

1. Problem:

Some snails have clockwise shells, and others have counter-clockwise shells. You want to construct a 98% confidence interval of the population proportion with clockwise shells. You hope the margin of error will be about 0.009. How many snails do you need?

Solution: We are given the confidence level and the margin of error.

$$\begin{aligned}\gamma &= 0.98 \\ ME &= 0.009\end{aligned}$$

Determine z^* such that $P(|Z| < z^*) = 0.98$. It is helpful to get the percentile of z^* by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.98}{2} = 0.99$. This lets you find z^* such that $P(Z < z^*) = 0.99$.

$$z^* = 2.33$$

Use the appropriate formula.

$$\begin{aligned}n &= \frac{1}{4} \left(\frac{z^*}{ME} \right)^2 \\ &= \frac{1}{4} \left(\frac{2.33}{0.009} \right)^2 \\ &= 16755.8641975\end{aligned}$$

When determining a necessary sample size, always round up (ceiling).

$$n = 16756$$

2. Problem:

A fair coin should have a 50% chance of landing on either side. Someone is mildly suspicious that a coin is unfair.

You are asked to judge the fairness of the coin by flipping it 200 times and counting how many heads are flipped. You are told to use a significance level of 0.1.

Then, you actually flip the coin 200 times and get 111 heads. Should we conclude this coin is unfair?

- (a) What kind of hypothesis test is appropriate?
- (b) State the hypotheses
- (c) Determine the p -value.
- (d) Decide whether we reject or retain the null hypothesis.
- (e) Do you think the coin is unfair?

Solution: We should use a two-tail proportion test.

State the hypotheses.

$$H_0 \text{ claims } p = 0.5$$

$$H_A \text{ claims } p \neq 0.5$$

Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(1-0.5)}{200}} = 0.0354$$

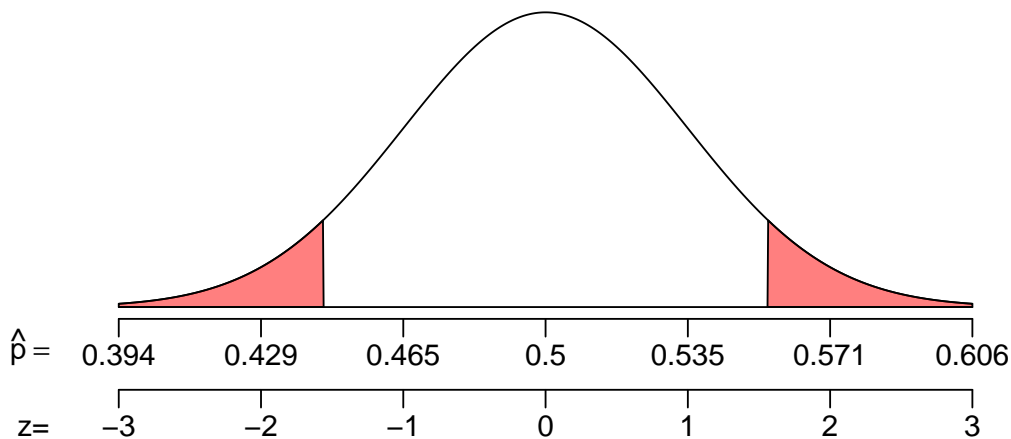
Determine the sample proportion.

$$\hat{p} = 0.555$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.555 - 0.5}{0.0354} = 1.56$$

Make a sketch of the null's sampling distribution. The p -value is a two-tail area.



To determine that two-tail area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(|Z| > 1.56) \\ &= 2 \cdot P(Z < -1.56) \\ &= 0.1188 \end{aligned}$$

Compare p -value to α (which is 0.1).

$$p\text{-value} > \alpha$$

Make the conclusion: we don't reject the null hypothesis.

We conclude the coin could be fair.

- (a) Two-tail proportion test
- (b) Hypotheses: H_0 claims $p = 0.5$ and H_A claims $p \neq 0.5$.
- (c) The p -value is 0.1188
- (d) We don't reject the null hypothesis.
- (e) We conclude the coin could be fair.

