

MATH 181 FIRST EXAM

SPRING 2019

Name: _____

- Write your **full name** on the line above.
- Show your work. Incorrect answers with work can receive partial credit.
- Attempt every question; showing you understand the question earns some credit.
- If you run out of room for an answer, continue on the back of the page. Before doing so, write “see back” with a circle around it.
- You can use 1 page (front and back) of notes.
- You can use (and probably need) a calculator.
- You can use the Geogebra Scientific Calculator instead of a calculator. You need to put your phone on **airplane mode** and then within the application, start **exam mode**; you should see a green bar with a timer counting up.
- If a question is confusing or ambiguous, please ask for clarification; however, you will not be told how to answer the question.
- **Box your final answer.**
- A formula sheet is attached to this test.

Do not write in this grade table.

Question:	1	2	3	4	5	6	Total
Points:	10	8	8	16	9	9	60
Score:							

Sample statistics: n = sample size x_i = the i th value in a sample \bar{x} = sample mean s = sample standard deviation

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

 Q_1 = first quartile m = median Q_3 = third quartileIQR = inter-quartile range = $Q_3 - Q_1$

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

Population parameters: μ = population mean σ = population standard deviation**Probability:** Ω = set of all possible equally likely outcomes A = event A , a set of outcomes A^c = The complement of A B = event B , another set of outcomes $\#(A)$ = size of set, number of outcomes in A $P(A)$ = probability of A $P(A \text{ AND } B)$ = probability of both A and B $P(A \text{ OR } B)$ = probability of either A or B (or both) $P(A|B)$ = probability of A given B \iff = "if and only if"

$$P(A) = \frac{\#(A)}{\#(\Omega)}$$

$$0 \leq P(A) \leq 1$$

$$P(A \text{ AND } B) = P(A) \cdot P(B|A)$$

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$$

$$P(A|B) = \frac{P(A \text{ AND } B)}{P(B)}$$

$$P(A^c) = 1 - P(A)$$

 A, B are disjoint (mutually exclusive) $\iff P(A \text{ AND } B) = 0$ A, B are non-disjoint $\iff P(A \text{ AND } B) > 0$ A, B are exhaustive $\iff P(A \text{ OR } B) = 1$ A, B are complements $\iff A, B$ are disjoint and exhaustive $\iff B = A^c$ A, B are independent $\iff P(A \text{ AND } B) = P(A) \times P(B) \iff P(A|B) = P(A)$ **Random variables and distributions:** X = random variable x_i = the i th possible value of X . (Notice different meaning here vs. sample statistics.) k = number of possible values of X . $E(X) = \mu$ = expected value of X σ = standard deviation of X

$$\mu = \sum_{i=1}^k x_i \cdot P(X = x_i)$$

$$\sigma = \sqrt{\sum_{i=1}^k (x_i - \mu)^2 \cdot P(X = x_i)}$$

1. (10 points) Samuel suspects that coffee impairs short-term memory. Samuel runs a study by asking random BHCC students to participate in a memory challenge: repeating back 7 random digits. Samuel marks whether the participant successfully repeated the digits. Then, Samuel asks the participant whether they had coffee in the last 3 hours. The results are summarized below.

	success	fail	total
coffee	16	7	23
no coffee	11	4	15
total	27	11	38

(a) What kind of study was this?

- ☐ experimental
☐ observational

(b) Which group performed better (had a higher proportion of success)?

- ☐ coffee
☐ no coffee

(c) Which hypothesis is the null hypothesis?

- ☐ The difference in proportions is due to chance.
☐ The difference in proportions is due to an association between coffee and memory.

(d) Which hypothesis is the alternative hypothesis?

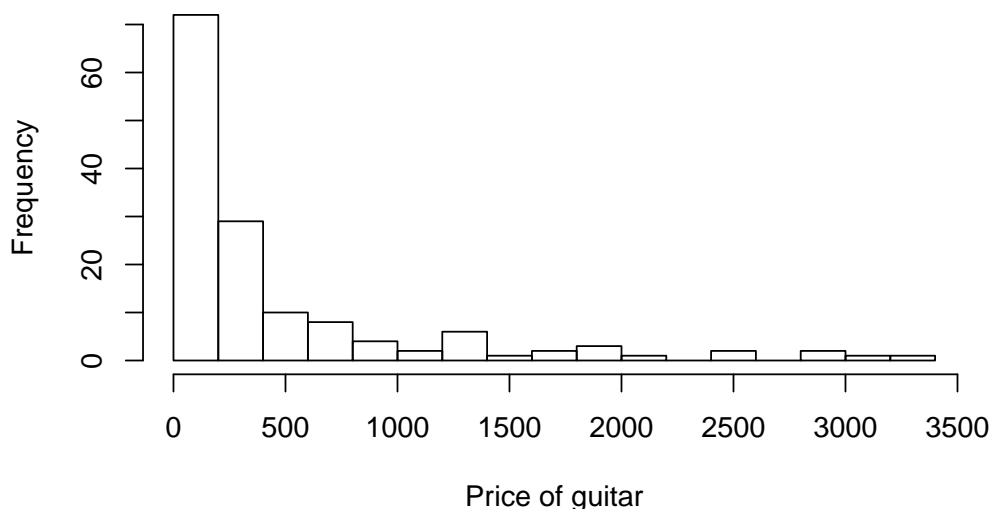
- ☐ The difference in proportions is due to chance.
☐ The difference in proportions is due to an association between coffee and memory.

