Confidence intervals

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- A plausible range of values for the population parameter is called a *confidence interval*.
- Using only a sample statistic to estimate a parameter is like fishing in a murky lake with a spear, and using a confidence interval is like fishing with a net.

We can throw a spear where we saw a fish but we will probably miss. If we toss a net in that area, we have a good chance of catching the fish.

- If we report a point estimate, we probably won't hit the exact population parameter. If we report a range of plausible values we have a good shot at capturing the parameter.

Photos by Mark Fischer (http://www.flickr.com/photos/fischerfotos/7439791462) and Chris Penny (http://www.flickr.com/photos/clearlydived/7029109617) on Flickr.

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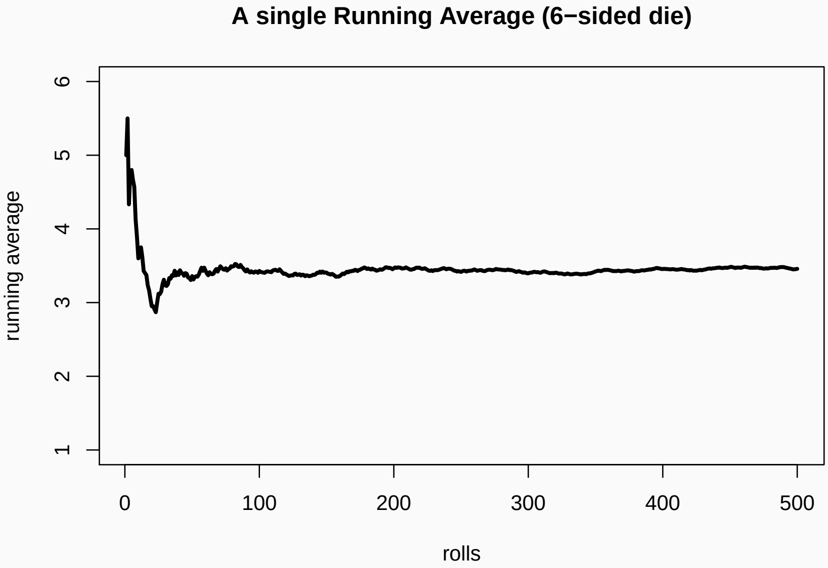
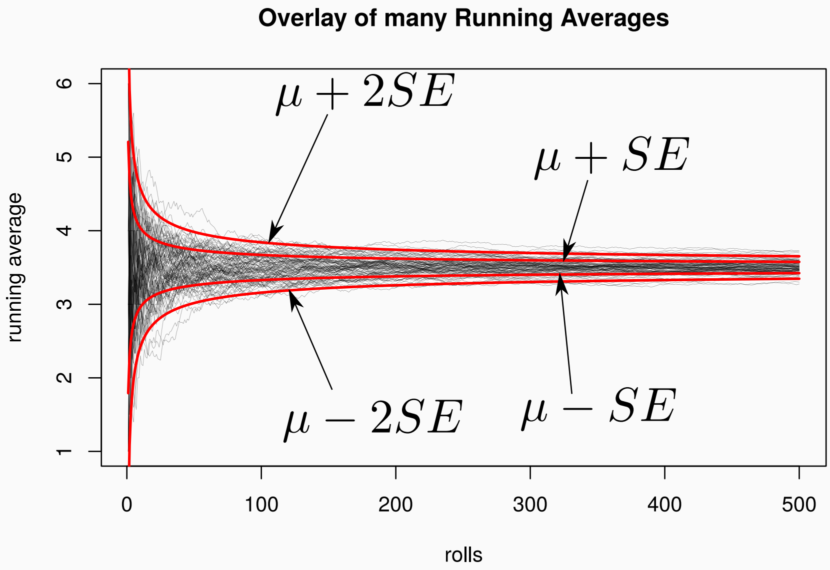
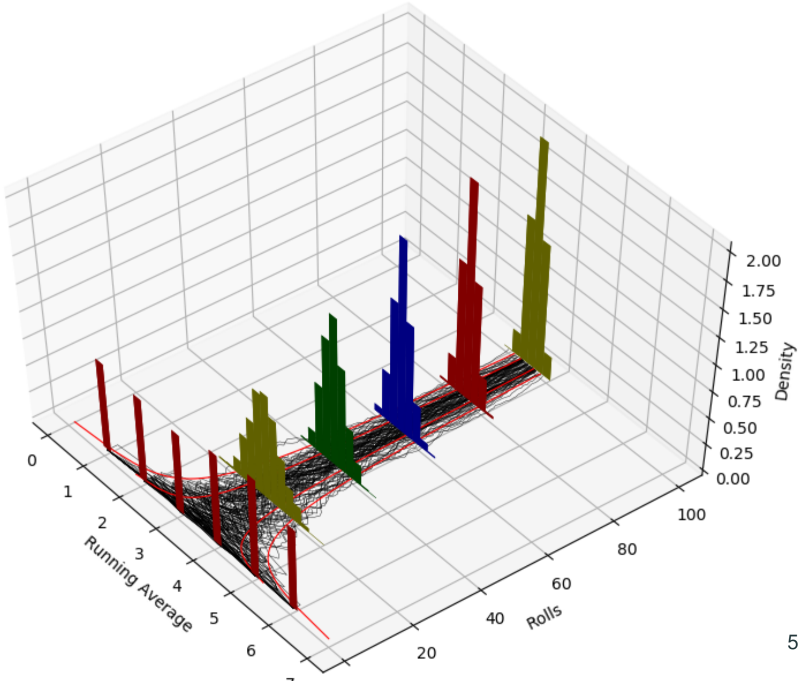
A sampling distribution (distribution of sample means) is approximately normal!

