

1. Problem:

A continuous random variable X was measured 11 times. The sorted data are shown below, along with each datum's index.

i	x
1	83.337
2	85.538
3	90.124
4	90.479
5	90.643
6	92.693
7	93.060
8	93.601
9	100.841
10	104.308
11	105.995

The total of the measurements is 1030.619.

- Determine the percentile rank of the value 90.479. In other words, determine what percent of data are less than or equal to 90.479.
- Determine the datum corresponding to a percentile rank of 0.182. In other words, determine x such that 18.2% of the data are less than or equal to x .
- Determine the mean of the measurements.
- Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 90.479$. This means $i = 4$. We know $n = 11$. Determine the percentile ℓ .

$$\ell = \frac{4}{11}$$

$$\ell = 0.364$$

So, the percentile rank is 0.364, or 36.4th percentile.

- (b) We are given $\ell = 0.182$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (11)(0.182)$$

$$i = 2$$

Determine the x associated with $i = 2$.

$$x = \text{85.538}$$

- (c) The mean: $\bar{x} = \frac{1030.619}{11} = \text{93.693}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 11$ and so n is odd.

$$\text{median} = x_{(11+1)/2} = x_6$$

So, median = 92.693.

2. Problem:

A continuous random variable X was measured 12 times. The sorted data are shown below, along with each datum's index.

i	x
1	77.964
2	79.566
3	87.387
4	93.218
5	97.001
6	99.068
7	100.373
8	103.564
9	103.946
10	104.551
11	107.786
12	108.626

The total of the measurements is 1163.05.

- (a) Determine the percentile rank of the value 107.786. In other words, determine what percent of data are less than or equal to 107.786.
- (b) Determine the datum corresponding to a percentile rank of 0.417. In other words, determine x such that 41.7% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 107.786$. This means $i = 11$. We know $n = 12$. Determine the percentile ℓ .

$$\ell = \frac{11}{12}$$

$$\ell = 0.917$$

So, the percentile rank is 0.917, or 91.7th percentile.

- (b) We are given $\ell = 0.417$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (12)(0.417)$$

$$i = 5$$

Determine the x associated with $i = 5$.

$$x = \text{97.001}$$

- (c) The mean: $\bar{x} = \frac{1163.05}{12} = \text{96.921}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 12$ and so n is even.

$$\text{median} = \frac{x_6 + x_7}{2} = \frac{99.068 + 100.373}{2}$$

So, median = 99.7205.

3. Problem:

A continuous random variable X was measured 6 times. The sorted data are shown below, along with each datum's index.

i	x
1	40.350
2	40.360
3	40.394
4	40.485
5	40.594
6	41.332

The total of the measurements is 243.515.

- (a) Determine the percentile rank of the value 40.394. In other words, determine what percent of data are less than or equal to 40.394.
- (b) Determine the datum corresponding to a percentile rank of 0.667. In other words, determine x such that 66.7% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 40.394$. This means $i = 3$. We know $n = 6$. Determine the percentile ℓ .

$$\ell = \frac{3}{6}$$

$$\ell = 0.5$$

So, the percentile rank is 0.5, or 50th percentile.

- (b) We are given $\ell = 0.667$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (6)(0.667)$$

$$i = 4$$

Determine the x associated with $i = 4$.

$$x = \text{40.485}$$

- (c) The mean: $\bar{x} = \frac{243.515}{6} = \text{40.586}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 6$ and so n is even.

$$\text{median} = \frac{x_3 + x_4}{2} = \frac{40.394 + 40.485}{2}$$

So, median = 40.4395.

4. Problem:

A continuous random variable X was measured 6 times. The sorted data are shown below, along with each datum's index.

i	x
1	45.098
2	45.668
3	48.140
4	48.370
5	49.065
6	54.272

The total of the measurements is 290.613.

- (a) Determine the percentile rank of the value 49.065. In other words, determine what percent of data are less than or equal to 49.065.
- (b) Determine the datum corresponding to a percentile rank of 0.167. In other words, determine x such that 16.7% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 49.065$. This means $i = 5$. We know $n = 6$. Determine the percentile ℓ .

$$\ell = \frac{5}{6}$$

$$\ell = 0.833$$

So, the percentile rank is $\boxed{0.833}$, or 83.3th percentile.

- (b) We are given $\ell = 0.167$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (6)(0.167)$$

$$i = 1$$

Determine the x associated with $i = 1$.

$$x = \boxed{45.098}$$

- (c) The mean: $\bar{x} = \frac{290.613}{6} = \boxed{48.436}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 6$ and so n is even.

$$\text{median} = \frac{x_3 + x_4}{2} = \frac{48.14 + 48.37}{2}$$

So, median = $\boxed{48.255}$.

