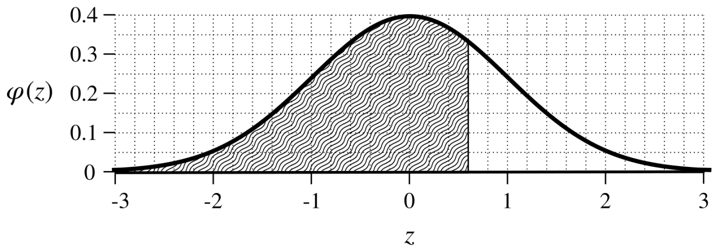
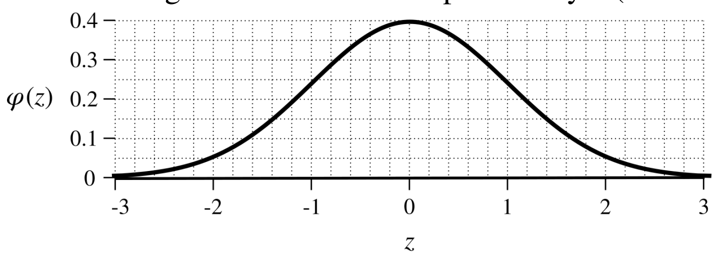
Q1: For each of the following, complete the diagram so it has a shaded region and a probability statement, like in the example below.



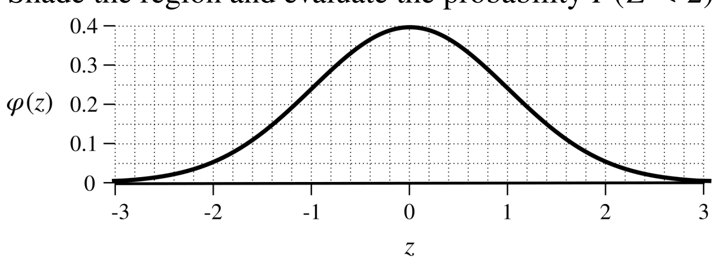
$P(Z < 0.6) = 0.7257$

a: Shade the region and evaluate the probability $P(Z < -1.4)$.



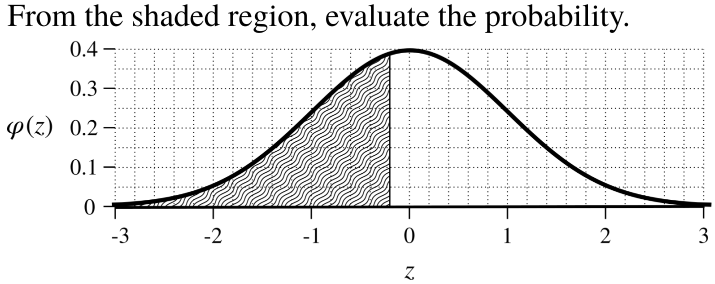
$P(Z < -1.4) =$

b: Shade the region and evaluate the probability $P(Z < 2)$.



$P(Z < 2) =$

c: From the shaded region, evaluate the probability.



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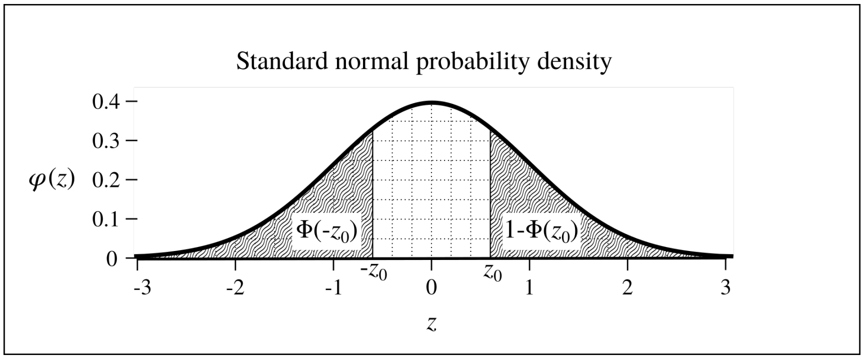
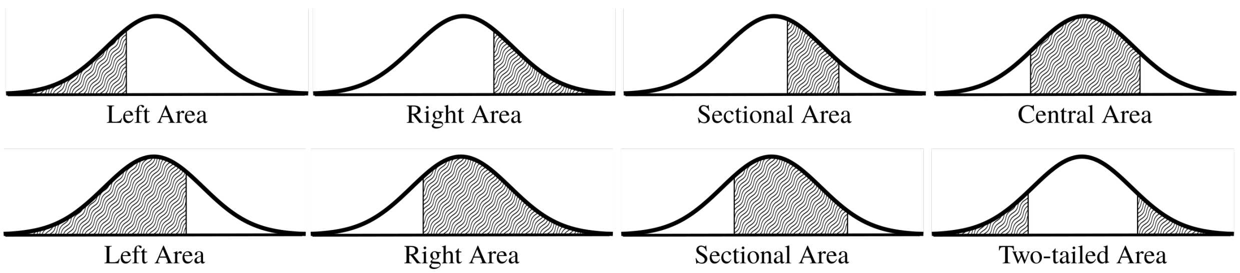
The area under $\varphi(z)$ from $-\infty$ to ∞ is 1. Also, the function $\varphi(z)$ is symmetric. This leads to a useful property:

