Bunker Hill Community College
imagine the possibilities

Math 181 2nd Exam SOLUTIONS

Spring 2019

Name: ______________________________

- Write your **full name** on the line above.
- Show your work. Incorrect answers with work can receive partial credit.
- Attempt every question; showing you understand the question earns some credit.
- If you run out of room for an answer, continue on the back of the page. Before doing so, write "see back" with a circle around it.
- You can use 1 page (front and back) of notes in addition to the formula sheet (on page 2) and z-table (last page).
- You can use (and probably need) a calculator.
- You can use the Geogebra Scientific Calculator instead of a calculator. You need to put your phone on **airplane mode** and then within the application, start **exam mode**; you should see a green bar with a timer counting up.
- If a question is confusing or ambiguous, please ask for clarification; however, you will not be told how to answer the question.
- **Box your final answer**.
- You can rip off the z-table, but please keep the rest of the test intact.

Do not write in this grade table.

Question:	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	Total
Points:	10	10	10	10	10	10	60
Score:							

Normal Distribution:
$X \sim \mathcal{N}(\mu,\ \sigma)$
$\mu =$ population mean
$\sigma =$ population standard deviation
$x =$ possible value of X
$\ell =$ percentile of x (left area)
$\Phi(z) =$ standard normal cumulative function

$$z = \frac{x-\mu}{\sigma}$$
$$P(X < x) = \Phi(z)$$
$$\ell = \Phi(z)$$
$$z = \Phi^{-1}(\ell)$$

Bernoulli Distribution:
$X \sim \texttt{Bern}(p)$
$X =$ 0 for fail or 1 for success
$p =$ probability of success

$$\begin{aligned} P(X=0) &= 1-p \\ P(X=1) &= p \\ \mu &= p \\ \sigma &= \sqrt{p(1-p)} \end{aligned}$$

Geometric Distribution:
$X \sim \texttt{Geo}(p)$
$X =$ number of trials until first success
$p =$ probability of success on each trial
$n =$ a possible number of trials

$$\begin{aligned} P(X=n) &= (1-p)^{n-1}(p) \\ \mu &= \frac{1}{p} \\ \sigma &= \sqrt{\frac{1-p}{p^2}} \end{aligned}$$

Mean-Sampling Distribution:
$\bar{X} =$ sample mean
$s =$ sample standard deviation
$n =$ sample size
$\mu =$ population mean
$\sigma =$ population standard deviation
$SE =$ standard error

$$SE = \frac{\sigma}{\sqrt{n}}$$

If $n \geq 30$ (or if population is normal) then:

$$\bar{X} \sim \mathcal{N}(\mu,\ SE)$$

Binomial Distribution:
$X \sim \mathcal{B}(n,\ p)$
$X =$ number of successes from n trials
$p =$ probability of success on each trial
$n =$ number of trials
$k =$ a possible number of successes

$$\begin{aligned} P(X=k) &= \binom{n}{k} p^k (1-p)^{n-k} \\ \mu &= np \\ \sigma &= \sqrt{np(1-p)} \end{aligned}$$

If $np \geq 10$ and $n(1-p) \geq 10$, then

$$X \sim \mathcal{N}(\mu,\ \sigma)$$

Continuity correction:

$$P(X \leq k) \approx \Phi\left(\frac{k+0.5-\mu}{\sigma}\right)$$

Confidence Interval:
$CI =$ confidence interval
$\gamma =$ confidence level
$\bar{x} =$ sample mean
$s =$ sample standard deviation

$$z^\star = \Phi^{-1}\left(\frac{\gamma+1}{2}\right)$$
$$SE \approx \frac{s}{\sqrt{n}}$$
$$CI = \bar{x} \pm z^\star SE$$

Hypothesis testing:
$H_0: \quad \mu = \mu_0$
$H_A: \quad \mu \neq \mu_0$
$\bar{x} =$ a possible/specific/observed sample mean
$s =$ sample standard deviation
$\alpha =$ significance level

$$\sigma \approx s$$
$$z = \frac{\bar{x}-\mu_0}{SE}$$

$$\begin{aligned} p\text{-value} &= P(|Z| > |z|) \\ &= 2 \cdot \Phi(-|z|) \end{aligned}$$

If p-value $< \alpha$, then reject H_0, else retain H_0.

