Bunker Hill Community College
imagine the possibilities

Math 181 2nd Exam Practice A SOLUTIONS

Spring 2019

Name: ____________________

- Write your **full name** on the line above.
- Show your work. Incorrect answers with work can receive partial credit.
- Attempt every question; showing you understand the question earns some credit.
- If you run out of room for an answer, continue on the back of the page. Before doing so, write "see back" with a circle around it.
- You can use 1 page (front and back) of notes.
- You can use (and probably need) a calculator.
- You can use the Geogebra Scientific Calculator instead of a calculator. You need to put your phone on **airplane mode** and then within the application, start **exam mode**; you should see a green bar with a timer counting up.
- If a question is confusing or ambiguous, please ask for clarification; however, you will not be told how to answer the question.
- **Box your final answer**.
- A formula sheet is attached to this test.

Do not write in this grade table.

Question:	**Q1**	**Q2**	**Q3**	**Q4**	Total
Points:	10	10	10	10	40
Score:					

Normal Distribution:

$X \sim \mathcal{N}(\mu, \sigma)$
$\mu =$ population mean
$\sigma =$ population standard deviation
$x =$ possible value of X
$\ell =$ percentile of x (left area)
$\Phi(z) =$ standard normal cumulative function

$$z = \frac{x-\mu}{\sigma}$$
$$P(X < x) = \Phi(z)$$
$$\ell = \Phi(z)$$
$$z = \Phi^{-1}(\ell)$$

Bernoulli Distribution:

$X \sim \texttt{Bern}(p)$
$X =$ 0 for fail or 1 for success
$p =$ probability of success

$$P(X=0) = 1-p$$
$$P(X=1) = p$$
$$\mu = p$$
$$\sigma = \sqrt{p(1-p)}$$

Geometric Distribution:

$X \sim \texttt{Geo}(p)$
$X =$ number of trials until first success
$p =$ probability of success on each trial
$n =$ a possible number of trials

$$P(X=n) = (1-p)^{n-1}(p)$$
$$\mu = \frac{1}{p}$$
$$\sigma = \sqrt{\frac{1-p}{p^2}}$$

Mean-Sampling Distribution:

$\bar{X} =$ sample mean
$s =$ sample standard deviation
$n =$ sample size
$\mu =$ population mean
$\sigma =$ population standard deviation
$SE =$ standard error

$$SE = \frac{\sigma}{\sqrt{n}}$$

If $n \geq 30$ (or if population is normal) then:

$$\bar{X} \sim \mathcal{N}(\mu, SE)$$

Binomial Distribution:

$X \sim \mathcal{B}(n, p)$
$X =$ number of successes from n trials
$p =$ probability of success on each trial
$n =$ number of trials
$k =$ a possible number of successes

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$
$$\mu = np$$
$$\sigma = \sqrt{np(1-p)}$$

If $np \geq 10$ and $n(1-p) \geq 10$, then

$$X \sim \mathcal{N}(\mu, \sigma)$$

Continuity correction:

$$P(X \leq k) \approx \Phi\left(\frac{k+0.5-\mu}{\sigma}\right)$$

Confidence Interval:

$CI =$ confidence interval
$\gamma =$ confidence level
$\bar{x} =$ sample mean
$s =$ sample standard deviation

$$z^\star = \Phi^{-1}\left(\frac{\gamma+1}{2}\right)$$
$$SE \approx \frac{s}{\sqrt{n}}$$
$$CI = \bar{x} \pm z^\star SE$$

Hypothesis testing:

$H_0: \quad \mu = \mu_0$
$H_A: \quad \mu \neq \mu_0$
$\bar{x} =$ a possible/specific/observed sample mean
$s =$ sample standard deviation
$\alpha =$ significance level

$$\sigma \approx s$$
$$z = \left|\frac{\bar{x}-\mu_0}{SE}\right|$$

$$p\text{-value} = P(|Z| > z) = 2\Phi(-z)$$

If p-value $< \alpha$, then reject H_0, else retain H_0.

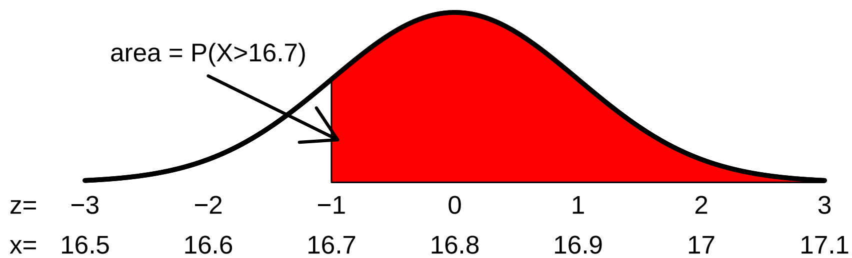
Q1. (10 points) Brood XIV is a population of 17-year cicadas in eastern United States, including Massachusetts. The juvenile lifespan is normally distributed with mean of 16.8 years and standard deviation of 0.1 years.

