1. **Problem:**
   A fair 8-sided die has a discrete uniform distribution with an expected value of $\mu = 4.5$ and a standard deviation $\sigma = 2.29$.

   You are told to check if a 8-sided die has an expected value different than 4.5. You are told to roll the die 220 times and do a significance test with a significance level of 0.05.

   You then roll the die 220 times and get a mean of 4.819. Should we conclude the die is unfair?

   (a) What kind of hypothesis test is appropriate?
   (b) State the hypotheses
   (c) Determine the p-value.
   (d) Decide whether we reject or retain the null hypothesis.
   (e) Do we conclude the die is unfair?

Solution: We should use a two-tail test of population mean.

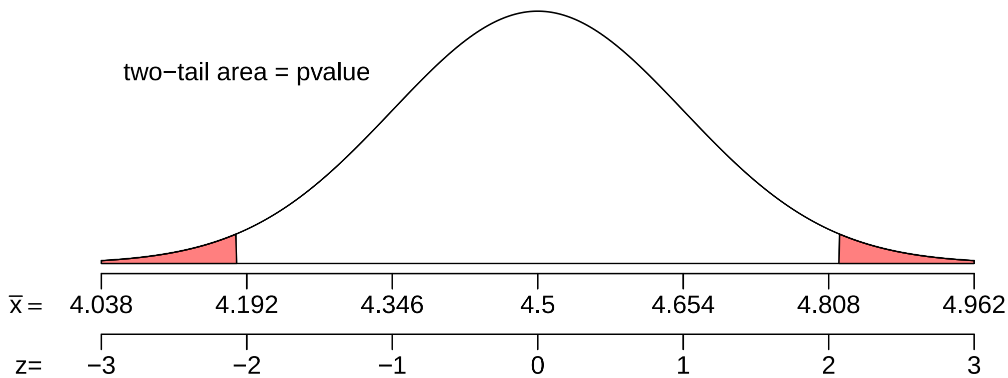
State the hypotheses:

$$H_0 \text{ claims } \mu = 4.5$$
$$H_A \text{ claims } \mu \neq 4.5$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.29}{\sqrt{220}} = 0.154$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{4.819 - 4.5}{0.154} = 2.07$$

Find the p-value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(|Z| > 2.07) \\ &= 2 \cdot P(Z < -2.07) \\ &= \boxed{0.0384} \end{aligned}$$

Compare the p-value and the significance level ($\alpha = 0.05$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

We conclude the die is unfair.

(a) Right-tail single mean test
(b) Hypotheses: H_0 claims $\mu = 4.5$ and H_A claims $\mu \neq 4.5$.
(c) p-value = 0.0384
(d) Yes, we reject the null hypothesis.
(e) We conclude the die is unfair.

2. **Problem:**
A fair 10-sided die has a discrete uniform distribution with an expected value of $\mu = 5.5$ and a standard deviation $\sigma = 2.87$.

You are told to check if a 10-sided die has an expected value different than 5.5. You are told to roll the die 211 times and do a significance test with a significance level of 0.1.

You then roll the die 211 times and get a mean of 5.227. Should we conclude the die is unfair?

(a) What kind of hypothesis test is appropriate?
(b) State the hypotheses
(c) Determine the *p*-value.
(d) Decide whether we reject or retain the null hypothesis.
(e) Do we conclude the die is unfair?

Solution: We should use a two-tail test of population mean.

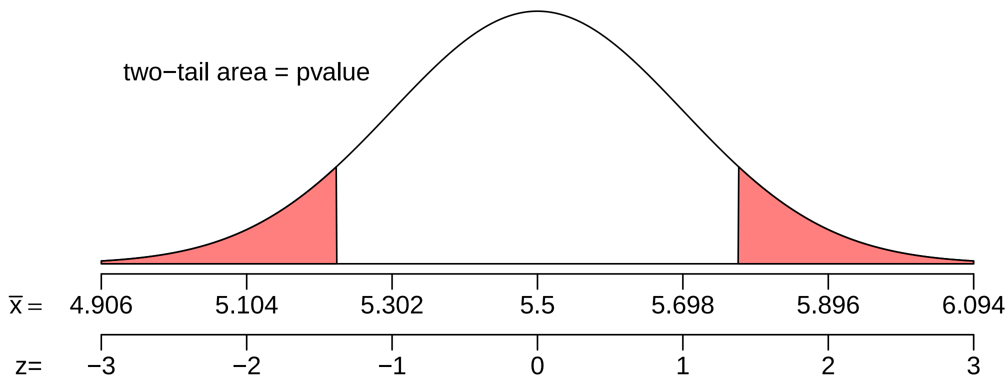
State the hypotheses:

$$H_0 \text{ claims } \mu = 5.5$$
$$H_A \text{ claims } \mu \neq 5.5$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.87}{\sqrt{211}} = 0.198$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{5.227 - 5.5}{0.198} = -1.38$$

Find the p-value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(|Z| > 1.38) \\ &= 2 \cdot P(Z < -1.38) \\ &= \boxed{0.1676} \end{aligned}$$

Compare the p-value and the significance level ($\alpha = 0.1$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

We conclude the die might be fair.

