Normal Approximation to Binomial Distribution

Central Limit Theorem

Let random variable W have mean μ_w and standard deviation σ_w.

Let random variable X represent the **sum** of n instances of W.

$$X = W_1 + W_2 + W_3 + \cdots + W_n$$

Then:

$$\mu_x = n\mu_w$$

$$\sigma_x = \sqrt{n}\sigma_w$$

and X is approximately normal.

$$X \sim \mathcal{N}(\mu_x, \sigma_x)$$

Bernoulli (review)

Let W be a Bernoulli random variable.

w	$P(w)$
0	q
1	p

$$\mu_w = p$$

$$\sigma_w = \sqrt{pq}$$

Binomial distribution is a case of Central Limit Theorem

Let W be a Bernoulli random variable.

w	$P(w)$
0	q
1	p

$$\mu_w = p$$

$$\sigma_w = \sqrt{pq}$$

Let X represent the **sum** of n instances of W.

$$\mu_x = np$$

$$\sigma_x = \sqrt{n}\sqrt{pq} = \sqrt{npq}$$

X is approximately normal.

Example

Let W be a Bernoulli random variable with 80% chance of success.

w	$P(w)$
0	0.2
1	0.8

$$\mu_w = 0.8$$
$$\sigma_w = \sqrt{(0.8)(0.2)} = 0.4$$

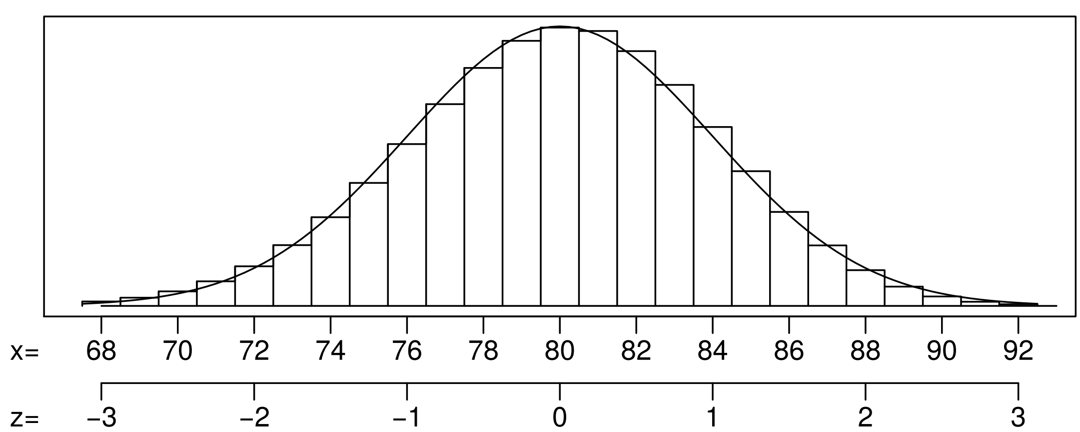
Let X represent 100 repetitions of W.

$$X = W_1 + W_2 + W_3 + \cdots + W_{100}$$

Thus,

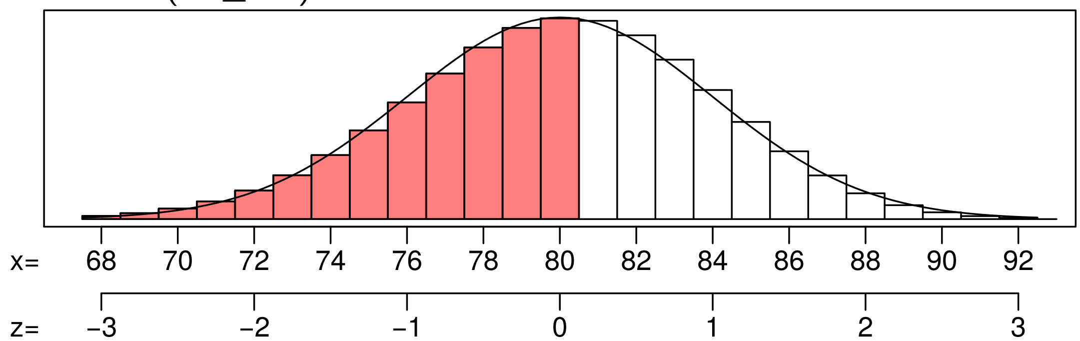
$$\mu_x = (100)(0.8) = 80$$
$$\sigma_x = (\sqrt{100})(0.4) = 4$$

Binomial and Normal Approx with $p = 0.8$ and $n = 100$.

