Bunker Hill Community College
imagine the possibilities

Math 181 First Exam

Spring 2019

Name: ANSWER KEY

- Write your **full name** on the line above.
- Show your work. Incorrect answers with work can receive partial credit.
- Attempt every question; showing you understand the question earns some credit.
- If you run out of room for an answer, continue on the back of the page. Before doing so, write "see back" with a circle around it.
- You can use 1 page (front and back) of notes.
- You can use (and probably need) a calculator.
- You can use the Geogebra Scientific Calculator instead of a calculator. You need to put your phone on **airplane mode** and then within the application, start **exam mode**; you should see a green bar with a timer counting up.
- If a question is confusing or ambiguous, please ask for clarification; however, you will not be told how to answer the question.
- **Box your final answer**.
- A formula sheet is attached to this test.

Do not write in this grade table.

Question:	**1**	**2**	**3**	**4**	**5**	**6**	Total
Points:	10	8	8	16	9	9	60
Score:							

Sample statistics:

n = sample size
x_i = the ith value in a sample
$\bar{x}$ = sample mean
s = sample standard deviation

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Q_1 = first quartile
m = median
Q_3 = third quartile
IQR = inter-quartile range = $Q3 - Q1$

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

Population parameters:

μ = population mean
σ = population standard deviation

Probability:

Ω = set of all possible equally likely outcomes
A = event A, a set of outcomes
A^c = The complement of A
B = event B, another set of outcomes
$\#(A)$ = size of set, number of outcomes in A
$P(A)$ = probability of A
$P(A \text{ AND } B)$ = probability of both A and B
$P(A \text{ OR } B)$ = probability of either A or B (or both)
$P(A|B)$ = probability of A given B
$\iff$ = "if and only if"

$$P(A) = \frac{\#(A)}{\#(\Omega)}$$

$$0 \le P(A) \le 1$$

$$P(A \text{ AND } B) = P(A) \cdot P(B|A)$$

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$$

$$P(A|B) = \frac{P(A \text{ AND } B)}{P(B)}$$

$$P(A^c) = 1 - P(A)$$

A, B are disjoint (mutually exclusive) $\iff$ $P(A \text{ AND } B) = 0$
A, B are non-disjoint $\iff$ $P(A \text{ AND } B) > 0$
A, B are exhaustive $\iff$ $P(A \text{ OR } B) = 1$
A, B are complements $\iff$ A, B are disjoint and exhaustive $\iff$ $B = A^c$
A, B are independent $\iff$ $P(A \text{ AND } B) = P(A) \times P(B)$ $\iff$ $P(A|B) = P(A)$

Random variables and distributions:

X = random variable
x_i = the ith possible value of X. (Notice different meaning here vs. sample statistics.)
k = number of possible values of X.
$E(X) = \mu$ = expected value of X
σ = standard deviation of X

$$\mu = \sum_{i=1}^{k} x_i \cdot P(X = x_i)$$

$$\sigma = \sqrt{\sum_{i=1}^{k} (x_i - \mu)^2 \cdot P(X = x_i)}$$

1. (10 points) Samuel suspects that coffee impairs short-term memory. Samuel runs a study by asking random BHCC students to participate in a memory challenge: repeating back 7 random digits. Samuel marks whether the participant successfully repeated the digits. Then, Samuel asks the participant whether they had coffee in the last 3 hours. The results are summarized below.

	success	fail	total
coffee	16	7	23
no coffee	11	4	15
total	27	11	38

(a) What kind of study was this?

◯ experimental

√ **observational**

Solution: There was no assignment to groups.

(b) Which group performed better (had a higher proportion of success)?

◯ coffee

√ **no coffee**

Solution: You need to find the proportion of each group.

$$\frac{16}{23} = 0.696$$

$$\frac{11}{15} = 0.733$$

(c) Which hypothesis is the null hypothesis?

√ **The difference in proportions is due to chance.**

◯ The difference in proportions is due to an association between coffee and memory.

(d) Which hypothesis is the alternative hypothesis?

◯ The difference in proportions is due to chance.