5. Problem:

A continuous random variable X was measured 9 times. The sorted data are shown below, along with each datum's index.

i	x
1	63.349
2	63.649
3	63.671
4	63.888
5	64.006
6	64.192
7	64.384
8	64.801
9	64.839

The total of the measurements is 576.779.

- (a) Determine the percentile rank of the value 63.671. In other words, determine what percent of data are less than or equal to 63.671.
- (b) Determine the datum corresponding to a percentile rank of 0.222. In other words, determine x such that 22.2% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 63.671$. This means $i = 3$. We know $n = 9$. Determine the percentile ℓ .

$$\ell = \frac{3}{9}$$

$$\ell = 0.333$$

So, the percentile rank is 0.333, or 33.3th percentile.

- (b) We are given $\ell = 0.222$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (9)(0.222)$$

$$i = 2$$

Determine the x associated with $i = 2$.

$$x = \text{63.649}$$

- (c) The mean: $\bar{x} = \frac{576.779}{9} = \text{64.087}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 9$ and so n is odd.

$$\text{median} = x_{(9+1)/2} = x_5$$

So, median = 64.006.

6. Problem:

A continuous random variable X was measured 12 times. The sorted data are shown below, along with each datum's index.

i	x
1	21.147
2	21.191
3	21.295
4	21.301
5	21.355
6	21.410
7	21.500
8	21.567
9	21.606
10	21.737
11	21.761
12	21.767

The total of the measurements is 257.637.

- (a) Determine the percentile rank of the value 21.5. In other words, determine what percent of data are less than or equal to 21.5.
- (b) Determine the datum corresponding to a percentile rank of 0.25. In other words, determine x such that 25% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 21.5$. This means $i = 7$. We know $n = 12$. Determine the percentile ℓ .

$$\ell = \frac{7}{12}$$

$$\ell = 0.583$$

So, the percentile rank is 0.583, or 58.3th percentile.

- (b) We are given $\ell = 0.25$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (12)(0.25)$$

$$i = 3$$

Determine the x associated with $i = 3$.

$$x = \text{21.295}$$

- (c) The mean: $\bar{x} = \frac{257.637}{12} = \text{21.47}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 12$ and so n is even.

$$\text{median} = \frac{x_6 + x_7}{2} = \frac{21.41 + 21.5}{2}$$

So, median = 21.455.

7. Problem:

A continuous random variable X was measured 7 times. The sorted data are shown below, along with each datum's index.

i	x
1	56.171
2	56.716
3	58.280
4	58.394
5	58.496
6	58.865
7	59.406

The total of the measurements is 406.328.

- (a) Determine the percentile rank of the value 58.28. In other words, determine what percent of data are less than or equal to 58.28.
- (b) Determine the datum corresponding to a percentile rank of 0.571. In other words, determine x such that 57.1% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 58.28$. This means $i = 3$. We know $n = 7$. Determine the percentile ℓ .

$$\ell = \frac{3}{7}$$

$$\ell = 0.429$$

So, the percentile rank is 0.429, or 42.9th percentile.

- (b) We are given $\ell = 0.571$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (7)(0.571)$$

$$i = 4$$

Determine the x associated with $i = 4$.

$$x = \text{58.394}$$

- (c) The mean: $\bar{x} = \frac{406.328}{7} = \text{58.047}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 7$ and so n is odd.

$$\text{median} = x_{(7+1)/2} = x_4$$

So, median = 58.394.

8. Problem:

A continuous random variable X was measured 63 times. The sorted data are shown below.

31.214	31.492	31.541	31.974	33.214	33.340	33.887	34.675	34.736
34.743	35.224	36.657	37.263	37.786	37.967	38.503	40.488	40.717
40.824	40.909	41.847	42.421	42.896	43.058	43.373	43.495	43.908
44.532	45.049	46.771	46.915	47.243	47.415	49.206	49.690	50.542
50.557	51.126	51.129	52.636	53.407	53.587	53.874	53.924	53.950
55.847	56.870	58.187	59.007	59.106	59.584	60.656	60.692	60.818
62.783	63.098	64.172	65.498	65.612	67.285	69.228	69.276	69.351

The total of the measurements is 3046.775.

