05$  by using a  $t$  table.

$$t^* = 2.02$$

Calculate the standard error.

$$\begin{aligned}SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.0297)^2}{20} + \frac{(0.0345)^2}{25}} \\ &= 0.01\end{aligned}$$

Determine the test statistic.

$$\begin{aligned}t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(0.998 - 1.02) - (0)}{0.01} \\ &= -2.3\end{aligned}$$

Compare  $|t_{\text{obs}}|$  and  $t^*$ .

$$|t_{\text{obs}}| > t^*$$

We can determine an interval for the  $p$ -value using the  $t$  table.

$$0.02 < p\text{-value} < 0.04$$

Compare  $p$ -value and  $\alpha$ .

$$p\text{-value} < \alpha$$

We conclude that we should reject the null hypothesis.

$$(a) H_0 : \mu_2 - \mu_1 = 0$$

- (b)  $H_A : \mu_2 - \mu_1 \neq 0$
- (c)  $t^* = 2.02$
- (d)  $SE = 0.01$
- (e)  $|t_{\text{obs}}| = 2.3$
- (f)  $0.02 < p\text{-value} < 0.04$
- (g) reject the null



7. From a very large population, a random sample of 5200 individuals was taken. In that sample, 86.3% were glowing. Determine a 98% confidence interval of the population proportion.
- (a) Find the lower bound of the confidence interval.
  - (b) Find the upper bound of the confidence interval.

**Solution**

Determine  $z^*$  such that  $P(|Z| < z^*) = 0.98$ .

$$z^* = 2.33$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \sqrt{\frac{(0.863)(1 - 0.863)}{5200}} = 0.00477$$

Calculate the margin of error.

$$ME = z^* SE = (2.33)(0.00477) = 0.0111$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Determine the interval.

$$(0.852, 0.874)$$

We are 98% confident that the true population proportion is between 85.2% and 87.4%.

- (a) The lower bound = 0.852, which can also be expressed as 85.2%.
- (b) The upper bound = 0.874, which can also be expressed as 87.4%.

8. An experiment is run with a treatment group of size 259 and a control group of size 227. The results are summarized in the table below.

	treatment	control
special	183	136
not special	76	91

Using a significance level of 0.02, determine whether the treatment causes an effect on the proportion of cases that are special.

- (a) State the null hypothesis.
- (b) State the alternative hypothesis.
- (c) Evaluate the critical value. (The critical value is either  $z^*$  or  $t^*$ . Determine its value.)
- (d) Determine the standard error of the relevant sampling distribution.
- (e) Evaluate the absolute value of the test statistic. (The test statistic is either  $z_{\text{obs}}$  or  $t_{\text{obs}}$ . Determine its absolute value.)
- (f) If possible, evaluate the  $p$ -value. Otherwise, describe an interval containing the  $p$ -value.
- (g) Do we reject or retain the null?

**Solution**

State the hypotheses.

$$H_0 : p_2 - p_1 = 0$$

$$H_A : p_2 - p_1 \neq 0$$

Find  $z^*$  such that  $P(|Z| > z^*) = 0.02$ .

$$z^* = \Phi^{-1} \left( 1 - \frac{\alpha}{2} \right) = 2.33$$

Determine the sample proportions.

$$\hat{p}_1 = \frac{183}{259} = 0.707$$

$$\hat{p}_2 = \frac{136}{227} = 0.599$$

Determine the difference of sample proportions.

$$\hat{p}_2 - \hat{p}_1 = 0.599 - 0.707 = -0.108$$

Determine the pooled proportion (because the null assumes the population proportions are equal).

$$\hat{p} = \frac{183 + 136}{259 + 227} = 0.656$$

Determine the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{\hat{p}(1 - \hat{p})}{n_1} + \frac{\hat{p}(1 - \hat{p})}{n_2}} \\ &= \sqrt{\frac{(0.656)(0.344)}{259} + \frac{(0.656)(0.344)}{227}} \\ &= 0.0432 \end{aligned}$$

We can be more specific about what the null hypothesis claims.

$$H_0 : \hat{P}_2 - \hat{P}_1 \sim \mathcal{N}(0, 0.0432)$$

We want to describe how unusual our observation is under the null by finding the  $p$ -value. To do so, first find the  $z$  score.

$$\begin{aligned} z &= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)_0}{SE} \\ &= \frac{(0.599 - 0.707) - 0}{0.0432} \\ &= -2.5 \end{aligned}$$

Determine the  $p$ -value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-|z|) \\ &= 2 \cdot \Phi(-2.5) \\ &= 0.0124 \end{aligned}$$

Compare the  $p$ -value to the significance level.

$$p\text{-value} < \alpha$$

So, we reject the null hypothesis. Thus the difference in proportions is significant.

(a)  $H_0 : p_2 - p_1 = 0$

(b)  $H_A : p_2 - p_1 \neq 0$

(c)  $z^* = 2.33$

(d)  $SE = 0.0432$

(e)  $|z_{\text{obs}}| = 2.5$

(f)  $p\text{-value} = 0.0124$

(g) reject the null