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The probability of exactly one 1 of 4 people refusing to administer the shock is the sum of all of these probabilities.

$$0.0961 + 0.0961 + 0.0961 + 0.0961 = 4 \times 0.0961 = 0.3844$$

Binomial distribution

The question from the prior slide asked for the probability of given number of successes, k, in a given number of trials, n, ($k = 1$ success in $n = 4$ trials), and we calculated this probability as

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The *Binomial distribution* describes the probability of having exactly k successes in n independent Bernouilli trials with probability of success p.

Counting the # of scenarios

Earlier we wrote out all possible scenarios that fit the condition of exactly one person refusing to administer the shock. If n was larger and/or k was different than 1, for example, $n = 9$ and $k = 2$:

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$\cdots$

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$\cdots$

SSSSSSS*RR*

writing out all possible scenarios would be incredibly tedious and prone to errors.

Calculating the # of scenarios

Choose function

The *choose function* is useful for calculating the number of ways to choose k successes in n trials.

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- $k = 1$, $n = 4$: $\binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4\times3\times2\times1}{1\times(3\times2\times1)} = 4$

Calculating the # of scenarios

Choose function

The *choose function* is useful for calculating the number of ways to choose k successes in n trials.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- $k = 1, n = 4$: $\binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4\times3\times2\times1}{1\times(3\times2\times1)} = 4$
- $k = 2, n = 9$: $\binom{9}{2} = \frac{9!}{2!(9-1)!} = \frac{9\times8\times7!}{2\times1\times7!} = \frac{72}{2} = 36$

Note: You can also use R for these calculations:

```
> choose(9,2)
[1] 36
```

Properties of the choose function

Which of the following is false?

(a) There are n ways of getting 1 success in n trials, $\binom{n}{1} = n$.

(b) There is only 1 way of getting n successes in n trials, $\binom{n}{n} = 1$.

(c) There is only 1 way of getting n failures in n trials, $\binom{n}{0} = 1$.

(d) There are $n - 1$ ways of getting $n - 1$ successes in n trials, $\binom{n}{n-1} = n - 1$.

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(d) *There are $n - 1$ ways of getting $n - 1$ successes in n trials, $\binom{n}{n-1} = n - 1$.*

Binomial distribution (cont.)

Binomial probabilities

If p represents probability of success, $(1 - p)$ represents probability of failure, n represents number of independent trials, and k represents number of successes

$$P(k\ successes\ in\ n\ trials) = \binom{n}{k} p^k\ (1-p)^{(n-k)}$$

Which of the following is not a condition that needs to be met for the binomial distribution to be applicable?

(a) the trials must be independent

(b) the number of trials, n, must be fixed

(c) each trial outcome must be classified as a *success* or a *failure*

(d) the number of desired successes, k, must be greater than the number of trials

(e) the probability of success, p, must be the same for each trial

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A 2012 Gallup survey suggests that 26.2% of Americans are obese. Among a random sample of 10 Americans, what is the probability that exactly 8 are obese?

(a) pretty high

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(a) $0.262^8 \times 0.738^2$

(b) $\binom{8}{10} \times 0.262^8 \times 0.738^2$

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(c) $\binom{10}{8} \times 0.262^8 \times 0.738^2 \ = 45 \times 0.262^8 \times 0.738^2 = 0.0005$

(d) $\binom{10}{8} \times 0.262^2 \times 0.738^8$

The birthday problem

What is the probability that 2 randomly chosen people share a birthday?

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Exactly 1! (Excluding the possibility of a leap year birthday.)

The birthday problem (cont.)

What is the probability that at least 2 people (1 match) out of 121 people share a birthday?

The birthday problem (cont.)

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Somewhat complicated to calculate, but we can think of it as the complement of the probability that there are no matches in 121 people.

$$P(no\ matches) = 1 \times \left(1 - \frac{1}{365}\right) \times \left(1 - \frac{2}{365}\right) \times \cdots \times \left(1 - \frac{120}{365}\right)$$

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&= \frac{365!}{365^{121} \times (365 - 121)!} \\
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P(\textit{at least } 1 \textit{ match}) &\approx 1
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Expected value

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Among a random sample of 100 Americans, how many would you expect to be obese?

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- Or more formally, $\mu = np = 100 \times 0.262 = 26.2$.

Expected value

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- Easy enough, $100 \times 0.262 = 26.2$.
- Or more formally, $\mu = np = 100 \times 0.262 = 26.2$.
- But this doesn't mean in every random sample of 100 people exactly 26.2 will be obese. In fact, that's not even possible. In some samples this value will be less, and in others more. How much would we expect this value to vary?

Expected value and its variability

Mean and standard deviation of binomial distribution

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}$$

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$$\sigma = \sqrt{np(1-p)} = \sqrt{100 \times 0.262 \times 0.738} \approx 4.4$$

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Mean and standard deviation of binomial distribution

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- Going back to the obesity rate:

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- We would expect 26.2 out of 100 randomly sampled Americans to be obese, with a standard deviation of 4.4.