- (a) Determine the percentile rank of the value 47.243. In other words, determine what percent of data are less than or equal to 47.243.
- (b) Determine the datum corresponding to a percentile rank of 0.397. In other words, determine x such that 39.7% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 47.243$. This means $i = 32$. We know $n = 63$. Determine the percentile ℓ .

$$\ell = \frac{32}{63}$$

$$\ell = 0.508$$

So, the percentile rank is 0.508, or 50.8th percentile.

- (b) We are given $\ell = 0.397$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (63)(0.397)$$

$$i = 25$$

Determine the x associated with $i = 25$.

$$x = \text{43.373}$$

- (c) The mean: $\bar{x} = \frac{3046.775}{63} = \text{48.362}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 63$ and so n is odd.

$$\text{median} = x_{(63+1)/2} = x_{32}$$

So, median = 47.243.

9. Problem:

A continuous random variable X was measured 32 times. The sorted data are shown below.

49.482	50.213	53.270	56.062	56.567	58.674	62.927	63.329
63.976	64.623	64.998	65.369	66.642	66.933	67.183	69.292
70.142	70.454	70.671	70.779	72.025	72.818	73.990	74.597
75.407	76.099	77.166	77.462	77.666	78.137	79.379	79.593

The total of the measurements is 2175.925.

- (a) Determine the percentile rank of the value 79.593. In other words, determine what percent of data are less than or equal to 79.593.
- (b) Determine the datum corresponding to a percentile rank of 0.312. In other words, determine x such that 31.2% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 79.593$. This means $i = 32$. We know $n = 32$. Determine the percentile ℓ .

$$\ell = \frac{32}{32}$$

$$\ell = 1$$

So, the percentile rank is $\boxed{1}$, or 100th percentile.

- (b) We are given $\ell = 0.312$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (32)(0.312)$$

$$i = 10$$

Determine the x associated with $i = 10$.

$$x = \boxed{64.623}$$

- (c) The mean: $\bar{x} = \frac{2175.925}{32} = \boxed{67.998}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 32$ and so n is even.

$$\text{median} = \frac{x_{16} + x_{17}}{2} = \frac{69.292 + 70.142}{2}$$

So, median = $\boxed{69.717}$.

10. Problem:

A continuous random variable X was measured 32 times. The sorted data are shown below.

30.007	30.197	30.314	30.334	30.351	30.359	30.482	30.502
30.598	30.607	30.643	30.670	30.690	30.771	30.822	30.863
30.951	30.975	31.016	31.246	31.287	31.435	31.457	31.492
31.706	31.895	32.043	32.044	32.343	33.141	33.263	33.703

The total of the measurements is 998.207.

- (a) Determine the percentile rank of the value 31.457. In other words, determine what percent of data are less than or equal to 31.457.
- (b) Determine the datum corresponding to a percentile rank of 0.906. In other words, determine x such that 90.6% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 31.457$. This means $i = 23$. We know $n = 32$. Determine the percentile ℓ .

$$\ell = \frac{23}{32}$$

$$\ell = 0.719$$

So, the percentile rank is $\boxed{0.719}$, or 71.9th percentile.

- (b) We are given $\ell = 0.906$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (32)(0.906)$$

$$i = 29$$

Determine the x associated with $i = 29$.

$$x = \boxed{32.343}$$

- (c) The mean: $\bar{x} = \frac{998.207}{32} = \boxed{31.194}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 32$ and so n is even.

$$\text{median} = \frac{x_{16} + x_{17}}{2} = \frac{30.863 + 30.951}{2}$$

So, median = $\boxed{30.907}$.

11. Problem:

A continuous random variable X was measured 32 times. The sorted data are shown below.

40.013	40.087	40.335	41.124	42.552	43.874	45.672	45.760
48.171	51.297	57.107	59.934	62.214	76.023	82.399	82.499
83.638	85.783	87.553	96.453	97.412	98.385	99.871	109.369
114.342	116.198	116.942	120.010	122.100	123.497	125.185	128.804

The total of the measurements is 2584.603.

- (a) Determine the percentile rank of the value 85.783. In other words, determine what percent of data are less than or equal to 85.783.
- (b) Determine the datum corresponding to a percentile rank of 0.875. In other words, determine x such that 87.5% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 85.783$. This means $i = 18$. We know $n = 32$. Determine the percentile ℓ .

$$\ell = \frac{18}{32}$$

$$\ell = 0.562$$

So, the percentile rank is $\boxed{0.562}$, or 56.2th percentile.

