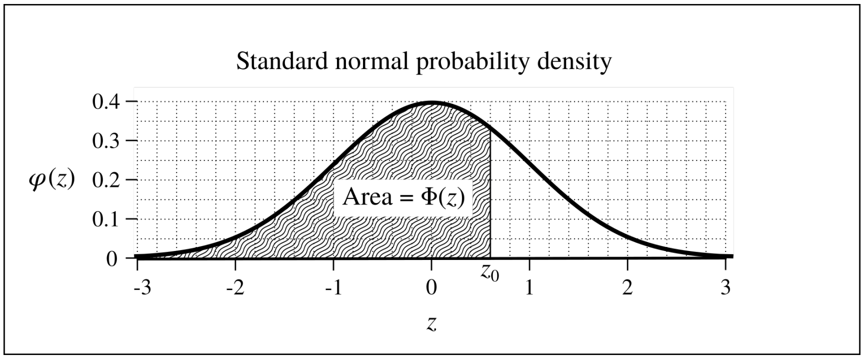
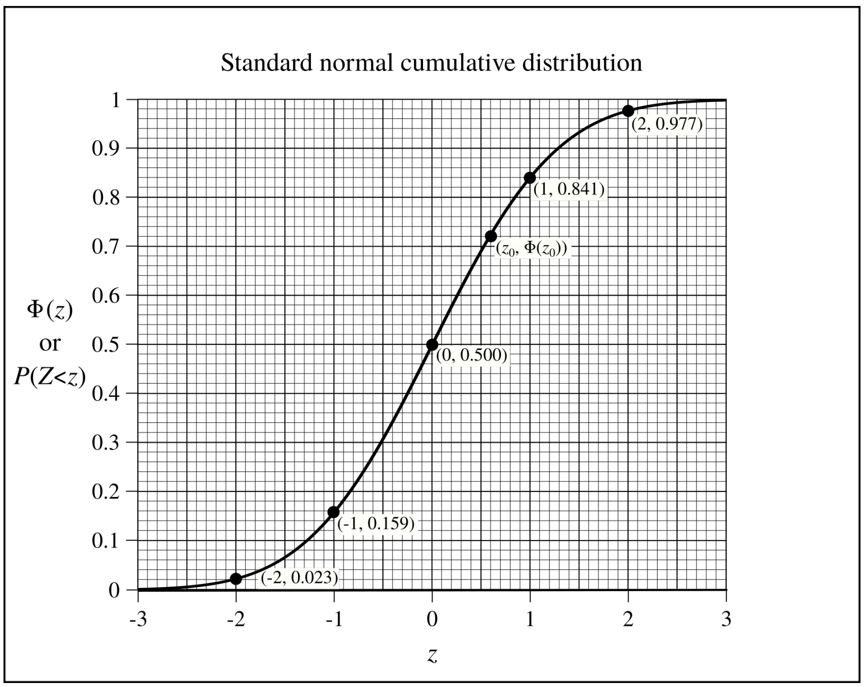
The random variable Z is normally distributed such that $\mu = 0$ and $\sigma = 1$. It has the following probability density function.

$$\varphi(z) = \frac{e^{-z^2/2}}{\sqrt{2\pi}}$$

This function gives us the bell-shaped curve we are accustomed to. To determine the probability that Z is less than z_0, we find the area under the curve from $-\infty$ to z_0.



If we repeat the process of finding the areas from $-\infty$ to any z, we get the cumulative distribution.

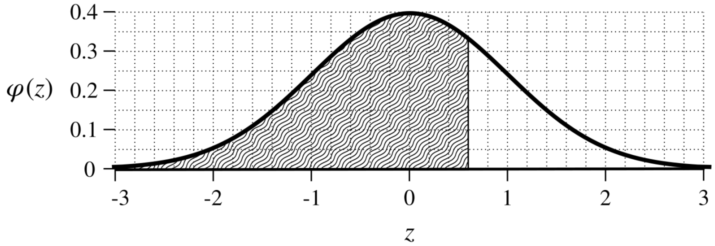


Notice the notation. We use a lower-case phi, φ, for the density function and an upper-case phi ,Φ, for the cumulative function.

The standard normal table gives precise values of Φ as a function of z. (See next page.)

