1. **Problem**
In a deck of strange cards, there are 1439 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	blue	green	indigo	yellow
dog	78	44	84	42	57
flower	14	34	48	19	99
pig	62	63	54	53	55
tree	27	97	67	25	39
wheel	64	90	85	60	79

(a) What is the probability a random card is a flower given it is black?
(b) What is the probability a random card is indigo?
(c) What is the probability a random card is a dog?
(d) What is the probability a random card is yellow given it is a wheel?
(e) What is the probability a random card is either a flower or green (or both)?
(f) What is the probability a random card is both a pig and yellow?

2. **Problem**
A farm produces 4 types of fruit: A, B, C, and D. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

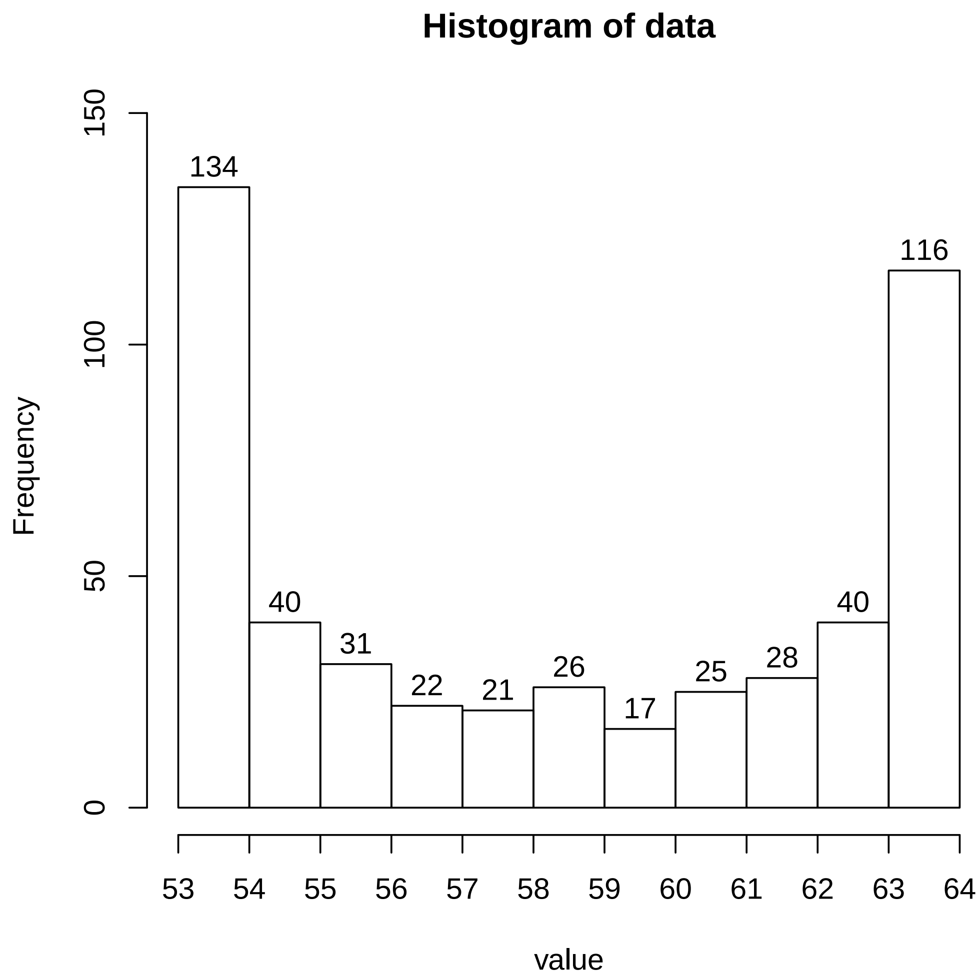
Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	68	7
B	119	15
C	120	10
D	71	4

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	58.27
B	125
C	108.3
D	73.44

Which specimen is the most unusually far from average (relative to others of its type)?

3. **Problem**
A continuous random variable was measured 500 times. The resulting histogram is shown below.



plot of chunk showhist

(a) Describe the overall shape of the distribution. (symmetric mound, skew left, skew right, uniform, or bimodal)

(b) Estimate the range of the distribution (range = max-min).

(c) What percent of the measurements are greater than 55?

