Estimations of Moon's Distance are Anchored Toward Earth's Radius or Sun's Distance

Abstract

Anchoring is demonstrated in a Bunker Hill Community College class of 23 students. Each student was asked to estimate the distance to the moon; group 1 was anchored with radius of Earth (3e3 miles) while group 2 was anchored on the distance to the sun (9e7 miles). Due to heavy skew of estimates, the normality-based failed to show significance (p-value $= 0.1$) while the more appropriate permutation-based methods allowed us to conclude that anchoring caused an effect (p-value $= 0.03$).

Background

Anchoring is a well-documented effect, where the presentation of a number effects a person's later numerical evaluations.

Statistical tests often assume sampling distributions are normal. This condition is easily met with moderate sample sizes from nearly any population (even with a population as unnormal as bimodal or exponential distributions). However, with small sample sizes, the normal approximation to sampling distributions easily fails. Luckily, permutation tests are more rubust and, with a modern version of R, intuitive to code.

Experimental Methods

Every participant was asked to estimate the distance to the moon. By having the instructions on cards, and asking the responses to be written on the cards, all participants could participate simultaneously. The participants were given approximately 30 seconds to write an answer. All participants responded.

Two types of card gave different hints. The groups (Earth and Sun) were randomly assigned by shuffling the cards. By using cards, it was easy to guarantee approximately equal sample sizes. Half of the participants were told the radius of Earth ($3,000$ miles) while the other half were told the distance to the Sun ($90,000,000$ miles). The cards were made with the printout that can be generated (or modified) with the R script below.

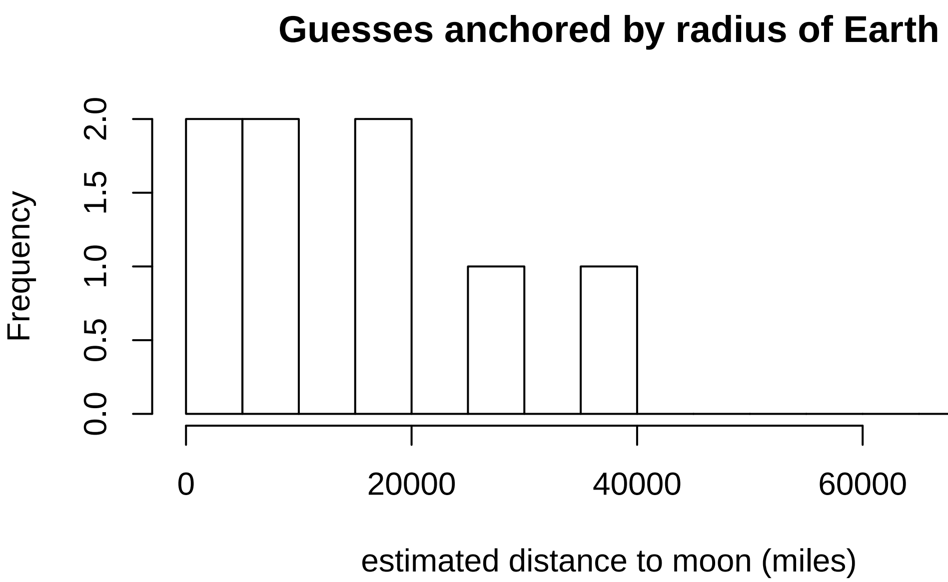
```
#This code will create a page that can be printed (saved in the current working directory).
#The resulting pdf/printout is included at the end of this article.
pdf("Moon_cards.pdf",width=8.5,height=11,paper="letter",title = "Moon cards printout")
par(mar = c(0,0,0,0))
plot.new()
fig.align="center"
for(xpos in seq(0,1,1/3)){ abline(v = xpos) }
for(ypos in seq(0,1,1/4)){ abline(h = ypos) }
for(x in seq(0,2/3,1/3)){
  for(y in seq(0,3/4,1/4)){
    text(x+0.16,y+0.20,"Please estimate the distance\nto the moon \nin miles.", cex=0.8)
  lines(c(x+0.05,x+0.3333-0.05),c(y+0.07,y+0.07))
  if (y %in% c(0,0.5)){
    text(x+0.16,y+0.03,"Hint:\nThe sun is 90,000,000 miles away.", cex=0.8)
  } else {
    text(x+0.16,y+0.03,"Hint:\nThe radius of Earth is 3,000 miles.", cex=0.8)
  }
```

```
  }
}
dev.off()
```