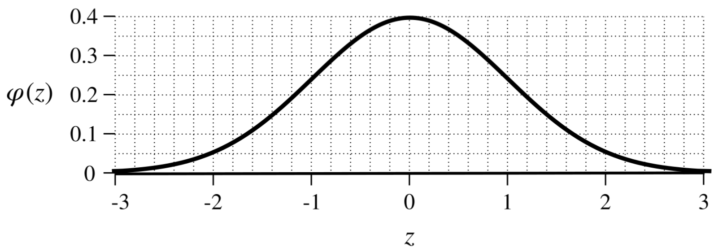
z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
-3.00	0.0013	-2.00	0.0228	-1.00	0.1587	0.00	0.5000	1.00	0.8413	2.00	0.9772
-2.99	0.0014	-1.99	0.0233	-0.99	0.1611	0.01	0.5040	1.01	0.8438	2.01	0.9778
-2.98	0.0014	-1.98	0.0239	-0.98	0.1635	0.02	0.5080	1.02	0.8461	2.02	0.9783
-2.97	0.0015	-1.97	0.0244	-0.97	0.1660	0.03	0.5120	1.03	0.8485	2.03	0.9788
-2.96	0.0015	-1.96	0.0250	-0.96	0.1685	0.04	0.5160	1.04	0.8508	2.04	0.9793
-2.95	0.0016	-1.95	0.0256	-0.95	0.1711	0.05	0.5199	1.05	0.8531	2.05	0.9798
-2.94	0.0016	-1.94	0.0262	-0.94	0.1736	0.06	0.5239	1.06	0.8554	2.06	0.9803
-2.93	0.0017	-1.93	0.0268	-0.93	0.1762	0.07	0.5279	1.07	0.8577	2.07	0.9808
-2.92	0.0018	-1.92	0.0274	-0.92	0.1788	0.08	0.5319	1.08	0.8599	2.08	0.9812
-2.91	0.0018	-1.91	0.0281	-0.91	0.1814	0.09	0.5359	1.09	0.8621	2.09	0.9817
-2.90	0.0019	-1.90	0.0287	-0.90	0.1841	0.10	0.5398	1.10	0.8643	2.10	0.9821
-2.89	0.0019	-1.89	0.0294	-0.89	0.1867	0.11	0.5438	1.11	0.8665	2.11	0.9826
-2.88	0.0020	-1.88	0.0301	-0.88	0.1894	0.12	0.5478	1.12	0.8686	2.12	0.9830
-2.87	0.0021	-1.87	0.0307	-0.87	0.1922	0.13	0.5517	1.13	0.8708	2.13	0.9834
-2.86	0.0021	-1.86	0.0314	-0.86	0.1949	0.14	0.5557	1.14	0.8729	2.14	0.9838
-2.85	0.0022	-1.85	0.0322	-0.85	0.1977	0.15	0.5596	1.15	0.8749	2.15	0.9842
-2.84	0.0023	-1.84	0.0329	-0.84	0.2005	0.16	0.5636	1.16	0.8770	2.16	0.9846
-2.83	0.0023	-1.83	0.0336	-0.83	0.2033	0.17	0.5675	1.17	0.8790	2.17	0.9850
-2.82	0.0024	-1.82	0.0344	-0.82	0.2061	0.18	0.5714	1.18	0.8810	2.18	0.9854
-2.81	0.0025	-1.81	0.0351	-0.81	0.2090	0.19	0.5753	1.19	0.8830	2.19	0.9857
-2.80	0.0026	-1.80	0.0359	-0.80	0.2119	0.20	0.5793	1.20	0.8849	2.20	0.9861
-2.79	0.0026	-1.79	0.0367	-0.79	0.2148	0.21	0.5832	1.21	0.8869	2.21	0.9864
-2.78	0.0027	-1.78	0.0375	-0.78	0.2177	0.22	0.5871	1.22	0.8888	2.22	0.9868
-2.77	0.0028	-1.77	0.0384	-0.77	0.2206	0.23	0.5910	1.23	0.8907	2.23	0.9871
-2.76	0.0029	-1.76	0.0392	-0.76	0.2236	0.24	0.5948	1.24	0.8925	2.24	0.9875
-2.75	0.0030	-1.75	0.0401	-0.75	0.2266	0.25	0.5987	1.25	0.8944	2.25	0.9878
-2.74	0.0031	-1.74	0.0409	-0.74	0.2296	0.26	0.6026	1.26	0.8962	2.26	0.9881
-2.73	0.0032	-1.73	0.0418	-0.73	0.2327	0.27	0.6064	1.27	0.8980	2.27	0.9884
-2.72	0.0033	-1.72	0.0427	-0.72	0.2358	0.28	0.6103	1.28	0.8997	2.28	0.9887
-2.71	0.0034	-1.71	0.0436	-0.71	0.2389	0.29	0.6141	1.29	0.9015	2.29	0.9890
-2.70	0.0035	-1.70	0.0446	-0.70	0.2420	0.30	0.6179	1.30	0.9032	2.30	0.9893
-2.69	0.0036	-1.69	0.0455	-0.69	0.2451	0.31	0.6217	1.31	0.9049	2.31	0.9896
-2.68	0.0037	-1.68	0.0465	-0.68	0.2483	0.32	0.6255	1.32	0.9066	2.32	0.9898
-2.67	0.0038	-1.67	0.0475	-0.67	0.2514	0.33	0.6293	1.33	0.9082	2.33	0.9901
-2.66	0.0039	-1.66	0.0485	-0.66	0.2546	0.34	0.6331	1.34	0.9099	2.34	0.9904
-2.65	0.0040	-1.65	0.0495	-0.65	0.2578	0.35	0.6368	1.35	0.9115	2.35	0.9906
-2.64	0.0041	-1.64	0.0505	-0.64	0.2611	0.36	0.6406	1.36	0.9131	2.36	0.9909
-2.63	0.0043	-1.63	0.0516	-0.63	0.2643	0.37	0.6443	1.37	0.9147	2.37	0.9911
-2.62	0.0044	-1.62	0.0526	-0.62	0.2676	0.38	0.6480	1.38	0.9162	2.38	0.9913
-2.61	0.0045	-1.61	0.0537	-0.61	0.2709	0.39	0.6517	1.39	0.9177	2.39	0.9916
-2.60	0.0047	-1.60	0.0548	-0.60	0.2743	0.40	0.6554	1.40	0.9192	2.40	0.9918
-2.59	0.0048	-1.59	0.0559	-0.59	0.2776	0.41	0.6591	1.41	0.9207	2.41	0.9920
-2.58	0.0049	-1.58	0.0571	-0.58	0.2810	0.42	0.6628	1.42	0.9222	2.42	0.9922
-2.57	0.0051	-1.57	0.0582	-0.57	0.2843	0.43	0.6664	1.43	0.9236	2.43	0.9925
-2.56	0.0052	-1.56	0.0594	-0.56	0.2877	0.44	0.6700	1.44	0.9251	2.44	0.9927
-2.55	0.0054	-1.55	0.0606	-0.55	0.2912	0.45	0.6736	1.45	0.9265	2.45	0.9929
-2.54	0.0055	-1.54	0.0618	-0.54	0.2946	0.46	0.6772	1.46	0.9279	2.46	0.9931
-2.53	0.0057	-1.53	0.0630	-0.53	0.2981	0.47	0.6808	1.47	0.9292	2.47	0.9932
-2.52	0.0059	-1.52	0.0643	-0.52	0.3015	0.48	0.6844	1.48	0.9306	2.48	0.9934
-2.51	0.0060	-1.51	0.0655	-0.51	0.3050	0.49	0.6879	1.49	0.9319	2.49	0.9936