√ **The difference in proportions is due to an association between coffee and memory.**

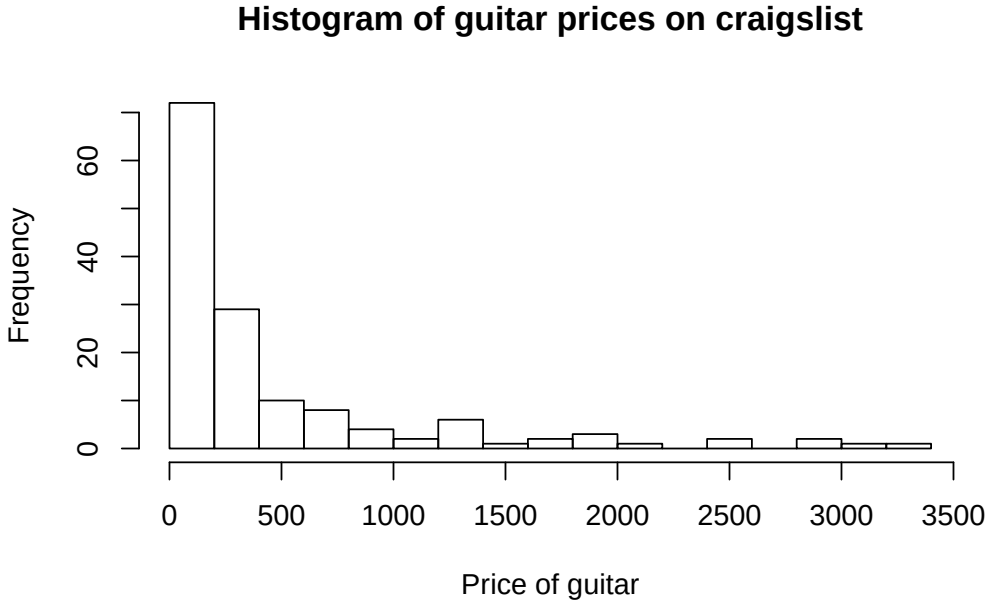
(e) Would you reject the null hypothesis? Why?

◯ Yes. The difference seems too large for chance.

√ **No. The difference seems small enough to be just from chance.**

Solution: With enough thought, these choices should give you the answers to the previous two questions...

2. (8 points) When "acoustic guitar" was searched on craigslist, there were 144 local postings that included a price. These prices are displayed as a histogram.



(a) Which of the following would be an appropriate estimate of the median?
◯ \$1 √ **\$200** ◯ \$500 ◯ \$3500

Solution: The class 0-200 has almost exactly 72 prices, which is half of the 144 prices. You could have also seen the 15-price sample has a similar median.

(b) Which of the following would be an appropriate estimate of the mean?
◯ \$1 ◯ \$200 √ **\$500** ◯ \$3500

Solution: We know the mean should be higher than the median because of the right skew. Also, you could have used the 15-price sample to inform this.

(c) Which of the following would be an appropriate estimate of the standard deviation?
◯ \$1 ◯ \$20 √ **\$700** ◯ \$3500

Solution: Standard deviation is approximately the average deviation from the mean. It can also be thought of as a typical amount of spread. The standard deviation is often about 1/6 of the range. You could have also calculated the standard deviation of the 15-price sample.

(d) Which option best describes this histogram?

