Name: ______________________________

1. **Problem**
An experiment has n_1 = 6 plants in the treatment group and n_2 = 8 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	0.85	1.01	1.02	1.29	1.2	0.87		
sample 2:	1.22	1.26	1.01	1.55	1.65	1.02	1.24	1.37

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 98% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.02$? (yes or no)

1. (a) 5.000

(b) 3.360

(c) 0.108

(d) -0.113

(e) 0.613

(f) 2.315

(g) 0.050

(h) 0.100

(i) no

1. **Solution**

These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1.04$$

$$\overline{x_2} = 1.29$$

$$s_1 = 0.176$$

$$s_2 = 0.228$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\ n_2) - 1 = \min(6, 8) - 1 = 5$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 3.36$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.176)^2}{6} + \frac{(0.228)^2}{8}} = 0.108$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$

$$CI = (-0.113,\ 0.613)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.29 - 1.04) - 0}{0.108} = 2.32$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 2.32$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.05 < p\text{-value} < 0.1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$

$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 5
(b) 3.36
(c) 0.108
(d) -0.113
(e) 0.613
(f) 2.315
(g) 0.05
(h) 0.1
(i) no

Name: ______________________________

1. **Problem**
An experiment has $n_1 = 7$ plants in the treatment group and $n_2 = 4$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	128	130	157	122	100	160	112
sample 2:	100	91	111	95			

(a) Determine degrees of freedom.
(b) Determine t^* for a 96% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 96% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 96% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.04$? (yes or no)

1. (a) 3 . 0 0 0

(b) 3 . 4 8 0

(c) 9 . 4 0 9

(d) - 6 3 . 5 4 3

(e) 1 . 9 4 3

(f) 3 . 2 7 4

(g) 0 . 0 4 0

(h) 0 . 0 5 0

(i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 130$$
$$\overline{x_2} = 99.2$$
$$s_1 = 22.1$$
$$s_2 = 8.66$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(7, 4) - 1 = 3$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.96$

$$t^\star = 3.48$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(22.1)^2}{7} + \frac{(8.66)^2}{4}} = 9.409$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-63.543,\ 1.943)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(99.2 - 130) - 0}{9.409} = -3.27$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 3.27$$

We use the table to determine bounds on p-value. Remember, $df = 3$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.04 < p\text{-value} < 0.05$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 3
(b) 3.48
(c) 9.409
(d) -63.543
(e) 1.943
(f) 3.274
(g) 0.04
(h) 0.05
(i) no

Name: ______________________________

1. **Problem**
An experiment has n_1 = 4 plants in the treatment group and n_2 = 3 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4
sample 1:	1.28	1.18	1.01	0.67
sample 2:	1.82	1.73	1.76	

(a) Determine degrees of freedom.

(b) Determine $t^{\star}$ for a 96% confidence interval.

(c) Determine SE.

(d) Determine a lower bound of the 96% confidence interval of $\mu_2 - \mu_1$.

(e) Determine an upper bound of the 96% confidence interval of $\mu_2 - \mu_1$.

(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.

(g) Determine a lower bound of the two-tail p-value.

(h) Determine an upper bound of two-tail p-value.

(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.04$? (yes or no)

1. (a) 2.000

(b) 4.850

(c) 0.137

(d) 0.076

(e) 1.404

(f) 5.418

(g) 0.020

(h) 0.040

(i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1.03$$
$$\overline{x_2} = 1.77$$
$$s_1 = 0.268$$
$$s_2 = 0.0458$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\, n_2) - 1 = \min(4, 3) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.96$

$$t^\star = 4.85$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.268)^2}{4} + \frac{(0.0458)^2}{3}} = 0.137$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (0.076,\ 1.404)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.77 - 1.03) - 0}{0.137} = 5.42$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 5.42$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.02 < p\text{-value} < 0.04$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 2
(b) 4.85
(c) 0.137
(d) 0.076
(e) 1.404
(f) 5.418
(g) 0.02
(h) 0.04
(i) yes

Name: ______________________________

1. **Problem**
An experiment has n_1 = 8 plants in the treatment group and n_2 = 4 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	109	108	114	115	103	114	109	110
sample 2:	98	108	124	89				

(a) Determine degrees of freedom.
(b) Determine t^* for a 90% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 90% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 90% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.1$? (yes or no)

1. (a) 3.000

(b) 2.350

(c) 7.632

(d) -22.935

(e) 12.935

(f) 0.655

(g) 0.200

(h) 1.000

(i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 110$$
$$\overline{x_2} = 105$$
$$s_1 = 3.99$$
$$s_2 = 15$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(8, 4) - 1 \;=\; 3$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.9$

$$t^\star = 2.35$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(3.99)^2}{8} + \frac{(15)^2}{4}} = 7.632$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI \;=\; (-22.935,\; 12.935)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(105 - 110) - 0}{7.632} = -0.66$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 0.66$$

