3.19: **(a):** Nope. There are way more than 2 possible hands in poker.

(b): There are more than 2 possible outcomes when rolling a die. Of course, you could define two mutually exclusive and exhaustive events...

3.20: **(a):** With replacement:

$$P(\text{1st female and 2nd female w/ replacement}) = \frac{5}{10} \times \frac{5}{10} = \frac{1}{4}$$

Without replacement:

$$P(\text{1st female and 2nd female w/out replacement}) = \frac{5}{10} \times \frac{4}{9} = \frac{20}{90} = \frac{2}{9} \approx 0.222$$

(b): With replacement:

$$P(\text{1st female and 2nd female w/ replacement}) = \frac{5000}{10000} \times \frac{5000}{10000} = \frac{1}{4}$$

Without replacement:

$$P(\text{1st female and 2nd female w/out replacement}) = \frac{5000}{10000} \times \frac{4999}{9999} = 0.249975 \approx 0.25$$

(c): This assumption is reasonable. We see that with a large population (3.20.b) there is a tiny error from the independence approximation.

3.21: **(a):** If we assume independence, this is a geometric random variable. Let $X \sim \texttt{Geo}(p = 0.471)$.

$$P(X = 3) \;=\; (1 - 0.471)^2(0.471) \;=\; 0.1318$$

(b): We assume independence. This is not geometric. We have not named repeated failures, but it is a special version of binomial distribution.

$$\begin{aligned} P(\text{1st AND 2nd AND 3rd}) &= P(\text{1st}) \cdot P(\text{2nd}) \cdot P(\text{3rd}) \\ &= 0.471^3 \\ &= 0.1044871 \end{aligned}$$

(c): We are back to geometric! Let $X \sim \texttt{Geo}(p = 0.471)$. We learned a formula for μ and σ for geometric distributions.

$$\begin{aligned} \mu &= \frac{1}{p} \\ &= \frac{1}{0.471} \\ &= 2.123142 \end{aligned}$$

$$\sigma = \frac{\sqrt{1-p}}{p}$$

$$= \frac{\sqrt{1-0.471}}{0.471}$$

$$= 1.544212$$

On average, we expect sampling 2.12 women before finding a married woman, give or take 1.5 women.

(d): We just change p.

$$\mu = \frac{1}{p}$$

$$= \frac{1}{0.3}$$

$$= 3.33333$$

$$\sigma = \frac{\sqrt{1-p}}{p}$$

$$= \frac{\sqrt{1-0.3}}{0.3}$$

$$= 2.788867$$

On average, we expect sampling 3.33 women before finding a married woman, give or take 2.79 women.

(e): Decreasing p increases both μ and σ.

3.22: **(a):** This is a geometric random variable. Let $X \sim \texttt{Geo}(p = 0.02)$.

$$P(X = 10) = (1 - 0.02)^9 (0.02) \approx \boxed{0.0167}$$

(b): This is a special case of a binomial random variable.

$$P(\text{no defects in } 100) = 0.98^{100} \approx \boxed{0.133}$$

(c): This is back to geometric.

$$\mu = \frac{1}{p} = \frac{1}{0.02} = 50$$

$$\sigma = \frac{\sqrt{1-p}}{p} = \frac{\sqrt{1-0.02}}{0.02} = 49.49747$$

On average, we expect to test 50 before we find a defective transistor, plus or minus 49.5.

(d): We just change p.

$$\mu = \frac{1}{p} = \frac{1}{0.05} = 20$$

$$\sigma = \frac{\sqrt{1-p}}{p} = \frac{\sqrt{1-0.05}}{0.05} = 19.49359$$

(e): Increasing p decreases both μ and σ. Also, we might conjecture that when p is small $\mu \approx \sigma + 0.5$.

3.23: **(a):** We are dealing with a geometric distribution, where each trial has a 0.125 chance of success. We let random variable X represent the number of trials until success.

$$X \sim \texttt{Geo}(0.125)$$

We are asked to calculate $P(X = 3)$.

$$P(X = 3) = (1 - 0.125)^2(0.125) \approx 0.09570$$

(b): We are still considering a geometric distribution, so we use the appropriate formulas for (population) mean and (population) standard deviation. (Remember, distributions are infinitely large populations.)

$$\mu = \frac{1}{p} = \frac{1}{0.125} = 8$$

$$\sigma = \frac{\sqrt{1-p}}{p} = \frac{\sqrt{1-0.125}}{0.125} \approx 7.483315$$

3.24: We first need to remember normal distributions. Let normal random variable X represent the speed of a car.

$$X \sim \mathcal{N}(72.6,\ 4.78)$$

We want to determine the probability that a single car is speeding.

$$P(X > 70) = 1 - \Phi\left(\frac{70 - 72.6}{4.78}\right) = 0.7068$$

I used precise software. If you are using the table, you can first estimate z.

$$z = \frac{70 - 72.6}{4.78} \approx -0.54$$

Then, use the table...

$$P(X > 70) \approx 1 - \Phi(-0.54) \approx 0.7054$$

We'll just go with that value; each car has a 70.5% chance of speeding.

(a): We will call speeding a "success" and not speeding a "failure" (even though speeding is bad). Thus, $p = 0.705$. Also, this question asks for the probability of 5 failures.

$$P(\text{0 out of 5 cars speeding}) = (1 - 0.705)^5 = 0.0022$$

(b): Now we are considering a geometric distribution, so we use the appropriate formulas.

$$\mu = \frac{1}{0.705} \approx 1.42$$

$$\sigma = \frac{\sqrt{1-p}}{p} = \frac{\sqrt{1-0.705}}{0.705} \approx 0.77$$