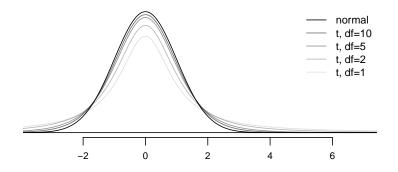
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Example confidence interval of single sample with small n, unknown σ .

A random sample of size n=10 was collected from a population which is believed to be approximately symmetric. The sample has a mean $\bar{x}=135.7$ and standard deviation s=24.6. Find the confidence interval with a confidence level $\gamma=0.95$.

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The formulas:

$$SE = \frac{s}{\sqrt{n}}$$

$$df = n - 1$$

$$P(|T| < t^*) = \gamma$$

$$CI = \bar{x} \pm t^* SE$$

► Calculate the standard error (same way as before).

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Calculate the degrees of freedom.

$$df = 10 - 1 = 9$$

▶ Determine t^* such that $P(|T| < t^*) = 0.95$. We use the T table.

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Calculate the confidence interval.

Calculate the standard error (same way as before).

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Calculate the confidence interval.

$$CI = 135.7 \pm (2.26)(7.78)$$

 $CI = (118.1, 153.3)$

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A random sample of size n=15 was collected from a population which is believed to be approximately symmetric. The sample has a mean $\bar{x}=11.1$ and standard deviation s=2.3. Find the confidence interval with a confidence level $\gamma=0.99$.

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$$SE = \frac{2.3}{\sqrt{15}} = 0.594$$

$$df = 15 - 1 = 14$$

$$t^* = 2.98$$

$$CI = 11.1 \pm (2.98)(0.594)$$

$$CI = (9.33, 12.87)$$

Working backwards: confidence intervals

A 90% confidence interval for a population mean, μ , is given as (43.84, 55.92). This confidence interval is based on a simple random sample of 12 observations. Calculate the sample mean and standard deviation. Assume that all conditions necessary for inference are satisfied. Use the T distribution in any calculations.

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The formulas:

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That margin of error is the product of t^* and SE. We find the t^* when $P(|T| < t^*) = 0.9$ and df = n - 1 = 11.

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We can calculate SE.

$$ME = t^*SE$$

$$6.04 = (1.8)SE$$

$$SE = \frac{6.04}{1.8} = 3.3564258$$

continued on next frame...

We can now calculate the sample standard deviation.

$$SE = \frac{S}{\sqrt{n}}$$

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We can now calculate the sample standard deviation.

$$SE = \frac{S}{\sqrt{n}}$$

$$3.3564258 = \frac{s}{\sqrt{12}}$$

$$s = (3.356)\sqrt{12} = 11.627$$

Thus, the sample mean is 49.88 and the sample standard deviation is 11.63.

$$\bar{x} = \frac{88.09 + 109.15}{2} = 98.62$$

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$$t^* = 2.31$$

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$$ME = \frac{109.15 - 88.09}{2} = 10.53$$

$$df = 8$$

$$t^* = 2.31$$

$$SE = \frac{10.53}{2.31} = 4.56$$

$$\bar{x} = \frac{88.09 + 109.15}{2} = 98.62$$

$$ME = \frac{109.15 - 88.09}{2} = 10.53$$

$$df = 8$$

$$t^* = 2.31$$

$$SE = \frac{10.53}{2.31} = 4.56$$

$$s = (4.56)\sqrt{9} = 13.68$$

You will perform a single-sample t test of the null hypothesis claiming $\mu=22$. Before collecting the sample, you decide to use a two-tail test with a significance level $\alpha=0.05$. The sample has the following attributes:

$$n = 11$$

$$\bar{x} = 15.49$$

$$s = 8.03$$

What is your conclusion?

You will perform a single-sample t test of the null hypothesis claiming $\mu=22$. Before collecting the sample, you decide to use a two-tail test with a significance level $\alpha=0.05$. The sample has the following attributes:

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What is your conclusion? We state the hypotheses:

$$H_0: \mu = 22$$

 $H_A: \mu \neq 22$

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What is your conclusion? We state the hypotheses:

$$H_0: \mu = 22$$

 $H_A: \mu \neq 22$

We estimate the standard error (same way as with z testing).

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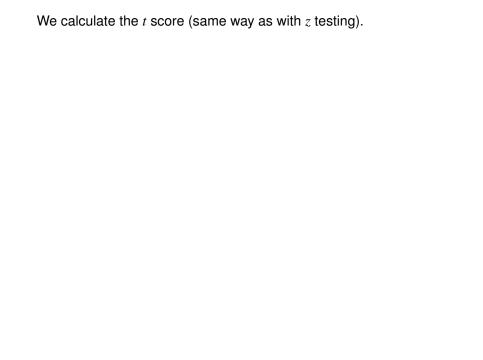
$$H_0: \mu = 22$$

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We estimate the standard error (same way as with z testing).

$$SE = \frac{s}{\sqrt{n}} = \frac{8.03}{\sqrt{11}} = 2.421$$

continued on next slide...



$$t = \frac{15.49 - 22}{2.421} = -2.69$$

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For the T table, we use the absolute value of t...

$$t = 2.69$$

We determine the degrees of freedom.

$$df=n-1=10$$

$$t = \frac{15.49 - 22}{2.421} = -2.69$$

For the T table, we use the absolute value of t...

