1. **Problem**
   In a very large population, 46.9% are cold. When a random sample of size 2000 is taken, what is the chance that at least 45.6% of the sample is cold?

Solution

Determine the standard error.

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.469(1-0.469)}{2000}} = 0.0112$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p}-p}{SE} = \frac{0.456-0.469}{0.0112} = -1.16$$

We are looking for a right area ("at least 45.6%").

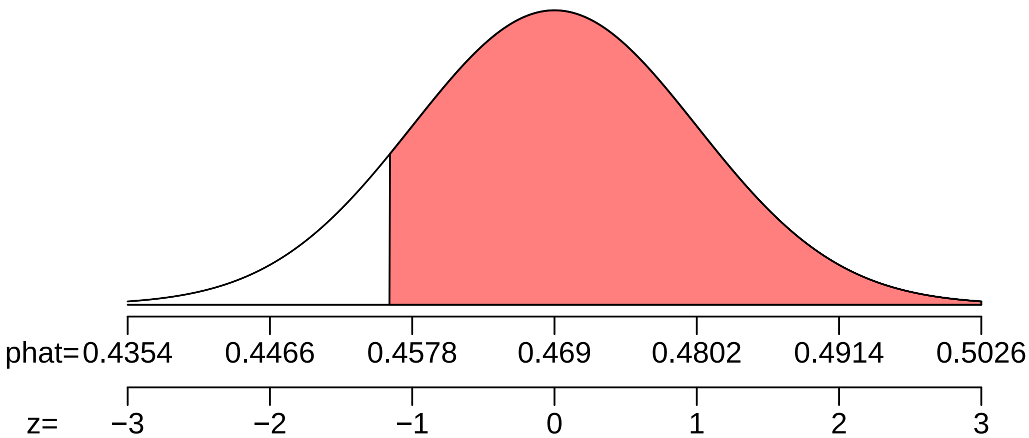


Figure 1:

To determine that right area, we use the z table.

$$\Pr\left(\hat{P} > 0.456\right) \quad = \quad 1-\Phi(-1.16) \quad = \quad 0.877$$

Thus, we conclude there is a 87.7% chance that the sample proportion is at least 45.6%.

2. **Problem**
   In a very large population, 35.8% are strong. When a random sample of size 4900 is taken, what is the chance that the sample proportion of strong individuals is beyond $\pm$ 1.2 percentage points from 35.8%?

Solution
Determine the standard error.

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.358(1-0.358)}{4900}} = 0.0068$$

Determine the upper and lower bounds on $\hat{p}$.

$$\hat{p}_{\text{lower}} = 0.358 - 0.012 = 0.346$$

$$\hat{p}_{\text{upper}} = 0.358 + 0.012 = 0.37$$

Determine the z scores. For simplicity, we ignore the continuity correction.

$$z_{\text{lower}} = \frac{\hat{p}_{\text{lower}} - p}{SE} = \frac{0.346 - 0.358}{0.0068} = \frac{-0.012}{0.0068} = -1.76$$

$$z_{\text{upper}} = \frac{\hat{p}_{\text{upper}} - p}{SE} = \frac{0.37 - 0.358}{0.0068} = \frac{0.012}{0.0068} = 1.76$$

We are looking for a two-tail area ("beyond ± 1.2 percentage points from 35.8%").

