Sample statistics:

n = sample size
x_i = the ith value in a sample
$\bar{x}$ = sample mean
s = sample standard deviation
Q1 = first quartile
m = median
Q3 = third quartile
IQR = inter-quartile range

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Population parameters:

μ = population mean
σ = population standard deviation

Probability:

Ω = set of all possible equally likely outcomes
A = event A, a set of outcomes
B = event B, another set of outcomes
$|A|$ = size of set, number of outcomes in A
$P(A)$ = probability of A
$P(A \cap B)$ = probability of both A and B
$P(A \cup B)$ = probability of either A or B (or both)
$P(A|B)$ = probability of A given B

$$P(A) = \frac{|A|}{|\Omega|}$$

$P(A \cap B) = P(A) \cdot P(B|A)$
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0 \le P(A) \le 1$
A, B are disjoint (mutually exclusive) $\iff$ $P(A \cap B) = 0$
A, B are non-disjoint $\iff$ $P(A \cap B) > 0$
A, B are exhaustive $\iff$ $P(A \cup B) = 1$
A, B are complements $\iff$ A, B are disjoint and exhaustive $\iff$ $B = A^c$
A, B are independent $\iff$ $P(A \cap B) = P(A) \times P(B)$ $\iff$ $P(A) = P(A|B)$ $\iff$ $P(B) = P(B|A)$

Random variables and distributions:

X = random variable
x_i = the ith possible value of X. (Notice different meaning here vs. sample statistics.)
k = number of possible values of X.
$E(X) = \mu$ = expected value of X
σ = standard deviation of X
$\mu = \sum_{i=1}^{k} x_i \cdot P(X = x_i)$
$\sigma = \sqrt{\sum_{i=1}^{k}(x_i - \mu)^2 \cdot P(X = x_i)}$

Q1: An urn contains 100 marbles with the following frequencies.

	red	green	blue	total
striped	11	12	2	25
checkered	13	14	3	30
dotted	7	9	29	45
total	31	35	34	100

a: What is the probability that a randomly selected marble is red?

b: What is the probability that a randomly selected marble is green **and** dotted?

c: What is the probability that a randomly selected marble is blue **or** striped?

d: **Given** that a randomly selected marble is checkered, what is the probability it is green?

e: **Given** that a randomly selected marble is green, what is the probability it is checkered?

Q2: American roulette involves spinning a wheel with 38 pockets. Jason is repeatedly betting on a number, such that he has a 1 in 38 chance to win each round. Each round he either loses \$1 or gains \$35. Jason figures if he plays 34 rounds, he only needs to win one round to end with more money (this is true).

a: What is the chance that Jason loses 34 rounds in a row?

b: What is the chance that Jason wins at least 1 round out of 34 rounds?

c: Is Jason more likely to be ahead or behind after 34 rounds?

Q3: Make Venn diagrams for the following events.

a: Two disjoint events.

b: Two exhaustive events that are non-disjoint.

c: Two events that are complements.

d: Two independent events.

e: Two dependent events that are non-disjoint.

f: A, B such that A implies B but B does not imply A.

g: A, B such that A implies B and B implies A.

Q4: Complete the following relative-frequency contingency table.

		Y true	Y false	total
X	true	0.05		
	false			0.4
	total	0.2		1

Q5: Complete the following relative-frequency contingency table such that X and Y are mutually exclusive and exhaustive.

		Y true	Y false	total
X	true			
	false	0.3		
	total			1

Q6: Complete the following relative-frequency contingency table such that X and Y are independent.

		Y true	Y false	total
X	true	0.28		
	false			
	total	0.7		1

Q7: Complete the following relative-frequency contingency table such that $P(Y) = 0.2$ and $P(X|Y) = 0.8$ and $P(X|Y^c) = 0.5$.

		Y true	Y false	total
X	true			
	false			
	total			1

Q8: Liam's friend, Owen, tells Liam that he has a biased coin that will land heads with probability 0.70. Liam is not sure he trusts Owen, because Owen sometimes lies and Liam can't imagine how a biased coin would work. Thus, Liam feels the likelihood that Owen is telling the truth is 0.10. Liam asks to see a flip of the coin, and feels the following contingency table explains his expectations.

		Coin is biased (B)		
		true	false	total
Heads (H)	true	0.07	0.45	0.52
	false	0.03	0.45	0.48
	total	0.1	0.9	1

a: If the coin is biased, what is the likelihood it lands heads?

$$P(H|B) =$$

b: If the coin is not biased, what is the likelihood it lands heads?

$$P(H|B^c) =$$

c: If the coin lands heads, how likely is the coin biased (based on Liam's prior beliefs)?

$$P(B|H) =$$

d: After one flip lands heads, Liam adjusts his expectations for the next flip.

		Coin is biased (B)		
		true	false	total
Heads (H)	true	0.091	0.435	0.526
	false	0.039	0.435	0.474
	total	0.13	0.87	1

Now, if this next flip is also heads, what is the likelihood the coin is biased?

$$P(B|H) =$$