(a) Right-tail single mean test
(b) Hypotheses: H_0 claims $\mu = 5.5$ and H_A claims $\mu \neq 5.5$.
(c) p-value = 0.1676
(d) No, we do not reject the null hypothesis.
(e) We conclude the die might be fair.

3. **Problem:**
A null hypothesis claims a population has a mean μ = 100. You decide to run two-tail test on a sample of size n = 151 using a significance level α = 0.02. You then collect the sample and find it has mean $\bar{x}$ = 105.39 and standard deviation s = 31.83.
   (a) Determine the p-value.
   (b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \text{ claims } \mu = 100$$
$$H_A \text{ claims } \mu \neq 100$$

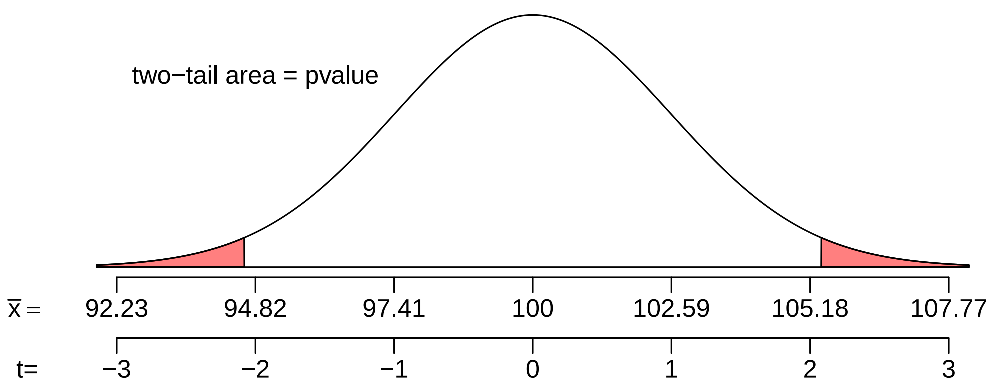
Determine the degrees of freedom.

$$df = 151 - 1 = 150$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{31.83}{\sqrt{151}} = 2.59$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{105.39 - 100}{2.59} = 2.08$$

Find the p-value.

$$p\text{-value} = P(|T| > 2.08)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 150$.)

$$P(|T| > 2.35) = 0.02$$

$$P(|T| > 2.07) = 0.04$$

Basically, because t is between 2.35 and 2.07, we know the p-value is between 0.02 and 0.04.

$$0.02 < p\text{-value} < 0.04$$

Compare the p-value and the significance level ($\alpha = 0.02$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) $0.02 < p$-value < 0.04

(b) No, we do not reject the null hypothesis.

4. **Problem:**
A null hypothesis claims a population has a mean $\mu = 150$. You decide to run two-tail test on a sample of size $n = 37$ using a significance level $\alpha = 0.05$. You then collect the sample and find it has mean $\bar{x} = 162.47$ and standard deviation $s = 35.45$.

   (a) Determine the p-value.

   (b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \text{ claims } \mu = 150$$
$$H_A \text{ claims } \mu \neq 150$$

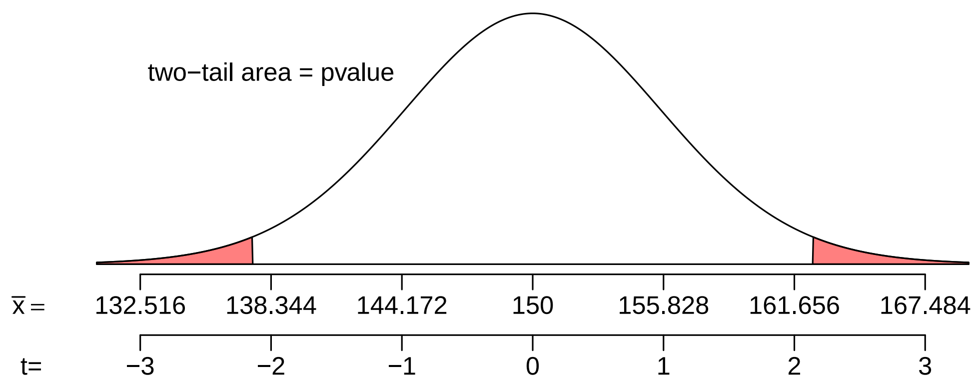
Determine the degrees of freedom.

$$df = 37 - 1 = 36$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{35.45}{\sqrt{37}} = 5.828$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{162.47 - 150}{5.828} = 2.14$$

Find the p-value.

$$p\text{-value} = P(|T| > 2.14)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 36$.)

$$P(|T| > 2.43) = 0.02$$

$$P(|T| > 2.13) = 0.04$$

Basically, because t is between 2.43 and 2.13, we know the p-value is between 0.02 and 0.04.

$$0.02 < p\text{-value} < 0.04$$

Compare the p-value and the significance level ($\alpha = 0.05$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) $0.02 < p$-value < 0.04
(b) Yes, we reject the null hypothesis.

