

MATH 181 2ND EXAM PRACTICE B SOLUTIONS

SPRING 2019

Name: _____

- Write your **full name** on the line above.
- Show your work. Incorrect answers with work can receive partial credit.
- Attempt every question; showing you understand the question earns some credit.
- If you run out of room for an answer, continue on the back of the page. Before doing so, write “see back” with a circle around it.
- You can use 1 page (front and back) of notes.
- You can use (and probably need) a calculator.
- You can use the Geogebra Scientific Calculator instead of a calculator. You need to put your phone on **airplane mode** and then within the application, start **exam mode**; you should see a green bar with a timer counting up.
- If a question is confusing or ambiguous, please ask for clarification; however, you will not be told how to answer the question.
- **Box your final answer.**
- A formula sheet is attached to this test.

Do not write in this grade table.

Question:	Q1	Q2	Q3	Q4	Q5	Q6	Total
Points:	10	10	10	10	10	10	60
Score:							

Normal Distribution:

$$X \sim \mathcal{N}(\mu, \sigma)$$

μ = population mean

σ = population standard deviation

x = possible value of X

ℓ = percentile of x (left area)

$\Phi(z)$ = standard normal cumulative function

$$z = \frac{x - \mu}{\sigma}$$

$$P(X < x) = \Phi(z)$$

$$\ell = \Phi(z)$$

$$z = \Phi^{-1}(\ell)$$

Bernoulli Distribution:

$$X \sim \text{Bern}(p)$$

X = 0 for fail or 1 for success

p = probability of success

$$P(X = 0) = 1 - p$$

$$P(X = 1) = p$$

$$\mu = p$$

$$\sigma = \sqrt{p(1 - p)}$$

Geometric Distribution:

$$X \sim \text{Geo}(p)$$

X = number of trials until first success

p = probability of success on each trial

n = a possible number of trials

$$P(X = n) = (1 - p)^{n-1}(p)$$

$$\mu = \frac{1}{p}$$

$$\sigma = \sqrt{\frac{1 - p}{p^2}}$$

Mean-Sampling Distribution:

\bar{X} = sample mean

s = sample standard deviation

n = sample size

μ = population mean

σ = population standard deviation

SE = standard error

$$SE = \frac{\sigma}{\sqrt{n}}$$

If $n \geq 30$ (or if population is normal) then:

$$\bar{X} \sim \mathcal{N}(\mu, SE)$$

Binomial Distribution:

$$X \sim \mathcal{B}(n, p)$$

X = number of successes from n trials

p = probability of success on each trial

n = number of trials

k = a possible number of successes

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu = np$$

$$\sigma = \sqrt{np(1 - p)}$$

If $np \geq 10$ and $n(1 - p) \geq 10$, then

$$X \sim \mathcal{N}(\mu, \sigma)$$

Continuity correction:

$$P(X \leq k) \approx \Phi\left(\frac{k + 0.5 - \mu}{\sigma}\right)$$

Confidence Interval:

CI = confidence interval

γ = confidence level

\bar{x} = sample mean

s = sample standard deviation

$$z^* = \Phi^{-1}\left(\frac{\gamma + 1}{2}\right)$$

$$SE \approx \frac{s}{\sqrt{n}}$$

$$CI = \bar{x} \pm z^* SE$$

Hypothesis testing:

$$H_0 : \mu = \mu_0$$

$$H_A : \mu \neq \mu_0$$

\bar{x} = a possible/specific/observed sample mean

s = sample standard deviation

α = significance level

$$\sigma \approx s$$

$$z = \left| \frac{\bar{x} - \mu_0}{SE} \right|$$

$$p\text{-value} = P(|Z| > z)$$

$$= 2\Phi(-z)$$

If $p\text{-value} < \alpha$, then reject H_0 , else retain H_0 .

