1. **Problem:**
   A continuous random variable X was measured 11 times. The sorted data are shown below, along with each datum's index.

i	x
1	90.998
2	92.429
3	92.893
4	93.067
5	93.492
6	93.630
7	94.382
8	94.595
9	94.727
10	94.753
11	94.948

   The total of the measurements is 1029.914.

   (a) Determine the percentile rank of the value 92.893. In other words, determine what percent of data are less than or equal to 92.893.

   (b) Determine the datum corresponding to a percentile rank of 0.818. In other words, determine x such that 81.8% of the data are less than or equal to x.

   (c) Determine the mean of the measurements.

   (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 92.893$. This means $i = 3$. We know $n = 11$. Determine the percentile ℓ.

$$\ell = \frac{3}{11}$$

$$\ell = 0.273$$

So, the answer is 0.273, or 27.3%.

(b) We are given $\ell = 0.818$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (11)(0.818)$$

$$i = 9$$

Determine the x associated with $i = 9$.

$$x = 94.727$$

(c) The mean is $\frac{1029.914}{11} = 93.6285455$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor \frac{n+1}{2} \rfloor}$ and $x_{\lceil \frac{n+1}{2} \rceil}$. So, median = 93.63.

2. **Problem:**

A continuous random variable X was measured 11 times. The sorted data are shown below, along with each datum's index.

i	x
1	62.215
2	75.423
3	90.304
4	100.741
5	103.763
6	104.337
7	117.313
8	119.937
9	120.921
10	121.267
11	143.891

The total of the measurements is 1160.112.

(a) Determine the percentile rank of the value 119.937. In other words, determine what percent of data are less than or equal to 119.937.

(b) Determine the datum corresponding to a percentile rank of 0.818. In other words, determine x such that 81.8% of the data are less than or equal to x.

(c) Determine the mean of the measurements.

(d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 119.937$. This means $i = 8$. We know $n = 11$. Determine the percentile ℓ.

$$\ell = \frac{8}{11}$$

$$\ell = 0.727$$

So, the answer is 0.727, or 72.7%.

(b) We are given $\ell = 0.818$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (11)(0.818)$$

$$i = 9$$

Determine the x associated with $i = 9$.

$$x = 120.921$$

(c) The mean is $\frac{1160.112}{11} = 105.4647273$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor \frac{n+1}{2} \rfloor}$ and $x_{\lceil \frac{n+1}{2} \rceil}$. So, median = 104.337.

3. **Problem:**
   A continuous random variable X was measured 9 times. The sorted data are shown below, along with each datum's index.

i	x
1	91.634
2	92.289
3	93.500
4	94.355
5	96.998
6	97.474
7	98.349
8	98.798
9	103.905

   The total of the measurements is 867.302.

   (a) Determine the percentile rank of the value 98.349. In other words, determine what percent of data are less than or equal to 98.349.

   (b) Determine the datum corresponding to a percentile rank of 0.222. In other words, determine x such that 22.2% of the data are less than or equal to x.

   (c) Determine the mean of the measurements.

   (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 98.349$. This means $i = 7$. We know $n = 9$. Determine the percentile ℓ.

$$\ell = \frac{7}{9}$$

$$\ell = 0.778$$

So, the answer is 0.778, or 77.8%.

(b) We are given $\ell = 0.222$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (9)(0.222)$$

$$i = 2$$

Determine the x associated with $i = 2$.

$$x = 92.289$$

(c) The mean is $\frac{867.302}{9} = 96.3668889$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor \frac{n+1}{2} \rfloor}$ and $x_{\lceil \frac{n+1}{2} \rceil}$. So, median = 96.998.

4. **Problem:**

A continuous random variable X was measured 8 times. The sorted data are shown below, along with each datum's index.

i	x
1	60.006
2	60.823
3	60.831
4	61.849
5	65.288
6	65.832
7	68.223
8	69.617

The total of the measurements is 512.469.

(a) Determine the percentile rank of the value 65.832. In other words, determine what percent of data are less than or equal to 65.832.

(b) Determine the datum corresponding to a percentile rank of 0.125. In other words, determine x such that 12.5% of the data are less than or equal to x.

