2.27: **(a):** $P(\text{1st is blue}) = \frac{3}{10} = 0.3$

(b): $P(\text{2nd is blue GIVEN 1st is blue... with replacement}) = \frac{3}{10} = 0.3$

(c): $P(\text{2nd is blue GIVEN 1st is orange... with replacement}) = \frac{3}{10} = 0.3$

(d): $P(\text{1st is blue AND 2nd is blue... with replacement}) = 0.3^2 = 0.09$

(e): When drawing with replacement, the draws are independent. The probabilities of the second draw do not change based on the result of the first draw.

2.28: **(a):** $\frac{4}{12} \times \frac{3}{11} \approx 0.0909$

(b): $\frac{7}{12} \times \frac{6}{11} = 0.318$

(c): We first calculate the probability of the *complement*.

$$P(\text{no black socks}) = \frac{9}{12} \times \frac{8}{11} = 0.545$$

Then, we use the complement rule.

$$P(\text{at least 1 black sock}) = 1 - 0.545 = \boxed{0.455}$$

(d): 0

(e): We are interested in the union of three mutually exclusive events.

$$\begin{aligned} P(\text{2 blues OR 2 grays OR 2 blacks}) &= P(\text{2 blues}) + P(\text{2 grays}) + P(\text{2 blacks}) \\ &= \frac{4}{12} \cdot \frac{3}{11} + \frac{5}{12} \cdot \frac{4}{11} + \frac{3}{12} \cdot \frac{2}{11} \\ &\approx \boxed{0.288} \end{aligned}$$

2.29: **(a):** When drawing without replacement, we can calculate conditional probabilities by considering which chips are left. After a blue is drawn, we have 5 reds, 2 blues, and 2 oranges.

$$P(B_2|B_1) = \frac{2}{9} = 0.22222$$

(b): After an orange is drawn, we have 5 reds, 3 blues, and 1 oranges.

$$P(B_2|O_1) = \frac{3}{9} = 0.3333$$

(c): We use the general rule for joint probabilities.

$$\begin{aligned} P(B_1 \text{ AND } B_2) &= P(B_1) \cdot P(B_2|B_1) \\ &= \frac{3}{10} \cdot \frac{2}{9} \\ &\approx \boxed{0.0666} \end{aligned}$$

(d): Nope. The first events differently change the probabilities of second events.

2.30: **(a):** To calculate this joint probability, we multiply a marginal and a conditional.

$$P(H_1 \text{ AND } PF_2) = P(H_1) \cdot P(PF_2|H_1)$$

$$= \frac{28}{95} \cdot \frac{59}{94}$$

$$\approx \boxed{0.185}$$

(b): This one is a bit more difficult because some fiction books are hardcovers, and we are sampling without replacement.

$$\begin{aligned} P(F_1 \text{ AND } H_2) &= P\Big([HF_1 \text{ AND } H_2] \text{ OR } [PF_1 \text{ AND } H_2]\Big) \\ &= P(HF_1 \text{ AND } H_2) \quad + \quad P(PF_1 \text{ AND } H_2) \\ &= P(HF_1) \cdot P(H_2|HF_1) \quad + \quad P(PF_1) \cdot P(H_2|PF_1) \\ &= \frac{13}{95} \cdot \frac{27}{94} \quad + \quad \frac{59}{95} \cdot \frac{28}{94} \\ &= \boxed{0.2243001} \end{aligned}$$

(c): This is easier; we are sampling with replacement, so the draws are independent.

$$P(F_1 \text{ AND } H_2) = P(F) \cdot P(H)$$

$$= \frac{72}{95} \cdot \frac{28}{95}$$

$$= \boxed{0.2233795}$$

(d): The answers to parts (b) and (c) are similar because we are only sampling 2 items from a relatively large population (population size = 95). A rule of thumb is when sample size is less than 10% of the population, an independence approximation is warranted.

