Bunker Hill Community College
imagine the possibilities

Math 181 2nd Exam Practice C SOLUTIONS

Spring 2019

Name: ______________________________

- Write your **full name** on the line above.
- Show your work. Incorrect answers with work can receive partial credit.
- Attempt every question; showing you understand the question earns some credit.
- If you run out of room for an answer, continue on the back of the page. Before doing so, write “see back” with a circle around it.
- You can use 1 page (front and back) of notes.
- You can use (and probably need) a calculator.
- You can use the Geogebra Scientific Calculator instead of a calculator. You need to put your phone on **airplane mode** and then within the application, start **exam mode**; you should see a green bar with a timer counting up.
- If a question is confusing or ambiguous, please ask for clarification; however, you will not be told how to answer the question.
- **Box your final answer**.
- A formula sheet is attached to this test.

Do not write in this grade table.

Question:	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**	Total
Points:	10	10	10	10	10	10	10	10	10	90
Score:										

Normal Distribution:

$X \sim \mathcal{N}(\mu,\ \sigma)$
$\mu =$ population mean
$\sigma =$ population standard deviation
$x =$ possible value of X
$\ell =$ percentile of x (left area)
$\Phi(z) =$ standard normal cumulative function

$$z = \frac{x-\mu}{\sigma}$$
$$P(X < x) = \Phi(z)$$
$$\ell = \Phi(z)$$
$$z = \Phi^{-1}(\ell)$$

Bernoulli Distribution:

$X \sim \texttt{Bern}(p)$
$X =$ 0 for fail or 1 for success
$p =$ probability of success

$$\begin{aligned} P(X=0) &= 1-p \\ P(X=1) &= p \\ \mu &= p \\ \sigma &= \sqrt{p(1-p)} \end{aligned}$$

Geometric Distribution:

$X \sim \texttt{Geo}(p)$
$X =$ number of trials until first success
$p =$ probability of success on each trial
$n =$ a possible number of trials

$$\begin{aligned} P(X=n) &= (1-p)^{n-1}(p) \\ \mu &= \frac{1}{p} \\ \sigma &= \sqrt{\frac{1-p}{p^2}} \end{aligned}$$

Mean-Sampling Distribution:

$\bar{X} =$ sample mean
$s =$ sample standard deviation
$n =$ sample size
$\mu =$ population mean
$\sigma =$ population standard deviation
$SE =$ standard error

$$SE = \frac{\sigma}{\sqrt{n}}$$

If $n \geq 30$ (or if population is normal) then:

$$\bar{X} \sim \mathcal{N}(\mu,\ SE)$$

Binomial Distribution:

$X \sim \mathcal{B}(n,\ p)$
$X =$ number of successes from n trials
$p =$ probability of success on each trial
$n =$ number of trials
$k =$ a possible number of successes

$$\begin{aligned} P(X=k) &= \binom{n}{k} p^k (1-p)^{n-k} \\ \mu &= np \\ \sigma &= \sqrt{np(1-p)} \end{aligned}$$

If $np \geq 10$ and $n(1-p) \geq 10$, then

$$X \sim \mathcal{N}(\mu,\ \sigma)$$

Continuity correction:

$$P(X \leq k) \approx \Phi\left(\frac{k+0.5-\mu}{\sigma}\right)$$

Confidence Interval:

$CI =$ confidence interval
$\gamma =$ confidence level
$\bar{x} =$ sample mean
$s =$ sample standard deviation

$$z^\star = \Phi^{-1}\left(\frac{\gamma+1}{2}\right)$$
$$SE \approx \frac{s}{\sqrt{n}}$$
$$CI = \bar{x} \pm z^\star SE$$

Hypothesis testing:

$H_0 : \quad \mu = \mu_0$
$H_A : \quad \mu \neq \mu_0$
$\bar{x} =$ a possible/specific/observed sample mean
$s =$ sample standard deviation
$\alpha =$ significance level

$$\sigma \approx s$$
$$z = \frac{\bar{x}-\mu_0}{SE}$$

$$\begin{aligned} p\text{-value} &= P(|Z| > |z|) \\ &= 2 \cdot \Phi(-|z|) \end{aligned}$$

If p-value $< \alpha$, then reject H_0, else retain H_0.

Q1. (10 points) Let random variable X be normally distributed with mean $\mu = 50$ and standard deviation $\sigma = 12$. What is the probability that X is between 46 and 54?

Solution:

$$z_{\text{LOWER}} = \frac{46 - 50}{12} = -0.33$$

$$z_{\text{UPPER}} = \frac{54 - 50}{12} = 0.33$$

$$\begin{aligned} P(46 < X < 54) &= \Phi(0.33) - \Phi(-0.33) \\ &= 0.6293 - 0.3707 \\ &= \boxed{0.2586} \end{aligned}$$

Q2. (10 points) Let random variable $\bar{X}$ be the sample mean of 36 draws from a normally distributed population with mean $\mu = 50$ and standard deviation $\sigma = 12$. What is the probability that $\bar{X}$ is between 46 and 54?

Solution:

$$SE = \frac{12}{\sqrt{36}} = 2$$

$$z_{\text{LOWER}} = \frac{46 - 50}{2} = -2$$

$$z_{\text{UPPER}} = \frac{54 - 50}{2} = 2$$

$$\begin{aligned} P(46 < \bar{X} < 54) &= \Phi(2) - \Phi(-2) \\ &= 0.9772 - 0.0228 \\ &= \boxed{0.9544} \end{aligned}$$

Q3. (10 points) Let random variable X be normally distributed with mean $\mu = 50$ and standard deviation $\sigma = 12$. What is the x-score of the 80th percentile?

