

**1. Problem**

From a very large population, a small sample of measurements was taken.

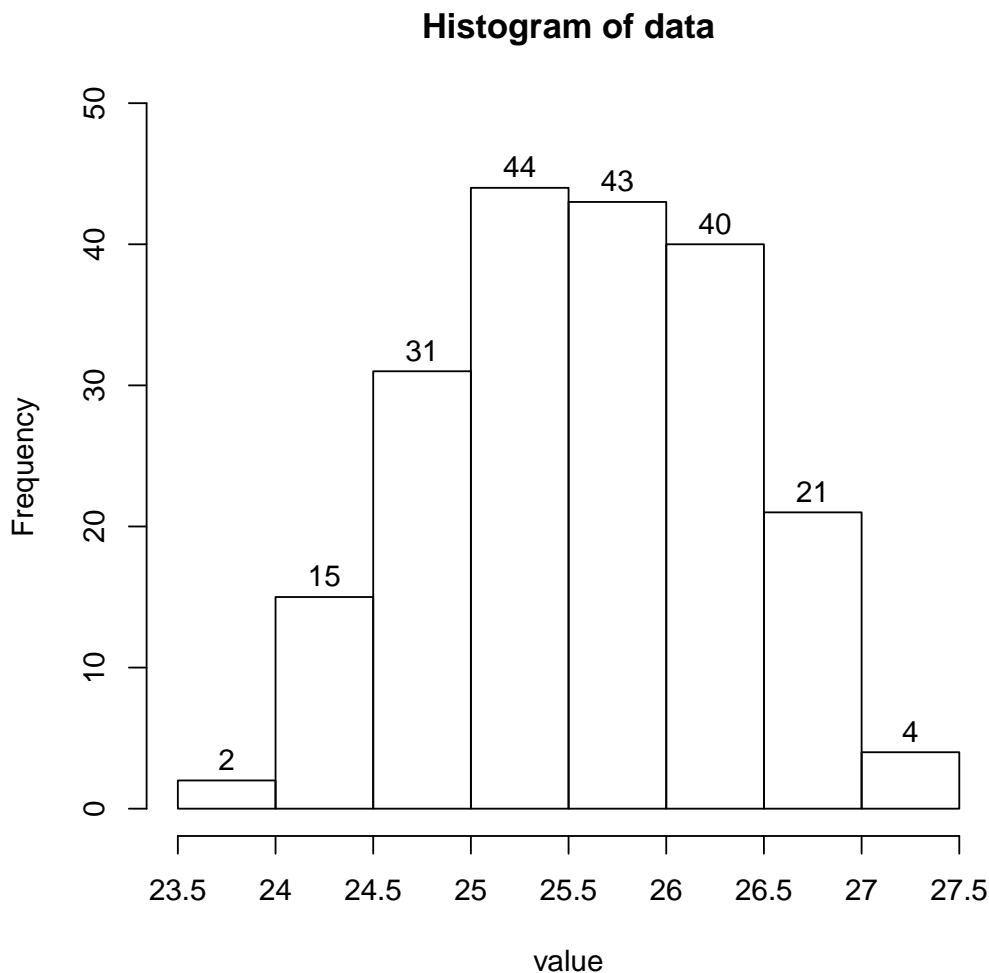
173	175	179	181
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Please calculate the Average Absolute Deviation (also called the Average Distance from Mean: ADM) using the following formula:

$$AAD = \frac{\sum |x - \bar{x}|}{n}$$

**2. Problem**

A continuous random variable was measured 200 times. The resulting histogram is shown below.



- (a) Describe the overall shape of the distribution. (symmetric mound, skew left, skew right, uniform, or bimodal)
- (b) Estimate the range of the distribution (range = max-min).
- (c) What percent of the measurements are less than 25.5?
- (d) What percent of the measurements are greater than 24.5?
- (e) What percent of the measurements are between 24.5 and 25.5?
- (f) What percent of the measurements are within 0.25 from 24.75? In other words, what percent of measurements satisfy  $|x - 24.75| \leq 0.25$ ?
- (g) Of the measurements less than 25.5, what percent are greater than 24.5?
- (h) Estimate the value of the 1th percentile. In other words, determine a value such that 1% of the measurements are less than or equal to it.

**3. Problem**

A continuous random variable  $X$  was measured 42 times. The sorted measurements are shown below.