We use the table to determine bounds on p-value. Remember, $df = 3$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.2 \;<\; p\text{-value} \;<\; 1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 3
(b) 2.35
(c) 7.632
(d) -22.935
(e) 12.935
(f) 0.655
(g) 0.2
(h) 1
(i) no

Name: ____________________

1. **Problem**
An experiment has n_1 = 7 plants in the treatment group and n_2 = 3 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	255	202	216	204	225	211	214
sample 2:	103	113	126				

(a) Determine degrees of freedom.
(b) Determine t^* for a 99% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 2.000

(b) 9.920

(c) 9.506

(d) -198.300

(e) -9.700

(f) 10.940

(g) 0.005

(h) 0.010

(i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 218$$
$$\overline{x_2} = 114$$
$$s_1 = 18$$
$$s_2 = 11.5$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(7, 3) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(18)^2}{7} + \frac{(11.5)^2}{3}} = 9.506$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-198.3, -9.7)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(114 - 218) - 0}{9.506} = -10.94$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 10.94$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.005 < p\text{-value} < 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 2
(b) 9.92
(c) 9.506
(d) -198.3
(e) -9.7
(f) 10.94
(g) 0.005
(h) 0.01
(i) yes

Name: ______________________________

1. **Problem**
An experiment has n_1 = 6 plants in the treatment group and n_2 = 6 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	14.1	11.2	12.8	15.1	13.8	14.1
sample 2:	9.8	8.1	9.8	11.2	11	10.8

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 99% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 5 . 0 0 0

(b) 4 . 0 3 0

(c) 0 . 7 2 7

(d) - 6 . 3 3 0

(e) - 0 . 4 7 0

(f) 4 . 6 7 9

(g) 0 . 0 0 5

(h) 0 . 0 1 0

(i) **yes**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 13.5$$
$$\overline{x_2} = 10.1$$
$$s_1 = 1.35$$
$$s_2 = 1.16$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(6, 6) - 1 \;=\; 5$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.99$

$$t^{\star} = 4.03$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(1.35)^2}{6} + \frac{(1.16)^2}{6}} = 0.727$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$
$$CI \;=\; (-6.33,\; -0.47)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(10.1 - 13.5) - 0}{0.727} = -4.68$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 4.68$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.005 \;<\; p\text{-value} \;<\; 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^{\star}$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 5
(b) 4.03
(c) 0.727
(d) -6.33
(e) -0.47
(f) 4.679
(g) 0.005
(h) 0.01
(i) yes

Name: ______________________________

1. **Problem**
An experiment has n_1 = 5 plants in the treatment group and n_2 = 3 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5
sample 1:	9	6.6	5.2	10.8	11.6
sample 2:	21.2	18.9	18.4		

(a) Determine degrees of freedom.
(b) Determine t^* for a 95% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 2.000
(b) 4.300
(c) 1.490
(d) 4.453
(e) 17.267
(f) 7.289
(g) 0.010
(h) 0.020
(i) yes

1. **Solution**
   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 8.64$$
$$\overline{x_2} = 19.5$$
$$s_1 = 2.72$$
$$s_2 = 1.49$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \; = \; \min(n_1,\, n_2) - 1 \; = \; \min(5, 3) - 1 \; = \; 2$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.95$

$$t^{\star} = 4.3$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(2.72)^2}{5} + \frac{(1.49)^2}{3}} = 1.49$$

We find the bounds of the confidence interval.

$$CI \; = \; (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$
$$CI \; = \; (4.453, \; 17.267)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(19.5 - 8.64) - 0}{1.49} = 7.29$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 7.29$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.01 \; < \; p\text{-value} \; < \; 0.02$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^{\star}$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

   (a) 2
   (b) 4.3
   (c) 1.49
   (d) 4.453
   (e) 17.267
   (f) 7.289
   (g) 0.01
   (h) 0.02
   (i) yes

Name: ____________________

1. **Problem**
   An experiment has n_1 = 6 plants in the treatment group and n_2 = 6 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	139	127	120	142	119	142
sample 2:	111	98	94	81	67	125

   (a) Determine degrees of freedom.
   (b) Determine $t^{\star}$ for a 95% confidence interval.
   (c) Determine *SE*.
   (d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail *p*-value.
   (h) Determine an upper bound of two-tail *p*-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 5.000
   (b) 2.570
   (c) 9.532
   (d) -60.497
   (e) -11.503
   (f) 3.777
   (g) 0.010
   (h) 0.020
   (i) yes

1. **Solution**
   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 132$$

$$\overline{x_2} = 96$$

$$s_1 = 10.8$$

$$s_2 = 20.7$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(6, 6) - 1 \;=\; 5$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.95$

$$t^{\star} = 2.57$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(10.8)^2}{6} + \frac{(20.7)^2}{6}} = 9.532$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$

$$CI \;=\; (-60.497,\; -11.503)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(96 - 132) - 0}{9.532} = -3.78$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 3.78$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.01 \;<\; p\text{-value} \;<\; 0.02$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^{\star}$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