Q1. (10 points) Let random variable X be normally distributed with mean $\mu = 33$ and standard deviation $\sigma = 4$.

(a) Evaluate $P(X < 37)$.

Solution: We find a z-score.

$$z = \frac{37 - 33}{4} = 1$$

We use the z table to evaluate the probability.

$$P(X < 37) = P(Z < 1) = \Phi(1) = \boxed{0.8413}$$

(b) Determine x such that $P(X < x) = 0.33$.

Solution: We find the z-score from the z-table.

$$z = \Phi^{-1}(0.33) = -0.44$$

We convert this z-score into an x-score.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 33 + (-0.44)(4) \\ &= \boxed{31.24} \end{aligned}$$

Q2. (10 points) Imagine a scratch-off lottery has a chance of success $p = 0.01$.

(a) What is the mean number of trials until the first success?

Solution: This situaton is described by a geometric distribution.

$$\mu = \frac{1}{p} = \frac{1}{0.01} = \boxed{100}$$

The mean number of trials is 100. The expected number of trials is 100.

(b) What is the probability of getting the first success on the twelfth trial?

Solution: This situation is still described by a geometric distribution. So, we use the geometric probability formula: $P(X = n) = (1 - p)^{n-1}p$.

$$\begin{aligned} P(X = 12) &= (1 - 0.01)^{12-1}(0.01) \\ &= (0.99)^{11}(0.01) \\ &= \boxed{0.00895} \end{aligned}$$

Q3. (10 points) Let each trial have a chance of success $p = 0.25$. We will predict what happens when we have 100 trials.

(a) What is the probability of getting exactly 26 successes?

Solution: This situation is described by a binomial distribution.

$$\begin{aligned} P(X = 26) &= \binom{100}{26}(0.25)^{26}(0.75)^{74} \\ &= \boxed{0.09} \end{aligned}$$

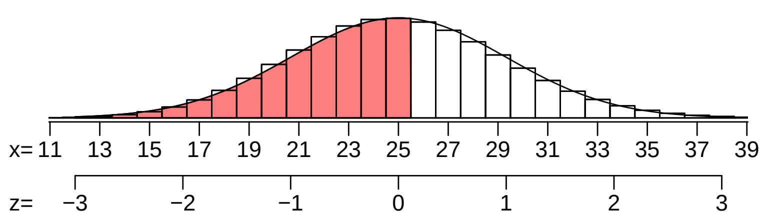
(b) What is the probability of getting fewer than 26 successes? (Use a normal approximation and continuity correction.)

Solution: We find the parameters needed to construct a normal approximation.

$$\mu = np = (100)(0.25) = 25$$

$$\begin{aligned} \sigma &= \sqrt{np(1-p)} \\ &= \sqrt{(100)(0.25)(0.75)} \\ &= 4.33 \end{aligned}$$

Here is a sketch:



We find the z score of the boundary.

$$z = \frac{25.5 - 25}{4.33} = 0.12$$

We calculate the probability.

$$P(X < 26) = P(X < 25.5) = P(Z < 0.12) = \boxed{0.55}$$

(c) What is the probability of getting more than 26 successes?

Solution: The easiest way to do this is by recognizing these three probabilities are mutually exclusive and exhaustive.

$$1 - 0.09 - 0.55 = \boxed{0.36}$$

Q4. (10 points) A uniformly distributed population has a mean $\mu = 250$ and a standard deviation $\sigma = 18$. When a sample of size $n = 144$ is taken, what is the probability of getting a sample mean between 247 and 253?

Solution: This situation is described by a mean-sampling distribution. We find the standard error.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{144}} = 1.5$$

Even though the population is uniformly distributed, the sampling distribution is approximately normal.