$$\Phi(-z) = 1 - \Phi(z)$$



There are five common areas we are asked to find: left, right, sectional, central (symmetric), and two-tailed (symmetric).



$$\begin{aligned}\texttt{Left area} &= P(Z < z)\\ &= \Phi(z)\end{aligned}$$

$$\begin{aligned}\texttt{Right area} &= P(Z > z)\\ &= 1 - \Phi(z)\\ &= \Phi(-z)\end{aligned}$$

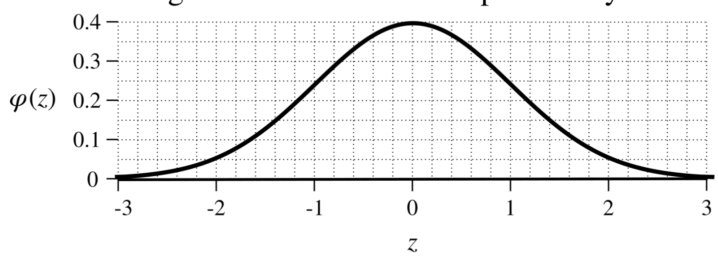
$$\begin{aligned}\texttt{Sectional area} &= P(z_1 < Z < z_2)\\ &= \Phi(z_2) - \Phi(z_1)\end{aligned}$$

$$\begin{aligned}\texttt{Central area} &= P(|Z| < z)\\ &= \Phi(z) - \Phi(-z)\\ &= 1 - 2\Phi(-z)\\ &= 2\Phi(z) - 1\end{aligned}$$

$$\begin{aligned}\texttt{Two-tailed area} &= P(|Z| > z)\\ &= 1 - \Phi(z) + \Phi(-z)\\ &= 2 - 2\Phi(z)\\ &= 2\Phi(-z)\end{aligned}$$

Q2: For each of the following, complete the diagram so it has a shaded region and a probability statement. Also, notice that you can estimate the probability by counting the number of shaded squares; each unit square is worth 1%.

a: Shade the region of and evaluate the probability that Z is more than 1.6.



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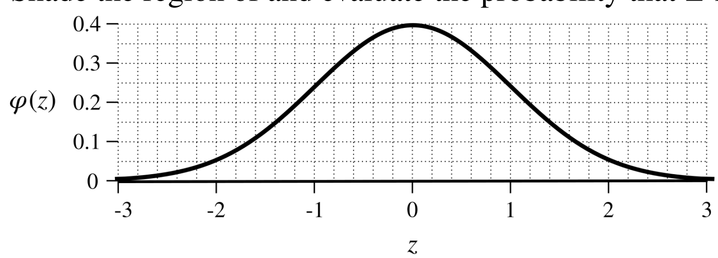
b: Shade the region of and evaluate the probability that Z is between 0.4 and 0.6.



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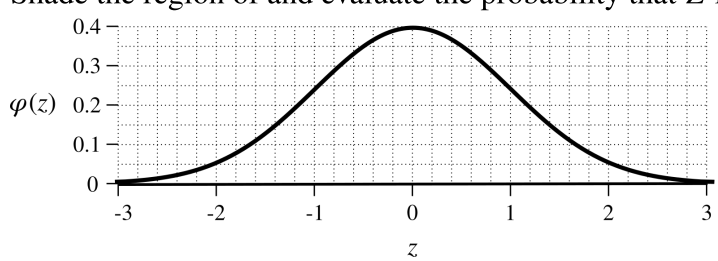
c: Shade the region of and evaluate the probability that Z is between 1 and 2.