   (a) 5
   (b) 2.57
   (c) 9.532
   (d) -60.497
   (e) -11.503
   (f) 3.777
   (g) 0.01
   (h) 0.02
   (i) yes

Name: ____________________

1. **Problem**
   An experiment has n_1 = 3 plants in the treatment group and n_2 = 7 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	21.3	21.8	27.2				
sample 2:	8.8	10.9	9.9	10.3	11.4	10.4	9

   (a) Determine degrees of freedom.
   (b) Determine $t^\star$ for a 99% confidence interval.
   (c) Determine *SE*.
   (d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 2.000

(b) 9.920

(c) 1.922

(d) -32.366

(e) 5.766

(f) 6.921

(g) 0.020

(h) 0.040

(i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 23.4$$

$$\overline{x_2} = 10.1$$

$$s_1 = 3.27$$

$$s_2 = 0.949$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(3, 7) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(3.27)^2}{3} + \frac{(0.949)^2}{7}} = 1.922$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$

$$CI = (-32.366, 5.766)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(10.1 - 23.4) - 0}{1.922} = -6.92$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 6.92$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.02 < p\text{-value} < 0.04$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$

$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 2
(b) 9.92
(c) 1.922
(d) -32.366
(e) 5.766
(f) 6.921
(g) 0.02
(h) 0.04
(i) no

Name: ______________________________

1. **Problem**
   An experiment has n_1 = 3 plants in the treatment group and n_2 = 6 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	10.3	8.6	10.8			
sample 2:	17.1	19.7	19.8	16.6	22.2	19.2

   (a) Determine degrees of freedom.
   (b) Determine $t^{\star}$ for a 99% confidence interval.
   (c) Determine SE.
   (d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 2.000

(b) 9.920

(c) 1.065

(d) -1.365

(e) 19.765

(f) 8.638

(g) 0.010

(h) 0.020

(i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 9.9$$
$$\overline{x_2} = 19.1$$
$$s_1 = 1.15$$
$$s_2 = 2.04$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(3, 6) - 1 \;=\; 2$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.99$

$$t^{\star} = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(1.15)^2}{3} + \frac{(2.04)^2}{6}} = 1.065$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$
$$CI \;=\; (-1.365,\, 19.765)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(19.1 - 9.9) - 0}{1.065} = 8.64$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 8.64$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.01 \;<\; p\text{-value} \;<\; 0.02$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^{\star}$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 2
(b) 9.92
(c) 1.065
(d) -1.365
(e) 19.765
(f) 8.638
(g) 0.01
(h) 0.02
(i) no

Name: ______________________

1. **Problem**
   An experiment has n_1 = 8 plants in the treatment group and n_2 = 7 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	9.1	11.4	9.7	8.9	11.1	8.1	9.2	13.3
sample 2:	12.4	12.4	18.6	14.5	13.9	12.1	10.6	

   (a) Determine degrees of freedom.
   (b) Determine $t^{\star}$ for a 95% confidence interval.
   (c) Determine *SE*.
   (d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail *p*-value.
   (h) Determine an upper bound of two-tail *p*-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 6.000
   (b) 2.450
   (c) 1.145
   (d) 0.595
   (e) 6.205
   (f) 2.968
   (g) 0.020
   (h) 0.040
   (i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 10.1$$
$$\overline{x_2} = 13.5$$
$$s_1 = 1.7$$
$$s_2 = 2.58$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(8, 7) - 1 \;=\; 6$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.95$

$$t^\star = 2.45$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(1.7)^2}{8} + \frac{(2.58)^2}{7}} = 1.145$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI \;=\; (0.595,\, 6.205)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(13.5 - 10.1) - 0}{1.145} = 2.97$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 2.97$$

We use the table to determine bounds on p-value. Remember, $df = 6$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.02 \;<\; p\text{-value} \;<\; 0.04$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 6
(b) 2.45
(c) 1.145
(d) 0.595
(e) 6.205
(f) 2.968
(g) 0.02
(h) 0.04
(i) yes

Name:

1. **Problem**
An experiment has n_1 = 8 plants in the treatment group and n_2 = 8 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	9.4	9.7	10.6	10.9	12.3	9.9	9.6	12
sample 2:	13.1	11.4	11.1	9.6	12.8	10.6	10.3	14.4

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 95% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail *p*-value.
(h) Determine an upper bound of two-tail *p*-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 7 . 0 0 0

