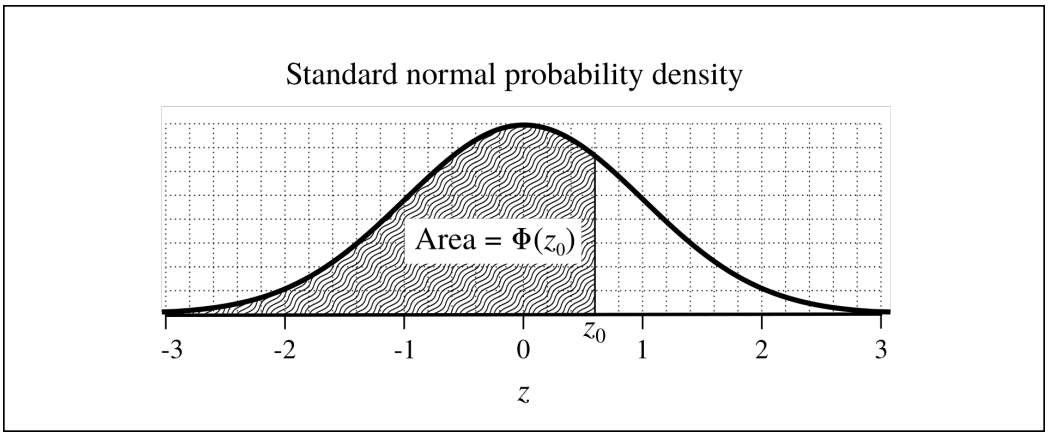
The random variable Z is normally distributed such that $\mu = 0$ and $\sigma = 1$.

$$Z \sim \mathcal{N}\left(0,\ 1\right)$$

To determine the probability that Z is less than z_0 (this is also the percentile of z_0), we find the area under the curve from $-\infty$ to z_0.

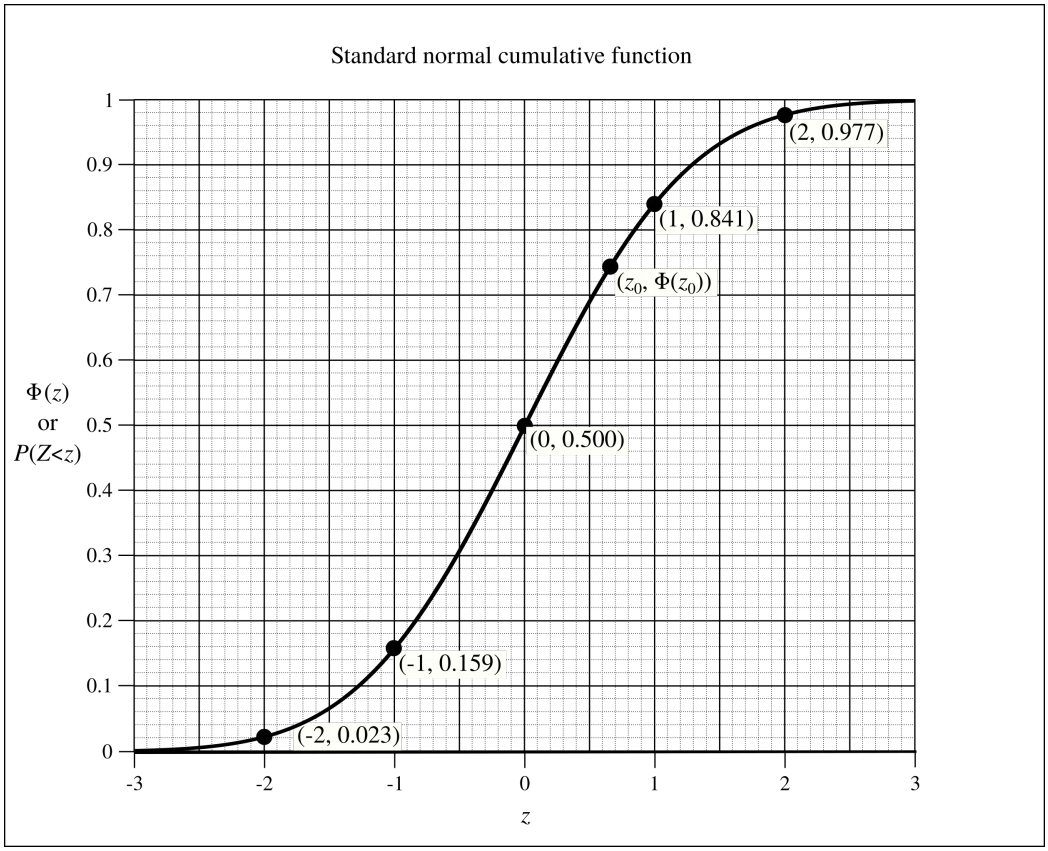
$$P(Z < z_0) \ = \ \text{Left area up to } z_0$$