Note: Mean and standard deviation of a binomial might not always be whole numbers, and that is alright, these values represent what we would expect to see on average.

Unusual observations

Using the notion that *observations that are more than 2 standard deviations away from the mean are considered unusual* and the mean and the standard deviation we just computed, we can calculate a range for the plausible number of obese Americans in random samples of 100.

$$26.2 \pm (2 \times 4.4) = (17.4, 35)$$

An August 2012 Gallup poll suggests that 13% of Americans think home schooling provides an excellent education for children. Would a random sample of 1,000 Americans where only 100 share this opinion be considered unusual?

(a) No

(b) Yes

	Excellent	Good	Only fair	Poor	Total excellent/ good
	%	%	%	%	%
Independent private school	31	47	13	2	78
Parochial or church-related schools	21	48	18	5	69
Charter schools	17	43	23	5	60
Home schooling	13	33	30	14	46
Public schools	5	32	42	19	37

Gallup, Aug. 9-12, 2012

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(a) No

(b) *Yes*

$$\mu = np = 1,000 \times 0.13 = 130$$
$$\sigma = \sqrt{np(1-p)} = \sqrt{1,000 \times 0.13 \times 0.87} \approx 10.6$$

http://www.gallup.com/poll/156974/private-schools-top-marks-educating-children.aspx

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Method 1: *Range of usual observations:* $130 \pm 2 \times 10.6 = (108.8, 151.2)$

100 is outside this range, so would be considered unusual.

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Method 1: *Range of usual observations:* $130 \pm 2 \times 10.6 = (108.8, 151.2)$
100 is outside this range, so would be considered unusual.

Method 2: *Z-score of observation:* $Z = \frac{x - mean}{SD} = \frac{100-130}{10.6} = -2.83$
100 is more than 2 SD below the mean, so would be considered unusual.

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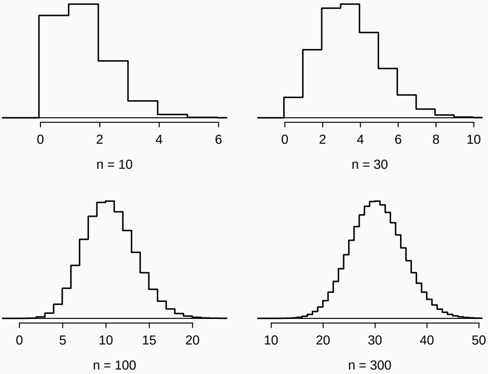
Shapes of binomial distributions

For this activity you will use a web applet. Go to *https://gallery.shinyapps.io/dist_calc/* and choose Binomial coin experiment in the drop down menu on the left.

- Set the number of trials to 20 and the probability of success to 0.15. Describe the shape of the distribution of number of successes.
- Keeping p constant at 0.15, determine the minimum sample size required to obtain a unimodal and symmetric distribution of number of successes. Please submit only one response per team.
- Further considerations:
  - What happens to the shape of the distribution as n stays constant and p changes?
  - What happens to the shape of the distribution as p stays constant and n changes?

Distributions of number of successes

Hollow histograms of samples from the binomial model where $p = 0.10$ and $n = 10$, 30, 100, and 300. What happens as n increases?



Low large is large enough?

The sample size is considered large enough if the expected number of successes and failures are both at least 10.

$$np \geq 10 \qquad \text{and} \qquad n(1-p) \geq 10$$

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$$np \geq 10 \qquad \text{and} \qquad n(1-p) \geq 10$$

$10 \times 0.13 = 1.3; 10 \times (1 - 0.13) = 8.7$

Below are four pairs of Binomial distribution parameters. Which distribution can be approximated by the normal distribution?

(a) $n = 100, p = 0.95$

(b) $n = 25, p = 0.45$

(c) $n = 150, p = 0.05$

(d) $n = 500, p = 0.015$

Below are four pairs of Binomial distribution parameters. Which distribution can be approximated by the normal distribution?

(a) $n = 100, p = 0.95$

(b) $n = 25, p = 0.45 \rightarrow 25 \times 0.45 = 11.25; 25 \times 0.55 = 13.75$

(c) $n = 150, p = 0.05$

(d) $n = 500, p = 0.015$

An analysis of Facebook users

A recent study found that "Facebook users get more than they give". For example:

- 40% of Facebook users in our sample made a friend request, but 63% received at least one request
- Users in our sample pressed the like button next to friends' content an average of 14 times, but had their content "liked" an average of 20 times
- Users sent 9 personal messages, but received 12
- 12% of users tagged a friend in a photo, but 35% were themselves tagged in a photo

Any guesses for how this pattern can be explained?

http://www.pewinternet.org/Reports/2012/Facebook-users/Summary.aspx

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Power users contribute much more content than the typical user.

http://www.pewinternet.org/Reports/2012/Facebook-users/Summary.aspx

This study also found that approximately 25% of Facebook users are considered power users. The same study found that the average Facebook user has 245 friends. What is the probability that the average Facebook user with 245 friends has 70 or more friends who would be considered power users? Note any assumptions you must make.

