1. **Problem:**
   A population has unknown μ but a known $\sigma = 5.2$. A sample of size 96 has a mean $\bar{x} = 118.14$. Determine the 80% confidence interval of the population mean.

Solution: We are given the sample size, sample mean, population standard deviation, and confidence level.

$$n = 96$$
$$\bar{x} = 118.14$$
$$\sigma = 5.2$$
$$\gamma = 0.8$$

Determine the critical z value, $z^\star$, such that $P(|Z| < z^\star) = 0.8$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.8+1}{2} = 0.9$

$$z^\star = 1.28$$

Use the formula for bounds (mean, σ known).

$$\begin{aligned} LB &= \bar{x} - z^\star \frac{\sigma}{\sqrt{n}} \\ &= 118.14 - 1.28 \times \frac{5.2}{\sqrt{96}} \\ &= 117.46 \end{aligned} \qquad \begin{aligned} UB &= \bar{x} + z^\star \frac{\sigma}{\sqrt{n}} \\ &= 118.14 + 1.28 \times \frac{5.2}{\sqrt{96}} \\ &= 118.82 \end{aligned}$$

We are 80% confident that the population mean is between 117.46 and 118.82.

$$CI = (117.46, 118.82)$$

2. **Problem:**
A population has unknown μ but a known $\sigma = 2.14$. A sample of size 128 has a mean $\bar{x} = 145.83$. Determine the 99% confidence interval of the population mean.

Solution: We are given the sample size, sample mean, population standard deviation, and confidence level.

$$n = 128$$
$$\bar{x} = 145.83$$
$$\sigma = 2.14$$
$$\gamma = 0.99$$

Determine the critical z value, $z^\star$, such that $P(|Z| < z^\star) = 0.99$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.99+1}{2} = 0.995$

$$z^\star = 2.58$$

Use the formula for bounds (mean, σ known).

$$\begin{aligned} LB &= \bar{x} - z^\star \frac{\sigma}{\sqrt{n}} \\ &= 145.83 - 2.58 \times \frac{2.14}{\sqrt{128}} \\ &= 145.34 \end{aligned} \qquad \begin{aligned} UB &= \bar{x} + z^\star \frac{\sigma}{\sqrt{n}} \\ &= 145.83 + 2.58 \times \frac{2.14}{\sqrt{128}} \\ &= 146.32 \end{aligned}$$

We are 99% confident that the population mean is between 145.34 and 146.32.

$$CI = (145.34,\ 146.32)$$

3. **Problem:**
A population has unknown μ and unknown σ. A sample of size 61 has a mean $\bar{x}$ = 136.07 and a standard deviation s = 2.53. Determine the 99.5% confidence interval of the population mean.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$
\begin{aligned}
n &= 61 \\
\bar{x} &= 136.07 \\
s &= 2.53 \\
\gamma &= 0.995
\end{aligned}
$$

Find the degrees of freedom.

$$
\begin{aligned}
df &= n - 1 \\
&= 61 - 1 \\
&= 60
\end{aligned}
$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.995$ and $df = 60$.

$$t^{\star} = 2.91$$

Use the formula for bounds (mean, σ unknown).

$$
\begin{aligned}
LB &= \bar{x} - t^{\star}\frac{s}{\sqrt{n}} & \qquad UB &= \bar{x} + t^{\star}\frac{s}{\sqrt{n}} \\
&= 136.07 - 2.91 \times \frac{2.53}{\sqrt{61}} & &= 136.07 + 2.91 \times \frac{2.53}{\sqrt{61}} \\
&= 135.13 & &= 137.01
\end{aligned}
$$

We are 99.5% confident that the population mean is between 135.13 and 137.01.

$$CI = (135.13,\ 137.01)$$

4. **Problem:**
As an ornithologist, you wish to determine the average body mass of *Hylocichla mustelina*. You randomly sample 26 adults of *Hylocichla mustelina*, resulting in a sample mean of 56.81 grams and a sample standard deviation of 6.41 grams. Determine a 98% confidence interval of the true population mean.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 26 \\ \bar{x} &= 56.81 \\ s &= 6.41 \\ \gamma &= 0.98 \end{aligned}$$

Find the degrees of freedom.

$$\begin{aligned} df &= n - 1 \\ &= 26 - 1 \\ &= 25 \end{aligned}$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.98$ and $df = 25$.

$$t^{\star} = 2.49$$

Use the formula for bounds (mean, σ unknown).

$$\begin{aligned} LB &= \bar{x} - t^{\star}\frac{s}{\sqrt{n}} \\ &= 56.81 - 2.49 \times \frac{6.41}{\sqrt{26}} \\ &= 53.7 \end{aligned} \qquad \begin{aligned} UB &= \bar{x} + t^{\star}\frac{s}{\sqrt{n}} \\ &= 56.81 + 2.49 \times \frac{6.41}{\sqrt{26}} \\ &= 59.9 \end{aligned}$$

We are 98% confident that the population mean is between 53.7 and 59.9.

$$CI = (53.7,\ 59.9)$$

5. **Problem:**
A population has unknown μ but a known σ = 9.1. You want to determine a 70% confidence interval of the population mean with a margin of error of approximately 3. How large of a sample is needed?

Solution: We are given the population standard deviation, confidence level, and margin of error.

$$\sigma = 9.1$$
$$\gamma = 0.7$$
$$ME = 3$$

Determine the critical z value, $z^\star$, such that $P(|Z| < z^\star) = 0.7$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.7+1}{2} = 0.85$

$$z^\star = 1.04$$

Use the formula for sample size.

$$n = \left(\frac{z^\star \sigma}{ME}\right)^2$$

$$= \left(\frac{(1.04)(9.1)}{3}\right)^2$$

$$= 9.9519218$$

Round up.

$$n = 10$$

Really, you should round up, retaining only about 2 significant figures.

$$n = 10$$

6. **Problem:**
   A population has unknown μ but a known $\sigma = 48$. You want to determine a 96% confidence interval of the population mean with a margin of error of approximately 10. How large of a sample is needed?

