1: A fair 6-sided die should have a population mean $\mu = 3.5$ and a population standard deviation $\sigma = 1.708$. To check the fairness of a die, you are asked to perform a sampling of size $n = 100$ and two-tailed hypothesis test with significance level $\alpha = 0.05$.

a: State the hypotheses.

b: Describe and sketch the null's population distribution. Use X_0 as the random variable.

c: Describe and sketch the null's sampling distribution (with $n = 100$). Let $\overline{X_0}$ be the random variable.

d: Determine r such that $P\left(\left|\overline{X_0} - 3.5\right| \geq r\right) = \alpha$. Then describe what r means in context.

e: Your sample yielded $\bar{x} = 3.3$. Determine the test statistic (z) and p-value. Also, make a conclusion. In this case, p-value $= P\left(\left|\overline{X_0} - 3.5\right| \geq 0.2\right)$.

2: Someone guessing on a 4-choice question has a 25% chance of success (worth 1 point) and a 75% chance of failure (worth 0 points). This means $\mu = 0.25$ and $\sigma = \sqrt{(0.25)(0.75)} = 0.433$ (Bernoulli distribution).

You wonder whether Jules will randomly guess on all 36 questions on a test. You decide to use a one-tailed test with $\alpha = 0.05$ to decide whether Jules is doing better than random guessing.

a: State the hypotheses.

b: Describe and sketch the null's population distribution (the probability distribution of a single random guess). Use X_0 as the random variable.

c: Describe and sketch the null's sampling distribution (with $n = 36$). Let $\overline{X_0}$ be the random variable.

d: Determine c such that $P\left(\overline{X_0} \geq c\right) = \alpha$. Then describe what c means in context.

e: Your sample yielded $\bar{x} = 0.389$ (because Jules got 14 questions right). Determine the test statistic (z) and p-value. Also, make a conclusion. In this case, p-value $= P\left(\overline{X_0} \geq 0.389\right)$.

3: Harold read that he has a 20% chance to win a scratch-off lottery each time he plays. Thus, on average he should only have to wait $\mu = 5$ times before winning, with a standard deviation of $\frac{\sqrt{1-0.2}}{0.2} = 4.47$ (geometric distribution). Harold wants to run a two-tail hypothesis test with a significance $\alpha = 0.02$ on the mean waiting time until success.

For the next 60 successes, Harold tracks how many tickets it takes until success.

a: State the hypotheses.

b: Describe and sketch the null's population distribution. Use X as the random variable.

c: Describe and sketch the null's sampling distribution (with $n = 60$). Let $\bar{X}$ be the random variable.

d: Determine r such that $P\left(\left|\bar{X} - \mu_0\right| \geq r\right) = \alpha$, where μ_0 is the null's mean. Then describe what r means in context.

e: Herold's sample yielded $\bar{x} = 5.6$. Determine the test statistic (z) and p-value. Also, make a conclusion.

4: A company claims the average weight of a trinket is 100 pounds. You decide to test their claim with a random sample and two-tail hypothesis test with a significance level of 0.05.

a: Describe the hypotheses.

b: You measure 40 trinkets, yielding a sample mean of 98.8 pounds with a standard deviation of 10 pounds. Using $\sigma \approx 10$, describe the sampling distribution **under the null hypothesis**. (Give the type of distribution and its parameters.)

c: Determine the test statistic (z) of the observation. In other words, determine a z score of the observed sample mean in the null's sampling distribution.

d: Determine a p-value. Also make a conclusion.

5: You wonder if μ is 888. You decide to do a 2-tail hypothesis test with a significance level of 0.05. A random sample of size 50 is taken, yielding $\bar{x} = 851$ and $s = 106$.

a: Describe the hypotheses.

b: Describe the null's sampling distribution by assuming $\sigma \approx s$.

c: Describe the p-value using a probability expression.

d: Find the test statistic and p-value.

e: Make a final judgement.

