

**1. Problem**

A continuous random variable  $X$  was measured 29 times. The sorted measurements are shown below.

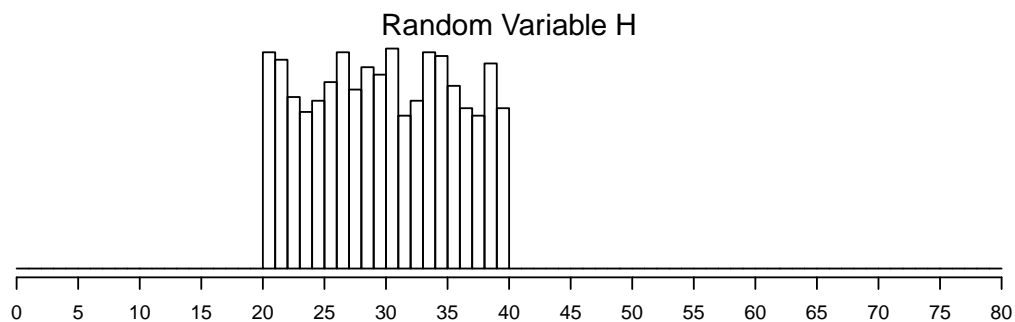
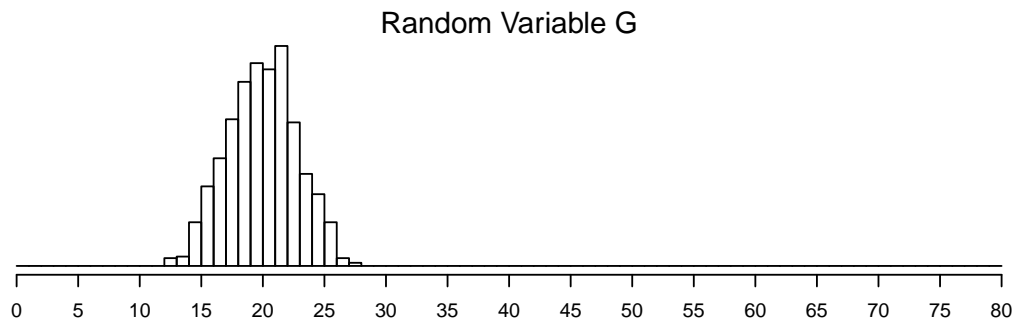
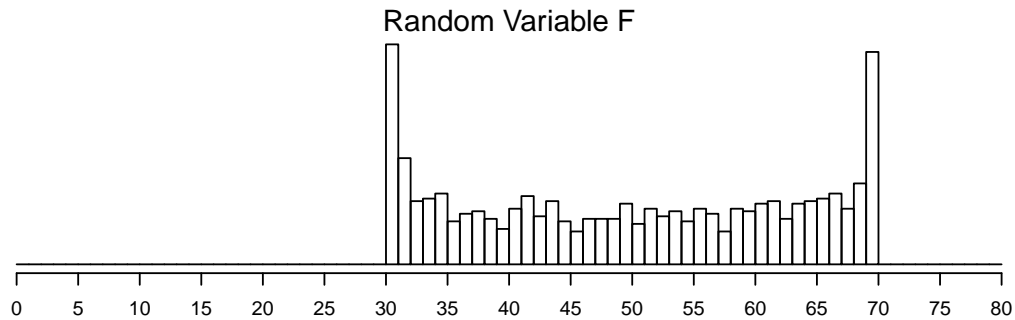
20	20.1	20.2	20.3	20.48	20.6	20.71	20.87	20.98	21.15
21.18	21.24	21.46	21.74	21.84	21.94	22.09	22.1	22.14	22.15
22.18	22.28	22.46	22.53	22.63	22.76	22.84	22.91	22.97	

The total of the measurements is 626.83.

- (a) Determine the percentile rank of the measurement 22.18. In other words, determine what percent of data are less than or equal to 22.18.
- (b) Determine the measurement corresponding to a percentile rank of 0.276. In other words, determine  $x$  such that 27.6% of the data are less than or equal to  $x$ .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

**2. Problem**

Three random variables (F, G, and H) were measured 1000 times each. The resulting histograms show the three distributions.