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43.28	43.49	43.72	43.95	44.1	44.11	44.11	44.19	44.24	44.37
44.37	44.4	44.42	44.45	44.46	44.48	44.53	44.72	44.81	44.81
44.82	44.83	44.84	45	45.11	45.12	45.19	45.24	45.27	45.38
45.39	45.41	45.46	45.52	45.57	45.59	45.69	45.81	45.87	46.02
46.04	46.25								

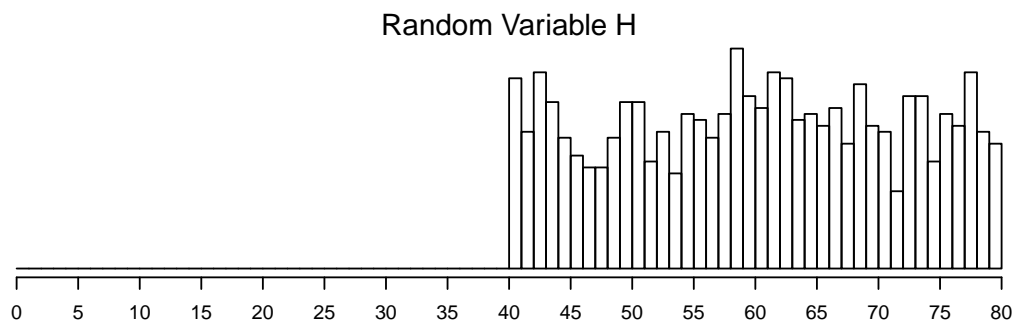
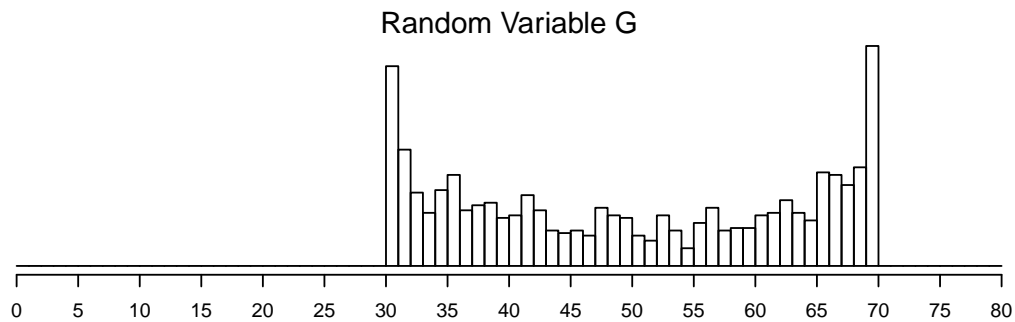
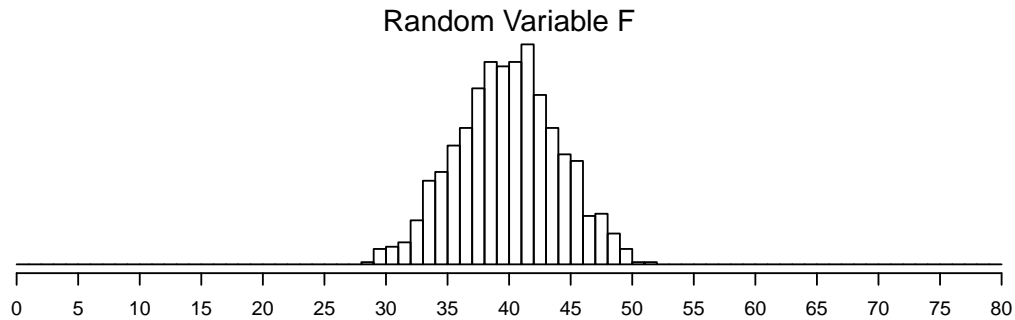
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The total of the measurements is 1884.43.

- (a) Determine the percentile rank of the measurement 45. In other words, determine what percent of data are less than or equal to 45.
- (b) Determine the measurement corresponding to a percentile rank of 0.548. In other words, determine  $x$  such that 54.8% of the data are less than or equal to  $x$ .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

**4. Problem**

Three random variables (F, G, and H) were measured 1000 times each. The resulting histograms show the three distributions.



- (a) Which distribution has the highest mean? (F, G, or H)
- (b) Which distribution has the lowest mean? (F, G, or H)
- (c) Which distribution has the largest standard deviation? (F, G, or H)
- (d) Which distribution has the smallest standard deviation? (F, G, or H)

**5. Problem**

From a very large population, a small sample of measurements was taken.