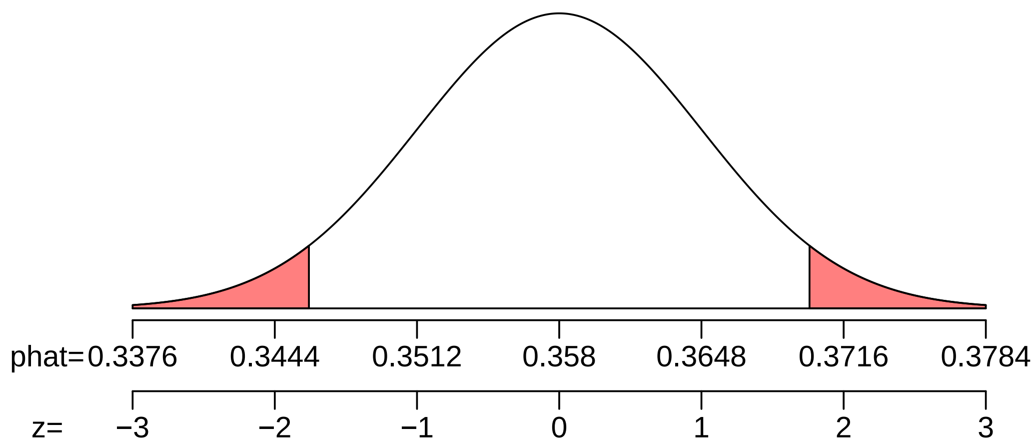


Figure 2:

To determine that central area, we use the z table.

$$\Pr\left(|\hat{P} - 0.358| > 0.012\right) \quad = \quad \Pr\left(|Z| > 1.76\right) \quad = \quad 2 \cdot \Phi(-1.76) \quad = \quad 0.0784$$

Thus, we conclude there is a 7.84% chance that the sample proportion is beyond ± 1.2 percentage points from 35.8%.

3. **Problem**
   A random sample of size 300 was found to have a sample proportion of 54.3%. Determine a 98% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.
   (b) Find the upper bound of the condifence interval.

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.543)(1-0.543)}{300}} = 0.0288$$

Calculate the margin of error.

$$ME = z^\star SE = (2.33)(0.0288) = 0.0671$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \;\approx\; \hat{p} \pm ME$$

Determine the interval.

$$(0.476\,,\ 0.61)$$

We are 98% confident that the true population proportion is between 47.6% and 61%.

(a) The lower bound = 0.476, which can also be expressed as 47.6%.

(b) The upper bound = 0.61, which can also be expressed as 61%.

4. **Problem**

   A random sample of size 42000 was found to have a sample proportion of 37.3%. Determine a 99% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.99$.

$$z^\star = 2.58$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.373)(1-0.373)}{42000}} = 0.00236$$

Calculate the margin of error.

$$ME = z^\star SE = (2.58)(0.00236) = 0.00609$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \;\approx\; \hat{p} \pm ME$$

Determine the interval.

$$(0.367\,,\;0.379)$$

We are 99% confident that the true population proportion is between 36.7% and 37.9%.

(a) The lower bound = 0.367, which can also be expressed as 36.7%.

(b) The upper bound = 0.379, which can also be expressed as 37.9%.

5. **Problem**
If you suspect that $\hat{p}$ will be near 0.77, how large of a sample is needed to guarantee a margin of error less than 0.009 when building a 99.5% confidence interval?

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.995$.

$$z^\star = 2.81$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$SE = \frac{ME}{z^\star} = \frac{0.009}{2.81} = 0.0032$$

Calculate n. Because we have no idea what p is, we will use a conservative approach and use $p = 0.5$.

$$SE = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{SE^2} = \frac{(0.77)(0.23)}{(0.0032)^2} = 17294.921875$$

When determining a necessary sample size, always round up (ceiling).

$$n = 17295$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 18000$$

6. **Problem**
   If you suspect that $\hat{p}$ will be near 0.27, how large of a sample is needed to guarantee a margin of error less than 0.06 when building a 96% confidence interval?

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.96$.

$$z^\star = 2.05$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$SE = \frac{ME}{z^\star} = \frac{0.06}{2.05} = 0.0293$$

Calculate n. Because we have no idea what p is, we will use a conservative approach and use $p = 0.5$.

$$SE = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{SE^2} = \frac{(0.27)(0.73)}{(0.0293)^2} = 229.5891624$$

When determining a necessary sample size, always round up (ceiling).

$$n = 230$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 230$$

7. **Problem**

   It is generally accepted that a population's proportion is 0.603. However, you think that maybe the population proportion is over 0.603, so you decide to run a one-tail hypothesis test with a significance level of 0.05 with a sample size of 10000.

   Then, when you collect the random sample, you find its proportion is 0.611. Do you reject or retain the null hypothesis?

