

1. Problem:

Brahim wants to estimate the average mass of the beans in a large bag. Somehow, Brahim is certain that the standard deviation of the beans in the bag is 57 milligrams. He takes a sample of size 80 and finds the sample mean to be 376 milligrams. What would be the 98% confidence interval?

Solution: We are given the sample size, sample mean, population standard deviation, and confidence level.

$$n = 80$$

$$\bar{x} = 376$$

$$\sigma = 57$$

$$\gamma = 0.98$$

Determine the critical z value, z^* , such that $P(|Z| < z^*) = 0.98$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.98+1}{2} = 0.99$

$$z^* = 2.33$$

Use the formula for bounds (mean, σ known).

$$LB = \bar{x} - z^* \frac{\sigma}{\sqrt{n}}$$

$$= 376 - 2.33 \times \frac{57}{\sqrt{80}}$$

$$= 361.15$$

$$UB = \bar{x} + z^* \frac{\sigma}{\sqrt{n}}$$

$$= 376 + 2.33 \times \frac{57}{\sqrt{80}}$$

$$= 390.85$$

We are 98% confident that the population mean is between 361.15 and 390.85 milligrams.

$$CI = (361.15, 390.85)$$

2. Problem:

A new virus has been devastating corn production. When exposed, 65.8% of common seedlings die within a week. We are trying to develop a resistant strain of corn.

When we expose 700 seedlings of our strain to the virus, 62.6% die within a week. Using a significance level of 0.05, can we conclude that our strain is significantly more resistant?

- (a) What kind of hypothesis test is appropriate?
- (b) State the hypotheses
- (c) Determine the p -value.
- (d) Decide whether we reject or retain the null hypothesis.
- (e) Do we think our strain is significantly more resistant?

Solution: This is a left-tail (one-tail) proportion test because we only care whether a lower percentage of seedlings will die.

State the hypotheses.

$$H_0 \text{ claims } p = 0.658$$

$$H_A \text{ claims } p < 0.658$$

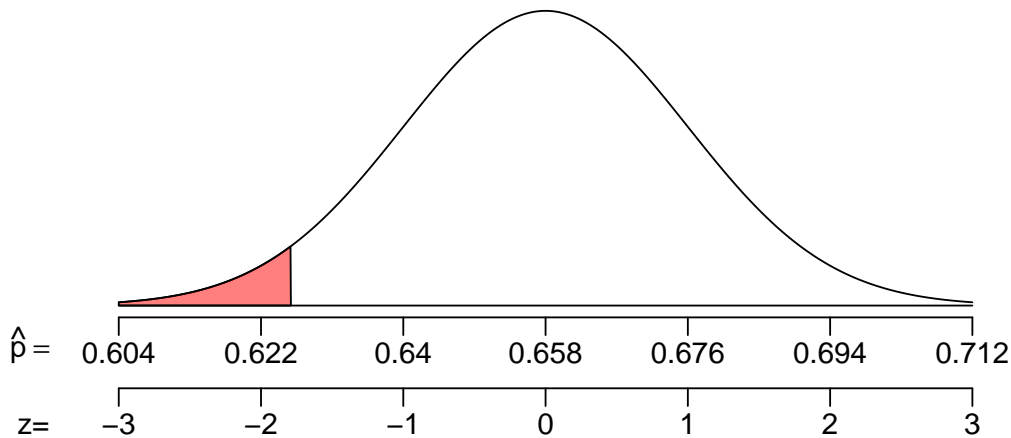
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.658(1-0.658)}{700}} = 0.0179$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.626 - 0.658}{0.0179} = -1.79$$

Make a sketch of the null's sampling distribution. The p -value is a left area.



To determine that left area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} < 0.626) \\ &= P(Z < -1.79) \\ &= 0.0367 \end{aligned}$$

Compare p -value to α (which is 0.05).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

We think our strain is more resistant than common corn.

- (a) Left-tail (one-tail) proportion test
- (b) Hypotheses: H_0 claims $p = 0.658$ and H_A claims $p < 0.658$.
- (c) The p -value is 0.0367
- (d) We reject the null hypothesis.
- (e) We think our strain is more resistant than common corn.

3. Problem:

A fair 8-sided die has a discrete uniform distribution with an expected value of $\mu = 4.5$ and a standard deviation $\sigma = 2.29$.

You are told to check if a 8-sided die has an expected value different than 4.5. You are told to roll the die 173 times and do a significance test with a significance level of 0.1.

You then roll the die 173 times and get a mean of 4.229. Should we conclude the die is unfair?