(c) Determine the mean of the measurements.

(d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 65.832$. This means $i = 6$. We know $n = 8$. Determine the percentile ℓ.

$$\ell = \frac{6}{8}$$

$$\ell = 0.75$$

So, the answer is 0.75, or 75%.

(b) We are given $\ell = 0.125$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (8)(0.125)$$

$$i = 1$$

Determine the x associated with $i = 1$.

$$x = 60.006$$

(c) The mean is $\frac{512.469}{8} = 64.058625$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor \frac{n+1}{2} \rfloor}$ and $x_{\lceil \frac{n+1}{2} \rceil}$. So, median = 63.5685.

5. **Problem:**
A continuous random variable X was measured 10 times. The sorted data are shown below, along with each datum's index.

i	x
1	40.014
2	40.405
3	40.753
4	41.113
5	41.146
6	41.194
7	44.217
8	45.298
9	46.881
10	56.737

The total of the measurements is 437.758.

(a) Determine the percentile rank of the value 41.194. In other words, determine what percent of data are less than or equal to 41.194.

(b) Determine the datum corresponding to a percentile rank of 0.7. In other words, determine x such that 70% of the data are less than or equal to x.

(c) Determine the mean of the measurements.

(d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 41.194$. This means $i = 6$. We know $n = 10$. Determine the percentile ℓ.

$$\ell = \frac{6}{10}$$

$$\ell = 0.6$$

So, the answer is 0.6, or 60%.

(b) We are given $\ell = 0.7$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (10)(0.7)$$

$$i = 7$$

Determine the x associated with $i = 7$.

$$x = 44.217$$

(c) The mean is $\frac{437.758}{10} = 43.7758$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor \frac{n+1}{2} \rfloor}$ and $x_{\lceil \frac{n+1}{2} \rceil}$. So, median = 41.17.

6. **Problem:**
A continuous random variable X was measured 10 times. The sorted data are shown below, along with each datum's index.

i	x
1	77.656
2	77.686
3	83.826
4	86.093
5	87.714
6	87.928
7	88.588
8	89.295
9	89.539
10	94.107

The total of the measurements is 862.432.

(a) Determine the percentile rank of the value 89.539. In other words, determine what percent of data are less than or equal to 89.539.

(b) Determine the datum corresponding to a percentile rank of 0.3. In other words, determine x such that 30% of the data are less than or equal to x.

(c) Determine the mean of the measurements.

(d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 89.539$. This means $i = 9$. We know $n = 10$. Determine the percentile ℓ.

$$\ell = \frac{9}{10}$$

$$\ell = 0.9$$

So, the answer is 0.9, or 90%.

(b) We are given $\ell = 0.3$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (10)(0.3)$$

$$i = 3$$

Determine the x associated with $i = 3$.

$$x = 83.826$$

(c) The mean is $\frac{862.432}{10} = 86.2432$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor \frac{n+1}{2} \rfloor}$ and $x_{\lceil \frac{n+1}{2} \rceil}$. So, median = 87.821.

7. **Problem:**

A continuous random variable X was measured 72 times. The sorted data are shown below.

20.315	20.783	21.418	21.724	21.756	21.910	21.943	22.006	22.250
22.257	22.264	22.441	22.541	22.551	22.574	22.617	22.676	22.691
22.767	22.794	22.845	22.873	23.032	23.042	23.121	23.133	23.174
23.356	23.409	23.543	23.648	23.751	23.908	23.938	23.985	24.007
24.009	24.096	24.144	24.146	24.159	24.192	24.225	24.230	24.270
24.296	24.323	24.349	24.376	24.426	24.451	24.486	24.487	24.530
24.564	24.579	24.581	24.605	24.611	24.617	24.636	24.639	24.734
24.743	24.764	24.776	24.805	24.842	24.887	24.935	24.937	24.949

The total of the measurements is 1699.442.

(a) Determine the percentile rank of the value 22.264. In other words, determine what percent of data are less than or equal to 22.264.

(b) Determine the datum corresponding to a percentile rank of 0.583. In other words, determine x such that 58.3% of the data are less than or equal to x.

