Question

A continuous random variable X was measured 20 times. The sorted data are shown below.

60.016	69.948	75.440	79.878	103.487
103.854	104.443	108.723	110.824	120.458
128.410	129.916	133.696	136.676	141.088
141.483	141.519	143.322	157.227	157.652

The total of the measurements is 2348.06.

Answerlist

- Determine the percentile rank of the value 75.44. In other words, determine what percent of data are less than or equal to 75.44.
- Determine the datum corresponding to a percentile rank of 0.45. In other words, determine x such that 45% of the data are less than or equal to x.
- Determine the mean of the measurements.
- Determine the median of the measurements.

Solution

Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

Answerlist

- We are given $x = 75.44$. This means $i = 3$. We know $n = 20$. Determine the percentile ℓ.

  $$\ell = \frac{3}{20}$$

  $$\ell = 0.15$$

  So, the percentile rank is $\boxed{0.15}$, or 15th percentile.
- We are given $\ell = 0.45$. We can use algebra to solve for i.

  $$\ell = \frac{i}{n}$$

  Multiply both sides by n.

  $$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

  Simplify both sides.

  $$n\ell = i$$

  To make me happy, switch the sides.

  $$i = n\ell$$

  Now, we can evaluate i.

  $$i = (20)(0.45)$$

  $$i = 9$$

  Determine the x associated with $i = 9$.

  $$x = \boxed{110.824}$$

- The mean is $\frac{2348.06}{20} = \boxed{117.4}$
- If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 20$ and so n is even.

$$\text{median} = \frac{x_{10} + x_{11}}{2} = \frac{120.458 + 128.41}{2}$$

. So, median = $\boxed{124.434}$.

Meta-information

extype: string exsolution: yup exname: reading hist extol: 0.01