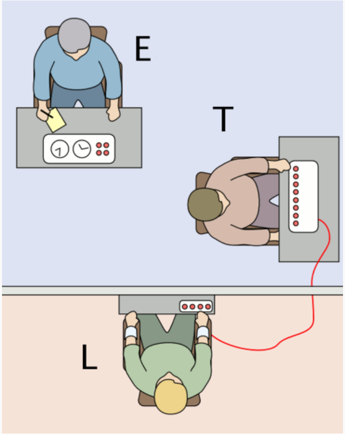
Geometric distribution

Milgram experiment

- Stanley Milgram, a Yale University psychologist, conducted a series of experiments on obedience to authority starting in 1963.
- Experimenter (E) orders the teacher (T), the subject of the experiment, to give severe electric shocks to a learner (L) each time the learner answers a question incorrectly.
- The learner is actually an actor, and the electric shocks are not real, but a prerecorded sound is played each time the teacher administers an electric shock.



http://en.wikipedia.org/wiki/File:

Milgram_Experiment_v2.png

Milgram experiment (cont.)

- These experiments measured the willingness of study participants to obey an authority figure who instructed them to perform acts that conflicted with their personal conscience.
- Milgram found that about 65% of people would obey authority and give such shocks.
- Over the years, additional research suggested this number is approximately consistent across communities and time.

Bernouilli random variables

- Each person in Milgram's experiment can be thought of as a *trial*.
- A person is labeled a *success* if she refuses to administer a severe shock, and *failure* if she administers such shock.
- Since only 35% of people refused to administer a shock, *probability of success* is $p = 0.35$.
- When an individual trial has only two possible outcomes, it is called a *Bernoulli random variable*.

Geometric distribution

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Geometric distribution (cont.)

Geometric distribution describes the waiting time until a success for *independent and identically distributed (iid)* Bernouilli random variables.

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Geometric probabilities

If p represents probability of success, $(1-p)$ represents probability of failure, and n represents number of independent trials

$$P(success\ on\ the\ n^{th}\ trial) = (1-p)^{n-1}p$$

Can we calculate the probability of rolling a 6 for the first time on the 6^{th} roll of a die using the geometric distribution? Note that what was a success (rolling a 6) and what was a failure (not rolling a 6) are clearly defined and one or the other must happen for each trial.

(a) no, on the roll of a die there are more than 2 possible outcomes

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(b) *yes, why not*

$$P(6 \text{ } on \text{ } the \text{ } 6^{th} \text{ } roll) = \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right) \approx 0.067$$

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But how can she test a non-whole number of people?

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Mean and standard deviation of geometric distribution

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- Dr. Smith is expected to test 2.86 people before finding the first one that refuses to administer the shock, give or take 2.3 people.
- These values only make sense in the context of repeating the experiment many many times.