3. Problem:

As an ornithologist, you wish to determine the average body mass of *Dolichonyx orizivorus*. You randomly sample 20 adults of *Dolichonyx orizivorus*, resulting in a sample mean of 37.23 grams and a sample standard deviation of 8.32 grams. Determine a 98% confidence interval of the true population mean.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$n = 20$$

$$\bar{x} = 37.23$$

$$s = 8.32$$

$$\gamma = 0.98$$

Find the degrees of freedom.

$$df = n - 1$$

$$= 20 - 1$$

$$= 19$$

Determine the critical t value, t^* , such that $P(|T| < t^*) = 0.98$ and $df = 19$.

$$t^* = 2.54$$

Use the formula for bounds (mean, σ unknown).

$$LB = \bar{x} - t^* \frac{s}{\sqrt{n}}$$

$$= 37.23 - 2.54 \times \frac{8.32}{\sqrt{20}}$$

$$= 32.5$$

$$UB = \bar{x} + t^* \frac{s}{\sqrt{n}}$$

$$= 37.23 + 2.54 \times \frac{8.32}{\sqrt{20}}$$

$$= 42$$

We are 98% confident that the population mean is between 32.5 and 42.

$$CI = (32.5, 42)$$

4. Problem:

You work at a lightbulb company. The basic bulbs currently have an average brightness of 7190 lumens with a standard deviation of 670 lumens. You are trying to engineer a brighter lightbulb.

Your newest model seems promising, so you decide to test, with a significance level of 0.025, whether your new bulbs have higher average brightness. A sample of 111 of these bulbs has an average brightness of 7320 lumens.

- (a) What kind of hypothesis test is appropriate?
- (b) State the hypotheses
- (c) Determine the p -value.
- (d) Decide whether we reject or retain the null hypothesis.
- (e) What can you conclude about your new model of lightbulb?

Solution: We should use a right-tail test of population mean.

State the hypotheses:

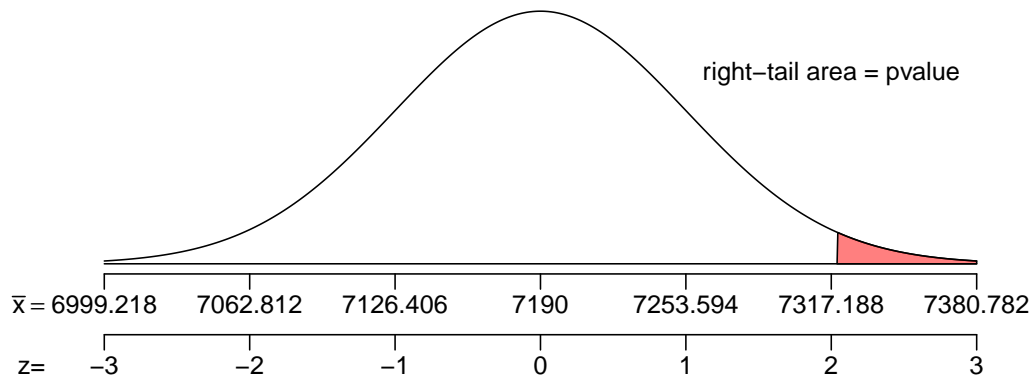
$$H_0 \text{ claims } \mu = 7190$$

$$H_A \text{ claims } \mu > 7190$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{670}{\sqrt{111}} = 63.594$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{7320 - 7190}{63.594} = 2.04$$

Find the p -value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(Z > 2.04) \\ &= 1 - P(Z < 2.04) \\ &= 1 - 0.9793 \\ &= \boxed{0.0207} \end{aligned}$$

Compare the p -value and the significance level ($\alpha = 0.025$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

We conclude your new bulbs have a higher average brightness than the basic bulbs.

- Right-tail single mean test
- Hypotheses: H_0 claims $\mu = 7190$ and H_A claims $\mu < 7190$.
- p -value = 0.0207
- Yes, we reject the null hypothesis.
- We conclude your new bulbs have a higher average brightness than the basic bulbs.

5. Problem:

Your boss wants to know what proportion of a very large population is golden. You already know the proportion approximately 0.24. But, your boss wants to guarantee that the margin of error of a 80% confidence interval will be less than 0.008 (which is 0.8 percentage points). How large of a sample is needed?

