Name:

1. **Problem**

   An experiment has $n_1 = 8$ plants in the treatment group and $n_2 = 4$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

   |          | value1 | value2 | value3 | value4 | value5 | value6 | value7 | value8 |
   | -------- | ------ | ------ | ------ | ------ | ------ | ------ | ------ | ------ |
   | sample 1: | 1.91  | 1.64   | 1.7    | 1.72   | 1.23   | 1.29   | 1.61   | 1.49   |
   | sample 2: | 0.89  | 0.96   | 1.12   | 0.8    |        |        |        |        |

   (a) Determine degrees of freedom.

   (b) Determine $t^*$ for a 98% confidence interval.

   (c) Determine $SE$.

   (d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.

   (e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.

   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.

   (g) Determine a lower bound of the two-tail $p$-value.

   (h) Determine an upper bound of two-tail $p$-value.

   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.02$? (yes or no)

1. (a)       **3** . **0** **0** **0**

   (b)       **4** . **5** **4** **0**

   (c)       **0** . **1** **0** **5**

   (d)      **-** **1** . **1** **0** **5**

   (e)      **-** **0** . **1** **5** **1**

   (f)       **5** . **9** **8** **8**

   (g)       **0** . **0** **0** **5**

   (h)       **0** . **0** **1** **0**

   (i) **yes**

1. **Solution**
   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

   $$\overline{x_1} = 1.57$$

   $$\overline{x_2} = 0.942$$

   $$s_1 = 0.227$$

   $$s_2 = 0.135$$

   We make a conservative estimate of the degrees of freedom using the appropriate formula.

   $$df = \min(n_1, n_2) - 1 = \min(8, 4) - 1 = 3$$

   We use the $t$ table to find $t^*$ such that $P(|T| < t^*) = 0.98$

   $$t^* = 4.54$$

   We use the $SE$ formula for unpaired data.

   $$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.227)^2}{8} + \frac{(0.135)^2}{4}} = 0.105$$

   We find the bounds of the confidence interval.

   $$CI = (\overline{x_2} - \overline{x_1}) \pm t^* SE$$

   $$CI = (-1.105, -0.151)$$

   We find $t_{obs}$.

   $$t_{obs} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(0.942 - 1.57) - 0}{0.105} = -5.99$$

   We find $|t_{obs}|$.

   $$|t_{obs}| = 5.99$$

   We use the table to determine bounds on $p$-value. Remember, $df = 3$ and $p$-value $= P(|T| > |t_{obs}|)$.

   $$0.005 < p\text{-value} < 0.01$$

   We should consider both comparisons to make our decision.

   $$|t_{obs}| > t^*$$

   $$p\text{-value} < \alpha$$

   Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

   (a) 3
   (b) 4.54
   (c) 0.105
   (d) -1.105
   (e) -0.151
   (f) 5.988
   (g) 0.005
   (h) 0.01
   (i) yes