6: When a fair coin is flipped, it lands tails 50% of the time. Kimberly has a coin, and she wonders if it is fair. She plans to flip the coin 100 times, record the proportion of tails, and perform a hypothesis test with a significance level of 0.05.

a: Describe the hypotheses.

b: Determine p and σ of a single flip under the null hypothesis. (Bernoulli trial)

c: Determine p and SE of the sampling distribution under the null hypothesis.

d: Kimberly flips the coin 100 times and gets 57 tails, giving $\hat{p} = 0.57$. **Determine the test statistic**, z, of this observation under the null's sampling distribution.

e: Determine a p-value, where p-value $= P(|\hat{p} - p_0| > 0.07)$ assuming H_0 is true.

f: What conclusion will Kimberly make?

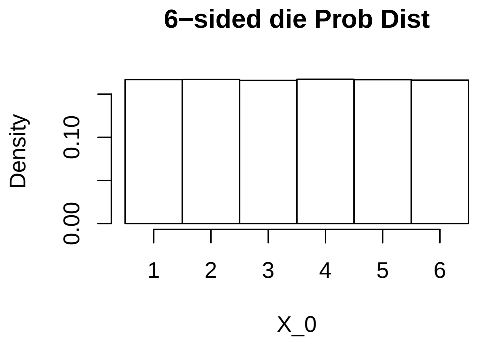
1:

a: $H_0: \quad \mu = 3.5$
$H_0: \quad \mu \neq 3.5$

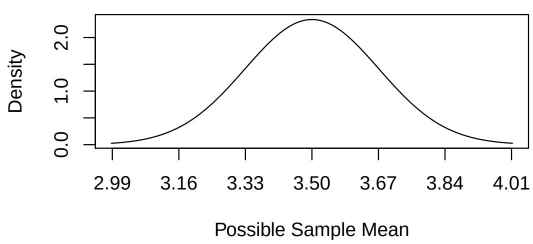
b:

x_{0i}	1	2	3	4	5	6
$P(X_0 = x_{0i})$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

X_0 is uniformly distributed across its 6 discrete possibilites (1 through 6).



c: We calculate $SE = \frac{1.708}{\sqrt{100}} = 0.1708$, so $\overline{X_0} \sim \mathcal{N}(\mu = 3.5,\ \sigma = 0.1708)$.



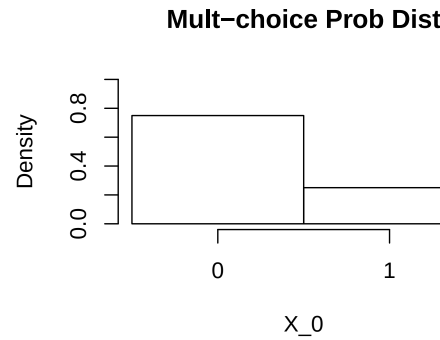
d: You should sketch a picture. We recognize we need the two-tail area to equal 0.05. We determine z_α such that $P(Z < z_\alpha) = 0.025$. That gives $z_\alpha = -1.96$. We convert this into a $\bar{x_\alpha}$ value. $\bar{x_\alpha} = 3.5 - (1.96)(0.1708)$, giving 3.17, which is 0.33 units from the mean. Thus, $r = 0.33$. In this context, r is how far an observed mean can be from 3.5 before we reject the null hypothesis.

e: You should sketch a picture. We want two-tail area below 3.3 and above 3.7. We find a z score. $z = \frac{3.3-3.5}{0.1708} = -1.17$. We determine the left area associated with $z = -1.17$.
$P(Z < -1.17) = 0.121$. We double this for the two-tailed area. p-value = 0.242. We retain the null hypothesis! This die seems fair to me.

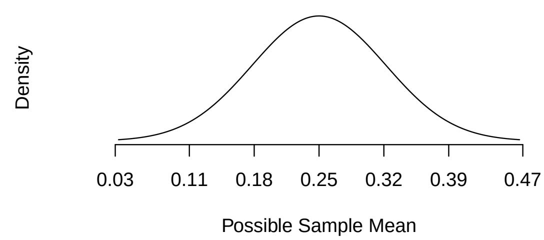
2:

a: $H_0: \quad \mu = 0.25$
$H_A: \quad \mu > 0.25$

b: $X_0 \sim Bernoulli(0.25)$.



c: We calculate $SE = \frac{0.433}{\sqrt{36}} = 0.072$, leading to $\overline{X_0} \sim \mathcal{N}(0.25,\ 0.072)$.