176	176	188	184
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Please calculate the (Bessel corrected) sample standard deviation using the following formula:

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

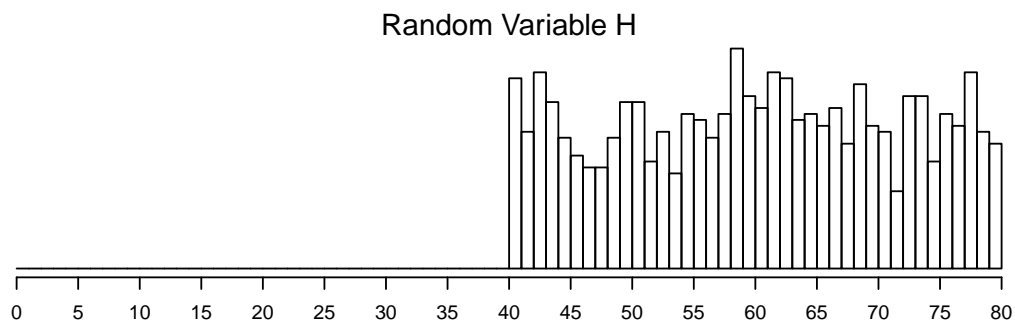
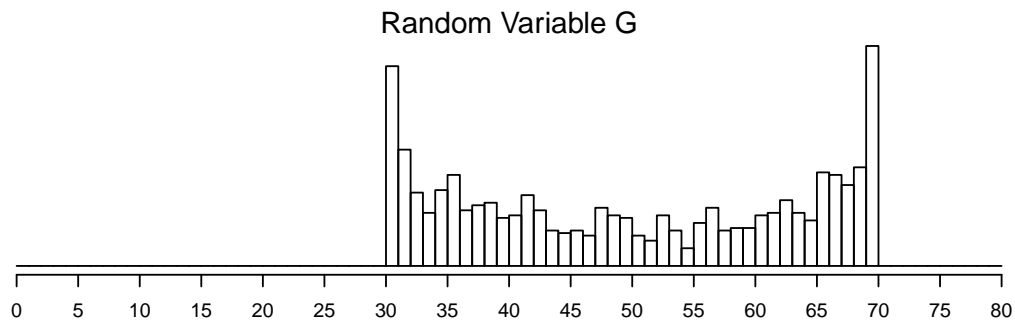
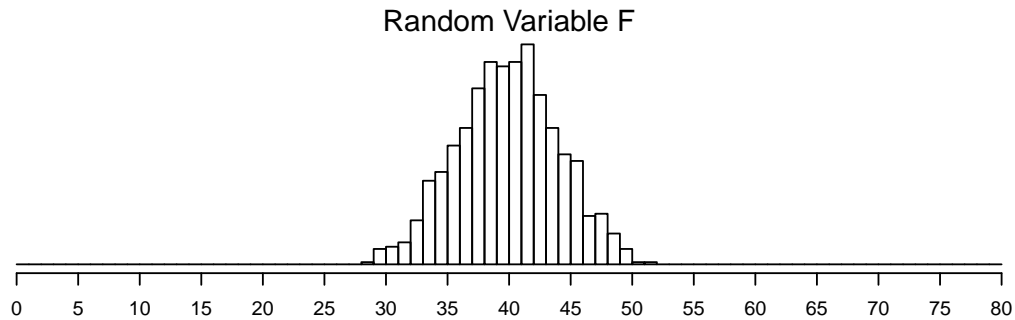
**6. Problem**

Please make a frequency table and a dot plot from the following (unsorted) data.

51	50	50	53	51	55
51	51	55	52	51	51
51	53	52	54	54	51
51	50	54	50	50	56
52	53	50	53	51	50

**7. Problem**

Three random variables (F, G, and H) were measured 1000 times each. The resulting histograms show the three distributions.



- (a) Which distribution has the highest mean? (F, G, or H)
- (b) Which distribution has the lowest mean? (F, G, or H)
- (c) Which distribution has the largest standard deviation? (F, G, or H)
- (d) Which distribution has the smallest standard deviation? (F, G, or H)

**8. Problem**

A continuous random variable was measured 300 times. The resulting frequency distribution is shown below.

class	frequency
50–60	10
60–70	26
70–80	33
80–90	28
90–100	28
100–110	28
110–120	31
120–130	28
130–140	29
140–150	33
150–160	26

- (a) Describe the overall shape of the distribution. (symmetric mound, skew left, skew right, uniform, or bimodal)
- (b) Estimate the range of the distribution (range = max-min).
- (c) What percent of the measurements are greater than 110?
- (d) What percent of the measurements are greater than 140?
- (e) What percent of the measurements are between 110 and 140?
- (f) What percent of the measurements are within 20 of 110? In other words, what percent of measurements satisfy  $|x - 110| \leq 20$ ?
- (g) Of the measurements greater than 110, what percent are greater than 140?
- (h) Estimate the value of the 3.333th percentile. In other words, determine a value such that 3.333% of the measurements are less than or equal to it.