- Q1.** (10 points) Hannah is curious about the expected number of rolls of a 6-sided die before getting every side, but Hannah forgets how to analyze it mathematically. So, she gets a 6-sided die and rolls it until she sees every number and writes down how many rolls it took. She repeats this over and over, getting the following sample:

18 19 26 23 17 12 11 12 23 16 13 8 10 8 7
 19 14 11 15 17 20 24 12 18 10 9 22 24 14 14

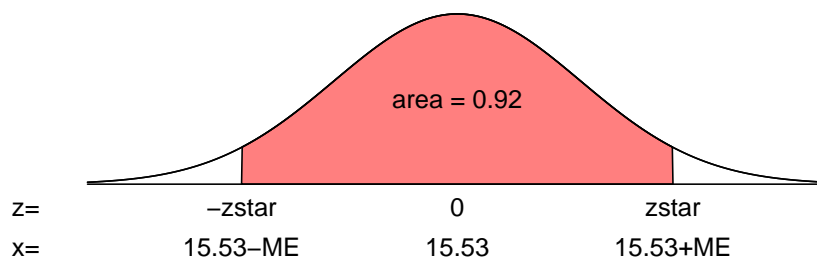
Hannah determines the sample size $n = 30$, sample mean $\bar{x} = 15.53$, and sample standard deviation $s = 5.41$.

- (a) Determine a 92% confidence interval for the expected number of rolls to get all sides. You can assume the sampling distribution is normal (even though the population is not normal).

Solution: We need to consider the sampling distribution. Because $n \geq 30$, we think the sampling distribution is approximately normal. We need standard error for sampling distributions.

$$SE = \frac{5.41}{\sqrt{30}} = 0.9877$$

We draw a sketch, where ME is the margin of error, defined as $ME = z^*SE$.



We can find z^* by recognizing it's left area is 0.96. In other words, $\Phi(z^*) = 0.96$. In other words, $P(Z < z^*) = 0.96$. In other words, $\ell = 0.96$. One way or another, we need to use the z table backwards to find z^* .

$$z^* = \Phi^{-1}(0.96) = 1.75$$

We now can create a confidence interval.

$$\begin{aligned} CI &= \bar{x} \pm ME \\ &= \bar{x} \pm z^*SE \\ &= 15.53 \pm (1.75)(0.9877) \\ &= (13.8, 17.3) \end{aligned}$$

- (b) After this study, would you believe a friend that suggests the expected number of rolls is 11? Why or why not?

Solution: Nope. 11 is outside the confidence interval. In fact, 11 would have a z -score of -4.59 , which makes it quite inconsistent with our study.

Q2. (10 points) Imagine each trial has a 42% chance of success. Let random variable W represent the result of a trial, where 0 means failure and 1 means success.

(a) What is the standard deviation of W ?

Solution: A single trial follows a Bernoulli distribution.

$$\sigma = \sqrt{p(1-p)} = \sqrt{(0.42)(0.58)} = \boxed{0.494}$$

(b) What is the expected number of trials until getting a success?

Solution: Trials until success follows a geometric distribution.

$$\mu = \frac{1}{p} = \frac{1}{0.42} = \boxed{2.38}$$

(c) What is the standard deviation of number of trials until getting a success?

Solution: Trials until success follows a geometric distribution.

$$\sigma = \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{0.58}{(0.42)^2}} = \boxed{1.81}$$

(d) What is the probability of getting 30 successes from 75 trials?

