

- 2.1:** (a): False. Each toss has a 50% chance of landing heads.
 (b): False. You can draw a red face card, like the queen of hearts.
 (c): True, because an ace card is not a face card (even though we pretended otherwise for the gender discrimination simulations).
- 2.2:** (a): $P(\text{red}) = \frac{18}{38} \approx 0.47$
 (b): Well, if we assume the wheel is fair, then $P(\text{red}) = \frac{18}{38} \approx 0.47$. (See part (c).)
 (c): Haha! Not really. 300 consecutive reds seems rather fishy.
- 2.3:** (a): 10. We want the sample proportion \hat{p} to be far from the probability, which is 0.5. Thus, we want a small sample size. Large samples will have sample proportions near the probability.
 (b): 100. We want the sample proportion to be near 0.5.
 (c): 100. We want the sample proportion to be near 0.5.
 (d): 10. We want the sample proportion far from 0.5.
- 2.4:** I rolled four 6s in a row. The probability of this can be determined using the product rule for independent events.

$$P(\text{four 6s}) = P(\text{first is 6}) \times P(\text{second is 6}) \times P(\text{third is 6}) \times P(\text{fourth is 6})$$

$$P(\text{four 6s}) = \left(\frac{1}{6}\right)^4 = \frac{1}{1296} \approx 0.00077$$

My friend rolled four 3s in a row.

$$P(\text{four 3s}) = P(\text{first is 3}) \times P(\text{second is 3}) \times P(\text{third is 3}) \times P(\text{fourth is 3})$$

$$P(\text{four 3s}) = \left(\frac{1}{6}\right)^4 = \frac{1}{1296} \approx 0.00077$$

You might also consider the following:

When rolling two dice, there are 36 possible equally likely outcomes.

		first die					
		1	2	3	4	5	6
second die	1	1,1	2,1	3,1	4,1	5,1	6,1
	2	1,2	2,2	3,2	4,2	5,2	6,2
	3	1,3	2,3	3,3	4,3	5,3	6,3
	4	1,4	2,4	3,4	4,4	5,4	6,4
	5	1,5	2,5	3,5	4,5	5,5	6,5
	6	1,6	2,6	3,6	4,6	5,6	6,6

This means that on each turn, a player has a $\frac{1}{36}$ chance to roll two 3s. On each turn, a player has the same chance to roll two 6s. Then, the chance to do two 3s twice can be determined using the multiplication rule. Let A mean a double on first turn and let B mean the same double on the second turn.

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

Because each turn is independent, we know $P(B|A) = P(B)$. Thus,

$$P(A \text{ and } B) = \frac{1}{36} \times \frac{1}{36}$$

2.5: (a): $P(\text{ten tails}) = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024} \approx 0.00098$

(b): $P(\text{ten heads}) = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024} \approx 0.00098$

(c): We can use complementary events here. The events “ten heads” and “at least one tails” are complementary, so they should sum to 1.

$$\begin{aligned} P(\text{at least one tails}) &= 1 - P(\text{ten heads}) \\ &= 1 - \frac{1}{1024} \\ &= \frac{1023}{1024} \\ &\approx 0.999 \end{aligned}$$

2.6: (a): 0. It is impossible for two dice to sum to 1.

(b): When rolling two dice, there are 36 equally-likely outcomes, 4 of which are favorable to the event:

		first die					
		1	2	3	4	5	6
second die	1	1,1	2,1	3,1	4,1	5,1	6,1
	2	1,2	2,2	3,2	4,2	5,2	6,2
	3	1,3	2,3	3,3	4,3	5,3	6,3
	4	1,4	2,4	3,4	4,4	5,4	6,4
	5	1,5	2,5	3,5	4,5	5,5	6,5
	6	1,6	2,6	3,6	4,6	5,6	6,6

