1. **Problem:**
   A random sample of size 1400 was found to have a sample proportion of 93% (because there were 1302 successes). Determine a 78% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$\begin{aligned} n &= 1400 \\ \hat{p} &= 0.93 \\ \gamma &= 0.78 \end{aligned}$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.78$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.78+1}{2} = 0.89$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.23$$

Estimate the standard error. (The standard error is the standard deviation of the sampling distribution.)

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.93)(1-0.93)}{1400}} = 0.0068$$

Calculate the margin of error.

$$ME \;=\; z^\star \sigma_{\hat{p}} \;=\; (1.23)(0.0068) \;=\; 0.0084$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \;\approx\; \hat{p} \pm ME$$

Thus,

$$\begin{aligned} LB &= \hat{p} - ME \\ &= 0.93 - 0.0084 \\ &= 0.922 \end{aligned}$$

and

$$\begin{aligned} UB &= \hat{p} + ME \\ &= 0.93 + 0.0084 \\ &= 0.938 \end{aligned}$$

Determine the interval.

$$CI \;=\; (0.922\,,\; 0.938)$$

We are 78% confident that the true population proportion is between 92.2% and 93.8%.

(a) The lower bound = 0.922, which can also be expressed as 92.2%.

(b) The upper bound = 0.938, which can also be expressed as 93.8%.

2. **Problem:**
   A random sample of size 7000 was found to have a sample proportion of 45% (because there were 3150 successes). Determine a 96% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.
   (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 7000$$
$$\hat{p} = 0.45$$
$$\gamma = 0.96$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.96$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.96+1}{2} = 0.98$. (Use the z-table to find $z^\star$.)

$$z^\star = 2.05$$

Estimate the standard error. (The standard error is the standard deviation of the sampling distribution.)

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.45)(1-0.45)}{7000}} = 0.006$$

Calculate the margin of error.

$$ME = z^\star \sigma_{\hat{p}} = (2.05)(0.006) = 0.0122$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Thus,

$$\begin{aligned} LB &= \hat{p} - ME \\ &= 0.45 - 0.0122 \\ &= 0.438 \end{aligned}$$

and

$$\begin{aligned} UB &= \hat{p} + ME \\ &= 0.45 + 0.0122 \\ &= 0.462 \end{aligned}$$

Determine the interval.

$$CI = (0.438\,,\ 0.462)$$

We are 96% confident that the true population proportion is between 43.8% and 46.2%.

(a) The lower bound = 0.438, which can also be expressed as 43.8%.

(b) The upper bound = 0.462, which can also be expressed as 46.2%.

3. **Problem:**
   A random sample of size 7300 was found to have a sample proportion of 65% (because there were 4745 successes). Determine a 73% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 7300$$
$$\hat{p} = 0.65$$
$$\gamma = 0.73$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.73$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.73+1}{2} = 0.865$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.1$$

Estimate the standard error. (The standard error is the standard deviation of the sampling distribution.)

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.65)(1-0.65)}{7300}} = 0.0056$$

Calculate the margin of error.

$$ME = z^\star \sigma_{\hat{p}} = (1.1)(0.0056) = 0.0061$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Thus,

$$\begin{aligned} LB &= \hat{p} - ME \\ &= 0.65 - 0.0061 \\ &= 0.644 \end{aligned}$$

and

$$\begin{aligned} UB &= \hat{p} + ME \\ &= 0.65 + 0.0061 \\ &= 0.656 \end{aligned}$$

Determine the interval.

$$CI = (0.644\,,\ 0.656)$$

We are 73% confident that the true population proportion is between 64.4% and 65.6%.

(a) The lower bound = 0.644, which can also be expressed as 64.4%.

(b) The upper bound = 0.656, which can also be expressed as 65.6%.

4. **Problem:**
A random sample of size 470 was found to have a sample proportion of 14.89% (because there were 70 successes). Determine a 87% confidence interval of the population proportion.

    (a) Find the lower bound of the confidence interval.

    (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 470$$
$$\hat{p} = 0.1489$$
$$\gamma = 0.87$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.87$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.87+1}{2} = 0.935$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.51$$

Estimate the standard error. (The standard error is the standard deviation of the sampling distribution.)

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.1489)(1-0.1489)}{470}} = 0.0164$$

Calculate the margin of error.

$$ME = z^\star \sigma_{\hat{p}} = (1.51)(0.0164) = 0.0248$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Thus,

$$\begin{aligned} LB &= \hat{p} - ME \\ &= 0.1489 - 0.0248 \\ &= 0.124 \end{aligned}$$

and

$$\begin{aligned} UB &= \hat{p} + ME \\ &= 0.1489 + 0.0248 \\ &= 0.174 \end{aligned}$$

Determine the interval.

$$CI = (0.124\,,\ 0.174)$$

We are 87% confident that the true population proportion is between 12.4% and 17.4%.

(a) The lower bound = 0.124, which can also be expressed as 12.4%.