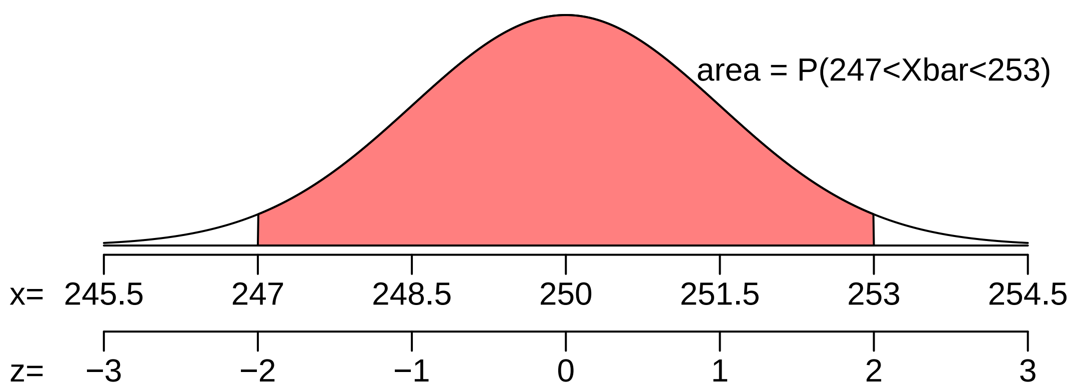
$$\bar{X} \sim \mathcal{N}(250,\ 1.5)$$

We find the z-scores.

$$z_{\text{LOWER}} = \frac{247-250}{1.5} = -2$$

$$z_{\text{UPPER}} = \frac{253-250}{1.5} = 2$$

Here is a sketch:



We find the probability.

$$\begin{aligned} P(247 < \bar{X} < 253) &= P(-2 < Z < 2) \\ &= \Phi(2) - \Phi(-2) \\ &= \boxed{0.9545} \end{aligned}$$

Q5. (10 points) A sample of size $n = 144$ has a mean $\bar{x} = 253.92$ and a standard deviation $s = 17.54$. Construct a confidence interval of the population's mean based on the sample using a confidence level $\gamma = 0.99$.

Solution: We find $z^\star$.

$$\begin{aligned} z^\star &= \Phi^{-1}\left(\frac{0.99 + 1}{2}\right) \\ &= \Phi^{-1}(0.995) \\ &= 2.58 \end{aligned}$$

We find the standard error.

$$SE = \frac{17.54}{\sqrt{144}} = 1.4617$$

We determine the confidence interval.

$$\begin{aligned} CI &= \bar{x} \pm z^\star SE \\ &= 253.92 \pm (2.58)(1.4617) \\ &= \boxed{(250.1\,,\ 257.7)} \end{aligned}$$

Q6. (10 points) Imagine you thought a uniformly distributed population had a mean $\mu = 250$ and standard deviation $s = 18$. However, you are skeptical, so you decide to run a two-tailed hypothesis test with a significance level $\alpha = 0.01$.

Your sample of size $n = 144$ results in a mean $\bar{x} = 253.92$.

(a) State the hypotheses.

Solution:

$$H_0: \quad \mu = 250$$
$$H_A: \quad \mu \neq 250$$

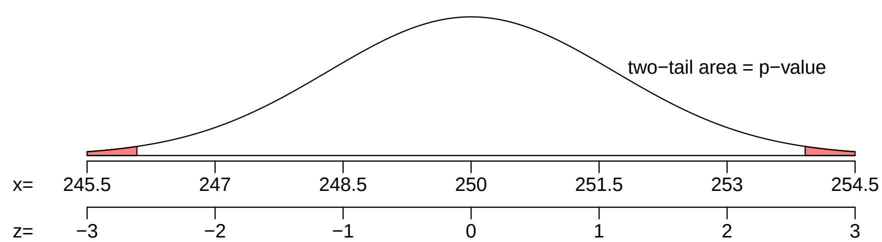
(b) Describe and sketch the null's sampling distribution.

Solution: We need the standard error for a sampling distribution.

$$SE = \frac{18}{\sqrt{144}} = 1.5$$

We think the null's sampling distribution is normal with mean $\mu = 250$ and standard deviation $\sigma = 1.5$.

$$\bar{X} \sim \mathcal{N}(250,\ 1.5)$$



(c) Find a z score of the sample mean.

Solution:

$$z = \frac{253.92 - 250}{1.5} = 2.61$$

(d) Calculate the p-value.

Solution:

$$p\text{-value} \ = \ 2 \cdot \Phi(-2.61) \ = \ (2)(0.0045) \ = \ 0.009$$

(e) Make your conclusion.

Solution: We reject the null hypothesis because $0.009 < 0.01$.