We use Φ, an upper-case "Phi", to represent a left area of the standard normal density. We know Φ depends on z_0, so we use function notation. Also, we call this function the **standard normal cumulative function**.

$$\Phi(z_0) \equiv P(Z < z_0)$$

If we repeat the process of finding the areas from $-\infty$ to any z, we get the cumulative function.



The standard normal table gives precise values of Φ as a function of z. (See next pages.)

Standard normal cumulative function z < 0

z	Phi(z)	z	Phi(z)	z	Phi(z)	z	Phi(z)	z	Phi(z)	z	Phi(z)
-3.00	0.0013	-2.50	0.0062	-2.00	0.0228	-1.50	0.0668	-1.00	0.1587	-0.50	0.3085
-2.99	0.0014	-2.49	0.0064	-1.99	0.0233	-1.49	0.0681	-0.99	0.1611	-0.49	0.3121
-2.98	0.0014	-2.48	0.0066	-1.98	0.0239	-1.48	0.0694	-0.98	0.1635	-0.48	0.3156
-2.97	0.0015	-2.47	0.0068	-1.97	0.0244	-1.47	0.0708	-0.97	0.1660	-0.47	0.3192
-2.96	0.0015	-2.46	0.0069	-1.96	0.0250	-1.46	0.0721	-0.96	0.1685	-0.46	0.3228
-2.95	0.0016	-2.45	0.0071	-1.95	0.0256	-1.45	0.0735	-0.95	0.1711	-0.45	0.3264
-2.94	0.0016	-2.44	0.0073	-1.94	0.0262	-1.44	0.0749	-0.94	0.1736	-0.44	0.3300
-2.93	0.0017	-2.43	0.0075	-1.93	0.0268	-1.43	0.0764	-0.93	0.1762	-0.43	0.3336
-2.92	0.0018	-2.42	0.0078	-1.92	0.0274	-1.42	0.0778	-0.92	0.1788	-0.42	0.3372
-2.91	0.0018	-2.41	0.0080	-1.91	0.0281	-1.41	0.0793	-0.91	0.1814	-0.41	0.3409
-2.90	0.0019	-2.40	0.0082	-1.90	0.0287	-1.40	0.0808	-0.90	0.1841	-0.40	0.3446
-2.89	0.0019	-2.39	0.0084	-1.89	0.0294	-1.39	0.0823	-0.89	0.1867	-0.39	0.3483
-2.88	0.0020	-2.38	0.0087	-1.88	0.0301	-1.38	0.0838	-0.88	0.1894	-0.38	0.3520
-2.87	0.0021	-2.37	0.0089	-1.87	0.0307	-1.37	0.0853	-0.87	0.1922	-0.37	0.3557
-2.86	0.0021	-2.36	0.0091	-1.86	0.0314	-1.36	0.0869	-0.86	0.1949	-0.36	0.3594
-2.85	0.0022	-2.35	0.0094	-1.85	0.0322	-1.35	0.0885	-0.85	0.1977	-0.35	0.3632
-2.84	0.0023	-2.34	0.0096	-1.84	0.0329	-1.34	0.0901	-0.84	0.2005	-0.34	0.3669
-2.83	0.0023	-2.33	0.0099	-1.83	0.0336	-1.33	0.0918	-0.83	0.2033	-0.33	0.3707
-2.82	0.0024	-2.32	0.0102	-1.82	0.0344	-1.32	0.0934	-0.82	0.2061	-0.32	0.3745
-2.81	0.0025	-2.31	0.0104	-1.81	0.0351	-1.31	0.0951	-0.81	0.2090	-0.31	0.3783
-2.80	0.0026	-2.30	0.0107	-1.80	0.0359	-1.30	0.0968	-0.80	0.2119	-0.30	0.3821
-2.79	0.0026	-2.29	0.0110	-1.79	0.0367	-1.29	0.0985	-0.79	0.2148	-0.29	0.3859
