

# MATH 181 2ND EXAM

## SPRING 2019

Name: \_\_\_\_\_

- Write your **full name** on the line above.
- Show your work. Incorrect answers with work can receive partial credit.
- Attempt every question; showing you understand the question earns some credit.
- If you run out of room for an answer, continue on the back of the page. Before doing so, write “see back” with a circle around it.
- You can use 1 page (front and back) of notes in addition to the formula sheet (on page 2) and z-table (last page).
- You can use (and probably need) a calculator.
- You can use the Geogebra Scientific Calculator instead of a calculator. You need to put your phone on **airplane mode** and then within the application, start **exam mode**; you should see a green bar with a timer counting up.
- If a question is confusing or ambiguous, please ask for clarification; however, you will not be told how to answer the question.
- **Box your final answer.**
- You can rip off the z-table, but please keep the rest of the test intact.

Do not write in this grade table.

Question:	Q1	Q2	Q3	Q4	Q5	Q6	Total
Points:	10	10	10	10	10	10	60
Score:							

**Normal Distribution:**

$$X \sim \mathcal{N}(\mu, \sigma)$$

$\mu$  = population mean

$\sigma$  = population standard deviation

$x$  = possible value of  $X$

$\ell$  = percentile of  $x$  (left area)

$\Phi(z)$  = standard normal cumulative function

$$z = \frac{x - \mu}{\sigma}$$

$$P(X < x) = \Phi(z)$$

$$\ell = \Phi(z)$$

$$z = \Phi^{-1}(\ell)$$

**Bernoulli Distribution:**

$$X \sim \text{Bern}(p)$$

$X = 0$  for fail or 1 for success

$p$  = probability of success

$$P(X = 0) = 1 - p$$

$$P(X = 1) = p$$

$$\mu = p$$

$$\sigma = \sqrt{p(1 - p)}$$

**Geometric Distribution:**

$$X \sim \text{Geo}(p)$$

$X$  = number of trials until first success

$p$  = probability of success on each trial

$n$  = a possible number of trials

$$P(X = n) = (1 - p)^{n-1}(p)$$

$$\mu = \frac{1}{p}$$

$$\sigma = \sqrt{\frac{1 - p}{p^2}}$$

**Mean-Sampling Distribution:**

$\bar{X}$  = sample mean

$s$  = sample standard deviation

$n$  = sample size

$\mu$  = population mean

$\sigma$  = population standard deviation

$SE$  = standard error

$$SE = \frac{\sigma}{\sqrt{n}}$$

If  $n \geq 30$  (or if population is normal) then:

$$\bar{X} \sim \mathcal{N}(\mu, SE)$$

**Binomial Distribution:**

$$X \sim \mathcal{B}(n, p)$$

$X$  = number of successes from  $n$  trials

$p$  = probability of success on each trial

$n$  = number of trials

$k$  = a possible number of successes

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu = np$$

$$\sigma = \sqrt{np(1 - p)}$$

If  $np \geq 10$  and  $n(1 - p) \geq 10$ , then

$$X \sim \mathcal{N}(\mu, \sigma)$$

Continuity correction:

$$P(X \leq k) \approx \Phi\left(\frac{k + 0.5 - \mu}{\sigma}\right)$$

**Confidence Interval:**

$CI$  = confidence interval

$\gamma$  = confidence level

$\bar{x}$  = sample mean

$s$  = sample standard deviation

$$z^* = \Phi^{-1}\left(\frac{\gamma + 1}{2}\right)$$

$$SE \approx \frac{s}{\sqrt{n}}$$

$$CI = \bar{x} \pm z^* SE$$

**Hypothesis testing:**

$$H_0 : \mu = \mu_0$$

$$H_A : \mu \neq \mu_0$$

$\bar{x}$  = a possible/specific/observed sample mean

$s$  = sample standard deviation

$\alpha$  = significance level

$$\sigma \approx s$$

$$z = \frac{\bar{x} - \mu_0}{SE}$$

$$p\text{-value} = P(|Z| > |z|)$$

$$= 2 \cdot \Phi(-|z|)$$

If  $p\text{-value} < \alpha$ , then reject  $H_0$ , else retain  $H_0$ .

