1. **Problem:**
   If you suspect that $\hat{p}$ will be near 0.86, how large of a sample is needed to guarantee a margin of error less than 0.02 when building a 95% confidence interval?

Solution: Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.95$.

$$z^{\star} = 1.96$$

Determine the maximal standard error.

$$ME = z^{\star} SE$$

$$\sigma_{\hat{p}} = \frac{ME}{z^{\star}} = \frac{0.02}{1.96} = 0.0102$$

Calculate n.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{\sigma_{\hat{p}}^2} = \frac{(0.86)(0.14)}{(0.0102)^2} = 1157.2472$$

When determining a necessary sample size, always round up (ceiling).

$$n = 1158$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 1200$$

2. **Problem:**
   If you suspect that $\hat{p}$ will be near 0.87, how large of a sample is needed to guarantee a margin of error less than 0.005 when building a 99.5% confidence interval?

Solution: Determine $z^{\star}$ such that $P(|Z| < z^{\star}) = 0.995$.

$$z^{\star} = 2.81$$

Determine the maximal standard error.

$$ME = z^{\star} SE$$

$$\sigma_{\hat{p}} = \frac{ME}{z^{\star}} = \frac{0.005}{2.81} = 0.0018$$

Calculate n.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{\sigma_{\hat{p}}^2} = \frac{(0.87)(0.13)}{(0.0018)^2} = 35696.2505$$

When determining a necessary sample size, always round up (ceiling).

$$n = 35697$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 36000$$

3. **Problem:**
If you suspect that $\hat{p}$ will be near 0.46, how large of a sample is needed to guarantee a margin of error less than 0.05 when building a 96% confidence interval?

Solution: Determine $z^\star$ such that $P(|Z| < z^\star) = 0.96$.

$$z^\star = 2.05$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$\sigma_{\hat{p}} = \frac{ME}{z^\star} = \frac{0.05}{2.05} = 0.0244$$

Calculate n.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{\sigma_{\hat{p}}^2} = \frac{(0.46)(0.54)}{(0.0244)^2} = 417.2266$$

When determining a necessary sample size, always round up (ceiling).

$$n = 418$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 420$$

4. **Problem:**
   If you suspect that $\hat{p}$ will be near 0.88, how large of a sample is needed to guarantee a margin of error less than 0.008 when building a 98% confidence interval?

Solution: Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$\sigma_{\hat{p}} = \frac{ME}{z^\star} = \frac{0.008}{2.33} = 0.0034$$

Calculate n.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{\sigma_{\hat{p}}^2} = \frac{(0.88)(0.12)}{(0.0034)^2} = 8975.8519$$

When determining a necessary sample size, always round up (ceiling).

$$n = 8976$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 9000$$

5. **Problem:**
   If you suspect that $\hat{p}$ will be near 0.93, how large of a sample is needed to guarantee a margin of error less than 0.003 when building a 90% confidence interval?

Solution: Determine $z^\star$ such that $P(|Z| < z^\star) = 0.9$.

$$z^\star = 1.64$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$\sigma_{\hat{p}} = \frac{ME}{z^\star} = \frac{0.003}{1.64} = 0.0018$$

Calculate n.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{\sigma_{\hat{p}}^2} = \frac{(0.93)(0.07)}{(0.0018)^2} = 19439.2188$$

When determining a necessary sample size, always round up (ceiling).

$$n = 19440$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 20000$$

6. **Problem:**
   If you suspect that $\hat{p}$ will be near 0.96, how large of a sample is needed to guarantee a margin of error less than 0.03 when building a 99.5% confidence interval?

Solution: Determine $z^\star$ such that $P(|Z| < z^\star) = 0.995$.

$$z^\star = 2.81$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$\sigma_{\hat{p}} = \frac{ME}{z^\star} = \frac{0.03}{2.81} = 0.0107$$

Calculate n.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{\sigma_{\hat{p}}^2} = \frac{(0.96)(0.04)}{(0.0107)^2} = 335.4005$$

When determining a necessary sample size, always round up (ceiling).

$$n = 336$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 340$$

7. **Problem:**
   If you suspect that $\hat{p}$ will be near 0.59, how large of a sample is needed to guarantee a margin of error less than 0.03 when building a 98% confidence interval?

Solution: Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$.

$$z^\star = 2.33$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$\sigma_{\hat{p}} = \frac{ME}{z^\star} = \frac{0.03}{2.33} = 0.0129$$

Calculate n.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{\sigma_{\hat{p}}^2} = \frac{(0.59)(0.41)}{(0.0129)^2} = 1453.6386$$

When determining a necessary sample size, always round up (ceiling).

$$n = 1454$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 1500$$

8. **Problem:**
   If you suspect that $\hat{p}$ will be near 0.88, how large of a sample is needed to guarantee a margin of error less than 0.02 when building a 95% confidence interval?

Solution: Determine $z^\star$ such that $P(|Z| < z^\star) = 0.95$.

$$z^\star = 1.96$$

Determine the maximal standard error.

$$ME = z^\star SE$$

$$\sigma_{\hat{p}} = \frac{ME}{z^\star} = \frac{0.02}{1.96} = 0.0102$$

Calculate n.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{\sigma_{\hat{p}}^2} = \frac{(0.88)(0.12)}{(0.0102)^2} = 1014.9942$$

When determining a necessary sample size, always round up (ceiling).

$$n = 1015$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 1100$$