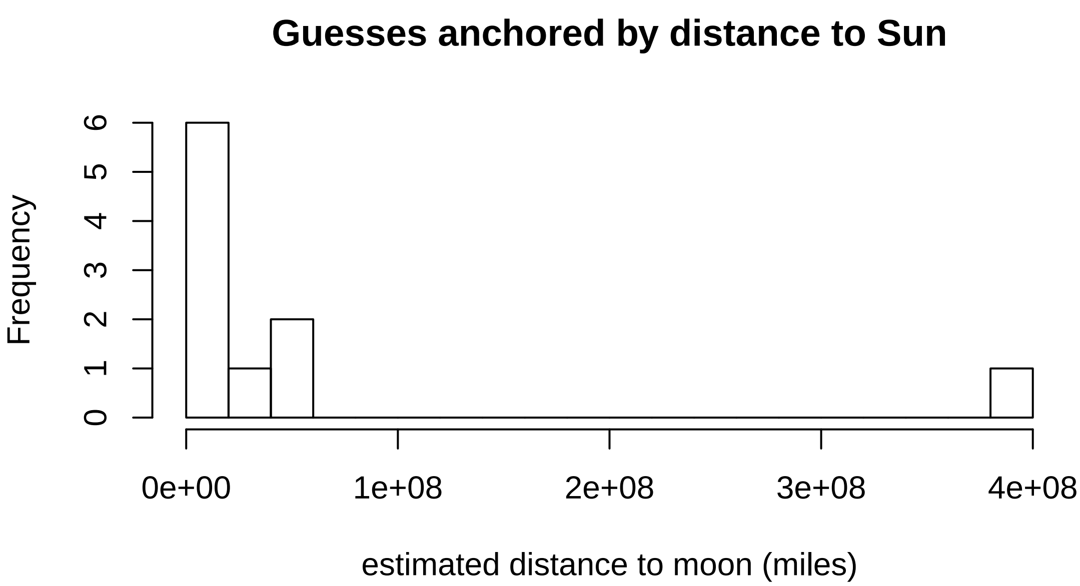
Results

The data showed a strong right skew in both groups.

```
earth = c(7.5e4,2e4,299,39590,3900,75000,30000,8197,15836,8100)
sun = c(1.23e7,1.85e5,2.2e5,45730003,1.2e5,6e5,4e8,2.05e7,48372213,3e5)
hist(earth,breaks=20, xlab = "estimated distance to moon (miles)",
     main = "Guesses anchored by radius of Earth")
```




```
hist(sun,breaks=20, xlab = "estimated distance to moon (miles)",
     main = "Guesses anchored by distance to Sun")
```



The heavy skew along with small sample size should make us wary of using an approach that assumes (repeated) small samples would be normally distributed. We continue with the Welch's t test as an exercise, even though we recognize it is not appropriate.

We find the means and sample standard deviations.

```
n1 = length(earth)
xbar1 = mean(earth)
s1 = sd(earth)
n2 = length(sun)
xbar2 = mean(sun)
s2 = sd(sun)
```

Group	Mean	Standard deviation
Earth	2.76×10^4	2.77×10^4
Sun	5.28×10^7	1.23×10^8

We state the hypotheses.

$$H_0: \quad \mu_2 - \mu_1 = 0$$
$$H_A: \quad \mu_2 - \mu_1 \neq 0$$

We find the degrees of freedom using the Welch-Satterthwaite equation.

```
df = floor((s1^2/n1+s2^2/n2)^2/(s1^4/n1^2/(n1-1)+s2^4/n2^2/(n2-1)))
```

$$df = \frac{\left(\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}\right)^2}{\frac{(s_1)^4}{(n_1)^2(n_1-1)} + \frac{(s_2)^4}{(n_2)^2(n_2-1)}} = 9$$