   (a) Determine the p-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution

State the hypotheses.

$$H_0: \quad p = 0.603$$

$$H_A: \quad p > 0.603$$

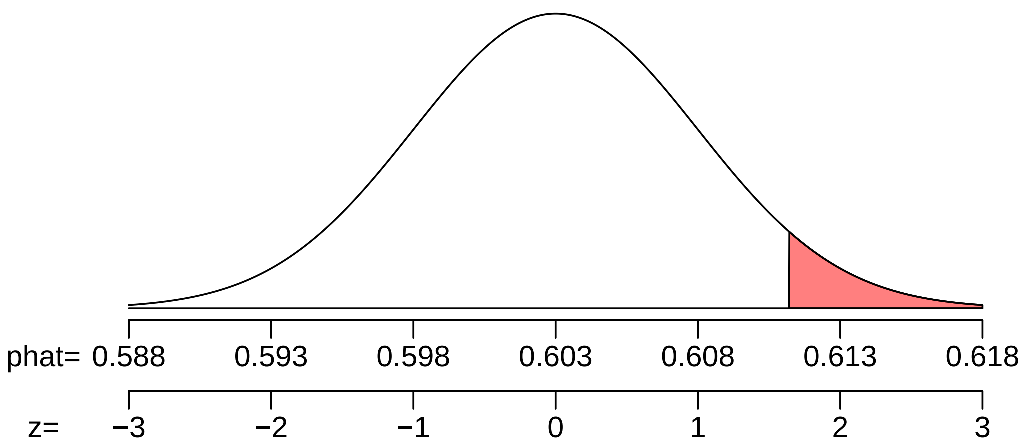
Determine the standard error.

$$SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.603(1-0.603)}{10000}} = 0.00489$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{SE} = \frac{0.611 - 0.603}{0.00489} = 1.64$$

The p-value is a right area ("over").



.image

Figure 3:

To determine that right area, we use the z table.

$$\Pr\left(\hat{P} > 0.611\right) \;=\; 1 - \Phi(1.64) \;=\; 0.0505$$

In other words:

$$p\text{-value} = 0.0505$$

Compare p-value to α (which is 0.05).

$$p\text{-value} > \alpha$$

Make the conclusion: we retain the null hypothesis.

(a) The p-value is 0.0505

(b) We retain the null hypothesis.

8. **Problem**

   It is generally accepted that a population's proportion is 0.739. However, you think that maybe the population proportion is below 0.739, so you decide to run a one-tail hypothesis test with a significance level of 0.02 with a sample size of 3000.

   Then, when you collect the random sample, you find its proportion is 0.757. Do you reject or retain the null hypothesis?

   (a) Determine the p-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution
State the hypotheses.

$$H_0: \quad p = 0.739$$
$$H_A: \quad p < 0.739$$

Determine the standard error.

$$SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.739(1-0.739)}{3000}} = 0.00802$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{SE} = \frac{0.757 - 0.739}{0.00802} = 2.24$$

The p-value is a left area ("below").

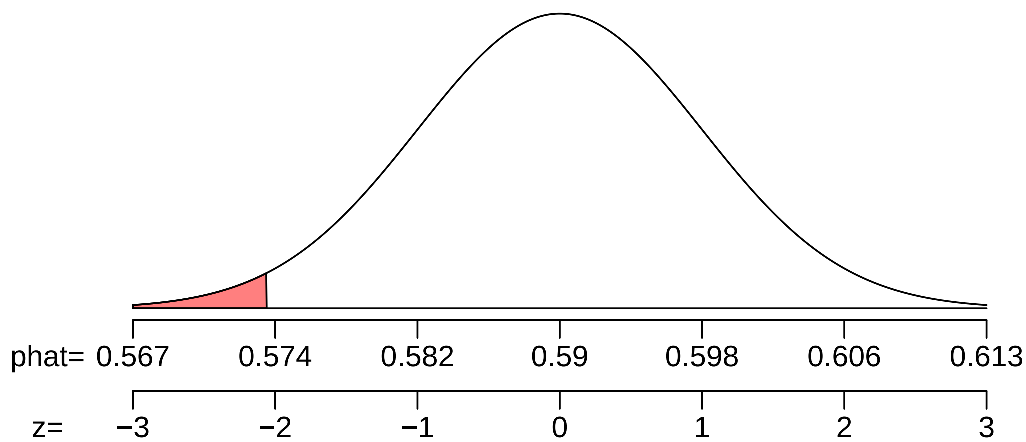


Figure 4:

To determine that left area, we use the z table.

$$\Pr\left(\hat{P} < 0.757\right) \; = \; \Phi(2.24) \; = \; 0.9875$$

In other words:

$$p\text{-value} = 0.9875$$

Compare p-value to α (which is 0.02).

$$p\text{-value} > \alpha$$

Make the conclusion: we retain the null hypothesis.

(a) The p-value is 0.9875
(b) We retain the null hypothesis.

9. **Problem**
In a very large population, 21.3% are great. When a random sample of size 3100 is taken, what is the chance that at most 23% of the sample is great?

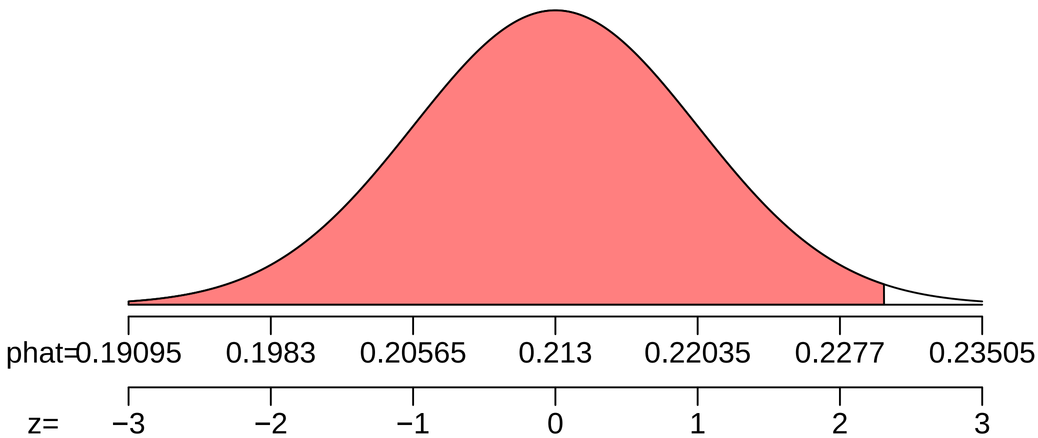
Solution
Determine the standard error.

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.213(1-0.213)}{3100}} = 0.00735$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p}-p}{SE} = \frac{0.23-0.213}{0.00735} = 2.31$$

We are looking for a left area ("at most 23%").



.image

Figure 5:

To determine that left area, we use the z table.

$$\Pr\left(\hat{P} < 0.23\right) \;=\; \Phi(2.31) \;=\; 0.9896$$

Thus, we conclude there is a 99% chance that the sample proportion is at most 23%.

10. **Problem**
    A random sample of size 65000 was found to have a sample proportion of 5.7%. Determine a 95% confidence interval of the population proportion.

    (a) Find the lower bound of the confidence interval.
    (b) Find the upper bound of the condifence interval.

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.95$.

$$z^\star = 1.96$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.057)(1-0.057)}{65000}} = 0.000909$$

Calculate the margin of error.

$$ME = z^\star SE = (1.96)(0.000909) = 0.00178$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \;\approx\; \hat{p} \pm ME$$

Determine the interval.

$$(0.0552\,,\;0.0588)$$

We are 95% confident that the true population proportion is between 5.52% and 5.88%.

(a) The lower bound = 0.0552, which can also be expressed as 5.52%.

(b) The upper bound = 0.0588, which can also be expressed as 5.88%.

11. **Problem**
If you suspect that $\hat{p}$ will be near 0.85, how large of a sample is needed to guarantee a margin of error less than 0.008 when building a 98% confidence interval?