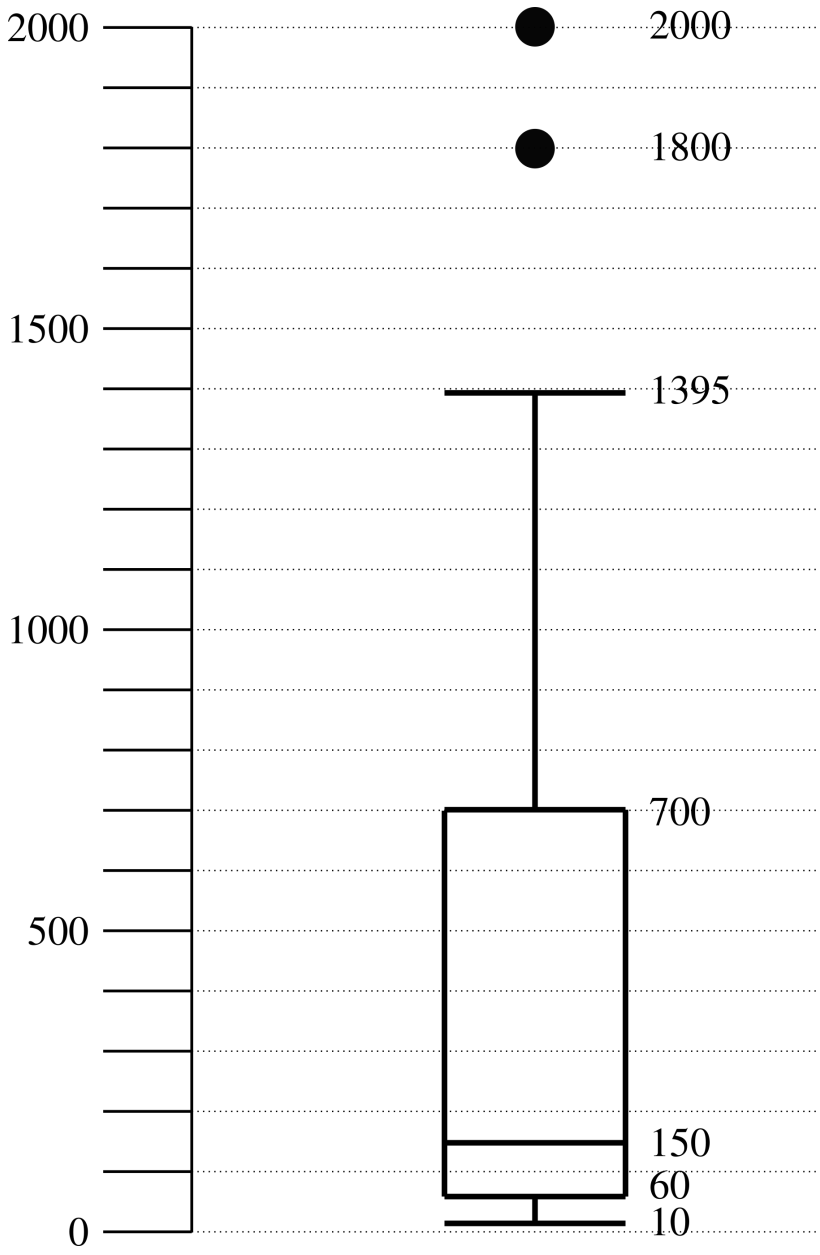
◯ Skew-left
√ **Skew-right**
◯ Superstitious
◯ Symmetric

3. (8 points) From the guitar prices, a random sample of size 15 was taken. Those 15 prices are listed below.

10 20 60 60 75 85 125 150 220 250 275 700 1395 1800 2000

Make a boxplot summarizing these data. Be sure to indicate Q_1, Q_3, median, outliers, and ends of whiskers. The axis below is meant to help you.

Solution: median = 150 ; Q1 = 60 ; Q3 = 700 ; IQR = 640
check for outliers:
60-1.5*640 = -900
700+1.5*640 = 1660



4. (16 points) A jar contains 99 marbles. Each marble has a color and a pattern. The frequencies are shown in the contingency table.

	red	green	blue	total
dotted	7	8	9	24
striped	10	11	12	33
checkered	13	14	15	42
total	30	33	36	99

(a) What is the probability that a random marble is green?

Solution: $\frac{33}{99} \approx 0.33$

(b) What is the probability that a random marble is checkered?

Solution: $\frac{42}{99} \approx 0.42$

(c) What is the probability that a random marble is either striped **or** green (or both)?

Solution: $\frac{33+33-11}{99} \approx 0.56$

(d) What is the probability that a random marble is both red **and** dotted?

Solution: $\frac{7}{99} \approx 0.07$

(e) What is the probability that a random marble is red **given** it is checkered?

Solution: $\frac{13}{42} \approx 0.31$

(f) What is the probability that a random marble is striped **given** it is blue?

Solution: $\frac{12}{36} \approx 0.33$

(g) When picking one random marble, which two events are disjoint (mutually exclusive)?
◯ red, checkered ◯ green, striped √ **blue, red**

Solution: $P(B \text{ AND } R) = 0$

(h) When picking one random marble, which two events are independent?
◯ red, checkered √ **green, striped** ◯ blue, red

Solution: $P(G \text{ AND } S) = P(G) \cdot P(S)$

5. (9 points) Let random variable X represent the number of tails showing when four fair coins are flipped. The probability distribution of X is shown below, where x_i represents the ith possible value of X.

x_i	$P(X = x_i)$
0	0.0625
1	0.25
2	0.375
3	0.25
4	0.0625

(a) What is the probability of 2 tails? In other words, evaluate $P(X = 2)$.