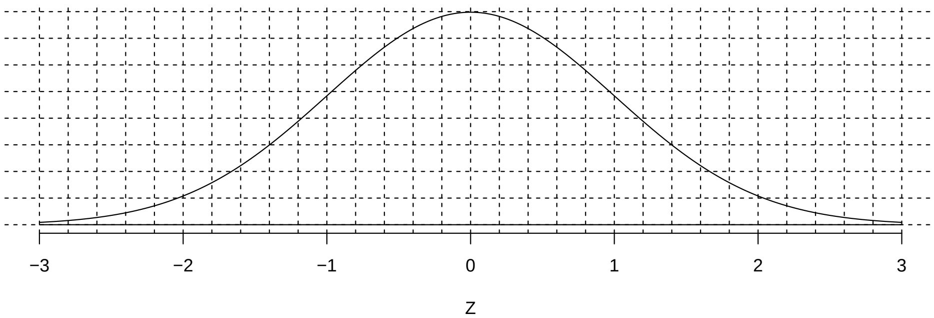
(d) What percent of the measurements are greater than 57?

(e) Of the measurements greater than 55, what percent are greater than 57?

(f) Estimate the value of the 63.2th percentile.

4. **Problem**

The figure below shows the standard normal density. Each grid square represents 1% of probability.



(a) Estimate $P(|Z| < 0.6)$ by shading and counting.

(b) Determine $P(|Z| < 0.6)$ by using the z-table.

5. **Problem**
   Let random variable W have mean $\mu_W = 21$ and standard deviation $\sigma_W = 6$. Let random variable X represent the **average** of $n = 144$ instances of W.

   (a) Determine the expected value of X. $\mu_X = ?$
   (b) Determine the standard deviation of X. $\sigma_X = ?$
   (c) Using normal approximation, determine $P(X < 21.73)$.
   (d) Using normal approximation, determine $P(X > 21.36)$.
   (e) Using normal approximation, determine $P(|X - \mu_X| < 0.105)$.
   (f) Using normal approximation, determine $P(|X - \mu_X| > 0.5)$.

6. **Problem**
   Brahim wants to estimate the average mass of the beans in a large bag. Somehow, Brahim is certain that the standard deviation of the beans in the bag is 66 milligrams. He takes a sample of size 132 and finds the sample mean to be 546 milligrams. What would be the 90% confidence interval?

7. **Problem**

You work at a lightbulb company. The basic bulbs currently have an average brightness of 3270 lumens with a standard deviation of 530 lumens. You are trying to engineer a brighter lightbulb.

Your newest model seems promising, so you decide to test, with a significance level of 0.05, whether your new bulbs have higher average brightness. A sample of 53 of these bulbs has an average brightness of 3378 lumens.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses

(c) Determine the p-value.

(d) Decide whether we reject or retain the null hypothesis.

(e) What can you conclude about your new model of lightbulb?

8. **Problem**

You have collected the following data:

x	y
7.3	19
2.5	60
5.6	40
7.7	19
5.4	37
9.6	1.2

Please plot the data and a corresponding regression line.

9. **Problem**

   Let each trial have a chance of success $p = 0.71$. If 60 trials occur, what is the probability of getting at least 37 but at most 47 successes?

   In other words, let $X \sim \text{Bin}(n = 60,\ p = 0.71)$ and find $P(37 \leq X \leq 47)$.

   Use a normal approximation along with the continuity correction.

10. **Problem**
A null hypothesis claims a population has a mean $\mu = 8.0$. You decide to run two-tail test on a sample of size $n = 8$ using a significance level $\alpha = 0.05$.

You then collect the sample:

8.2	9	10.2	7.9	7.7
8.4	9.1	9.2		

(a) Determine the p-value.

(b) Do you reject the null hypothesis?

1. (a) $P(\text{flower given black}) = \frac{14}{78+14+62+27+64} = 0.0571$
   (b) $P(\text{indigo}) = \frac{42+19+53+25+60}{1439} = 0.138$
   (c) $P(\text{dog}) = \frac{78+44+84+42+57}{1439} = 0.212$
   (d) $P(\text{yellow given wheel}) = \frac{79}{64+90+85+60+79} = 0.209$
   (e) $P(\text{flower or green}) = \frac{14+34+48+19+99+84+48+54+67+85-48}{1439} = 0.35$
   (f) $P(\text{pig and yellow}) = \frac{55}{1439} = 0.0382$

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2. We compare the absolute z-scores. The largest absolute z-score corresponds to the specimen that is most unusually far from average.

