Bunker Hill Community College
imagine the possibilities

Math 181 First Exam Practice D

Spring 2019

Name: ______________________________

- Write your **full name** on the line above.
- Show your work. Incorrect answers with work can receive partial credit.
- Attempt every question; showing you understand the question earns some credit.
- If you run out of room for an answer, continue on the back of the page. Before doing so, write “see back” with a circle around it.
- You can use 1 page (front and back) of notes.
- You can use (and probably need) a calculator.
- You can use the Geogebra Scientific Calculator instead of a calculator. You need to put your phone on **airplane mode** and then within the application, start **exam mode**; you should see a green bar with a timer counting up.
- If a question is confusing or ambiguous, please ask for clarification; however, you will not be told how to answer the question.
- **Box your final answer**.
- A formula sheet is attached to this test.

Do not write in this grade table.

Question:	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	Total
Points:	10	10	10	10	10	10	60
Score:							

Sample statistics:

n = sample size
x_i = the ith value in a sample
$\bar{x}$ = sample mean
s = sample standard deviation

Q_1 = first quartile
m = median
Q_3 = third quartile
IQR = inter-quartile range = $Q3 - Q1$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Population parameters:

μ = population mean
σ = population standard deviation

Probability:

Ω = set of all possible equally likely outcomes
A = event A, a set of outcomes
A^c = The complement of A
B = event B, another set of outcomes
$|A|$ = size of set, number of outcomes in A
$P(A)$ = probability of A
$P(A \text{ AND } B)$ = probability of both A and B
$P(A \text{ OR } B)$ = probability of either A or B (or both)
$P(A|B)$ = probability of A given B

$$P(A) = \frac{|A|}{|\Omega|}$$

$0 \leq P(A) \leq 1$
$P(A \text{ AND } B) = P(A) \cdot P(B|A)$
$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$
$P(A^c) = 1 - P(A)$

A, B are disjoint (mutually exclusive) $\iff$ $P(A \text{ AND } B) = 0$
A, B are non-disjoint $\iff$ $P(A \text{ AND } B) > 0$
A, B are exhaustive $\iff$ $P(A \text{ OR } B) = 1$
A, B are complements $\iff$ A, B are disjoint and exhaustive $\iff$ $B = A^c$
A, B are independent $\iff$ $P(A \text{ AND } B) = P(A) \times P(B)$ $\iff$ $P(A|B) = P(A)$

Random variables and distributions:

X = random variable
x_i = the ith possible value of X. (Notice different meaning here vs. sample statistics.)
k = number of possible values of X.
$E(X) = \mu$ = expected value of X
σ = standard deviation of X
$\mu = \sum_{i=1}^{k} x_i \cdot P(X = x_i)$
$\sigma = \sqrt{\sum_{i=1}^{k}(x_i - \mu)^2 \cdot P(X = x_i)}$

Q1. (10 points) For simplicity, pretend a lie detector **beeps** when it detects a lie. Assume 30% of people lie on a test. If someone lies, the detector beeps about 80% of the time. If someone tells the truth, the detector stays silent about 70% of the time.

If a detector beeps, what is the chance the person is lying?

(a) Make a tree diagram.

(b) Make a contingency table.

(c) $P(\text{lying}|\text{beep}) = \ ?$

Q2. (10 points) In Boston, a study was performed to investigate the effectiveness of charter schools in raising MCAS (math test) scores. There were 18,000 students who entered a lottery to go to a charter school. About 9000 "won" the lottery, so they went to charter schools. The other 9000 continued in public schools. After 2 years, these students then took the MCAS test, which each student either passed or failed.

	pass	fail
charter	5850	3150
public	3600	5400

(a) Is this study observational or experimental?

(b) Can causal relationships be established?

(c) Describe the null hypothesis.

(d) Describe the alternative hypothesis.

(e) Which treatment (charter or public) had a higher proportion of students pass the test?

(f) Do you think this difference in proportion happened just due to chance? Why?

Q3. (10 points) The wildlife museum suggests that in the US, your chance of dying in a car accident is 1 in 84. That seems a bit high, so let's check with a back-of-the-envelope calculation.

In the United States, about 33,000 people are killed by cars each year from a population of about 330,000,000. If we assume these deaths are randomly distributed, that means each year a US resident has a 0.01% chance of dying by car. Let's assume this chance remains constant and each year is independent.

(a) What is the probability of a US resident not dying by car this year?

(b) What is the probability of a US resident not dying by car for 100 years?

(c) What is the probability that a US resident will die by car at least once over the next 100 years?

Q4. (10 points) Seven randomly sampled drivers in Massachusetts calculated their yearly cost of owning and operating an automobile (in thousand USD). (The US average is about $9000 per year.)

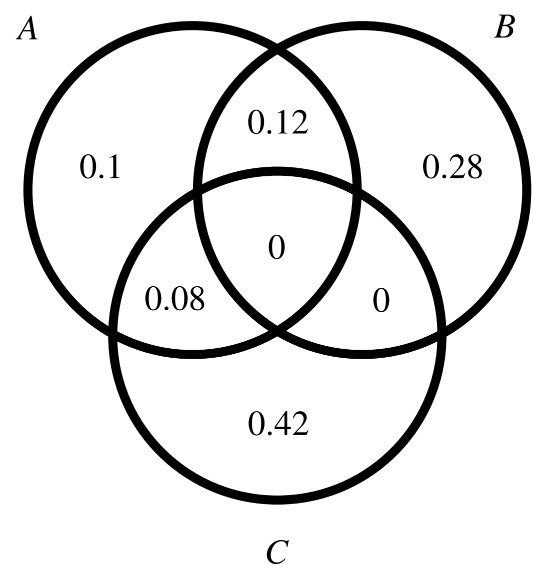
5.7 13.1 9.8 7.1 15.5 6.3 4.1

(a) Calculate the sample mean.

(b) Calculate the sample standard deviation.

(c) Construct a boxplot of these data.

Q5. (10 points) Consider the three events depicted in the Venn diagram below.



(a) Evaluate $P(A)$.

(b) Evaluate $P(B)$.

(c) Evaluate $P(C^c)$.

(d) Evaluate $P(A|B)$.

(e) Evaluate $P(B^c|C)$.

(f) Evaluate $P(A \text{ AND } B)$.

(g) Evaluate $P(A \text{ OR } C)$.

(h) Which two events are independent? How do you know?

(i) Which two events are disjoint (mutually exclusive)? How do you know?

(j) Are these three events exhaustive? How do you know?

Q6. (10 points) Pip rolls a 4-sided die (marked 1, 2, 3, 4) and flips a coin (marked 1, 2). Pip takes the sum. This process results in the probability distribution below, where the random variable Y represents the sum.

y_i	$P(Y = y_i)$
2	0.125
3	0.25
4	0.25
5	0.25
6	0.125

(a) Evaluate $P(Y = 5)$.

(b) Evaluate $P(Y = 1)$.

(c) Evaluate $P(2 \leq Y \leq 4)$.

(d) Find the expected value (mean) of this probability distribution.

(e) Find the standard deviation of this probability distribution.