**3.9:** **(a):** $\mu = (77 - 32) \times \frac{5}{9} = 25$
$\Delta C = \Delta F \times \frac{5}{9}$
$\sigma = 5 \times \frac{5}{9} = 2.78$
$X \sim \mathcal{N}(25,\ 2.78)$

**(b):** $z = (28 - 25)/2.78 = 1.08$
$\Phi(1.08) = 0.8599$
$P(X > 28) = 1 - 0.8599 = 0.1401$

**(c):** The answers are very close; they only differ due to rounding. The temperature scale (F vs C) should not change the probability of events.

**(d):** Determine the z-scores of $Q_1$ and $Q_3$.

$$z_{\text{LOWER}} = \Phi^{-1}(0.25) = -0.6745$$

$$z_{\text{UPPER}} = \Phi^{-1}(0.75) = 0.6745$$

Determine the temperatures.

$$x_{\text{LOWER}} = (-0.6745)(2.78) + 25 = 23.12$$

$$x_{\text{UPPER}} = (0.6745)(2.78) + 25 = 26.88$$

Find the IQR.

$$26.88 - 23.12 = \boxed{3.76 \text{ C}}$$

**3.10:** Just for my notes... $X \sim \mathcal{N}(55,\ 6)$

**(a):** We want to find a left area. Find $z$.

$$z = \frac{48 - 55}{6} = -1.17$$

Use the $z$ table.

$$P(X < 48) = \Phi(-1.17) = 0.1210$$

**(b):** We want to find a sectional area. Find both $z$ scores.

$$z_{\text{LOWER}} = \frac{60 - 55}{6} = 0.83$$

$$z_{\text{UPPER}} = \frac{65 - 55}{6} = 1.67$$

We take a difference of the areas.

$$P(60 < X < 65) = \Phi(1.67) - \Phi(0.83) = 0.1558$$

**(c):** We are given that right area = 0.10. This corresponds to 90th percentile. To convert percentile to $z$ score, we use the table backwards.

$$z = \Phi^{-1}(0.9) = 1.28$$

From this $z$ score we calculate a height.

$$x = 1.28 \times 6 + 55 = \boxed{62.7 \text{ in}}$$

**(d):** We want to calculate a left area. Determine the $z$ score.

$$z = \frac{54 - 55}{6} = -0.17$$

We get a percentile from this $z$ score.

$$P(X < 54) = \Phi(-0.17) = \boxed{0.4325}$$

**3.11:** Just for my notes... $X \sim \mathcal{N}(1650,\ \sigma)$

**(a):** $z = \Phi^{-1}(0.75) = 0.67$

**(b):** $\mu = \$1650$. The cutoff is \$1800.

**(c):** We use $\sigma = (x - \mu)/z$

$$\sigma = \frac{1800 - 1650}{0.67} = \boxed{\$223.88}$$

**3.12:** Just for my notes... $X \sim \mathcal{N}(72.6,\ 4.78)$

**(a):** We are looking for a left area. Find $z$ of the cutoff.

$$z = \frac{80 - 72.6}{4.78} = 1.55$$

From this $z$ we look up the left area.

$$P(X < 80) = \Phi(1.55) = \boxed{0.9394}$$

**(b):** We are looking for a sectional area. Find the $z$s of the boundaries.

$$z_{\text{LOWER}} = \frac{60 - 72.6}{4.78} = -2.64$$

$$z_{\text{UPPER}} = \frac{80 - 72.6}{4.78} = 1.55$$

We find the area between these $z$ scores by taking a difference of left areas.

$$P(60 < X < 80) = \Phi(1.55) - \Phi(-2.64) = \boxed{0.935}$$

**(c):** We are told the right area is 0.05. This means we are dealing with a 95th percentile, which we convert to a $z$ score by using the table in reverse.

$$z = \Phi^{-1}(0.95) = 1.64$$

We convert this $z$ score into a speed.

$$x = z\sigma + \mu = (1.64)(4.78) + 72.6 = 80.4 \text{ miles/hour}$$

The fastest 5% of vehicles travel **faster than 80.4 miles/hour**.

