

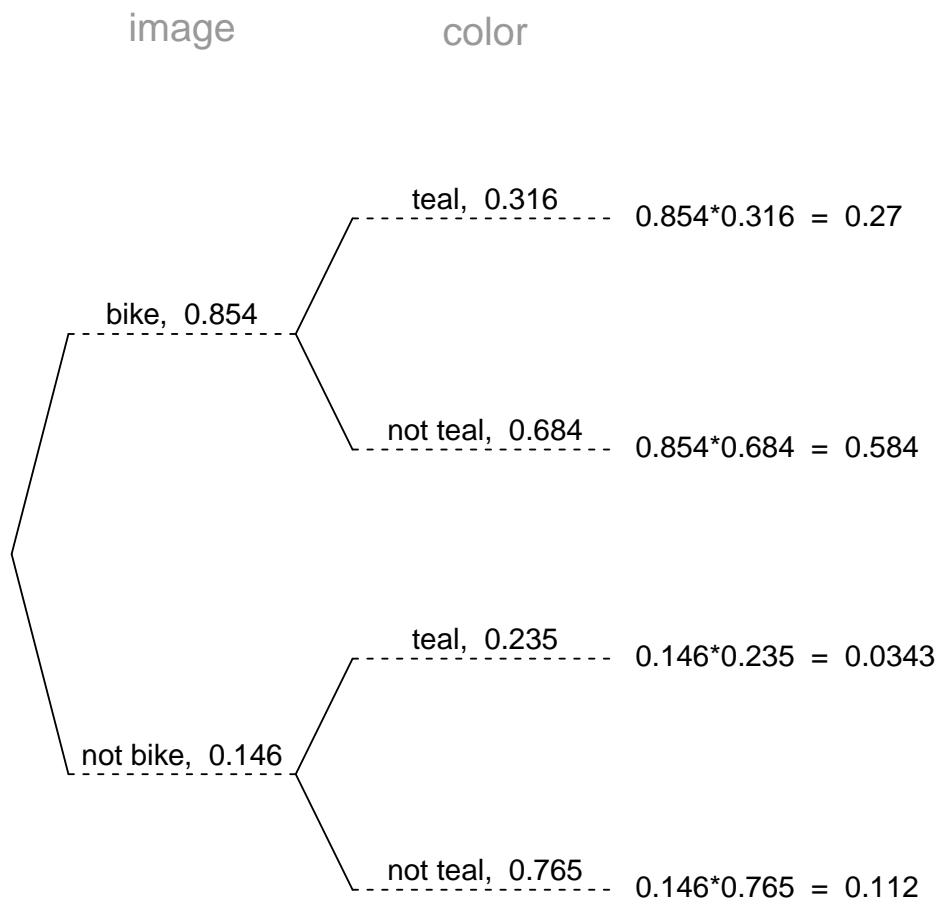
1. Problem

In a deck of strange cards, each card has an image and a color. The chance of drawing a bike is 85.4%. If a bike is drawn, there is a 31.6% chance that it is teal. If a card that is not a bike is drawn, there is a 23.5% chance that it is teal.

Now, someone draws a random card and reveals it is teal. What is the chance the card is a bike?

Solution

I'd recommend making a tree. Remember, on the first branch, we put simple probabilities. On the second branches we put conditional probabilities. The results (products) are joint probabilities.



Determine the appropriate conditional probability.

$$P(\text{"bike" given "teal"}) = \frac{0.27}{0.27 + 0.0343} = 0.887$$

2. Problem

A treatment group of size 29 has a mean of 1.11 and standard deviation of 0.239. A control group of size 13 has a mean of 0.953 and standard deviation of 0.254. If you decided to use a significance level of 0.05, is there sufficient evidence to conclude the treatment causes an effect?

By using the Welch-Satterthwaite equation, I've calculated the degrees of freedom should be 21.

- (a) State the null hypothesis.
- (b) State the alternative hypothesis.
- (c) Evaluate the critical value. (The critical value is either z^* or t^* . Determine its value.)
- (d) Determine the standard error of the relevant sampling distribution.
- (e) Evaluate the absolute value of the test statistic. (The test statistic is either z_{obs} or t_{obs} . Determine its absolute value.)
- (f) If possible, evaluate the p -value. Otherwise, describe an interval containing the p -value.
- (g) Do we reject or retain the null?