Solution: Number of successes in n trials follows a binomial distribution.

$$P(X = 30) = \binom{75}{30} (0.42)^{30} (0.58)^{45}$$

$$P(X = 30) = \boxed{0.088}$$

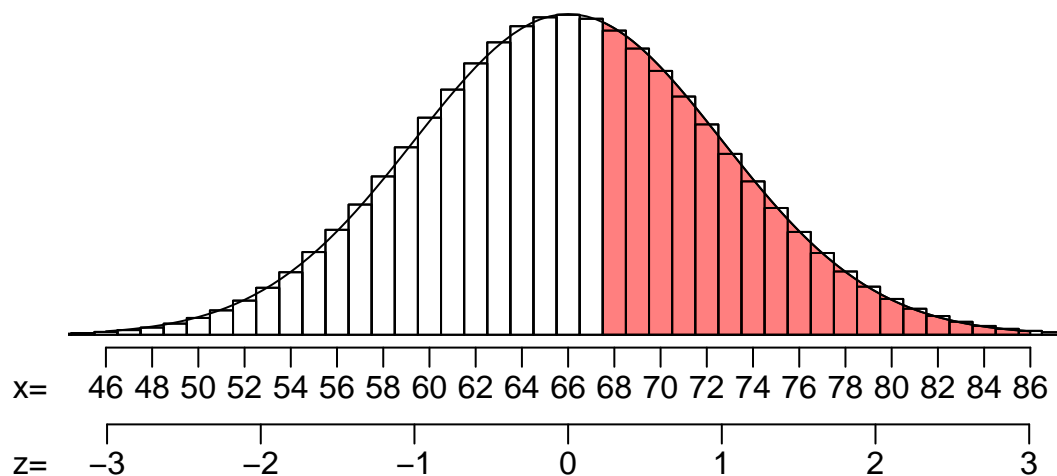
- Q3.** (10 points) If each trial has a 33% chance of success and there are 200 trials, what is the probability that the number of successes is more than 67? Please use a normal approximation with the continuity correction.

Solution: We determine the mean and standard deviation of the binomial distribution.

$$\begin{aligned}\mu &= np \\ &= (200)(0.33) \\ &= 66\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{np(1-p)} \\ &= \sqrt{(200)(0.33)(0.67)} \\ &= 6.65\end{aligned}$$

A sketch is helpful... your sketch can be simpler. If you look closely, the continuity correction should be clear.



In order to estimate the area of the bars higher than 67, we find the area under the curve when $x > 67.5$. That extra 0.5 is the continuity correction, necessary because these bars have width of 1 and are centered on the integers. We find the z-score.

$$z = \frac{67.5 - 66}{6.65} = 0.23$$

We find the right area.

$$\begin{aligned}P(X > 67) &\approx P(Z > 0.23) \\ &= 1 - \Phi(0.23) \\ &= 1 - 0.5910 \\ &= \boxed{0.4090}\end{aligned}$$

- Q4.** (10 points) Perform a two-tail hypothesis test with $\mu_0 = 100$, $n = 50$, $\bar{x} = 103.2$, $s = 14.4$, and $\alpha = 0.10$.

Solution: State the hypotheses.

$$H_0 : \mu = 100$$

$$H_A : \mu \neq 100$$

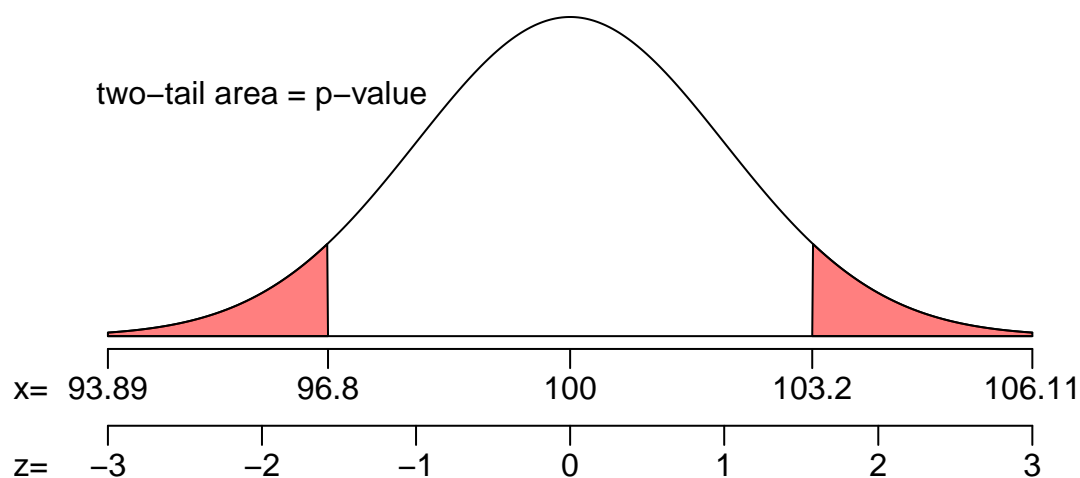
Calculate the standard error of the null's sampling distribution by assuming $\sigma \approx s$.

