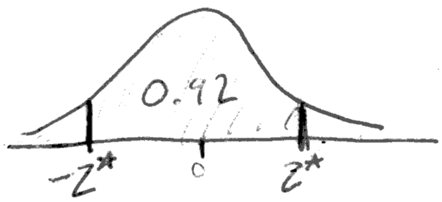
Q1. (10 points) Hannah is curious about the expected number of rolls of a 6-sided die before getting every side, but Hannah forgets how to analyze it mathematically. So, she gets a 6-sided die and rolls it until she sees every number and writes down how many rolls it took. She repeats this over and over, getting the following sample:

18	19	26	23	17	12	11	12	23	16	13	8	10	8	7
19	14	11	15	17	20	24	12	18	10	9	22	24	14	14

Hannah determines the sample size $n = 30$, sample mean $\bar{x} = 15.53$, and sample standard deviation $s = 5.41$.

(a) Determine a 92% confidence interval for the expected number of rolls to get all sides. You can assume the sampling distribution is normal (even though the population is not normal).



$$SE = \frac{5.41}{\sqrt{30}} = 0.988$$

$$P(|Z| < z^*) = 0.92$$

$$P(Z < z^*) = 0.96$$

$$CI = \bar{x} \pm z^* SE$$

$$= 15.53 \pm (1.75)(0.988)$$

$$= (13.8, 17.3)$$

$$z^* = \Phi^{-1}(0.96) = 1.75$$

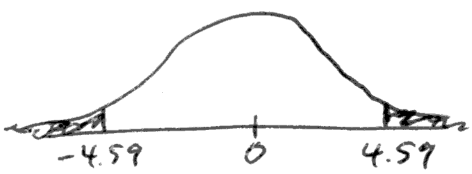
(b) After this study, would you believe a friend that suggests the expected number of rolls is 11? Why or why not?

No. 11 is outside the confidence interval.

$$Z = \frac{11 - 15.53}{0.988} = -4.59$$

extra

that z-score is far from 0!



$$\text{p-value} = 2 \cdot P(Z < -4.59) << 0.05$$

$p=0.42$

Q2. (10 points) Imagine each trial has a 42% chance of success. Let random variable W represent the result of a trial, where 0 means failure and 1 means success.

(a) What is the standard deviation of W?

Bernoulli:

$$\sigma = \sqrt{p(1-p)}$$

$$\sigma = \sqrt{(0.42)(0.58)}$$

$$\boxed{\sigma = 0.49}$$

(b) What is the expected number of trials until getting a success?

Geometric

$$\mu = \frac{1}{p} = \frac{1}{0.42} = \boxed{2.38}$$

(c) What is the standard deviation of number of trials until getting a success?

Geometric

$$\sigma = \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{0.58}{(0.42)^2}} = \boxed{1.81}$$

(d) What is the probability of getting 30 successes from 75 trials?

Bernoulli

$$P(X=30) = \binom{75}{30}(0.42)^{30}(0.58)^{45}$$

$$\boxed{= 0.088}$$

Q3. (10 points) If each trial has a 33% chance of success and there are 200 trials, what is the probability that the number of successes is more than 67? Please use a normal approximation with the continuity correction.

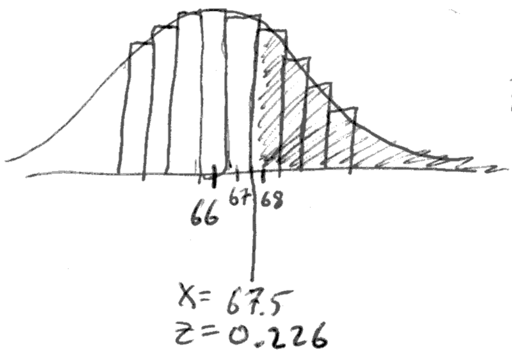
$n = 200$

$p = 0.33$

$\mu = 66$

$\sigma = \sqrt{np(1-p)} = 6.65$

$X \sim N(66, 6.65)$



$z = \frac{67.5 - 66}{6.65} = 0.226$

$P(X > 67) \approx P(Z > 0.23) = 1 - P(Z < 0.23)$

$= 1 - 0.5910$

$\boxed{= 0.409}$

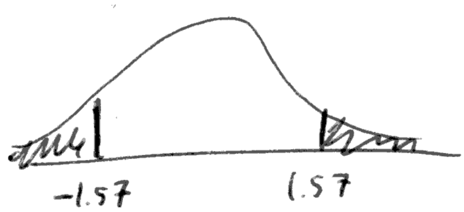
Q4. (10 points) Perform a two-tail hypothesis test with $\mu_0 = 100$, $n = 50$, $\bar{x} = 103.2$, $s = 14.4$, and $\alpha = 0.10$.

