Bunker Hill Community College
imagine the possibilities

MATH 181 FIRST EXAM PRACTICE

SPRING 2019

Name: ANSWER KEY

- Write your **full name** on the line above.
- Show your work. Incorrect answers with work can receive partial credit.
- Attempt every question; showing you understand the question earns some credit.
- If you run out of room for an answer, continue on the back of the page. Before doing so, write "see back" with a circle around it.
- You can use 1 page (front and back) of notes.
- You can use (and probably need) a calculator.
- You can use the Geogebra Scientific Calculator instead of a calculator. You need to put your phone on **airplane mode** and then within the application, start **exam mode**; you should see a green bar with a timer counting up.
- If a question is confusing or ambiguous, please ask for clarification; however, you will not be told how to answer the question.
- **Box your final answer**.
- A formula sheet is attached to this test.

Do not write in this grade table.

Question:	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	Total
Points:	10	10	10	10	10	10	10	10	80
Score:									

Sample statistics:

n = sample size
x_i = the ith value in a sample
$\bar{x}$ = sample mean
s = sample standard deviation

Q_1 = first quartile
m = median
Q_3 = third quartile
IQR = inter-quartile range = $Q3 - Q1$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

Population parameters:

μ = population mean
σ = population standard deviation

Probability:

Ω = set of all possible equally likely outcomes
A = event A, a set of outcomes
A^c = The complement of A
B = event B, another set of outcomes
$|A|$ = size of set, number of outcomes in A
$P(A)$ = probability of A
$P(A \text{ AND } B)$ = probability of both A and B
$P(A \text{ OR } B)$ = probability of either A or B (or both)
$P(A|B)$ = probability of A given B

$$P(A) = \frac{|A|}{|\Omega|}$$

$0 \le P(A) \le 1$
$P(A \text{ AND } B) = P(A) \cdot P(B|A)$
$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$
$P(A^c) = 1 - P(A)$

A, B are disjoint (mutually exclusive) $\iff$ $P(A \text{ AND } B) = 0$
A, B are non-disjoint $\iff$ $P(A \text{ AND } B) > 0$
A, B are exhaustive $\iff$ $P(A \text{ OR } B) = 1$
A, B are complements $\iff$ A, B are disjoint and exhaustive $\iff$ $B = A^c$
A, B are independent $\iff$ $P(A \text{ AND } B) = P(A) \times P(B)$ $\iff$ $P(A|B) = P(A)$

Random variables and distributions:

X = random variable
x_i = the ith possible value of X. (Notice different meaning here vs. sample statistics.)
k = number of possible values of X.
$E(X) = \mu$ = expected value of X
σ = standard deviation of X
$\mu = \sum_{i=1}^{k} x_i \cdot P(X = x_i)$
$\sigma = \sqrt{\sum_{i=1}^{k} (x_i - \mu)^2 \cdot P(X = x_i)}$

Q1. (10 points) An urn contains marbles. Each marble has a color and a pattern. The frequencies are shown in the contingency table.

		R red	G green	B blue	total
D	dotted	89	31	22	142
S	striped	22	41	97	160
C	checkered	16	36	42	92
	total	127	106	161	394

(a) What is the probability that a random marble is green?

$$P(G) = \frac{106}{394} \approx 0.269$$

(b) What is the probability that a random marble is striped and green?

$$P(S \text{ and } G) = \frac{41}{394} \approx 0.104$$

(c) What is the probability that a random marble is striped or green?

$$P(S \text{ or } G) = \frac{22+41+97+31+36}{394} = \frac{160+106-41}{394} \approx 0.571$$

(d) What is the probability that a random marble is striped given it is green?

$$P(S|G) = \frac{41}{106} \approx 0.387$$

(e) What is the probability that a random marble is green given it is striped?

$$P(G|S) = \frac{41}{160} \approx 0.256$$

Q2. (10 points) Amira ran a study to see whether a rooting hormone increased the survival rate of roma-tomato cuttings (some plants can propagate by slicing off a branch and planting it). After collecting 18 cuttings, she dipped 9 (randomly selected) cuttings in rooting hormone and the others in water. After 4 weeks, she noted how many from each group survived.

	survived	died
hormone	6	3
water	5	4

(a) Is this study observational or experimental?

Experimental

(b) What is the sample?

18 cuttings

(c) What are the cases (individuals)?

the cuttings

(d) What are the variables?

survival and dipping substance

(e) Could a causal relationship be established by the study design?

yes

(f) Was blinding used in this study?

no

(g) Which treatment (hormone or water) had a higher proportion surviving after 4 weeks?