- (a) What kind of hypothesis test is appropriate?
- (b) State the hypotheses
- (c) Determine the p -value.
- (d) Decide whether we reject or retain the null hypothesis.
- (e) Do we conclude the die is unfair?

Solution: We should use a two-tail test of population mean.

State the hypotheses:

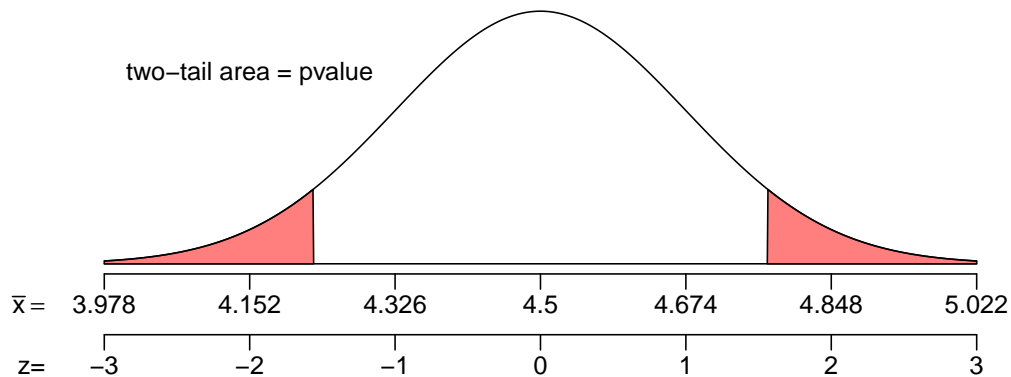
$$H_0 \text{ claims } \mu = 4.5$$

$$H_A \text{ claims } \mu \neq 4.5$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.29}{\sqrt{173}} = 0.174$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{4.229 - 4.5}{0.174} = -1.56$$

Find the p -value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(|Z| > 1.56) \\ &= 2 \cdot P(Z < -1.56) \\ &= \boxed{0.1188} \end{aligned}$$

Compare the p -value and the significance level ($\alpha = 0.1$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

We conclude the die might be fair.

- (a) Right-tail single mean test
- (b) Hypotheses: H_0 claims $\mu = 4.5$ and H_A claims $\mu \neq 4.5$.
- (c) p -value = 0.1188
- (d) No, we do not reject the null hypothesis.
- (e) We conclude the die might be fair.

4. Problem:

A traffic engineer wants to determine the average speed of cars on the Longfellow Bridge. She knows the population standard deviation of speeds is 10 mph. To determine a 90% confidence interval with a margin of error of 0.2 mph, what sample size is needed?

Solution: We are given the population standard deviation, confidence level, and margin of error.

$$\sigma = 10$$

$$\gamma = 0.9$$

$$ME = 0.2$$

Determine the critical z value, z^* , such that $P(|Z| < z^*) = 0.9$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.9+1}{2} = 0.95$

$$z^* = 1.64$$

Use the formula for sample size.

$$n = \left(\frac{z^* \sigma}{ME} \right)^2$$

$$= \left(\frac{(1.64)(10)}{0.2} \right)^2$$

$$= 6724$$

Round up.

$$n = 6724$$

5. Problem:

A fair coin should have a 50% chance of landing on either side. Someone is mildly suspicious that a coin is unfair.

You are asked to judge the fairness of the coin by flipping it 10000 times and counting how many heads are flipped. You are told to use a significance level of 0.05.

Then, you actually flip the coin 10000 times and get 5110 heads. Should we conclude this coin is unfair?

- (a) What kind of hypothesis test is appropriate?
- (b) State the hypotheses
- (c) Determine the p -value.
- (d) Decide whether we reject or retain the null hypothesis.
- (e) Do you think the coin is unfair?

Solution: We should use a two-tail proportion test.

State the hypotheses.