Type of fruit	formula	absolute z-score
A	$z = \frac{\lvert 58.27-68\rvert}{7}$	1.39
B	$z = \frac{\lvert 125-119\rvert}{15}$	0.4
C	$z = \frac{\lvert 108.3-120\rvert}{10}$	1.17
D	$z = \frac{\lvert 73.44-71\rvert}{4}$	0.61

Thus, the specimen of type A is the most unusually far from average.

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3. (a) bimodal
   (b) 11
   (c) 65.2%
   (d) 54.6%
   (e) 83.74%
   (f) 61

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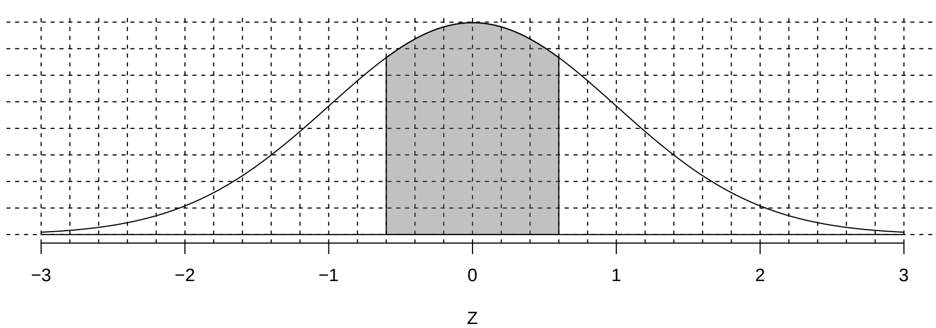
4. (a) The shaded region is shown below.



You should count about 45 shaded squares, giving a probability of about 0.45.

(b) The probability is 0.4515.

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5. (a) 21
   (b) 0.5
   (c) 0.9265
   (d) 0.2358
   (e) 0.1663
   (f) 0.3173

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6. We are given the sample size, sample mean, population standard deviation, and confidence level.

$$n = 132$$
$$\bar{x} = 546$$
$$\sigma = 66$$
$$\gamma = 0.9$$

Determine the critical z value, $z^{\star}$, such that $P(|Z| < z^{\star}) = 0.9$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.9+1}{2} = 0.95$

$$z^{\star} = 1.64$$

Use the formula for bounds (mean, σ known).

$$\begin{aligned} LB &= \bar{x} - z^{\star}\frac{\sigma}{\sqrt{n}} \\ &= 546 - 1.64 \times \frac{66}{\sqrt{132}} \\ &= 536.58 \end{aligned} \qquad \begin{aligned} UB &= \bar{x} + z^{\star}\frac{\sigma}{\sqrt{n}} \\ &= 546 + 1.64 \times \frac{66}{\sqrt{132}} \\ &= 555.42 \end{aligned}$$

We are 90% confident that the population mean is between 536.58 and 555.42 milligrams.

$$CI = (536.58,\ 555.42)$$

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7. We should use a right-tail test of population mean.

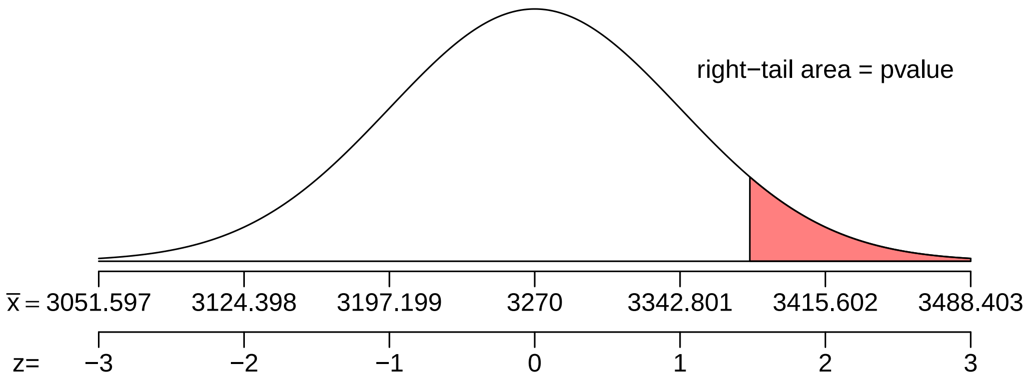
State the hypotheses:

$$H_0 \text{ claims } \mu = 3270$$
$$H_A \text{ claims } \mu > 3270$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{530}{\sqrt{53}} = 72.801$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{3378 - 3270}{72.801} = 1.48$$