-2.78	0.0027	-2.28	0.0113	-1.78	0.0375	-1.28	0.1003	-0.78	0.2177	-0.28	0.3897
-2.77	0.0028	-2.27	0.0116	-1.77	0.0384	-1.27	0.1020	-0.77	0.2206	-0.27	0.3936
-2.76	0.0029	-2.26	0.0119	-1.76	0.0392	-1.26	0.1038	-0.76	0.2236	-0.26	0.3974
-2.75	0.0030	-2.25	0.0122	-1.75	0.0401	-1.25	0.1056	-0.75	0.2266	-0.25	0.4013
-2.74	0.0031	-2.24	0.0125	-1.74	0.0409	-1.24	0.1075	-0.74	0.2296	-0.24	0.4052
-2.73	0.0032	-2.23	0.0129	-1.73	0.0418	-1.23	0.1093	-0.73	0.2327	-0.23	0.4090
-2.72	0.0033	-2.22	0.0132	-1.72	0.0427	-1.22	0.1112	-0.72	0.2358	-0.22	0.4129
-2.71	0.0034	-2.21	0.0136	-1.71	0.0436	-1.21	0.1131	-0.71	0.2389	-0.21	0.4168
-2.70	0.0035	-2.20	0.0139	-1.70	0.0446	-1.20	0.1151	-0.70	0.2420	-0.20	0.4207
-2.69	0.0036	-2.19	0.0143	-1.69	0.0455	-1.19	0.1170	-0.69	0.2451	-0.19	0.4247
-2.68	0.0037	-2.18	0.0146	-1.68	0.0465	-1.18	0.1190	-0.68	0.2483	-0.18	0.4286
-2.67	0.0038	-2.17	0.0150	-1.67	0.0475	-1.17	0.1210	-0.67	0.2514	-0.17	0.4325
-2.66	0.0039	-2.16	0.0154	-1.66	0.0485	-1.16	0.1230	-0.66	0.2546	-0.16	0.4364
-2.65	0.0040	-2.15	0.0158	-1.65	0.0495	-1.15	0.1251	-0.65	0.2578	-0.15	0.4404
-2.64	0.0041	-2.14	0.0162	-1.64	0.0505	-1.14	0.1271	-0.64	0.2611	-0.14	0.4443
-2.63	0.0043	-2.13	0.0166	-1.63	0.0516	-1.13	0.1292	-0.63	0.2643	-0.13	0.4483
-2.62	0.0044	-2.12	0.0170	-1.62	0.0526	-1.12	0.1314	-0.62	0.2676	-0.12	0.4522
-2.61	0.0045	-2.11	0.0174	-1.61	0.0537	-1.11	0.1335	-0.61	0.2709	-0.11	0.4562
-2.60	0.0047	-2.10	0.0179	-1.60	0.0548	-1.10	0.1357	-0.60	0.2743	-0.10	0.4602
-2.59	0.0048	-2.09	0.0183	-1.59	0.0559	-1.09	0.1379	-0.59	0.2776	-0.09	0.4641
-2.58	0.0049	-2.08	0.0188	-1.58	0.0571	-1.08	0.1401	-0.58	0.2810	-0.08	0.4681
-2.57	0.0051	-2.07	0.0192	-1.57	0.0582	-1.07	0.1423	-0.57	0.2843	-0.07	0.4721
-2.56	0.0052	-2.06	0.0197	-1.56	0.0594	-1.06	0.1446	-0.56	0.2877	-0.06	0.4761
-2.55	0.0054	-2.05	0.0202	-1.55	0.0606	-1.05	0.1469	-0.55	0.2912	-0.05	0.4801
-2.54	0.0055	-2.04	0.0207	-1.54	0.0618	-1.04	0.1492	-0.54	0.2946	-0.04	0.4840
-2.53	0.0057	-2.03	0.0212	-1.53	0.0630	-1.03	0.1515	-0.53	0.2981	-0.03	0.4880
-2.52	0.0059	-2.02	0.0217	-1.52	0.0643	-1.02	0.1539	-0.52	0.3015	-0.02	0.4920
-2.51	0.0060	-2.01	0.0222	-1.51	0.0655	-1.01	0.1562	-0.51	0.3050	-0.01	0.4960
-2.50	0.0062	-2.00	0.0228	-1.50	0.0668	-1.00	0.1587	-0.50	0.3085	0.00	0.5000

