Hypothesis testing

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		Promoted	Not Promoted	Total
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	Total	35	13	48

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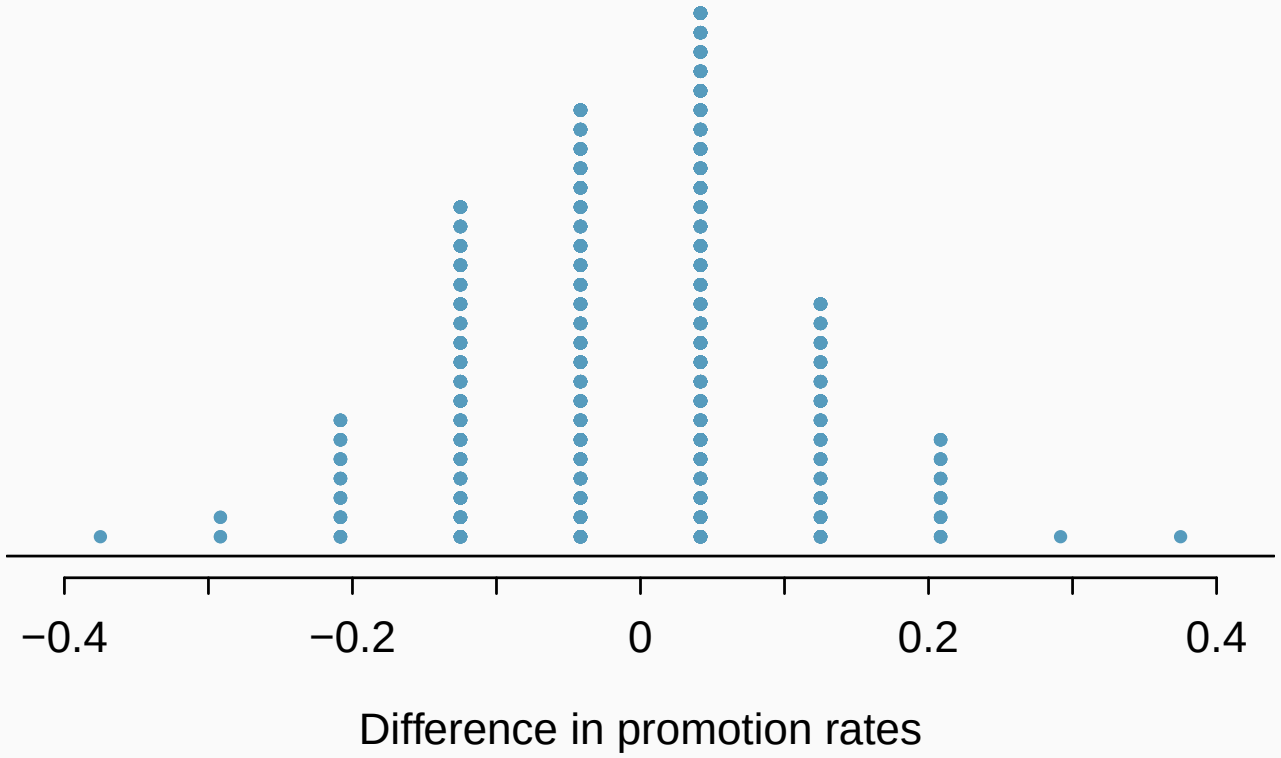
Possible explanations:

- Promotion and gender are *independent*; there is no gender discrimination; observed difference in proportions is simply due to chance. $\rightarrow$ *null* - (nothing is going on)
- Promotion and gender are *dependent*; there is gender discrimination, observed difference in proportions is not due to chance. $\rightarrow$ *alternative* - (something is going on)