Solution: We are given the confidence level and the margin of error.

$$\begin{aligned}\gamma &= 0.8 \\ ME &= 0.008\end{aligned}$$

Determine z^* such that $P(|Z| < z^*) = 0.8$. It is helpful to get the percentile of z^* by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.8}{2} = 0.9$. This lets you find z^* such that $P(Z < z^*) = 0.9$.

$$z^* = 1.28$$

Use the appropriate formula.

$$\begin{aligned}n &= \hat{p}(1 - \hat{p}) \left(\frac{z^*}{ME} \right)^2 \\ &= (0.24)(0.76) \left(\frac{1.28}{0.008} \right)^2 \\ &= 4669.44\end{aligned}$$

When determining a necessary sample size, always round up (ceiling).

$$n = 4670$$

6. Problem:

A new virus has been devastating corn production. When exposed, 24.7% of common seedlings die within a week. We are trying to develop a resistant strain of corn.

When we expose 700 seedlings of our strain to the virus, 22% die within a week. Using a significance level of 0.05, can we conclude that our strain is significantly more resistant?

- (a) What kind of hypothesis test is appropriate?
- (b) State the hypotheses
- (c) Determine the p -value.
- (d) Decide whether we reject or retain the null hypothesis.
- (e) Do we think our strain is significantly more resistant?

Solution: This is a left-tail (one-tail) proportion test because we only care whether a lower percentage of seedlings will die.

State the hypotheses.

$$H_0 \text{ claims } p = 0.247$$

$$H_A \text{ claims } p < 0.247$$

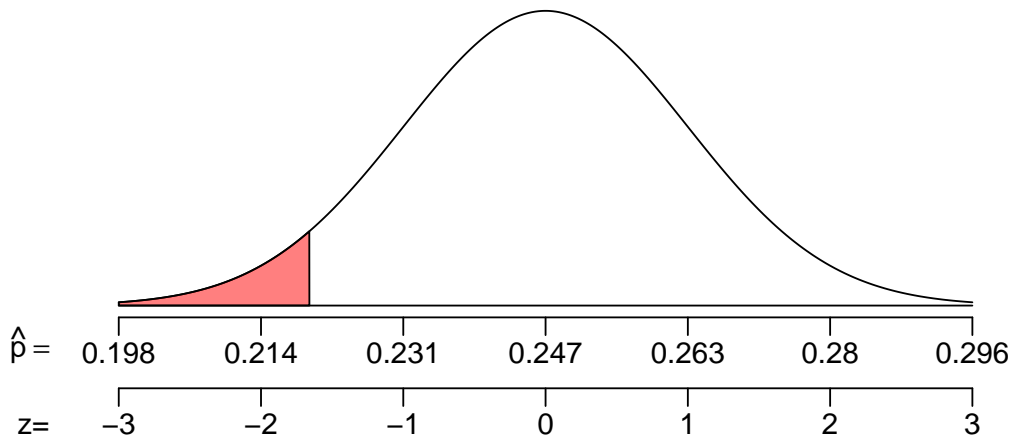
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.247(1-0.247)}{700}} = 0.0163$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.22 - 0.247}{0.0163} = -1.66$$

Make a sketch of the null's sampling distribution. The p -value is a left area.



To determine that left area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} < 0.22) \\ &= P(Z < -1.66) \\ &= 0.0485 \end{aligned}$$

Compare p -value to α (which is 0.05).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

We think our strain is more resistant than common corn.

- (a) Left-tail (one-tail) proportion test
- (b) Hypotheses: H_0 claims $p = 0.247$ and H_A claims $p < 0.247$.
- (c) The p -value is 0.0485
- (d) We reject the null hypothesis.
- (e) We think our strain is more resistant than common corn.

7. Problem:

A null hypothesis claims a population has a mean $\mu = 250$. You decide to run two-tail test on a sample of size $n = 401$ using a significance level $\alpha = 0.01$. You then collect the sample and find it has mean $\bar{x} = 258.94$ and standard deviation $s = 74.26$.