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We determine the degrees of freedom.

$$df = n - 1 = 10$$

We estimate the p-value from the T table.

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$$t = 2.69$$

We determine the degrees of freedom.

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We estimate the p-value from the T table.

$$0.02 < p$$
-value < 0.04

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$$0.02 < p$$
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We compare the p-value to α .

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$$0.02 < p$$
-value < 0.04

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$$p$$
-value $< \alpha$

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We determine the degrees of freedom.

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We estimate the p-value from the T table.

$$0.02 < p$$
-value < 0.04

We compare the *p*-value to α .

$$p$$
-value < α

We make our conclusion: we reject the null.

Practice:

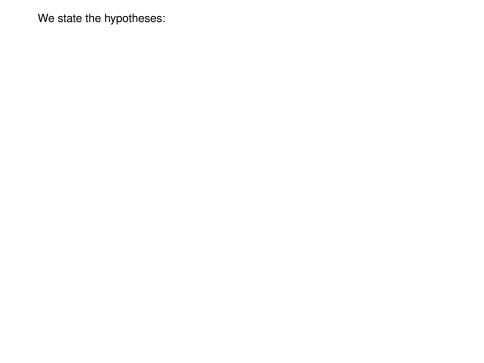
You will perform a single-sample t test of the null hypothesis claiming $\mu=140$. Before collecting the sample, you decide to use a two-tail test with a significance level $\alpha=0.1$. The sample has the following attributes:

$$n = 3$$

$$\bar{x} = 193.06$$

$$s = 52.22$$

What is your conclusion?



$$H_0: \ \mu = 140$$

$$H_A: \mu \neq 140$$

$$H_0: \mu = 140$$

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We estimate the standard error (same way as with z testing).

$$H_0: \mu = 140$$

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We estimate the standard error (same way as with z testing).

$$SE = \frac{s}{\sqrt{n}} = \frac{52.22}{\sqrt{3}} = 30.149$$

$$H_0: \mu = 140$$

 $H_A: \mu \neq 140$

We estimate the standard error (same way as with z testing).

$$SE = \frac{s}{\sqrt{n}} = \frac{52.22}{\sqrt{3}} = 30.149$$

We calculate the t score (same way as with z testing).

$$H_0: \mu = 140$$

 $H_A: \mu \neq 140$

We estimate the standard error (same way as with z testing).

$$SE = \frac{s}{\sqrt{n}} = \frac{52.22}{\sqrt{3}} = 30.149$$

We calculate the t score (same way as with z testing).

$$t = \frac{193.06 - 140}{30.149} = 1.76$$

$$H_0: \mu = 140$$

 $H_A: \mu \neq 140$

We estimate the standard error (same way as with z testing).

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We calculate the t score (same way as with z testing).

$$t = \frac{193.06 - 140}{30.149} = 1.76$$

We determine the degrees of freedom.

$$df = n - 1 = 2$$

$$H_0: \mu = 140$$

 $H_A: \mu \neq 140$

We estimate the standard error (same way as with z testing).

$$SE = \frac{s}{\sqrt{n}} = \frac{52.22}{\sqrt{3}} = 30.149$$

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$$t = \frac{193.06 - 140}{30.149} = 1.76$$

We determine the degrees of freedom.

$$df = n - 1 = 2$$

We estimate the p-value from the T table.

$$H_0: \mu = 140$$

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We calculate the t score (same way as with z testing).

$$t = \frac{193.06 - 140}{30.149} = 1.76$$

We determine the degrees of freedom.

$$df = n - 1 = 2$$

We estimate the p-value from the T table.

$$0.2 < p$$
-value < 0.5

$$H_0: \mu = 140$$

 $H_A: \mu \neq 140$

We estimate the standard error (same way as with z testing).

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$$0.2 < p$$
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We compare the p-value to α .

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$$p$$
-value > α

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-value < 0.5

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$$p$$
-value > α

We make our conclusion:

$$H_0: \mu = 140$$

 $H_A: \mu \neq 140$

We estimate the standard error (same way as with z testing).

$$SE = \frac{s}{\sqrt{n}} = \frac{52.22}{\sqrt{3}} = 30.149$$

We calculate the t score (same way as with z testing).

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We determine the degrees of freedom.

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We estimate the p-value from the T table.

$$0.2 < p$$
-value < 0.5

We compare the p-value to α .

$$p$$
-value > α

We make our conclusion: we retain the null.