- (a) Which distribution has the highest mean? (F, G, or H)
- (b) Which distribution has the lowest mean? (F, G, or H)
- (c) Which distribution has the largest standard deviation? (F, G, or H)
- (d) Which distribution has the smallest standard deviation? (F, G, or H)

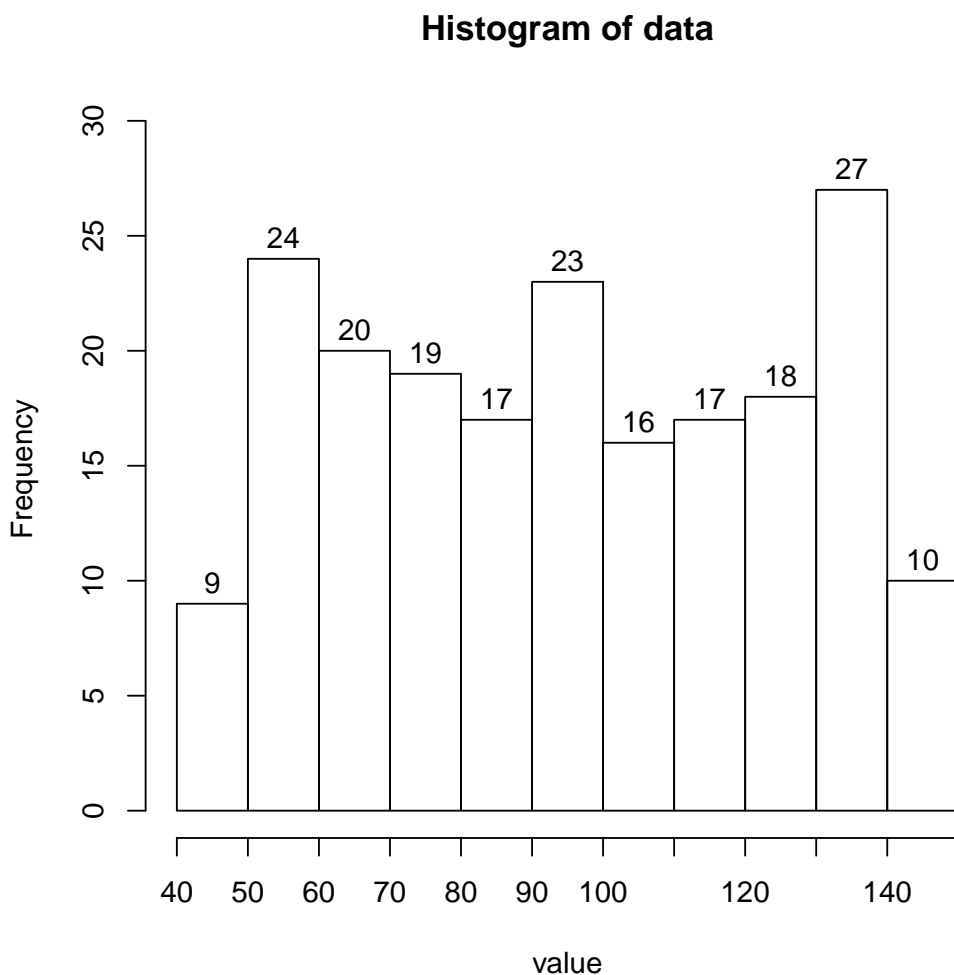
**3. Problem**

Please make a frequency table and a dot plot from the following (unsorted) data.

30	30	30	30	28	30
30	30	30	30	29	28
30	28	30	26	30	25
30	25	27	30	26	29

**4. Problem**

A continuous random variable was measured 200 times. The resulting histogram is shown below.



- (a) Describe the overall shape of the distribution. (symmetric mound, skew left, skew right, uniform, or bimodal)
- (b) Estimate the range of the distribution (range = max-min).
- (c) What percent of the measurements are less than 80?
- (d) What percent of the measurements are less than 50?
- (e) What percent of the measurements are between 50 and 80?
- (f) What percent of the measurements are within 20 from 90? In other words, what percent of measurements satisfy  $|x - 90| \leq 20$ ?
- (g) Of the measurements less than 80, what percent are less than 50?
- (h) Estimate the value of the 81.5th percentile. In other words, determine a value such that 81.5% of the measurements are less than or equal to it.

**5. Problem**

From a very large population, a small sample of measurements was taken.

176	176	188	184
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Please calculate the (Bessel corrected) sample standard deviation using the following formula:

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

**6. Problem**

A continuous random variable was measured 500 times. The resulting frequency distribution is shown below.

class	frequency
50–51	2
51–52	4
52–53	12
53–54	41
54–55	94
55–56	109
56–57	116
57–58	76
58–59	33
59–60	11
60–61	2

- (a) Describe the overall shape of the distribution. (symmetric mound, skew left, skew right, uniform, or bimodal)
- (b) Estimate the range of the distribution (range = max-min).
- (c) What percent of the measurements are greater than 58?
- (d) What percent of the measurements are greater than 59?
- (e) What percent of the measurements are between 58 and 59?
- (f) What percent of the measurements are within 0.5 of 54.5? In other words, what percent of measurements satisfy  $|x - 54.5| \leq 0.5$ ?
- (g) Of the measurements greater than 58, what percent are greater than 59?
- (h) Estimate the value of the 30.6th percentile. In other words, determine a value such that 30.6% of the measurements are less than or equal to it.