(b) 2 . 3 6 0

(c) 0 . 6 9 4

(d) - 0 . 5 3 8

(e) 2 . 7 3 8

(f) 1 . 5 8 4

(g) 0 . 1 0 0

(h) 0 . 2 0 0

(i) **no**

1. **Solution**

These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 10.6$$
$$\overline{x_2} = 11.7$$
$$s_1 = 1.11$$
$$s_2 = 1.62$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\, n_2) - 1 = \min(8, 8) - 1 = 7$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.95$

$$t^{\star} = 2.36$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(1.11)^2}{8} + \frac{(1.62)^2}{8}} = 0.694$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$
$$CI = (-0.538,\, 2.738)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(11.7 - 10.6) - 0}{0.694} = 1.58$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 1.58$$

We use the table to determine bounds on p-value. Remember, $df = 7$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.1 < p\text{-value} < 0.2$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^{\star}$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 7
(b) 2.36
(c) 0.694
(d) -0.538
(e) 2.738
(f) 1.584
(g) 0.1
(h) 0.2
(i) no

Name: ______________________________

1. **Problem**
   An experiment has n_1 = 5 plants in the treatment group and n_2 = 3 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5
sample 1:	9	14.7	8.3	11.4	8.5
sample 2:	18.4	17.9	15.4		

   (a) Determine degrees of freedom.
   (b) Determine $t^{\star}$ for a 96% confidence interval.
   (c) Determine *SE*.
   (d) Determine a lower bound of the 96% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 96% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail *p*-value.
   (h) Determine an upper bound of two-tail *p*-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.04$? (yes or no)

1. (a) 2.000

   (b) 4.850

   (c) 1.527

   (d) -0.606

   (e) 14.206

   (f) 4.452

   (g) 0.040

   (h) 0.050

   (i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 10.4$$
$$\overline{x_2} = 17.2$$
$$s_1 = 2.71$$
$$s_2 = 1.61$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(5, 3) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.96$

$$t^\star = 4.85$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(2.71)^2}{5} + \frac{(1.61)^2}{3}} = 1.527$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-0.606,\ 14.206)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(17.2 - 10.4) - 0}{1.527} = 4.45$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 4.45$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.04 < p\text{-value} < 0.05$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 2
(b) 4.85
(c) 1.527
(d) -0.606
(e) 14.206
(f) 4.452
(g) 0.04
(h) 0.05
(i) no

Name: ____________________

1. **Problem**
   An experiment has $n_1 = 4$ plants in the treatment group and $n_2 = 3$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4
sample 1:	10.6	12.7	10.6	10.8
sample 2:	12.4	13	11.9	

   (a) Determine degrees of freedom.
   (b) Determine $t^{\star}$ for a 95% confidence interval.
   (c) Determine SE.
   (d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 2.000
   (b) 4.300
   (c) 0.601
   (d) -1.384
   (e) 3.784
   (f) 1.996
   (g) 0.100
   (h) 0.200
   (i) **no**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 11.2$$
$$\overline{x_2} = 12.4$$
$$s_1 = 1.02$$
$$s_2 = 0.551$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(4, 3) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.95$

$$t^\star = 4.3$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(1.02)^2}{4} + \frac{(0.551)^2}{3}} = 0.601$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-1.384, 3.784)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(12.4 - 11.2) - 0}{0.601} = 2$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 2$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.1 < p\text{-value} < 0.2$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 2
(b) 4.3
(c) 0.601
(d) -1.384
(e) 3.784
(f) 1.996
(g) 0.1
(h) 0.2
(i) no

Name:

1. **Problem**
An experiment has n_1 = 6 plants in the treatment group and n_2 = 7 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	91	118	144	104	118	141	
sample 2:	97	120	81	87	97	91	112

(a) Determine degrees of freedom.
(b) Determine t^* for a 90% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 90% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 90% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail *p*-value.
(h) Determine an upper bound of two-tail *p*-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.1$? (yes or no)

1. (a) 5.000
(b) 2.020
(c) 9.896
(d) -41.090
(e) -1.110
(f) 2.132
(g) 0.050
(h) 0.100
(i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 119$$

$$\overline{x_2} = 97.9$$

$$s_1 = 20.6$$

$$s_2 = 13.8$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(6, 7) - 1 = 5$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.9$

$$t^\star = 2.02$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(20.6)^2}{6} + \frac{(13.8)^2}{7}} = 9.896$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$

$$CI = (-41.09, -1.11)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(97.9 - 119) - 0}{9.896} = -2.13$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 2.13$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.05 < p\text{-value} < 0.1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 5
(b) 2.02
(c) 9.896
(d) -41.09
(e) -1.11
(f) 2.132
(g) 0.05
(h) 0.1
(i) yes

Name: ______________________________

1. **Problem**
An experiment has $n_1 = 8$ plants in the treatment group and $n_2 = 6$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	137	134	157	141	128	114	166	134
sample 2:	92	102	96	97	89	101		

(a) Determine degrees of freedom.
(b) Determine t^* for a 98% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.02$? (yes or no)