Result



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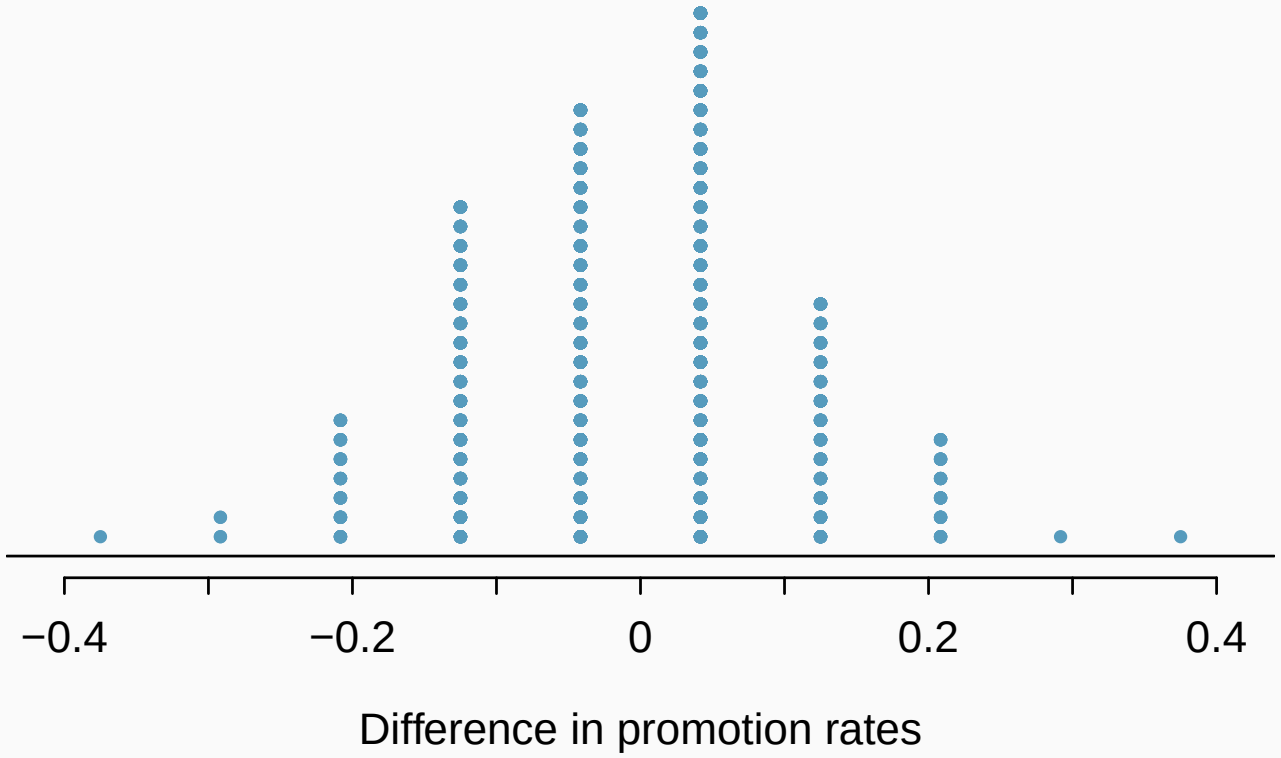

Since it was quite unlikely to obtain results like the actual data or something more extreme in the simulations (male promotions being at least 30 percentage points higher than female promotions), we decided to reject the null hypothesis in favor of the alternative.

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We'll formally introduce the hypothesis testing framework using an example on testing a claim about a population mean.

Testing hypotheses using confidence intervals

Earlier we calculated a 95% confidence interval for the average number of exclusive relationships college students have been in to be (2.7, 3.7). Based on this confidence interval, do these data support the hypothesis that college students on average have been in more than 3 exclusive relationships.

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- Since the null value is included in the interval, we do not reject the null hypothesis in favor of the alternative.
- This is a quick-and-dirty approach for hypothesis testing. However it doesn't tell us the likelihood of certain outcomes under the null hypothesis, i.e. the p-value, based on which we can make a decision on the hypotheses.

example study

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So, we want to know how common/uncommon this difference is from random fluctuation alone.

Luckily, sampling distributions are normal! So if we estimate that $\sigma \approx 15$ mph, we have everything we need to answer that question.