(a) What is the probability of a random juvenile's lifespan being more than 16.7 years? In other words, let $X \sim \mathcal{N}(16.8,\ 0.1)$ and find $P(X > 16.7)$.

Solution: Find the z-score.

$$z = \frac{16.7 - 16.8}{0.1} = -1$$

Draw a sketch.



Find the area.

$$\begin{aligned} P(X > 16.7) &= P(Z > -1) \\ &= 1 - P(Z < -1) \\ &= 1 - \Phi(-1) \\ &= 1 - 0.1587 \\ &= \boxed{0.8413} \end{aligned}$$

(b) What is the IQR of juvenile lifespans?

Solution: We find the z-scores of 25th percentile and 75th percentile. So, let's find z_{LOW} such that $P(Z < z_{\text{LOW}}) = 0.25$.

$$z_{\text{LOW}} = \Phi^{-1}(0.25) = -0.67$$

Let's find z_{HIGH} such that $P(Z < z_{\text{HIGH}}) = 0.75$.

$$z_{\text{HIGH}} = \Phi^{-1}(0.75) = 0.67$$

We find the associated x scores.

$$x_{\text{LOW}} = 16.8 + (-0.67)(0.1) = 16.733$$

$$x_{\text{HIGH}} = 16.8 + (0.67)(0.1) = 16.867$$

To find IQR, we find the difference.

$$IQR = 16.867 - 16.733 = \boxed{0.134}$$

Q2. (10 points) A 20-sided die (icosahedron) has a 5% chance of landing on each side. Imagine that only one side is a success and the rest are fails.

(a) What is the chance the first success happens on the third roll?

Solution: We use a geometric model. $p = 0.05$ and $n = 3$.

$$P(\text{Fail, Fail, Success}) = (0.95)^2(0.05) = \boxed{0.045}$$

(b) What is the chance of getting exactly 5 successes in 100 rolls?

Solution: We use a binomial model. $p = 0.05$ and $n = 100$ and $k = 5$. Let X represent the number of successes when 100 of these dice are thrown.

$$P(X = 5) = \binom{100}{5}(0.05)^5(0.95)^{95}$$

$$= \boxed{0.18}$$

(c) What is the chance of getting between at least 10 and less than 30 successes in 300 rolls?

Solution: We hope to use the normal approximation to the binomial distribution. We first check to make sure we can use the normal approximation.

$$np = (300)(0.05) = 15 > 10$$

$$n(1 - p) = (300)(0.95) = 285 > 10$$

Great, we can. We determine the mean and standard deviation of the binomial distribution.

$$\mu = np = (300)(0.05) = 15$$

$$\sigma = \sqrt{np(1 - p)} = \sqrt{(300)(0.05)(0.95)} = 3.7749$$

We find the z-scores. REMEMBER THE CONTINUITY CORRECTIONS!

$$z_{\text{LOWER}} = \frac{10 - 0.5 - 15}{3.77} = -1.46$$

$$z_{\text{UPPER}} = \frac{30 - 0.5 - 15}{3.77} = 3.84$$

Because z_{UPPER} is larger than 3.5, we can ignore that upper bound (and just find a right area instead.)

$$P(10 \le X < 30) \approx P(Z > -1.46) = 1 - P(Z < -1.46) = \boxed{0.93}$$

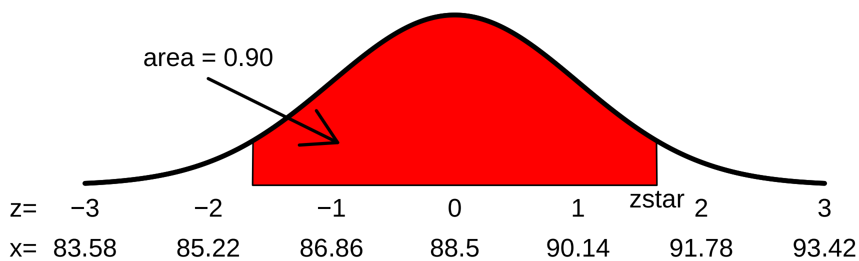
Q3. (10 points) You collect 45 measurements with a mean of 88.5 mm and a standard deviation of 11.0 mm.

(a) Determine a 90% confidence interval.

Solution: We calculate the standard error.

$$SE = \frac{11}{\sqrt{45}} = 1.63978$$

We determine $z^\star$ such that $P(|Z| < z^\star) = 0.90$.



