

Context	Standard error	Confidence interval	Two-tail $H_0$	Test statistic	Distribution
One mean					
— $\sigma$ known	$SE = \frac{\sigma}{\sqrt{n}}$	$\bar{x} \pm (z^*)(SE)$	$\mu = \mu_0$	$z = \frac{\bar{x} - \mu_0}{SE}$	$\mathcal{N}(0, 1)$
— $\sigma$ unknown	$\widehat{SE} = \frac{s}{\sqrt{n}}$	$\bar{x} \pm (t^*)(\widehat{SE})$	$\mu = \mu_0$	$t = \frac{\bar{x} - \mu_0}{\widehat{SE}}$	$t(n - 1)$
Two means					
— $\sigma_1$ and $\sigma_2$ known	$SE = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	$\bar{x}_2 - \bar{x}_1 \pm (z^*)(SE)$	$\mu_1 = \mu_2$	$z = \frac{\bar{x}_2 - \bar{x}_1}{SE}$	$\mathcal{N}(0, 1)$
— $\sigma_1$ and $\sigma_2$ unknown	$\widehat{SE} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$\bar{x}_2 - \bar{x}_1 \pm (t^*)(\widehat{SE})$	$\mu_1 = \mu_2$	$t = \frac{\bar{x}_2 - \bar{x}_1}{\widehat{SE}}$	$t(\min(n_1, n_2) - 1)$
One proportion					
— confidence intervals	$\widehat{SE} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$\hat{p} \pm (z^*)(\widehat{SE})$			
— hypothesis tests	$\widehat{SE} = \sqrt{\frac{p_0(1-p_0)}{n}}$		$p = p_0$	$z = \frac{\hat{p} - p_0}{\widehat{SE}}$	$\mathcal{N}(0, 1)$
Two proportions					
— confidence intervals	$\widehat{SE} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	$\hat{p}_2 - \hat{p}_1 \pm (z^*)(\widehat{SE})$			
— hypothesis tests	$\widehat{SE} = \sqrt{\bar{p}(1-\bar{p}) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$		$p_1 = p_2$	$z = \frac{\hat{p}_2 - \hat{p}_1}{\widehat{SE}}$	$\mathcal{N}(0, 1)$

- $\mu_0$  is the mean suggested by  $H_0$ , the null hypothesis.
- $p_0$  is the proportion suggested by  $H_0$ , the null hypothesis.
- $\bar{p} = \frac{\hat{p}_1 n_1 + \hat{p}_2 n_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$  is the pooled proportion.
- The confidence level  $\gamma$  is typically 0.95 and the significance level  $\alpha = 1 - \gamma$  is typically 0.05.
- In two-tailed tests (= as  $H_0$ ;  $\neq$  as  $H_a$ ),  $z^*$  is the value such that the area between  $-z^*$  and  $z^*$  under the standard normal distribution is  $\gamma$ .
- In two-tailed tests,  $t^*$  is the value such that the area between  $-t^*$  and  $t^*$  under the  $t(df)$  distribution is  $\gamma$ , where  $df$  is the degrees of freedom. Typically  $df = n - 1$ .
- In left-tailed tests ( $H_0 : \mu \geq \mu_0$ ), the value of  $z^*$  or  $t^*$  will represent upper limit of left area equal to  $\alpha$ .
- In right-tailed tests ( $H_0 : \mu \leq \mu_0$ ), the value of  $z^*$  or  $t^*$  will represent lower limit of right area equal to  $\alpha$ .
- Some common values of  $z^*$  are listed below. For a given  $\gamma$  or  $\alpha$ , we know  $|t^*| \geq |z^*|$ .

$\gamma$	$\alpha$	two-tailed $z^*$	right-tail $z^*$	left-tail $z^*$
0.8	0.2	1.28	0.84	-0.84
0.9	0.1	1.64	1.28	-1.28
0.95	0.05	1.96	1.64	-1.64
0.99	0.01	2.58	2.33	-2.33
0.999	0.001	3.29	3.09	-3.09

- For two-tailed hypothesis testing:

if  $|z| > z^*$  then reject  $H_0$ , otherwise retain  $H_0$

if  $|t| > t^*$  then reject  $H_0$ , otherwise retain  $H_0$

$$p\text{-value} = 2 \cdot \Phi(-|z|)$$

if  $p\text{-value} < \alpha$  then reject  $H_0$ , otherwise retain  $H_0$