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$SE = \frac{ME}{z^\star} = \frac{0.008}{2.33} = 0.00343$$

Calculate n. Because we have no idea what p is, we will use a conservative approach and use $p = 0.5$.

$$SE = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{SE^2} = \frac{(0.85)(0.15)}{(0.00343)^2} = 10837.3211842$$

When determining a necessary sample size, always round up (ceiling).

$$n = 10838$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 11000$$

12. **Problem**

It is generally accepted that a population's proportion is 0.34. However, you think that maybe the population proportion is different than 0.34, so you decide to run a two-tail hypothesis test with a significance level of 0.04 with a sample size of 900.

Then, when you collect the random sample, you find its proportion is 0.305. Do you reject or retain the null hypothesis?

(a) Determine the p-value.

(b) Decide whether we reject or retain the null hypothesis.

Solution
State the hypotheses.

$$H_0: \quad p = 0.34$$
$$H_A: \quad p \neq 0.34$$

Determine the standard error.

$$SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.34(1-0.34)}{900}} = 0.0158$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{SE} = \frac{0.305 - 0.34}{0.0158} = -2.22$$

The p-value is a two-tail area.

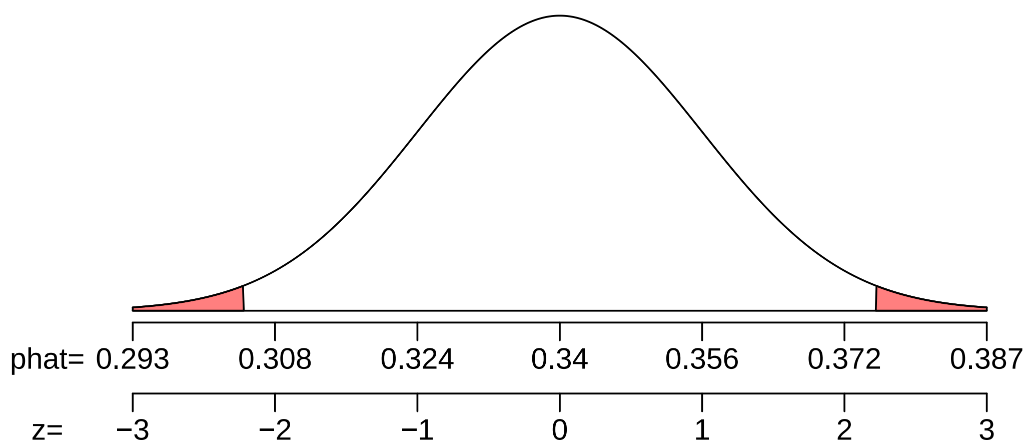


Figure 6:

To determine that two-tail area, we use the z table.

$$\Pr\left(\hat{P} > 0.305\right) \;=\; 2 \cdot \Phi(-2.22) \;=\; 0.0264$$

In other words:

$$p\text{-value} = 0.0264$$

Compare p-value to α (which is 0.04).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) The p-value is 0.0264
(b) We reject the null hypothesis.

13. **Problem**
    In a very large population, 75.3% are happy. When a random sample of size 400 is taken, what is the chance that the sample proportion of happy individuals is farther than $\pm$ 3.5 percentage points from 75.3%?

Solution
Determine the standard error.

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.753(1-0.753)}{400}} = 0.022$$

Determine the upper and lower bounds on $\hat{p}$.

$$\hat{p}_{\text{lower}} = 0.753 - 0.035 = 0.718$$

$$\hat{p}_{\text{upper}} = 0.753 + 0.035 = 0.788$$

Determine the z scores. For simplicity, we ignore the continuity correction.

$$z_{\text{lower}} = \frac{\hat{p}_{\text{lower}} - p}{SE} = \frac{0.718 - 0.753}{0.022} = \frac{-0.035}{0.022} = -1.59$$

$$z_{\text{upper}} = \frac{\hat{p}_{\text{upper}} - p}{SE} = \frac{0.788 - 0.753}{0.022} = \frac{0.035}{0.022} = 1.59$$

We are looking for a two-tail area ("farther than ± 3.5 percentage points from 75.3%").