1. (a) 5 . 0 0 0

(b) 3 . 3 6 0

(c) 6 . 1 1 9

(d) -6 3 . 3 6 0

(e) -2 2 . 2 4 0

(f) 6 . 9 9 4

(g) 0 . 0 0 0

(h) 0 . 0 0 2

(i) **yes**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 139$$
$$\overline{x_2} = 96.2$$
$$s_1 = 16.3$$
$$s_2 = 5.04$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(8, 6) - 1 = 5$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 3.36$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(16.3)^2}{8} + \frac{(5.04)^2}{6}} = 6.119$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-63.36, -22.24)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(96.2 - 139) - 0}{6.119} = -6.99$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 6.99$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0 < p\text{-value} < 0.002$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 5
(b) 3.36
(c) 6.119
(d) -63.36
(e) -22.24
(f) 6.994
(g) 0
(h) 0.002
(i) yes

Name: ____________________

1. **Problem**
An experiment has n_1 = 3 plants in the treatment group and n_2 = 3 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3
sample 1:	13.9	9.8	7.9
sample 2:	23.3	22.7	21.3

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 99% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 2.000

(b) 9.920

(c) 1.870

(d) -6.650

(e) 30.450

(f) 6.365

(g) 0.020

(h) 0.040

(i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 10.5$$
$$\overline{x_2} = 22.4$$
$$s_1 = 3.07$$
$$s_2 = 1.03$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(3, 3) - 1 \;=\; 2$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.99$

$$t^{\star} = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(3.07)^2}{3} + \frac{(1.03)^2}{3}} = 1.87$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$
$$CI \;=\; (-6.65,\, 30.45)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(22.4 - 10.5) - 0}{1.87} = 6.37$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 6.37$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.02 \;<\; p\text{-value} \;<\; 0.04$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^{\star}$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 2
(b) 9.92
(c) 1.87
(d) -6.65
(e) 30.45
(f) 6.365
(g) 0.02
(h) 0.04
(i) no

Name: ____________________

1. **Problem**
   An experiment has n_1 = 3 plants in the treatment group and n_2 = 7 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	110	106	106				
sample 2:	266	278	266	270	234	250	287

   (a) Determine degrees of freedom.
   (b) Determine t^* for a 99% confidence interval.
   (c) Determine *SE*.
   (d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 2.000

   (b) 9.920

   (c) 6.785

   (d) 89.693

   (e) 224.307

   (f) 23.141

   (g) 0.000

   (h) 0.002

   (i) yes

1. **Solution**
   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 107$$
$$\overline{x_2} = 264$$
$$s_1 = 2.31$$
$$s_2 = 17.6$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(3, 7) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(2.31)^2}{3} + \frac{(17.6)^2}{7}} = 6.785$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (89.693,\ 224.307)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(264 - 107) - 0}{6.785} = 23.14$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 23.14$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0 < p\text{-value} < 0.002$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

   (a) 2
   (b) 9.92
   (c) 6.785
   (d) 89.693
   (e) 224.307
   (f) 23.141
   (g) 0
   (h) 0.002
   (i) yes

Name: ______________________________

1. **Problem**
   An experiment has n_1 = 6 plants in the treatment group and n_2 = 6 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	1.22	1.37	1.33	1.14	1.22	1.29
sample 2:	1.21	1.63	1.1	1.01	1.07	0.81

   (a) Determine degrees of freedom.
   (b) Determine t^* for a 95% confidence interval.
   (c) Determine SE.
   (d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 5.000

   (b) 2.570

   (c) 0.117

   (d) -0.421

   (e) 0.181

   (f) 1.022

   (g) 0.200

   (h) 1.000

   (i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1.26$$
$$\overline{x_2} = 1.14$$
$$s_1 = 0.0842$$
$$s_2 = 0.275$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\, n_2) - 1 = \min(6, 6) - 1 = 5$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.95$

$$t^{\star} = 2.57$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.0842)^2}{6} + \frac{(0.275)^2}{6}} = 0.117$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$
$$CI = (-0.421,\, 0.181)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.14 - 1.26) - 0}{0.117} = -1.02$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 1.02$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.2 < p\text{-value} < 1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^{\star}$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 5
(b) 2.57
(c) 0.117
(d) -0.421
(e) 0.181
(f) 1.022
(g) 0.2
(h) 1
(i) no

Name: ______________________________

1. **Problem**
An experiment has n_1 = 8 plants in the treatment group and n_2 = 8 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	12.1	12.5	10	10.8	7.4	11.2	8.2	12.1
sample 2:	10.9	14.2	10.8	12.6	8.7	13.7	16.2	13.8

(a) Determine degrees of freedom.
(b) Determine $t^\star$ for a 95% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 95% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 95% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.05$? (yes or no)

1. (a) 7.000

(b) 2.360

(c) 1.072

(d) -0.430

(e) 4.630

(f) 1.958

(g) 0.050

(h) 0.100

(i) no

1. **Solution**

   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

   $$\overline{x_1} = 10.5$$
   $$\overline{x_2} = 12.6$$
   $$s_1 = 1.88$$
   $$s_2 = 2.38$$

