1. **Problem:**
   It is generally accepted that a population's proportion is 0.736. However, you think that maybe the population proportion is below 0.736, so you decide to run a one-tail hypothesis test with a significance level of 0.05 with a sample size of 5000.

   Then, when you collect the random sample, you find its proportion is 0.724. Do you reject or retain the null hypothesis?

   (a) Determine the p-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.736$$

$$H_A \quad \text{claims} \quad p < 0.736$$

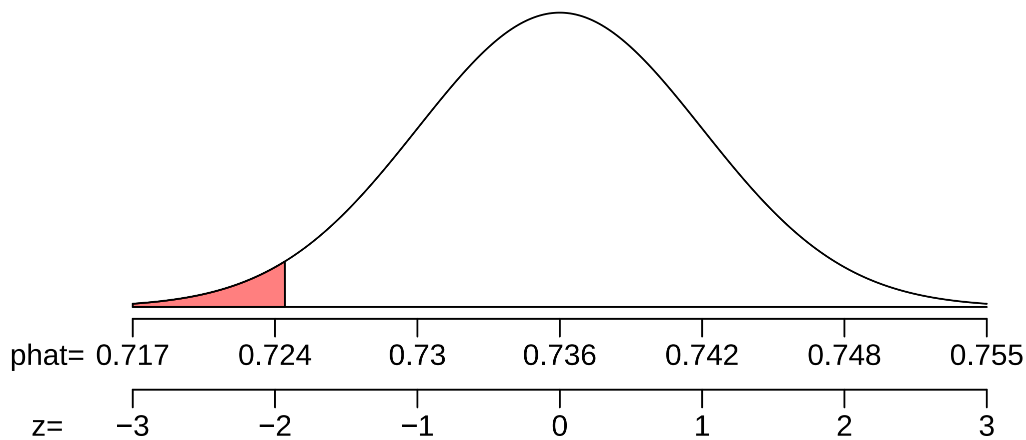
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.736(1-0.736)}{5000}} = 0.00623$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.724 - 0.736}{0.00623} = -1.93$$

The p-value is a left area.



To determine that left area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} < 0.724) \\ &= P(Z < -1.93) \\ &= 0.0268 \end{aligned}$$

Compare p-value to α (which is 0.05).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) The p-value is 0.0268

(b) We reject the null hypothesis.

2. **Problem:**
   It is generally accepted that a population's proportion is 0.359. However, you think that maybe the population proportion is under 0.359, so you decide to run a one-tail hypothesis test with a significance level of 0.025 with a sample size of 104.

   Then, when you collect the random sample, you find its proportion is 0.349. Do you reject or retain the null hypothesis?

   (a) Determine the *p*-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.359$$

$$H_A \quad \text{claims} \quad p < 0.359$$

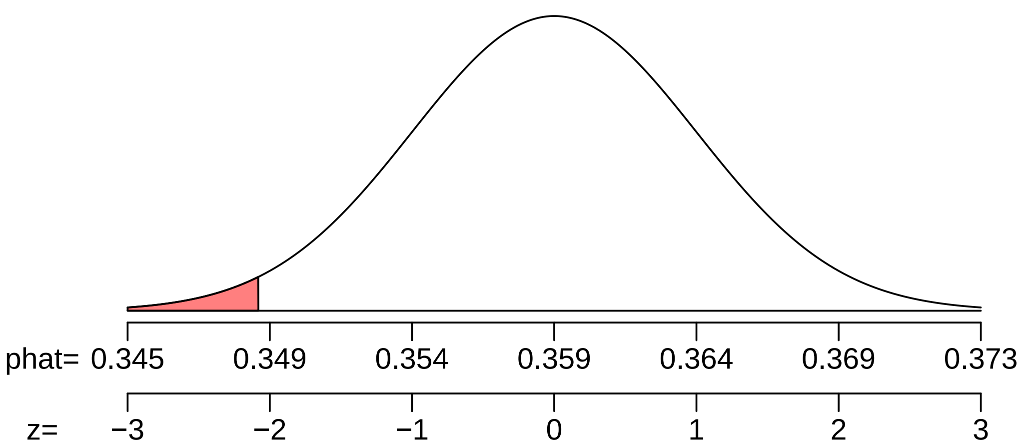
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.359(1-0.359)}{10 < sup > 4 < /sup >}} = 0.0048$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.349 - 0.359}{0.0048} = -2.08$$

The p-value is a left area.



