1. **Problem:**
   Some snails have clockwise shells, and others have counter-clockwise shells. In a random sample of 670 snails, there were 456 snails with clockwise shells. Construct a 98% confidence interval of the population proportion with clockwise shells.

Solution: Identify the givens.

$$n = 670$$
$$\hat{p} = 0.6806$$
$$\gamma = 0.98$$

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.98$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.98+1}{2} = 0.99$. (Use the z-table to find $z^{\star}$.)

$$z^{\star} = 2.33$$

Use the formula (proportion) for the bounds.

$$LB = \hat{p} - z^{\star}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad UB = \hat{p} + z^{\star}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.6806 - 2.33\sqrt{\frac{(0.6806)(0.3194)}{670}} \qquad = 0.6806 + 2.33\sqrt{\frac{(0.6806)(0.3194)}{670}}$$

$$= 0.639 \qquad = 0.723$$

Determine the interval.

$$CI = (0.639\,,\ 0.723)$$

We are 98% confident that the true population proportion is between 63.9% and 72.3%.

2. **Problem:**
   Brahim wants to estimate the average mass of the beans in a large bag. Somehow, Brahim is certain that the standard deviation of the beans in the bag is 25 milligrams. He takes a sample of size 61 and finds the sample mean to be 341. What would be the 90% confidence interval?

Solution: We are given the sample size, sample mean, population standard deviation, and confidence level.

$$n = 61$$
$$\bar{x} = 341$$
$$\sigma = 25$$
$$\gamma = 0.9$$

Determine the critical z value, $z^\star$, such that $P(|Z| < z^\star) = 0.9$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.9+1}{2} = 0.95$

$$z^\star = 1.64$$

Use the formula for bounds (mean, σ known).

$$\begin{aligned} LB &= \bar{x} - z^\star \frac{\sigma}{\sqrt{n}} & UB &= \bar{x} + z^\star \frac{\sigma}{\sqrt{n}} \\ &= 341 - 1.64 \times \frac{25}{\sqrt{61}} & &= 341 + 1.64 \times \frac{25}{\sqrt{61}} \\ &= 335.75 & &= 346.25 \end{aligned}$$

We are 90% confident that the population mean is between 335.75 and 346.25.

$$CI = (335.75, 346.25)$$

3. **Problem:**
   Your boss wants to know what proportion of a very large population is shiny. She also wants to guarantee that the margin of error of a 80% confidence interval will be less than 0.03 (which is 3 percentage points). How large of a sample is needed?

Solution: We are given the confidence level and the margin of error.

$$\gamma = 0.8$$
$$ME = 0.03$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.8$. It is helpful to get the percentile of $z^\star$ by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.8}{2} = 0.9$. This lets you find $z^\star$ such that $P(Z < z^\star) = 0.9$.

$$z^\star = 1.28$$

Use the appropriate formula. We have no knowledge of $\hat{p}$, so we are conservative by using $\hat{p} = 0.5$.

$$\begin{aligned} n &= \frac{1}{4}\left(\frac{z^\star}{ME}\right)^2 \\ &= \frac{1}{4}\left(\frac{1.28}{0.03}\right)^2 \\ &= 455.1111 \end{aligned}$$

When determining a necessary sample size, always round up (ceiling).

$$n = 456$$

4. **Problem:**
   Some snails have clockwise shells, and others have counter-clockwise shells. You want to construct a 95% confidence interval of the population proportion with clockwise shells. You hope the margin of error will be about 0.007. You already know that the proportion with clockwise shells is near 0.1. How many snails do you need?

Solution: We are given the confidence level, sample proportion, and the margin of error.

$$\gamma = 0.95$$
$$\hat{p} = 0.1$$
$$ME = 0.007$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.95$. It is helpful to get the percentile of $z^\star$ by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.95}{2} = 0.975$. This lets you find $z^\star$ such that $P(Z < z^\star) = 0.975$.

$$z^\star = 1.96$$

Use the appropriate formula.

$$\begin{aligned} n &= \hat{p}(1-\hat{p})\left(\frac{z^\star}{ME}\right)^2 \\ &= (0.1)(0.9)\left(\frac{1.96}{0.007}\right)^2 \\ &= 7056 \end{aligned}$$

When determining a necessary sample size, always round up (ceiling).

$$n = 7056$$

5. **Problem:**
   As an ornithologist, you wish to determine the average body mass of *Setophaga ruticilla*. You randomly sample 25 adults of *Setophaga ruticilla*, resulting in a sample mean of 9.09 grams and a sample standard deviation of 1.18 grams. Determine a 95% confidence interval of the true population mean.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 25 \\ \bar{x} &= 9.09 \\ s &= 1.18 \\ \gamma &= 0.95 \end{aligned}$$

