1. **Problem:**
   It is generally accepted that a population's proportion is 0.523. However, you think that maybe the population proportion is under 0.523, so you decide to run a one-tail hypothesis test with a significance level of 0.025 with a sample size of 600.

   Then, when you collect the random sample, you find its proportion is 0.482. Do you reject or retain the null hypothesis?

   (a) What type of hypothesis test is appropriate?

   (b) State the hypotheses.

   (c) Determine the p-value.

   (d) Decide whether we reject or retain the null hypothesis.

Solution: A left-tail proportion test is appropriate. State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.523$$

$$H_A \quad \text{claims} \quad p < 0.523$$

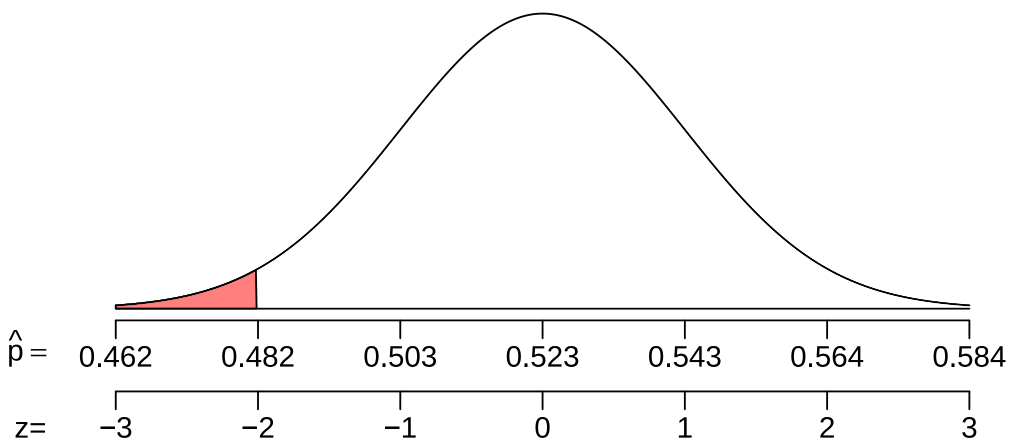
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.523(1-0.523)}{600}} = 0.0204$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.482 - 0.523}{0.0204} = -2.01$$

Make a sketch of the null's sampling distribution. The p-value is a left area.



To determine that left area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} < 0.482) \\ &= P(Z < -2.01) \\ &= 0.0222 \end{aligned}$$

Compare p-value to α (which is 0.025).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) A left-tail (one-tail) proportion test is appropriate.

(b) Hypotheses: H_0 claims $p = 0.523$ and H_A claims $p < 0.523$.

(c) The p-value is 0.0222

(d) We reject the null hypothesis.

2. **Problem:**
   A new virus has been devastating corn production. When exposed, 17.4% of common seedlings die within a week. We are trying to develop a resistant strain of corn.

   When we expose 500 seedlings of our strain to the virus, 14.8% die within a week. Using a significance level of 0.1, can we conclude that our strain is significantly more resistant?

   (a) What kind of hypothesis test is appropriate?
   (b) State the hypotheses
   (c) Determine the *p*-value.
   (d) Decide whether we reject or retain the null hypothesis.
   (e) Do we think our strain is significantly more resistant?

Solution: This is a left-tail (one-tail) proportion test because we only care whether a lower percentage of seedlings will die.

State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.174$$
$$H_A \quad \text{claims} \quad p < 0.174$$

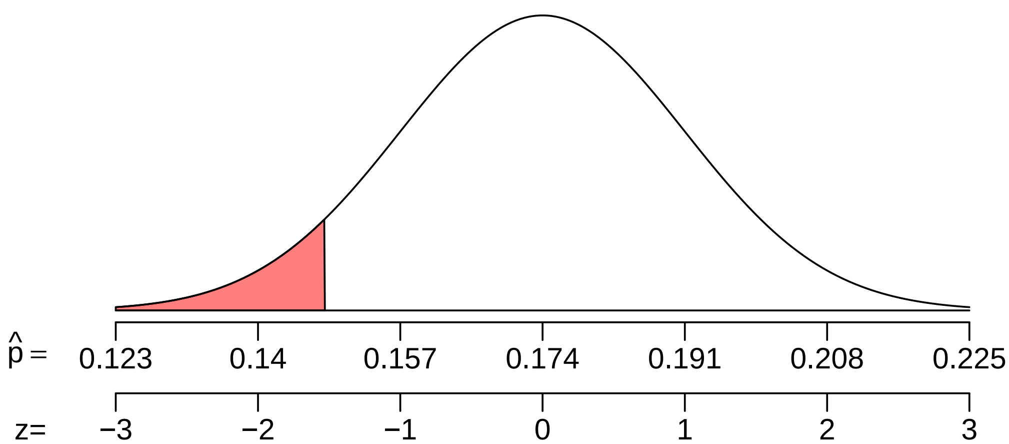
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.174(1-0.174)}{500}} = 0.017$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.148 - 0.174}{0.017} = -1.53$$

