2.9: **(a):** Independent, not disjoint.

(b): Dependent, not disjoint.

(c): No.

2.10: **(a):** Let event X_i represent "Nancy gets the ith question right", where i is an integer such that $1 \leq i \leq 5$. On each question, Nancy has a 25% chance of success.

$$\begin{array}{ccccc} P(X_1) = 0.25 & P(X_2) = 0.25 & P(X_3) = 0.25 & P(X_4) = 0.25 & P(X_5) = 0.25 \\ P(X_1^c) = 0.75 & P(X_2^c) = 0.75 & P(X_3^c) = 0.75 & P(X_4^c) = 0.75 & P(X_5^c) = 0.75 \end{array}$$

We are asked to consider the possibility of Nancy missing the first four questions and getting the fifth question.

$$P\left(\begin{array}{c}\text{“The first question she}\\ \text{gets right is the 5th question”}\end{array}\right) = P(X_1^c \text{ AND } X_2^c \text{ AND } X_3^c \text{ AND } X_4^c \text{ AND } X_5)$$

The five elementary events are independent, so we can find the joint probability by multiplying the marginal probabilities. (Refer to the Multiplication Rule for independent processes on page 86.)

$$\begin{aligned} P(X_1^c \text{ AND } X_2^c \text{ AND } X_3^c \text{ AND } X_4^c \text{ AND } X_5) &= P(X_1^c) \cdot P(X_2^c) \cdot P(X_3^c) \cdot P(X_4^c) \cdot P(X_5) \\ &= 0.75 \cdot 0.75 \cdot 0.75 \cdot 0.75 \cdot 0.25 \\ &= 0.75^4 \cdot 0.25 \\ &\approx \boxed{0.0791} \end{aligned}$$

Thus, the probability is about 7.9%.

(b): We hope to determine the probability that Nancy gets all five correct. Again, the elementary events are independent, so we can use the Multiplication Rule.

$$\begin{aligned} P(X_1 \text{ AND } X_2 \text{ AND } X_3 \text{ AND } X_4 \text{ AND } X_5) &= P(X_1) \cdot P(X_2) \cdot P(X_3) \cdot P(X_4) \cdot P(X_5) \\ &= 0.25 \cdot 0.25 \cdot 0.25 \cdot 0.25 \cdot 0.25 \\ &= 0.25^5 \\ &\approx \boxed{0.000977} \end{aligned}$$

Thus, the probability is about 0.098%.

(c): The event "at least one right" is the complement of "all wrong". It is easy to calculate the probability of "all wrong".

$$\begin{aligned} P(\text{“all wrong”}) &= P(X_1^c \text{ AND } X_2^c \text{ AND } X_3^c \text{ AND } X_4^c \text{ AND } X_5^c) \\ &= P(X_1^c) \cdot P(X_2^c) \cdot P(X_3^c) \cdot P(X_4^c) \cdot P(X_5^c) \\ &= 0.75^5 \\ &\approx 0.237 \end{aligned}$$

Then we can use the Complement Rule (see page 84), which states, "For any event A and its complement A^c, the probabilities add to 1."

$$\begin{aligned} P(A) &= 1 - P(A^c) \\ P(\text{“at least one right”}) &= 1 - P(\text{“all wrong”}) \\ &\approx 1 - 0.237 \\ &= \boxed{0.763} \end{aligned}$$

2.11: **(a):** $0.16 + 0.09 = \boxed{0.25}$

(b): $0.17 + 0.09 = \boxed{0.26}$

(c): Let event A represent "random man has at least Bachelor's degree". Let event B represent "random woman has at least Bachelor's degree". Let's assume that the man and woman are each selected randomly, such that the simple events are **independent**. This independence allows us to use the Multiplication Rule for independent processes on page 86.

$$\begin{aligned} P(A \text{ AND } B) &= P(A) \cdot P(B) \\ &= 0.25 \cdot 0.26 \\ &= \boxed{0.065} \end{aligned}$$

(d): The assumption of independence is not reasonable. Marriage tends to occur between people of similar academic achievement levels.

2.12: **(a):** "Missing 0 days" is the only other possible outcome. The probabilities of all four outcomes should add to 1.

$$\begin{aligned} P(\text{"a student misses 0 days"}) &= 1 - 0.25 - 0.15 - 0.28 \\ &= \boxed{0.32} \end{aligned}$$

(b): These outcomes are disjoint (mutually exclusive), so we can use the Addition Rule of disjoint outcomes (page 79).

$$\begin{aligned} P(\text{"student misses no more than 1 day"}) &= P(\text{"misses zero days" OR "misses one day"}) \\ &= P(\text{"misses zero days"}) + P(\text{"misses one day"}) \\ &= 0.32 + 0.25 \\ &= \boxed{0.57} \end{aligned}$$

(c): We can use the Addition Rule of disjoint outcomes.

$$\begin{aligned} P(\text{"at least 1 day"}) &= P(\text{"1 day" OR "2 days" OR "at least 3 days"}) \\ &= P(\text{"1 day"}) + P(\text{"2 days"}) + P(\text{"at least 3 days"}) \\ &= 0.25 + 0.15 + 0.28 \\ &= \boxed{0.68} \end{aligned}$$

We could have also used the Complement Rule.

$$\begin{aligned} P(\text{"at least 1 day"}) &= 1 - P(\text{"zero days"}) \\ &= 1 - 0.32 \\ &= \boxed{0.68} \end{aligned}$$

(d): Let's assume the absences of each child are independent so we can use the Multiplication Rule for independent processes.

$$\begin{aligned} P(\text{"2 kids have 0 absences"}) &= P(\text{"1st kid misses 0" AND "2nd kid misses 0"}) \\ &= P(\text{"1st kid misses 0"}) \cdot P(\text{"2nd kid misses 0"}) \\ &= 0.32 \cdot 0.32 \\ &= 0.32^2 \\ &= \boxed{0.1024} \end{aligned}$$

(e): We continue the assumption that each child's attendance is independent of the other, such that the probability above is correct. Each child has a 0.68 chance of missing some school.

$$\begin{aligned} P(\text{"both miss some"}) &= P(\text{"1st misses some" AND "2nd misses some"}) \\ &= P(\text{"1st misses some"}) \cdot P(\text{"2nd misses some"}) \\ &= 0.68 \cdot 0.68 \\ &= 0.68^2 \\ &= \boxed{0.4624} \end{aligned}$$

(f): The assumption of independence is not very reasonable. Siblings often get each other sick. Some parents are more lenient about missing school.

2.13: **(a):** Invalid. The probabilities sum to 1.2, which is more than 1.

(b): Valid. The outcomes are disjoint. Each probability is between 0 and 1. They sum to 1.

(c): Invalid. The probabilities sum to 0.9, which is less than 1.

(d): Invalid. There are negative probabilities.

(e): Valid. The outcomes are disjoint. Each probability is between 0 and 1. They sum to 1.

(f): Invalid. There are negative probabilities.

2.14: **(a):** This is a joint probability. $\frac{459}{20000} = \boxed{0.2295}$

(b): This is a disjoint probability. $\frac{4657+2524-459}{20000} = \boxed{0.3361}$
Or, you can add up all the relevant numbers.
$\frac{4198+459+727+854+385+99}{20000} = \frac{6722}{20000} = \boxed{0.3361}$