Confidence intervals

- A plausible range of values for the population parameter is called a *confidence interval*.
- Using only a sample statistic to estimate a parameter is like fishing in a murky lake with a spear, and using a confidence interval is like fishing with a net.

We can throw a spear where we saw a fish but we will probably miss. If we toss a net in that area, we have a good chance of catching the fish.

- If we report a point estimate, we probably won't hit the exact population parameter. If we report a range of plausible values we have a good shot at capturing the parameter.

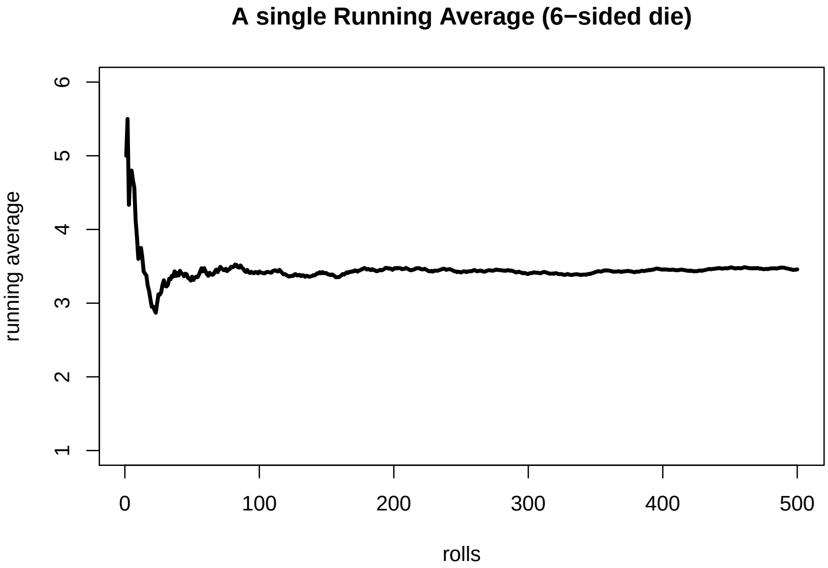
Photos by Mark Fischer (http://www.flickr.com/photos/fischerfotos/7439791462) and Chris Penny (http://www.flickr.com/photos/clearlydived/7029109617) on Flickr.

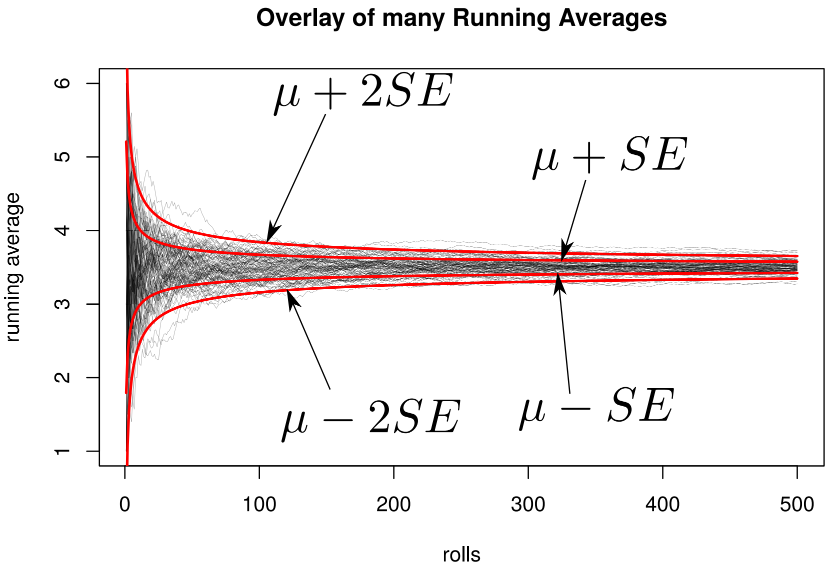
Remember **standard error**.
We have a population with a mean μ and a standard deviation σ.
We take a sample of size n and find the sample mean $\bar{x}$.
The standard error is calculated.