Make a sketch of the null's sampling distribution. The p-value is a left area.



To determine that left area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} < 0.148) \\ &= P(Z < -1.53) \\ &= 0.063 \end{aligned}$$

Compare p-value to α (which is 0.1).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

We think our strain is more resistant than common corn.

(a) Left-tail (one-tail) proportion test
(b) Hypotheses: H_0 claims $p = 0.174$ and H_A claims $p < 0.174$.
(c) The p-value is 0.063
(d) We reject the null hypothesis.
(e) We think our strain is more resistant than common corn.

3. **Problem:**
   It is generally accepted that a population's proportion is 0.628. However, you think that maybe the population proportion is over 0.628, so you decide to run a one-tail hypothesis test with a significance level of 0.1 with a sample size of 3000.

   Then, when you collect the random sample, you find its proportion is 0.64. Do you reject or retain the null hypothesis?

   (a) What type of hypothesis test is appropriate?
   (b) State the hypotheses.
   (c) Determine the *p*-value.
   (d) Decide whether we reject or retain the null hypothesis.

Solution: A right-tail proportion test is appropriate. State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.628$$

$$H_A \quad \text{claims} \quad p > 0.628$$

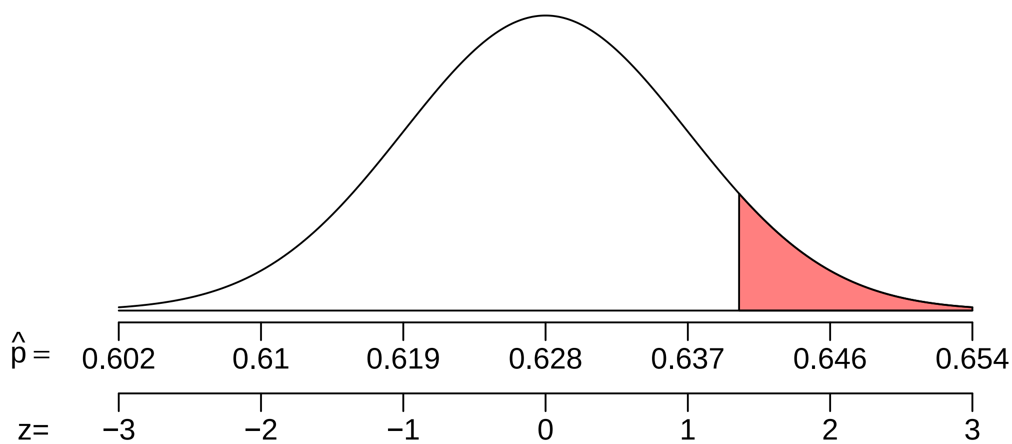
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.628(1-0.628)}{3000}} = 0.00882$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.64 - 0.628}{0.00882} = 1.36$$

Make a sketch of the null's sampling distribution. The p-value is a right area.



To determine that right area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.64) \\ &= P(Z > 1.36) \\ &= 1 - P(Z < 1.36) \\ &= 0.0869 \end{aligned}$$

Compare p-value to α (which is 0.1).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) A right-tail (one-tail) proportion test is appropriate.
(b) Hypotheses: H_0 claims $p = 0.628$ and H_A claims $p > 0.628$.
(c) The p-value is 0.0869
(d) We reject the null hypothesis.

4. **Problem:**
A student is taking a multiple choice test with 700 questions. Each question has 5 choices. You want to detect whether the student does better than random guessing, so you decide to run a hypothesis test with a significance level of 0.025.

Then, the student takes the test and gets 158 questions correct.