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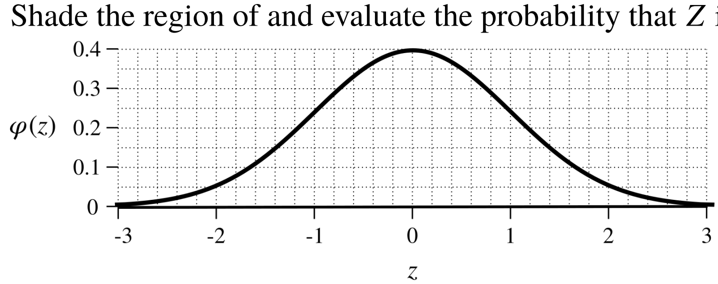
d: Shade the region of and evaluate the probability that Z is between -0.4 and 0.4.



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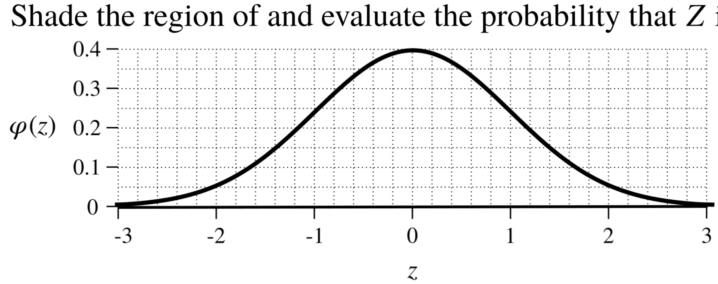
e: Shade the region of and evaluate the probability that Z is less than -0.4 or more than 0.4.



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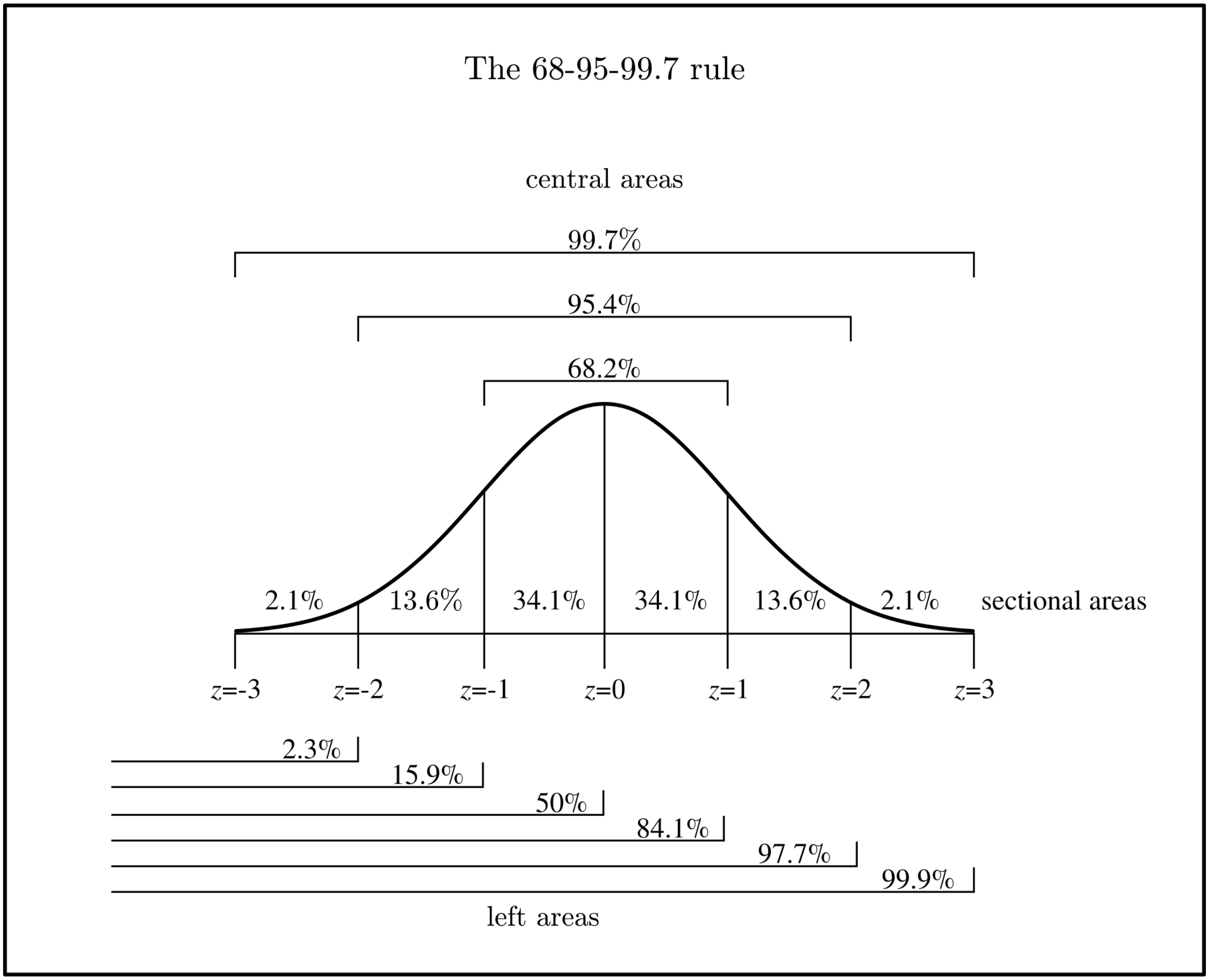
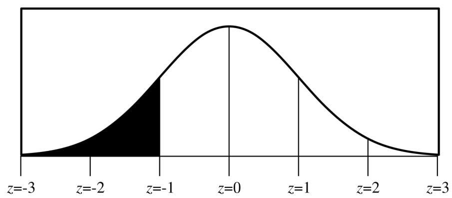
This diagram might be useful. Some of the areas seem to add imperfectly because these numbers are all rounded. Also, it should be noted that $\Phi(-3) = 0.00135 \neq 0$.