Build the sampling distribution of the null hypothesis

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$$SE = \frac{15}{\sqrt{80}} = 1.68$$

Test statistic

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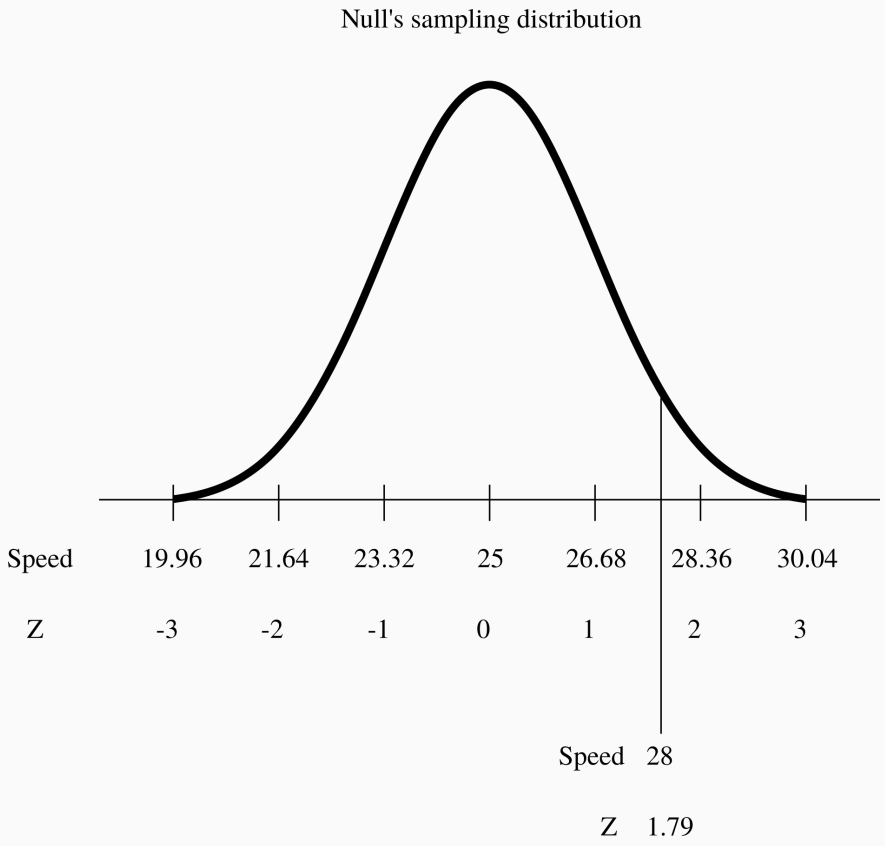

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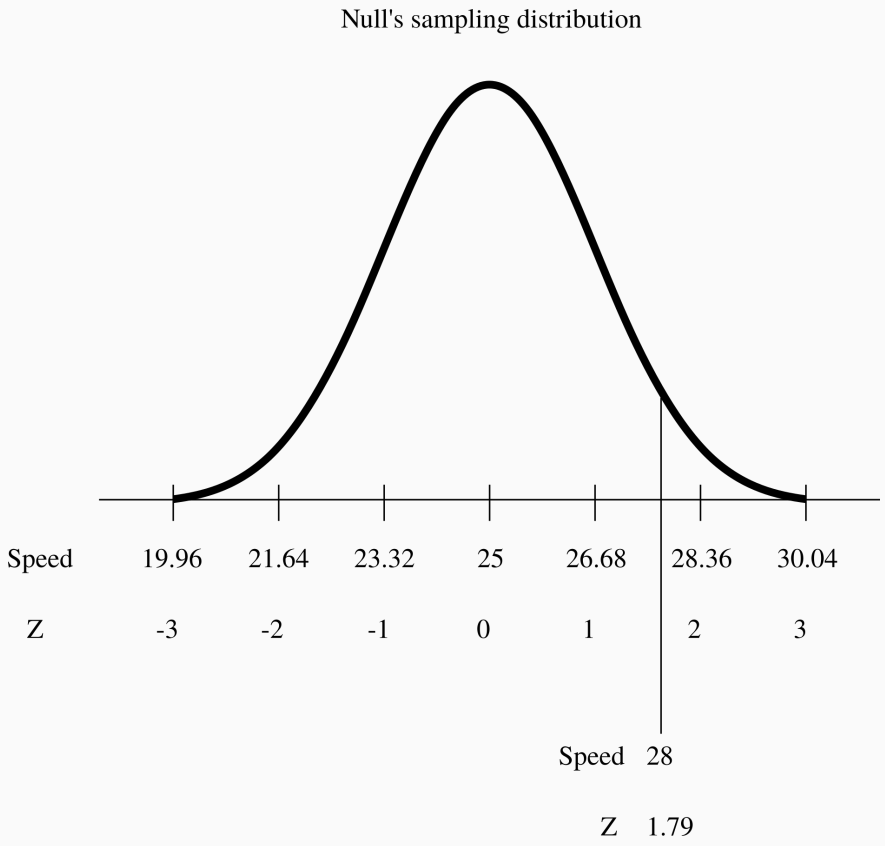