-2.50	0.0062	-1.50	0.0668	-0.50	0.3085	0.50	0.6915	1.50	0.9332	2.50	0.9938
-2.49	0.0064	-1.49	0.0681	-0.49	0.3121	0.51	0.6950	1.51	0.9345	2.51	0.9940
-2.48	0.0066	-1.48	0.0694	-0.48	0.3156	0.52	0.6985	1.52	0.9357	2.52	0.9941
-2.47	0.0068	-1.47	0.0708	-0.47	0.3192	0.53	0.7019	1.53	0.9370	2.53	0.9943
-2.46	0.0069	-1.46	0.0721	-0.46	0.3228	0.54	0.7054	1.54	0.9382	2.54	0.9945
-2.45	0.0071	-1.45	0.0735	-0.45	0.3264	0.55	0.7088	1.55	0.9394	2.55	0.9946
-2.44	0.0073	-1.44	0.0749	-0.44	0.3300	0.56	0.7123	1.56	0.9406	2.56	0.9948
-2.43	0.0075	-1.43	0.0764	-0.43	0.3336	0.57	0.7157	1.57	0.9418	2.57	0.9949
-2.42	0.0078	-1.42	0.0778	-0.42	0.3372	0.58	0.7190	1.58	0.9429	2.58	0.9951
-2.41	0.0080	-1.41	0.0793	-0.41	0.3409	0.59	0.7224	1.59	0.9441	2.59	0.9952
-2.40	0.0082	-1.40	0.0808	-0.40	0.3446	0.60	0.7257	1.60	0.9452	2.60	0.9953
-2.39	0.0084	-1.39	0.0823	-0.39	0.3483	0.61	0.7291	1.61	0.9463	2.61	0.9955
-2.38	0.0087	-1.38	0.0838	-0.38	0.3520	0.62	0.7324	1.62	0.9474	2.62	0.9956
-2.37	0.0089	-1.37	0.0853	-0.37	0.3557	0.63	0.7357	1.63	0.9484	2.63	0.9957
-2.36	0.0091	-1.36	0.0869	-0.36	0.3594	0.64	0.7389	1.64	0.9495	2.64	0.9959
-2.35	0.0094	-1.35	0.0885	-0.35	0.3632	0.65	0.7422	1.65	0.9505	2.65	0.9960
-2.34	0.0096	-1.34	0.0901	-0.34	0.3669	0.66	0.7454	1.66	0.9515	2.66	0.9961
-2.33	0.0099	-1.33	0.0918	-0.33	0.3707	0.67	0.7486	1.67	0.9525	2.67	0.9962
-2.32	0.0102	-1.32	0.0934	-0.32	0.3745	0.68	0.7517	1.68	0.9535	2.68	0.9963
-2.31	0.0104	-1.31	0.0951	-0.31	0.3783	0.69	0.7549	1.69	0.9545	2.69	0.9964
-2.30	0.0107	-1.30	0.0968	-0.30	0.3821	0.70	0.7580	1.70	0.9554	2.70	0.9965
-2.29	0.0110	-1.29	0.0985	-0.29	0.3859	0.71	0.7611	1.71	0.9564	2.71	0.9966
-2.28	0.0113	-1.28	0.1003	-0.28	0.3897	0.72	0.7642	1.72	0.9573	2.72	0.9967
-2.27	0.0116	-1.27	0.1020	-0.27	0.3936	0.73	0.7673	1.73	0.9582	2.73	0.9968
-2.26	0.0119	-1.26	0.1038	-0.26	0.3974	0.74	0.7704	1.74	0.9591	2.74	0.9969
-2.25	0.0122	-1.25	0.1056	-0.25	0.4013	0.75	0.7734	1.75	0.9599	2.75	0.9970
-2.24	0.0125	-1.24	0.1075	-0.24	0.4052	0.76	0.7764	1.76	0.9608	2.76	0.9971
-2.23	0.0129	-1.23	0.1093	-0.23	0.4090	0.77	0.7794	1.77	0.9616	2.77	0.9972
-2.22	0.0132	-1.22	0.1112	-0.22	0.4129	0.78	0.7823	1.78	0.9625	2.78	0.9973
-2.21	0.0136	-1.21	0.1131	-0.21	0.4168	0.79	0.7852	1.79	0.9633	2.79	0.9974
-2.20	0.0139	-1.20	0.1151	-0.20	0.4207	0.80	0.7881	1.80	0.9641	2.80	0.9974
-2.19	0.0143	-1.19	0.1170	-0.19	0.4247	0.81	0.7910	1.81	0.9649	2.81	0.9975
-2.18	0.0146	-1.18	0.1190	-0.18	0.4286	0.82	0.7939	1.82	0.9656	2.82	0.9976
-2.17	0.0150	-1.17	0.1210	-0.17	0.4325	0.83	0.7967	1.83	0.9664	2.83	0.9977
-2.16	0.0154	-1.16	0.1230	-0.16	0.4364	0.84	0.7995	1.84	0.9671	2.84	0.9977
-2.15	0.0158	-1.15	0.1251	-0.15	0.4404	0.85	0.8023	1.85	0.9678	2.85	0.9978
-2.14	0.0162	-1.14	0.1271	-0.14	0.4443	0.86	0.8051	1.86	0.9686	2.86	0.9979
-2.13	0.0166	-1.13	0.1292	-0.13	0.4483	0.87	0.8078	1.87	0.9693	2.87	0.9979
-2.12	0.0170	-1.12	0.1314	-0.12	0.4522	0.88	0.8106	1.88	0.9699	2.88	0.9980
-2.11	0.0174	-1.11	0.1335	-0.11	0.4562	0.89	0.8133	1.89	0.9706	2.89	0.9981
-2.10	0.0179	-1.10	0.1357	-0.10	0.4602	0.90	0.8159	1.90	0.9713	2.90	0.9981
-2.09	0.0183	-1.09	0.1379	-0.09	0.4641	0.91	0.8186	1.91	0.9719	2.91	0.9982
-2.08	0.0188	-1.08	0.1401	-0.08	0.4681	0.92	0.8212	1.92	0.9726	2.92	0.9982
-2.07	0.0192	-1.07	0.1423	-0.07	0.4721	0.93	0.8238	1.93	0.9732	2.93	0.9983
-2.06	0.0197	-1.06	0.1446	-0.06	0.4761	0.94	0.8264	1.94	0.9738	2.94	0.9984
-2.05	0.0202	-1.05	0.1469	-0.05	0.4801	0.95	0.8289	1.95	0.9744	2.95	0.9984
-2.04	0.0207	-1.04	0.1492	-0.04	0.4840	0.96	0.8315	1.96	0.9750	2.96	0.9985
-2.03	0.0212	-1.03	0.1515	-0.03	0.4880	0.97	0.8340	1.97	0.9756	2.97	0.9985
-2.02	0.0217	-1.02	0.1539	-0.02	0.4920	0.98	0.8365	1.98	0.9761	2.98	0.9986
-2.01	0.0222	-1.01	0.1562	-0.01	0.4960	0.99	0.8389	1.99	0.9767	2.99	0.9986
-2.00	0.0228	-1.00	0.1587	0.00	0.5000	1.00	0.8413	2.00	0.9772	3.00	0.9987