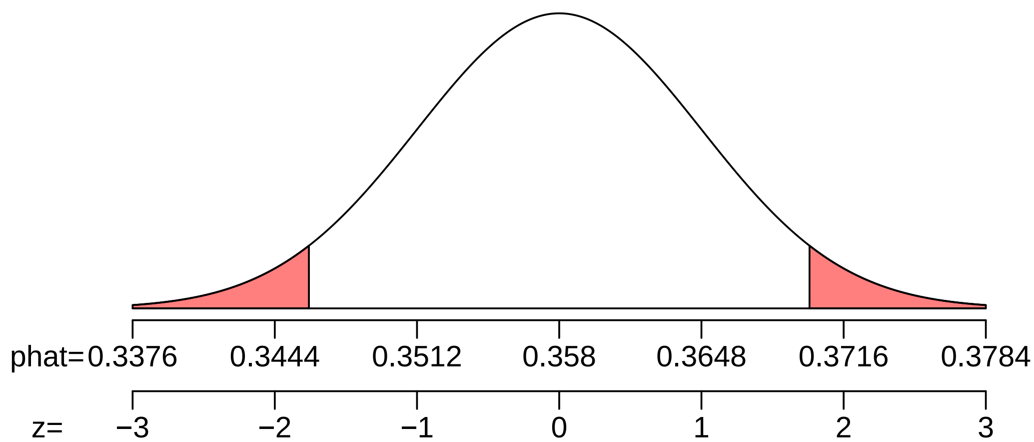


Figure 7:

To determine that central area, we use the z table.

$$\Pr\left(|\hat{P} - 0.753| > 0.035\right) \quad = \quad \Pr\left(|Z| > 1.59\right) \quad = \quad 2 \cdot \Phi(-1.59) \quad = \quad 0.1118$$

Thus, we conclude there is a 11.2% chance that the sample proportion is farther than ± 3.5 percentage points from 75.3%.

14. **Problem**
A random sample of size 65000 was found to have a sample proportion of 29.9%. Determine a 99% confidence interval of the population proportion.

(a) Find the lower bound of the confidence interval.

(b) Find the upper bound of the condifence interval.

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.99$.

$$z^\star = 2.58$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.299)(1-0.299)}{65000}} = 0.0018$$

Calculate the margin of error.

$$ME = z^\star SE = (2.58)(0.0018) = 0.00464$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \ \approx \ \hat{p} \pm ME$$

Determine the interval.

$$(0.294\,,\ 0.304)$$

We are 99% confident that the true population proportion is between 29.4% and 30.4%.

(a) The lower bound = 0.294, which can also be expressed as 29.4%.

(b) The upper bound = 0.304, which can also be expressed as 30.4%.

15. **Problem**
How large of a sample is needed to guarantee a margin of error less than 0.05 when building a 98% confidence interval?

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$SE = \frac{ME}{z^\star} = \frac{0.05}{2.33} = 0.0215$$

Calculate n. Because we have no idea what p is, we will use a conservative approach and use $p = 0.5$.

$$SE = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{SE^2} = \frac{(0.5)(0.5)}{(0.0215)^2} = 540.8328826$$

When determining a necessary sample size, always round up (ceiling).

$$n = 541$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 550$$

16. **Problem**

It is generally accepted that a population's proportion is 0.59. However, you think that maybe the population proportion is below 0.59, so you decide to run a one-tail hypothesis test with a significance level of 0.02 with a sample size of 4000.

Then, when you collect the random sample, you find its proportion is 0.574. Do you reject or retain the null hypothesis?

(a) Determine the p-value.

(b) Decide whether we reject or retain the null hypothesis.

Solution

State the hypotheses.

$$H_0: \quad p = 0.59$$

$$H_A: \quad p < 0.59$$

Determine the standard error.

$$SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.59(1-0.59)}{4000}} = 0.00778$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{SE} = \frac{0.574 - 0.59}{0.00778} = -2.06$$

The p-value is a left area ("below").