$$SE = \frac{\sigma}{\sqrt{n}}$$

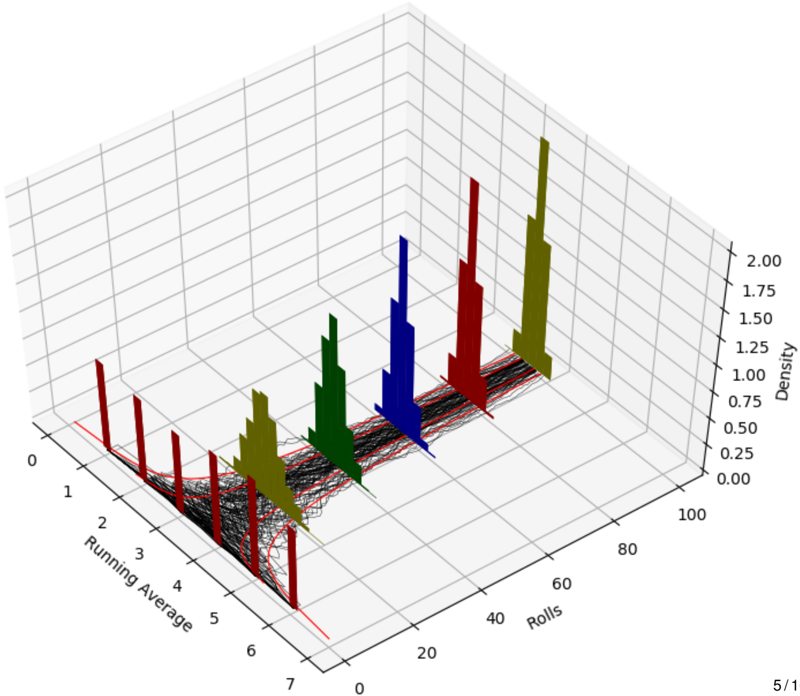
There is a 68% chance that $|\bar{x} - \mu| < SE$ and a 95% chance that $|\bar{x} - \mu| < 2SE$.
A sampling distribution (distribution of sample means) is approximately normal!

A running average



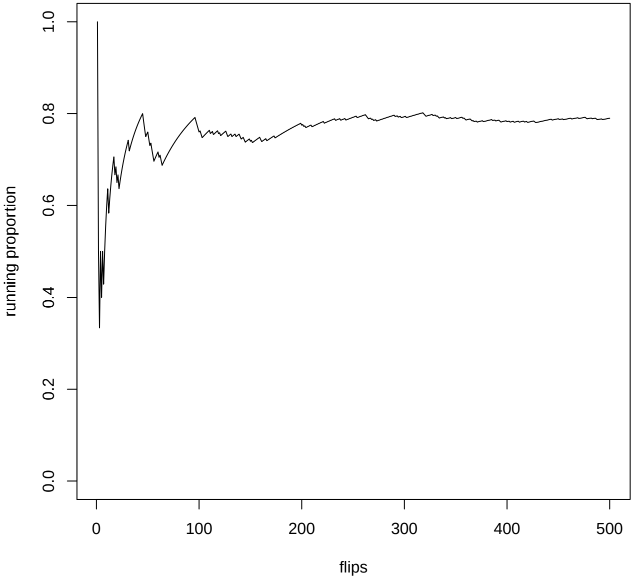


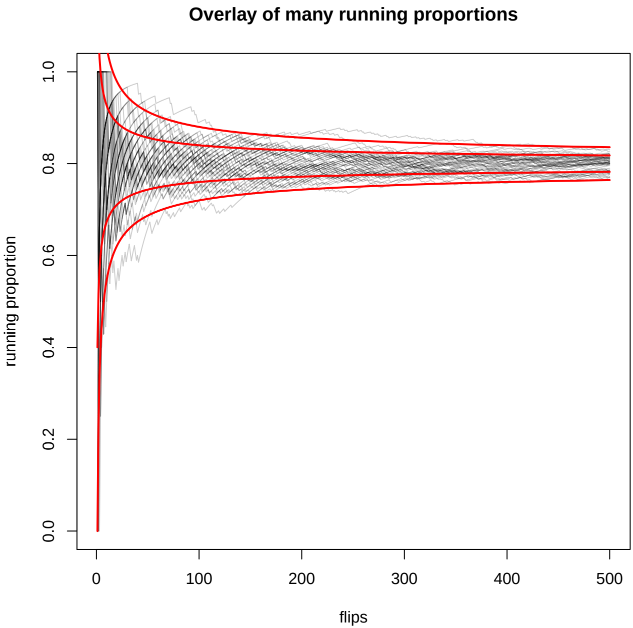
68% of data is within $\mu \pm SE$. 95% of data is between $\mu \pm 2SE$.



Running Average
0
1
2
3
4
5
6
7
Rolls
0
20
40
60
80
100
Density
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

A single running proportion




Overlay of many running proportions
running proportion
flips
0.0
0.2
0.4
0.6
0.8
1.0
0
100
200
300
400
500

Standard error in practice: confidence interval

In practice, we do not know μ or σ. We only have a single sample, which is characterized by $\bar{x}$ and s.

We estimate that $\sigma \approx s$.

We know that if we did repeat the sampling procedure many times, then 95% of the time $|\bar{x} - \mu| < 2SE$.

Thus, we say we are 95% confident that μ is within 2 SEs of $\bar{x}$.

To make frequentists happy, we do not say there is a 95% chance that μ is within 2 SEs of $\bar{x}$.

Confidence intervals only try to capture the population parameter. A confidence interval says nothing about the confidence of capturing individual observations, a proportion of the observations, or about capturing point estimates. Confidence intervals only attempt to capture population parameters.

