t-table practice

- If $df = 12$, estimate $P(T < -3.2)$.
- If $df = 12$, determine t such that $P(T > t) = 0.99$.
- If $df = 18$, determine $t^\star$ of a 95% confidence interval.
- If a two-tail hypothesis test has a significance level of 0.05 and a sample size $n = 10$, what is the critical value $t^\star$?

t-table practice

- If the alternative hypothesis states $\mu < 100$ with a significance level 0.01 and a sample size $n = 15$, what is the critical value $t^\star$?

- If the alternative hypothesis states $\mu \neq 55.5$ with a significance level 0.1 and a sample size $n = 17$, what is the critical value $t^\star$?

lower-tail t test

You will perform a single-sample t test of the alternative hypothesis claiming $\mu < 158$. Before collecting the sample, you decide to use a significance level $\alpha = 0.05$. The sample has the following attributes:

$$\begin{aligned} n &= 3 \\ \bar{x} &= 67.31 \\ s &= 25.54 \end{aligned}$$

What is your conclusion?

We state the hypotheses:

$$H_0: \quad \mu = 158$$

$$H_A: \quad \mu < 158$$

We estimate the standard error (same way as with z testing).

$$SE = \frac{s}{\sqrt{n}} = \frac{25.54}{\sqrt{3}} = 14.746$$

We calculate the t score (same way as with z testing).

$$t = \frac{67.31 - 158}{14.746} = -6.15$$

We determine the degrees of freedom.

$$df = n - 1 = 2$$

We estimate the p-value from the T table.

$$0.01 < p\text{-value} < 0.02$$

We compare the p-value to α.

$$p\text{-value} < \alpha$$

We make our conclusion: we reject the null.

You are given the following hypotheses:

$$H_0 : \quad \mu = 140 \qquad\qquad H_A : \quad \mu > 140$$

We know that the sample standard deviation is 124 and the sample size is 10. For what sample mean would the *p*-value be equal to 0.001? Assume that all conditions necessary for inference are satisfied.
Determine the degrees of freedom.

$$df = 9$$

From the *p*-value we find a *t* score from the *t* table. In this case, our *p*-value is a one-tail probability.

$$t = 4.3$$

We calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{124}{\sqrt{10}} = 39.2$$

We calculate the sample mean that would give *p*-value $= 0.001$.

$$\bar{x} = \mu + t \cdot SE = 140 + (4.3)(39.2) = 309$$

Practice

You are given the following hypotheses:

$$H_0: \ \mu = 12$$

$$H_A: \ \mu < 12$$

We know that the sample standard deviation is 0.205 and the sample size is 20. For what sample mean would the *p*-value be equal to 0.005? Assume that all conditions necessary for inference are satisfied.

A population is known to have a standard deviation $\sigma = 12$. What is the sample size *n* needed to build a 96% confidence interval with a margin of error $ME = 1$?

Solution: Let's remember the formulas for confidence intervals (with known σ):

$$SE = \frac{\sigma}{\sqrt{n}}$$

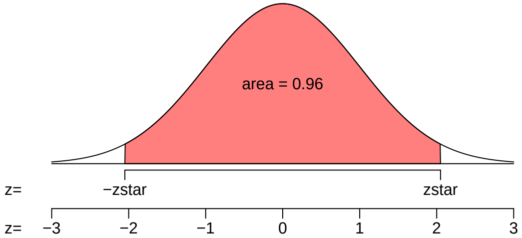
$$CL = P(|Z| < z^\star)$$

$$ME = z^\star SE$$

$$CI = \bar{x} \pm ME$$

From the confidence level $CL = 0.96$, we determine $z^\star$.

$$P(|Z| < z^\star) = 0.96$$



You can use a z table or the last row of the t-table (where $df = \infty$).

$$z^\star = 2.05$$

We know that $ME = z^\star SE$, so

$$SE = \frac{ME}{z^\star} = \frac{1}{2.05} = 0.488$$

We know that $SE = \frac{\sigma}{\sqrt{n}}$. Let's solve for n.

$$SE = \frac{\sigma}{\sqrt{n}}$$

Multiply both sides by $\sqrt{n}$.

$$SE\sqrt{n} = \sigma$$

Divide both sides by SE.

$$\sqrt{n} = \frac{\sigma}{SE}$$

Square both sides. (Raise both sides to the power of 2.)

$$n = \left(\frac{\sigma}{SE}\right)^2$$

$$n = \left(\frac{12}{0.4878049}\right)^2 = 605.16$$

We round n up.

$$n = 606$$

A population is known to have a standard deviation $\sigma = 1.6$. What is the sample size *n* needed to build a 80% confidence interval with a margin of error $ME = 0.2$?

You will perform a single-sample t test of the alternative hypothesis claiming $\mu < 94$. Before collecting the sample, you decide to use a significance level $\alpha = 0.05$. The sample has the following attributes:

$$\begin{aligned} n &= 7 \\ \bar{x} &= 103.4 \\ s &= 16.5 \end{aligned}$$

What is your conclusion?