Solution

We are given unpaired data. We are considering a difference of means. Label the given information.

$$n_1 = 29$$

$$\bar{x}_1 = 1.11$$

$$s_1 = 0.239$$

$$n_2 = 13$$

$$\bar{x}_2 = 0.953$$

$$s_2 = 0.254$$

$$\alpha = 0.05$$

$$df = 21$$

State the hypotheses.

$$H_0 : \mu_2 - \mu_1 = 0$$

$$H_A : \mu_2 - \mu_1 \neq 0$$

We are using a two-tail test. Find t^* such that $P(|T| > t^*) = 0.05$ by using a t table.

$$t^* = 2.08$$

Calculate the standard error.

$$\begin{aligned} SE &= \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} \\ &= \sqrt{\frac{(0.239)^2}{29} + \frac{(0.254)^2}{13}} \\ &= 0.083 \end{aligned}$$

Determine the test statistic.

$$\begin{aligned} t_{\text{obs}} &= \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} \\ &= \frac{(0.953 - 1.11) - (0)}{0.083} \\ &= -1.89 \end{aligned}$$

Compare $|t_{\text{obs}}|$ and t^* .

$$|t_{\text{obs}}| < t^*$$

We can determine an interval for the p -value using the t table.

$$0.05 < p\text{-value} < 0.1$$

Compare p -value and α .

$$p\text{-value} > \alpha$$

We conclude that we should retain the null hypothesis.

3. Problem

A roughly symmetric population has a mean $\mu = 220$ and standard deviation $\sigma = 63$. What is the probability that a sample of size $n = 75$ has a mean more than 225.6?

Solution

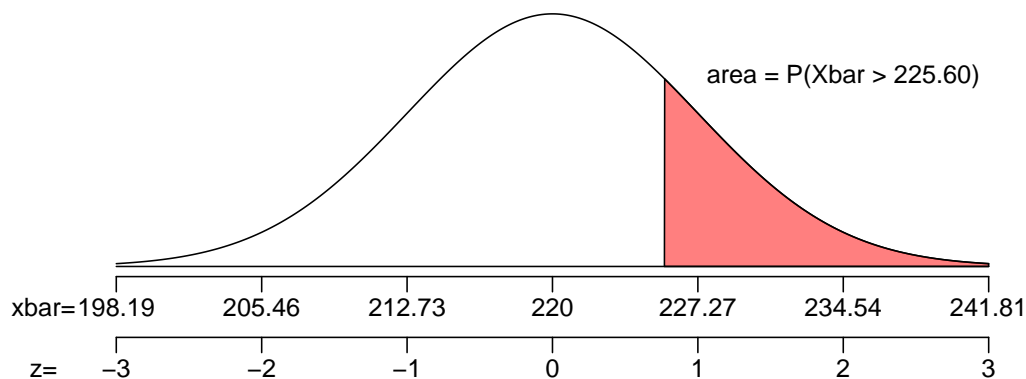
Find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{63}{\sqrt{75}} = 7.27$$

Describe the sampling distribution.

$$\bar{X} \sim \mathcal{N}(220, 7.27)$$

Draw a sketch.



Calculate a z score.

$$z = \frac{225.6 - 220}{7.27} = 0.77$$

Determine the probability.

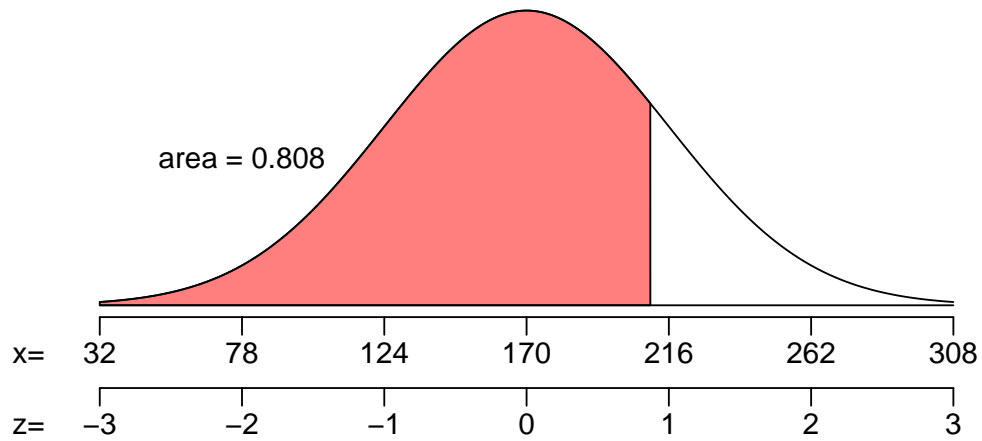
$$P(X > 225.6) = 0.221$$

4. Problem

Let $X \sim \mathcal{N}(170, 46)$. Determine x such that $P(X < x) = 0.808$.

Solution

We draw a sketch.