- (a) Determine the p -value.
- (b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \text{ claims } \mu = 250$$

$$H_A \text{ claims } \mu \neq 250$$

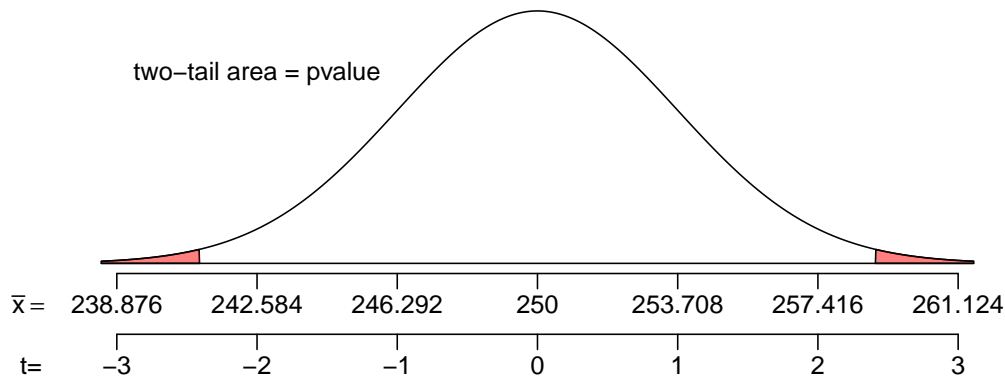
Determine the degrees of freedom.

$$df = 401 - 1 = 400$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{74.26}{\sqrt{401}} = 3.708$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{258.94 - 250}{3.708} = 2.41$$

Find the p -value.

$$p\text{-value} = P(|T| > 2.41)$$

We can't get an exact value with our table, but we can determine an interval that contains the p -value. (Look at row with $df = 400$.)

$$P(|T| > 2.59) = 0.01$$

$$P(|T| > 2.34) = 0.02$$

Basically, because t is between 2.59 and 2.34, we know the p -value is between 0.01 and 0.02.

$$0.01 < p\text{-value} < 0.02$$

Compare the p -value and the significance level ($\alpha = 0.01$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) $0.01 < p\text{-value} < 0.02$

(b) No, we do not reject the null hypothesis.

8. Problem:

You are tasked with estimating the proportion of widgets that are defective. In a sample of 3100 widgets, you determine that 44% were defective. Determine a 99.9% confidence interval of the population proportion. (You are making an inference about the proportion of all widgets that are defective, which you'd only know from a census.)

Solution: Identify the givens.

$$n = 3100$$

$$\hat{p} = 0.44$$

$$\gamma = 0.999$$

Determine z^* such that $P(|Z| < z^*) = 0.999$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.999+1}{2} = 0.9995$. (Use the z-table to find z^* .)

$$z^* = 3.29$$

Use the formula (proportion) for the bounds.

$$LB = \hat{p} - z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$UB = \hat{p} + z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$= 0.44 - 3.29 \sqrt{\frac{(0.44)(0.56)}{3100}}$$

$$= 0.44 + 3.29 \sqrt{\frac{(0.44)(0.56)}{3100}}$$

$$= 0.411$$

$$= 0.469$$

Determine the interval.

$$CI = (0.411, 0.469)$$

We are 99.9% confident that the true population proportion is between 41.1% and 46.9%.

9. Problem:

A student is taking a multiple choice test with 500 questions. Each question has 2 choices. You want to detect whether the student does better than random guessing, so you decide to run a hypothesis test with a significance level of 0.01.

Then, the student takes the test and gets 277 questions correct.

- (a) What kind of hypothesis test is appropriate?
- (b) State the hypotheses
- (c) Determine the p -value.
- (d) Decide whether we reject or retain the null hypothesis.
- (e) Do we think the student did significantly better than random guessing?

Solution: This is a right-tail (one-tail) proportion test because we only care whether the student does better than random.

Determine the null population proportion.

$$p_0 = \frac{1}{2} = 0.5$$

State the hypotheses.

$$H_0 \text{ claims } p = 0.5$$

$$H_A \text{ claims } p > 0.5$$

Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(1-0.5)}{500}} = 0.0224$$

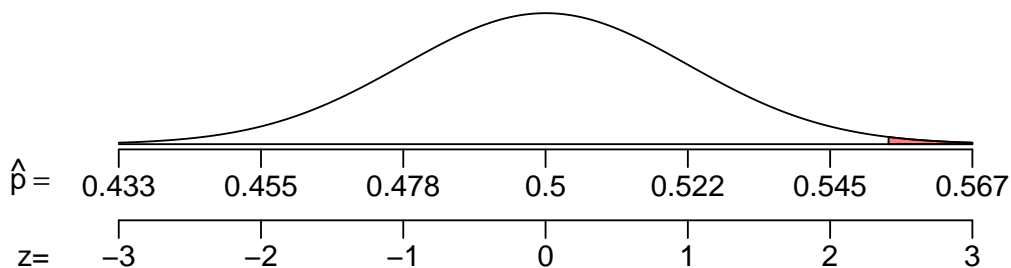
Determine the sample proportion.

$$\hat{p} = \frac{277}{500} = 0.554$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.554 - 0.5}{0.0224} = 2.41$$

Make a sketch of the null's sampling distribution. The p -value is a right area.