Using symmetry, we recognize how to find $z^\star$.

$$z^\star = \Phi^{-1}(0.95) = 1.64$$

We find the confidence interval.

$$\begin{aligned} CI &= \bar{x} \pm z^\star SE \\ &= 88.5 \pm (1.64)(1.63978) \\ &= (85.81,\ 91.19) \end{aligned}$$

(b) Determine a 99% confidence interval.

Solution: The standard error is the same as above. We calculate a new $z^\star$.

$$z^\star = \Phi^{-1}(0.995) = 2.58$$

We find the confidence interval.

$$\begin{aligned} CI &= \bar{x} \pm z^\star SE \\ &= 88.5 \pm (2.58)(1.63978) \\ &= (84.27,\ 92.73) \end{aligned}$$

(c) If a normally distributed population has a mean of 90 and a standard deviation of 11, what is the chance that 45 measurements will have a mean lower than 88.5?

Solution: Let X represent a single measurement.

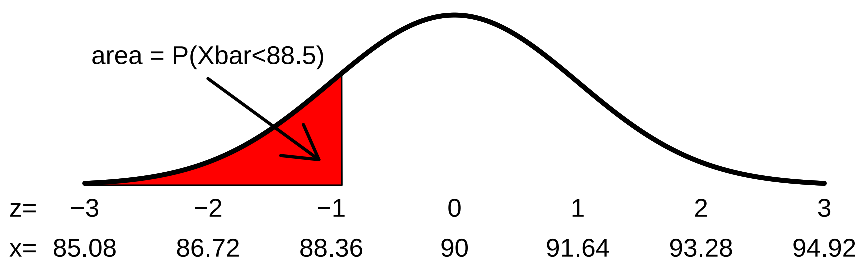
$$X \sim \mathcal{N}(90,\ 11)$$

Let $\bar{X}$ represent the mean of 45 measurements.

$$\bar{X} \sim \mathcal{N}\left(90,\ \frac{11}{\sqrt{45}}\right)$$

$$\bar{X} \sim \mathcal{N}(90,\ 1.64)$$

We hope to calculate $P(\bar{X} < 88.5)$. We draw a sketch.



We calculate the z-score.

$$z = \frac{88.5 - 90}{1.64} = -0.91$$

We calculate the probability.

$$P(\bar{X} < 88.5) \quad = \quad P(Z < -0.91) \quad = \quad \boxed{0.1814}$$

Q4. (10 points) You had been told that adult elephants have a mean weight of 255 kg. You decided to measure the weights of 50 random elephants and run a hypothesis test with a significance level of 0.05.

Your sample has a mean of 249.8 kg and a standard deviation of 12.34 kg. What is your conclusion and why? Show your work for full credit.

Solution: We state the hypotheses.

$$H_0 : \quad \mu = 255$$

$$H_A : \quad \mu \neq 255$$

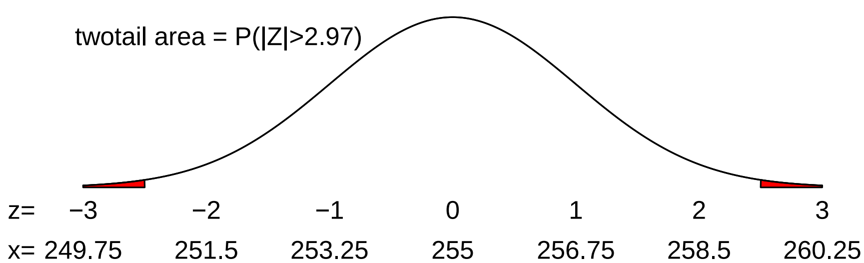
We describe the null's sampling distribution by assuming $\sigma \approx 12.34$. We calculate the standard error: $SE = 12.34/\sqrt{50} = 1.75$

$$\bar{X}_0 \sim \mathcal{N}(255,\ 1.75)$$

We find the z-score of the actual sample's mean (249.8) under the null's sampling distribution.

$$z = \frac{249.8 - 255}{1.75} = -2.97$$

We sketch the null's sampling distribution, along with a two-tailed area using 249.8 (the actual sample's mean) as a boundary.



We determine the probability.

$$P(|Z| > 2.97) \quad = \quad 2P(Z < -2.97) \quad = \quad (2)(0.0015) \quad = \quad 0.003$$

$$p\text{-value} \quad = \quad 0.003$$

We compare the p-value to the significance level.

$$0.003 < 0.05$$

$$p\text{-value} < \alpha$$

We reject the null hypothesis. We conclude the true mean weight of elephants is not 255 kg.