(a) What kind of hypothesis test is appropriate?
(b) State the hypotheses
(c) Determine the p-value.
(d) Decide whether we reject or retain the null hypothesis.
(e) Do we think the student did significantly better than random guessing?

Solution: This is a right-tail (one-tail) proportion test because we only care whether the student does better than random.

Determine the null population proportion.

$$p_0 = \frac{1}{5} = 0.2$$

State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.2$$
$$H_A \quad \text{claims} \quad p > 0.2$$

Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.2(1-0.2)}{700}} = 0.0151$$

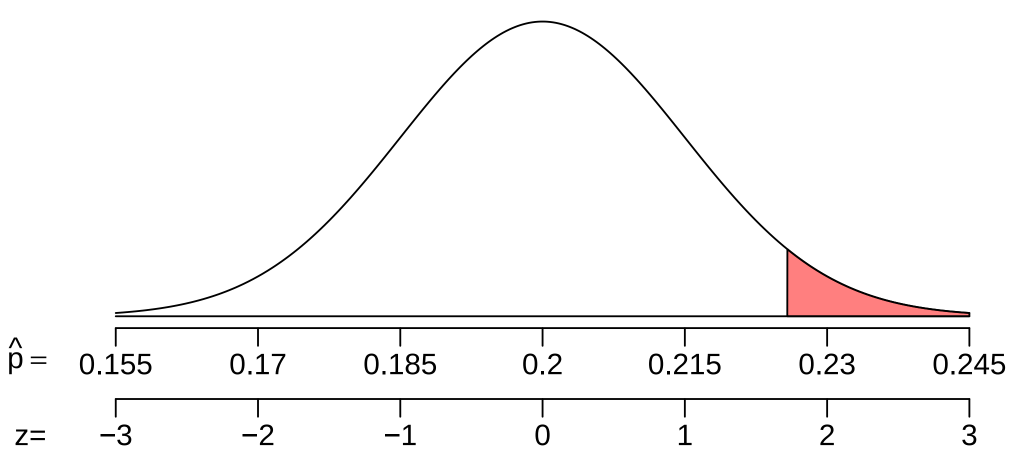
Determine the sample proportion.

$$\hat{p} = \frac{158}{700} = 0.226$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.226 - 0.2}{0.0151} = 1.72$$

Make a sketch of the null's sampling distribution. The p-value is a right area.



To determine that right area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.226) \\ &= P(Z > 1.72) \\ &= 1 - P(Z < 1.72) \\ &= 0.0427 \end{aligned}$$

Compare p-value to α (which is 0.025).

$$p\text{-value} > \alpha$$

Make the conclusion: we retain the null hypothesis.

We think the student might just be guessing.

(a) Right tail (one-tail) proportion test

(b) Hypotheses: H_0 claims $p = 0.2$ and H_A claims $p > 0.2$.

(c) The p-value is 0.0427

(d) We retain the null hypothesis.

(e) We think the student might just be guessing.

5. **Problem:**
   It is generally accepted that a population's proportion is 0.606. However, you think that maybe the population proportion is different than 0.606, so you decide to run a two-tail hypothesis test with a significance level of 0.2 with a sample size of 900.

   Then, when you collect the random sample, you find its proportion is 0.624. Do you reject or retain the null hypothesis?

   (a) What type of hypothesis test is appropriate?
   (b) State the hypotheses.
   (c) Determine the *p*-value.
   (d) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.606$$

$$H_A \quad \text{claims} \quad p \neq 0.606$$

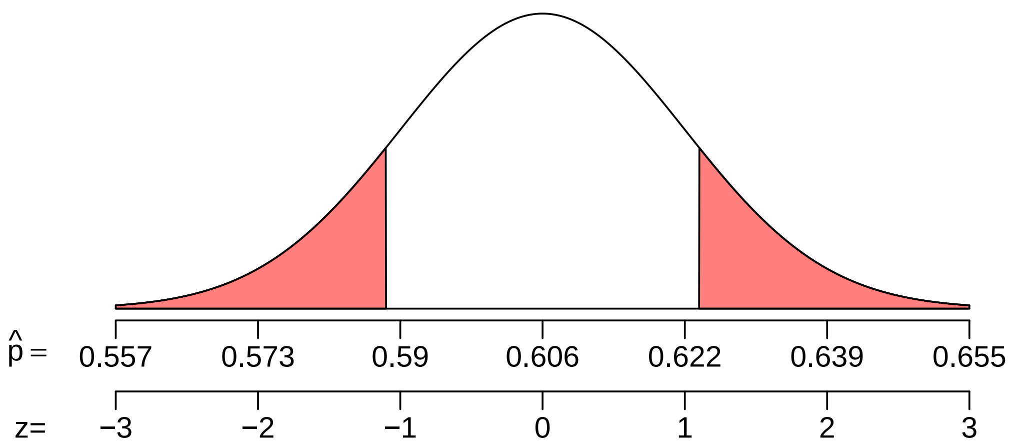
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.606(1-0.606)}{900}} = 0.0163$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.624 - 0.606}{0.0163} = 1.1$$