1. We fill out the table column by column.

$x$	$x - \bar{x}$	$ x - \bar{x} $
173	-4	4
175	-2	2
179	2	2
181	4	4
=====	=====	=====
$\sum x = 708$		$\sum  x - \bar{x}  = 12$
$\bar{x} = 177$		

We are ready for the formula.

$$s = \frac{\sum |x - \bar{x}|}{n}$$

$$= \frac{12}{4}$$

$$= \boxed{3}$$

2. (a) symmetric mound

(b) 4

(c) 46%

(d) 91.5%

(e) 37.5%

(f) 15.5%

(g) 81.52%

(h) 24

3. Let  $x$  represent a measurement of interest. Let  $i$  represent that measurement's index. Let  $\ell$  represent that measurement's percentile. Let  $n$  represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given  $x = 45$ . This means  $i = 24$ . We know  $n = 42$ . Determine the percentile  $\ell$ .

$$\ell = \frac{24}{42}$$

$$\ell = 0.571$$

So, the percentile rank is  $\boxed{0.571}$ , or 57.1th percentile.

(b) We are given  $\ell = 0.548$ . We can use algebra to solve for  $i$ .

$$\ell = \frac{i}{n}$$

Multiply both sides by  $n$ .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate  $i$ .

$$i = (42)(0.548)$$

$$i = 23$$

Determine the  $x$  associated with  $i = 23$ .

$$x = \boxed{44.84}$$

(c) The mean:  $\bar{x} = \frac{1884.43}{42} = \boxed{44.867}$

(d) If  $n$  is odd, then median is  $x_{i=\frac{n+1}{2}}$ , the value of  $x$  when  $i = \frac{n+1}{2}$ . Otherwise, if  $n$  is even, the median is mean of  $x_{i=\frac{n}{2}}$  and  $x_{i=\frac{n}{2}+1}$ . In this case,  $n = 42$  and so  $n$  is even.

$$\text{median} = \frac{x_{21} + x_{22}}{2} = \frac{44.82 + 44.83}{2}$$

So, median =  $\boxed{44.825}$ .

4. (a) H

(b) F

(c) G

(d) F

5. We fill out the table column by column.

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
176	-5	25
176	-5	25
188	7	49
184	3	9
=====	=====	=====
$\sum x = 724$		$\sum (x - \bar{x})^2 = 108$
$\bar{x} = 181$		

We are ready for the formula.

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

$$= \sqrt{\frac{108}{4 - 1}}$$

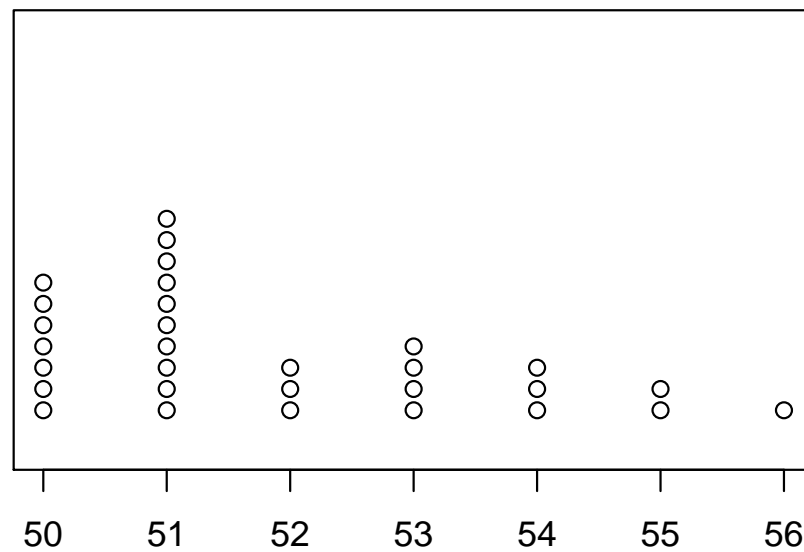
$$= \sqrt{36}$$

$$= \boxed{6}$$

6. Make a frequency table.

value	frequency
50	7
51	10
52	3
53	4
54	3
55	2
56	1

Make the dot plot.



.image

7. (a) F  
(b) H  
(c) H  
(d) F
8. (a) uniform  
(b) 110  
(c) 49%  
(d) 19.67%  
(e) 29.33%  
(f) 38.33%  
(g) 40.14%  
(h) 60