$$SE = \frac{14.4}{\sqrt{50}} = 2.036468$$

Find a z -score.

$$z = \frac{103.2 - 100}{2.036} = 1.57$$

Draw a sketch of the null's sampling distribution.



We calculate the p -value.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-1.57) \\ &= (2)(0.0582) \\ &= 0.1164 \end{aligned}$$

Compare p -value to α .

$$0.1164 > 0.10$$

$$p\text{-value} > \alpha$$

So, we retain the null.

Q5. (10 points) Let $X \sim \mathcal{N}(500, 20)$.

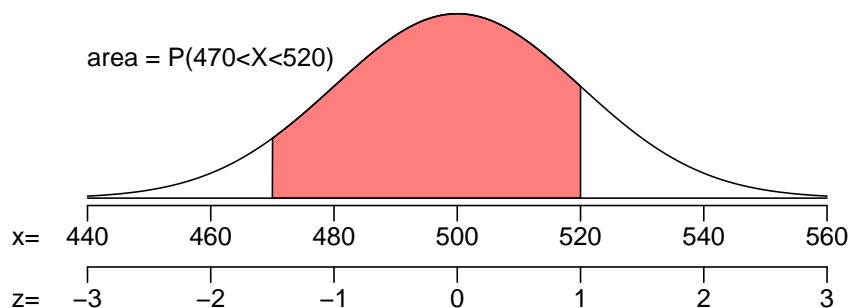
(a) Evaluate $P(470 < X < 520)$.

Solution: Find the z -scores.

$$z_{\text{LOWER}} = \frac{470 - 500}{20} = -1.5$$

$$z_{\text{UPPER}} = \frac{520 - 500}{20} = 1$$

Make a sketch.

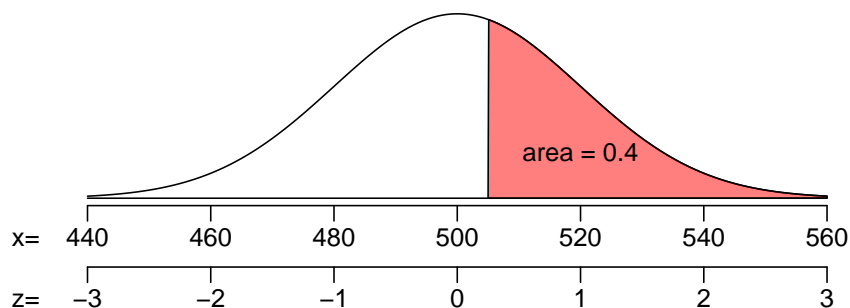


Calculate the probability.

$$\begin{aligned} P(470 < X < 520) &= \Phi(1) - \Phi(-1.5) \\ &= 0.8413 - 0.0668 \\ &= \boxed{0.7745} \end{aligned}$$

(b) Determine x such that $P(X > x) = 0.40$.

Solution: Draw a sketch.



We want the horizontal coordinate of the boundary. Find the z .

$$z = \Phi^{-1}(0.6) = 0.25$$

Find x .

$$x = \mu + z\sigma = 500 + (0.25)(20) = \boxed{505}$$

- Q6.** (10 points) There is a continuous population with $\mu = 500$ and $\sigma = 20$. What is the probability that a sample of size 100 has a mean between 500 and 503?