To determine that left area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} < 0.349) \\ &= P(Z < -2.08) \\ &= 0.0188 \end{aligned}$$

Compare p-value to α (which is 0.025).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) The p-value is 0.0188

(b) We reject the null hypothesis.

3. **Problem:**
   It is generally accepted that a population's proportion is 0.211. However, you think that maybe the population proportion is over 0.211, so you decide to run a one-tail hypothesis test with a significance level of 0.02 with a sample size of 104.

   Then, when you collect the random sample, you find its proportion is 0.22. Do you reject or retain the null hypothesis?

   (a) Determine the *p*-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.211$$

$$H_A \quad \text{claims} \quad p > 0.211$$

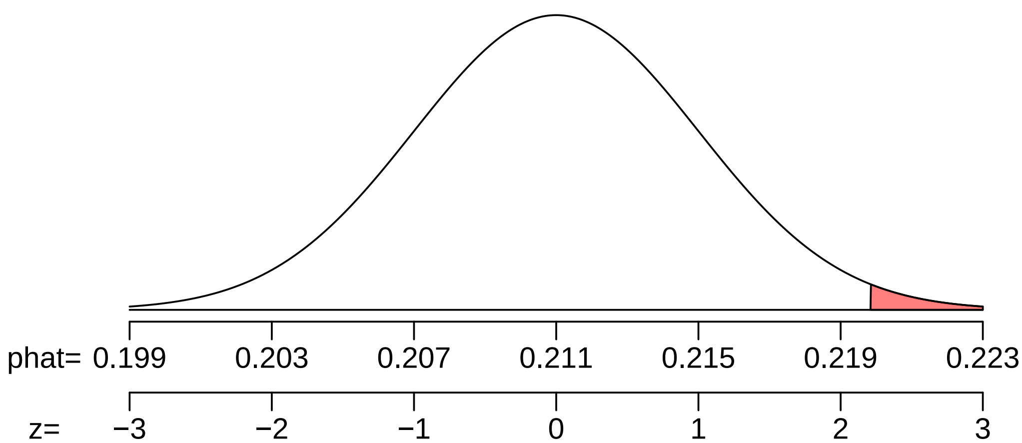
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.211(1-0.211)}{10 < sup > 4 < /sup >}} = 0.00408$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.22 - 0.211}{0.00408} = 2.21$$

The p-value is a right area.



To determine that right area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.22) \\ &= P(Z > 2.21) \\ &= 1 - P(Z < 2.21) \\ &= 0.0136 \end{aligned}$$

Compare p-value to α (which is 0.02).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) The p-value is 0.0136
(b) We reject the null hypothesis.

4. **Problem:**
   It is generally accepted that a population's proportion is 0.259. However, you think that maybe the population proportion is more than 0.259, so you decide to run a one-tail hypothesis test with a significance level of 0.02 with a sample size of 2000.

   Then, when you collect the random sample, you find its proportion is 0.282. Do you reject or retain the null hypothesis?

   (a) Determine the *p*-value.
   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.259$$

$$H_A \quad \text{claims} \quad p > 0.259$$

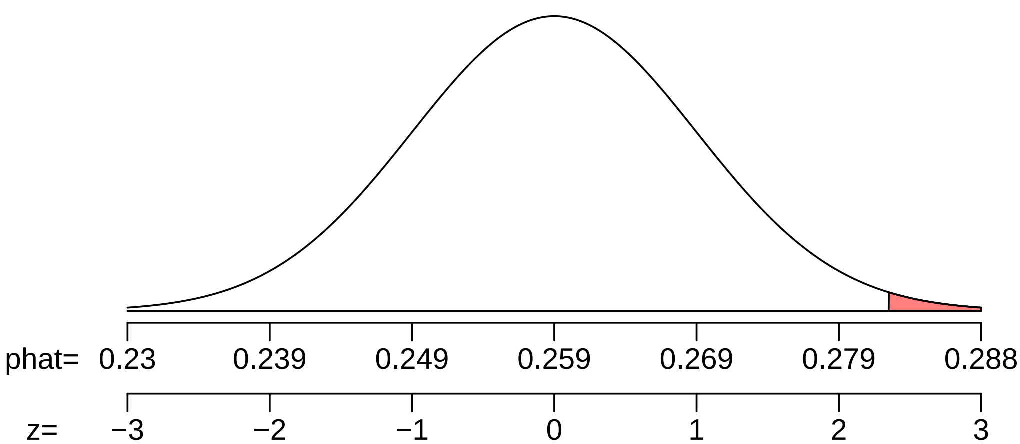
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.259(1-0.259)}{2000}} = 0.0098$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.282 - 0.259}{0.0098} = 2.35$$

The p-value is a right area.