$$H_0 \text{ claims } p = 0.5$$

$$H_A \text{ claims } p \neq 0.5$$

Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(1-0.5)}{10000}} = 0.005$$

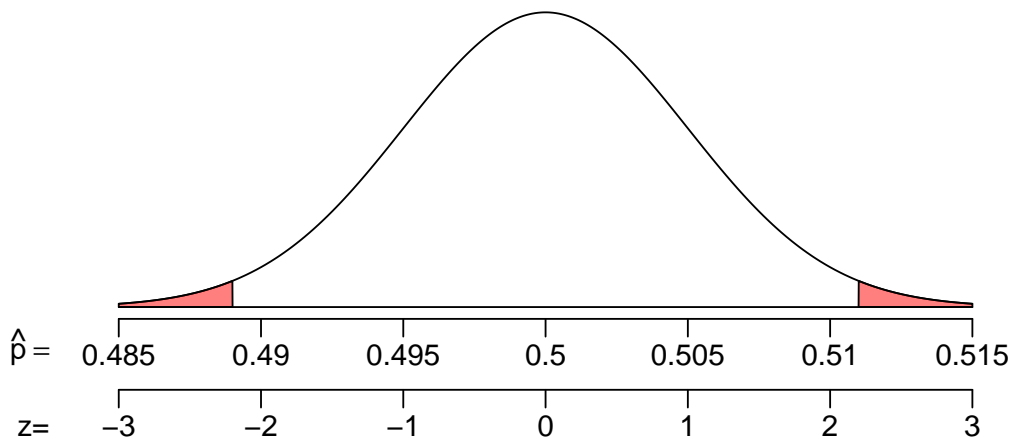
Determine the sample proportion.

$$\hat{p} = 0.511$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.511 - 0.5}{0.005} = 2.2$$

Make a sketch of the null's sampling distribution. The p -value is a two-tail area.



To determine that two-tail area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(|Z| > 2.2) \\ &= 2 \cdot P(Z < -2.2) \\ &= 0.0278 \end{aligned}$$

Compare p -value to α (which is 0.05).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

We conclude the coin is unfair.

- (a) Two-tail proportion test
- (b) Hypotheses: H_0 claims $p = 0.5$ and H_A claims $p \neq 0.5$.
- (c) The p -value is 0.0278
- (d) We reject the null hypothesis.
- (e) We conclude the coin is unfair.

6. Problem:

A null hypothesis claims a population has a mean $\mu = 80$. You decide to run two-tail test on a sample of size $n = 8$ using a significance level $\alpha = 0.1$.

You then collect the sample:

78.9	81.5	100.7	80.2	86.3
98.3	74	99.7		

- (a) Determine the p -value.
- (b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \text{ claims } \mu = 80$$

$$H_A \text{ claims } \mu \neq 80$$

Find the mean and standard deviation of the sample.

$$\bar{x} = 87.45$$

$$s = 10.601$$

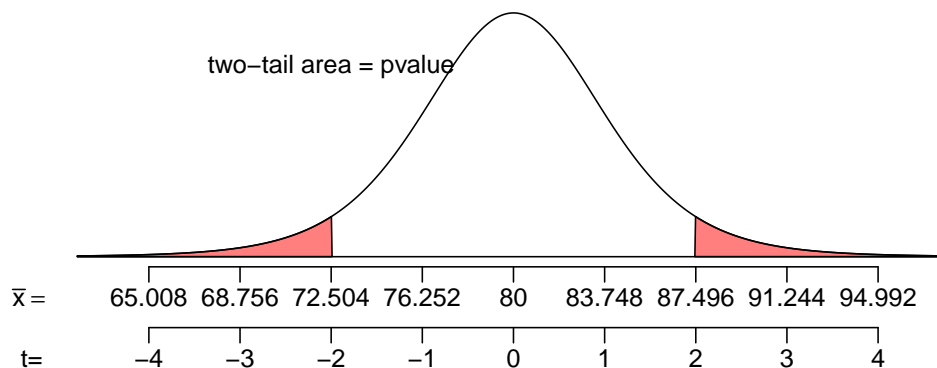
Determine the degrees of freedom.

$$df = 8 - 1 = 7$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{10.601}{\sqrt{8}} = 3.748$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{87.45 - 80}{3.748} = 1.99$$

Find the p -value.

$$p\text{-value} = P(|T| > 1.99)$$

We can't get an exact value with our table, but we can determine an interval that contains the p -value. (Look at row with $df = 7$.)

$$P(|T| > 2.36) = 0.05$$

$$P(|T| > 1.89) = 0.1$$

Basically, because t is between 2.36 and 1.89, we know the p -value is between 0.05 and 0.1.

$$0.05 < p\text{-value} < 0.1$$

Compare the p -value and the significance level ($\alpha = 0.1$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) $0.05 < p\text{-value} < 0.1$

(b) Yes, we reject the null hypothesis.