Standard normal cumulative function z > 0

z	Phi(z)	z	Phi(z)	z	Phi(z)	z	Phi(z)	z	Phi(z)	z	Phi(z)
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772	2.50	0.9938
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.01	0.9778	2.51	0.9940
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.02	0.9783	2.52	0.9941
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.03	0.9788	2.53	0.9943
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.04	0.9793	2.54	0.9945
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.05	0.9798	2.55	0.9946
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.06	0.9803	2.56	0.9948
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.07	0.9808	2.57	0.9949
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.08	0.9812	2.58	0.9951
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.09	0.9817	2.59	0.9952
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.10	0.9821	2.60	0.9953
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463	2.11	0.9826	2.61	0.9955
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474	2.12	0.9830	2.62	0.9956
0.13	0.5517	0.63	0.7357	1.13	0.8708	1.63	0.9484	2.13	0.9834	2.63	0.9957
0.14	0.5557	0.64	0.7389	1.14	0.8729	1.64	0.9495	2.14	0.9838	2.64	0.9959
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.15	0.9842	2.65	0.9960
0.16	0.5636	0.66	0.7454	1.16	0.8770	1.66	0.9515	2.16	0.9846	2.66	0.9961
0.17	0.5675	0.67	0.7486	1.17	0.8790	1.67	0.9525	2.17	0.9850	2.67	0.9962
0.18	0.5714	0.68	0.7517	1.18	0.8810	1.68	0.9535	2.18	0.9854	2.68	0.9963
0.19	0.5753	0.69	0.7549	1.19	0.8830	1.69	0.9545	2.19	0.9857	2.69	0.9964
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.20	0.9861	2.70	0.9965
0.21	0.5832	0.71	0.7611	1.21	0.8869	1.71	0.9564	2.21	0.9864	2.71	0.9966
0.22	0.5871	0.72	0.7642	1.22	0.8888	1.72	0.9573	2.22	0.9868	2.72	0.9967
0.23	0.5910	0.73	0.7673	1.23	0.8907	1.73	0.9582	2.23	0.9871	2.73	0.9968
0.24	0.5948	0.74	0.7704	1.24	0.8925	1.74	0.9591	2.24	0.9875	2.74	0.9969
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.25	0.9878	2.75	0.9970
0.26	0.6026	0.76	0.7764	1.26	0.8962	1.76	0.9608	2.26	0.9881	2.76	0.9971
0.27	0.6064	0.77	0.7794	1.27	0.8980	1.77	0.9616	2.27	0.9884	2.77	0.9972
0.28	0.6103	0.78	0.7823	1.28	0.8997	1.78	0.9625	2.28	0.9887	2.78	0.9973
0.29	0.6141	0.79	0.7852	1.29	0.9015	1.79	0.9633	2.29	0.9890	2.79	0.9974
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.30	0.9893	2.80	0.9974
0.31	0.6217	0.81	0.7910	1.31	0.9049	1.81	0.9649	2.31	0.9896	2.81	0.9975
0.32	0.6255	0.82	0.7939	1.32	0.9066	1.82	0.9656	2.32	0.9898	2.82	0.9976
0.33	0.6293	0.83	0.7967	1.33	0.9082	1.83	0.9664	2.33	0.9901	2.83	0.9977
0.34	0.6331	0.84	0.7995	1.34	0.9099	1.84	0.9671	2.34	0.9904	2.84	0.9977
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678	2.35	0.9906	2.85	0.9978
0.36	0.6406	0.86	0.8051	1.36	0.9131	1.86	0.9686	2.36	0.9909	2.86	0.9979
0.37	0.6443	0.87	0.8078	1.37	0.9147	1.87	0.9693	2.37	0.9911	2.87	0.9979
0.38	0.6480	0.88	0.8106	1.38	0.9162	1.88	0.9699	2.38	0.9913	2.88	0.9980
0.39	0.6517	0.89	0.8133	1.39	0.9177	1.89	0.9706	2.39	0.9916	2.89	0.9981
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713	2.40	0.9918	2.90	0.9981
0.41	0.6591	0.91	0.8186	1.41	0.9207	1.91	0.9719	2.41	0.9920	2.91	0.9982
0.42	0.6628	0.92	0.8212	1.42	0.9222	1.92	0.9726	2.42	0.9922	2.92	0.9982
0.43	0.6664	0.93	0.8238	1.43	0.9236	1.93	0.9732	2.43	0.9925	2.93	0.9983
0.44	0.6700	0.94	0.8264	1.44	0.9251	1.94	0.9738	2.44	0.9927	2.94	0.9984
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744	2.45	0.9929	2.95	0.9984
0.46	0.6772	0.96	0.8315	1.46	0.9279	1.96	0.9750	2.46	0.9931	2.96	0.9985
0.47	0.6808	0.97	0.8340	1.47	0.9292	1.97	0.9756	2.47	0.9932	2.97	0.9985
0.48	0.6844	0.98	0.8365	1.48	0.9306	1.98	0.9761	2.48	0.9934	2.98	0.9986
0.49	0.6879	0.99	0.8389	1.49	0.9319	1.99	0.9767	2.49	0.9936	2.99	0.9986
0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772	2.50	0.9938	3.00	0.9987