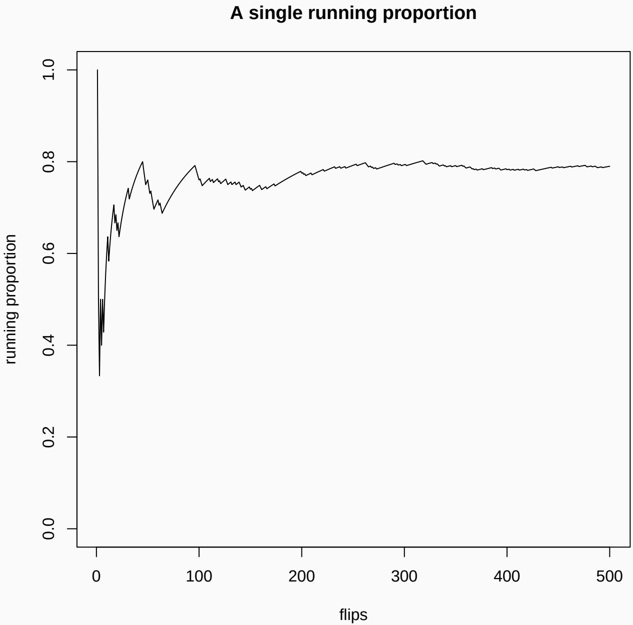
A running average



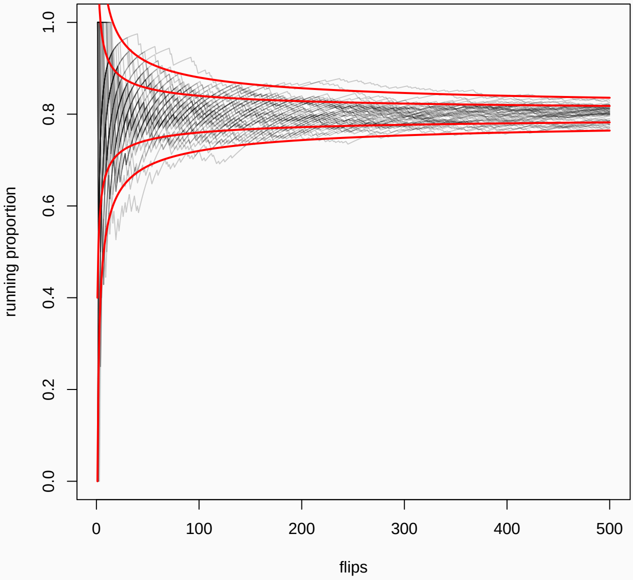


68% of data is within $\mu \pm SE$. 95% of data is between $\mu \pm 2SE$.


Running Average
Rolls
Density
0
1
2
3
4
5
6
7
20
40
60
80
100
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00


A single running proportion
running proportion
0.0
0.2
0.4
0.6
0.8
1.0
flips
0
100
200
300
400
500

Overlay of many running proportions



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Confidence intervals only try to capture the population parameter. A confidence interval says nothing about the confidence of capturing individual observations, a proportion of the observations, or about capturing point estimates. Confidence intervals only attempt to capture population parameters.

Average number of exclusive relationships

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$$\begin{aligned} \bar{x} \pm 2 \times SE &= 3.2 \pm 2 \times 0.25 \\ &= (3.2 - 0.5, 3.2 + 0.5) \\ &= (2.7, 3.7) \end{aligned}$$

Which of the following is the correct interpretation of this confidence interval?