7. Problem:

As an ornithologist, you wish to determine the average body mass of *Setophaga ruticilla*. You randomly sample 23 adults of *Setophaga ruticilla*, resulting in a sample mean of 8.66 grams and a sample standard deviation of 1.13 grams. Determine a 95% confidence interval of the true population mean.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 23$$

$$\bar{x} = 8.66$$

$$s = 1.13$$

$$\gamma = 0.95$$

Find the degrees of freedom.

$$df = n - 1$$

$$= 23 - 1$$

$$= 22$$

Determine the critical t value, t^* , such that $P(|T| < t^*) = 0.95$ and $df = 22$.

$$t^* = 2.07$$

Use the formula for bounds (mean, σ unknown).

$$LB = \bar{x} - t^* \frac{s}{\sqrt{n}}$$

$$= 8.66 - 2.07 \times \frac{1.13}{\sqrt{23}}$$

$$= 8.17$$

$$UB = \bar{x} + t^* \frac{s}{\sqrt{n}}$$

$$= 8.66 + 2.07 \times \frac{1.13}{\sqrt{23}}$$

$$= 9.15$$

We are 95% confident that the population mean is between 8.17 and 9.15.

$$CI = (8.17, 9.15)$$

8. Problem:

You are tasked with estimating the proportion of widgets that are defective. In a sample of 960 widgets, you determine that 41.04% were defective. Determine a 80% confidence interval of the population proportion. (You are making an inference about the proportion of all widgets that are defective, which you'd only know from a census.)

Solution: Identify the givens.

$$n = 960$$

$$\hat{p} = 0.4104$$

$$\gamma = 0.8$$

Determine z^* such that $P(|Z| < z^*) = 0.8$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.8+1}{2} = 0.9$. (Use the z-table to find z^* .)

$$z^* = 1.28$$

Use the formula (proportion) for the bounds.

$$LB = \hat{p} - z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$UB = \hat{p} + z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$= 0.4104 - 1.28 \sqrt{\frac{(0.4104)(0.5896)}{960}}$$

$$= 0.4104 + 1.28 \sqrt{\frac{(0.4104)(0.5896)}{960}}$$

$$= 0.39$$

$$= 0.431$$

Determine the interval.

$$CI = (0.39, 0.431)$$

We are 80% confident that the true population proportion is between 39% and 43.1%.

9. Problem:

Some snails have clockwise shells, and others have counter-clockwise shells. You want to construct a 90% confidence interval of the population proportion with clockwise shells. You hope the margin of error will be about 0.005. How many snails do you need?

Solution: We are given the confidence level and the margin of error.

$$\begin{aligned}\gamma &= 0.9 \\ ME &= 0.005\end{aligned}$$

Determine z^* such that $P(|Z| < z^*) = 0.9$. It is helpful to get the percentile of z^* by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.9}{2} = 0.95$. This lets you find z^* such that $P(Z < z^*) = 0.95$.

$$z^* = 1.64$$

Use the appropriate formula.

$$\begin{aligned}n &= \frac{1}{4} \left(\frac{z^*}{ME} \right)^2 \\ &= \frac{1}{4} \left(\frac{1.64}{0.005} \right)^2 \\ &= 26896\end{aligned}$$

When determining a necessary sample size, always round up (ceiling).

$$n = 26896$$

10. Problem:

You work at a lightbulb company. The basic bulbs currently have an average brightness of 5580 lumens with a standard deviation of 520 lumens. You are trying to engineer a brighter lightbulb.

Your newest model seems promising, so you decide to test, with a significance level of 0.02, whether your new bulbs have higher average brightness. A sample of 152 of these bulbs has an average brightness of 5661 lumens.

- (a) What kind of hypothesis test is appropriate?
- (b) State the hypotheses
- (c) Determine the p -value.
- (d) Decide whether we reject or retain the null hypothesis.
- (e) What can you conclude about your new model of lightbulb?

Solution: We should use a right-tail test of population mean.

State the hypotheses:

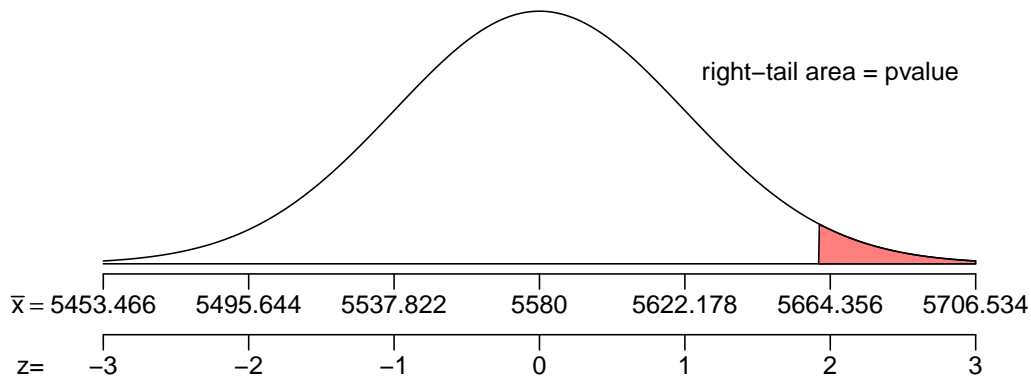
$$H_0 \text{ claims } \mu = 5580$$

$$H_A \text{ claims } \mu > 5580$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{520}{\sqrt{152}} = 42.178$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{5661 - 5580}{42.178} = 1.92$$

Find the p -value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(Z > 1.92) \\ &= 1 - P(Z < 1.92) \\ &= 1 - 0.9726 \\ &= \boxed{0.0274} \end{aligned}$$

Compare the p -value and the significance level ($\alpha = 0.02$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

We conclude your new bulbs could be just as bright on average as the basic bulbs.