The hormone treatment had a higher proportion. $\frac{6}{9} > \frac{5}{9}$

(h) Describe the null hypothesis in the context of this study.

H_0: The difference is just due to chance.

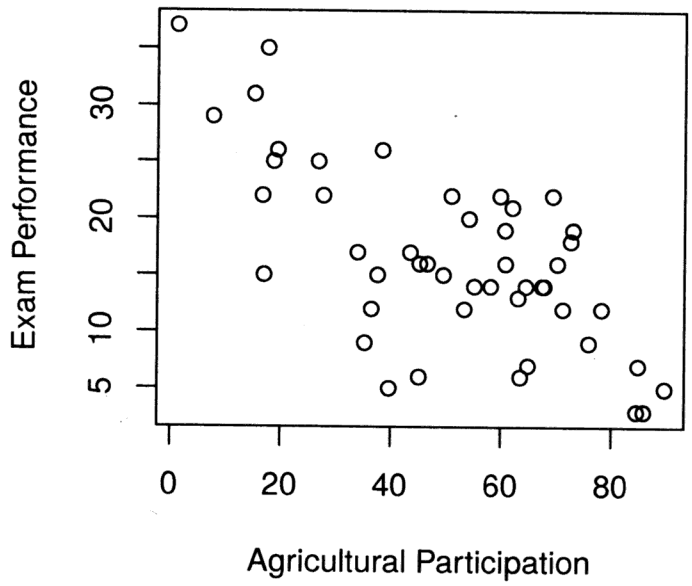
(i) Describe the alternative hypothesis in the context of this study.

H_A: Hormones cause a different survival rate than water.

(j) Has Amira shown, beyond a reasonable doubt, that hormone is more effective than water? Why or why not?

No. It seems like chance could easily account for the difference.

Q3. (10 points) In 1888, the Swiss government collected data on 47 French-speaking provinces. Two of the variables were the percent of men engaged in agriculture and the percent of draftees earning top marks on an examination.



(a) Which variable is the implied explanatory variable?

Agricultural participation

(b) Which variable is the implied response variable?

Exam performance

(c) What kind of association (positive, negative, or none) exists between these two variables?

Negative

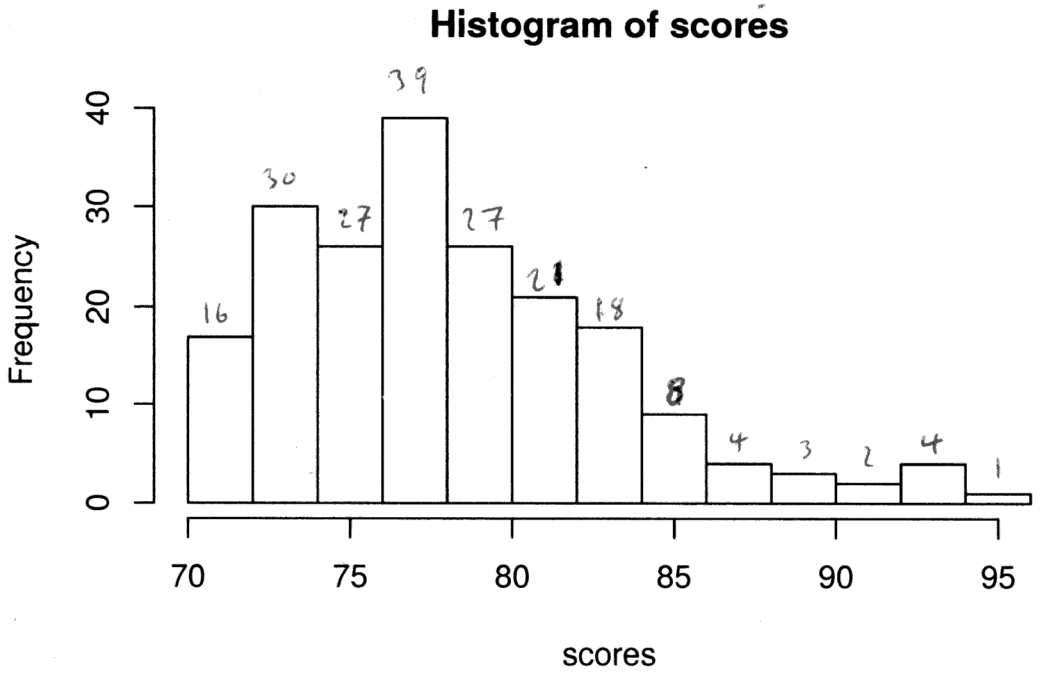
(d) A friend suggests this study shows that agricultural participation causes lower exam performance. Is this a reasonable conclusion? Why or why not?