- (b) We are given $\ell = 0.875$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (32)(0.875)$$

$$i = 28$$

Determine the x associated with $i = 28$.

$$x = \boxed{120.01}$$

- (c) The mean: $\bar{x} = \frac{2584.603}{32} = \boxed{80.769}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 32$ and so n is even.

$$\text{median} = \frac{x_{16} + x_{17}}{2} = \frac{82.499 + 83.638}{2}$$

So, median = $\boxed{83.0685}$.

12. Problem:

A continuous random variable X was measured 63 times. The sorted data are shown below.

64.053	68.789	72.504	74.505	77.795	78.259	82.731	90.003	90.821
93.674	96.205	97.786	99.844	101.204	101.736	102.604	102.949	103.563
103.784	103.820	104.551	104.621	107.599	108.571	110.190	113.506	115.767
115.801	117.828	118.921	119.925	119.943	120.211	121.776	121.883	122.545
122.897	123.071	125.921	127.089	127.273	127.378	127.535	128.104	128.117
129.198	129.394	130.576	131.334	131.448	131.667	131.945	132.367	133.290
133.520	134.002	134.125	134.383	134.496	134.726	136.104	136.377	138.770

The total of the measurements is 7185.374.

- (a) Determine the percentile rank of the value 103.82. In other words, determine what percent of data are less than or equal to 103.82.
- (b) Determine the datum corresponding to a percentile rank of 0.794. In other words, determine x such that 79.4% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 103.82$. This means $i = 20$. We know $n = 63$. Determine the percentile ℓ .

$$\ell = \frac{20}{63}$$

$$\ell = 0.317$$

So, the percentile rank is $\boxed{0.317}$, or 31.7th percentile.

- (b) We are given $\ell = 0.794$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (63)(0.794)$$

$$i = 50$$

Determine the x associated with $i = 50$.

$$x = \boxed{131.448}$$

- (c) The mean: $\bar{x} = \frac{7185.374}{63} = \boxed{114.05}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 63$ and so n is odd.

$$\text{median} = x_{(63+1)/2} = x_{32}$$

So, median = $\boxed{119.943}$.

13. Problem:

A continuous random variable X was measured 20 times. The sorted data are shown below.

90.251	90.609	90.907	90.942	90.964
91.084	91.262	91.280	91.315	91.445
91.503	91.986	92.017	92.146	92.202
92.497	92.538	92.591	92.824	92.862

The total of the measurements is 1833.225.

- (a) Determine the percentile rank of the value 90.251. In other words, determine what percent of data are less than or equal to 90.251.
- (b) Determine the datum corresponding to a percentile rank of 0.65. In other words, determine x such that 65% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 90.251$. This means $i = 1$. We know $n = 20$. Determine the percentile ℓ .

$$\ell = \frac{1}{20}$$

$$\ell = 0.05$$

So, the percentile rank is $\boxed{0.05}$, or 5th percentile.

- (b) We are given $\ell = 0.65$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (20)(0.65)$$

$$i = 13$$

Determine the x associated with $i = 13$.

$$x = \boxed{92.017}$$

- (c) The mean: $\bar{x} = \frac{1833.225}{20} = \boxed{91.661}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 20$ and so n is even.

$$\text{median} = \frac{x_{10} + x_{11}}{2} = \frac{91.445 + 91.503}{2}$$

So, median = $\boxed{91.474}$.

14. Problem:

A continuous random variable X was measured 30 times. The sorted data are shown below.

10.045	10.156	10.420	10.575	11.130	11.306
11.318	11.383	11.510	11.646	11.698	12.119
12.171	12.265	12.528	12.586	12.933	13.140
13.410	13.562	13.650	13.668	14.262	14.327
14.357	14.470	14.576	14.686	14.718	14.906

The total of the measurements is 379.521.

- (a) Determine the percentile rank of the value 10.575. In other words, determine what percent of data are less than or equal to 10.575.
- (b) Determine the datum corresponding to a percentile rank of 0.333. In other words, determine x such that 33.3% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 10.575$. This means $i = 4$. We know $n = 30$. Determine the percentile ℓ .

$$\ell = \frac{4}{30}$$

$$\ell = 0.133$$

So, the percentile rank is 0.133, or 13.3th percentile.

- (b) We are given $\ell = 0.333$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (30)(0.333)$$

$$i = 10$$

Determine the x associated with $i = 10$.

$$x = \text{11.646}$$

- (c) The mean: $\bar{x} = \frac{379.521}{30} = \text{12.651}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 30$ and so n is even.

$$\text{median} = \frac{x_{15} + x_{16}}{2} = \frac{12.528 + 12.586}{2}$$

So, median = 12.557.