

Q7. (10 points) The random variable X follows the probability distribution below.

x_i	$P(X = x_i)$	$x \cdot P(x)$	$(x - 68.5)^2 \cdot P(x)$
1	0.50	0.5	2278.125
10	0.30	3	1026.675
100	0.15	15	148.8375
1000	0.05	50	43384.6125

(a) Evaluate $P(X = 100)$.

$$\boxed{0.15}$$

$$\mu = 68.5$$

$$\sigma^2 = 46838.25$$

$$\sigma = 216.42$$

(b) Evaluate $P(10 \leq X \leq 100)$.

$$0.3 + 0.15 = \boxed{0.45}$$

(c) Evaluate the mean of the probability distribution.

$$\mu = \boxed{68.5}$$

(d) Evaluate the standard deviation of the probability distribution.

$$\sigma = \boxed{216.42}$$

(e) Assume multiple draws are independent, where X_i is the result of the i th draw. Evaluate the probability $P(X_1 = 10 \text{ AND } X_2 = 100)$. In other words, what is the chance of drawing a 10 and then a 100?

$$(0.3)(0.15) = \boxed{0.045}$$

(f) Evaluate $P(X_1 \neq 1000 \text{ AND } X_2 \neq 1000 \text{ AND } X_3 \neq 1000)$. In other words, what is the chance of drawing thrice and getting no 1000s?

$$(0.95)^3 \approx 0.857$$

(g) Evaluate $P(X_1 = 1000 \text{ OR } X_2 = 1000 \text{ OR } X_3 = 1000)$. In other words, what is the chance of drawing thrice and getting at least one 1000?

$$1 - 0.857 = 0.143$$