- Q1.** (10 points) Let random variable  $X$  be normally distributed with mean  $\mu = 33$  and standard deviation  $\sigma = 4$ .
- (a) Evaluate  $P(X < 37)$ .

- (b) Determine  $x$  such that  $P(X < x) = 0.33$ .

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**Q2.** (10 points) Imagine a scratch-off lottery has a chance of success  $p = 0.01$ .

(a) What is the mean number of trials until the first success?

(b) What is the probability of getting the first success on the twelfth trial?

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(a) What is the probability of getting exactly 26 successes?

(b) What is the probability of getting fewer than 26 successes? (Use a normal approximation and continuity correction.)

(c) What is the probability of getting more than 26 successes?

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- Q4.** (10 points) A uniformly distributed population has a mean  $\mu = 250$  and a standard deviation  $\sigma = 18$ . When a sample of size  $n = 144$  is taken, what is the probability of getting a sample mean between 247 and 253?

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- Q5.** (10 points) A sample of size  $n = 144$  has a mean  $\bar{x} = 253.92$  and a standard deviation  $s = 17.54$ . Construct a confidence interval of the population's mean based on the sample using a confidence level  $\gamma = 0.99$ .

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**Q6.** (10 points) Imagine you thought a uniformly distributed population had a mean  $\mu = 250$  and standard deviation  $\sigma = 18$ . However, you are skeptical, so you decide to run a two-tailed hypothesis test with a significance level  $\alpha = 0.01$ .

Your sample of size  $n = 144$  results in a mean  $\bar{x} = 253.92$ .

(a) State the hypotheses.

(b) Describe and sketch the null's sampling distribution.

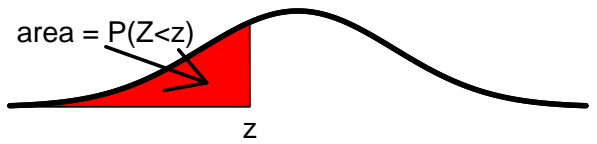
(c) Find a  $z$  score of the sample mean.

(d) Calculate the  $p$ -value.

(e) Make your conclusion.

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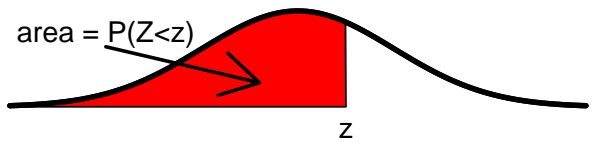
## STANDARD NORMAL CUMULATIVE FUNCTION

