1. **Problem:**
   A population has unknown μ but a known $\sigma = 5.2$. A sample of size 96 has a mean $\bar{x} = 118.14$. Determine the 80% confidence level of the population mean.

Solution: We are given the sample size, sample mean, population standard deviation, and confidence level.

$$\begin{aligned} n &= 96 \\ \bar{x} &= 118.14 \\ \sigma &= 5.2 \\ \gamma &= 0.8 \end{aligned}$$

Determine the critical z value, $z^\star$, such that $P(|Z| < z^\star) = 0.8$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.8+1}{2} = 0.9$

$$z^\star = 1.28$$

Calculate the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5.2}{\sqrt{96}} = 0.531$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm z^\star \sigma_{\bar{x}}$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - z^\star\sigma_{\bar{x}},\ \bar{x} + z^\star\sigma_{\bar{x}}) \\ &= (118.14 - 1.28 \times 0.531,\ 118.14 + 1.28 \times 0.531) \\ &= (117.46,\ 118.82) \end{aligned}$$

We are 80% confident that the population mean is between 117.46 and 118.82.

2. **Problem:**
A population has unknown μ but a known $\sigma = 2.14$. A sample of size 128 has a mean $\bar{x} = 145.83$. Determine the 99% confidence level of the population mean.

Solution: We are given the sample size, sample mean, population standard deviation, and confidence level.

$$\begin{aligned} n &= 128 \\ \bar{x} &= 145.83 \\ \sigma &= 2.14 \\ \gamma &= 0.99 \end{aligned}$$

Determine the critical z value, $z^\star$, such that $P(|Z| < z^\star) = 0.99$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.99+1}{2} = 0.995$

$$z^\star = 2.58$$

Calculate the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.14}{\sqrt{128}} = 0.189$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm z^\star \sigma_{\bar{x}}$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - z^\star\sigma_{\bar{x}},\ \bar{x} + z^\star\sigma_{\bar{x}}) \\ &= (145.83 - 2.58 \times 0.189,\ 145.83 + 2.58 \times 0.189) \\ &= (145.34,\ 146.32) \end{aligned}$$

We are 99% confident that the population mean is between 145.34 and 146.32.

3. **Problem:**
   As an ornithologist, you wish to determine the average body mass of *Catharus guttatus*. You randomly sample 13 adults of *Catharus guttatus*, resulting in a sample mean of 29.35 grams and a sample standard deviation of 0.909 grams. Determine a 98% confidence interval of the true population mean.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 13$$
$$\bar{x} = 29.35$$
$$s = 0.909$$
$$\gamma = 0.98$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 12$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.98$.

$$t^\star = 2.68$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{0.909}{\sqrt{13}} = 0.252$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (29.35 - 2.68 \times 0.252,\ 29.35 + 2.68 \times 0.252) \\ &= (28.7,\ 30) \end{aligned}$$

We are 98% confident that the population mean is between 28.7 and 30.

4. **Problem:**
As an ornithologist, you wish to determine the average body mass of *Dendroica coronata*. You randomly sample 31 adults of *Dendroica coronata*, resulting in a sample mean of 11.75 grams and a sample standard deviation of 1.25 grams. Determine a 80% confidence interval of the true population mean.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 31$$
$$\bar{x} = 11.75$$
$$s = 1.25$$
$$\gamma = 0.8$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 30$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.8$.

$$t^\star = 1.31$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.25}{\sqrt{31}} = 0.225$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^\star SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^\star SE,\ \bar{x} + t^\star SE) \\ &= (11.75 - 1.31 \times 0.225,\ 11.75 + 1.31 \times 0.225) \\ &= (11.5,\ 12) \end{aligned}$$

We are 80% confident that the population mean is between 11.5 and 12.

5. **Problem:**
   Your boss wants to know what proportion of a very large population is cold. She also wants to guarantee that the margin of error of a 99% confidence interval will be less than 0.02 (which is 2 percentage points). How large of a sample is needed? Please round up, using only 2 significant digits.

Solution: Determine $z^\star$ such that $P(|Z| < z^\star) = 0.99$.

$$z^\star = 2.58$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$SE = \frac{ME}{z^\star} = \frac{0.02}{2.58} = 0.0078$$

Calculate n. Because we have no idea what p is, we will use a conservative approach and use $p = 0.5$.

$$SE = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{SE^2} = \frac{(0.5)(0.5)}{(0.0078)^2} = 4162.3309$$

When determining a necessary sample size, always round up (ceiling).

$$n = 4163$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 4200$$

6. **Problem:**
   If you suspect that $\hat{p}$ will be near 0.69, how large of a sample is needed to guarantee a margin of error less than 0.008 when building a 95% confidence interval?

Solution: Determine $z^\star$ such that $P(|Z| < z^\star) = 0.95$.

$$z^\star = 1.96$$

Determine the maximal standard error.

$$ME = z^\star \sigma_{\hat{p}}$$

$$\sigma_{\hat{p}} = \frac{ME}{z^\star} = \frac{0.008}{1.96} = 0.0041$$

Calculate n.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{\sigma_{\hat{p}}^2} = \frac{(0.69)(0.31)}{(0.0041)^2} = 12849.6251$$

When determining a necessary sample size, always round up (ceiling).

$$n = 12850$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 13000$$

7. **Problem:**
   A random sample of size 2200 was found to have a sample proportion of 11% (because there were 242 successes). Determine a 81% confidence interval of the population proportion.

   (a) Find the lower bound of the confidence interval.

   (b) Find the upper bound of the condifence interval.

Solution: Identify the givens.

$$n = 2200$$
$$\hat{p} = 0.11$$
$$\gamma = 0.81$$

Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.81$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.81+1}{2} = 0.905$. (Use the z-table to find $z^{\star}$.)

$$z^{\star} = 1.31$$

Estimate the standard error. (The standard error is the standard deviation of the sampling distribution.)

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.11)(1-0.11)}{2200}} = 0.0067$$

Calculate the margin of error.

$$ME = z^{\star}\sigma_{\hat{p}} = (1.31)(0.0067) = 0.0087$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Thus,

$$\begin{aligned} LB &= \hat{p} - ME \\ &= 0.11 - 0.0087 \\ &= 0.101 \end{aligned}$$

and

$$\begin{aligned} UB &= \hat{p} + ME \\ &= 0.11 + 0.0087 \\ &= 0.119 \end{aligned}$$

Determine the interval.

$$CI = (0.101\ ,\ 0.119)$$

We are 81% confident that the true population proportion is between 10.1% and 11.9%.

(a) The lower bound = 0.101, which can also be expressed as 10.1%.

(b) The upper bound = 0.119, which can also be expressed as 11.9%.