To determine that right area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.282) \\ &= P(Z > 2.35) \\ &= 1 - P(Z < 2.35) \\ &= 0.0094 \end{aligned}$$

Compare p-value to α (which is 0.02).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) The p-value is 0.0094

(b) We reject the null hypothesis.

5. **Problem:**
   It is generally accepted that a population's proportion is 0.754. However, you think that maybe the population proportion is not equal to 0.754, so you decide to run a two-tail hypothesis test with a significance level of 0.05 with a sample size of 9000.

   Then, when you collect the random sample, you find its proportion is 0.763. Do you reject or retain the null hypothesis?

   (a) Determine the *p*-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.754$$

$$H_A \quad \text{claims} \quad p \neq 0.754$$

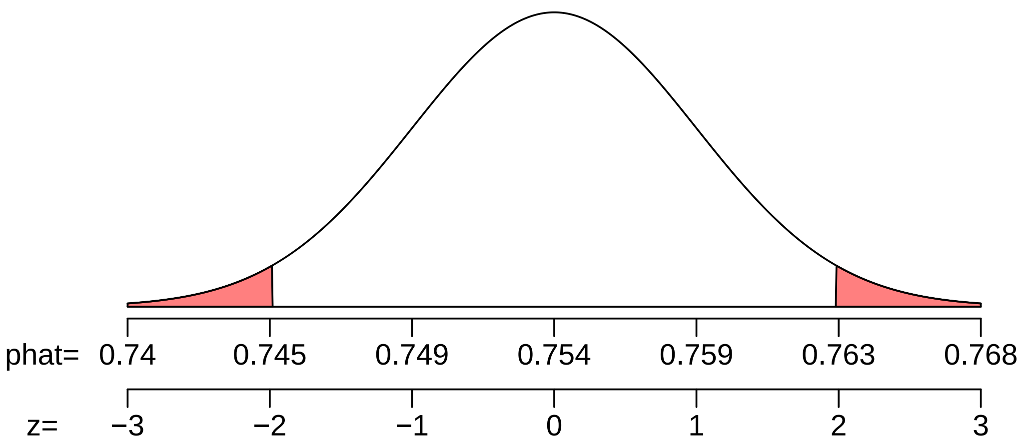
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.754(1-0.754)}{9000}} = 0.00454$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.763 - 0.754}{0.00454} = 1.98$$

The p-value is a two-tail area.



To determine that two-tail area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(|Z| > 1.98) \\ &= 2 \cdot P(Z < -1.98) \\ &= 0.0478 \end{aligned}$$

Compare p-value to α (which is 0.05).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) The p-value is 0.0478

(b) We reject the null hypothesis.

6. **Problem:**
   It is generally accepted that a population's proportion is 0.447. However, you think that maybe the population proportion is not 0.447, so you decide to run a two-tail hypothesis test with a significance level of 0.02 with a sample size of 104.

   Then, when you collect the random sample, you find its proportion is 0.434. Do you reject or retain the null hypothesis?

   (a) Determine the *p*-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.447$$

$$H_A \quad \text{claims} \quad p \neq 0.447$$

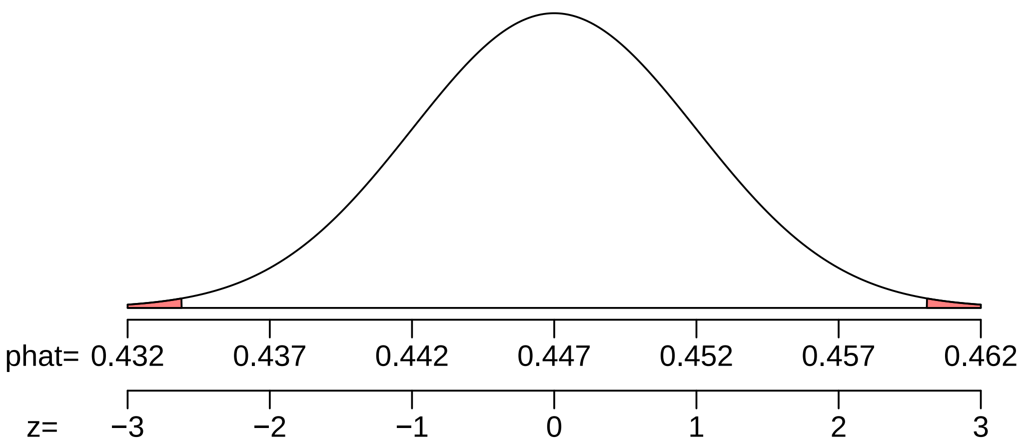
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.447(1-0.447)}{10 < sup > 4 < /sup >}} = 0.00497$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.434 - 0.447}{0.00497} = -2.62$$

The p-value is a two-tail area.