(e) Would you reject the null hypothesis? Why?

- ☐ Yes. The difference seems too large for chance.
☐ No. The difference seems small enough to be just from chance.

2. (8 points) When “acoustic guitar” was searched on craigslist, there were 144 local postings that included a price. These prices are displayed as a histogram.

Histogram of guitar prices on craigslist

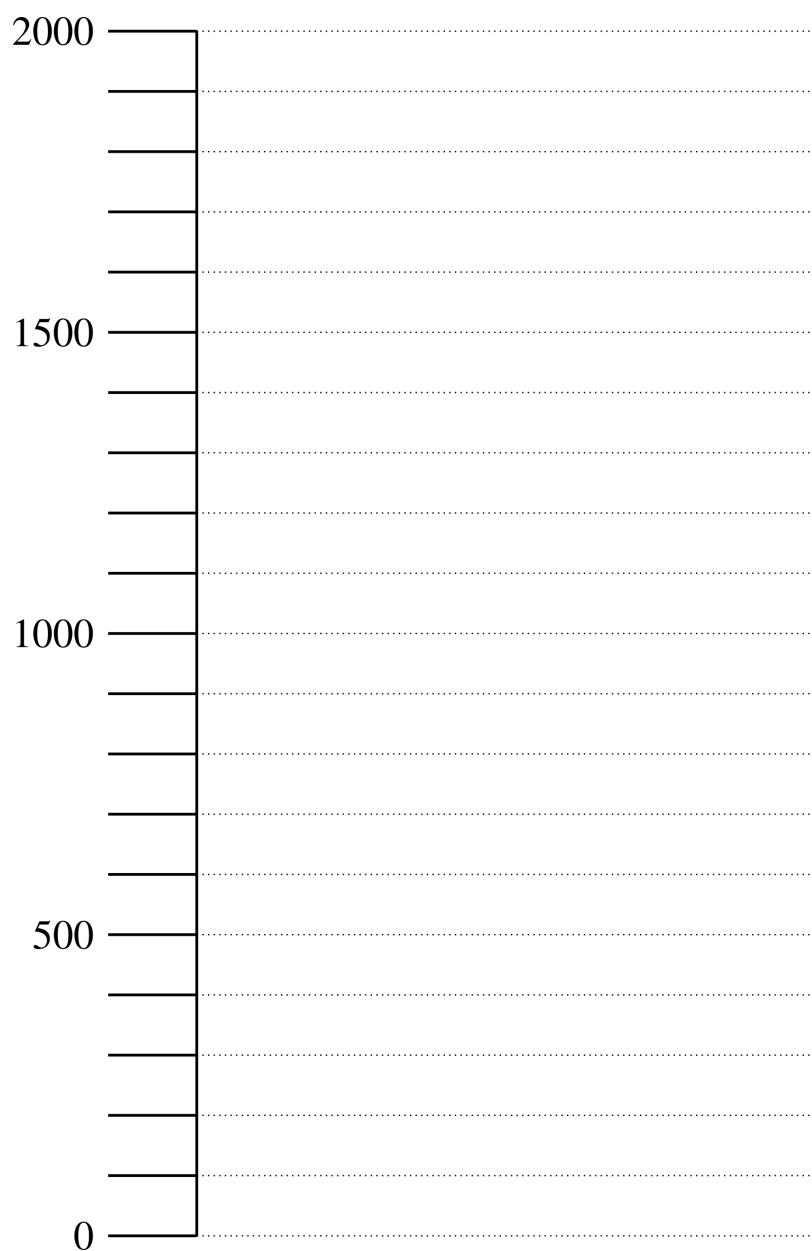


- (a) Which of the following would be an appropriate estimate of the median?
- ☐ \$1
 - ☐ \$200
 - ☐ \$500
 - ☐ \$3500
- (b) Which of the following would be an appropriate estimate of the mean?
- ☐ \$1
 - ☐ \$200
 - ☐ \$500
 - ☐ \$3500
- (c) Which of the following would be an appropriate estimate of the standard deviation?
- ☐ \$1
 - ☐ \$20
 - ☐ \$700
 - ☐ \$3500
- (d) Which option best describes this histogram?
- ☐ Skew-left
 - ☐ Skew-right
 - ☐ Superstitious
 - ☐ Symmetric

3. (8 points) From the guitar prices, a random sample of size 15 was taken. Those 15 prices are listed below.

