

MATH 181 2ND EXAM

SPRING 2019

Name: _____

- Write your **full name** on the line above.
- Show your work. Incorrect answers with work can receive partial credit.
- Attempt every question; showing you understand the question earns some credit.
- If you run out of room for an answer, continue on the back of the page. Before doing so, write “see back” with a circle around it.
- You can use 1 page (front and back) of notes in addition to the formula sheet (on page 2) and z-table (last page).
- You can use (and probably need) a calculator.
- You can use the Geogebra Scientific Calculator instead of a calculator. You need to put your phone on **airplane mode** and then within the application, start **exam mode**; you should see a green bar with a timer counting up.
- If a question is confusing or ambiguous, please ask for clarification; however, you will not be told how to answer the question.
- **Box your final answer.**
- You can rip off the z-table, but please keep the rest of the test intact.

Do not write in this grade table.

Question:	Q1	Q2	Q3	Q4	Q5	Q6	Total
Points:	10	10	10	10	10	10	60
Score:							

Normal Distribution:

$$X \sim \mathcal{N}(\mu, \sigma)$$

μ = population mean

σ = population standard deviation

x = possible value of X

ℓ = percentile of x (left area)

$\Phi(z)$ = standard normal cumulative function

$$z = \frac{x - \mu}{\sigma}$$

$$P(X < x) = \Phi(z)$$

$$\ell = \Phi(z)$$

$$z = \Phi^{-1}(\ell)$$

Bernoulli Distribution:

$$X \sim \text{Bern}(p)$$

$X = 0$ for fail or 1 for success

p = probability of success

$$P(X = 0) = 1 - p$$

$$P(X = 1) = p$$

$$\mu = p$$

$$\sigma = \sqrt{p(1 - p)}$$

Geometric Distribution:

$$X \sim \text{Geo}(p)$$

X = number of trials until first success

p = probability of success on each trial

n = a possible number of trials

$$P(X = n) = (1 - p)^{n-1}(p)$$

$$\mu = \frac{1}{p}$$

$$\sigma = \sqrt{\frac{1 - p}{p^2}}$$

Mean-Sampling Distribution:

\bar{X} = sample mean

s = sample standard deviation

n = sample size

μ = population mean

σ = population standard deviation

SE = standard error

$$SE = \frac{\sigma}{\sqrt{n}}$$

If $n \geq 30$ (or if population is normal) then:

$$\bar{X} \sim \mathcal{N}(\mu, SE)$$

Binomial Distribution:

$$X \sim \mathcal{B}(n, p)$$

X = number of successes from n trials

p = probability of success on each trial

n = number of trials

k = a possible number of successes

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu = np$$

$$\sigma = \sqrt{np(1 - p)}$$

If $np \geq 10$ and $n(1 - p) \geq 10$, then

$$X \sim \mathcal{N}(\mu, \sigma)$$

Continuity correction:

$$P(X \leq k) \approx \Phi\left(\frac{k + 0.5 - \mu}{\sigma}\right)$$

Confidence Interval:

CI = confidence interval

γ = confidence level

\bar{x} = sample mean

s = sample standard deviation

$$z^* = \Phi^{-1}\left(\frac{\gamma + 1}{2}\right)$$

$$SE \approx \frac{s}{\sqrt{n}}$$

$$CI = \bar{x} \pm z^* SE$$

Hypothesis testing:

$$H_0 : \mu = \mu_0$$

$$H_A : \mu \neq \mu_0$$

\bar{x} = a possible/specific/observed sample mean

s = sample standard deviation

α = significance level

$$\sigma \approx s$$

$$z = \frac{\bar{x} - \mu_0}{SE}$$

$$\begin{aligned} p\text{-value} &= P(|Z| > |z|) \\ &= 2 \cdot \Phi(-|z|) \end{aligned}$$

If $p\text{-value} < \alpha$, then reject H_0 , else retain H_0 .

Q1. (10 points) Let random variable X be normally distributed with mean $\mu = 33$ and standard deviation $\sigma = 4$.

(a) Evaluate $P(X < 37)$.