Average number of exclusive relationships

A random sample of 50 college students was asked how many exclusive relationships they have been in so far. This sample yielded a mean of 3.2 and a standard deviation of 1.74. Estimate the true average number of exclusive relationships using this sample.

$$\bar{x} = 3.2 \qquad s = 1.74$$

The approximate 95% confidence interval is defined as

$$\textit{point estimate} \pm 2 \times SE$$

$$SE = \frac{s}{\sqrt{n}} = \frac{1.74}{\sqrt{50}} \approx 0.25$$

$$\begin{aligned} \bar{x} \pm 2 \times SE &= 3.2 \pm 2 \times 0.25 \\ &= (3.2 - 0.5, 3.2 + 0.5) \\ &= (2.7, 3.7) \end{aligned}$$

Which of the following is the correct interpretation of this confidence interval?

We are 95% confident that

(a) the average number of exclusive relationships college students in this sample have been in is between 2.7 and 3.7.

(b) college students on average have been in between 2.7 and 3.7 exclusive relationships.

(c) a randomly chosen college student has been in 2.7 to 3.7 exclusive relationships.

(d) 95% of college students have been in 2.7 to 3.7 exclusive relationships.

A more accurate interval

Confidence interval, a general formula

$$point\ estimate \pm z^{\star} \times SE$$

Conditions when the point estimate = $\bar{x}$:

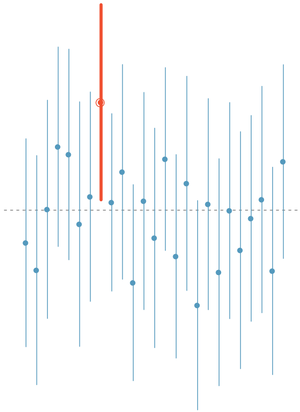
1. *Independence:* Observations in the sample must be independent
   - random sample/assignment
   - if sampling without replacement, $n <$ 10% of population
2. *Sample size / skew:* $n \geq 30$ and population distribution should not be extremely skewed

Note: We will discuss working with samples where $n < 30$ in the next chapter.

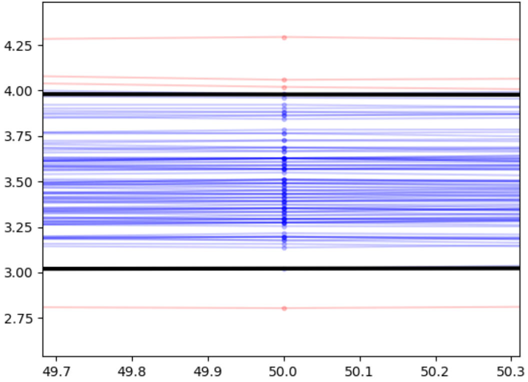
What does 95% confident mean?

- Suppose we took many samples and built a confidence interval from each sample using the equation *point estimate* $\pm 2 \times SE$.
- Then about 95% of those intervals would contain the true population mean (μ).

- The figure shows this process with 25 samples, where 24 of the resulting confidence intervals contain the true average number of exclusive relationships, and one does not.



Same idea shown from running averages view.



About 95% of the time the population mean is withing 2 SE of the sample mean.

Width of an interval

If we want to be more certain that we capture the population parameter, i.e. increase our confidence level, should we use a wider interval or a smaller interval?

Can you see any drawbacks to using a wider interval?

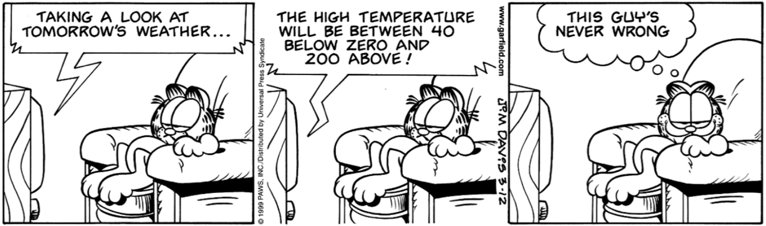


Image source: http://web.as.uky.edu/statistics/users/earo227/misc/garfield_weather.gif

Changing the confidence level

$$\textit{point estimate} \pm z^\star \times SE$$

- In a confidence interval, $z^\star \times SE$ is called the *margin of error*, and for a given sample, the margin of error changes as the confidence level changes.
- In order to change the confidence level we need to adjust $z^\star$ in the above formula.
- Commonly used confidence levels in practice are 90%, 95%, 98%, and 99%.
- For a 95% confidence interval, $z^\star = 1.96$.
- However, using the standard normal (z) distribution, it is possible to find the appropriate $z^\star$ for any confidence level.

Which of the below Z scores is the appropriate $z^{\star}$ when calculating a 98% confidence interval?

(a) $Z = 2.05$

(b) $Z = 1.96$

(c) $Z = 2.33$

(d) $Z = -2.33$

(e) $Z = -1.65$