Today's key words

- point estimate
- standard error
- sampling distribution

Point estimates

- sample proportion
  - Each measurement is a 0 or 1.
  - 0 usually means “no” or “false” or “fail”.
  - 1 usually means “yes” or “true” or “success”.
  - The proportion is the average of the 0s and 1s.
- sample mean
  - Each measurement is a weight, height, mass, volume, count, etc...
  - The sample mean is the average of the measurements.

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Based on all 24 students, what is the point estimate of the population proportion?

$$\frac{5+7}{24} = 0.5$$

Our point estimate is about 50% of BHCC students like coconut water.

Consider the probability distribution (infinite population) of rolling a fair 4-sided die.

x	1	2	3	4
$P(x)$	0.25	0.25	0.25	0.25

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$$\mu = (1)(0.25) + (2)(0.25) + (3)(0.25) + (4)(0.25) = \boxed{2.5}$$

Let's sample from this population.

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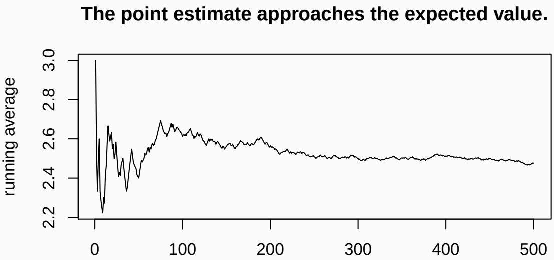
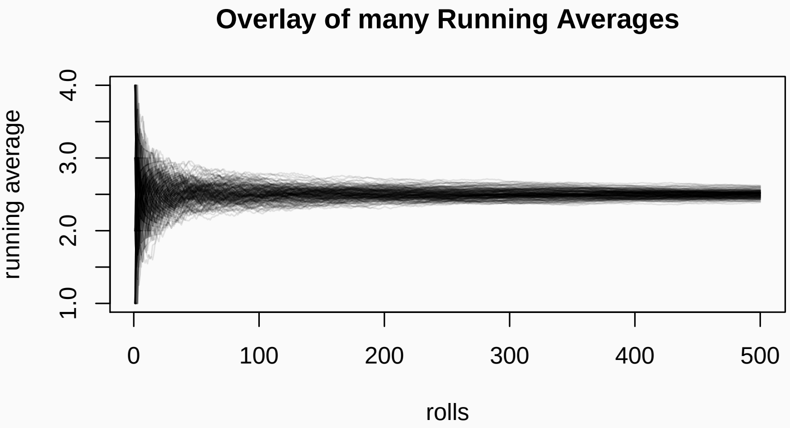
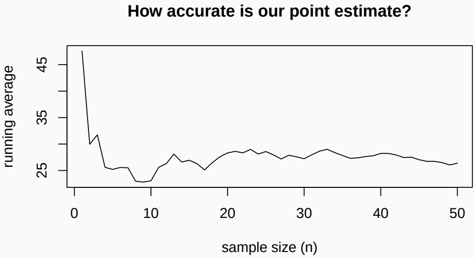


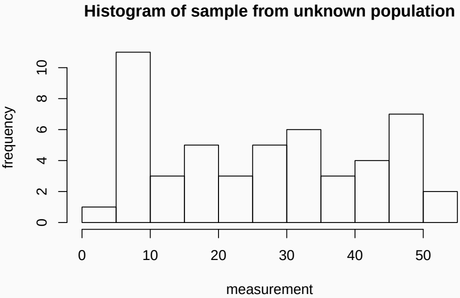


Notice the uncertainty gets smaller with larger sample size. However, there are diminishing returns...

The accuracy improves drastically from $n = 1$ to $n = 100$, but not nearly as drastically from $n = 401$ to $n = 500$.

Now, imagine we sample from a new population/distribution, but we don't know the population parameters. What can we conclude?





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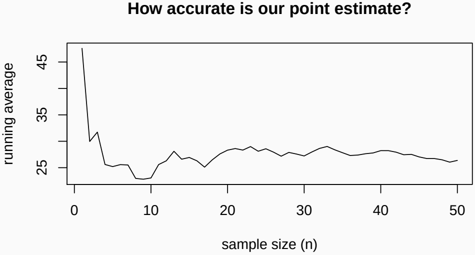

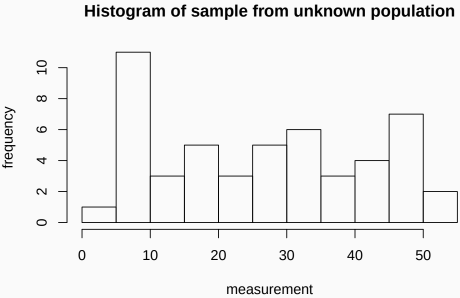

Well, our point estimate is

$$\mu \approx \bar{x} = 26.4$$

However, we want to also describe our uncertainty. To me, based on the previous slide, I'd guess the uncertainty is about $\pm 1/10$ *of the range?*

$$\mu = 26.4 \pm 5$$

Standard error

Standard error quantifies our uncertainty of a point estimate.