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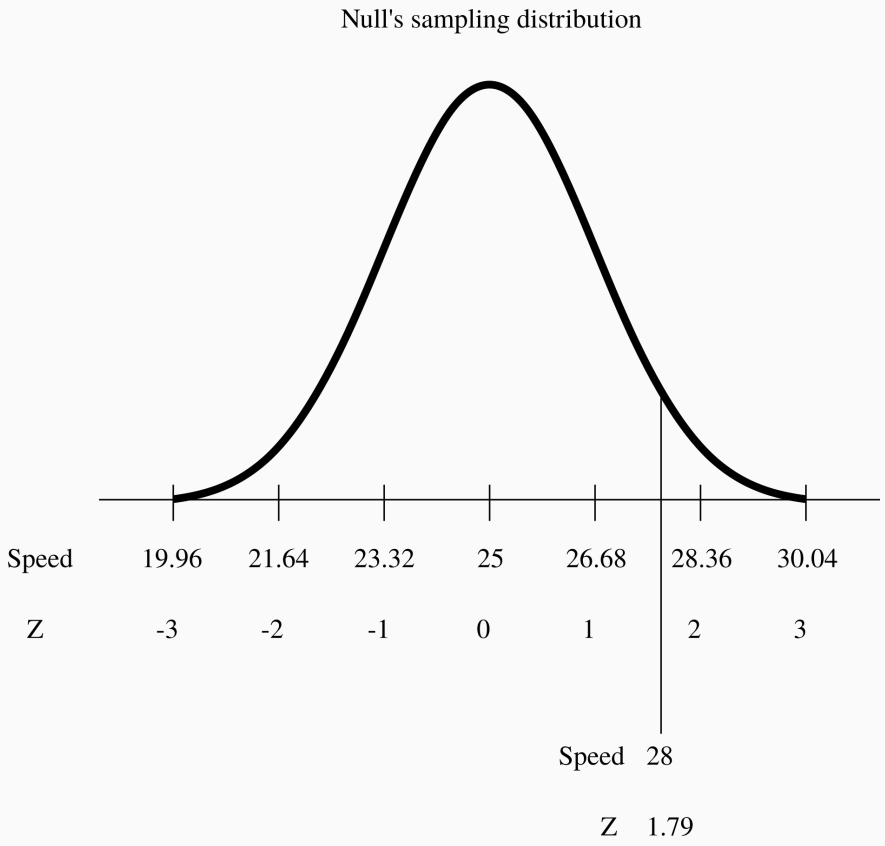

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We quantify how unusual this is with a p-value.

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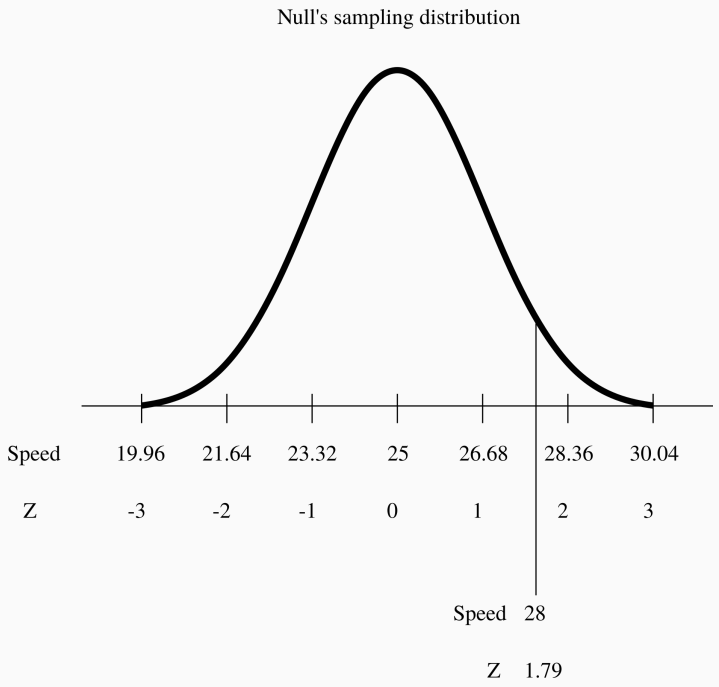
- If the p-value is *high* (higher than α) we say that it is likely to observe the data even if the null hypothesis were true, and hence *do not reject* H_0.