Make a sketch of the null's sampling distribution. The p-value is a two-tail area.



To determine that two-tail area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(|Z| > 1.1) \\ &= 2 \cdot P(Z < -1.1) \\ &= 0.2714 \end{aligned}$$

Compare p-value to α (which is 0.2).

$$p\text{-value} > \alpha$$

Make the conclusion: we don't reject the null hypothesis.

(a) A two-tail proportion test is appropriate.
(b) Hypotheses: H_0 claims $p = 0.606$ and H_A claims $p \neq 0.606$.
(c) The p-value is 0.2714
(d) We don't reject the null hypothesis.

6. **Problem:**

   A fair coin should have a 50% chance of landing on either side. Someone is mildly suspicious that a coin is unfair.

   You are asked to judge the fairness of the coin by flipping it 600 times and counting how many heads are flipped. You are told to use a significance level of 0.04.

   Then, you actually flip the coin 600 times and get 327 heads. Should we conclude this coin is unfair?

   (a) What kind of hypothesis test is appropriate?

   (b) State the hypotheses

   (c) Determine the *p*-value.

   (d) Decide whether we reject or retain the null hypothesis.

   (e) Do you think the coin is unfair?

Solution: We should use a two-tail proportion test.

State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.5$$
$$H_A \quad \text{claims} \quad p \neq 0.5$$

Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(1-0.5)}{600}} = 0.0204$$

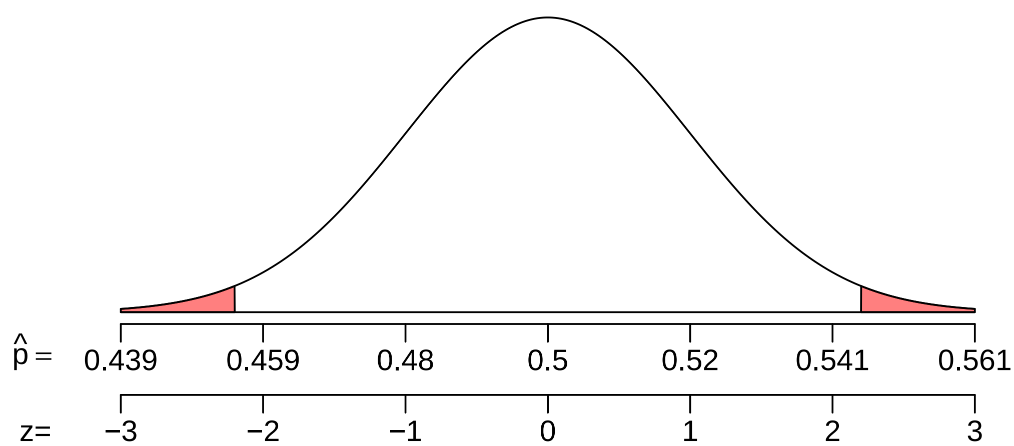
Determine the sample proportion.

$$\hat{p} = 0.545$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.545 - 0.5}{0.0204} = 2.2$$

Make a sketch of the null's sampling distribution. The p-value is a two-tail area.



To determine that two-tail area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(|Z| > 2.2) \\ &= 2 \cdot P(Z < -2.2) \\ &= 0.0278 \end{aligned}$$

Compare p-value to α (which is 0.04).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

We conclude the coin is unfair.

(a) Two-tail proportion test
(b) Hypotheses: H_0 claims $p = 0.5$ and H_A claims $p \neq 0.5$.
(c) The p-value is 0.0278
(d) We reject the null hypothesis.
(e) We conclude the coin is unfair.