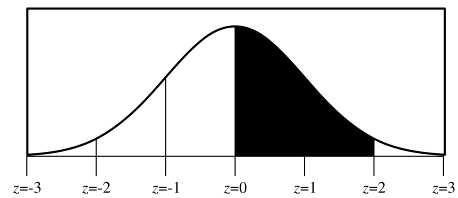
https://en.wikipedia.org/wiki/68-95-99.7_rule

Q3: By using the standard normal table (or the 68-95-99.7 rule), you should be able to determine the following probabilities. For each question, determine the probability (area) of the shaded region or regions. In cases where the bound could be -3 or 3, use $-\infty$ or ∞ instead. Write the answer using the "$P(\texttt{condition}) = \texttt{number}$" format.

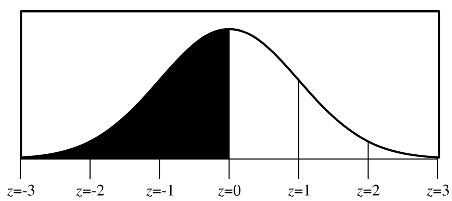
a:



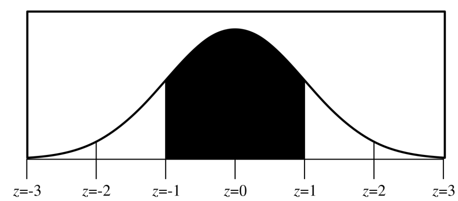
f:



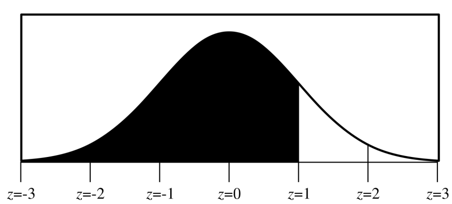
b:



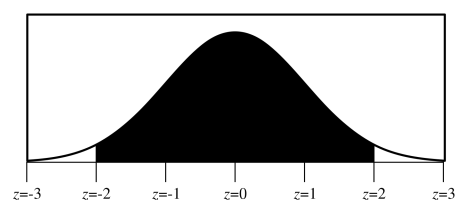
g:



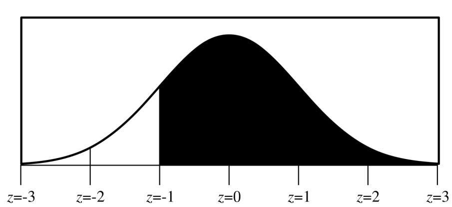
c:



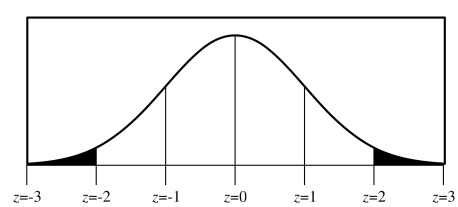
h:



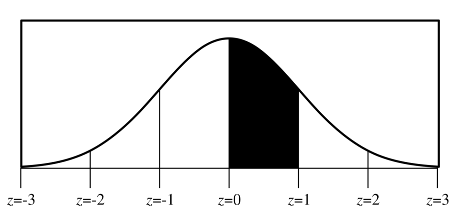
d:



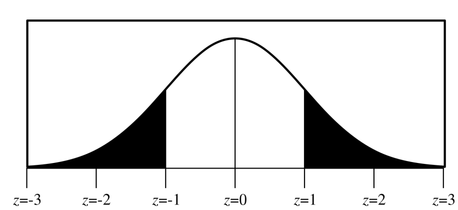
i:



e:



j:



We have practiced finding areas from z-scores. We might also want to find z-scores from areas. You'll need to use your standard normal table backwards.