$H_0: \mu = 100$

$H_1: \mu \neq 100$

$$SE = \frac{14.4}{\sqrt{50}} = 2.036$$

$$Z = \frac{103.2 - 100}{2.036} = 1.57$$



$$\text{p-value} = P(|Z| > 1.57) = 2\cdot\Phi(-1.57)$$

$$= 2\cdot(0.0582)$$

$$\text{p-value} = 0.1164$$

$0.1164 > 0.1$

$\text{p-value} > \alpha$

$\boxed{\text{retain the null hypothesis}}$

Q5. (10 points) Let $X \sim \mathcal{N}(500, 20)$.

(a) Evaluate $P(470 < X < 520)$.

$$Z_{low} = \frac{470-500}{20} = -1.5$$

$$Z_{high} = \frac{520-500}{20} = 1$$

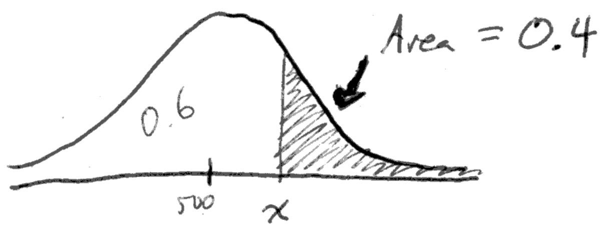
$$P(470 < X < 520) = P(-1.5 < Z < 1)$$

$$= \Phi(1) - \Phi(-1.5)$$

$$= 0.8413 - 0.0668$$

$$= 0.7745$$

(b) Determine x such that $P(X > x) = 0.40$.



$$Z = \Phi^{-1}(0.6) = 0.25$$

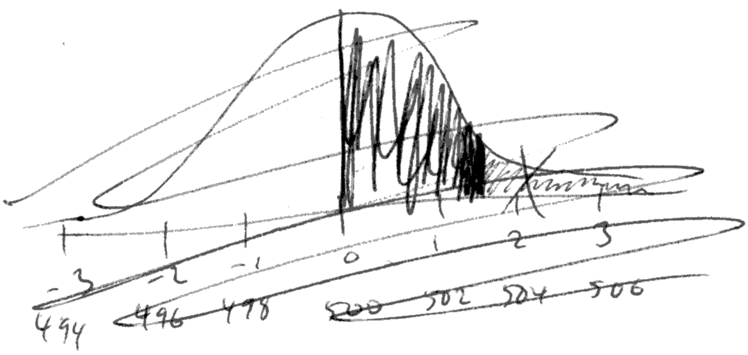
$$X = 500 + (0.25)(20)$$

$$\boxed{X = 505}$$

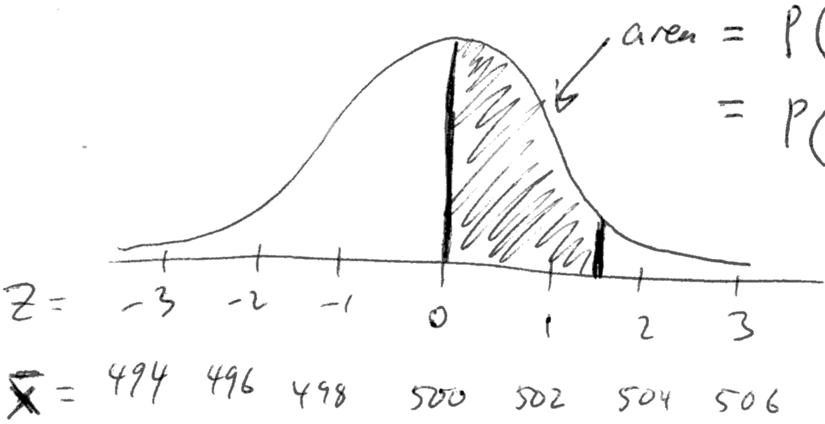
Q6. (10 points) There is a continuous population with $\mu = 500$ and $\sigma = 20$. What is the probability that a sample of size 100 has a mean between 500 and 503?

$$SE = \frac{20}{\sqrt{100}} = 2$$

sampling distribution $\sim N(500, 2)$



area $= P(500 < X < 503)$
$= P(0 < Z < 1.5)$



$$Z_{low} = \frac{500-500}{2} = 0$$

$$Z_{high} = \frac{503-500}{2} = 1.5$$

$$P(0 < Z < 1.5) = \Phi(1.5) - \Phi(0)$$
$$= 0.9332 - 0.5$$
$$\boxed{= 0.4332}$$