Q1: For each of the following, complete the diagram so it has a shaded region and a probability statement, like the example below.



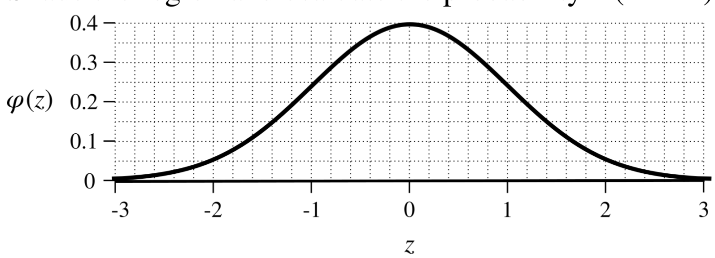
$P(Z < 0.6) = 0.7257$

a: Shade the region and evaluate the probability $P(Z < -1.4)$.



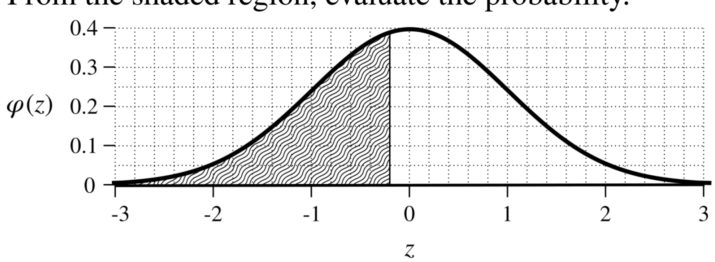
$P(Z < -1.4) =$

b: Shade the region and evaluate the probability $P(Z < 2)$.



$P(Z < 2) =$

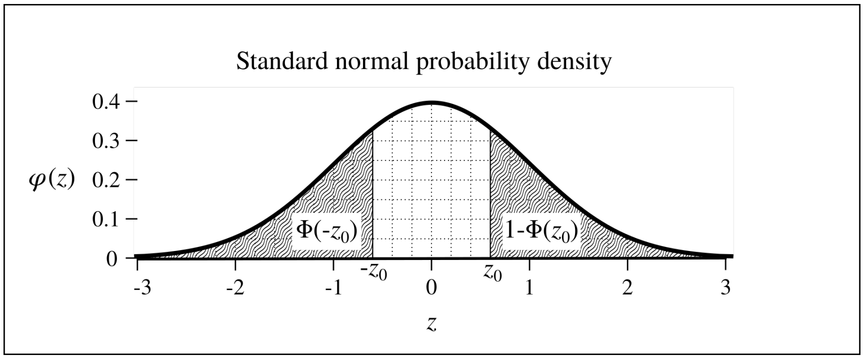
c: From the shaded region, evaluate the probability.



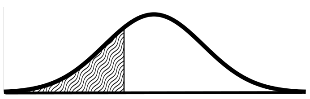
=

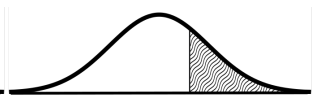
The area under $\varphi(z)$ from $-\infty$ to ∞ is 1. Also, the function $\varphi(z)$ is symmetric. This leads to a useful property:

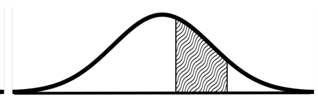
$$\Phi(-z) = 1 - \Phi(z)$$

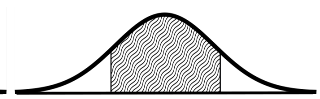


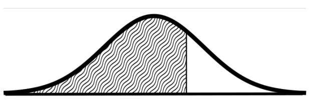
There are five common areas we are asked to find: left, right, sectional, central (symmetric), and two-tailed (symmetric).