Here are some useful rules. But, really, you should come up with these by sketching areas under curves.

Left area up to z_0 :

$$P(Z < z_0) = \Phi(z_0)$$

Right area from z_1 :

$$\begin{aligned} P(Z > z_1) &= 1 - \Phi(z_1) \\ &= \Phi(-z_1) \end{aligned}$$

Sector from z_2 **to** z_3 :

$$P(z_2 < Z < z_3) = \Phi(z_3) - \Phi(z_2)$$

Central area from $-z_4$ **to** z_4 :

$$\begin{aligned} P(|Z| < z_4) &= \Phi(z_4) - \Phi(-z_4) \\ &= 1 - 2\Phi(-z_4) \end{aligned}$$

Two-tail area below $-z_5$ **and above** z_5 :

$$P(|Z| > z_5) = 2\Phi(-z_5)$$

Let $X \sim \mathcal{N}\left(\mu,\ \sigma\right)$ and let x be a specific value of X. You may want to convert the x value into a z score.

$$z = \frac{x - \mu}{\sigma}$$

You also might want to convert a z score into an x value.

$$x = z\sigma + \mu$$

You also might want to find μ or σ when you know the other quantities.

$$\mu = x - z\sigma$$

$$\sigma = \frac{x - \mu}{z}$$

Notice all four of these equations represent the same relationship; we've just algebraically solved for different variables.