Find the degrees of freedom.

$$\begin{aligned} df &= n - 1 \\ &= 25 - 1 \\ &= 24 \end{aligned}$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.95$ and $df = 24$.

$$t^\star = 2.06$$

Use the formula for bounds (mean, σ unknown).

$$\begin{aligned} LB &= \bar{x} - t^\star \frac{s}{\sqrt{n}} \\ &= 9.09 - 2.06 \times \frac{1.18}{\sqrt{25}} \\ &= 8.6 \end{aligned} \qquad \begin{aligned} UB &= \bar{x} + t^\star \frac{s}{\sqrt{n}} \\ &= 9.09 + 2.06 \times \frac{1.18}{\sqrt{25}} \\ &= 9.58 \end{aligned}$$

We are 95% confident that the population mean is between 8.6 and 9.58.

$$CI = (8.6,\ 9.58)$$

6. **Problem:**
   Some snails have clockwise shells, and others have counter-clockwise shells. You want to construct a 80% confidence interval of the population proportion with clockwise shells. You hope the margin of error will be about 0.02. How many snails do you need?

Solution: We are given the confidence level and the margin of error.

$$\gamma = 0.8$$
$$ME = 0.02$$

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.8$. It is helpful to get the percentile of $z^{\star}$ by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.8}{2} = 0.9$. This lets you find $z^{\star}$ such that $P(Z < z^{\star}) = 0.9$.

$$z^{\star} = 1.28$$

Use the appropriate formula.

$$n = \frac{1}{4}\left(\frac{z^{\star}}{ME}\right)^2$$

$$= \frac{1}{4}\left(\frac{1.28}{0.02}\right)^2$$

$$= 1024$$

When determining a necessary sample size, always round up (ceiling).

$$n = 1024$$

7. **Problem:**
A random sample of size 950 was found to have a sample proportion of 22% (because there were 209 successes). Determine a 74% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.
   (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 950$$
$$\hat{p} = 0.22$$
$$\gamma = 0.74$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.74$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.74+1}{2} = 0.87$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.13$$

Use the formula (proportion) for the bounds.

$$LB = \hat{p} - z^\star\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad UB = \hat{p} + z^\star\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.22 - 1.13\sqrt{\frac{(0.22)(0.78)}{950}} \qquad = 0.22 + 1.13\sqrt{\frac{(0.22)(0.78)}{950}}$$

$$= 0.205 \qquad = 0.235$$

Determine the interval.

$$CI \;=\; (0.205\,,\; 0.235)$$

We are 74% confident that the true population proportion is between 20.5% and 23.5%.

(a) The lower bound = 0.205, which can also be expressed as 20.5%.

(b) The upper bound = 0.235, which can also be expressed as 23.5%.

8. **Problem:**
   Your boss wants to know what proportion of a very large population is broken. You already know the proportion approximately 0.75. But, your boss wants to guarantee that the margin of error of a 96% confidence interval will be less than 0.001 (which is 0.1 percentage points). How large of a sample is needed?

Solution: We are given the confidence level and the margin of error.

$$\gamma = 0.96$$
$$ME = 0.001$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.96$. It is helpful to get the percentile of $z^\star$ by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.96}{2} = 0.98$. This lets you find $z^\star$ such that $P(Z < z^\star) = 0.98$.

$$z^\star = 2.05$$

Use the appropriate formula.

$$n = \hat{p}(1 - \hat{p})\left(\frac{z^\star}{ME}\right)^2$$

$$= (0.75)(0.25)\left(\frac{2.05}{0.001}\right)^2$$

$$= 787968.75$$

When determining a necessary sample size, always round up (ceiling).

$$n = 787969$$

9. **Problem:**
   Brahim wants to estimate the average mass of the beans in a large bag. Somehow, Brahim is certain that the standard deviation of the beans in the bag is 54 milligrams. He knows he wants the margin of error to be 0.1 when determining a 73% confidence interval. How large of a sample does Brahim need?

Solution: We are given the population standard deviation, confidence level, and margin of error.

$$\sigma = 54$$
$$\gamma = 0.73$$
$$ME = 0.1$$

Determine the critical z value, $z^\star$, such that $P(|Z| < z^\star) = 0.73$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.73+1}{2} = 0.865$

$$z^\star = 1.1$$

Use the formula for sample size.

$$n = \left(\frac{z^\star \sigma}{ME}\right)^2$$

$$= \left(\frac{(1.1)(54)}{0.1}\right)^2$$

$$= 352836$$

Round up.

$$n = 352836$$

10. **Problem:**
A traffic engineer wants to determine the average speed of cars on the Longfellow Bridge. She finds that a simple random sample of 81 cars has a mean of 42.04 mph. She also knows the population standard deviation of speeds is 14.27 mph. Determine a 99.5% confidence interval of the average speed of cars on the Longfellow Bridge.