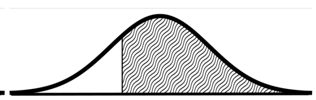

Left Area

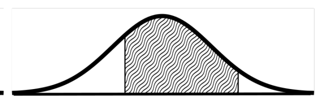

Right Area

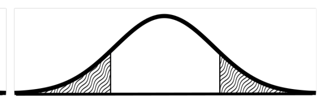

Sectional Area


Central Area


Left Area


Right Area


Sectional Area


Two-tailed Area

$$\begin{aligned}\texttt{Left area} &= P(Z < z)\\ &= \Phi(z)\end{aligned}$$

$$\begin{aligned}\texttt{Right area} &= P(Z > z)\\ &= 1 - \Phi(z)\\ &= \Phi(-z)\end{aligned}$$

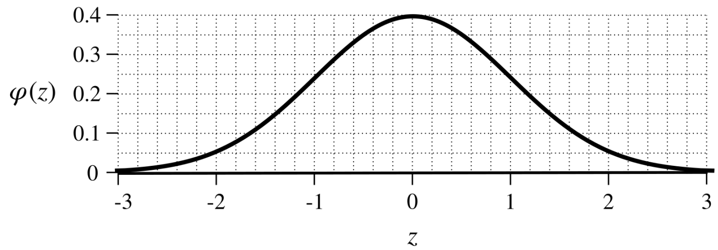
$$\begin{aligned}\texttt{Sectional area} &= P(z_1 < Z < z_2)\\ &= \Phi(z_2) - \Phi(z_1)\end{aligned}$$

$$\begin{aligned}\texttt{Central area} &= P(|Z| < z)\\ &= \Phi(z) - \Phi(-z)\\ &= 1 - 2\Phi(-z)\\ &= 2\Phi(z) - 1\end{aligned}$$

$$\begin{aligned}\texttt{Two-tailed area} &= P(|Z| > z)\\ &= 1 - \Phi(z) + \Phi(-z)\\ &= 2 - 2\Phi(z)\\ &= 2\Phi(-z)\end{aligned}$$

Q2: For each of the following, complete the diagram so it has a shaded region and a probability statement. Also, notice that you can estimate the probability by counting the number of shaded squares; each unit square is worth 1%.

a: Shade the region of and evaluate the probability that Z is more than 1.6.



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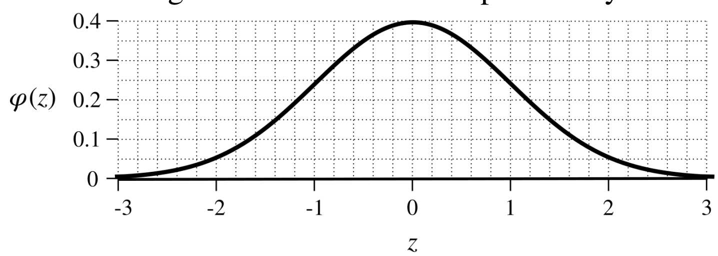
b: Shade the region of and evaluate the probability that Z is between 0.4 and 0.6.



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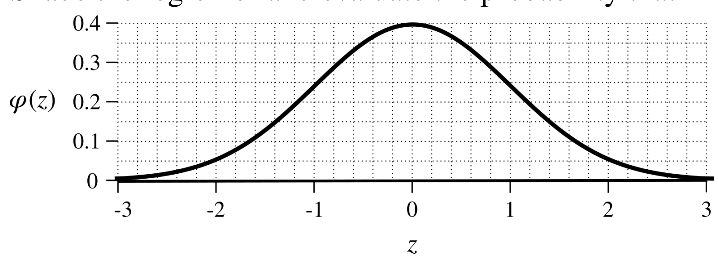
c: Shade the region of and evaluate the probability that Z is between 1 and 2.



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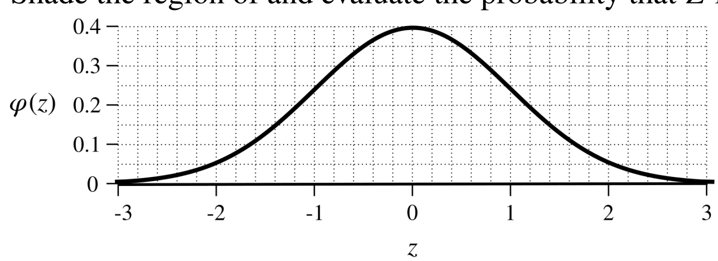
d: Shade the region of and evaluate the probability that Z is between -0.4 and 0.4.



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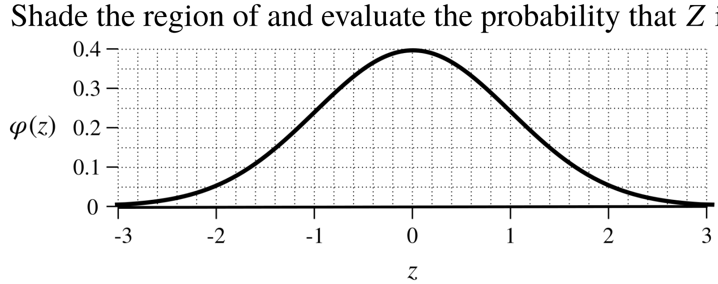
e: Shade the region of and evaluate the probability that Z is less than -0.4 or more than 0.4.