Solution: Calculate the standard error.

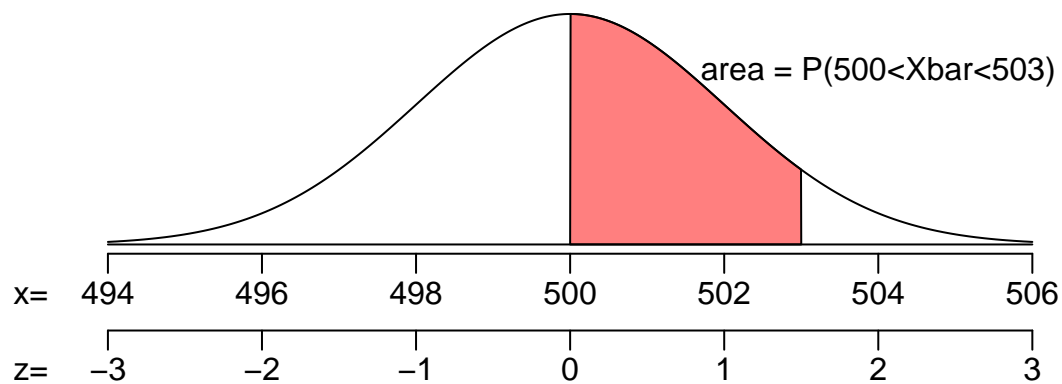
$$SE = \frac{20}{\sqrt{100}} = 2$$

Find the z-scores.

$$z_{\text{LOWER}} = \frac{500 - 500}{2} = 0$$

$$z_{\text{UPPER}} = \frac{503 - 500}{2} = 1.5$$

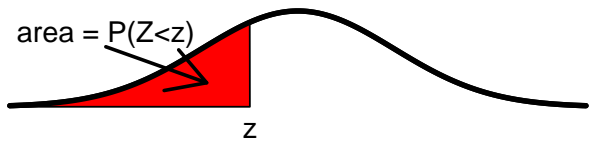
Sketch the sampling distribution with the shaded region.



Calculate the probability.

$$\begin{aligned} P(500 < \bar{X} < 503) &= \Phi(1.5) - \Phi(0) \\ &= 0.9332 - 0.5000 \\ &= \boxed{0.4332} \end{aligned}$$