Solution: We are given the population standard deviation, confidence level, and margin of error.

$$\sigma = 48$$
$$\gamma = 0.96$$
$$ME = 10$$

Determine the critical z value, $z^\star$, such that $P(|Z| < z^\star) = 0.96$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.96+1}{2} = 0.98$

$$z^\star = 2.05$$

Use the formula for sample size.

$$n = \left(\frac{z^\star \sigma}{ME}\right)^2$$

$$= \left(\frac{(2.05)(48)}{10}\right)^2$$

$$= 96.8256$$

Round up.

$$n = 97$$

Really, you should round up, retaining only about 2 significant figures.

$$n = 97$$

7. **Problem:**
   A random sample of size 6800 was found to have a sample proportion of 20% (because there were 1360 successes). Determine a 83% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 6800$$
$$\hat{p} = 0.2$$
$$\gamma = 0.83$$

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.83$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.83+1}{2} = 0.915$. (Use the z-table to find $z^{\star}$.)

$$z^{\star} = 1.37$$

Use the formula (proportion) for the bounds.

$$LB = \hat{p} - z^{\star}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad UB = \hat{p} + z^{\star}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.2 - 1.37\sqrt{\frac{(0.2)(0.8)}{6800}} \qquad = 0.2 + 1.37\sqrt{\frac{(0.2)(0.8)}{6800}}$$

$$= 0.193 \qquad = 0.207$$

Determine the interval.

$$CI = (0.193\,,\ 0.207)$$

We are 83% confident that the true population proportion is between 19.3% and 20.7%.

(a) The lower bound = 0.193, which can also be expressed as 19.3%.

(b) The upper bound = 0.207, which can also be expressed as 20.7%.

8. **Problem:**
A random sample of size 950 was found to have a sample proportion of 58.95% (because there were 560 successes). Determine a 83% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 950$$
$$\hat{p} = 0.5895$$
$$\gamma = 0.83$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.83$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.83+1}{2} = 0.915$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.37$$

Use the formula (proportion) for the bounds.

$$LB = \hat{p} - z^\star\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad UB = \hat{p} + z^\star\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.5895 - 1.37\sqrt{\frac{(0.5895)(0.4105)}{950}} \qquad = 0.5895 + 1.37\sqrt{\frac{(0.5895)(0.4105)}{950}}$$

$$= 0.568 \qquad = 0.611$$

Determine the interval.

$$CI = (0.568\,,\ 0.611)$$

We are 83% confident that the true population proportion is between 56.8% and 61.1%.

(a) The lower bound = 0.568, which can also be expressed as 56.8%.

(b) The upper bound = 0.611, which can also be expressed as 61.1%.

9. **Problem:**
   If you suspect that $\hat{p}$ will be near 0.91, how large of a sample is needed to guarantee a margin of error less than 0.02 when building a 95% confidence interval?

Solution: We are given the confidence level and the margin of error.

$$\gamma = 0.95$$
$$ME = 0.02$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.95$. It is helpful to get the percentile of $z^\star$ by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.95}{2} = 0.975$. This lets you find $z^\star$ such that $P(Z < z^\star) = 0.975$.

$$z^\star = 1.96$$

Use the appropriate formula.

$$\begin{aligned} n &= \hat{p}(1-\hat{p})\left(\frac{z^\star}{ME}\right)^2 \\ &= (0.91)(0.09)\left(\frac{1.96}{0.02}\right)^2 \\ &= 786.5676 \end{aligned}$$

When determining a necessary sample size, always round up (ceiling).

$$n = 787$$

10. **Problem:**
    Your boss wants to know what proportion of a very large population is angry. You already know the proportion approximately 0.36. But, your boss wants to guarantee that the margin of error of a 90% confidence interval will be less than 0.01 (which is 1 percentage points). How large of a sample is needed? Please round up, using only 2 significant digits.

Solution: We are given the confidence level and the margin of error.

$$\gamma = 0.9$$
$$ME = 0.01$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.9$. It is helpful to get the percentile of $z^\star$ by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.9}{2} = 0.95$. This lets you find $z^\star$ such that $P(Z < z^\star) = 0.95$.

$$z^\star = 1.64$$

Use the appropriate formula.

$$n = \hat{p}(1 - \hat{p}) \left(\frac{z^\star}{ME} \right)^2$$

$$= (0.36)(0.64) \left(\frac{1.64}{0.01} \right)^2$$

$$= 6196.8384$$

When determining a necessary sample size, always round up (ceiling).

$$n = 6197$$

11. **Problem:**
    Your boss wants to know what proportion of a very large population is angry. She also wants to guarantee that the margin of error of a 99% confidence interval will be less than 0.07 (which is 7 percentage points). How large of a sample is needed? Please round up, using only 2 significant digits.

Solution: We are given the confidence level and the margin of error.

$$\gamma = 0.99$$
$$ME = 0.07$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.99$. It is helpful to get the percentile of $z^\star$ by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.99}{2} = 0.995$. This lets you find $z^\star$ such that $P(Z < z^\star) = 0.995$.

$$z^\star = 2.58$$

Use the appropriate formula. We have no knowledge of $\hat{p}$, so we are conservative by using $\hat{p} = 0.5$.

$$n = \frac{1}{4}\left(\frac{z^\star}{ME}\right)^2$$

$$= \frac{1}{4}\left(\frac{2.58}{0.07}\right)^2$$

$$= 339.6122449$$

When determining a necessary sample size, always round up (ceiling).

$$n = 340$$