   We make a conservative estimate of the degrees of freedom using the appropriate formula.

   $$df \ = \ \min(n_1,\, n_2) - 1 \ = \ \min(8, 8) - 1 \ = \ 7$$

   We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.95$

   $$t^{\star} = 2.36$$

   We use the SE formula for unpaired data.

   $$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(1.88)^2}{8} + \frac{(2.38)^2}{8}} = 1.072$$

   We find the bounds of the confidence interval.

   $$CI \ = \ (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$
   $$CI \ = \ (-0.43,\, 4.63)$$

   We find t_{obs}.

   $$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(12.6 - 10.5) - 0}{1.072} = 1.96$$

   We find $|t_{\text{obs}}|$.

   $$|t_{\text{obs}}| = 1.96$$

   We use the table to determine bounds on p-value. Remember, $df = 7$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

   $$0.05 \ < \ p\text{-value} \ < \ 0.1$$

   We should consider both comparisons to make our decision.

   $$|t_{\text{obs}}| < t^{\star}$$
   $$p\text{-value} > \alpha$$

   Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

   (a) 7
   (b) 2.36
   (c) 1.072
   (d) -0.43
   (e) 4.63
   (f) 1.958
   (g) 0.05
   (h) 0.1
   (i) no

Name: ____________________

1. **Problem**
An experiment has n_1 = 8 plants in the treatment group and n_2 = 4 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	1.91	1.64	1.7	1.72	1.23	1.29	1.61	1.49
sample 2:	0.89	0.96	1.12	0.8				

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 98% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail *p*-value.
(h) Determine an upper bound of two-tail *p*-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.02$? (yes or no)

1. (a) 3.000

(b) 4.540

(c) 0.105

(d) -1.105

(e) -0.151

(f) 5.988

(g) 0.005

(h) 0.010

(i) **yes**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1.57$$
$$\overline{x_2} = 0.942$$
$$s_1 = 0.227$$
$$s_2 = 0.135$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(8, 4) - 1 = 3$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 4.54$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.227)^2}{8} + \frac{(0.135)^2}{4}} = 0.105$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-1.105, \ -0.151)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(0.942 - 1.57) - 0}{0.105} = -5.99$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 5.99$$

We use the table to determine bounds on p-value. Remember, $df = 3$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.005 < p\text{-value} < 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 3
(b) 4.54
(c) 0.105
(d) -1.105
(e) -0.151
(f) 5.988
(g) 0.005
(h) 0.01
(i) yes

Name: ______________________________

1. **Problem**
An experiment has n_1 = 6 plants in the treatment group and n_2 = 5 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	10.6	9.4	9.9	9.4	11.6	9.8
sample 2:	19.4	14.2	14	15.9	15.7	

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 99% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 4.000

(b) 4.600

(c) 1.031

(d) 0.957

(e) 10.443

(f) 5.531

(g) 0.005

(h) 0.010

(i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 10.1$$
$$\overline{x_2} = 15.8$$
$$s_1 = 0.85$$
$$s_2 = 2.17$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\ n_2) - 1 = \min(6, 5) - 1 = 4$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 4.6$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.85)^2}{6} + \frac{(2.17)^2}{5}} = 1.031$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (0.957,\ 10.443)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(15.8 - 10.1) - 0}{1.031} = 5.53$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 5.53$$

We use the table to determine bounds on p-value. Remember, $df = 4$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.005 < p\text{-value} < 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 4
(b) 4.6
(c) 1.031
(d) 0.957
(e) 10.443
(f) 5.531
(g) 0.005
(h) 0.01
(i) yes

Name: ______________________________

1. **Problem**
   An experiment has $n_1 = 3$ plants in the treatment group and $n_2 = 6$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	1.07	0.98	1			
sample 2:	1.86	2.77	2.8	1.91	2.37	2.21

   (a) Determine degrees of freedom.
   (b) Determine t^* for a 99% confidence interval.
   (c) Determine SE.
   (d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 2.000

   (b) 9.920

   (c) 0.168

   (d) -0.367

   (e) 2.967

   (f) 7.720

   (g) 0.010

   (h) 0.020

   (i) no

1. **Solution**
   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1.02$$
$$\overline{x_2} = 2.32$$
$$s_1 = 0.0473$$
$$s_2 = 0.407$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\, n_2) - 1 = \min(3, 6) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.0473)^2}{3} + \frac{(0.407)^2}{6}} = 0.168$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-0.367,\, 2.967)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(2.32 - 1.02) - 0}{0.168} = 7.72$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 7.72$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.01 < p\text{-value} < 0.02$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 2
(b) 9.92
(c) 0.168
(d) -0.367
(e) 2.967
(f) 7.72
(g) 0.01
(h) 0.02
(i) no