10 20 60 60 75 85 125 150 220 250 275 700 1395 1800 2000

Make a boxplot summarizing these data. Be sure to indicate Q_1 , Q_3 , median, outliers, and ends of whiskers. The axis below is meant to help you.



4. (16 points) A jar contains 99 marbles. Each marble has a color and a pattern. The frequencies are shown in the contingency table.

	red	green	blue	total
dotted	7	8	9	24
striped	10	11	12	33
checkered	13	14	15	42
total	30	33	36	99

- (a) What is the probability that a random marble is green?
- (b) What is the probability that a random marble is checkered?
- (c) What is the probability that a random marble is either striped **or** green (or both)?
- (d) What is the probability that a random marble is both red **and** dotted?
- (e) What is the probability that a random marble is red **given** it is checkered?
- (f) What is the probability that a random marble is striped **given** it is blue?
- (g) When picking one random marble, which two events are disjoint (mutually exclusive)?
- ☐ red, checkered
 - ☐ green, striped
 - ☐ blue, red
- (h) When picking one random marble, which two events are independent?
- ☐ red, checkered
 - ☐ green, striped
 - ☐ blue, red

5. (9 points) Let random variable X represent the number of tails showing when four fair coins are flipped. The probability distribution of X is shown below, where x_i represents the i th possible value of X .

x_i	$P(X = x_i)$
0	0.0625
1	0.25
2	0.375
3	0.25
4	0.0625

(a) What is the probability of 2 tails? In other words, evaluate $P(X = 2)$.

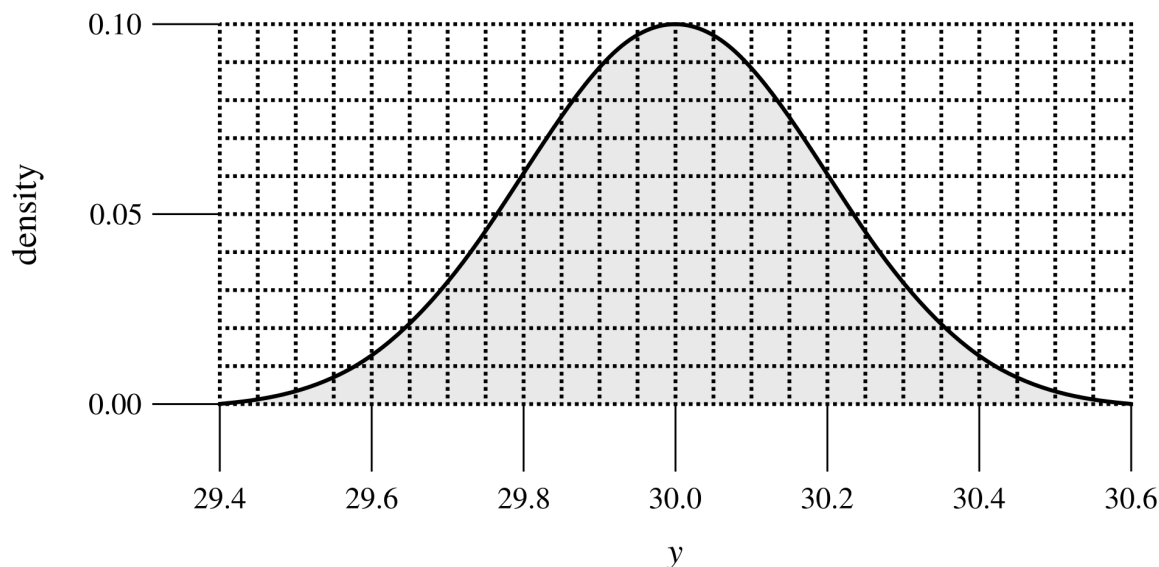
(b) What is the probability of at least 2 tails? In other words, evaluate $P(X \geq 2)$.

(c) What is the probability of more than 2 tails? In other words, evaluate $P(X > 2)$.

(d) (2 points (bonus)) Determine x such that $P(X < x) = 0.9375$ and $P(X > x) = 0$.

6. (9 points) A machine cuts rods to 30 centimeters. However, the machine is not perfect, so the actual lengths have variability.

Let the continuous random variable Y represent the length of a rod. An engineer determines Y approximately follows the distribution shown by the density function below. The entire area under the curve is 100%, and each square is worth 1%.



- (a) Estimate the probability a rod is cut to exactly 30.2 centimeters.
In other words, estimate $P(Y = 30.2)$.
- (b) Estimate the probability a rod is cut to a length between 30.2 centimeters and 30.4 centimeters?
In other words, evaluate $P(30.2 < Y < 30.4)$.
- (c) Estimate Q_1 , the 25th percentile. Answers within ± 0.02 will count.
In other words, estimate Q_1 such that $P(Y < Q_1) = 0.25$.