Name: ____________________

1. **Problem**
An experiment has $n_1 = 8$ plants in the treatment group and $n_2 = 4$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	109	108	114	115	103	114	109	110
sample 2:	98	108	124	89				

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 90% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 90% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 90% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.1$? (yes or no)

1. (a) 3 . 0 0 0

(b) 2 . 3 5 0

(c) 7 . 6 3 2

(d) - 2 2 . 9 3 5

(e) 1 2 . 9 3 5

(f) 0 . 6 5 5

(g) 0 . 2 0 0

(h) 1 . 0 0 0

(i) no

1. **Solution**
   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 110$$
$$\overline{x_2} = 105$$
$$s_1 = 3.99$$
$$s_2 = 15$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(8, 4) - 1 \;=\; 3$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.9$

$$t^{\star} = 2.35$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(3.99)^2}{8} + \frac{(15)^2}{4}} = 7.632$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$
$$CI \;=\; (-22.935,\, 12.935)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(105 - 110) - 0}{7.632} = -0.66$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 0.66$$

We use the table to determine bounds on p-value. Remember, $df = 3$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.2 \;<\; p\text{-value} \;<\; 1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^{\star}$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

   (a) 3
   (b) 2.35
   (c) 7.632
   (d) -22.935
   (e) 12.935
   (f) 0.655
   (g) 0.2
   (h) 1
   (i) no