We will use the notation $X \sim \mathcal{B}(n,\ p)$ to say X is binomially distributed with n trials and chance of success p (on each trial).

3.31: In all cases, the sides are all equally likely, so we can just say 1 in 4 sides corresponds to "success", but which side is "success" depends on the case. So, we can just define random variable $X \sim \mathcal{B}(3,\ 0.25)$, where X is the number of successes. We make a table, where we use k to represent specific (possible) outcomes.

k	$P(X = k)$ before simplification	$P(X = k)$
0	$\binom{3}{0}(0.25)^0(0.75)^3$	0.421875
1	$\binom{3}{1}(0.25)^1(0.75)^2$	0.421875
2	$\binom{3}{2}(0.25)^2(0.75)^1$	0.140625
3	$\binom{3}{3}(0.25)^3(0.75)^0$	0.015625

(a): Reread as "at least one success". This is the complement of "no successes".

$$P(X \geq 1) \quad = \quad 1 - P(X = 0) \quad = \quad 1 - 0.422 \quad = \quad \boxed{0.578}$$

(b): Reread as "exactly two successes", and look at the table.

$$P(X = 2) \quad = \quad \boxed{0.141}$$

(c): Reread as "exactly one success", and look at the table.

$$P(X = 1) \quad = \quad \boxed{0.422}$$

(d): Reread as "at most two success". This is the complement of "exactly three successes".

$$P(X \leq 2) \quad = \quad 1 - P(X = 3) \quad = \quad 1 - 0.0156 \quad = \quad \boxed{0.9844}$$

3.32: Let $X \sim \mathcal{N}(10,\ 0.07)$, where X represents the number of teenagers suffering from arachnophobia.

(a): We want to calculate $P(X \geq 1)$. This is the complement of $P(X = 0)$.

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - \binom{10}{0}(0.07)^0(0.93)^{10} \\ &= \boxed{0.516} \end{aligned}$$

(b): We want to calculate $P(X = 2)$.

$$\begin{aligned} P(X = 2) &= \binom{10}{2}(0.07)^2(0.93)^8 \\ &= \boxed{0.123} \end{aligned}$$

(c): We want to calculate $P(X \leq 1)$.

$$\begin{aligned} P(X \leq 1) &= \binom{10}{0}(0.07)^0(0.93)^{10} + \binom{10}{1}(0.07)^1(0.93)^9 \\ &= \boxed{0.848} \end{aligned}$$

(d): No. There is a 15% chance that, in the tent, more than 1 teenager is afraid of spiders.

3.33: **(a):** $0.125 \times (1 - 0.125) = \boxed{0.109}$

(b): $\binom{2}{1}(0.125)^1(1 - 0.125)^1 = \boxed{0.219}$

(c): $\binom{6}{2}(0.125)^2(1 - 0.125)^4 = \boxed{0.137}$

(d): Complement. $1 - \binom{6}{0}(0.125)^0(1 - 0.125)^6 = \boxed{0.551}$

(e): Geometric. $(1 - 0.125)^3(0.125) = \boxed{0.0837}$

(f): We can calculate a z score. First we need μ and σ of the binomial distribution $\mathcal{B}(6,\ 0.75)$.

$$\mu = (6)(0.75) = 4.5$$

$$\sigma = \sqrt{(6)(0.75)(0.25)} = 1.06$$

$$z = \frac{2 - 4.5}{1.06} = -2.36$$

This z score is considered unusual because $|-2.36| > 2$.

We could also calculate the probability of having 2 **or fewer** children with brown eyes.

$$\begin{aligned} P(X \leq 2) &= \binom{6}{0}(0.75)^0(0.25)^6 + \binom{6}{1}(0.75)^1(0.25)^5 + \binom{6}{2}(0.75)^2(0.25)^4 \\ &= 0.0376 \end{aligned}$$

So, having 2 **or fewer** kids with brown eyes only happens about 4% of the time. This is low enough to be unusual.

3.34: **(a):** Let $X_a \sim \mathcal{B}(3,\ 0.25)$. We are asked for $P(X_a = 2)$.

$$P(X_a = 2) = \binom{3}{2}(0.25)^2(0.75)^1 = \boxed{0.14}$$

(b): Let $X_b \sim \mathcal{B}(3,\ 0.25)$. We are asked for $P(X_b = 0)$.

$$P(X_b = 0) = \binom{3}{0}(0.25)^0(0.75)^3 = \boxed{0.42}$$

(c): $X_c \sim \mathcal{B}(3,\ 0.25)$.

$$\begin{aligned} P(X_c \geq 1) &= 1 - P(X_c = 0) \\ &= 1 - \binom{3}{0}(0.25)^0(0.75)^3 \\ &= \boxed{0.578} \end{aligned}$$

(d): Geometric.

$$(1 - 0.25)^2(0.25) = \boxed{0.14}$$

3.35: Let X represent the number of games won. $X \sim \mathcal{B}(3,\ 18/38)$. We use k to represent possible values of X.

k	$P(X = k)$ before simplification	$P(X = k)$
0	$\binom{3}{0}(18/38)^0(20/38)^3$	0.1457938
1	$\binom{3}{1}(18/38)^1(20/38)^2$	0.3936434
2	$\binom{3}{2}(18/38)^2(20/38)^1$	0.3542790
3	$\binom{3}{3}(18/38)^3(20/38)^0$	0.1062837

The above probabilities are used for the distribution of Y. Let's use y to represent possible values of Y (where USD means \$). If the player loses three times, they will lose \$3. If someone loses twice but wins once, they net -1 USD... etc...

y	$P(Y = y)$
-3 USD	0.1457938
-1 USD	0.3936434
1 USD	0.3542790
3 USD	0.1062837

3.36: **(a):** Geometric. $P(3) = (0.75)^2(0.25) = 0.140625$

(b): Binomial.

$$P(3 \text{ OR } 4) = \binom{5}{3}(0.25)^3(0.75)^2 + \binom{5}{4}(0.25)^4(0.75)^1 = \boxed{0.1025}$$

(c): Binomial.

$$P(3 \text{ OR } 4 \text{ OR } 5) = \binom{5}{3}(0.25)^3(0.75)^2 + \binom{5}{4}(0.25)^4(0.75)^1 + \binom{5}{5}(0.25)^5(0.75)^0 = \boxed{0.1035}$$

3.37: **(a):** We have 5 **dependent** events connected with logical AND, so we multiply.

$$\begin{aligned} P(A_1B_2C_3D_4E_5) &= P(A_1) \cdot P(B_2|A_1) \cdot P(C_3|A_1B_2) \cdot P(D_4|A_1B_2C_3) \cdot P(E_5|A_1B_2C_3D_4) \\ &= \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} \\ &= \frac{1}{5!} \\ &= 1/120 \\ &= \boxed{0.008333} \end{aligned}$$

(b): If each arrangement is equally likely, and the probability of alphabetical arrangement is $1/120$, then there must be $\boxed{120}$ arrangements possible.

(c): $8! = 40320$

3.38: **(a):** Let $X \sim \mathcal{B}(3,\ 0.51)$.

$$P(X = 2) \quad = \quad \binom{3}{2}(0.51)^2(0.49)^1 \quad = \quad \boxed{0.3823}$$

(b): bbg bgb gbb. Three ways, each of which has probability of $(0.51)^2(0.49)^1$. Then, multiply by three (add 3 copies of the probability). $\boxed{0.3823}$

(c): Because 8 choose 3 is 56... there are 56 different ways.