5. **Problem:**
A null hypothesis claims a population has a mean $\mu = 180$. You decide to run two-tail test on a sample of size $n = 10$ using a significance level $\alpha = 0.02$.

You then collect the sample:

184.5	184.5	179.6	180	188.9
186.8	182.5	181.7	183.4	178.4

(a) Determine the *p*-value.

(b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \text{ claims } \mu = 180$$
$$H_A \text{ claims } \mu \neq 180$$

Find the mean and standard deviation of the sample.

$$\bar{x} = 183.03$$

$$s = 3.292$$

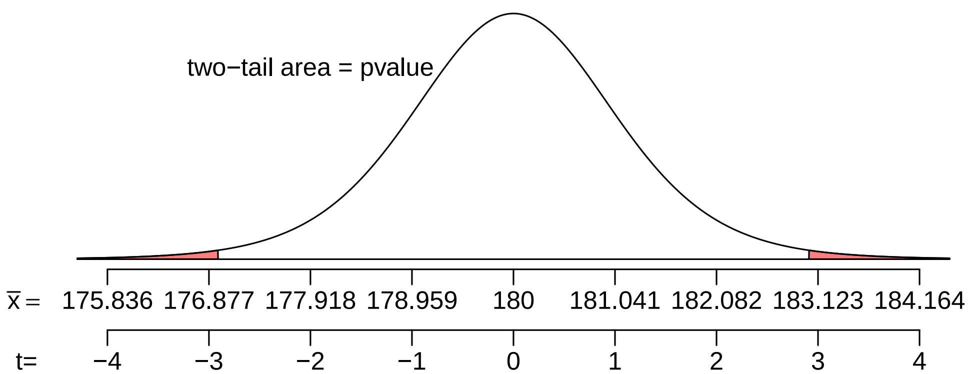
Determine the degrees of freedom.

$$df = 10 - 1 = 9$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{3.292}{\sqrt{10}} = 1.041$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{183.03 - 180}{1.041} = 2.91$$

Find the p-value.

$$p\text{-value} = P(|T| > 2.91)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 9$.)

$$P(|T| > 3.25) = 0.01$$

$$P(|T| > 2.82) = 0.02$$

Basically, because t is between 3.25 and 2.82, we know the p-value is between 0.01 and 0.02.

$$0.01 < p\text{-value} < 0.02$$

Compare the p-value and the significance level ($\alpha = 0.02$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) $0.01 < p$-value < 0.02

(b) Yes, we reject the null hypothesis.

6. **Problem:**
A null hypothesis claims a population has a mean $\mu = 90$. You decide to run two-tail test on a sample of size $n = 12$ using a significance level $\alpha = 0.1$.

You then collect the sample:

92.5	88.4	100.2	91.2	88.3
93.8	91.5	89.5	88	93.4
92.9	92.1			

(a) Determine the p-value.

(b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \text{ claims } \mu = 90$$
$$H_A \text{ claims } \mu \neq 90$$

Find the mean and standard deviation of the sample.

$$\bar{x} = 91.817$$
$$s = 3.343$$

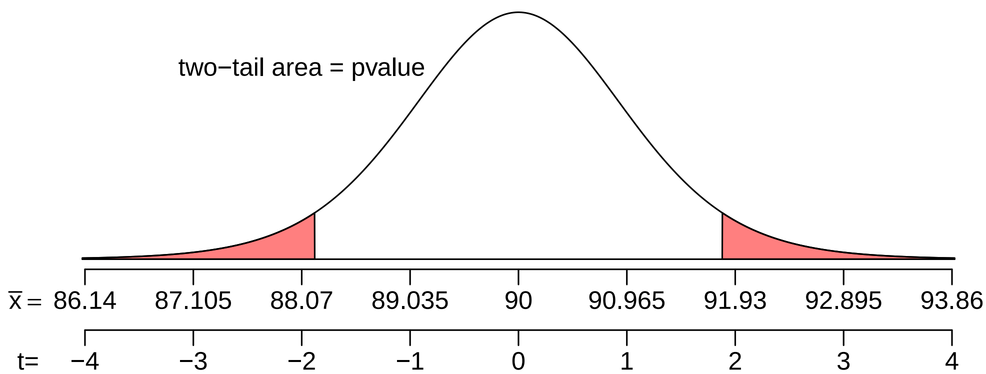
Determine the degrees of freedom.

$$df = 12 - 1 = 11$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{3.343}{\sqrt{12}} = 0.965$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{91.817 - 90}{0.965} = 1.88$$

Find the p-value.

$$p\text{-value} = P(|T| > 1.88)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 11$.)

$$P(|T| > 2.2) = 0.05$$
$$P(|T| > 1.8) = 0.1$$

Basically, because t is between 2.2 and 1.8, we know the p-value is between 0.05 and 0.1.

$$0.05 < p\text{-value} < 0.1$$

Compare the p-value and the significance level ($\alpha = 0.1$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) $0.05 < p$-value < 0.1

(b) Yes, we reject the null hypothesis.