Solution: We are given the sample size, sample mean, population standard deviation, and confidence level.

$$n = 81$$
$$\bar{x} = 42.04$$
$$\sigma = 14.27$$
$$\gamma = 0.995$$

Determine the critical z value, $z^{\star}$, such that $P(|Z| < z^{\star}) = 0.995$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.995+1}{2} = 0.9975$

$$z^{\star} = 2.81$$

Use the formula for bounds (mean, σ known).

$$\begin{aligned} LB &= \bar{x} - z^{\star}\frac{\sigma}{\sqrt{n}} \\ &= 42.04 - 2.81 \times \frac{14.27}{\sqrt{81}} \\ &= 37.58 \end{aligned} \qquad \begin{aligned} UB &= \bar{x} + z^{\star}\frac{\sigma}{\sqrt{n}} \\ &= 42.04 + 2.81 \times \frac{14.27}{\sqrt{81}} \\ &= 46.5 \end{aligned}$$

We are 99.5% confident that the population mean is between 37.58 and 46.5.

$$CI = (37.58,\ 46.5)$$

11. **Problem:**
A traffic engineer wants to determine the average speed of cars on the Longfellow Bridge. She knows the population standard deviation of speeds is 32 mph. To determine a 67% confidence interval with a margin of error of 2 mph, what sample size is needed?

Solution: We are given the population standard deviation, confidence level, and margin of error.

$$\sigma = 32$$
$$\gamma = 0.67$$
$$ME = 2$$

Determine the critical z value, $z^\star$, such that $P(|Z| < z^\star) = 0.67$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.67+1}{2} = 0.835$

$$z^\star = 0.97$$

Use the formula for sample size.

$$\begin{aligned} n &= \left(\frac{z^\star \sigma}{ME}\right)^2 \\ &= \left(\frac{(0.97)(32)}{2}\right)^2 \\ &= 240.8704 \end{aligned}$$

Round up.

$$n = 241$$

12. **Problem:**
A traffic engineer wants to determine the average speed of cars on the Longfellow Bridge. She finds that a simple random sample of 38 cars has a mean of 41.53 mph and a standard deviation of 14.91 mph. Determine a 98% confidence interval of the average speed (in mph) of cars on the Longfellow Bridge.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 38 \\ \bar{x} &= 41.53 \\ s &= 14.91 \\ \gamma &= 0.98 \end{aligned}$$

Find the degrees of freedom.

$$\begin{aligned} df &= n - 1 \\ &= 38 - 1 \\ &= 37 \end{aligned}$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.98$ and $df = 37$.

$$t^\star = 2.43$$

Use the formula for bounds (mean, σ unknown).

$$\begin{aligned} LB &= \bar{x} - t^\star \frac{s}{\sqrt{n}} \\ &= 41.53 - 2.43 \times \frac{14.91}{\sqrt{38}} \\ &= 35.65 \end{aligned} \qquad \begin{aligned} UB &= \bar{x} + t^\star \frac{s}{\sqrt{n}} \\ &= 41.53 + 2.43 \times \frac{14.91}{\sqrt{38}} \\ &= 47.41 \end{aligned}$$

We are 98% confident that the population mean is between 35.65 and 47.41. In this scenario, we are 98% confident that the true average speed of cars on Longfellow is between 35.65 mph and 47.41 mph.

$$CI = (35.65,\ 47.41)$$

13. **Problem:**
Brahim wants to estimate the average mass of the beans in a large bag. He takes a sample of size 49 and finds the sample mean to be 431 and the sample standard deviation to be 50. What would be the 99.5% confidence interval?

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 49 \\ \bar{x} &= 431 \\ s &= 50 \\ \gamma &= 0.995 \end{aligned}$$

Find the degrees of freedom.

$$\begin{aligned} df &= n - 1 \\ &= 49 - 1 \\ &= 48 \end{aligned}$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.995$ and $df = 48$.

$$t^\star = 2.94$$

Use the formula for bounds (mean, σ unknown).

$$\begin{aligned} LB &= \bar{x} - t^\star \frac{s}{\sqrt{n}} & \qquad UB &= \bar{x} + t^\star \frac{s}{\sqrt{n}} \\ &= 431 - 2.94 \times \frac{50}{\sqrt{49}} & &= 431 + 2.94 \times \frac{50}{\sqrt{49}} \\ &= 410 & &= 452 \end{aligned}$$

We are 99.5% confident that the population mean is between 410 and 452.

$$CI = (410,\ 452)$$