2.31: First, how many people are wearing leggings?

$$24 - 7 - 4 - 8 = 5$$

We need to consider that there are multiple ways to end up with #leggings = 1 and #jeans = 2.

$$\begin{aligned} P(\text{2 jeans and 1 leggings}) &= P(L_1 J_2 J_3 \text{ OR } J_1 L_2 J_3 \text{ OR } J_1 J_2 L_3) \\ &= P(L_1 J_2 J_3) + P(J_1 L_2 J_3) + P(J_1 J_2 L_3) \\ &= \frac{5 \cdot 7 \cdot 6}{24 \cdot 23 \cdot 22} + \frac{7 \cdot 5 \cdot 6}{24 \cdot 23 \cdot 22} + \frac{7 \cdot 6 \cdot 5}{24 \cdot 23 \cdot 22} \\ &= \boxed{0.05187747} \end{aligned}$$

2.32: There are 365 days in a year.

(a): We just need to consider the chance that the second birthday matches the first birthday.

$$P(\text{first two share a birthday}) = \frac{1}{365} \approx 0.002739726$$

(b): We recognize "at least two share" is the complement of "nobody shares".

$$P(\text{nobody shares}) = 1 \times \frac{364}{365} \times \frac{363}{365} = 0.9917958$$

$$P(\text{at least 2 share}) = 1 - P(\text{nobody shares}) = \boxed{0.0082}$$

2.33: **(a):** $0.13 \times 100 = \boxed{13}$

(b): No. Students at a gym are probably more health conscious than average students.

2.34: **(a):** The four events each have a different payoff and probability.

i	x_i	$P(X = x_i)$	$x_i \cdot P(X = x_i)$	$(x_i - \mu)^2 \cdot P(X = x_i)$
1	\$0	0.5	0	8.55
2	\$5	1/4	1.25	0.19
3	\$10	12/52	2.308	7.94
4	\$30	1/52	0.577	12.87
		Totals:	$\mu = 4.135$	$\sigma^2 = 29.54$

We calculate the standard deviation from the variance.

$$\sigma = \sqrt{29.54} \approx 5.44$$

The expected winnings are \$4.14 with a standard deviation of \$5.44.

(b): I'm rather risk neutral with these small amounts of money. So, I'm willing to pay \$4.14 to play this game. This would give me a \$0 expected payoff (better than a casino!).

2.35: **(a):** Let's first calculate the probabilities.

$$P(H_1 \text{ AND } H_2 \text{ AND } H_3) = \frac{13 \cdot 12 \cdot 11}{52 \cdot 51 \cdot 50} = 0.01294118$$

$$P(B_1 \text{ AND } B_2 \text{ AND } B_3) = \frac{26 \cdot 25 \cdot 24}{52 \cdot 51 \cdot 50} = 0.1176471$$

$$P(\text{other}) = 1 - 0.0129 - 0.1176 = 0.8695$$

We make a table. Notice the textbook uses rows where I use columns.

i	x_i	$P(X = x_i)$	$x_i \cdot P(X = x_i)$	$(x_i - \mu)^2 \cdot P(X = x_i)$
1	\$0	0.8695	0	11.18
2	\$50	0.0129	0.645	27.79
3	\$25	0.1176	2.940	53.93
		Totals:	$\mu = 3.585$	$\sigma^2 = 92.9$

We calculate the standard deviation from the variance.

$$\sigma = \sqrt{92.9} \approx 9.64$$

The expected winnings are \$3.59 with a standard deviation of 9.64.

(b): The expected winnings would be -1.41 USD.

$$3.59 - 5 = -1.41$$

The variability does not change. Every value under x_i is decreased by 5, but so is the mean, so the last column will be unchanged.

i	x_i	$P(X = x_i)$	$x_i \cdot P(X = x_i)$	$(x_i - \mu)^2 \cdot P(X = x_i)$
1	-5	0.8695	-4.35	11.18
2	45	0.0129	0.58	27.79
3	20	0.1176	2.35	53.93
		Totals:	$\mu = -1.41$	$\sigma^2 = 92.9$

We can also use rules in chapter 2.4 (where I've replaced a and b with 1s).

$$E(X + Y) = E(X) + E(Y)$$

$$Var(X + Y) = Var(X) + Var(Y)$$

The expected value of a constant is the constant. The variance of a constant is 0.

$$E(X - 5) = E(X) - 5 = 3.59 - 5 = -1.41$$

$$Var(X - 5) = Var(X) + Var(-5) = 92.9 - 0 = 92.9$$

(c): No. It has a negative expected value.