## Sample statistics:

$n$ = sample size
$x_i$ = the $i$th value in a sample
$\bar{x}$ = sample mean
$s$ = sample standard deviation

Q1 = first quartile
$m$ = median
Q3 = third quartile
IQR = inter-quartile range

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

## Population parameters:

$\mu$ = population mean
$\sigma$ = population standard deviation

## Probability:

$\Omega$ = set of all possible equally likely outcomes
$A$ = event A, a set of outcomes
$B$ = event B, another set of outcomes
$|A|$ = size of set, number of outcomes in $A$
$P(A)$ = probability of $A$
$P(A \cap B)$ = probability of both $A$ and $B$
$P(A \cup B)$ = probability of either $A$ or $B$ (or both)
$P(A|B)$ = probability of $A$ given $B$

$$P(A) = \frac{|A|}{|\Omega|}$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$0 \leq P(A) \leq 1$

$A$, $B$ are disjoint (mutually exclusive) $\iff P(A \cap B) = 0$

$A$, $B$ are non-disjoint $\iff P(A \cap B) > 0$

$A$, $B$ are exhaustive $\iff P(A \cup B) = 1$

$A$, $B$ are complements $\iff A$, $B$ are disjoint and exhaustive $\iff B = A^c$

$A$, $B$ are independent $\iff P(A \cap B) = P(A) \times P(B) \iff P(A) = P(A|B) \iff P(B) = P(B|A)$

**Random variables and distributions:**

$X$ = random variable

$x_i$ = the $i$th possible value of $X$. (Notice different meaning here vs. sample statistics.)

$k$ = number of possible values of $X$.

$E(X) = \mu$ = expected value of $X$

$\sigma$ = standard deviation of $X$

$\mu = \sum_{i=1}^{k} x_i \cdot P(X = x_i)$

$\sigma = \sqrt{\sum_{i=1}^{k} (x_i - \mu)^2 \cdot P(X = x_i)}$

**Q1:** An urn contains 100 marbles with the following frequencies.

| | red | green | blue | total |
|---|---|---|---|---|
| striped | 11 | 12 | 2 | 25 |
| checkered | 13 | 14 | 3 | 30 |
| dotted | 7 | 9 | 29 | 45 |
| total | 31 | 35 | 34 | 100 |

**a:** What is the probability that a randomly selected marble is red?

**b:** What is the probability that a randomly selected marble is green **and** dotted?

**c:** What is the probability that a randomly selected marble is blue **or** striped?

**d:** **Given** that a randomly selected marble is checkered, what is the probability it is green?

**e:** **Given** that a randomly selected marble is green, what is the probability it is checkered?

**Q2:** American roulette involves spinning a wheel with 38 pockets. Jason is repeatedly betting on a number, such that he has a 1 in 38 chance to win each round. Each round he either loses \$1 or gains \$35. Jason figures if he plays 34 rounds, he only needs to win one round to end with more money (this is true).

**a:** What is the chance that Jason loses 34 rounds in a row?

**b:** What is the chance that Jason wins at least 1 round out of 34 rounds?

**c:** Is Jason more likely to be ahead or behind after 34 rounds?

**Q3:** Make Venn diagrams for the following events.

**a:** Two disjoint events.

**b:** Two exhaustive events that are non-disjoint.

**c:** Two events that are complements.

**d:** Two independent events.

**e:** Two dependent events that are non-disjoint.

**f:** $A$, $B$ such that $A$ implies $B$ but $B$ does not imply $A$.

**g:** $A$, $B$ such that $A$ implies $B$ and $B$ implies $A$.

**Q4:** Complete the following relative-frequency contingency table.

| | | $Y$ true | $Y$ false | total |
|---|---|---|---|---|
| $X$ | true | 0.05 | | |
| | false | | | 0.4 |
| | total | 0.2 | | 1 |

**Q5:** Complete the following relative-frequency contingency table such that $X$ and $Y$ are mutually exclusive and exhaustive.

| | | $Y$ true | $Y$ false | total |
|---|---|---|---|---|
| $X$ | true | | | |
| | false | 0.3 | | |
| | total | | | 1 |

**Q6:** Complete the following relative-frequency contingency table such that $X$ and $Y$ are independent.

| | | $Y$ true | $Y$ false | total |
|---|---|---|---|---|
| $X$ | true | 0.28 | | |
| | false | | | |
| | total | 0.7 | | 1 |

**Q7:** Complete the following relative-frequency contingency table such that $P(Y) = 0.2$ and $P(X|Y) = 0.8$ and $P(X|Y^c) = 0.5$.

| | | $Y$ true | $Y$ false | total |
|---|---|---|---|---|
| $X$ | true | | | |
| | false | | | |
| | total | | | 1 |

**Q8:** Liam's friend, Owen, tells Liam that he has a biased coin that will land heads with probability 0.70. Liam is not sure he trusts Owen, because Owen sometimes lies and Liam can't imagine how a biased coin would work. Thus, Liam feels the likelihood that Owen is telling the truth is 0.10. Liam asks to see a flip of the coin, and feels the following contingency table explains his expectations.

| | | Coin is biased ($B$) | | |
|---|---|---|---|---|
| | | true | false | total |
| Heads ($H$) | true | 0.07 | 0.45 | 0.52 |
| | false | 0.03 | 0.45 | 0.48 |
| | total | 0.1 | 0.9 | 1 |

**a:** If the coin is biased, what is the likelihood it lands heads?

$$P(H|B) =$$

**b:** If the coin is not biased, what is the likelihood it lands heads?

$$P(H|B^c) =$$

**c:** If the coin lands heads, how likely is the coin biased (based on Liam's prior beliefs)?

$$P(B|H) =$$

**d:** After one flip lands heads, Liam adjusts his expectations for the next flip.

| | | Coin is biased ($B$) | | |
|---|---|---|---|---|
| | | true | false | total |
| Heads ($H$) | true | 0.091 | 0.435 | 0.526 |
| | false | 0.039 | 0.435 | 0.474 |
| | total | 0.13 | 0.87 | 1 |

Now, if this next flip is also heads, what is the likelihood the coin is biased?

$$P(B|H) =$$