So, $P(\text{sum of 5}) = \frac{4}{36} \approx 0.1111$

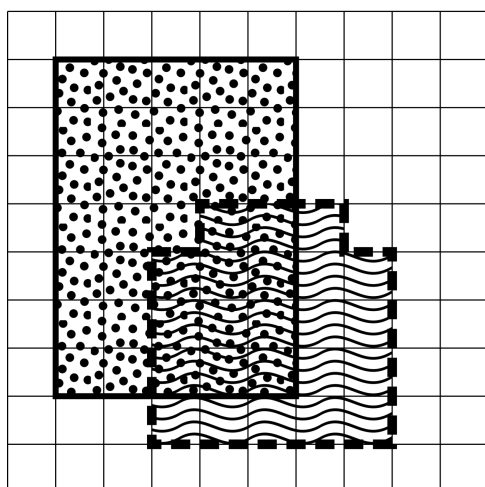
(c): There is only one (of 36) ways to roll a sum of 12.



$$P(\text{sum of 12}) = \frac{1}{36} \approx 0.0278$$

2.7: (a): Nope. Some voters are both.

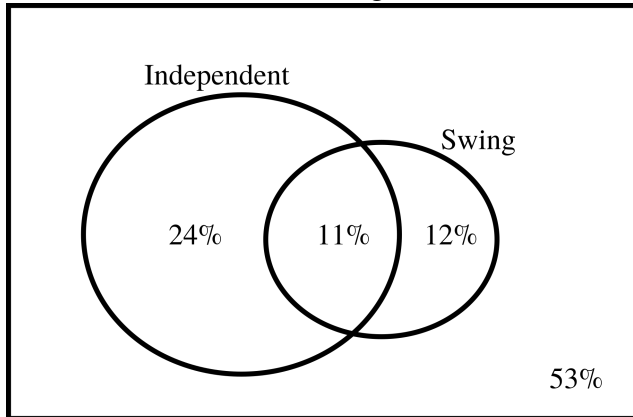
(b):

Venn diagram



 Independent
 Swing

Or, we could do the following:



Also, if you interpreted the prompt to mean 35% were only independent, 23% were only swing, and 11% were both, I'd understand why you interpreted it that way.

(c): 24%

(d): 47%

(e): 53%

(f): Wow, this is confusing... we are using “Independent” for political affiliation and “independent” for probability concept.

$$P(\text{swing}|\text{Independent}) = \frac{11}{35} \approx 0.314$$

$$P(\text{swing}|\text{not Independent}) = \frac{12}{63} \approx 0.185$$

$$P(\text{swing}) = \frac{23}{100} = 0.23$$

Because $P(\text{swing}|\text{Independent}) \neq P(\text{swing})$, swingness is dependent on Independentness.

Also, we could have used the other test of independence.

$$\text{independence} \iff P(A \text{ and } B) = P(A) \times P(B)$$

$$P(\text{Indy and swing}) = 0.11$$

$$P(\text{Indy}) = 0.35$$

$$P(\text{swing}) = 0.23$$

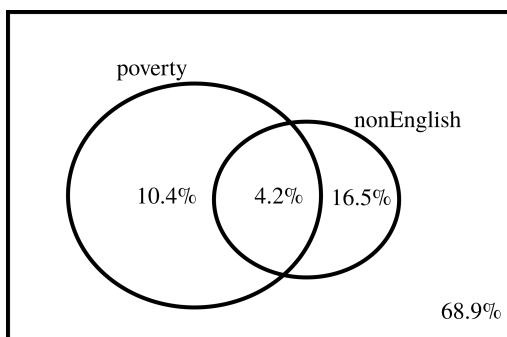
$$P(\text{Indy}) \times P(\text{swing}) = 0.0805$$

$$P(\text{Indy and swing}) \neq P(\text{Indy}) \times P(\text{swing})$$

So, swingness is dependent on Independentness.

2.8: (a): Nope.

(b):



(c): 10.4%

(d): 31.1%

(e): 68.9%

(f):

$$P(\text{poverty}) = 0.146$$

$$P(\text{nonEnglish}) = 0.207$$

$$P(\text{poverty and nonEnglish}) = 0.042$$

$$P(\text{poverty}) \times P(\text{nonEnglish}) \approx 0.146 \times 0.207 \approx 0.03 \neq 0.042$$

No. The events are dependent.