Central Limit Theorem

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$$Y \sim \mathcal{N}\left(\mu_{w}, \frac{\sigma_{w}}{\sqrt{n}}\right)$$

► Let *W* be a random variable with the following probability distribution

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W	P(w)
26	0.52
27	0.43
29	0.05

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Notice W has mean $\mu_{w}=26.58$ and standard deviation $\sigma_{w}=0.737$.

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- \blacktriangleright Let X be the sum of 12 instances of W.

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27	0.43
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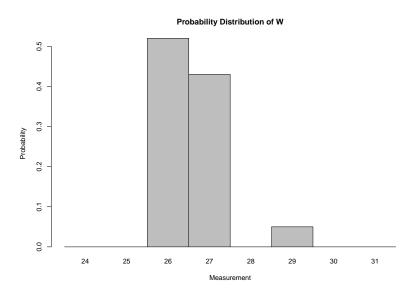
- Notice W has mean $\mu_{w}=26.58$ and standard deviation $\sigma_{w}=0.737$.
- ▶ Let X be the sum of 12 instances of W.
- We predict X is approximately normal, with mean and standard deviation from formulas.

$$\mu_X = n\mu_W = (12)(26.58) = 318.96$$

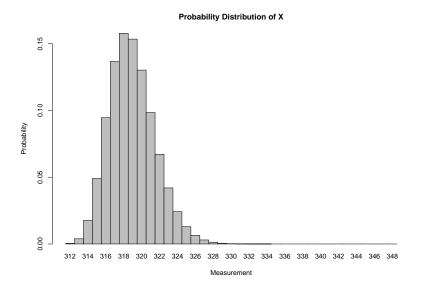
$$\sigma_W = \sigma_W \sqrt{n} = (0.737)(\sqrt{12}) = 2.55$$

$$X \sim \mathcal{N}(318.96, 2.55)$$

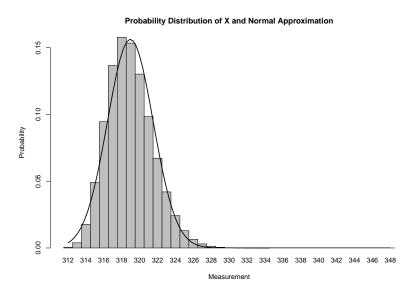
Example 1 continued...



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Let random variable W represent a 6-sided die.

w	P(w)
1	0.1667
2	0.1667
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- Notice W has mean $\mu_{\rm w}=3.5$ and standard deviation $\sigma_{\rm w}=1.708$.
- ightharpoonup Let X be the sum of 100 instances of W.

Let random variable W represent a 6-sided die.

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1	0.1667
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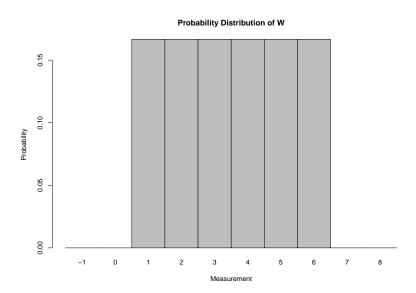
- Notice W has mean $\mu_w = 3.5$ and standard deviation $\sigma_w = 1.708$.
- Let X be the sum of 100 instances of W.
- ► We predict *X* is approximately normal, with mean and standard deviation from formulas.

$$\mu_{x} = n\mu_{w} = (100)(3.5) = 350$$

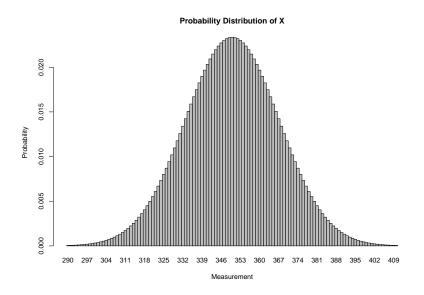
$$\sigma_{w} = \sigma_{w}\sqrt{n} = (1.708)(\sqrt{100}) = 17.08$$

$$X \sim \mathcal{N}(350, 17.08)$$

Probability distribution of standard 6-sided die



Probability distribution of sum of 100 6-sided dice



How to roll 100 dice: spin this once