This is not reasonable. Observational studies do not show causal relationships.

(e) What is a possible confounding variable?

Education, wealth, socioeconomic status, cultural values...

Q4. (10 points) A teacher has given an exam to 200 students, whose scores are shown in the histogram below. Please assume that the scores are almost all non-integer (this is meant to simplify the analysis by removing inclusive/exclusive concerns).



(a) Estimate the proportion of students who scored between 70 and 74.

$\approx \frac{46}{200}$, so about 23%

(b) Estimate the median.

≈ 77

(c) Is the mean higher or lower than the median? Why?

The mean is higher than the mean; the distribution is skewed right.

(d) Estimate the first quartile, Q_1.

≈ 74.5

(e) Estimate the third quartile, Q_3.

≈ 81

(f) Which of the following choices is the best estimate of the standard deviation.

- ○ 1
- ● 5
- ○ 30
- ○ 80
- ○ 95

Q5. (10 points) Gary measured the masses (in grams) of 8 random oranges.

259 254 259 267 244 263 242 300

(a) Find the sample mean ($\bar{x}$).

$$\frac{259+254+259+267+244+263+242+300}{8} = 261$$

$\bar{X} = 261$

(b) Find the sample standard deviation (s).

x	$(x-\bar{x})^2$
259	4
254	49
259	4
267	36
244	289
263	4
242	361
300	1521

$\Sigma (x-\bar{x})^2 = 2268$

$$s = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n-1}} = \sqrt{\frac{2268}{7}} = 18$$

$s = 18$

(c) Make a box plot. Label the features (with numbers).

sort

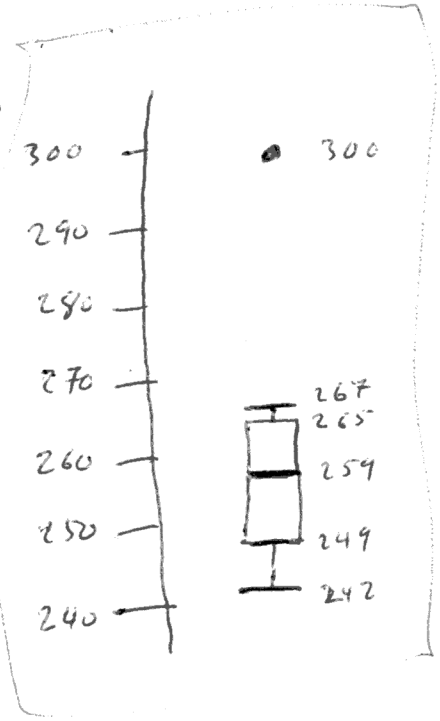
242
244 ← $Q_1 = 249$
254
259 ← median = 259
259
263 ← $Q_3 = 265$
267
300

$IQR = Q_3 - Q_1$
$IQR = 265 - 249$
$IQR = 16$

$Q_1 - 1.5 \times IQR$
$249 - 24$
225

$Q_3 + 1.5 \times IQR$
$265 + 24$
289

to check for outliers



Q6. (10 points) Jenn has a weighted die that rolls with probability distribution below. Let random variable X represent the result of a roll.

x_i	$P(X = x_i)$	$x_i \cdot P(X=x_i)$	$(x_i-\mu)^2 \cdot P(X=x_i)$
1	0.10	0.1	0.9
2	0.15	0.3	0.6
3	0.15	0.45	0.15
4	0.15	0.6	0
5	0.15	0.75	0.15
6	0.30	1.8	1.2

$\sum_{i=1}^{6} x_i \cdot P(X=x_i) = 4$

$\mu = 4$

$\sigma^2 = 3$

$\sigma \approx 1.73$

(a) What is $P(X = 6)$? 0.30

(b) Evaluate $P(2 \le X \le 5)$.

$0.15 + 0.15 + 0.15 + 0.15 = 0.6$

(c) Evaluate the mean of the probability distribution.

$\mu = 4 = 0.1 + 0.3 + 0.45 + 0.6 + 0.75 + 1.8$

(d) Evaluate the standard deviation of the probability distribution.

$\sigma \approx 1.73$

(e) Assume multiple rolls are independent, where X_i is the result of the ith roll. Evaluate the probability $P(X_1 = 6 \text{ AND } X_2 = 6)$. In other words, what is the chance of rollings two 6s in a row?

$(0.3)(0.3) = 0.09$

(f) Evaluate $P(X_1 \neq 6 \text{ AND } X_2 \neq 6 \text{ AND } X_3 \neq 6)$. In other words, what is the chance of rolling thrice and getting no 6s?