$$SE = \frac{\sigma}{\sqrt{n}}$$

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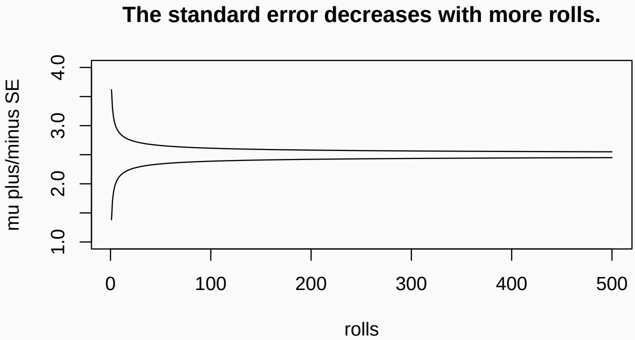
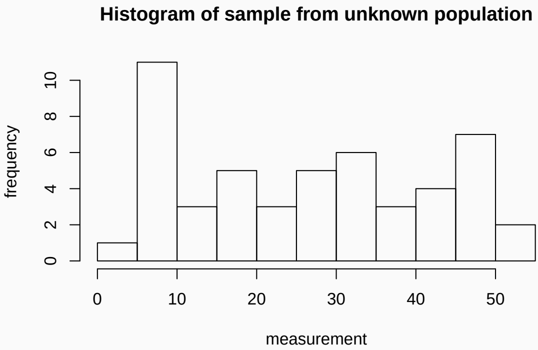
Remember the 4-sided die. That distribution has $\sigma = 1.118$. We can plot $\mu \pm SE$ as a function of n.



Let's return to our sample from an unknown population.



We do not know σ.

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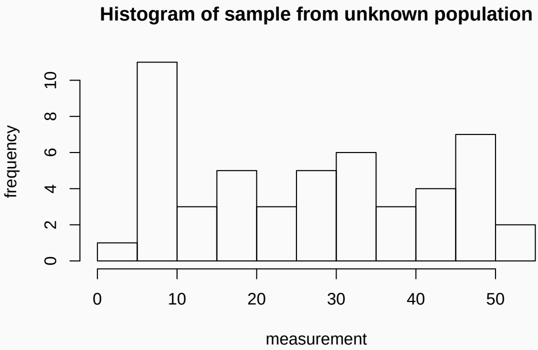

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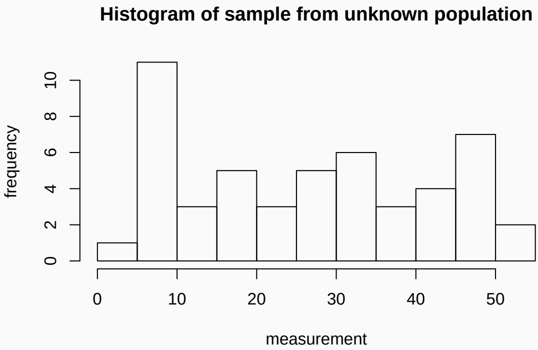

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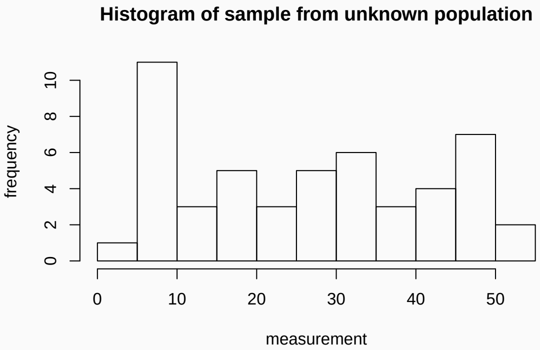

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$$SE \approx \frac{15.5}{\sqrt{50}} = 2.19$$

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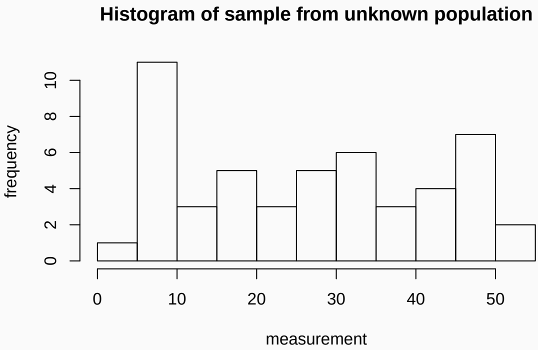

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So we think our estimate $\mu \approx \bar{x} = 26.4$ has an "uncertainty" of 2.19.