Q4: **a:** Determine z_0 such that $\Phi(z_0) = 0.0505$.

b: Determine z_1 such that $\Phi(z_1) \approx 0.99$.

c: Determine z_2 such that $P(Z < z_2) = 55.57\%$

d: Determine z_3 such that $P(Z > z_3) = 15.87\%$

e: Determine z_4 such that $P(-z_4 < Z < z_4) = 68.2\%$

f: Determine z_5 such that $P(|Z| < z_5) = 95\%$

g: Determine z_6 such that $P(|Z| < z_6) = 90\%$

h: Determine z_7 such that $P(|Z| > z_7) = 10\%$

Q5: If the scores on a test are normally distributed with a mean of 80 and a standard deviation of 10, what score is the 84.1th percentile? (Hint: check out the 68-95-99.7 rule.)

Q6: If the scores on a test are normally distributed with a mean of 80 and a standard deviation of 10, what score is the 97.7th percentile?

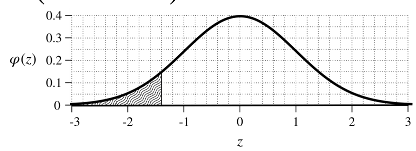
Q7: If the scores on a test are normally distributed with a mean of 80 and a standard deviation of 10, what score is the 90th percentile?

Q8: What is the z-score such that 68.2% of the area lies between $-z$ and z? (Hint: check out the 68-95-99.7 rule.)

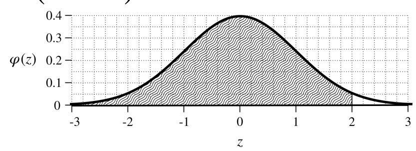
Q9: What is the z-score such that 95.4% of the area lies between $-z$ and z?

Q10: What is the z-score such that 80% of the area lies between $-z$ and z?

A1: **a:** $P(Z < -1.4) = 0.0808$

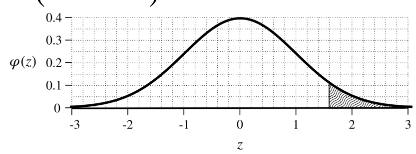


b: $P(Z < 2) = 0.9772$

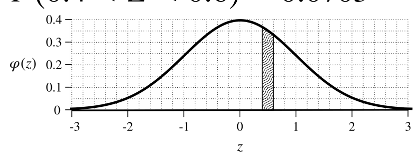


c: $P(Z < -0.2) = 0.4207$

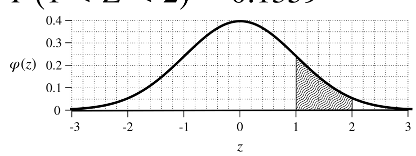
A2: **a:** $P(Z > 1.6) = 0.0548$



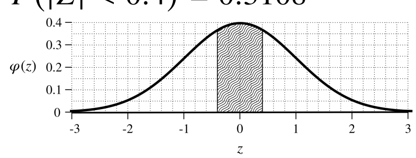
b: $P(0.4 < Z < 0.6) = 0.0703$



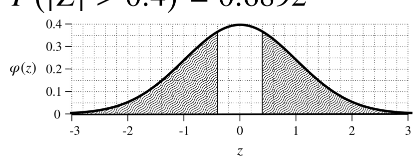
c: $P(1 < Z < 2) = 0.1359$



d: $P(|Z| < 0.4) = 0.3108$



e: $P(|Z| > 0.4) = 0.6892$



A3: **a:** $P(Z < -1) = 0.159$

b: $P(Z < 0) = 0.5$

c: $P(Z < 1) = 0.841$

d: $P(-1 < Z) = 0.841$

e: $P(0 < Z < 1) = 0.341$

f: $P(0 < Z < 2) = 0.477$

g: $P(|Z| < 1) = 0.682$

h: $P(|Z| < 2) = 0.954$

i: $P(|Z| > 2) = 0.046$

j: $P(|Z| > 1) = 0.318$

A4: **a:** $z_0 = -1.64$

b: $z_1 = 2.33$

c: $z_2 = 0.14$

d: $z_3 = 1$

e: $z_4 = 1$

f: $z_5 = 1.96$

g: $z_6 = 1.64$

h: $z_7 = 1.64$

A5: 90.0

A6: 100.0

A7: $z = 1.28$
$(1.28)(10) + 80 \approx \boxed{92.8}$

A8: $z = 1$

A9: $z = 2$

A10: $z = 1.28$