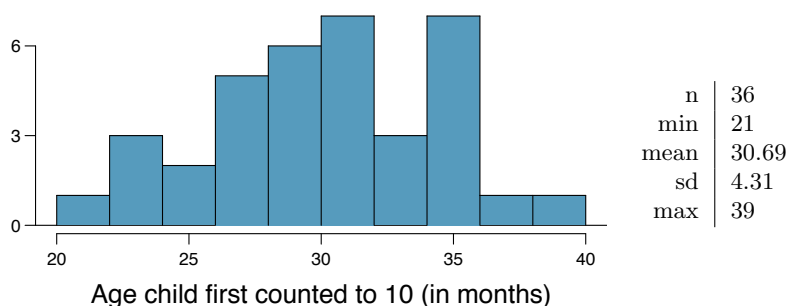


**4.24 Gifted children, Part I.** Researchers investigating characteristics of gifted children collected data from schools in a large city on a random sample of thirty-six children who were identified as gifted children soon after they reached the age of four. The following histogram shows the distribution of the ages (in months) at which these children first counted to 10 successfully. Also provided are some sample statistics.<sup>43</sup>



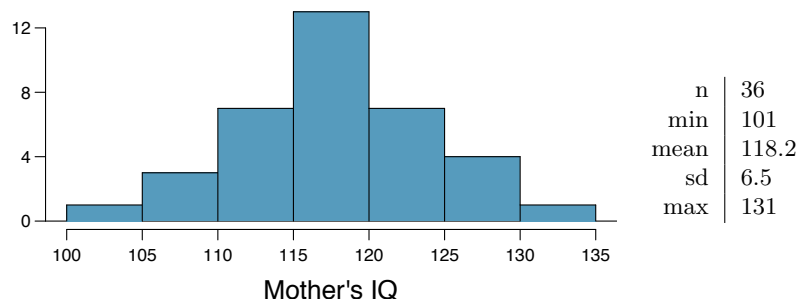
- Are conditions for inference satisfied?
- Suppose you read online that children first count to 10 successfully when they are 32 months old, on average. Perform a hypothesis test to evaluate if these data provide convincing evidence that the average age at which gifted children first count to 10 successfully is less than the general average of 32 months. Use a significance level of 0.10.
- Interpret the p-value in context of the hypothesis test and the data.
- Calculate a 90% confidence interval for the average age at which gifted children first count to 10 successfully.
- Do your results from the hypothesis test and the confidence interval agree? Explain.

**4.25 Waiting at an ER, Part III.** The hospital administrator mentioned in Exercise 4.13 randomly selected 64 patients and measured the time (in minutes) between when they checked in to the ER and the time they were first seen by a doctor. The average time is 137.5 minutes and the standard deviation is 39 minutes. She is getting grief from her supervisor on the basis that the wait times in the ER has increased greatly from last year's average of 127 minutes. However, she claims that the increase is probably just due to chance.

- Are conditions for inference met? Note any assumptions you must make to proceed.
- Using a significance level of  $\alpha = 0.05$ , is the change in wait times statistically significant? Use a two-sided test since it seems the supervisor had to inspect the data before she suggested an increase occurred.
- Would the conclusion of the hypothesis test change if the significance level was changed to  $\alpha = 0.01$ ?

<sup>43</sup>F.A. Graybill and H.K. Iyer. *Regression Analysis: Concepts and Applications*. Duxbury Press, 1994, pp. 511–516.

**4.26 Gifted children, Part II.** Exercise 4.24 describes a study on gifted children. In this study, along with variables on the children, the researchers also collected data on the mother's and father's IQ of the 36 randomly sampled gifted children. The histogram below shows the distribution of mother's IQ. Also provided are some sample statistics.



- Perform a hypothesis test to evaluate if these data provide convincing evidence that the average IQ of mothers of gifted children is different than the average IQ for the population at large, which is 100. Use a significance level of 0.10.
- Calculate a 90% confidence interval for the average IQ of mothers of gifted children.
- Do your results from the hypothesis test and the confidence interval agree? Explain.

**4.27 Working backwards, one-sided.** You are given the following hypotheses:

$$H_0 : \mu = 30$$

$$H_A : \mu > 30$$

We know that the sample standard deviation is 10 and the sample size is 70. For what sample mean would the p-value be equal to 0.05? Assume that all conditions necessary for inference are satisfied.

**4.28 Working backwards, two-sided.** You are given the following hypotheses:

$$H_0 : \mu = 30$$

$$H_A : \mu \neq 30$$

We know that the sample standard deviation is 10 and the sample size is 70. For what sample mean would the p-value be equal to 0.05? Assume that all conditions necessary for inference are satisfied.

**4.29 Testing for Fibromyalgia.** A patient named Diana was diagnosed with Fibromyalgia, a long-term syndrome of body pain, and was prescribed anti-depressants. Being the skeptic that she is, Diana didn't initially believe that anti-depressants would help her symptoms. However after a couple months of being on the medication she decides that the anti-depressants are working, because she feels like her symptoms are in fact getting better.

- Write the hypotheses in words for Diana's skeptical position when she started taking the anti-depressants.
- What is a Type 1 Error in this context?
- What is a Type 2 Error in this context?

**4.30 Testing for food safety.** A food safety inspector is called upon to investigate a restaurant with a few customer reports of poor sanitation practices. The food safety inspector uses a hypothesis testing framework to evaluate whether regulations are not being met. If he decides the restaurant is in gross violation, its license to serve food will be revoked.

- (a) Write the hypotheses in words.
- (b) What is a Type 1 Error in this context?
- (c) What is a Type 2 Error in this context?
- (d) Which error is more problematic for the restaurant owner? Why?
- (e) Which error is more problematic for the diners? Why?
- (f) As a diner, would you prefer that the food safety inspector requires strong evidence or very strong evidence of health concerns before revoking a restaurant's license? Explain your reasoning.

**4.31 Which is higher?** In each part below, there is a value of interest and two scenarios (I and II). For each part, report if the value of interest is larger under scenario I, scenario II, or whether the value is equal under the scenarios.

- (a) The standard error of  $\bar{x}$  when  $s = 120$  and (I)  $n = 25$  or (II)  $n = 125$ .
- (b) The margin of error of a confidence interval when the confidence level is (I) 90% or (II) 80%.
- (c) The p-value for a Z-statistic of 2.5 when (I)  $n = 500$  or (II)  $n = 1000$ .
- (d) The probability of making a Type 2 Error when the alternative hypothesis is true and the significance level is (I) 0.05 or (II) 0.10.

**4.32 True or false.** Determine if the following statements are true or false, and explain your reasoning. If false, state how it could be corrected.

- (a) If a given value (for example, the null hypothesized value of a parameter) is within a 95% confidence interval, it will also be within a 99% confidence interval.
- (b) Decreasing the significance level ( $\alpha$ ) will increase the probability of making a Type 1 Error.
- (c) Suppose the null hypothesis is  $\mu = 5$  and we fail to reject  $H_0$ . Under this scenario, the true population mean is 5.
- (d) If the alternative hypothesis is true, then the probability of making a Type 2 Error and the power of a test add up to 1.
- (e) With large sample sizes, even small differences between the null value and the true value of the parameter, a difference often called the effect size, will be identified as statistically significant.