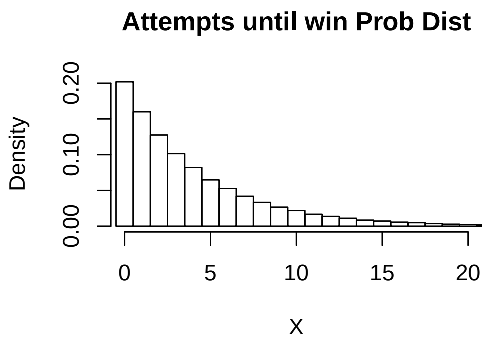


d: $c = 0.369$. In this context, c is the cutoff mean for us deciding whether or not Jules is merely guessing.

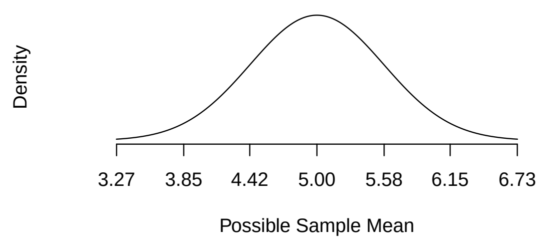
e: $z = \frac{0.389-0.25}{0.072} = 1.93$. We use the z table to find $P(Z > 1.93) = 0.0268$. So, p-value = 0.0268. We reject the null hypothesis. Jules is NOT merely guessing!

3: **a:** $H_0: \quad \mu = 5$
$H_A: \quad \mu \neq 5$

b: $X \sim \mathsf{Geo}(0.20)$.



c: We calculate $SE = \frac{4.47}{\sqrt{60}} = 0.577$. So, $\bar{X} \sim \mathcal{N}(5,\ 0.577)$.



d: We find z from $P(Z > z) = 0.01$, giving $z = 2.32$. We can find the corresponding distance from mean, $r = z \cdot SE = 2.32 \cdot 0.577 = 1.34$. In this context r represents a cutoff distance, between observed mean and μ_0, for rejecting the null.

e: We find $z^\star = \frac{5.6-5}{0.577} = 1.04$. We find $P(|Z| > z^\star) = 0.298$. We retain the null.

4: **a:** $H_0: \quad \mu = 100$
$H_A: \quad \mu \neq 100$

b: The sampling distribution is normal. We calculate standard error,
$SE = \frac{10}{\sqrt{40}} = 1.58$.
So, $\bar{X} \sim \mathcal{N}(100,\ 1.58)$.

c: $z = \frac{98.8-100}{1.58} = -0.759$.

d: For this two-tailed test, we determine $P(|Z| > 0.759) = 0.447$. We retain the null hypothesis.

5: **a:** $H_0: \quad \mu = 888$
$H_A: \quad \mu \neq 888$

b: We find $SE = \frac{106}{\sqrt{50}} = 15$.
So, $\bar{X} \sim \mathcal{N}(888,\ 15)$.

c: Let $\bar{X}$ represent a random draw from the null's sampling distribution.
p-value $= P(|\bar{X} - \mu_0| > 37)$.
I got 37 from the absolute difference between 888 and 851.

d: $z = \frac{851-888}{15} = -2.47$.
$P(|Z| > 2.47) = 0.0136$.
p-value is 0.0136.

e: We reject the null hypothesis.

6: **a:** $H_0: \quad p = 0.5$
$H_A: \quad p \neq 0.5$

b: Under the null, $p = 0.5$ and (Bernoulli) $\sigma = \sqrt{(0.5)(0.5)} = 0.5$. This population distribution is a Bernoulli distribution.

c: The sampling distribution is normal, with the same proportion as the population. $p = 0.5$. However, the SE is smaller than σ.
$SE = \frac{\sigma}{\sqrt{n}} = \frac{0.5}{\sqrt{100}} = 0.05$

d: $z = \frac{\hat{p}-p_0}{SE} = \frac{0.57-0.5}{0.05} = \boxed{1.4}$.

e: We find $P(|Z| > 1.4) = 0.1615$.

f: Kimberly retains the null hypothesis. For now she is still satisfied that the coin seems fair.