We find the standard error.

```
SE = sqrt(s1^2/n1+s2^2/n2)
```

$$SE = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = 3.9 \times 10^7$$

We determine the observed test statistic.

```
tobs = (xbar2-xbar1)/SE
```

$$t_{\text{obs}} = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{SE} = 1.35$$

We can determine the two-tail p-value.

```
pvalue = 2*pt(-abs(tobs),df)
```

$$p\text{-value} = 0.21$$

Notice, we can also check our work with R's built-in function.

```
t.test(earth,sun)
```

```
##
##  Welch Two Sample t-test
##
```

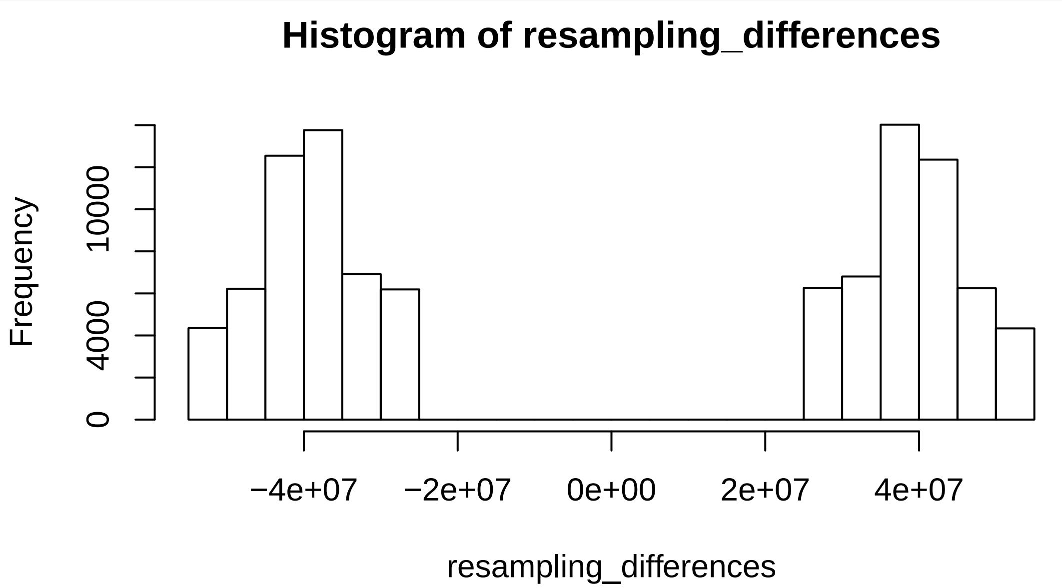
```
## data:  earth and sun
## t = -1.353, df = 9, p-value = 0.2091
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -141093758   35483499
## sample estimates:
##  mean of x  mean of y
##    27592.2 52832721.6
```

The other test of significance is based on an exploration of the possible reshufflings of the given data.

```
num_resamples = 100000
original_difference = xbar2-xbar1
resampling_differences = vector("numeric",num_resamples)
for(i in 1:num_resamples)
{
  tot1 = sum(sample(c(earth,sun),n1))
  tot2 = sum(c(earth,sun))-tot1
  mean1 = tot1/n1
  mean2 = tot2/n2
  resampling_differences[i] = mean2-mean1
}
signif(original_difference,2)
```

```
## [1] 5.3e+07
```

```
hist(resampling_differences)
```




```
pvalue2 = sum(abs(resampling_differences)>=abs(original_difference))/num_resamples
```

$$p\text{-value} = 2 \times 10^{-5}$$

Discussion

This experiment showed a causal link between the hint and the response. The exact mechanism is unclear, but behavior is often labelled anchoring.

Please estimate the distance to the moon in miles. ____________________ Hint: The radius of Earth is 3,000 miles.	Please estimate the distance to the moon in miles. ____________________ Hint: The radius of Earth is 3,000 miles.	Please estimate the distance to the moon in miles. ____________________ Hint: The radius of Earth is 3,000 miles.
Please estimate the distance to the moon in miles. ____________________ Hint: The sun is 90,000,000 miles away.	Please estimate the distance to the moon in miles. ____________________ Hint: The sun is 90,000,000 miles away.	Please estimate the distance to the moon in miles. ____________________ Hint: The sun is 90,000,000 miles away.
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