

1. Problem:

A population has unknown μ but a known $\sigma = 5.2$. A sample of size 96 has a mean $\bar{x} = 118.14$. Determine the 80% confidence interval of the population mean.

Solution: We are given the sample size, sample mean, population standard deviation, and confidence level.

$$n = 96$$

$$\bar{x} = 118.14$$

$$\sigma = 5.2$$

$$\gamma = 0.8$$

Determine the critical z value, z^* , such that $P(|Z| < z^*) = 0.8$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.8+1}{2} = 0.9$

$$z^* = 1.28$$

Use the formula for bounds (mean, σ known).

$$\begin{aligned} CI &= \left(\bar{x} - z^* \frac{\sigma}{\sqrt{n}}, \bar{x} + z^* \frac{\sigma}{\sqrt{n}} \right) \\ &= \left(118.14 - 1.28 \times \frac{5.2}{\sqrt{96}}, 118.14 + 1.28 \times \frac{5.2}{\sqrt{96}} \right) \\ &= (117.46, 118.82) \end{aligned}$$

We are 80% confident that the population mean is between 117.46 and 118.82.

2. Problem:

A population has unknown μ but a known $\sigma = 2.14$. A sample of size 128 has a mean $\bar{x} = 145.83$. Determine the 99% confidence interval of the population mean.

Solution: We are given the sample size, sample mean, population standard deviation, and confidence level.

$$n = 128$$

$$\bar{x} = 145.83$$

$$\sigma = 2.14$$

$$\gamma = 0.99$$

Determine the critical z value, z^* , such that $P(|Z| < z^*) = 0.99$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.99+1}{2} = 0.995$

$$z^* = 2.58$$

Use the formula for bounds (mean, σ known).

$$\begin{aligned} CI &= \left(\bar{x} - z^* \frac{\sigma}{\sqrt{n}}, \bar{x} + z^* \frac{\sigma}{\sqrt{n}} \right) \\ &= \left(145.83 - 2.58 \times \frac{2.14}{\sqrt{128}}, 145.83 + 2.58 \times \frac{2.14}{\sqrt{128}} \right) \\ &= (145.34, 146.32) \end{aligned}$$

We are 99% confident that the population mean is between 145.34 and 146.32.

3. Problem:

As an ornithologist, you wish to determine the average body mass of *Catharus guttatus*. You randomly sample 13 adults of *Catharus guttatus*, resulting in a sample mean of 29.35 grams and a sample standard deviation of 0.909 grams. Determine a 98% confidence interval of the true population mean.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 13$$

$$\bar{x} = 29.35$$

$$s = 0.909$$

$$\gamma = 0.98$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 12$$

Determine the critical t value, t^* , such that $P(|T| < t^*) = 0.98$.

$$t^* = 2.68$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{0.909}{\sqrt{13}} = 0.252$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^* SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^* SE, \bar{x} + t^* SE) \\ &= (29.35 - 2.68 \times 0.252, 29.35 + 2.68 \times 0.252) \\ &= (28.7, 30) \end{aligned}$$

We are 98% confident that the population mean is between 28.7 and 30.

4. Problem:

As an ornithologist, you wish to determine the average body mass of *Dendroica coronata*. You randomly sample 31 adults of *Dendroica coronata*, resulting in a sample mean of 11.75 grams and a sample standard deviation of 1.25 grams. Determine a 80% confidence interval of the true population mean.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 31$$

$$\bar{x} = 11.75$$

$$s = 1.25$$

$$\gamma = 0.8$$

Determine the degrees of freedom (because we don't know σ and we are doing inference so we need to use the t distribution).

$$df = n - 1 = 30$$

Determine the critical t value, t^* , such that $P(|T| < t^*) = 0.8$.

$$t^* = 1.31$$

Calculate the standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{1.25}{\sqrt{31}} = 0.225$$

We want to make an inference about the population mean.

$$\mu \approx \bar{x} \pm t^* SE$$

Determine the bounds.

$$\begin{aligned} CI &= (\bar{x} - t^* SE, \bar{x} + t^* SE) \\ &= (11.75 - 1.31 \times 0.225, 11.75 + 1.31 \times 0.225) \\ &= (11.5, 12) \end{aligned}$$

We are 80% confident that the population mean is between 11.5 and 12.

5. Problem:

Your boss wants to know what proportion of a very large population is cold. She also wants to guarantee that the margin of error of a 99% confidence interval will be less than 0.02 (which is 2 percentage points). How large of a sample is needed? Please round up, using only 2 significant digits.

Solution: Determine z^* such that $P(|Z| < z^*) = 0.99$.

$$z^* = 2.58$$

Determine the maximal standard error.

$$ME = z^* SE$$

$$SE = \frac{ME}{z^*} = \frac{0.02}{2.58} = 0.00775$$

Calculate n . Because we have no idea what p is, we will use a conservative approach and use $p = 0.5$.

$$SE = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{SE^2} = \frac{(0.5)(0.5)}{(0.00775)^2} = 4162.3309053$$

When determining a necessary sample size, always round up (ceiling).

$$n = 4163$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 4200$$

6. Problem:

If you suspect that \hat{p} will be near 0.69, how large of a sample is needed to guarantee a margin of error less than 0.008 when building a 95% confidence interval?

Solution: Determine z^* such that $P(|Z| < z^*) = 0.95$.

$$z^* = 1.96$$

Determine the maximal standard error.

$$ME = z^* \sigma_{\hat{p}}$$

$$\sigma_{\hat{p}} = \frac{ME}{z^*} = \frac{0.008}{1.96} = 0.00408$$

Calculate n .

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$n = \frac{p(1-p)}{\sigma_{\hat{p}}^2} = \frac{(0.69)(0.31)}{(0.00408)^2} = 12849.6251442$$

When determining a necessary sample size, always round up (ceiling).

$$n = 12850$$

Also, for simplicity, you should probably only use about 2 significant digits, while still rounding up.

$$n = 13000$$

7. Problem:

A random sample of size 2200 was found to have a sample proportion of 11% (because there were 242 successes). Determine a 81% confidence interval of the population proportion.

- (a) Find the lower bound of the confidence interval.
- (b) Find the upper bound of the confidence interval.

Solution: Identify the givens.

$$n = 2200$$

$$\hat{p} = 0.11$$

$$\gamma = 0.81$$

Determine z^* such that $P(|Z| < z^*) = 0.81$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.81+1}{2} = 0.905$. (Use the z-table to find z^* .)

$$z^* = 1.31$$

Estimate the standard error. (The standard error is the standard deviation of the sampling distribution.)

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \sqrt{\frac{(0.11)(1 - 0.11)}{2200}} = 0.00667$$

Calculate the margin of error.

$$ME = z^* \sigma_{\hat{p}} = (1.31)(0.00667) = 0.00874$$

To find the confidence interval's bounds, find the sample proportion plus or minus the margin of error.

$$p \approx \hat{p} \pm ME$$

Thus,

$$\begin{aligned} LB &= \hat{p} - ME \\ &= 0.11 - 0.0087377 \\ &= 0.101 \end{aligned}$$

and

$$\begin{aligned} UB &= \hat{p} + ME \\ &= 0.11 + 0.0087377 \\ &= 0.119 \end{aligned}$$

Determine the interval.

$$CI = (0.101, 0.119)$$

We are 81% confident that the true population proportion is between 10.1% and 11.9%.

(a) The lower bound = 0.101, which can also be expressed as 10.1%.

(b) The upper bound = 0.119, which can also be expressed as 11.9%.