But we need to define SE better...

Sampling Distributions

Let X_i be the ith draw from a population. Let n represent the number of draws. Let Y be the average of those draws.

$$Y = \frac{\sum_{i=1}^{n} X_i}{n}$$

By using the rules of Ch 2.4 we can show

$$\mu_Y = \mu_X$$

$$\sigma_Y = \frac{\sigma_X}{\sqrt{n}}$$

We say Y is determined by a sampling distribution. That sampling distribution has the same mean as the population, but it has a smaller standard deviation (and its SD shrinks as n increases). The SD of Y is the SE.

$$SE = \sigma_Y$$

Sampling Distributions

Let X_i be the ith draw from a population. Let n represent the number of draws. Let $\bar{X}$ be the average of those draws.

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

By using the rules of Ch 2.4 we can show

$$E(\bar{X}) = E(X)$$

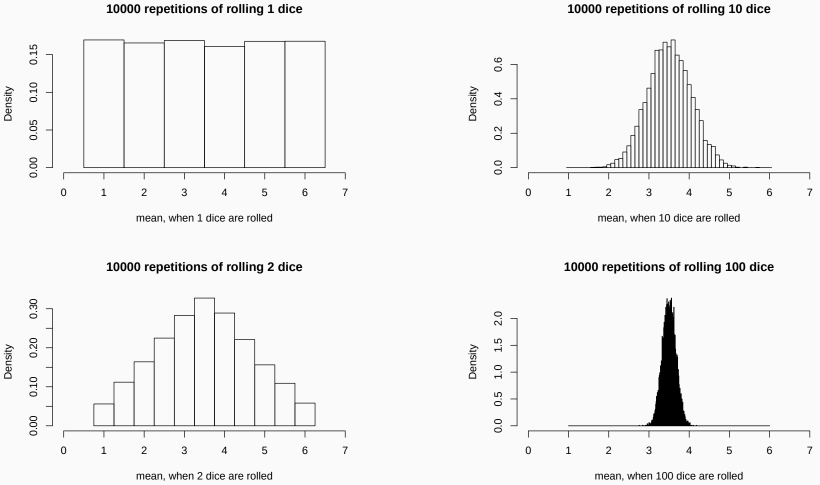
$$SD(\bar{X}) = \frac{SD(X)}{\sqrt{n}}$$

The book also uses $SD_{\bar{x}}$ to represent standard error.

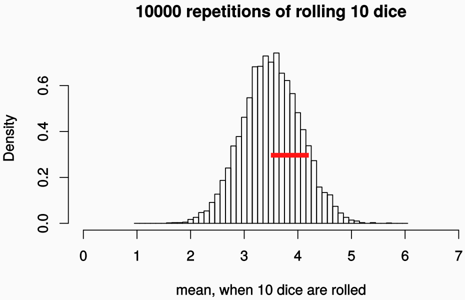
$$SE = SD(\bar{X}) = SD_{\bar{x}}$$

Sampling Simulations

Let's roll 6-sided dice (on a computer to save time).

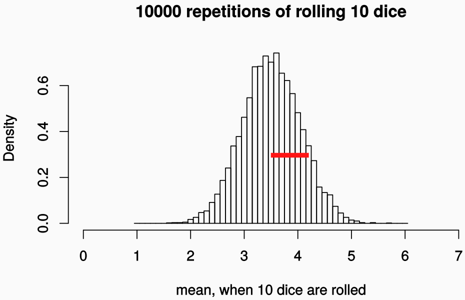


Practice



Estimate the standard error when rolling 10 dice at a time.

Practice



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Notice the sampling distribution looks nearly normal. I estimate that the standard deviation looks to be about **0.7***?*

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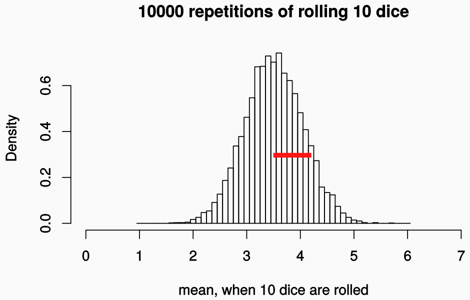

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Notice the sampling distribution looks nearly normal. I estimate that the standard deviation looks to be about **0.7**?

I calculated that for rolling a single die, $\sigma = 2.92$. Calculate the standard error when rolling 10 dice.

$$SE = \frac{2.92}{\sqrt{10}} = 0.922$$