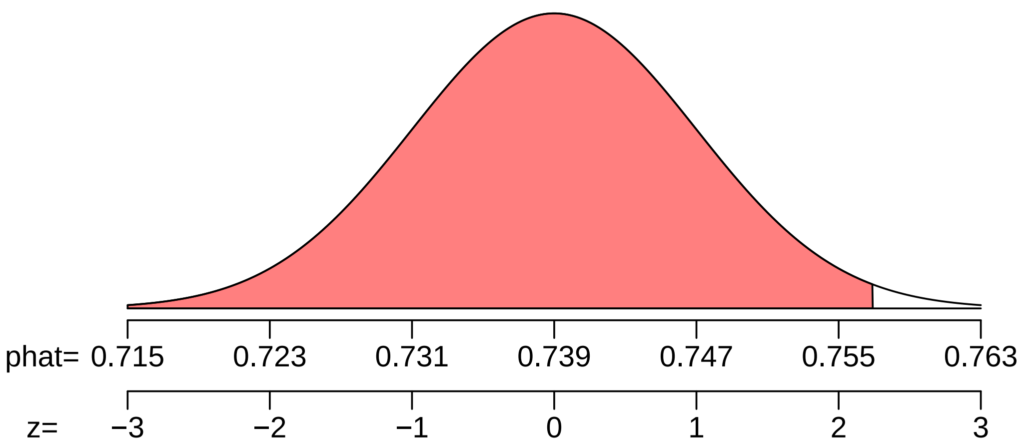


Figure 8:

To determine that left area, we use the z table.

$$\Pr\left(\hat{P} < 0.574\right) \; = \; \Phi(-2.06) \; = \; 0.0197$$

In other words:

$$p\text{-value} = 0.0197$$

Compare p-value to α (which is 0.02).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) The p-value is 0.0197

(b) We reject the null hypothesis.

17. **Problem**
In a very large population, 35.3% are strong. When a random sample of size 2800 is taken, what is the chance that over 34.1% of the sample is strong?

Solution
Determine the standard error.

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.353(1-0.353)}{2800}} = 0.00903$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p}-p}{SE} = \frac{0.341-0.353}{0.00903} = -1.33$$

We are looking for a right area ("over 34.1%").

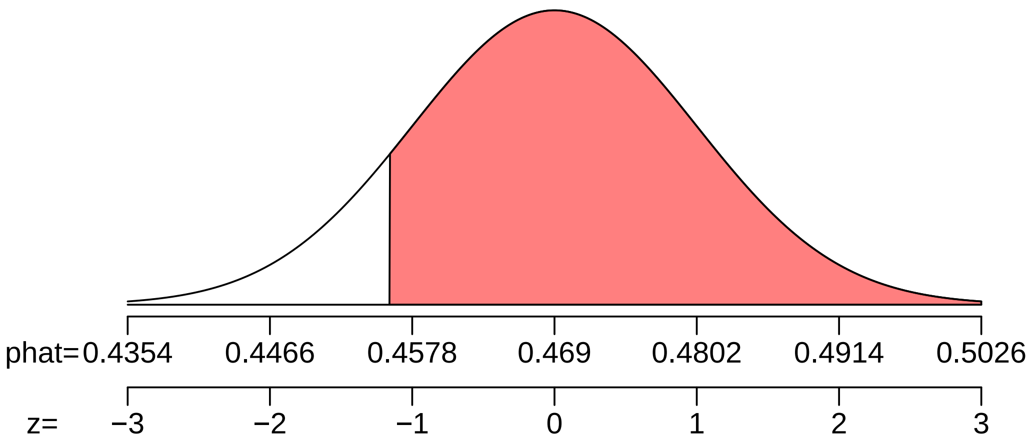


Figure 9:

To determine that right area, we use the z table.

$$\Pr\left(\hat{P} > 0.341\right) \quad = \quad 1-\Phi(-1.33) \quad = \quad 0.9082$$

Thus, we conclude there is a 90.8% chance that the sample proportion is over 34.1%.