$0.7 \times 0.7 \times 0.7 = 0.343$

(g) Evaluate $P(X_1 = 6 \text{ OR } X_2 = 6 \text{ OR } X_3 = 6)$. In other words, what is the chance of rolling thrice and getting at least one 6?

$1 - 0.343 = 0.657$

(h) If you want a series of rolls to average between 3 and 5, and you get to choose the number of rolls before starting, should you choose 10 rolls or 100 rolls? Why?

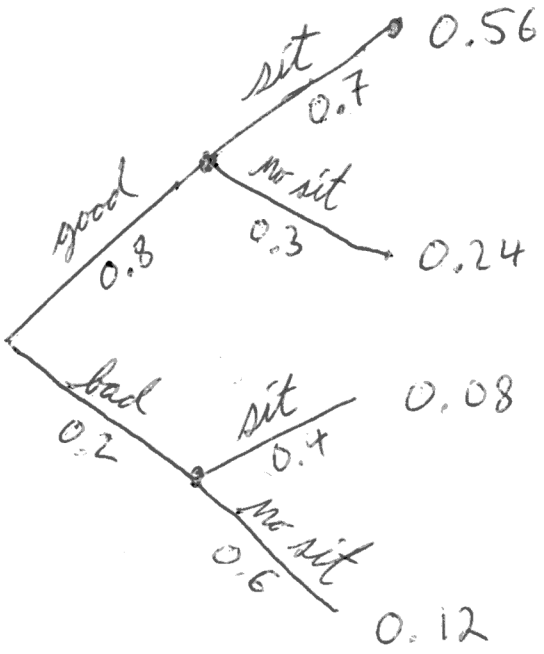
100 rolls will usually make sample mean $\approx \mu = 4$.

(i) If you want a series of rolls to average between 1 and 3, and you get to choose the number of rolls before starting, should you choose 10 rolls or 100 rolls? Why?

10 rolls, we want the variability of ~~small~~ small samples to help us get a sample mean far from the expected value (4).

Q7. (10 points) In a kennel, 80% of dogs are good dogs; the others are bad dogs. If a dog is good, it has a 70% chance of sitting on command. If a dog is bad, it has a 40% chance of sitting on command. You meet a dog, and it **does not** sit on command. What is the chance it is a good dog given it didn't sit on command?

(a) Draw a tree diagram.



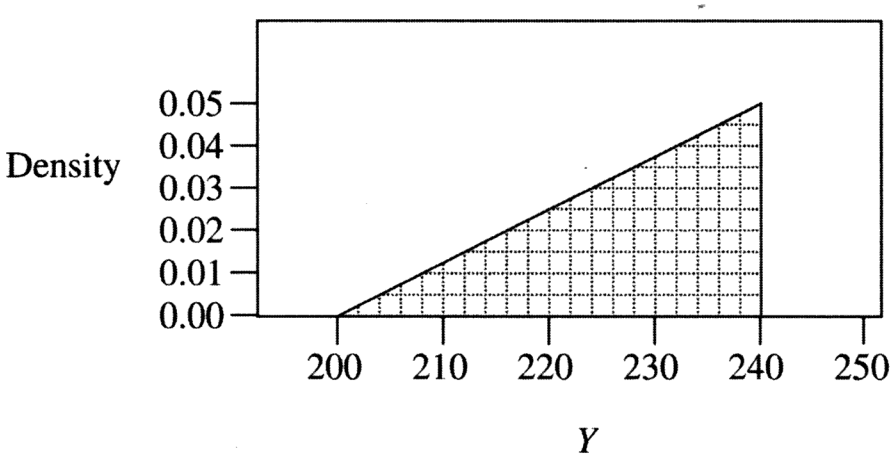
(b) Make a contingency table.

	good	bad	
sit	0.56	0.08	0.64
no sit	0.24	0.12	0.36
	0.8	0.2	

(c) Determine the probability the dog is good given it **did not** sit on command.

$$P(\text{good} \mid \text{no sit}) = \frac{0.24}{0.36} = \frac{2}{3} \approx 0.666$$

Q8. (10 points) Let random variable Y be continuously distributed by the probability density function shown below. Notice the entire area is split into 100 percentile squares.



(a) Evaluate $P(Y = 212)$. In other words, what is the probability that Y is exactly 212?

0

(b) Evaluate $P(Y < 212)$. In other words, what is the probability that Y is less than 212?

0.09

(c) Evaluate $P(220 < Y < 224)$. In other words, what is the probability that Y is between 220 and 224?

0.11

(d) Estimate Q_1.

220

(e) Estimate the median.

228.1

(f) Is the mean lower than, equal to, or greater than the median?

The mean is lower than the median.

(Left skew)