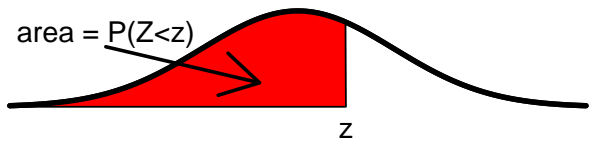
STANDARD NORMAL CUMULATIVE FUNCTION

 $z < 0$ 

z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$
-3.00	0.0013	-2.50	0.0062	-2.00	0.0228	-1.50	0.0668	-1.00	0.1587	-0.50	0.3085
-2.99	0.0014	-2.49	0.0064	-1.99	0.0233	-1.49	0.0681	-0.99	0.1611	-0.49	0.3121
-2.98	0.0014	-2.48	0.0066	-1.98	0.0239	-1.48	0.0694	-0.98	0.1635	-0.48	0.3156
-2.97	0.0015	-2.47	0.0068	-1.97	0.0244	-1.47	0.0708	-0.97	0.1660	-0.47	0.3192
-2.96	0.0015	-2.46	0.0069	-1.96	0.0250	-1.46	0.0721	-0.96	0.1685	-0.46	0.3228
-2.95	0.0016	-2.45	0.0071	-1.95	0.0256	-1.45	0.0735	-0.95	0.1711	-0.45	0.3264
-2.94	0.0016	-2.44	0.0073	-1.94	0.0262	-1.44	0.0749	-0.94	0.1736	-0.44	0.3300
-2.93	0.0017	-2.43	0.0075	-1.93	0.0268	-1.43	0.0764	-0.93	0.1762	-0.43	0.3336
-2.92	0.0018	-2.42	0.0078	-1.92	0.0274	-1.42	0.0778	-0.92	0.1788	-0.42	0.3372
-2.91	0.0018	-2.41	0.0080	-1.91	0.0281	-1.41	0.0793	-0.91	0.1814	-0.41	0.3409
-2.90	0.0019	-2.40	0.0082	-1.90	0.0287	-1.40	0.0808	-0.90	0.1841	-0.40	0.3446
-2.89	0.0019	-2.39	0.0084	-1.89	0.0294	-1.39	0.0823	-0.89	0.1867	-0.39	0.3483
-2.88	0.0020	-2.38	0.0087	-1.88	0.0301	-1.38	0.0838	-0.88	0.1894	-0.38	0.3520
-2.87	0.0021	-2.37	0.0089	-1.87	0.0307	-1.37	0.0853	-0.87	0.1922	-0.37	0.3557
-2.86	0.0021	-2.36	0.0091	-1.86	0.0314	-1.36	0.0869	-0.86	0.1949	-0.36	0.3594
-2.85	0.0022	-2.35	0.0094	-1.85	0.0322	-1.35	0.0885	-0.85	0.1977	-0.35	0.3632
-2.84	0.0023	-2.34	0.0096	-1.84	0.0329	-1.34	0.0901	-0.84	0.2005	-0.34	0.3669
-2.83	0.0023	-2.33	0.0099	-1.83	0.0336	-1.33	0.0918	-0.83	0.2033	-0.33	0.3707
-2.82	0.0024	-2.32	0.0102	-1.82	0.0344	-1.32	0.0934	-0.82	0.2061	-0.32	0.3745
-2.81	0.0025	-2.31	0.0104	-1.81	0.0351	-1.31	0.0951	-0.81	0.2090	-0.31	0.3783
-2.80	0.0026	-2.30	0.0107	-1.80	0.0359	-1.30	0.0968	-0.80	0.2119	-0.30	0.3821
-2.79	0.0026	-2.29	0.0110	-1.79	0.0367	-1.29	0.0985	-0.79	0.2148	-0.29	0.3859
-2.78	0.0027	-2.28	0.0113	-1.78	0.0375	-1.28	0.1003	-0.78	0.2177	-0.28	0.3897
-2.77	0.0028	-2.27	0.0116	-1.77	0.0384	-1.27	0.1020	-0.77	0.2206	-0.27	0.3936
-2.76	0.0029	-2.26	0.0119	-1.76	0.0392	-1.26	0.1038	-0.76	0.2236	-0.26	0.3974
