Confidence Interval

- The confidence level, γ, represents how confident we are the interval will contain the population parameter (population proportion or population mean).
- To get $z^\star$, find $z^\star$ such that $P(|Z| < z^\star) = \gamma$. To do that, first get percentile, ℓ, from confidence level (γ):

$$\ell = \frac{\gamma + 1}{2}$$

  then, use the z-table to find $z^\star$ such that $P(Z < z^\star) = \ell$.

Proportion

The population proportion, p, is estimated with an interval (to indicate uncertainty) based on a sample proportion, $\hat{p}$.

- Bounds:

$$\hat{p} \pm z^\star \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- Necessary sample size for a given margin of error:
  - If $\hat{p}$ is known:

$$n = \hat{p}(1-\hat{p})\left(\frac{z^\star}{ME}\right)^2$$

  - If $\hat{p}$ is unknown, assume it is 0.5 to be conservative

$$n = \frac{1}{4}\left(\frac{z^\star}{ME}\right)^2$$

Mean

The population mean, μ, is estimated with an interval (to indicate uncertainty) based on a sample mean, $\bar{x}$.

- Bounds:
  - If σ is known:

$$\bar{x} \pm z^\star \cdot \frac{\sigma}{\sqrt{n}}$$

  - If σ is unknown, use the sample standard deviation (and $t^\star$). Remember, $df = n - 1$. To get $t^\star$, find $t^\star$ such that $P(|T| < t^\star) = \gamma$ and $df = n - 1$.

$$\bar{x} \pm t^\star \cdot \frac{s}{\sqrt{n}}$$

- Necessary sample size for a given margin of error:

$$n = \left(\frac{z^\star \sigma}{ME}\right)^2$$

Hypothesis Testing (Single-Sample)

$$H_0 = \text{null hypothesis}$$
$$H_A = \text{alternative hypothesis}$$
$$p\text{-value} = \text{probability of sample at least as extreme as observed, } \textbf{given } H_0$$
$$\alpha = \text{significance level = chance of type II error given } H_0$$

- Calculate the p-value.
  - "at least as extreme" can mean "as large or larger", "as small or smaller", or "as far from expected in either direction".
- If p-value is small enough, we reject the null hypothesis. (This logic is similar to *reductio ad absurdum* or proof by contradiction.)

$$\text{If} \quad p\text{-value} < \alpha \quad \text{then} \quad \text{reject } H_0$$

$$\text{If} \quad p\text{-value} \geq \alpha \quad \text{then} \quad \text{do not reject } H_0$$

Single-sample proportion testing

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Right tail (one tail)

- H_0 claims $p = p_0$
- H_A claims $p > p_0$
- $p\text{-value} = P(Z > z_0)$

Left tail (one tail)

- H_0 claims $p = p_0$
- H_A claims $p < p_0$
- $p\text{-value} = P(Z < z_0)$

Two tail

- H_0 claims $p = p_0$
- H_A claims $p \neq p_0$
- $p\text{-value} = P(|Z| > |z_0|)$

Single-sample mean testing, σ known

$$z_0 = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

Right tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu > \mu_0$
- $p\text{-value} = P(Z > z_0)$

Left tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu < \mu_0$
- $p\text{-value} = P(Z < z_0)$

Two tail

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu \neq \mu_0$
- $p\text{-value} = P(|Z| > |z_0|)$

Single-sample mean testing, σ unknown

$$t_0 = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Right tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu > \mu_0$
- $p\text{-value} = P(T > t_0)$

Left tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu < \mu_0$
- $p\text{-value} = P(T < t_0)$

Two tail

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu \neq \mu_0$
- $p\text{-value} = P(|T| > |t_0|)$