Find the p-value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(Z > 1.48) \\ &= 1 - P(Z < 1.48) \\ &= 1 - 0.9306 \\ &= \boxed{0.0694} \end{aligned}$$

Compare the p-value and the significance level ($\alpha = 0.05$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

We conclude your new bulbs could be just as bright on average as the basic bulbs.

(a) Right-tail single mean test
(b) Hypotheses: H_0 claims $\mu = 3270$ and H_A claims $\mu < 3270$.
(c) p-value = 0.0694
(d) No, we do not reject the null hypothesis.
(e) We conclude your new bulbs could be just as bright on average as the basic bulbs.

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8. Remember the formula for the correlation coefficient.

$$r = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

We calculate the necessary values.

x	y	xy
7.3	19	138.7
2.5	60	150
5.6	40	224
7.7	19	146.3
5.4	37	199.8
9.6	1.2	11.52
$\sum x = 38.1$	$\sum y = 176.2$	$\sum x_i y_i = 870.32$
$\bar{x} = 6.35$	$\bar{y} = 29.4$	
$s_x = 2.43$	$s_y = 20.6$	

The regression line has the form

$$y = a + bx$$

So, a is the y-intercept and b is the slope. We have formulas to determine them:

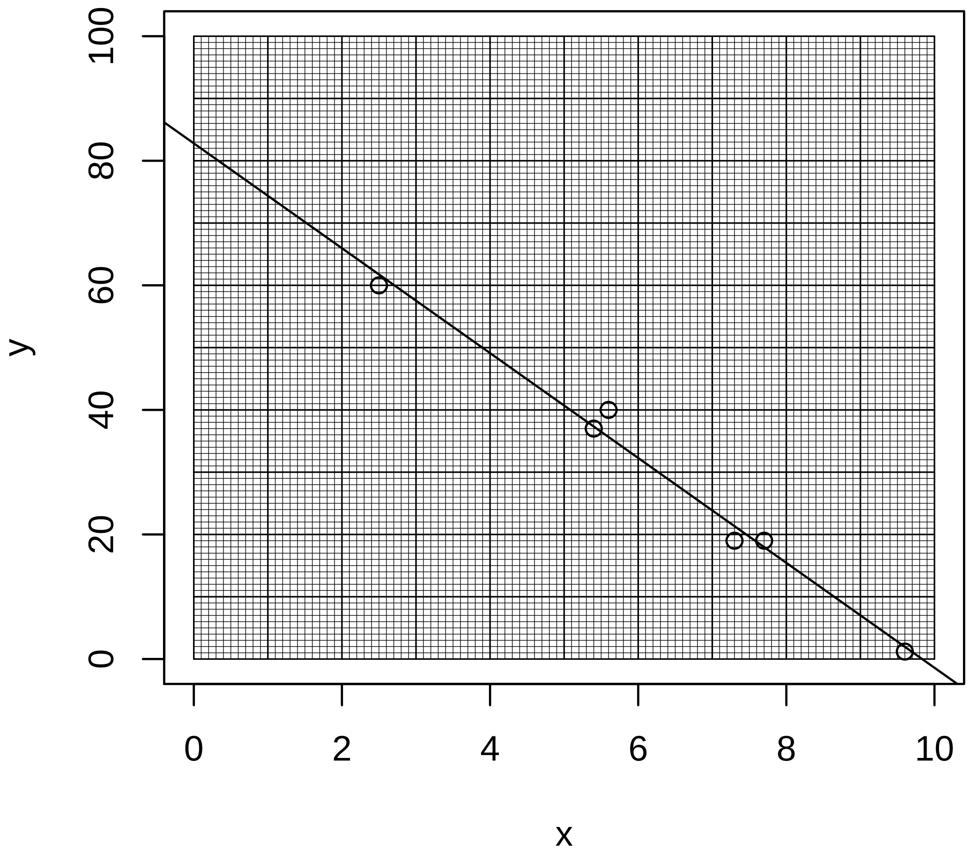
$$b = r\frac{s_y}{s_x} = -0.993 \cdot \frac{20.6}{2.43} = -8.42$$

$$a = \bar{y} - b\bar{x} = 29.4 - -8.42 \cdot 6.35 = 82.8$$

Our regression line:

$$y = 82.8 + -8.42x$$

Make a plot.