18. **Problem**
A random sample of size 1300 was found to have a sample proportion of 76.9%. Determine a 99% confidence interval of the population proportion.
    (a) Find the lower bound of the confidence interval.
    (b) Find the upper bound of the condifence interval.

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.99$.

$$z^\star = 2.58$$

Calculate the standard error.

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.769)(1-0.769)}{1300}} = 0.0117$$

Calculate the margin of error.

$$ME = z^\star SE = (2.58)(0.0117) = 0.0302$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \;\approx\; \hat{p} \pm ME$$

Determine the interval.

$$(0.739\,,\; 0.799)$$

We are 99% confident that the true population proportion is between 73.9% and 79.9%.

(a) The lower bound = 0.739, which can also be expressed as 73.9%.

(b) The upper bound = 0.799, which can also be expressed as 79.9%.

19. **Problem**
    How large of a sample is needed to guarantee a margin of error less than 0.06 when building a 99% confidence interval?

Solution
Determine $z^\star$ such that $P(|Z| < z^\star) = 0.99$.

$$z^\star = 2.58$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$SE = \frac{ME}{z^\star} = \frac{0.06}{2.58} = 0.0233$$

Calculate n. Because we have no idea what p is, we will use a conservative approach and use $p = 0.5$.

$$SE = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{SE^2} = \frac{(0.5)(0.5)}{(0.0233)^2} = 460.4984435$$

When determining a necessary sample size, always round up (ceiling).

$$n = 461$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 470$$

20. **Problem**

It is generally accepted that a population's proportion is 0.752. However, you think that maybe the population proportion is above 0.752, so you decide to run a one-tail hypothesis test with a significance level of 0.01 with a sample size of 600.

Then, when you collect the random sample, you find its proportion is 0.791. Do you reject or retain the null hypothesis?

(a) Determine the p-value.

(b) Decide whether we reject or retain the null hypothesis.

Solution
State the hypotheses.

$$H_0: \quad p = 0.752$$
$$H_A: \quad p > 0.752$$

Determine the standard error.

$$SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.752(1-0.752)}{600}} = 0.0176$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{SE} = \frac{0.791 - 0.752}{0.0176} = 2.22$$

The p-value is a right area ("above").

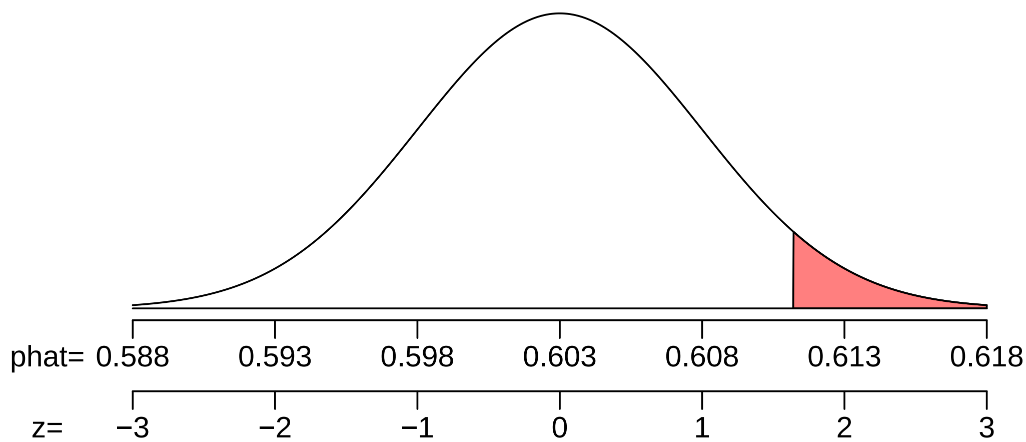


Figure 10:

To determine that right area, we use the z table.

$$\Pr\left(\hat{P} > 0.791\right) \;=\; 1 - \Phi(2.22) \;=\; 0.0132$$

In other words:

$$p\text{-value} = 0.0132$$

Compare p-value to α (which is 0.01).

$$p\text{-value} > \alpha$$

Make the conclusion: we retain the null hypothesis.

(a) The p-value is 0.0132
(b) We retain the null hypothesis.