-2.75	0.0030	-2.25	0.0122	-1.75	0.0401	-1.25	0.1056	-0.75	0.2266	-0.25	0.4013
-2.74	0.0031	-2.24	0.0125	-1.74	0.0409	-1.24	0.1075	-0.74	0.2296	-0.24	0.4052
-2.73	0.0032	-2.23	0.0129	-1.73	0.0418	-1.23	0.1093	-0.73	0.2327	-0.23	0.4090
-2.72	0.0033	-2.22	0.0132	-1.72	0.0427	-1.22	0.1112	-0.72	0.2358	-0.22	0.4129
-2.71	0.0034	-2.21	0.0136	-1.71	0.0436	-1.21	0.1131	-0.71	0.2389	-0.21	0.4168
-2.70	0.0035	-2.20	0.0139	-1.70	0.0446	-1.20	0.1151	-0.70	0.2420	-0.20	0.4207
-2.69	0.0036	-2.19	0.0143	-1.69	0.0455	-1.19	0.1170	-0.69	0.2451	-0.19	0.4247
-2.68	0.0037	-2.18	0.0146	-1.68	0.0465	-1.18	0.1190	-0.68	0.2483	-0.18	0.4286
-2.67	0.0038	-2.17	0.0150	-1.67	0.0475	-1.17	0.1210	-0.67	0.2514	-0.17	0.4325
-2.66	0.0039	-2.16	0.0154	-1.66	0.0485	-1.16	0.1230	-0.66	0.2546	-0.16	0.4364
-2.65	0.0040	-2.15	0.0158	-1.65	0.0495	-1.15	0.1251	-0.65	0.2578	-0.15	0.4404
-2.64	0.0041	-2.14	0.0162	-1.64	0.0505	-1.14	0.1271	-0.64	0.2611	-0.14	0.4443
-2.63	0.0043	-2.13	0.0166	-1.63	0.0516	-1.13	0.1292	-0.63	0.2643	-0.13	0.4483
-2.62	0.0044	-2.12	0.0170	-1.62	0.0526	-1.12	0.1314	-0.62	0.2676	-0.12	0.4522
-2.61	0.0045	-2.11	0.0174	-1.61	0.0537	-1.11	0.1335	-0.61	0.2709	-0.11	0.4562
-2.60	0.0047	-2.10	0.0179	-1.60	0.0548	-1.10	0.1357	-0.60	0.2743	-0.10	0.4602
-2.59	0.0048	-2.09	0.0183	-1.59	0.0559	-1.09	0.1379	-0.59	0.2776	-0.09	0.4641
-2.58	0.0049	-2.08	0.0188	-1.58	0.0571	-1.08	0.1401	-0.58	0.2810	-0.08	0.4681
-2.57	0.0051	-2.07	0.0192	-1.57	0.0582	-1.07	0.1423	-0.57	0.2843	-0.07	0.4721
-2.56	0.0052	-2.06	0.0197	-1.56	0.0594	-1.06	0.1446	-0.56	0.2877	-0.06	0.4761
-2.55	0.0054	-2.05	0.0202	-1.55	0.0606	-1.05	0.1469	-0.55	0.2912	-0.05	0.4801
-2.54	0.0055	-2.04	0.0207	-1.54	0.0618	-1.04	0.1492	-0.54	0.2946	-0.04	0.4840
-2.53	0.0057	-2.03	0.0212	-1.53	0.0630	-1.03	0.1515	-0.53	0.2981	-0.03	0.4880
-2.52	0.0059	-2.02	0.0217	-1.52	0.0643	-1.02	0.1539	-0.52	0.3015	-0.02	0.4920
-2.51	0.0060	-2.01	0.0222	-1.51	0.0655	-1.01	0.1562	-0.51	0.3050	-0.01	0.4960
-2.50	0.0062	-2.00	0.0228	-1.50	0.0668	-1.00	0.1587	-0.50	0.3085	0.00	0.5000