 $z < 0$ 

$z$	$P(Z < z)$	$z$	$P(Z < z)$	$z$	$P(Z < z)$	$z$	$P(Z < z)$	$z$	$P(Z < z)$	$z$	$P(Z < z)$
-3.00	0.0013	-2.50	0.0062	-2.00	0.0228	-1.50	0.0668	-1.00	0.1587	-0.50	0.3085
-2.99	0.0014	-2.49	0.0064	-1.99	0.0233	-1.49	0.0681	-0.99	0.1611	-0.49	0.3121
-2.98	0.0014	-2.48	0.0066	-1.98	0.0239	-1.48	0.0694	-0.98	0.1635	-0.48	0.3156
-2.97	0.0015	-2.47	0.0068	-1.97	0.0244	-1.47	0.0708	-0.97	0.1660	-0.47	0.3192
-2.96	0.0015	-2.46	0.0069	-1.96	0.0250	-1.46	0.0721	-0.96	0.1685	-0.46	0.3228
-2.95	0.0016	-2.45	0.0071	-1.95	0.0256	-1.45	0.0735	-0.95	0.1711	-0.45	0.3264
-2.94	0.0016	-2.44	0.0073	-1.94	0.0262	-1.44	0.0749	-0.94	0.1736	-0.44	0.3300
-2.93	0.0017	-2.43	0.0075	-1.93	0.0268	-1.43	0.0764	-0.93	0.1762	-0.43	0.3336
-2.92	0.0018	-2.42	0.0078	-1.92	0.0274	-1.42	0.0778	-0.92	0.1788	-0.42	0.3372
-2.91	0.0018	-2.41	0.0080	-1.91	0.0281	-1.41	0.0793	-0.91	0.1814	-0.41	0.3409
-2.90	0.0019	-2.40	0.0082	-1.90	0.0287	-1.40	0.0808	-0.90	0.1841	-0.40	0.3446
-2.89	0.0019	-2.39	0.0084	-1.89	0.0294	-1.39	0.0823	-0.89	0.1867	-0.39	0.3483
-2.88	0.0020	-2.38	0.0087	-1.88	0.0301	-1.38	0.0838	-0.88	0.1894	-0.38	0.3520
-2.87	0.0021	-2.37	0.0089	-1.87	0.0307	-1.37	0.0853	-0.87	0.1922	-0.37	0.3557
-2.86	0.0021	-2.36	0.0091	-1.86	0.0314	-1.36	0.0869	-0.86	0.1949	-0.36	0.3594
-2.85	0.0022	-2.35	0.0094	-1.85	0.0322	-1.35	0.0885	-0.85	0.1977	-0.35	0.3632
-2.84	0.0023	-2.34	0.0096	-1.84	0.0329	-1.34	0.0901	-0.84	0.2005	-0.34	0.3669
-2.83	0.0023	-2.33	0.0099	-1.83	0.0336	-1.33	0.0918	-0.83	0.2033	-0.33	0.3707
-2.82	0.0024	-2.32	0.0102	-1.82	0.0344	-1.32	0.0934	-0.82	0.2061	-0.32	0.3745
-2.81	0.0025	-2.31	0.0104	-1.81	0.0351	-1.31	0.0951	-0.81	0.2090	-0.31	0.3783
-2.80	0.0026	-2.30	0.0107	-1.80	0.0359	-1.30	0.0968	-0.80	0.2119	-0.30	0.3821
-2.79	0.0026	-2.29	0.0110	-1.79	0.0367	-1.29	0.0985	-0.79	0.2148	-0.29	0.3859
-2.78	0.0027	-2.28	0.0113	-1.78	0.0375	-1.28	0.1003	-0.78	0.2177	-0.28	0.3897
-2.77	0.0028	-2.27	0.0116	-1.77	0.0384	-1.27	0.1020	-0.77	0.2206	-0.27	0.3936
-2.76	0.0029	-2.26	0.0119	-1.76	0.0392	-1.26	0.1038	-0.76	0.2236	-0.26	0.3974