To determine that right area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.554) \\ &= P(Z > 2.41) \\ &= 1 - P(Z < 2.41) \\ &= 0.008 \end{aligned}$$

Compare p -value to α (which is 0.01).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

We think the student did better than random guessing typically allows.

- Right tail (one-tail) proportion test
- Hypotheses: H_0 claims $p = 0.5$ and H_A claims $p > 0.5$.
- The p -value is 0.008
- We reject the null hypothesis.
- We think the student did better than random guessing typically allows.

10. Problem:

A null hypothesis claims a population has a mean $\mu = 170$. You decide to run two-tail test on a sample of size $n = 12$ using a significance level $\alpha = 0.05$.

You then collect the sample:

213.3	189.4	189.1	221.7	169.2
173.7	180.3	174.1	163.4	171.6
166.7	167.5			

- (a) Determine the p -value.
- (b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \text{ claims } \mu = 170$$

$$H_A \text{ claims } \mu \neq 170$$

Find the mean and standard deviation of the sample.

$$\bar{x} = 181.667$$

$$s = 18.723$$

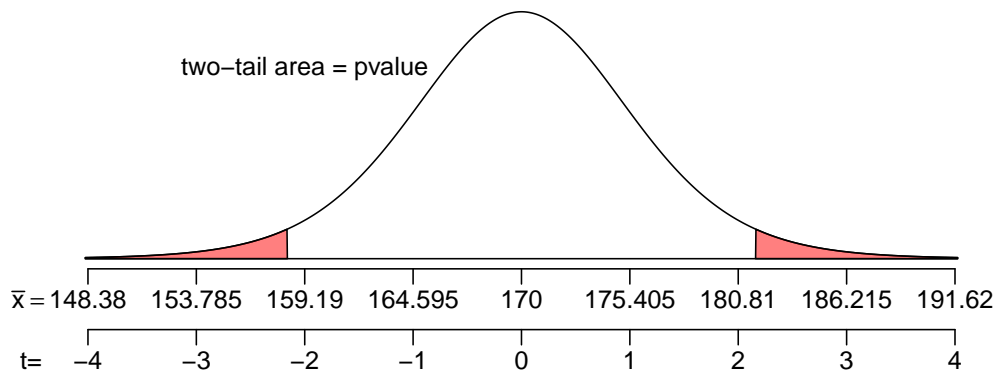
Determine the degrees of freedom.

$$df = 12 - 1 = 11$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{18.723}{\sqrt{12}} = 5.405$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{181.667 - 170}{5.405} = 2.16$$

Find the p -value.

$$p\text{-value} = P(|T| > 2.16)$$

We can't get an exact value with our table, but we can determine an interval that contains the p -value. (Look at row with $df = 11$.)

$$P(|T| > 2.2) = 0.05$$

$$P(|T| > 1.8) = 0.1$$

Basically, because t is between 2.2 and 1.8, we know the p -value is between 0.05 and 0.1.

$$0.05 < p\text{-value} < 0.1$$

Compare the p -value and the significance level ($\alpha = 0.05$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) $0.05 < p\text{-value} < 0.1$

(b) No, we do not reject the null hypothesis.

11. Problem:

Brahim wants to estimate the average mass of the beans in a large bag. Somehow, Brahim is certain that the standard deviation of the beans in the bag is 23 milligrams. He takes a sample of size 34 and finds the sample mean to be 456 milligrams. What would be the 99% confidence interval?