We are 95% confident that

(a) the average number of exclusive relationships college students in this sample have been in is between 2.7 and 3.7.

(b) college students on average have been in between 2.7 and 3.7 exclusive relationships.

(c) a randomly chosen college student has been in 2.7 to 3.7 exclusive relationships.

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Confidence interval, a general formula

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Conditions when the point estimate = $\bar{x}$:

1. *Independence:* Observations in the sample must be independent
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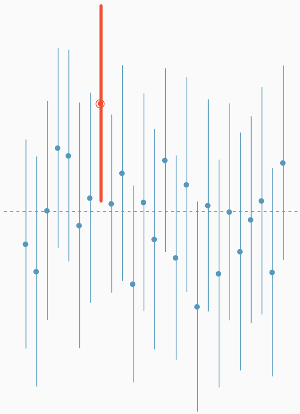
Note: We will discuss working with samples where $n < 30$ in the next chapter.

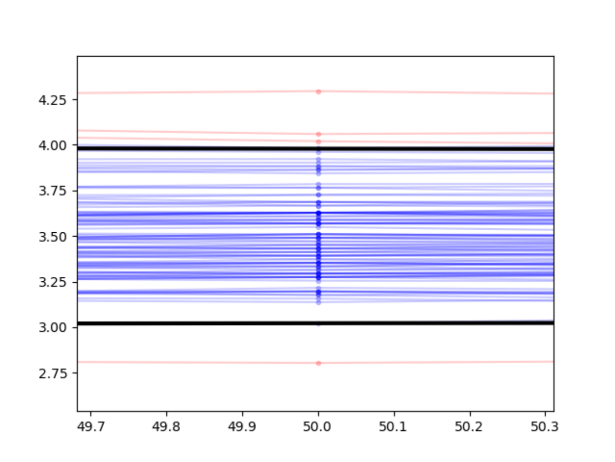
What does 95% confident mean?

- Suppose we took many samples and built a confidence interval from each sample using the equation $point\ estimate \pm 2 \times SE$.
- Then about 95% of those intervals would contain the true population mean (μ).

- The figure shows this process with 25 samples, where 24 of the resulting confidence intervals contain the true average number of exclusive relationships, and one does not.



Same idea shown from running averages view.



About 95% of the time the population mean is withing 2 SE of the sample mean.

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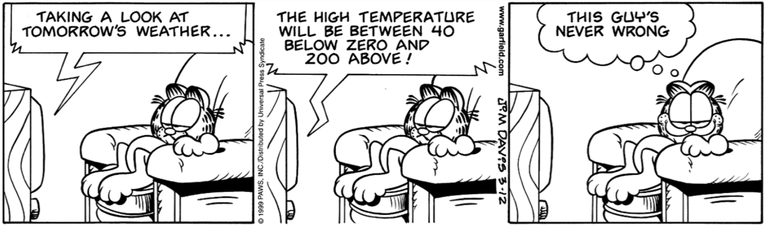

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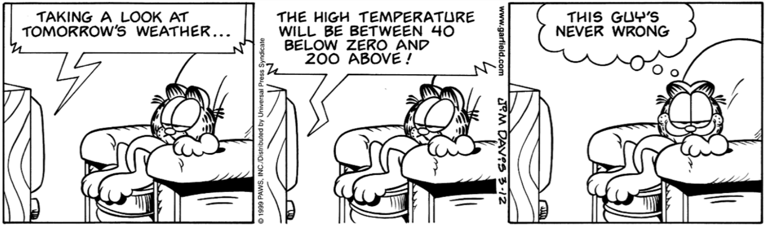

If the interval is too wide it may not be very informative. Image source: http://web.as.uky.edu/statistics/users/earo227/misc/garfield_weather.gif

Changing the confidence level

$$point\ estimate \pm z^{\star} \times SE$$

- In a confidence interval, $z^{\star} \times SE$ is called the *margin of error*, and for a given sample, the margin of error changes as the confidence level changes.
- In order to change the confidence level we need to adjust $z^{\star}$ in the above formula.
- Commonly used confidence levels in practice are 90%, 95%, 98%, and 99%.
- For a 95% confidence interval, $z^{\star} = 1.96$.
- However, using the standard normal (z) distribution, it is possible to find the appropriate $z^{\star}$ for any confidence level.

Which of the below Z scores is the appropriate $z^\star$ when calculating a 98% confidence interval?

(a) $Z = 2.05$

(b) $Z = 1.96$

(c) $Z = 2.33$

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