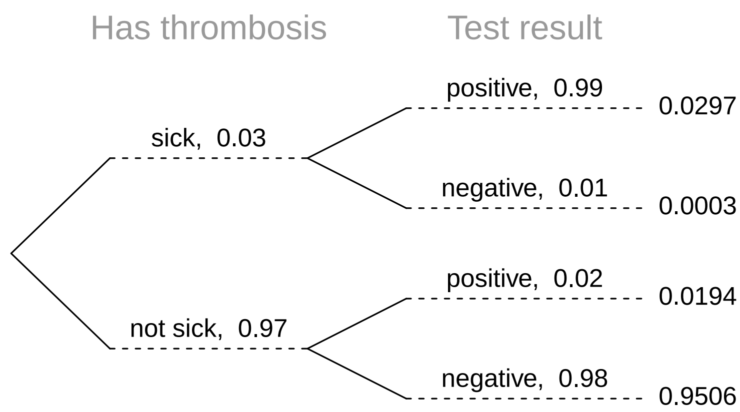
2.22: Let event S represent "sick" and T represent "positive test". We are told:

$$P(S) = 0.03$$
$$P(T|S) = 0.99$$
$$P(T^c|S^c) = 0.98$$

We are asked to determine $P(S|T)$. To do so, we can make a tree.



I like to make a contingency table.

	pos	neg	total
sick	0.0297	0.0003	0.03
not sick	0.0194	0.9506	0.97
total	0.0491	0.9509	1

Then, it is easy to calculate the conditional probability.

$$P(\text{sick}|\text{pos}) = \frac{P(\text{sick AND pos})}{P(\text{pos})}$$

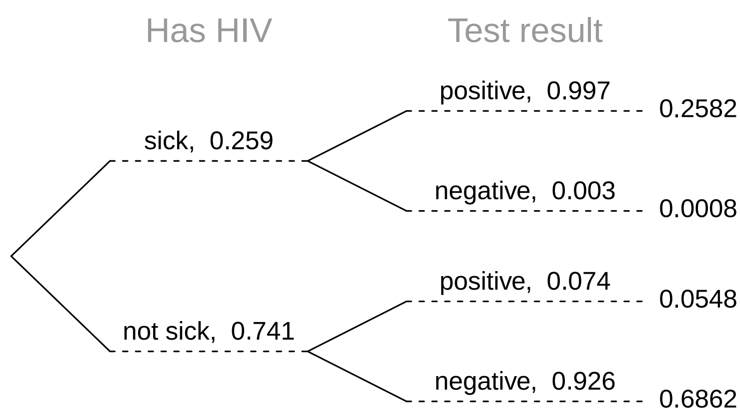
$$= \frac{0.0297}{0.0491}$$

$$\approx \boxed{0.605}$$

2.23: We are told:

$$P(\text{sick}) = 0.259$$
$$P(\text{pos}|\text{sick}) = 0.997$$
$$P(\text{neg}|\text{not sick}) = 0.926$$

We are asked to determine $P(\text{sick}|\text{pos})$. To do so, we can make a tree.



I like to make a contingency table.

	pos	neg	total
sick	0.2582	0.0008	0.259
not sick	0.0548	0.6862	0.741
total	0.313	0.687	1

Then, it is easy to calculate the conditional probability.

$$P(\text{sick}|\text{pos}) = \frac{P(\text{sick AND pos})}{P(\text{pos})}$$

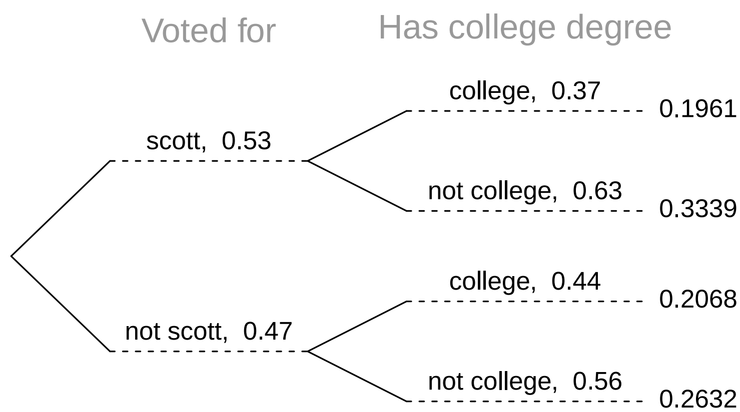
$$= \frac{0.2582}{0.313}$$

$$\approx \boxed{0.825}$$

2.24: We are told:

$$P(\text{scott}) = 0.53$$
$$P(\text{college}|\text{scott}) = 0.37$$
$$P(\text{college}|\text{not scott}) = 0.44$$

We are asked to determine $P(\text{scott}|\text{college})$. To do so, we can make a tree.



I like to make a contingency table.

	college	not college	total
scott	0.1961	0.3339	0.53
not scott	0.2068	0.2632	0.47
total	0.4029	0.5971	1

Then, it is easy to calculate the conditional probability.

$$P(\text{scott}|\text{college}) = \frac{P(\text{scott AND college})}{P(\text{college})}$$

$$= \frac{0.1961}{0.0.4029}$$

$$\approx \boxed{0.825}$$

2.25: We are told:

$$\begin{aligned} P(\text{lupus}) &= 0.02 \\ P(\text{pos}|\text{lupus}) &= 0.98 \\ P(\text{neg}|\text{not lupus}) &= 0.74 \end{aligned}$$

We want to determine $P(\text{lupus}|\text{pos})$. To do so, we could make a tree, but I'll just use the formula.

$$\begin{aligned} P(\text{lupus}|\text{pos}) &= \frac{P(\text{lupus AND pos})}{P(\text{pos})} \\ &= \frac{P(\text{pos}|\text{lupus}) \cdot P(\text{lupus})}{P(\text{pos}|\text{lupus}) \cdot P(\text{lupus}) \quad + \quad P(\text{pos}|\text{not lupus}) \cdot P(\text{not lupus})} \\ &= \frac{0.98 \times 0.02}{0.98 \times 0.02 \quad + \quad 0.26 \times 0.98} \\ &= 0.071 \end{aligned}$$

So, even when someone tests positive for lupus, we only think there is about a 7% chance of them actually having lupus. This kind of supports the notion that often when you think it might be lupus, it actually is not. Of course, about 2% of the time overall, it really is lupus...

2.26: We are told that for twins,

$$\begin{aligned} P(\text{identical}) &= 0.3 \\ P(\text{2 girls}|\text{identical}) &= 0.5 \\ P(\text{2 girls}|\text{not identical}) &= 0.25 \end{aligned}$$

We want to determine $P(\text{identical}|\text{2 girls})$. To do so, we could make a tree, but I'll just use the formula.

$$\begin{aligned} P(\text{identical}|\text{2 girls}) &= \frac{P(\text{identical AND 2 girls})}{P(\text{2 girls})} \\ &= \frac{P(\text{2 girls AND identical})}{P(\text{2 girls AND identical}) \quad + \quad P(\text{2 girls AND not identical})} \\ &= \frac{P(\text{2 girls}|\text{identical}) \cdot P(\text{identical})}{P(\text{2 girls}|\text{identical}) \cdot P(\text{identical}) \quad + \quad P(\text{2 girls}|\text{not identical}) \cdot P(\text{not identical})} \\ &= \frac{0.5 \times 0.3}{0.5 \times 0.3 \quad + \quad 0.25 \times 0.7} \\ &= \boxed{0.46} \end{aligned}$$