We are given that $n = 245, p = 0.25$, and we are asked for the probability $P(K \geq 70)$. To proceed, we need independence, which we'll assume but could check if we had access to more Facebook data.

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We are given that $n = 245, p = 0.25$, and we are asked for the probability $P(K \geq 70)$. To proceed, we need independence, which we'll assume but could check if we had access to more Facebook data.

$$
\begin{aligned}
P(X \geq 70) &= P(K = 70 \text{ or } K = 71 \text{ or } K = 72 \text{ or } \cdots \text{ or } K = 245) \\
&= P(K = 70) + P(K = 71) + P(K = 72) + \cdots + P(K = 245)
\end{aligned}
$$

This study also found that approximately 25% of Facebook users are considered power users. The same study found that the average Facebook user has 245 friends. What is the probability that the average Facebook user with 245 friends has 70 or more friends who would be considered power users? Note any assumptions you must make.

We are given that $n = 245, p = 0.25$, and we are asked for the probability $P(K \geq 70)$. To proceed, we need independence, which we'll assume but could check if we had access to more Facebook data.

$$
\begin{aligned}
P(X \geq 70) &= P(K = 70 \text{ or } K = 71 \text{ or } K = 72 \text{ or } \cdots \text{ or } K = 245) \\
&= P(K = 70) + P(K = 71) + P(K = 72) + \cdots + P(K = 245)
\end{aligned}
$$

This seems like an awful lot of work...

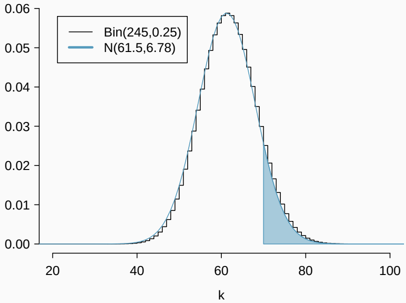
Normal approximation to the binomial

When the sample size is large enough, the binomial distribution with parameters n and p can be approximated by the normal model with parameters $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.

- In the case of the Facebook power users, $n = 245$ and $p = 0.25$.

$$\mu = 245 \times 0.25 = 61.25 \qquad \sigma = \sqrt{245 \times 0.25 \times 0.75} = 6.78$$

- $Bin(n = 245, p = 0.25) \approx N(\mu = 61.25, \sigma = 6.78)$.



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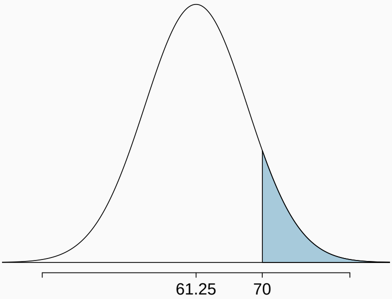

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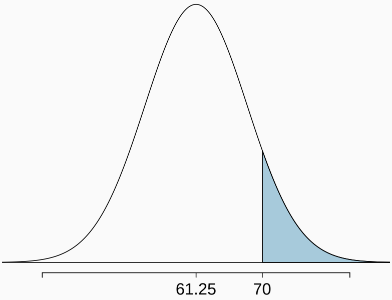

$$Z = \frac{obs - mean}{SD} = \frac{70 - 61.25}{6.78} = 1.29$$

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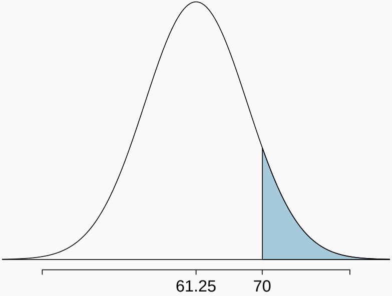

$$Z = \frac{obs - mean}{SD} = \frac{70 - 61.25}{6.78} = 1.29$$

	Second decimal place of Z				
Z	0.05	0.06	0.07	0.08	0.09
1.0	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8944	0.8962	0.8980	0.8997	0.9015

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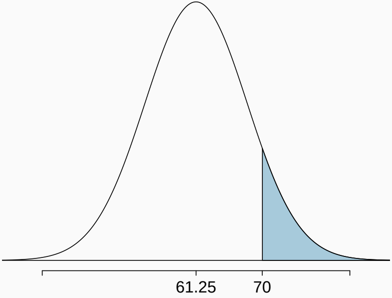

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$$P(Z > 1.29) = 1 - 0.9015 = 0.0985$$