7. **Problem:**
A null hypothesis claims a population has a mean μ = 220 and a standard deviation σ = 35. You decide to run one-tail test on a sample of size n = 83 using a significance level α = 0.1 to detect if the actual population mean is less than 220. You then collect the sample and find it has mean $\bar{x}$ = 214.35.

   (a) What kind of hypothesis test is appropriate?
   (b) State the hypotheses
   (c) Determine the p-value.
   (d) Decide whether we reject or retain the null hypothesis.

Solution: We should use a left-tail test of population mean.

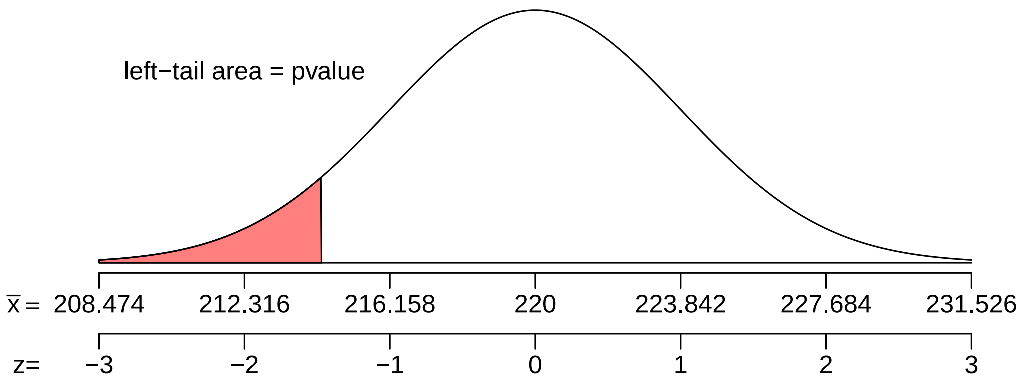
State the hypotheses:

$$H_0 \quad \text{claims} \quad \mu = 220$$
$$H_A \quad \text{claims} \quad \mu < 220$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{35}{\sqrt{83}} = 3.842$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{214.35 - 220}{3.842} = -1.47$$

Find the p-value (using formula for left-tail test of mean).

$$p\text{-value} = P(Z < -1.47)$$
$$= \boxed{0.0708}$$

Compare the p-value and the significance level ($\alpha = 0.1$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) Left-tail single mean test

(b) Hypotheses: H_0 claims $\mu = 220$ and H_A claims $\mu < 220$.

(c) p-value = 0.0708

(d) Yes, we reject the null hypothesis.

8. **Problem:**
A fair 20-sided die has a discrete uniform distribution with an expected value of $\mu = 10.5$ and a standard deviation $\sigma = 5.77$.

You are told to check if a 20-sided die has an expected value less than 10.5. You are told to roll the die 206 times and do a one-tail significance test with a significance level of 0.01.

You then roll the die 206 times and get a mean of 9.587. Should we conclude the die is unfair?

(a) What kind of hypothesis test is appropriate?
(b) State the hypotheses
(c) Determine the *p*-value.
(d) Decide whether we reject or retain the null hypothesis.
(e) Do we conclude the die is unfair?

Solution: We should use a left-tail test of population mean.

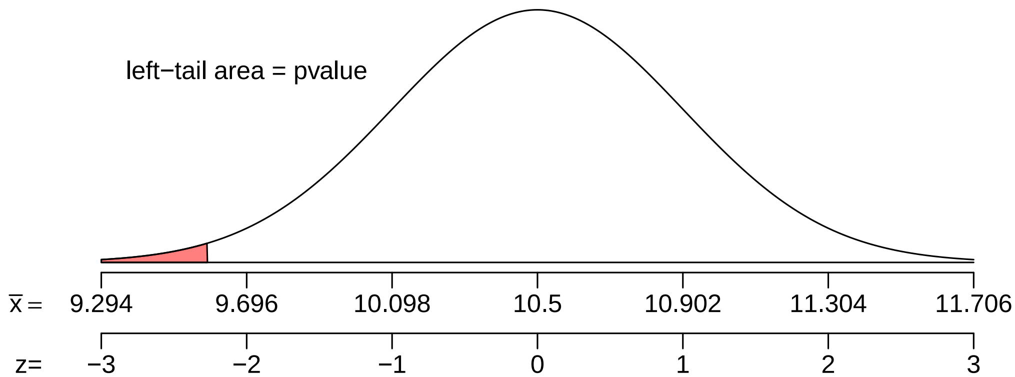
State the hypotheses:

$$H_0 \text{ claims } \mu = 10.5$$
$$H_A \text{ claims } \mu < 10.5$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5.77}{\sqrt{206}} = 0.402$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{9.587 - 10.5}{0.402} = -2.27$$