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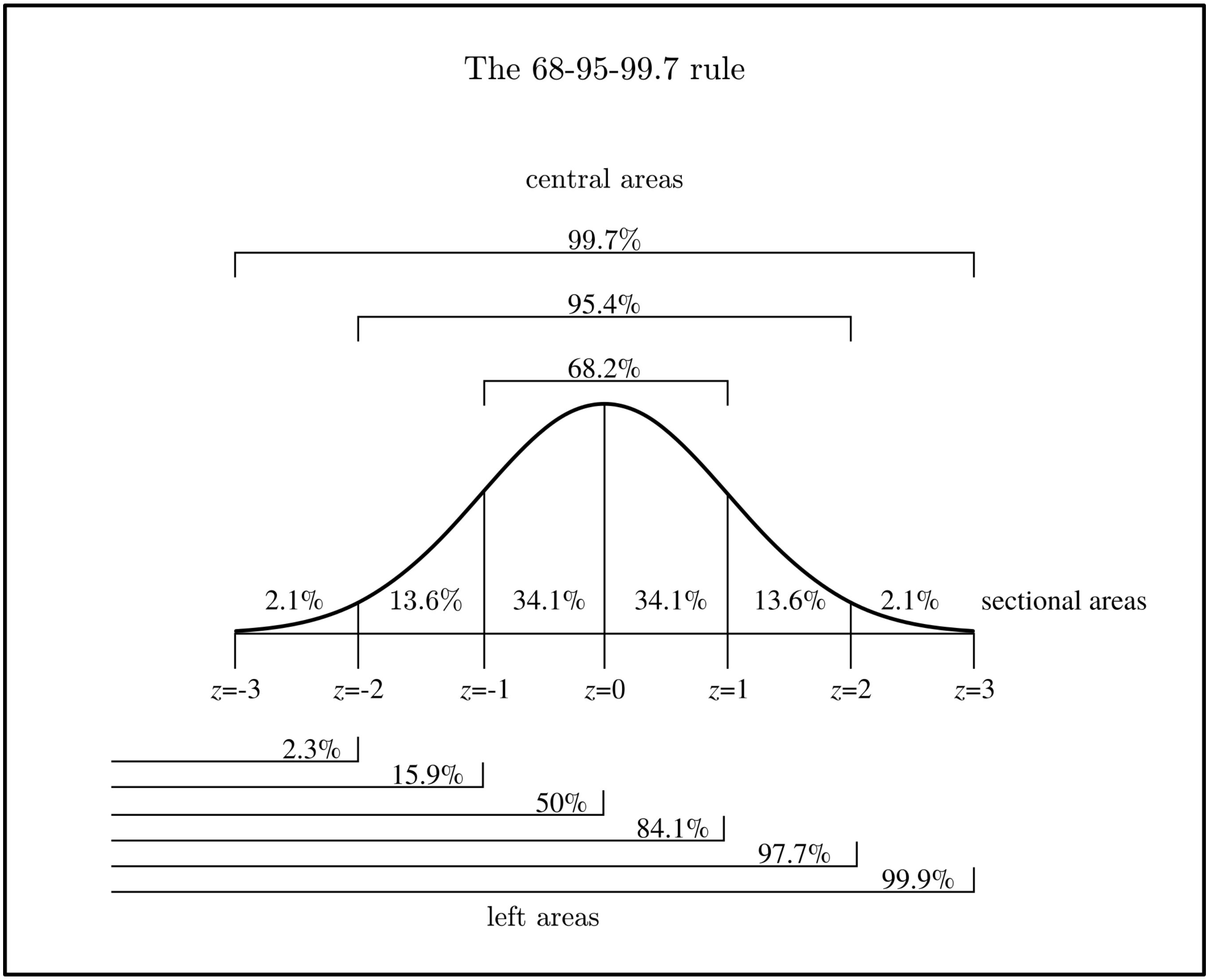
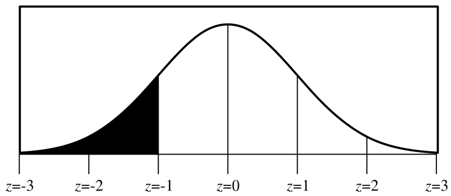
This diagram might be useful. Some of the areas seem to add imperfectly because these numbers are all rounded. Also, it should be noted that $\Phi(-3) = 0.00135 \neq 0$.



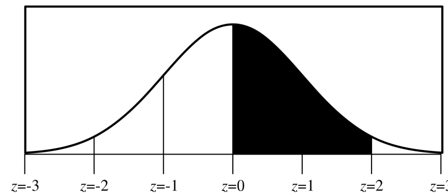
https://en.wikipedia.org/wiki/68-95-99.7_rule

Q3: By using the standard normal table (or the 68-95-99.7 rule), you should be able to determine the following probabilities. For each question, determine the probability (area) of the shaded region or regions. In cases where the bound could be -3 or 3, use $-\infty$ or ∞ instead. Write the answer using the "$P(\texttt{condition}) = \texttt{number}$" format.

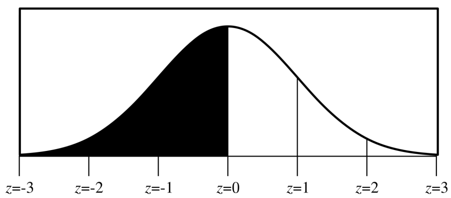
a:



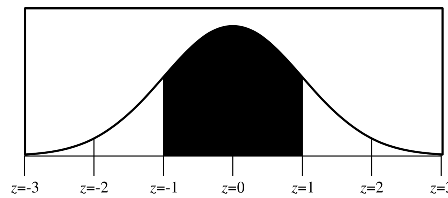
f:



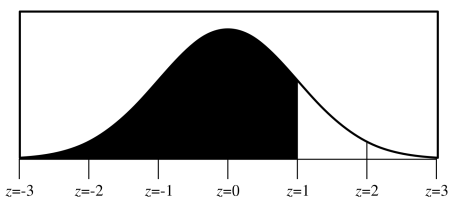
b:



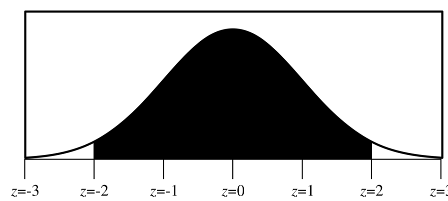
g:



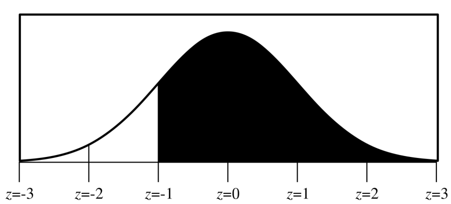
c:



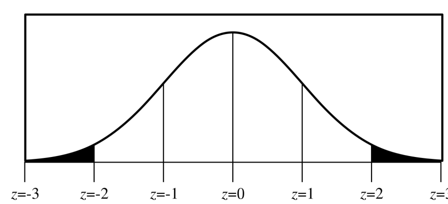
h:



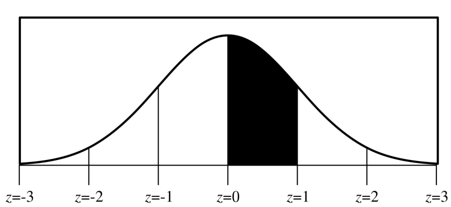
d:



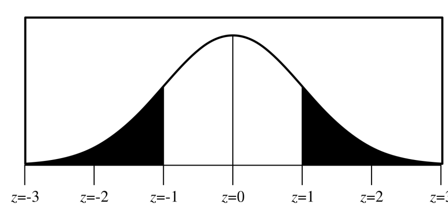
i:



e:



j:



Central Limit Theorem

Let the random variable $\bar{X}$ be the mean of a sample of size n taken from a distribution with mean μ and standard deviation σ. If $n > 30$ then $\bar{X}$ is approximately normally distributed with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$. (If the original distribution is normal, then n can be any number, and the distribution of $\bar{X}$ is exactly normally distributed with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$.)

A typical problem will provide values for μ, σ, n, and boundaries on $\bar{X}$. In these situations, we convert boundaries of $\bar{X}$ into boundaries of Z to calculate the probability.

$$P\left(\bar{x}_1 < \bar{X} < \bar{x}_2\right) = ?$$

$$z_1 = \frac{\bar{x}_1 - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$z_2 = \frac{\bar{x}_2 - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$P\left(\bar{x}_1 < \bar{X} < \bar{x}_2\right) = P\left(z_1 < Z < z_2\right)$$

Normal approximation of binomial distribution

A binomial distribution can often be approximated by a normal distribution. This de Moivre–Laplace theorem was known before the central limit theorem, but can now be understood as a special case of the central limit theorem.

Let the random variable X represent the number of successes from n trials, each with chance p. If $np \geq 5$ and $n(1-p) \geq 5$, then we can approximate the binomial distribution of X as a normal distribution of Y with $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.

Because a normal distribution is continuous while a binomial distribution is discrete, a continuity correction is made. Some examples:

$$\begin{aligned}
P\left(X < x\right) &= P\left(Y < x{-}0.5\right) = P\left(Z < \frac{x - 0.5 - \mu}{\sigma}\right) \\
P\left(X \leq x\right) &= P\left(Y < x{+}0.5\right) = P\left(Z < \frac{x + 0.5 - \mu}{\sigma}\right) \\
P\left(X > x\right) &= P\left(Y > x{+}0.5\right) = P\left(Z > \frac{x + 0.5 - \mu}{\sigma}\right) \\
P\left(X \geq x\right) &= P\left(Y > x{-}0.5\right) = P\left(Z > \frac{x - 0.5 - \mu}{\sigma}\right) \\
P\left(x_1 < X < x_2\right) &= P\left(x_1 + 0.5 < Y < x_2 - 0.5\right) = P\left(\frac{x_1 + 0.5 - \mu}{\sigma} < Z < \frac{x_2 - 0.5 - \mu}{\sigma}\right) \\
P\left(x_1 \leq X \leq x_2\right) &= P\left(x_1 - 0.5 < Y < x_2 + 0.5\right) = P\left(\frac{x_1 - 0.5 - \mu}{\sigma} < Z < \frac{x_2 + 0.5 - \mu}{\sigma}\right) \\
P\left(X = x\right) &= P\left(x - 0.5 < Y < x + 0.5\right) = P\left(\frac{x - 0.5 - \mu}{\sigma} < Z < \frac{x + 0.5 - \mu}{\sigma}\right)
\end{aligned}$$

Q4: An individual is measured from a normal distribution with $\mu = 500$ and $\sigma = 100$. What is the probability that the individual has a measurement greater than 530?

Q5: If 64 individuals are measured from a continuous distribution with $\mu = 500$ and $\sigma = 100$, then what is the probability that their mean is greater than 530?

Q6: If 15 trials each have a 45% chance of success, then what is the probability of getting exactly 7 successes? Use the normal approximation.