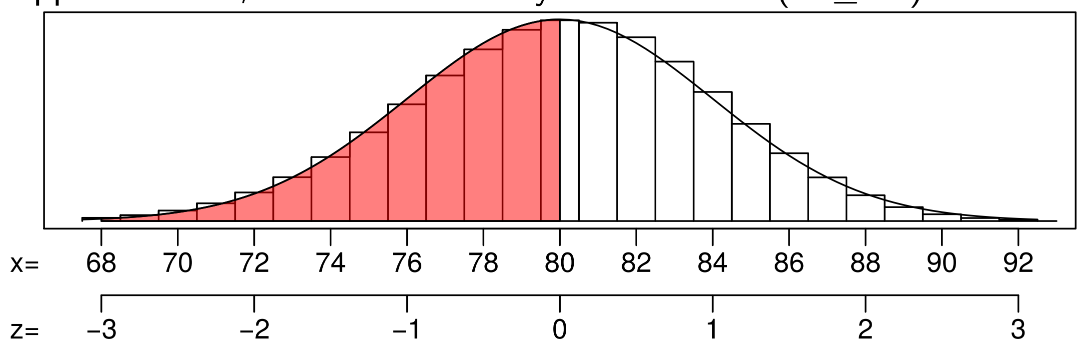


Actual vs Approx... $P(X \leq 80)$

Actual: $P(X \leq 80) = 0.5398386$

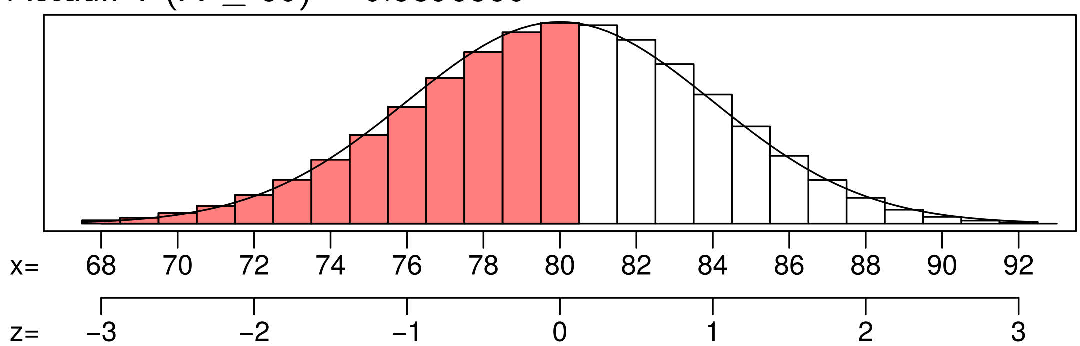


Approximation, without continuity correction: $P(X \leq 80) \approx 0.5$

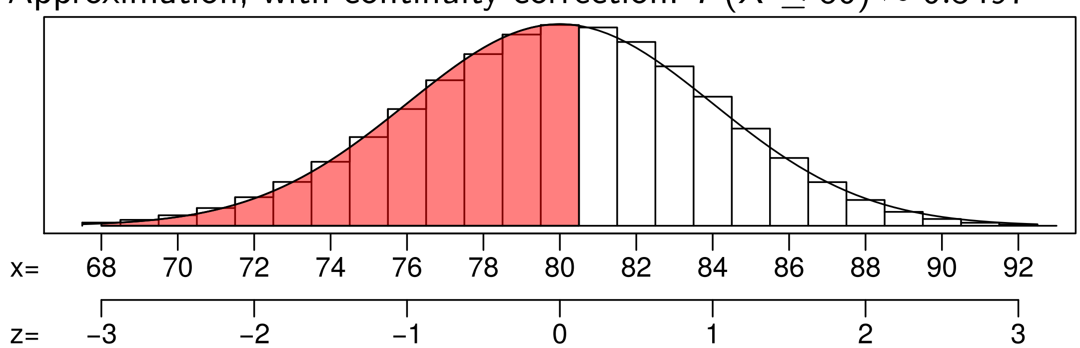


Actual vs Approx with continuity correction... $P(X \leq 80)$

Actual: $P(X \leq 80) = 0.5398386$



Approximation, with continuity correction: $P(X \leq 80) \approx 0.5497$



When to use normal approximation to Binomial Distribution

- When n is large.
- When p is not near 0 or 1.

- If both $np \geq 10$ and $nq \geq 10$ then normal approximation to binomial distribution is cool.

Bad approximation... $n = 7$ and $p = 0.2$