(c) Determine the mean of the measurements.

(d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 22.264$. This means $i = 11$. We know $n = 72$. Determine the percentile ℓ.

$$\ell = \frac{11}{72}$$

$$\ell = 0.153$$

So, the answer is 0.153, or 15.3%.

(b) We are given $\ell = 0.583$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (72)(0.583)$$

$$i = 42$$

Determine the x associated with $i = 42$.

$$x = 24.192$$

(c) The mean is $\frac{1699.442}{72} = 23.603$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor \frac{n+1}{2} \rfloor}$ and $x_{\lceil \frac{n+1}{2} \rceil}$. So, median = 24.008.

8. **Problem:**
A continuous random variable X was measured 45 times. The sorted data are shown below.

20.026	20.052	20.104	20.114	20.342	20.502	20.561	20.624	21.017
21.060	21.563	21.753	22.034	22.186	22.236	22.723	22.875	23.109
23.991	24.247	24.729	25.349	25.545	25.901	25.972	27.092	27.212
27.571	27.701	27.735	27.848	28.115	28.319	28.915	28.917	29.364
29.496	29.509	29.656	29.791	29.797	29.809	29.837	29.859	29.901

The total of the measurements is 1135.059.

(a) Determine the percentile rank of the value 22.723. In other words, determine what percent of data are less than or equal to 22.723.

(b) Determine the datum corresponding to a percentile rank of 0.844. In other words, determine x such that 84.4% of the data are less than or equal to x.

(c) Determine the mean of the measurements.

(d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 22.723$. This means $i = 16$. We know $n = 45$. Determine the percentile ℓ.

$$\ell = \frac{16}{45}$$

$$\ell = 0.356$$

So, the answer is 0.356, or 35.6%.

(b) We are given $\ell = 0.844$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (45)(0.844)$$

$$i = 38$$

Determine the x associated with $i = 38$.

$$x = 29.509$$

(c) The mean is $\frac{1135.059}{45} = 25.224$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor\frac{n+1}{2}\rfloor}$ and $x_{\lceil\frac{n+1}{2}\rceil}$. So, median = 25.545.

9. **Problem:**

A continuous random variable X was measured 30 times. The sorted data are shown below.

90.206	90.383	90.494	90.763	91.142	91.339
91.428	91.439	91.462	91.638	91.639	91.939
92.419	92.459	92.464	92.514	92.671	92.767
92.964	93.083	93.111	93.197	93.218	93.487
93.626	93.761	94.038	94.489	94.720	94.855

The total of the measurements is 2773.715.

(a) Determine the percentile rank of the value 92.671. In other words, determine what percent of data are less than or equal to 92.671.

(b) Determine the datum corresponding to a percentile rank of 0.0333. In other words, determine x such that 3.33% of the data are less than or equal to x.

(c) Determine the mean of the measurements.

(d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 92.671$. This means $i = 17$. We know $n = 30$. Determine the percentile ℓ.

$$\ell = \frac{17}{30}$$

$$\ell = 0.567$$

So, the answer is 0.567, or 56.7%.

(b) We are given $\ell = 0.0333$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (30)(0.0333)$$

$$i = 1$$

Determine the x associated with $i = 1$.

$$x = 90.206$$

(c) The mean is $\frac{2773.715}{30} = 92.457$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor \frac{n+1}{2} \rfloor}$ and $x_{\lceil \frac{n+1}{2} \rceil}$. So, median = 92.489.

10. **Problem:**

A continuous random variable X was measured 35 times. The sorted data are shown below.

70.001	70.013	70.014	70.045	70.097	70.221	70.258
70.418	70.424	70.648	70.833	71.100	71.254	71.861
72.093	72.394	72.556	73.065	73.161	73.283	73.355
73.370	73.606	73.869	74.215	74.399	74.693	74.696
74.735	74.752	74.791	74.924	74.947	74.950	74.951

The total of the measurements is 2539.992.

(a) Determine the percentile rank of the value 74.696. In other words, determine what percent of data are less than or equal to 74.696.

(b) Determine the datum corresponding to a percentile rank of 0.286. In other words, determine x such that 28.6% of the data are less than or equal to x.