Name: ____________________

1. **Problem**
An experiment has n_1 = 4 plants in the treatment group and n_2 = 8 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7	value8
sample 1:	143	134	145	151				
sample 2:	108	109	101	110	94	81	96	96

(a) Determine degrees of freedom.
(b) Determine t^* for a 99% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1$ = 0.
(g) Determine a lower bound of the two-tail *p*-value.
(h) Determine an upper bound of two-tail *p*-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level α = 0.01? (yes or no)

1. (a) 3.000

(b) 5.840

(c) 4.939

(d) -72.444

(e) -14.756

(f) 8.827

(g) 0.002

(h) 0.004

(i) yes

1. **Solution**

These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 143$$
$$\overline{x_2} = 99.4$$
$$s_1 = 7.04$$
$$s_2 = 9.8$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(4, 8) - 1 = 3$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 5.84$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(7.04)^2}{4} + \frac{(9.8)^2}{8}} = 4.939$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-72.444, -14.756)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(99.4 - 143) - 0}{4.939} = -8.83$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 8.83$$

We use the table to determine bounds on p-value. Remember, $df = 3$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.002 < p\text{-value} < 0.004$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 3
(b) 5.84
(c) 4.939
(d) -72.444
(e) -14.756
(f) 8.827
(g) 0.002
(h) 0.004
(i) yes

Name: ____________________

1. **Problem**
An experiment has $n_1 = 7$ plants in the treatment group and $n_2 = 4$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	1.07	1.03	1.39	0.76	0.82	0.83	0.74
sample 2:	1.05	0.84	1.62	1.19			

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 90% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 90% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 90% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.1$? (yes or no)

1. (a) 3 . 0 0 0

(b) 2 . 3 5 0

(c) 0 . 1 8 7

(d) - 0 . 2 0 8

(e) 0 . 6 7 0

(f) 1 . 2 3 5

(g) 0 . 2 0 0

(h) 1 . 0 0 0

(i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 0.949$$
$$\overline{x_2} = 1.18$$
$$s_1 = 0.233$$
$$s_2 = 0.33$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\, n_2) - 1 = \min(7, 4) - 1 = 3$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.9$

$$t^\star = 2.35$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.233)^2}{7} + \frac{(0.33)^2}{4}} = 0.187$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-0.208,\, 0.67)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.18 - 0.949) - 0}{0.187} = 1.24$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 1.24$$

We use the table to determine bounds on p-value. Remember, $df = 3$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.2 < p\text{-value} < 1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 3
(b) 2.35
(c) 0.187
(d) -0.208
(e) 0.67
(f) 1.235
(g) 0.2
(h) 1
(i) no

Name: ____________________

1. **Problem**
   An experiment has n_1 = 6 plants in the treatment group and n_2 = 3 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	215	232	210	204	217	215
sample 2:	76	104	92			

   (a) Determine degrees of freedom.
   (b) Determine t^* for a 98% confidence interval.
   (c) Determine SE.
   (d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.02$? (yes or no)

1. (a) 2.000
   (b) 6.960
   (c) 8.939
   (d) -187.515
   (e) -63.085
   (f) 14.017
   (g) 0.005
   (h) 0.010
   (i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 216$$

$$\overline{x_2} = 90.7$$

$$s_1 = 9.35$$

$$s_2 = 14$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(6, 3) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 6.96$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(9.35)^2}{6} + \frac{(14)^2}{3}} = 8.939$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$

$$CI = (-187.515, -63.085)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(90.7 - 216) - 0}{8.939} = -14.02$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 14.02$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.005 < p\text{-value} < 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 2
(b) 6.96
(c) 8.939
(d) -187.515
(e) -63.085
(f) 14.017
(g) 0.005
(h) 0.01
(i) yes

Name: ____________________

1. **Problem**
An experiment has $n_1 = 7$ plants in the treatment group and $n_2 = 5$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	1.1	1.05	1.08	1.05	0.86	1.29	0.6
sample 2:	1.62	1.4	1.51	1.17	1.46		

(a) Determine degrees of freedom.
(b) Determine $t^{\star}$ for a 98% confidence interval.
(c) Determine *SE*.
(d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail *p*-value.
(h) Determine an upper bound of two-tail *p*-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.02$? (yes or no)

1. (a) 4 . 0 0 0

(b) 3 . 7 5 0

(c) 0 . 1 1 1

(d) 0 . 0 1 4

(e) 0 . 8 4 6

(f) 3 . 8 6 7

(g) 0 . 0 1 0

(h) 0 . 0 2 0

(i) **yes**

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1$$

$$\overline{x_2} = 1.43$$

$$s_1 = 0.218$$

$$s_2 = 0.167$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(7, 5) - 1 \;=\; 4$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 3.75$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.218)^2}{7} + \frac{(0.167)^2}{5}} = 0.111$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$

$$CI \;=\; (0.014,\, 0.846)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.43 - 1) - 0}{0.111} = 3.87$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 3.87$$