-2.75	0.0030	-2.25	0.0122	-1.75	0.0401	-1.25	0.1056	-0.75	0.2266	-0.25	0.4013
-2.74	0.0031	-2.24	0.0125	-1.74	0.0409	-1.24	0.1075	-0.74	0.2296	-0.24	0.4052
-2.73	0.0032	-2.23	0.0129	-1.73	0.0418	-1.23	0.1093	-0.73	0.2327	-0.23	0.4090
-2.72	0.0033	-2.22	0.0132	-1.72	0.0427	-1.22	0.1112	-0.72	0.2358	-0.22	0.4129
-2.71	0.0034	-2.21	0.0136	-1.71	0.0436	-1.21	0.1131	-0.71	0.2389	-0.21	0.4168
-2.70	0.0035	-2.20	0.0139	-1.70	0.0446	-1.20	0.1151	-0.70	0.2420	-0.20	0.4207
-2.69	0.0036	-2.19	0.0143	-1.69	0.0455	-1.19	0.1170	-0.69	0.2451	-0.19	0.4247
-2.68	0.0037	-2.18	0.0146	-1.68	0.0465	-1.18	0.1190	-0.68	0.2483	-0.18	0.4286
-2.67	0.0038	-2.17	0.0150	-1.67	0.0475	-1.17	0.1210	-0.67	0.2514	-0.17	0.4325
-2.66	0.0039	-2.16	0.0154	-1.66	0.0485	-1.16	0.1230	-0.66	0.2546	-0.16	0.4364
-2.65	0.0040	-2.15	0.0158	-1.65	0.0495	-1.15	0.1251	-0.65	0.2578	-0.15	0.4404
-2.64	0.0041	-2.14	0.0162	-1.64	0.0505	-1.14	0.1271	-0.64	0.2611	-0.14	0.4443
-2.63	0.0043	-2.13	0.0166	-1.63	0.0516	-1.13	0.1292	-0.63	0.2643	-0.13	0.4483
-2.62	0.0044	-2.12	0.0170	-1.62	0.0526	-1.12	0.1314	-0.62	0.2676	-0.12	0.4522
-2.61	0.0045	-2.11	0.0174	-1.61	0.0537	-1.11	0.1335	-0.61	0.2709	-0.11	0.4562
-2.60	0.0047	-2.10	0.0179	-1.60	0.0548	-1.10	0.1357	-0.60	0.2743	-0.10	0.4602
-2.59	0.0048	-2.09	0.0183	-1.59	0.0559	-1.09	0.1379	-0.59	0.2776	-0.09	0.4641
-2.58	0.0049	-2.08	0.0188	-1.58	0.0571	-1.08	0.1401	-0.58	0.2810	-0.08	0.4681
-2.57	0.0051	-2.07	0.0192	-1.57	0.0582	-1.07	0.1423	-0.57	0.2843	-0.07	0.4721
-2.56	0.0052	-2.06	0.0197	-1.56	0.0594	-1.06	0.1446	-0.56	0.2877	-0.06	0.4761
-2.55	0.0054	-2.05	0.0202	-1.55	0.0606	-1.05	0.1469	-0.55	0.2912	-0.05	0.4801
-2.54	0.0055	-2.04	0.0207	-1.54	0.0618	-1.04	0.1492	-0.54	0.2946	-0.04	0.4840
-2.53	0.0057	-2.03	0.0212	-1.53	0.0630	-1.03	0.1515	-0.53	0.2981	-0.03	0.4880
-2.52	0.0059	-2.02	0.0217	-1.52	0.0643	-1.02	0.1539	-0.52	0.3015	-0.02	0.4920
-2.51	0.0060	-2.01	0.0222	-1.51	0.0655	-1.01	0.1562	-0.51	0.3050	-0.01	0.4960
-2.50	0.0062	-2.00	0.0228	-1.50	0.0668	-1.00	0.1587	-0.50	0.3085	0.00	0.5000