We have thoroughly practiced finding areas from z-scores. We might also want to find z-scores from areas. For example, if we wanted to find the value of z_0 that satisfies $P(Z < z_0) = 0.86$, we scan the right-columns for $\Phi(z) \approx 0.86$. We find $\Phi(1.08) = 0.8599$.

z	$\Phi(z)$
$\vdots$	$\vdots$
1.08	0.8599
$\vdots$	$\vdots$

Thus, we estimate $z_0 \approx 1.08$. We can also write this as $\Phi^{-1}(0.8599) = 1.08$. Here are some formulas for determining z from various areas. These are all derived from equations on page 4.

$$\begin{aligned}\texttt{Left area} &= A_{\text{L}} \\ z &= \Phi^{-1}(A_{\text{L}})\end{aligned}$$

$$\begin{aligned}\texttt{Right area} &= A_{\text{R}} \\ z &= \Phi^{-1}(1 - A_{\text{R}})\end{aligned}$$

$$\begin{aligned}\texttt{Central area} &= A_{\text{C}} \\ z &= \Phi^{-1}\left(\frac{A_{\text{C}} + 1}{2}\right)\end{aligned}$$

$$\begin{aligned}\texttt{Two-tailed area} &= A_{\text{T}} \\ z &= \Phi^{-1}\left(1 - \frac{A_{\text{T}}}{2}\right)\end{aligned}$$

Remember, these areas also correspond to probabilities. Also, you don't need to memorize these formulas, as you can figure them out by drawing a quick sketch.

Q7: **a:** Determine z_0 such that $\Phi(z_0) = 0.0505$. In other words, evaluate $\Phi^{-1}(0.0505)$.

b: Determine z_1 such that $\Phi(z_1) \approx 0.99$.

c: Determine z_2 such that $P(Z < z_2) = 55.57\%$

d: Determine z_3 such that $P(Z > z_3) = 15.87\%$

e: Determine z_4 such that $P(-z_4 < Z < z_4) = 68.2\%$

f: Determine z_5 such that $P(|Z| < z_5) = 95\%$

g: Determine z_6 such that $P(|Z| < z_6) = 90\%$

h: Determine z_7 such that $P(|Z| > z_7) = 10\%$

Q8: If the scores on a test are normally distributed with a mean of 80 and a standard deviation of 10, what score is the 84.1th percentile? (Hint: check out the 68-95-99.7 rule.)

Q9: If the scores on a test are normally distributed with a mean of 80 and a standard deviation of 10, what score is the 97.7th percentile?

Q10: If the scores on a test are normally distributed with a mean of 80 and a standard deviation of 10, what score is the 90th percentile?

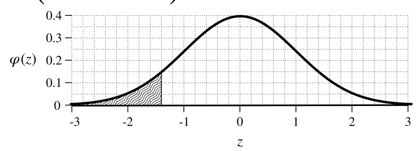
Q11: What is the z-score such that 68.2% of the area lies between $-z$ and z? (Hint: check out the 68-95-99.7 rule.)

Q12: What is the z-score such that 95.4% of the area lies between $-z$ and z?

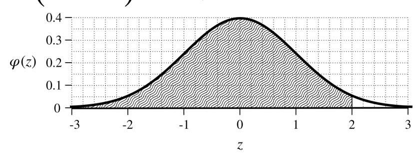
Q13: What is the z-score such that 80% of the area lies between $-z$ and z?

A1:

a: $P(Z < -1.4) = 0.0808$



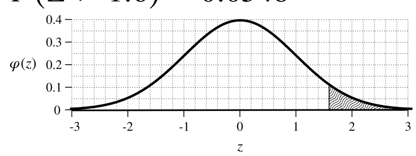
b: $P(Z < 2) = 0.9772$



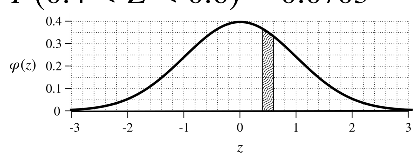
c: $P(Z < -0.2) = 0.4207$

A2:

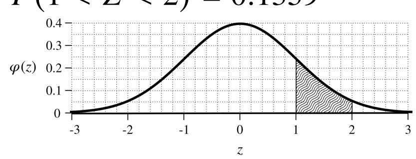
a: $P(Z > 1.6) = 0.0548$



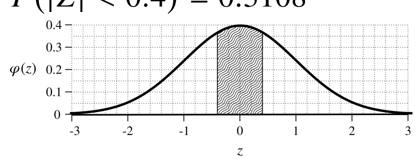
b: $P(0.4 < Z < 0.6) = 0.0703$



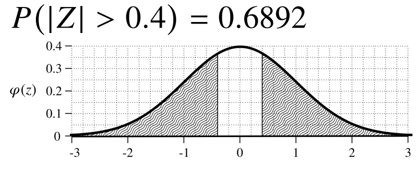
c: $P(1 < Z < 2) = 0.1359$



d: $P(|Z| < 0.4) = 0.3108$



e: $P(|Z| > 0.4) = 0.6892$



A3:

a: $P(Z < -1) = 0.159$

b: $P(Z < 0) = 0.5$

c: $P(Z < 1) = 0.841$

d: $P(-1 < Z) = 0.841$

e: $P(0 < Z < 1) = 0.341$

f: $P(0 < Z < 2) = 0.477$

g: $P(|Z| < 1) = 0.682$

h: $P(|Z| < 2) = 0.954$

i: $P(|Z| > 2) = 0.046$

j: $P(|Z| > 1) = 0.318$

A4: 0.3821

A5: 0.0082

A6: 0.2031

A7:

a: $z_0 = -1.64$

b: $z_1 = 2.33$

c: $z_2 = 0.14$

d: $z_3 = 1$

e: $z_4 = 1$

f: $z_5 = 1.96$

g: $z_6 = 1.64$

h: $z_7 = 1.64$

A8: 90.0

A9: 100.0

A10: $z = \Phi^{-1}(0.9) = 1.2815$
$(1.2815)(10) + 80 \approx \boxed{92.8}$

A11: $z = 1$

A12: $z = 2$

A13: $z = \Phi^{-1}\left(\frac{0.8+1}{2}\right) = \Phi^{-1}(0.9) = 1.282$