(b) The upper bound = 0.174, which can also be expressed as 17.4%.

5. **Problem:**
   A random sample of size 290 was found to have a sample proportion of 47.93% (because there were 139 successes). Determine a 81% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.
   (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 290$$
$$\hat{p} = 0.4793$$
$$\gamma = 0.81$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.81$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.81+1}{2} = 0.905$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.31$$

Estimate the standard error. (The standard error is the standard deviation of the sampling distribution.)

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.4793)(1-0.4793)}{290}} = 0.0293$$

Calculate the margin of error.

$$ME = z^\star \sigma_{\hat{p}} = (1.31)(0.0293) = 0.0384$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Thus,

$$\begin{aligned} LB &= \hat{p} - ME \\ &= 0.4793 - 0.0384 \\ &= 0.441 \end{aligned}$$

and

$$\begin{aligned} UB &= \hat{p} + ME \\ &= 0.4793 + 0.0384 \\ &= 0.518 \end{aligned}$$

Determine the interval.

$$CI = (0.441\ ,\ 0.518)$$

We are 81% confident that the true population proportion is between 44.1% and 51.8%.

(a) The lower bound = 0.441, which can also be expressed as 44.1%.

(b) The upper bound = 0.518, which can also be expressed as 51.8%.

6. **Problem:**
A random sample of size 680 was found to have a sample proportion of 13.97% (because there were 95 successes). Determine a 88% confidence interval of the population proportion.

    (a) Find the lower bound of the confidence interval.

    (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 680$$
$$\hat{p} = 0.1397$$
$$\gamma = 0.88$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.88$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.88+1}{2} = 0.94$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.55$$

Estimate the standard error. (The standard error is the standard deviation of the sampling distribution.)

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.1397)(1-0.1397)}{680}} = 0.0133$$

Calculate the margin of error.

$$ME = z^\star \sigma_{\hat{p}} = (1.55)(0.0133) = 0.0206$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Thus,

$$\begin{aligned} LB &= \hat{p} - ME \\ &= 0.1397 - 0.0206 \\ &= 0.119 \end{aligned}$$

and

$$\begin{aligned} UB &= \hat{p} + ME \\ &= 0.1397 + 0.0206 \\ &= 0.16 \end{aligned}$$

Determine the interval.

$$CI = (0.119\,,\ 0.16)$$

We are 88% confident that the true population proportion is between 11.9% and 16%.

(a) The lower bound = 0.119, which can also be expressed as 11.9%.

(b) The upper bound = 0.16, which can also be expressed as 16%.

7. **Problem:**
   A random sample of size 5200 was found to have a sample proportion of 10% (because there were 520 successes). Determine a 89% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 5200$$
$$\hat{p} = 0.1$$
$$\gamma = 0.89$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.89$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.89+1}{2} = 0.945$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.6$$

Estimate the standard error. (The standard error is the standard deviation of the sampling distribution.)

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.1)(1-0.1)}{5200}} = 0.0042$$

Calculate the margin of error.

$$ME = z^\star \sigma_{\hat{p}} = (1.6)(0.0042) = 0.0067$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Thus,

$$\begin{aligned} LB &= \hat{p} - ME \\ &= 0.1 - 0.0067 \\ &= 0.0933 \end{aligned}$$

and

$$\begin{aligned} UB &= \hat{p} + ME \\ &= 0.1 + 0.0067 \\ &= 0.107 \end{aligned}$$

Determine the interval.

$$CI = (0.0933\,,\ 0.107)$$

We are 89% confident that the true population proportion is between 9.33% and 10.7%.

(a) The lower bound = 0.0933, which can also be expressed as 9.33%.

(b) The upper bound = 0.107, which can also be expressed as 10.7%.

8. **Problem:**
A random sample of size 9500 was found to have a sample proportion of 55% (because there were 5225 successes). Determine a 87% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 9500$$
$$\hat{p} = 0.55$$
$$\gamma = 0.87$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.87$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.87+1}{2} = 0.935$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.51$$

Estimate the standard error. (The standard error is the standard deviation of the sampling distribution.)

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.55)(1-0.55)}{9500}} = 0.0051$$

Calculate the margin of error.

$$ME = z^\star \sigma_{\hat{p}} = (1.51)(0.0051) = 0.0077$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Thus,

$$\begin{aligned} LB &= \hat{p} - ME \\ &= 0.55 - 0.0077 \\ &= 0.542 \end{aligned}$$

and

$$\begin{aligned} UB &= \hat{p} + ME \\ &= 0.55 + 0.0077 \\ &= 0.558 \end{aligned}$$

Determine the interval.

$$CI = (0.542\,,\ 0.558)$$

We are 87% confident that the true population proportion is between 54.2% and 55.8%.

(a) The lower bound = 0.542, which can also be expressed as 54.2%.

(b) The upper bound = 0.558, which can also be expressed as 55.8%.