Solution:

$$0.375$$

(b) What is the probability of at least 2 tails? In other words, evaluate $P(X \geq 2)$.

Solution:

$$0.375 + 0.25 + 0.0625 = \boxed{0.6875}$$

(c) What is the probability of more than 2 tails? In other words, evaluate $P(X > 2)$.

Solution:

$$0.25 + 0.0625 = \boxed{0.3125}$$

(d) (2 points (bonus)) Determine x such that $P(X < x) = 0.9375$ and $P(X > x) = 0$.

Solution: Notice that

$$P(X < 4) = 0.0625 + 0.25 + 0.375 + 0.25 = 0.9375$$

and

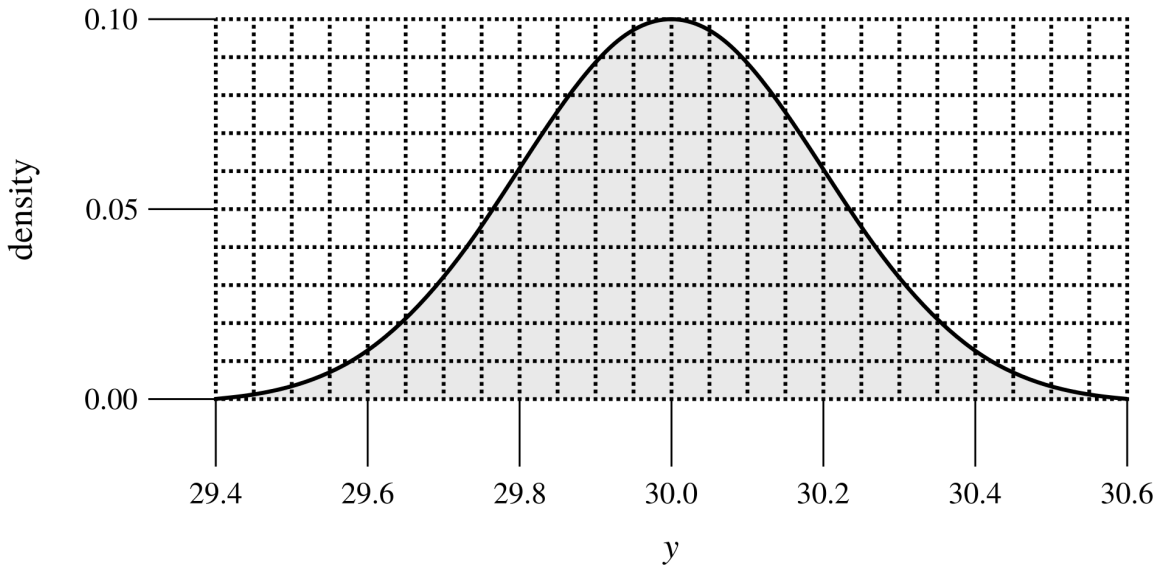
$$P(X > 4) = 0$$

Thus,

$$x = 4$$

6. (9 points) A machine cuts rods to 30 centimeters. However, the machine is not perfect, so the actual lengths have variability.

Let the continuous random variable Y represent the length of a rod. An engineer determines Y approximately follows the distribution shown by the density function below. The entire area under the curve is 100%, and each square is worth 1%.



(a) Estimate the probability a rod is cut to exactly 30.2 centimeters.
In other words, estimate $P(Y = 30.2)$.

Solution: A continuous random variable will never exactly hit a specific number. Remember, we find areas to find probabilities. A rectangle with no width has no area.

0

(b) Estimate the probability a rod is cut to a length between 30.2 centimeters and 30.4 centimeters?
In other words, evaluate $P(30.2 < Y < 30.4)$.

Solution: If you count the squares under the curve from 30.2 until 30.4, you should count about 13 or 14 squares (I guess 12 is fine too).

13% or 14%

(c) Estimate Q_1, the 25th percentile. Answers within ± 0.02 will count.
In other words, estimate Q_1 such that $P(Y < Q_1) = 0.25$.

Solution: Start counting from the left until you reach 25%. Your answer should be between 29.85 and 29.90.