To determine that two-tail area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(|Z| > 2.62) \\ &= 2 \cdot P(Z < -2.62) \\ &= 0.0088 \end{aligned}$$

Compare p-value to α (which is 0.02).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) The p-value is 0.0088

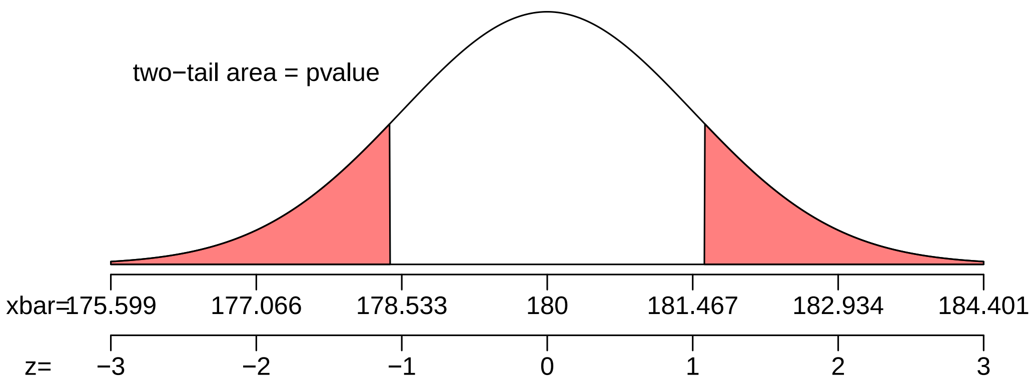
(b) We reject the null hypothesis.

7. **Problem:**
   A null hypothesis claims a roughly symmetric population has a mean $\mu = 180$ and a standard deviation $\sigma = 22$. Determine the *p*-value of a two-tail test if your sample of size $n = 225$ has mean $\bar{x} = 178.42$.

Solution: Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{22}{\sqrt{225}} = 1.467$$

Make a sketch.



Find the z score.

$$z_0 = \frac{178.42 - 180}{1.467} = -1.08$$

Find the p-value.

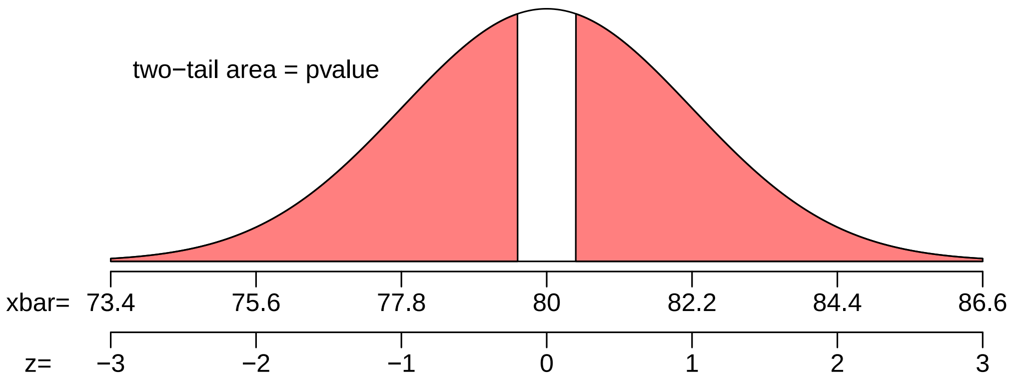
$$\begin{aligned} p\text{-value} &= P(|Z| > 1.08) \\ &= 2 \cdot P(Z < -1.08) \\ &= \boxed{0.2802} \end{aligned}$$

8. **Problem:**
   A null hypothesis claims a roughly symmetric population has a mean $\mu = 80$ and a standard deviation $\sigma = 22$. Determine the *p*-value of a two-tail test if your sample of size $n = 100$ has mean $\bar{x} = 79.56$.

Solution: Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{22}{\sqrt{100}} = 2.2$$

Make a sketch.



Find the z score.

$$z_0 = \frac{79.56 - 80}{2.2} = -0.2$$

Find the p-value.

$$\begin{aligned} p\text{-value} &= P(|Z| > 0.2) \\ &= 2 \cdot P(Z < -0.2) \\ &= \boxed{0.8414} \end{aligned}$$