Solution: We find a z -score.

$$z = \frac{37 - 33}{4} = 1$$

We use the z table to evaluate the probability.

$$P(X < 37) = P(Z < 1) = \Phi(1) = \boxed{0.8413}$$

(b) Determine x such that $P(X < x) = 0.33$.

Solution: We find the z -score from the z -table.

$$z = \Phi^{-1}(0.33) = -0.44$$

We convert this z -score into an x -score.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 33 + (-0.44)(4) \\ &= \boxed{31.24} \end{aligned}$$

Q2. (10 points) Imagine a scratch-off lottery has a chance of success $p = 0.01$.

(a) What is the mean number of trials until the first success?

Solution: This situation is described by a geometric distribution.

$$\mu = \frac{1}{p} = \frac{1}{0.01} = \boxed{100}$$

The mean number of trials is 100. The expected number of trials is 100.

(b) What is the probability of getting the first success on the twelfth trial?

Solution: This situation is still described by a geometric distribution. So, we use the geometric probability formula: $P(X = n) = (1 - p)^{n-1}p$.

$$\begin{aligned} P(X = 12) &= (1 - 0.01)^{12-1}(0.01) \\ &= (0.99)^{11}(0.01) \\ &= \boxed{0.00895} \end{aligned}$$

Q3. (10 points) Let each trial have a chance of success $p = 0.25$. We will predict what happens when we have 100 trials.

(a) What is the probability of getting exactly 26 successes?

Solution: This situation is described by a binomial distribution.

$$\begin{aligned} P(X = 26) &= \binom{100}{26} (0.25)^{26} (0.75)^{74} \\ &= \boxed{0.0883} \end{aligned}$$

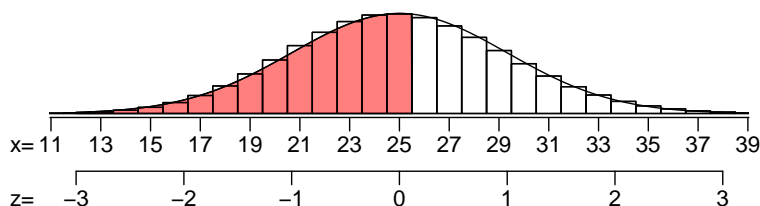
(b) What is the probability of getting fewer than 26 successes? (Use a normal approximation and continuity correction.)

Solution: We find the parameters needed to construct a normal approximation.

$$\mu = np = (100)(0.25) = 25$$

$$\begin{aligned} \sigma &= \sqrt{np(1-p)} \\ &= \sqrt{(100)(0.25)(0.75)} \\ &= 4.3863 \end{aligned}$$

Here is a sketch:



We find the z score of the boundary.

$$z = \frac{25.5 - 25}{4.3863} = 0.11$$

We calculate the probability.

$$P(X < 26) = P(X < 25.5) = P(Z < 0.11) = \boxed{0.5438}$$

(c) What is the probability of getting more than 26 successes?

Solution: The easiest way to do this is by recognizing these three probabilities are mutually exclusive and exhaustive.

$$1 - 0.0883 - 0.5438 = \boxed{0.3679}$$

- Q4.** (10 points) A uniformly distributed population has a mean $\mu = 250$ and a standard deviation $\sigma = 18$. When a sample of size $n = 144$ is taken, what is the probability of getting a sample mean between 247 and 253?

Solution: This situation is described by a mean-sampling distribution. We find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{144}} = 1.5$$

Even though the population is uniformly distributed, the sampling distribution is approximately normal.

