Name: ______________________

1. **Problem**
An experiment has n_1 = 8 plants in the treatment group and n_2 = 6 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	137	134	157	141	128	114	166	134
sample 2:	92	102	96	97	89	101		

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 98% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail *p*-value.
(h) Determine an upper bound of two-tail *p*-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level α = 0.02? (yes or no)

1. (a) 5.000
(b) 3.360
(c) 6.119
(d) -63.360
(e) -22.240
(f) 6.994
(g) 0.000
(h) 0.002
(i) **yes**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 139$$
$$\overline{x_2} = 96.2$$
$$s_1 = 16.3$$
$$s_2 = 5.04$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\ n_2) - 1 = \min(8, 6) - 1 = 5$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 3.36$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(16.3)^2}{8} + \frac{(5.04)^2}{6}} = 6.119$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-63.36,\ -22.24)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(96.2 - 139) - 0}{6.119} = -6.99$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 6.99$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0 < p\text{-value} < 0.002$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 5
(b) 3.36
(c) 6.119
(d) -63.36
(e) -22.24
(f) 6.994
(g) 0
(h) 0.002
(i) yes