

### 1. Problem

A population is known to have a standard deviation  $\sigma = 250$ . What is the sample size  $n$  needed to build a 80% confidence interval with a margin of error  $ME = 20$ ?

### Solution

Let's remember the formulas for confidence intervals (with known  $\sigma$ ):

$$SE = \frac{\sigma}{\sqrt{n}}$$

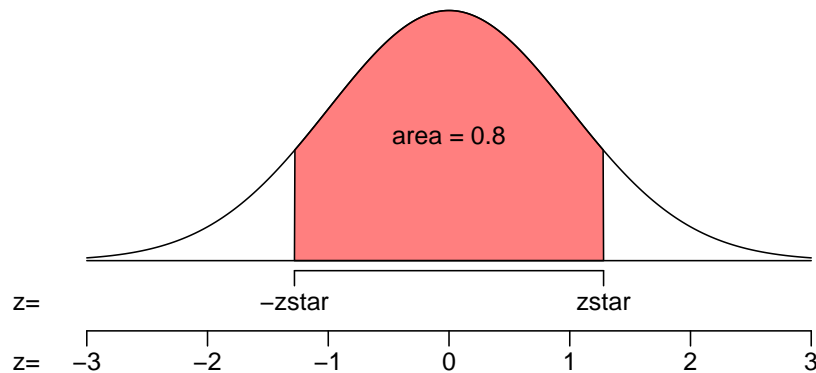
$$CL = P(|Z| < z^*)$$

$$ME = z^* SE$$

$$CI = \bar{x} \pm ME$$

From the confidence level  $CL = 0.8$ , we determine  $z^*$ .

$$P(|Z| < z^*) = 0.8$$



You can use a  $z$  table or the last row of the  $t$ -table (where  $df = \infty$ ).

$$z^* = 1.28$$

We know that  $ME = z^* SE$ , so

$$SE = \frac{ME}{z^*} = \frac{20}{1.28} = 15.6$$

We know that  $SE = \frac{\sigma}{\sqrt{n}}$ . Let's solve for  $n$ .

$$SE = \frac{\sigma}{\sqrt{n}}$$

Multiply both sides by  $\sqrt{n}$ .

$$SE\sqrt{n} = \sigma$$

Divide both sides by  $SE$ .

$$\sqrt{n} = \frac{\sigma}{SE}$$

Square both sides. (Raise both sides to the power of 2.)

$$n = \left( \frac{\sigma}{SE} \right)^2$$

$$n = \left( \frac{250}{15.625} \right)^2 = 256$$

We round  $n$  up.

$$n = 256$$