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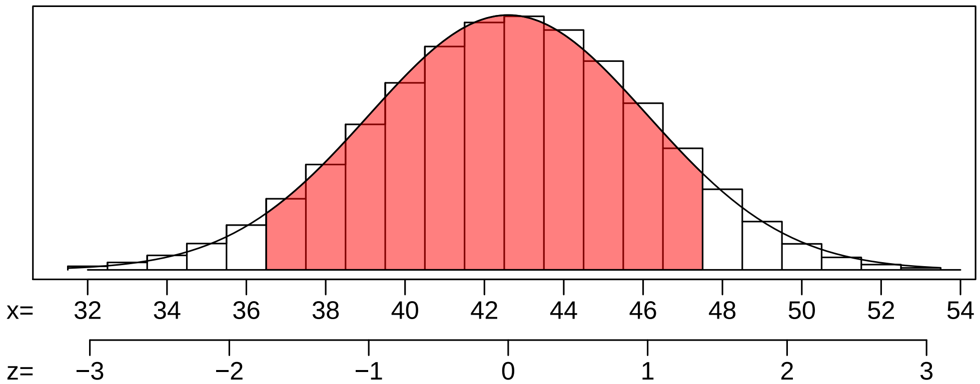
9. Find the mean.

$$\mu = np = (60)(0.71) = 42.6$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(60)(0.71)(1-0.71)} = 3.5148$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{36.5 - 42.6}{3.5148} = -1.74$$

$$z_2 = \frac{47.5 - 42.6}{3.5148} = 1.39$$

Find the percentiles (from z-table).

$$\ell_1 = 0.0409$$

$$\ell_2 = 0.9177$$

Calculate the probability.

$$P(37 \leq X \leq 47) = 0.9177 - 0.0409 = 0.8768$$

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10. State the hypotheses.

$$H_0 \quad \text{claims} \quad \mu = 8$$
$$H_A \quad \text{claims} \quad \mu \neq 8$$

Find the mean and standard deviation of the sample.

$$\bar{x} = 8.713$$

$$s = 0.822$$

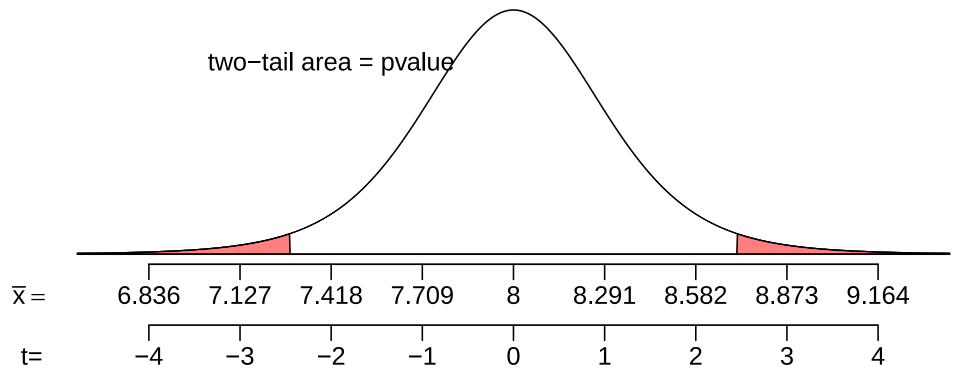
Determine the degrees of freedom.

$$df = 8 - 1 = 7$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{0.822}{\sqrt{8}} = 0.291$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{8.713 - 8}{0.291} = 2.45$$

Find the p-value.

$$p\text{-value} = P(|T| > 2.45)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 7$.)

$$P(|T| > 2.52) = 0.04$$

$$P(|T| > 2.36) = 0.05$$

Basically, because t is between 2.52 and 2.36, we know the p-value is between 0.04 and 0.05.

$$0.04 < p\text{-value} < 0.05$$

Compare the p-value and the significance level ($\alpha = 0.05$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) $0.04 < p$-value < 0.05

(b) Yes, we reject the null hypothesis.