We use the table to determine bounds on p-value. Remember, $df = 4$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.01 \;<\; p\text{-value} \;<\; 0.02$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$

$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 4
(b) 3.75
(c) 0.111
(d) 0.014
(e) 0.846
(f) 3.867
(g) 0.01
(h) 0.02
(i) yes

Name:

1. **Problem**
An experiment has n_1 = 3 plants in the treatment group and n_2 = 4 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4
sample 1:	112	114	100	
sample 2:	211	204	233	223

(a) Determine degrees of freedom.
(b) Determine t^* for a 98% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.02$? (yes or no)

1. (a) 2.000
(b) 6.960
(c) 7.750
(d) 55.060
(e) 162.940
(f) 14.065
(g) 0.005
(h) 0.010
(i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 109$$
$$\overline{x_2} = 218$$
$$s_1 = 7.57$$
$$s_2 = 12.8$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(3, 4) - 1 \;=\; 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 6.96$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(7.57)^2}{3} + \frac{(12.8)^2}{4}} = 7.75$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI \;=\; (55.06,\ 162.94)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(218 - 109) - 0}{7.75} = 14.06$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 14.06$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.005 \;<\; p\text{-value} \;<\; 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 2
(b) 6.96
(c) 7.75
(d) 55.06
(e) 162.94
(f) 14.065
(g) 0.005
(h) 0.01
(i) yes

Name: ____________________

1. **Problem**
   An experiment has n_1 = 6 plants in the treatment group and n_2 = 6 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	1.25	1.13	1.31	1.16	1.13	1.14
sample 2:	1.13	0.96	0.98	1.14	1.26	1.07

   (a) Determine degrees of freedom.
   (b) Determine $t^{\star}$ for a 90% confidence interval.
   (c) Determine *SE*.
   (d) Determine a lower bound of the 90% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 90% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail *p*-value.
   (h) Determine an upper bound of two-tail *p*-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.1$? (yes or no)

1. (a) 5.000

   (b) 2.020

   (c) 0.055

   (d) -0.211

   (e) 0.011

   (f) 1.813

   (g) 0.100

   (h) 0.200

   (i) **no**

1. **Solution**

These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1.19$$

$$\overline{x_2} = 1.09$$

$$s_1 = 0.0755$$

$$s_2 = 0.112$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(6, 6) - 1 \;=\; 5$$

We use the t table to find $t^{\star}$ such that $P(|T| < t^{\star}) = 0.9$

$$t^{\star} = 2.02$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.0755)^2}{6} + \frac{(0.112)^2}{6}} = 0.055$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^{\star} SE$$

$$CI \;=\; (-0.211,\, 0.011)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.09 - 1.19) - 0}{0.055} = -1.81$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 1.81$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.1 \;<\; p\text{-value} \;<\; 0.2$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^{\star}$$

$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 5
(b) 2.02
(c) 0.055
(d) -0.211
(e) 0.011
(f) 1.813
(g) 0.1
(h) 0.2
(i) no

Name: ______________________________

1. **Problem**
   An experiment has n_1 = 6 plants in the treatment group and n_2 = 6 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6
sample 1:	0.98	1.03	1.12	1.02	1.3	1.02
sample 2:	1.18	1.34	1.19	1.13	1.22	1.22

   (a) Determine degrees of freedom.
   (b) Determine $t^{\star}$ for a 98% confidence interval.
   (c) Determine *SE*.
   (d) Determine a lower bound of the 98% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 98% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail *p*-value.
   (h) Determine an upper bound of two-tail *p*-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level α = 0.02? (yes or no)

1. (a) 5 . 0 0 0

(b) 3 . 3 6 0

(c) 0 . 0 5 6

(d) - 0 . 0 5 8

(e) 0 . 3 1 8

(f) 2 . 3 1 8

(g) 0 . 0 5 0

(h) 0 . 1 0 0

(i) no

1. **Solution**
   These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 1.08$$
$$\overline{x_2} = 1.21$$
$$s_1 = 0.118$$
$$s_2 = 0.0703$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df \;=\; \min(n_1,\, n_2) - 1 \;=\; \min(6, 6) - 1 \;=\; 5$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.98$

$$t^\star = 3.36$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(0.118)^2}{6} + \frac{(0.0703)^2}{6}} = 0.056$$

We find the bounds of the confidence interval.

$$CI \;=\; (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI \;=\; (-0.058,\, 0.318)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(1.21 - 1.08) - 0}{0.056} = 2.32$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 2.32$$

We use the table to determine bounds on p-value. Remember, $df = 5$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.05 \;<\; p\text{-value} \;<\; 0.1$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

   (a) 5
   (b) 3.36
   (c) 0.056
   (d) -0.058
   (e) 0.318
   (f) 2.318
   (g) 0.05
   (h) 0.1
   (i) no