# STANDARD NORMAL CUMULATIVE FUNCTION

$z > 0$



z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772	2.50	0.9938
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.01	0.9778	2.51	0.9940
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.02	0.9783	2.52	0.9941
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.03	0.9788	2.53	0.9943
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.04	0.9793	2.54	0.9945
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.05	0.9798	2.55	0.9946
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.06	0.9803	2.56	0.9948
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.07	0.9808	2.57	0.9949
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.08	0.9812	2.58	0.9951
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.09	0.9817	2.59	0.9952
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.10	0.9821	2.60	0.9953
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463	2.11	0.9826	2.61	0.9955
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474	2.12	0.9830	2.62	0.9956
0.13	0.5517	0.63	0.7357	1.13	0.8708	1.63	0.9484	2.13	0.9834	2.63	0.9957
0.14	0.5557	0.64	0.7389	1.14	0.8729	1.64	0.9495	2.14	0.9838	2.64	0.9959
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.15	0.9842	2.65	0.9960
0.16	0.5636	0.66	0.7454	1.16	0.8770	1.66	0.9515	2.16	0.9846	2.66	0.9961
0.17	0.5675	0.67	0.7486	1.17	0.8790	1.67	0.9525	2.17	0.9850	2.67	0.9962
0.18	0.5714	0.68	0.7517	1.18	0.8810	1.68	0.9535	2.18	0.9854	2.68	0.9963
0.19	0.5753	0.69	0.7549	1.19	0.8830	1.69	0.9545	2.19	0.9857	2.69	0.9964
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.20	0.9861	2.70	0.9965
0.21	0.5832	0.71	0.7611	1.21	0.8869	1.71	0.9564	2.21	0.9864	2.71	0.9966
0.22	0.5871	0.72	0.7642	1.22	0.8888	1.72	0.9573	2.22	0.9868	2.72	0.9967
0.23	0.5910	0.73	0.7673	1.23	0.8907	1.73	0.9582	2.23	0.9871	2.73	0.9968
0.24	0.5948	0.74	0.7704	1.24	0.8925	1.74	0.9591	2.24	0.9875	2.74	0.9969
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.25	0.9878	2.75	0.9970
0.26	0.6026	0.76	0.7764	1.26	0.8962	1.76	0.9608	2.26	0.9881	2.76	0.9971
0.27	0.6064	0.77	0.7794	1.27	0.8980	1.77	0.9616	2.27	0.9884	2.77	0.9972
0.28	0.6103	0.78	0.7823	1.28	0.8997	1.78	0.9625	2.28	0.9887	2.78	0.9973
0.29	0.6141	0.79	0.7852	1.29	0.9015	1.79	0.9633	2.29	0.9890	2.79	0.9974
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.30	0.9893	2.80	0.9974
0.31	0.6217	0.81	0.7910	1.31	0.9049	1.81	0.9649	2.31	0.9896	2.81	0.9975
0.32	0.6255	0.82	0.7939	1.32	0.9066	1.82	0.9656	2.32	0.9898	2.82	0.9976
0.33	0.6293	0.83	0.7967	1.33	0.9082	1.83	0.9664	2.33	0.9901	2.83	0.9977
0.34	0.6331	0.84	0.7995	1.34	0.9099	1.84	0.9671	2.34	0.9904	2.84	0.9977
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678	2.35	0.9906	2.85	0.9978
0.36	0.6406	0.86	0.8051	1.36	0.9131	1.86	0.9686	2.36	0.9909	2.86	0.9979
0.37	0.6443	0.87	0.8078	1.37	0.9147	1.87	0.9693	2.37	0.9911	2.87	0.9979
0.38	0.6480	0.88	0.8106	1.38	0.9162	1.88	0.9699	2.38	0.9913	2.88	0.9980
0.39	0.6517	0.89	0.8133	1.39	0.9177	1.89	0.9706	2.39	0.9916	2.89	0.9981
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713	2.40	0.9918	2.90	0.9981
0.41	0.6591	0.91	0.8186	1.41	0.9207	1.91	0.9719	2.41	0.9920	2.91	0.9982
0.42	0.6628	0.92	0.8212	1.42	0.9222	1.92	0.9726	2.42	0.9922	2.92	0.9982
0.43	0.6664	0.93	0.8238	1.43	0.9236	1.93	0.9732	2.43	0.9925	2.93	0.9983
0.44	0.6700	0.94	0.8264	1.44	0.9251	1.94	0.9738	2.44	0.9927	2.94	0.9984
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744	2.45	0.9929	2.95	0.9984
0.46	0.6772	0.96	0.8315	1.46	0.9279	1.96	0.9750	2.46	0.9931	2.96	0.9985
0.47	0.6808	0.97	0.8340	1.47	0.9292	1.97	0.9756	2.47	0.9932	2.97	0.9985
0.48	0.6844	0.98	0.8365	1.48	0.9306	1.98	0.9761	2.48	0.9934	2.98	0.9986
0.49	0.6879	0.99	0.8389	1.49	0.9319	1.99	0.9767	2.49	0.9936	2.99	0.9986
0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772	2.50	0.9938	3.00	0.9987