Name: _____

1. **Problem**

   An experiment has $n_1 = 4$ plants in the treatment group and $n_2 = 3$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4
sample 1:	1.28	1.18	1.01	0.67
sample 2:	1.82	1.73	1.76	

   (a) Determine degrees of freedom.

   (b) Determine t^* for a 96% confidence interval.

   (c) Determine SE.

   (d) Determine a lower bound of the 96% confidence interval of $\mu_2 - \mu_1$.

   (e) Determine an upper bound of the 96% confidence interval of $\mu_2 - \mu_1$.

   (f) Determine $|t_{obs}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.

   (g) Determine a lower bound of the two-tail p-value.

   (h) Determine an upper bound of two-tail p-value.

   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.04$? (yes or no)

1. (a) | | | | | **2** . **0** **0** **0** |

   (b) | | | | | **4** . **8** **5** **0** |

   (c) | | | | | **0** . **1** **3** **7** |

   (d) | | | | | **0** . **0** **7** **6** |

   (e) | | | | | **1** . **4** **0** **4** |

   (f) | | | | | **5** . **4** **1** **8** |

   (g) | | | | | **0** . **0** **2** **0** |

   (h) | | | | | **0** . **0** **4** **0** |

   (i) **yes**

1. **Solution**
   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1.03$$

$$\overline{x_2} = 1.77$$

$$s_1 = 0.268$$

$$s_2 = 0.0458$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\, n_2) - 1 = \min(4, 3) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.96$

$$t^\star = 4.85$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.268)^2}{4} + \frac{(0.0458)^2}{3}} = 0.137$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$

$$CI = (0.076,\ 1.404)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.77 - 1.03) - 0}{0.137} = 5.42$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 5.42$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.02 < p\text{-value} < 0.04$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

   (a) 2
   (b) 4.85
   (c) 0.137
   (d) 0.076
   (e) 1.404
   (f) 5.418
   (g) 0.02
   (h) 0.04
   (i) yes