**7. Problem**

From a very large population, a small sample of measurements was taken.

36	53	54	45	23	50
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Please calculate the average absolute deviation using the following formula:

$$\text{AAD} = \frac{\sum |x - \bar{x}|}{n}$$

1. Let  $x$  represent a measurement of interest. Let  $i$  represent that measurement's index. Let  $\ell$  represent that measurement's percentile. Let  $n$  represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given  $x = 22.18$ . This means  $i = 21$ . We know  $n = 29$ . Determine the percentile  $\ell$ .

$$\ell = \frac{21}{29}$$

$$\ell = 0.724$$

So, the percentile rank is 0.724, or 72.4th percentile.

- (b) We are given  $\ell = 0.276$ . We can use algebra to solve for  $i$ .

$$\ell = \frac{i}{n}$$

Multiply both sides by  $n$ .

$$n \cdot (\ell) = n \cdot \left( \frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate  $i$ .

$$i = (29)(0.276)$$

$$i = 8$$

Determine the  $x$  associated with  $i = 8$ .

$$x = \text{20.87}$$

- (c) The mean:  $\bar{x} = \frac{626.83}{29} = \text{21.615}$

- (d) If  $n$  is odd, then median is  $x_{i=\frac{n+1}{2}}$ , the value of  $x$  when  $i = \frac{n+1}{2}$ . Otherwise, if  $n$  is even, the median is mean of  $x_{i=\frac{n}{2}}$  and  $x_{i=\frac{n}{2}+1}$ . In this case,  $n = 29$  and so  $n$  is odd.

$$\text{median} = x_{(29+1)/2} = x_{15}$$

So, median = 21.84.

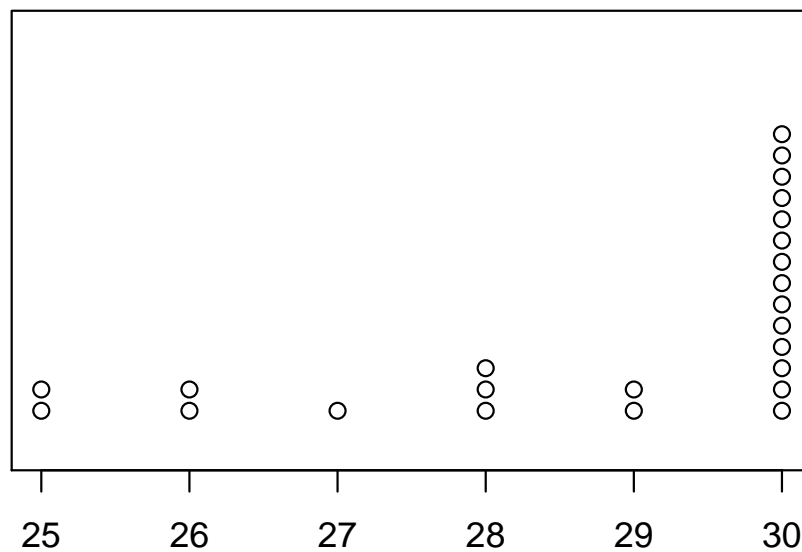
2. (a) F  
(b) G  
(c) F  
(d) G

3. Make a frequency table.



value	frequency
25	2
26	2
27	1
28	3
29	2
30	14

Make the dot plot.



4. (a) uniform  
(b) 110  
(c) 36%  
(d) 4.5%  
(e) 44.5%  
(f) 37.5%  
(g) 12.5%  
(h) 130

5. We fill out the table column by column.

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
176	-5	25
176	-5	25
188	7	49
184	3	9
=====	=====	=====
$\sum x = 724$		$\sum (x - \bar{x})^2 = 108$
$\bar{x} = 181$		

We are ready for the formula.

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$= \sqrt{\frac{108}{4 - 1}}$$

$$= \sqrt{36}$$

$$= \boxed{6}$$

6. (a) symmetric mound

(b) 11

(c) 9.2%

(d) 2.6%

(e) 6.6%

(f) 18.8%

(g) 28.26%

(h) 55

7. We fill out the table column by column.

$x$	$x - \bar{x}$	$ x - \bar{x} $
36	-7.5	7.5
53	9.5	9.5
54	10.5	10.5
45	1.5	1.5
23	-20.5	20.5
50	6.5	6.5
=====	=====	=====
$\sum x = 261$		$\sum  x - \bar{x}  = 56$
$\bar{x} = 43.5$		

We are ready for the formula.

$$s = \frac{\sum |x - \bar{x}|}{n}$$

$$= \frac{56}{6}$$

$$= 9.3333333$$