Name: ______________________________

1. **Problem**
   An experiment has $n_1 = 3$ plants in the treatment group and $n_2 = 3$ plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3
sample 1:	13.9	9.8	7.9
sample 2:	23.3	22.7	21.3

   (a) Determine degrees of freedom.
   (b) Determine t^* for a 99% confidence interval.
   (c) Determine SE.
   (d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
   (f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
   (g) Determine a lower bound of the two-tail p-value.
   (h) Determine an upper bound of two-tail p-value.
   (i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 2.000

   (b) 9.920

   (c) 1.870

   (d) -6.650

   (e) 30.450

   (f) 6.365

   (g) 0.020

   (h) 0.040

   (i) no

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 10.5$$
$$\overline{x_2} = 22.4$$
$$s_1 = 3.07$$
$$s_2 = 1.03$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1,\, n_2) - 1 = \min(3, 3) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(3.07)^2}{3} + \frac{(1.03)^2}{3}} = 1.87$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-6.65,\, 30.45)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(22.4 - 10.5) - 0}{1.87} = 6.37$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 6.37$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.02 < p\text{-value} < 0.04$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| < t^\star$$
$$p\text{-value} > \alpha$$

Thus, we retain the null hypothesis. Also notice the confidence interval does contain 0.

(a) 2
(b) 9.92
(c) 1.87
(d) -6.65
(e) 30.45
(f) 6.365
(g) 0.02
(h) 0.04
(i) no