Find the p-value (using formula for left-tail test of mean).

$$p\text{-value} = P(Z < -2.27)$$
$$= \boxed{0.0116}$$

Compare the p-value and the significance level ($\alpha = 0.01$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

We conclude the die might be fair.

(a) Left-tail single mean test
(b) Hypotheses: H_0 claims $\mu = 10.5$ and H_A claims $\mu < 10.5$.
(c) p-value = 0.0116
(d) No, we do not reject the null hypothesis.
(e) We conclude the die might be fair.

9. **Problem:**
A null hypothesis claims a population has a mean $\mu = 180$ and a standard deviation $\sigma = 26$. You decide to run one-tail test on a sample of size $n = 294$ using a significance level $\alpha = 0.02$ to detect if the actual population mean is more than 180. You then collect the sample and find it has mean $\bar{x} = 183.03$.

   (a) What kind of hypothesis test is appropriate?
   (b) State the hypotheses
   (c) Determine the p-value.
   (d) Decide whether we reject or retain the null hypothesis.

Solution: We should use a right-tail test of population mean.

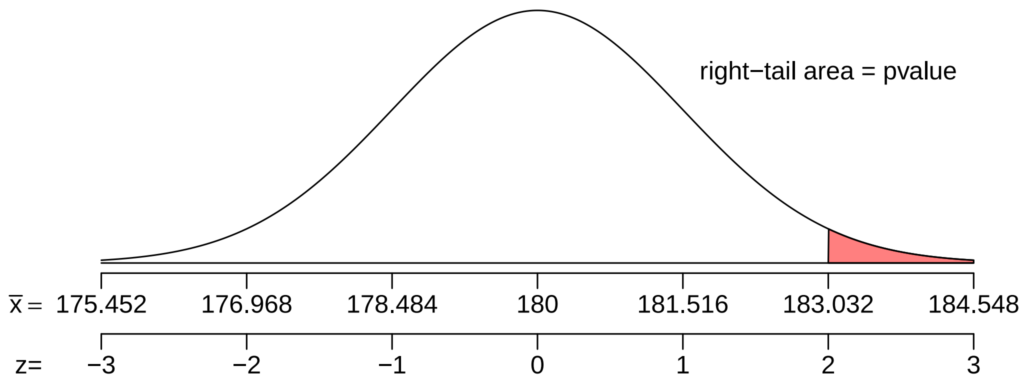
State the hypotheses:

$$H_0 \text{ claims } \mu = 180$$
$$H_A \text{ claims } \mu > 180$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{26}{\sqrt{294}} = 1.516$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{183.03 - 180}{1.516} = 2$$

Find the p-value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(Z > 2) \\ &= 1 - P(Z < 2) \\ &= 1 - 0.9772 \\ &= \boxed{0.0228} \end{aligned}$$

Compare the p-value and the significance level ($\alpha = 0.02$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) Right-tail single mean test
(b) Hypotheses: H_0 claims $\mu = 180$ and H_A claims $\mu < 180$.
(c) p-value = 0.0228
(d) No, we do not reject the null hypothesis.

10. **Problem:**
A fair 4-sided die has a discrete uniform distribution with an expected value of $\mu = 2.5$ and a standard deviation $\sigma = 1.12$.

You are told to check if a 4-sided die has an expected value higher than 2.5. You are told to roll the die 189 times and do a one-tail significance test with a significance level of 0.02.

You then roll the die 189 times and get a mean of 2.689. Should we conclude the die is unfair?

(a) What kind of hypothesis test is appropriate?
(b) State the hypotheses
(c) Determine the p-value.
(d) Decide whether we reject or retain the null hypothesis.
(e) Do we conclude the die is unfair?

Solution: We should use a right-tail test of population mean.