Speeding example

p-value: The probability of observing data at least as favorable to H_A as our current data set (a sample mean greater than 28) if in fact H_0 were true (the population mean was 25).



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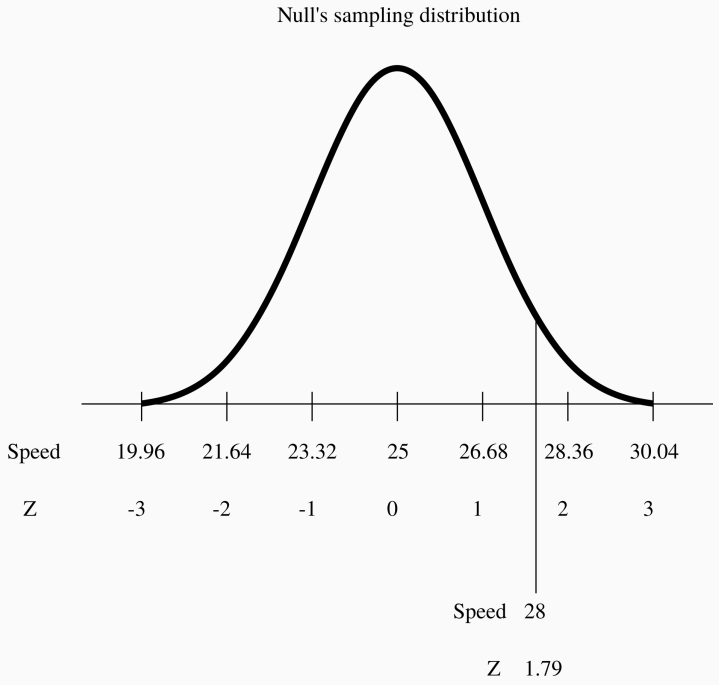

$$P(\bar{x} > 28 | \mu = 25) = P(Z > 1.79) = 0.0367$$

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- Since p-value is *low* (lower than 5%) we *reject* H_0.
- The data provide convincing evidence that cars speed.
- The difference between the null value of 25 mph and observed sample mean of 28 mph is *not due to chance* or sampling variability.

A poll by the National Sleep Foundation found that college students average about 7 hours of sleep per night. A sample of 169 college students taking an introductory statistics class yielded an average of 6.88 hours, with a standard deviation of 0.94 hours. Assuming that this is a random sample representative of all college students *(bit of a leap of faith?)*, a hypothesis test was conducted to evaluate if college students on average sleep less than 7 hours per night. The p-value for this hypothesis test is 0.0485. Which of the following is correct?

(a) Fail to reject H_0, the data provide convincing evidence that college students sleep less than 7 hours on average.
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(c) Reject H_0, the data prove that college students sleep more than 7 hours on average.
(d) Fail to reject H_0, the data do not provide convincing evidence that college students sleep less than 7 hours on average.
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Two-sided hypothesis testing with p-values

- If the research question was “Do the data provide convincing evidence that the average amount of sleep college students get per night is *different* than the national average?”, the alternative hypothesis would be different.

