Name: ______________________________

1. **Problem**
An experiment has $n_1 = 3$ plants in the treatment group and $n_2 = 7$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	110	106	106				
sample 2:	266	278	266	270	234	250	287

(a) Determine degrees of freedom.
(b) Determine t^* for a 99% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail *p*-value.
(h) Determine an upper bound of two-tail *p*-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 2 . 0 0 0

(b) 9 . 9 2 0

(c) 6 . 7 8 5

(d) 8 9 . 6 9 3

(e) 2 2 4 . 3 0 7

(f) 2 3 . 1 4 1

(g) 0 . 0 0 0

(h) 0 . 0 0 2

(i) yes

1. **Solution**

These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 107$$
$$\overline{x_2} = 264$$
$$s_1 = 2.31$$
$$s_2 = 17.6$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(3, 7) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(2.31)^2}{3} + \frac{(17.6)^2}{7}} = 6.785$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (89.693,\ 224.307)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(264 - 107) - 0}{6.785} = 23.14$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 23.14$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0 < p\text{-value} < 0.002$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 2
(b) 9.92
(c) 6.785
(d) 89.693
(e) 224.307
(f) 23.141
(g) 0
(h) 0.002
(i) yes