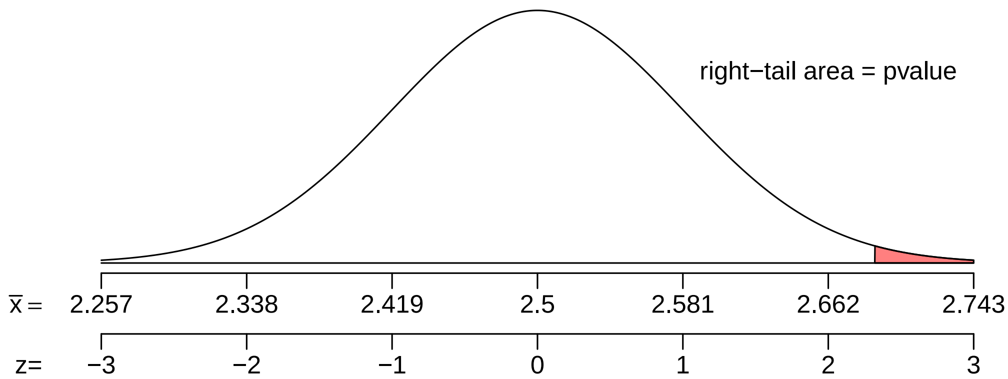
State the hypotheses:

$$H_0 \text{ claims } \mu = 2.5$$
$$H_A \text{ claims } \mu > 2.5$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.12}{\sqrt{189}} = 0.081$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{2.689 - 2.5}{0.081} = 2.32$$

Find the p-value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(Z > 2.32) \\ &= 1 - P(Z < 2.32) \\ &= 1 - 0.9898 \\ &= \boxed{0.0102} \end{aligned}$$

Compare the p-value and the significance level ($\alpha = 0.02$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

We conclude the die is unfair, with a higher than fair expected value.

(a) Right-tail single mean test
(b) Hypotheses: H_0 claims $\mu = 2.5$ and H_A claims $\mu > 2.5$.
(c) p-value = 0.0102
(d) Yes, we reject the null hypothesis.
(e) We conclude the die is unfair, with a higher than fair expected value.

11. **Problem:**
A null hypothesis claims a population has a mean $\mu = 180$ and a standard deviation $\sigma = 30$. You decide to run two-tail test on a sample of size $n = 278$ using a significance level $\alpha = 0.025$. You then collect the sample and find it has mean $\bar{x} = 183.72$.

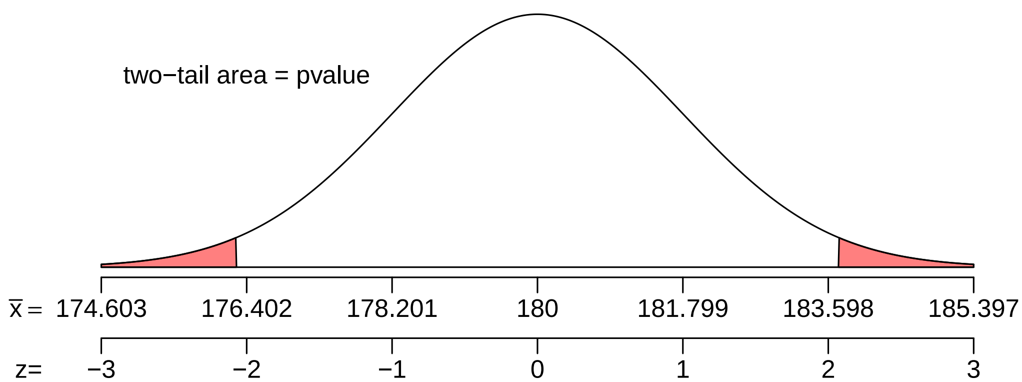
   (a) Determine the p-value.

   (b) Do you reject the null hypothesis?

Solution: Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{30}{\sqrt{278}} = 1.799$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{183.72 - 180}{1.799} = 2.07$$

Find the p-value.

$$\begin{aligned} p\text{-value} &= P(|Z| > 2.07) \\ &= 2 \cdot P(Z < -2.07) \\ &= \boxed{0.0384} \end{aligned}$$

Compare the p-value and the significance level.

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) p-value = 0.0384

(b) No, we do not reject the null hypothesis.

12. **Problem:**
A fair 12-sided die has a discrete uniform distribution with an expected value of $\mu = 6.5$ and a standard deviation $\sigma = 3.45$.

You are told to check if a 12-sided die has an expected value different than 6.5. You are told to roll the die 244 times and do a significance test with a significance level of 0.05.