$$H_0 : \mu = 7$$

$$H_A : \mu \neq 7$$

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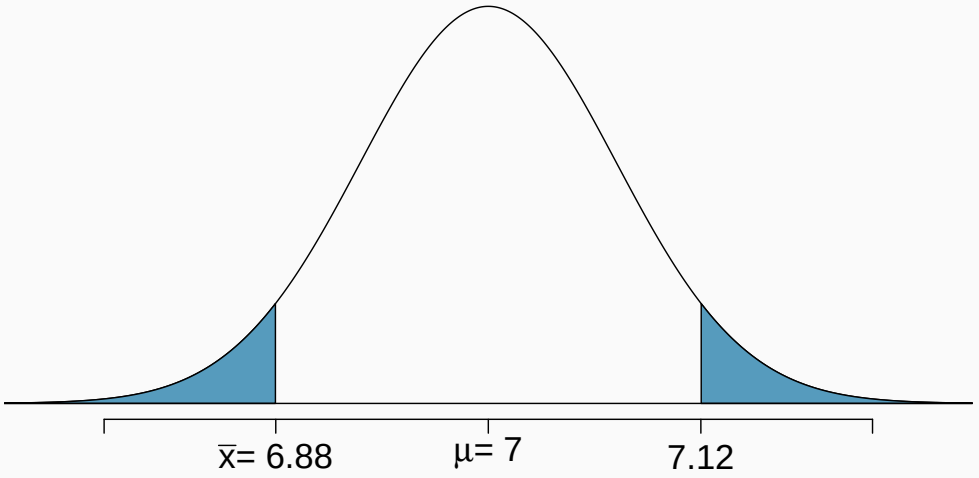
- Hence the p-value would change as well:



p-value
$= 0.0485 \times 2$
$= 0.097$

Decision errors

- Hypothesis tests are not flawless.
- In the court system innocent people are sometimes wrongly convicted and the guilty sometimes walk free.
- Similarly, we can make a wrong decision in statistical hypothesis tests as well.
- The difference is that we have the tools necessary to quantify how often we make errors in statistics.

Decision errors (cont.)

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- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true.

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- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true.
- A *Type 2 Error* is failing to reject the null hypothesis when H_A is true.
- We (almost) never know if H_0 or H_A is true, but we need to consider all possibilities.

Hypothesis Test as a trial

If we again think of a hypothesis test as a criminal trial then it makes sense to frame the verdict in terms of the null and alternative hypotheses:

$$H_0: \text{ Defendant is innocent}$$
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Which type of error is being committed in the following circumstances?

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$$P(\text{Type 1 error} | H_0 \text{ true}) = \alpha$$

- This is why we prefer small values of α – increasing α increases the Type 1 error rate.

Choosing a significance level

- Choosing a significance level for a test is important in many contexts, and the traditional level is 0.05. However, it is often helpful to adjust the significance level based on the application.
- We may select a level that is smaller or larger than 0.05 depending on the consequences of any conclusions reached from the test.
- If making a Type 1 Error is dangerous or especially costly, we should choose a small significance level (e.g. 0.01). Under this scenario we want to be very cautious about rejecting the null hypothesis, so we demand very strong evidence favoring H_A before we would reject H_0.
- If a Type 2 Error is relatively more dangerous or much more costly than a Type 1 Error, then we should choose a higher significance level (e.g. 0.10). Here we want to be cautious about failing to reject H_0 when the null is actually false.

the next two slides are provided as a brief summary of hypothesis testing...

Recap: Hypothesis testing framework

1. Set the hypotheses.
2. Check assumptions and conditions.
3. Calculate a *test statistic* and a p-value.
4. Make a decision, and interpret it in context of the research question.

Recap: Hypothesis testing for a population mean

1. Set the hypotheses
   - $H_0 : \mu = null\ value$
   - $H_A : \mu <$ or $>$ or $\neq null\ value$
2. Calculate the point estimate
3. Check assumptions and conditions
   - Independence: random sample/assignment, 10% condition when sampling without replacement
   - Normality: nearly normal population or $n \geq 30$, no extreme skew – or use the t distribution
4. Calculate a *test statistic* and a p-value (draw a picture!)

$$Z = \frac{\bar{x} - \mu}{SE},\ where\ SE = \frac{s}{\sqrt{n}}$$

5. Make a decision, and interpret it in context
   - If p-value $< \alpha$, reject H_0, data provide evidence for H_A
   - If p-value $> \alpha$, do not reject H_0, data do not provide evidence for H_A