- (a) Right-tail single mean test
- (b) Hypotheses: H_0 claims $\mu = 5580$ and H_A claims $\mu < 5580$.
- (c) p -value = 0.0274
- (d) No, we do not reject the null hypothesis.
- (e) We conclude your new bulbs could be just as bright on average as the basic bulbs.

11. Problem:

A student is taking a multiple choice test with 400 questions. Each question has 2 choices. You want to detect whether the student does better than random guessing, so you decide to run a hypothesis test with a significance level of 0.01.

Then, the student takes the test and gets 222 questions correct.

- (a) What kind of hypothesis test is appropriate?
- (b) State the hypotheses
- (c) Determine the p -value.
- (d) Decide whether we reject or retain the null hypothesis.
- (e) Do we think the student did significantly better than random guessing?

Solution: This is a right-tail (one-tail) proportion test because we only care whether the student does better than random.

Determine the null population proportion.

$$p_0 = \frac{1}{2} = 0.5$$

State the hypotheses.

$$H_0 \text{ claims } p = 0.5$$

$$H_A \text{ claims } p > 0.5$$

Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(1-0.5)}{400}} = 0.025$$

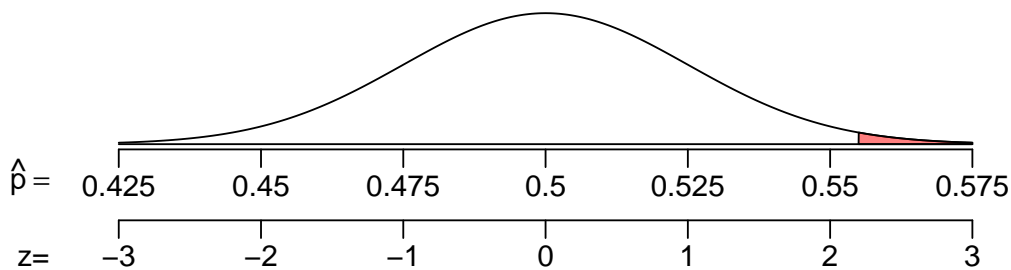
Determine the sample proportion.

$$\hat{p} = \frac{222}{400} = 0.555$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.555 - 0.5}{0.025} = 2.2$$

Make a sketch of the null's sampling distribution. The p -value is a right area.



To determine that right area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.555) \\ &= P(Z > 2.2) \\ &= 1 - P(Z < 2.2) \\ &= 0.0139 \end{aligned}$$

Compare p -value to α (which is 0.01).

$$p\text{-value} > \alpha$$

Make the conclusion: we retain the null hypothesis.

We think the student might just be guessing.

- (a) Right tail (one-tail) proportion test
- (b) Hypotheses: H_0 claims $p = 0.5$ and H_A claims $p > 0.5$.
- (c) The p -value is 0.0139
- (d) We retain the null hypothesis.
- (e) We think the student might just be guessing.

12. Problem:

Your boss wants to know what proportion of a very large population is happy. You already know the proportion approximately 0.46. But, your boss wants to guarantee that the margin of error of a 99.5% confidence interval will be less than 0.01 (which is 1 percentage points). How large of a sample is needed?

Solution: We are given the confidence level and the margin of error.

$$\begin{aligned}\gamma &= 0.995 \\ ME &= 0.01\end{aligned}$$

Determine z^* such that $P(|Z| < z^*) = 0.995$. It is helpful to get the percentile of z^* by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.995}{2} = 0.9975$. This lets you find z^* such that $P(Z < z^*) = 0.9975$.

$$z^* = 2.81$$

Use the appropriate formula.

$$\begin{aligned}n &= \hat{p}(1 - \hat{p}) \left(\frac{z^*}{ME} \right)^2 \\ &= (0.46)(0.54) \left(\frac{2.81}{0.01} \right)^2 \\ &= 19613.9124\end{aligned}$$

When determining a necessary sample size, always round up (ceiling).

$$n = 19614$$