(c) Determine the mean of the measurements.

(d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 74.696$. This means $i = 28$. We know $n = 35$. Determine the percentile ℓ.

$$\ell = \frac{28}{35}$$

$$\ell = 0.8$$

So, the answer is 0.8, or 80%.

(b) We are given $\ell = 0.286$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (35)(0.286)$$

$$i = 10$$

Determine the x associated with $i = 10$.

$$x = 70.648$$

(c) The mean is $\frac{2539.992}{35} = 72.571$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor\frac{n+1}{2}\rfloor}$ and $x_{\lceil\frac{n+1}{2}\rceil}$. So, median = 73.065.

11. **Problem:**

A continuous random variable X was measured 48 times. The sorted data are shown below.

60.001	60.002	60.005	60.029	60.058	60.126	60.199	60.214
60.222	60.266	60.322	60.354	60.359	60.457	60.514	60.522
60.540	60.615	60.709	60.744	60.745	60.875	60.973	60.975
61.082	61.435	61.514	61.554	61.583	61.677	61.699	61.780
61.816	61.849	61.889	61.923	61.980	62.324	62.593	62.651
62.663	62.745	62.776	62.910	62.921	62.944	62.956	62.964

The total of the measurements is 2942.054.

(a) Determine the percentile rank of the value 60.001. In other words, determine what percent of data are less than or equal to 60.001.

(b) Determine the datum corresponding to a percentile rank of 0.708. In other words, determine x such that 70.8% of the data are less than or equal to x.

(c) Determine the mean of the measurements.

(d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 60.001$. This means $i = 1$. We know $n = 48$. Determine the percentile ℓ.

$$\ell = \frac{1}{48}$$

$$\ell = 0.0208$$

So, the answer is 0.0208, or 2.08%.

(b) We are given $\ell = 0.708$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (48)(0.708)$$

$$i = 34$$

Determine the x associated with $i = 34$.

$$x = 61.849$$

(c) The mean is $\frac{2942.054}{48} = 61.293$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor \frac{n+1}{2} \rfloor}$ and $x_{\lceil \frac{n+1}{2} \rceil}$. So, median = 61.028.

12. **Problem:**

A continuous random variable X was measured 72 times. The sorted data are shown below.

30.005	37.063	45.864	47.128	48.361	57.817	58.204	63.970	65.605
69.161	71.536	72.088	72.494	75.794	77.697	77.979	80.362	80.781
84.155	84.626	85.614	86.152	86.153	88.343	88.532	88.823	89.565
90.926	91.327	91.718	92.106	92.225	93.358	93.880	94.434	95.072
96.742	97.064	97.075	98.568	98.940	99.357	100.062	100.073	100.252
100.775	101.709	102.181	102.202	102.432	102.913	103.129	105.658	105.824
106.723	107.939	108.401	108.773	108.785	109.676	109.867	110.016	111.426
113.157	113.785	114.183	114.831	114.879	115.815	116.642	117.613	118.844

The total of the measurements is 6585.229.

(a) Determine the percentile rank of the value 98.94. In other words, determine what percent of data are less than or equal to 98.94.

(b) Determine the datum corresponding to a percentile rank of 0.889. In other words, determine x such that 88.9% of the data are less than or equal to x.

(c) Determine the mean of the measurements.

(d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 98.94$. This means $i = 41$. We know $n = 72$. Determine the percentile ℓ.

$$\ell = \frac{41}{72}$$

$$\ell = 0.569$$

So, the answer is 0.569, or 56.9%.

(b) We are given $\ell = 0.889$. We can use algebra to solve for i.

$$\ell = \frac{i}{n}$$

Multiply both sides by n.

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i.

$$i = (72)(0.889)$$

$$i = 64$$

Determine the x associated with $i = 64$.

$$x = 113.157$$

(c) The mean is $\frac{6585.229}{72} = 91.462$

(d) If n is odd, then median is $x_{\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise median is mean of $x_{\lfloor \frac{n+1}{2} \rfloor}$ and $x_{\lceil \frac{n+1}{2} \rceil}$. So, median = 95.907.