Solution: We are given the sample size, sample mean, population standard deviation, and confidence level.

$$n = 34$$

$$\bar{x} = 456$$

$$\sigma = 23$$

$$\gamma = 0.99$$

Determine the critical z value, z^* , such that $P(|Z| < z^*) = 0.99$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.99+1}{2} = 0.995$

$$z^* = 2.58$$

Use the formula for bounds (mean, σ known).

$$\begin{aligned} LB &= \bar{x} - z^* \frac{\sigma}{\sqrt{n}} & UB &= \bar{x} + z^* \frac{\sigma}{\sqrt{n}} \\ &= 456 - 2.58 \times \frac{23}{\sqrt{34}} & &= 456 + 2.58 \times \frac{23}{\sqrt{34}} \\ &= 445.82 & &= 466.18 \end{aligned}$$

We are 99% confident that the population mean is between 445.82 and 466.18 milligrams.

$$CI = (445.82, 466.18)$$

12. Problem:

A fair 4-sided die has a discrete uniform distribution with an expected value of $\mu = 2.5$ and a standard deviation $\sigma = 1.12$.

You are told to check if a 4-sided die has an expected value different than 2.5. You are told to roll the die 170 times and do a significance test with a significance level of 0.025.

You then roll the die 170 times and get a mean of 2.309. Should we conclude the die is unfair?

- (a) What kind of hypothesis test is appropriate?
- (b) State the hypotheses
- (c) Determine the p -value.
- (d) Decide whether we reject or retain the null hypothesis.
- (e) Do we conclude the die is unfair?

Solution: We should use a two-tail test of population mean.

State the hypotheses:

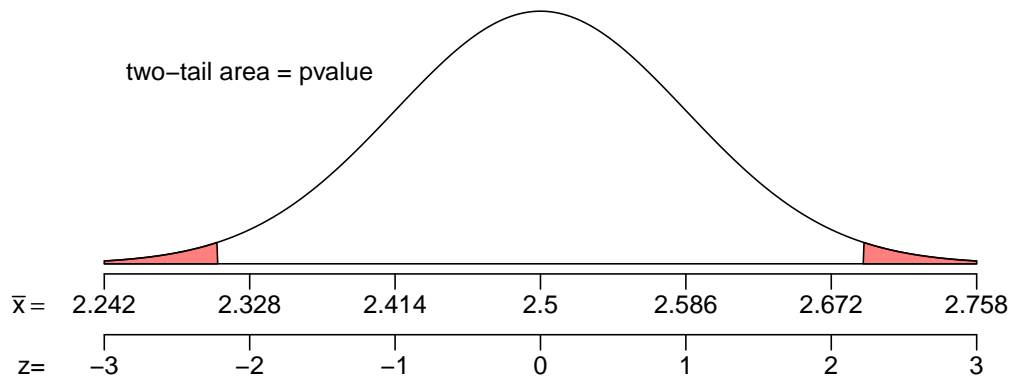
$$H_0 \text{ claims } \mu = 2.5$$

$$H_A \text{ claims } \mu \neq 2.5$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.12}{\sqrt{170}} = 0.086$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{2.309 - 2.5}{0.086} = -2.22$$

Find the p -value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(|Z| > 2.22) \\ &= 2 \cdot P(Z < -2.22) \\ &= \boxed{0.0264} \end{aligned}$$

Compare the p -value and the significance level ($\alpha = 0.025$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

We conclude the die might be fair.

- (a) Two-tail single mean test
- (b) Hypotheses: H_0 claims $\mu = 2.5$ and H_A claims $\mu \neq 2.5$.
- (c) p -value = 0.0264
- (d) No, we do not reject the null hypothesis.
- (e) We conclude the die might be fair.

13. Problem:

A traffic engineer wants to determine the average speed of cars on the Longfellow Bridge. She knows the population standard deviation of speeds is 5 mph. To determine a 90% confidence interval with a margin of error of 1 mph, what sample size is needed?

Solution: We are given the population standard deviation, confidence level, and margin of error.

$$\sigma = 5$$

$$\gamma = 0.9$$

$$ME = 1$$

Determine the critical z value, z^* , such that $P(|Z| < z^*) = 0.9$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.9+1}{2} = 0.95$

$$z^* = 1.64$$

Use the formula for sample size.

$$n = \left(\frac{z^* \sigma}{ME} \right)^2$$

$$= \left(\frac{(1.64)(5)}{1} \right)^2$$

$$= 67.24$$

Round up.

$$n = 68$$