Name: ______________________________

1. **Problem**
An experiment has n_1 = 7 plants in the treatment group and n_2 = 3 plants in the control group. After some time, the plants' heights (in cm) are measured, resulting in the following data:

	value1	value2	value3	value4	value5	value6	value7
sample 1:	255	202	216	204	225	211	214
sample 2:	103	113	126				

(a) Determine degrees of freedom.
(b) Determine t^* for a 99% confidence interval.
(c) Determine SE.
(d) Determine a lower bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(e) Determine an upper bound of the 99% confidence interval of $\mu_2 - \mu_1$.
(f) Determine $|t_{\text{obs}}|$ under the null hypothesis $\mu_2 - \mu_1 = 0$.
(g) Determine a lower bound of the two-tail p-value.
(h) Determine an upper bound of two-tail p-value.
(i) Do you reject the null hypothesis with a two-tail test using a significance level $\alpha = 0.01$? (yes or no)

1. (a) 2.000

(b) 9.920

(c) 9.506

(d) -198.300

(e) -9.700

(f) 10.940

(g) 0.005

(h) 0.010

(i) yes

1. **Solution**
These data are unpaired. We might as well find the sample means and sample standard deviations (use a calculator's built-in function for standard deviation).

$$\overline{x_1} = 218$$
$$\overline{x_2} = 114$$
$$s_1 = 18$$
$$s_2 = 11.5$$

We make a conservative estimate of the degrees of freedom using the appropriate formula.

$$df = \min(n_1, n_2) - 1 = \min(7, 3) - 1 = 2$$

We use the t table to find $t^\star$ such that $P(|T| < t^\star) = 0.99$

$$t^\star = 9.92$$

We use the SE formula for unpaired data.

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{\frac{(18)^2}{7} + \frac{(11.5)^2}{3}} = 9.506$$

We find the bounds of the confidence interval.

$$CI = (\overline{x_2} - \overline{x_1}) \pm t^\star SE$$
$$CI = (-198.3, -9.7)$$

We find t_{obs}.

$$t_{\text{obs}} = \frac{(\overline{x_2} - \overline{x_1}) - (\mu_2 - \mu_1)_0}{SE} = \frac{(114 - 218) - 0}{9.506} = -10.94$$

We find $|t_{\text{obs}}|$.

$$|t_{\text{obs}}| = 10.94$$

We use the table to determine bounds on p-value. Remember, $df = 2$ and p-value $= P(|T| > |t_{\text{obs}}|)$.

$$0.005 < p\text{-value} < 0.01$$

We should consider both comparisons to make our decision.

$$|t_{\text{obs}}| > t^\star$$
$$p\text{-value} < \alpha$$

Thus, we reject the null hypothesis. Also notice the confidence interval does not contain 0.

(a) 2
(b) 9.92
(c) 9.506
(d) -198.3
(e) -9.7
(f) 10.94
(g) 0.005
(h) 0.01
(i) yes