STANDARD NORMAL CUMULATIVE FUNCTION

$z > 0$



z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772	2.50	0.9938
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.01	0.9778	2.51	0.9940
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.02	0.9783	2.52	0.9941
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.03	0.9788	2.53	0.9943
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.04	0.9793	2.54	0.9945
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.05	0.9798	2.55	0.9946
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.06	0.9803	2.56	0.9948
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.07	0.9808	2.57	0.9949
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.08	0.9812	2.58	0.9951
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.09	0.9817	2.59	0.9952
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.10	0.9821	2.60	0.9953
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463	2.11	0.9826	2.61	0.9955
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474	2.12	0.9830	2.62	0.9956
0.13	0.5517	0.63	0.7357	1.13	0.8708	1.63	0.9484	2.13	0.9834	2.63	0.9957
0.14	0.5557	0.64	0.7389	1.14	0.8729	1.64	0.9495	2.14	0.9838	2.64	0.9959
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.15	0.9842	2.65	0.9960
0.16	0.5636	0.66	0.7454	1.16	0.8770	1.66	0.9515	2.16	0.9846	2.66	0.9961
0.17	0.5675	0.67	0.7486	1.17	0.8790	1.67	0.9525	2.17	0.9850	2.67	0.9962
0.18	0.5714	0.68	0.7517	1.18	0.8810	1.68	0.9535	2.18	0.9854	2.68	0.9963
0.19	0.5753	0.69	0.7549	1.19	0.8830	1.69	0.9545	2.19	0.9857	2.69	0.9964
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.20	0.9861	2.70	0.9965
0.21	0.5832	0.71	0.7611	1.21	0.8869	1.71	0.9564	2.21	0.9864	2.71	0.9966
0.22	0.5871	0.72	0.7642	1.22	0.8888	1.72	0.9573	2.22	0.9868	2.72	0.9967
0.23	0.5910	0.73	0.7673	1.23	0.8907	1.73	0.9582	2.23	0.9871	2.73	0.9968
0.24	0.5948	0.74	0.7704	1.24	0.8925	1.74	0.9591	2.24	0.9875	2.74	0.9969
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.25	0.9878	2.75	0.9970
0.26	0.6026	0.76	0.7764	1.26	0.8962	1.76	0.9608	2.26	0.9881	2.76	0.9971
0.27	0.6064	0.77	0.7794	1.27	0.8980	1.77	0.9616	2.27	0.9884	2.77	0.9972
0.28	0.6103	0.78	0.7823	1.28	0.8997	1.78	0.9625	2.28	0.9887	2.78	0.9973
0.29	0.6141	0.79	0.7852	1.29	0.9015	1.79	0.9633	2.29	0.9890	2.79	0.9974
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.30	0.9893	2.80	0.9974
0.31	0.6217	0.81	0.7910	1.31	0.9049	1.81	0.9649	2.31	0.9896	2.81	0.9975
0.32	0.6255	0.82	0.7939	1.32	0.9066	1.82	0.9656	2.32	0.9898	2.82	0.9976
0.33	0.6293	0.83	0.7967	1.33	0.9082	1.83	0.9664	2.33	0.9901	2.83	0.9977
0.34	0.6331	0.84	0.7995	1.34	0.9099	1.84	0.9671	2.34	0.9904	2.84	0.9977
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678	2.35	0.9906	2.85	0.9978
0.36	0.6406	0.86	0.8051	1.36	0.9131	1.86	0.9686	2.36	0.9909	2.86	0.9979
0.37	0.6443	0.87	0.8078	1.37	0.9147	1.87	0.9693	2.37	0.9911	2.87	0.9979
0.38	0.6480	0.88	0.8106	1.38	0.9162	1.88	0.9699	2.38	0.9913	2.88	0.9980
0.39	0.6517	0.89	0.8133	1.39	0.9177	1.89	0.9706	2.39	0.9916	2.89	0.9981
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713	2.40	0.9918	2.90	0.9981
0.41	0.6591	0.91	0.8186	1.41	0.9207	1.91	0.9719	2.41	0.9920	2.91	0.9982
0.42	0.6628	0.92	0.8212	1.42	0.9222	1.92	0.9726	2.42	0.9922	2.92	0.9982
0.43	0.6664	0.93	0.8238	1.43	0.9236	1.93	0.9732	2.43	0.9925	2.93	0.9983
0.44	0.6700	0.94	0.8264	1.44	0.9251	1.94	0.9738	2.44	0.9927	2.94	0.9984
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744	2.45	0.9929	2.95	0.9984
0.46	0.6772	0.96	0.8315	1.46	0.9279	1.96	0.9750	2.46	0.9931	2.96	0.9985
0.47	0.6808	0.97	0.8340	1.47	0.9292	1.97	0.9756	2.47	0.9932	2.97	0.9985
0.48	0.6844	0.98	0.8365	1.48	0.9306	1.98	0.9761	2.48	0.9934	2.98	0.9986
0.49	0.6879	0.99	0.8389	1.49	0.9319	1.99	0.9767	2.49	0.9936	2.99	0.9986
0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772	2.50	0.9938	3.00	0.9987