**(d):** We are given a speed cutoff and asked for the right area.

$$z = \frac{70 - 72.6}{4.78} = -0.54$$

We find the right area.

$$P(X > 70) = 1 - \phi(-0.54) = \boxed{0.7054}$$

$\boxed{\text{About 70.5\% of cars travel faster than the speed limit.}}$ Slow down please.

**3.13:** $X \sim \mathcal{N}(45,\ 3.2)$

$$P(X > 50) = 1 - \Phi\left(\frac{50 - 45}{3.2}\right) = \boxed{0.059}$$

About 6% of passengers incur this fee.

**3.14:** **(a):** We are told the mean, a specific value, and that value's right area:

$$X \sim \mathcal{N}(100,\ \sigma)$$

$$P(X > 132) = 0.02$$

$$P(X < 132) = 0.98$$

An IQ of 132 is a 98th percentile IQ. We determine $z$.

$$z = \Phi^{-1}(0.98) = 2.05$$

We calculate $\sigma$.

$$\sigma = \frac{x - \mu}{z} = \frac{132 - 100}{2.05} = \boxed{15.6\ \text{ IQ points}}$$

**(b):** We are told the population mean, a specific value, and (implicitly) that value's percentile.

$$X \sim \mathcal{N}(185,\ \sigma)$$

$$P(X > 220) = 0.185$$

$$P(X < 220) = 1 - 0.185 = 0.815$$

We determine the $z$ score from the percentile by using the table in reverse.

$$z = \Phi^{-1}(0.815) = 0.90$$

We calculate $\sigma$.

$$\sigma = \frac{x - \mu}{z} = \frac{220 - 185}{0.9} = \boxed{38.9\ \text{ mg/dl}}$$

**3.15:** For my notes: $X \sim \mathcal{N}(89,\ 15)$.

**(a):**

$$P(X > 100) = 1 - \Phi\left(\frac{100 - 89}{15}\right) = \boxed{0.2317}$$

**(b):** With a bid price too low, you will never win. With a big price too high, you might pay too much, and if you have multiple textbook bids, you might win more than one.

**(c):** Let's just make it so we have a 10% chance on each one, giving an expected value of 1 textbook. The chance of losing all ten bids would be $0.9^{10} = 0.35$. Hmm... I guess that is "reasonably sure". (Using a binomial distribution we can find there is a 39% of exactly one, 19% chance of getting two textbooks, and a 5% chance of getting three textbooks...)

**(d):** We want $x$ such that $P(X < x) = 0.1$.

$$z = \Phi^{-1}(0.1) = -1.28$$

$$x = (-1.28)(15) + 89 = \boxed{\$69.65}$$

**3.16:** Okay... this one is a bit harder... $X \sim \mathcal{N}(1500,\ 300)$. Let's first find the percentile of a student who gets 1900.

$$P(X < 1900) = \Phi\left(\frac{1900 - 1500}{300}\right) = 0.9088$$

Thus, only 9.12% of students get above 1900.
Let's find the percentile of a student who gets a 2100.

$$P(X < 2100) = \Phi\left(\frac{2100 - 1500}{300}\right) = 0.9772$$

Thus, only 2.28% of students get above 2100.
We are asked to determine a conditional probability.

$$P(\text{over 2100 GIVEN over 1900}) = \frac{P(\text{over 2100 AND over 1900})}{P(\text{over 1900})}$$

"Over 2100" is a proper subset of "over 1900", thus:

$$P(\text{over 2100 AND over 1900}) = P(\text{over 2100})$$

So,

$$P(\text{over 2100 GIVEN over 1900}) = \frac{2.28}{9.12} = \boxed{0.25}$$

A random student over 1900 has a 0.25 chance of also being over 2100.