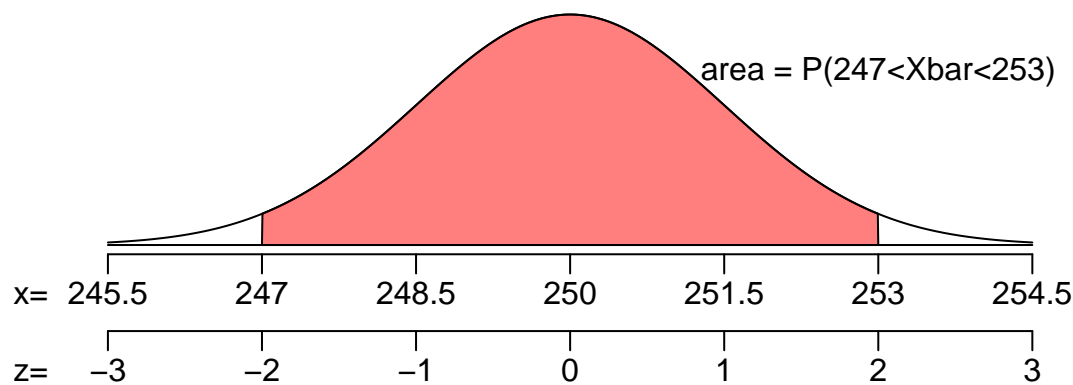
$$\bar{X} \sim N(250, 1.5)$$

We find the z -scores.

$$z_{\text{LOWER}} = \frac{247 - 250}{1.5} = -2$$

$$z_{\text{UPPER}} = \frac{253 - 250}{1.5} = 2$$

Here is a sketch:



We find the probability.

$$\begin{aligned} P(247 < \bar{X} < 253) &= P(-2 < Z < 2) \\ &= \Phi(2) - \Phi(-2) \\ &= \boxed{0.9545} \end{aligned}$$

- Q5.** (10 points) A sample of size $n = 144$ has a mean $\bar{x} = 253.92$ and a standard deviation $s = 17.54$. Construct a confidence interval of the population's mean based on the sample using a confidence level $\gamma = 0.99$.

Solution: We find z^* .

$$\begin{aligned} z^* &= \Phi^{-1}\left(\frac{0.99 + 1}{2}\right) \\ &= \Phi^{-1}(0.995) \\ &= 2.58 \end{aligned}$$

We find the standard error.

$$SE = \frac{17.54}{\sqrt{144}} = 1.4617$$

We determine the confidence interval. (Old \bar{x} on first print)

$$\begin{aligned} CI &= \bar{x} \pm z^* SE \\ &= 254.87 \pm (2.58)(1.4617) \\ &= \boxed{(251.1, 258.6)} \end{aligned}$$

We determine the confidence interval. (Answer to updated problem.)

$$\begin{aligned} CI &= \bar{x} \pm z^* SE \\ &= 253.92 \pm (2.58)(1.4617) \\ &= \boxed{(250.1, 257.7)} \end{aligned}$$

Q6. (10 points) Imagine you thought a uniformly distributed population had a mean $\mu = 250$ and standard deviation $s = 18$. However, you are skeptical, so you decide to run a two-tailed hypothesis test with a significance level $\alpha = 0.01$.

Your sample of size $n = 144$ results in a mean $\bar{x} = 253.92$.

(a) State the hypotheses.

Solution:

$$H_0 : \mu = 250$$

$$H_A : \mu \neq 250$$

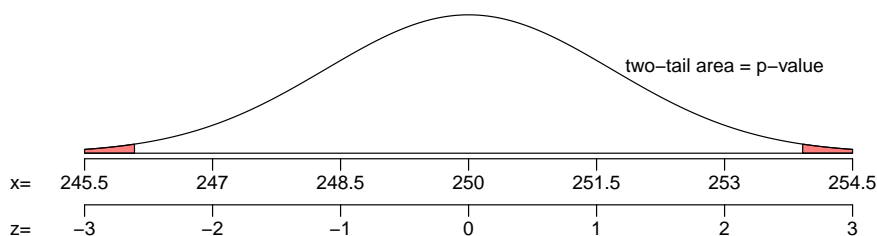
(b) Describe and sketch the null's sampling distribution.

Solution: We need the standard error for a sampling distribution.

$$SE = \frac{18}{\sqrt{144}} = 1.5$$

We think the null's sampling distribution is normal with mean $\mu = 250$ and standard deviation $\sigma = 1.5$.

$$\bar{X} \sim N(250, 1.5)$$



(c) Find a z score of the sample mean.

Solution:

$$z = \frac{253.92 - 250}{1.5} = 2.61$$

(d) Calculate the p -value.

Solution:

$$p\text{-value} = 2 \cdot \Phi(-2.61) = (2)(0.0045) = 0.009$$

(e) Make your conclusion.

Solution: We reject the null hypothesis because $0.009 < 0.01$.