Solution:

$$z = \Phi^{-1}(0.80) = 0.84$$

$$0.84 = \frac{x - 50}{12}$$

$$x = 50 + (0.84)(12) = \boxed{60.08}$$

Q4. (10 points) Let random variable Y be normally distributed with mean $\mu = 72$ and an unknown standard deviation σ. However, you know the 90th percentile is $y = 79$. What is the distribution's standard deviation?

Solution:

$$z = \Phi^{-1}(0.90) = 1.28$$

$$1.28 = \frac{79 - 72}{\sigma}$$

$$\sigma = \frac{79 - 72}{1.28} = \boxed{5.47}$$

Q5. (10 points) Let random variable W be normally distributed with an unknown mean μ and standard deviation $\sigma = 0.5$. However, you know the 30th percentile is $w = 8$. What is the distribution's mean?

Solution:

$$z = \Phi^{-1}(0.30) = -0.52$$

$$-0.52 = \frac{8 - \mu}{0.5}$$

$$\mu = 8 + (0.5)(0.52) = \boxed{8.26}$$

Q6. (10 points) Let each trial have a probability of success $p = 0.61$.

(a) What is the probability that in 400 trials there are 250 successes?

Solution:

$$P(X = 250) = \binom{400}{250}(0.61)^{250}(0.39)^{150}$$

$$P(X = 250) = \boxed{0.034}$$

(b) What is the probability that in 400 trials there are at least 250 successes? (Please use a normal approximation and continuity correction. Also, remember that $p = 0.61$.)

Solution: Find the mean and standard deviation (binomial distribution).

$$\mu = (400)(0.61) = 244$$

$$\sigma = \sqrt{(400)(0.61)(0.39)} = 9.75$$

Find the z-score (with appropriate continuity correction).

$$z = \frac{249.5 - 244}{9.75} = 0.56$$

Find the probability.

$$\begin{aligned} P(X \geq 250) &= P(X > 249.5) \\ &= P(Z > 0.56) \\ &= 1 - P(Z < 0.56) \\ &= 1 - \Phi(0.56) \\ &= 1 - 0.7123 \\ &= \boxed{0.288} \end{aligned}$$

Q7. (10 points) A random sample of size $n = 89$ has a mean $\bar{x} = 23.4$ and a sample standard deviation $s = 5.6$ (and no apparent skew). Determine a confidence interval of the population's mean using a confidence level of 75%.

Solution: Determine $z^\star$.

$$\begin{aligned} z^\star &= \Phi^{-1}\left(\frac{0.75+1}{2}\right) \\ z^\star &= \Phi^{-1}(0.875) \\ z^\star &= 1.15 \end{aligned}$$

Determine SE.

$$SE = \frac{5.6}{\sqrt{89}} = 0.625$$

Find the confidence interval.

$$\begin{aligned} CI &= \bar{x} \pm z^\star SE \\ &= 23.4 \pm (1.15)(0.625) \\ &= \boxed{(22.7\,,\,24.1)} \end{aligned}$$

Q8. (10 points) A population is claimed to have a mean $\mu = 678$. However, you are skeptical, so you decide you'll take a random sample and run a two-tail hypothesis test with a significance level $\alpha = 0.05$.

Your random sample of size $n = 211$ results in a sample mean of $\bar{x} = 664.4$ and a sample standard deviation $s = 101.3$. What do you conclude?

Solution: State the hypotheses.

$$H_0: \quad \mu = 678$$
$$H_A: \quad \mu \neq 678$$

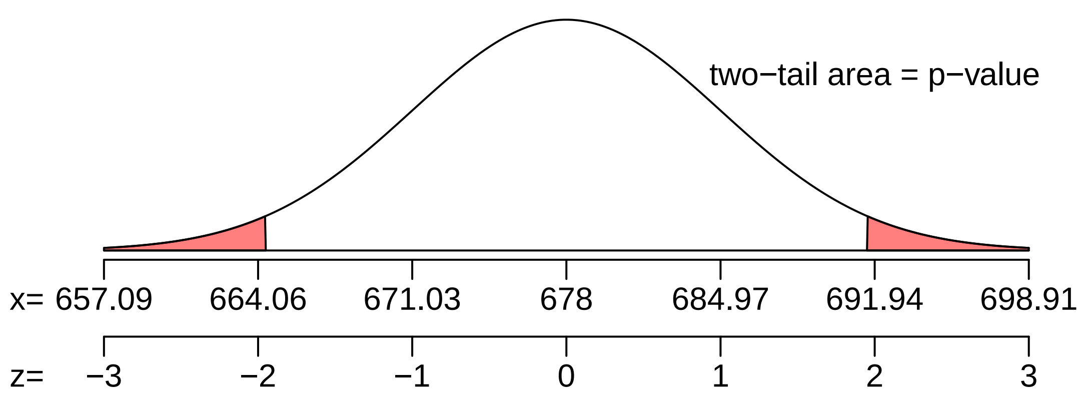
Find the standard error.

$$SE = \frac{101.3}{\sqrt{211}} = 6.974$$

Determine the z-score.

$$z = \frac{664.4 - 678}{6.974} = -1.95$$

Draw a sketch of the two-tail area of the null's sampling distribution.



Find the area.

$$\begin{aligned} p\text{-value} &= 2 \cdot \Phi(-1.95) \\ &= (2)(0.0256) \\ &= 0.0512 \end{aligned}$$

Compare p-value and α.

$$0.0512 > 0.05$$
$$p\text{-value} > \alpha$$

Make the conclusion. In this case we $\boxed{\text{retain the null}}$.

Q9. (10 points) What is a sampling distribution?

Solution: A sampling distribution describes our expectations about what might happen (and with what kind of variability) when we sample from a population and determine a statistic (often the sample mean).