We find the z score.

$$z = \Phi^{-1}(0.808) = 0.87$$

We find x .

$$x = \mu + z\sigma = 170 + (0.87)(46) = 210.02$$

5. Problem

Let each trial have a chance of success $p = 0.14$. If 157 trials occur, what is the probability of getting at least 15 but less than 26 successes?

In other words, let $X \sim \text{Bin}(n = 157, p = 0.14)$ and find $P(15 \leq X < 26)$.

Use a normal approximation along with the continuity correction.

Solution

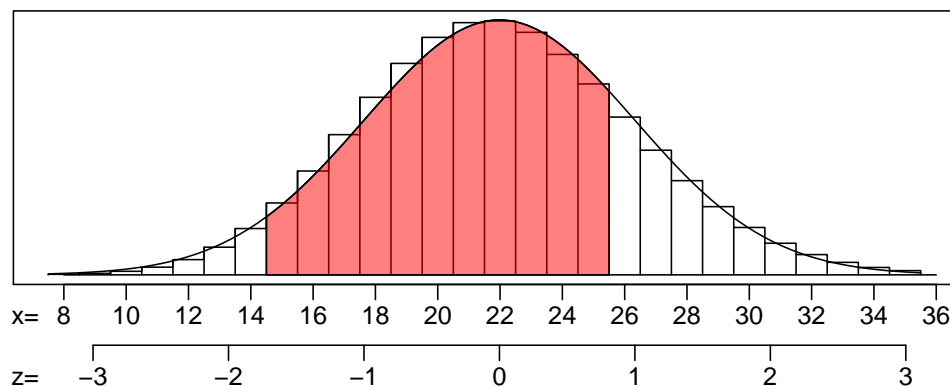
Find the mean.

$$\mu = np = (157)(0.14) = 21.98$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(157)(0.14)(1-0.14)} = 4.3477$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{14.5 - 21.98}{4.3477} = -1.72$$

$$z_2 = \frac{25.5 - 21.98}{4.3477} = 0.81$$

Calculate the probability.

$$P(15 \leq X < 26) = \Phi(0.81) - \Phi(-1.72) = 0.748$$

6. Problem

As an ornithologist, you wish to determine the average body mass of *Catharus guttatus*. You randomly capture 17 adults of *Catharus guttatus*, resulting in a sample mean of 29.52 grams and a sample standard deviation of 1.6 grams. You decide to report a 90% confidence interval.

- (a) Determine the lower bound of the confidence interval.
- (b) Determine the upper bound of the confidence interval.

Solution

We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 17$$

$$\bar{x} = 29.52$$

$$s = 1.6$$

$$CL = 0.9$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 16$$

Determine the critical t value, t^* , such that $P(|T| < t^*) = 0.9$.

$$t^* = 1.75$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.6}{\sqrt{17}} = 0.388$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^* SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^* SE, \bar{x} + t^* SE) \\ &= (29.52 - 1.75 \times 0.388, 29.52 + 1.75 \times 0.388) \\ &= (28.8, 30.2) \end{aligned}$$

We are 90% confident that the population mean is between 28.8 and 30.2.

(a) Lower bound = 28.8

(b) Upper bound = 30.2

7. Problem

Let each trial have a chance of success $p = 0.62$. If 150 trials occur, what is the probability of getting exactly 106 successes?

In other words, let $X \sim \text{Bin}(n = 150, p = 0.62)$ and find $P(X = 106)$.

Solution

We use the formula for binomial probabilities.

$$P(X = k) = \binom{n}{k} (p)^k (1 - p)^{n-k}$$

$$P(X = 106) = \binom{150}{106} (0.62)^{106} (1 - 0.62)^{150-106}$$

$$P(X = 106) = \binom{150}{106} (0.62)^{106} (0.38)^{44}$$

In a TI calculator, the expression will look like:

$$(150 \text{ nCr } 106) * 0.62^{106} * (1 - 0.62)^{(150 - 106)}$$

In Geogebra:

$$\text{nCr}(150, 106) \cdot 0.62^{106} (1 - 0.62)^{150 - 106}$$

$$P(X = 106) = 0.006$$

1. $P(\text{"bike" given "teal"}) = 0.887$

2. (a) $H_0 : \mu_2 - \mu_1 = 0$

(b) $H_0 : \mu_2 - \mu_1 \neq 0$

(c) $t^* = 2.08$

(d) $SE = 0.083$

(e) $|t_{\text{obs}}| = 1.89$

(f) $0.05 < p\text{-value} < 0.1$

(g) **retain**

3. $P(X > 225.6) = 0.221$

4. $x = 210.02$

5. $P(15 \leq X < 26) = 0.748$

6. (a) **LB = 28.8**

(b) **UB = 30.2**

7. $P(X = 106) = 0.006$