You then roll the die 244 times and get a mean of 6.051. Should we conclude the die is unfair?

(a) What kind of hypothesis test is appropriate?
(b) State the hypotheses
(c) Determine the *p*-value.
(d) Decide whether we reject or retain the null hypothesis.
(e) Do we conclude the die is unfair?

Solution: We should use a two-tail test of population mean.

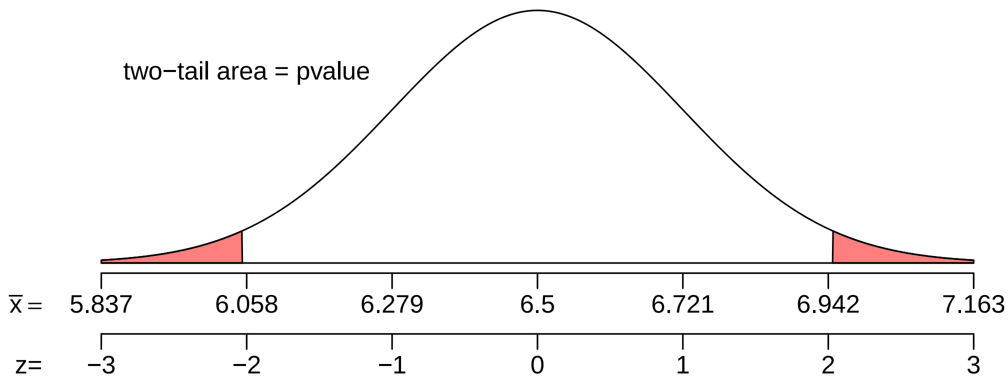
State the hypotheses:

$$H_0 \text{ claims } \mu = 6.5$$
$$H_A \text{ claims } \mu \neq 6.5$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.45}{\sqrt{244}} = 0.221$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{6.051 - 6.5}{0.221} = -2.03$$

Find the p-value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(|Z| > 2.03) \\ &= 2 \cdot P(Z < -2.03) \\ &= \boxed{0.0424} \end{aligned}$$

Compare the p-value and the significance level ($\alpha = 0.05$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

We conclude the die is unfair.

(a) Right-tail single mean test
(b) Hypotheses: H_0 claims $\mu = 6.5$ and H_A claims $\mu \neq 6.5$.
(c) p-value = 0.0424
(d) Yes, we reject the null hypothesis.
(e) We conclude the die is unfair.

13. **Problem:**
A null hypothesis claims a population has a mean μ = 140. You decide to run two-tail test on a sample of size n = 326 using a significance level α = 0.05. You then collect the sample and find it has mean $\bar{x}$ = 143.76 and standard deviation s = 31.47.

 (a) Determine the p-value.

 (b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad \mu = 140$$

$$H_A \quad \text{claims} \quad \mu \neq 140$$

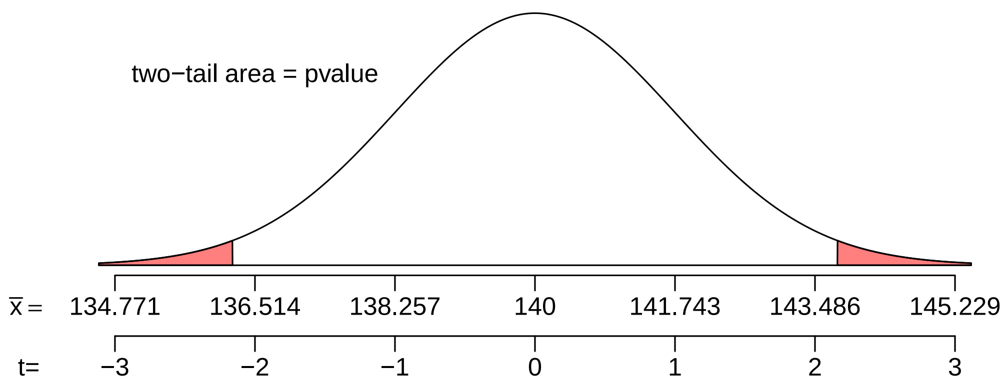
Determine the degrees of freedom.

$$df = 326 - 1 = 325$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{31.47}{\sqrt{326}} = 1.743$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{143.76 - 140}{1.743} = 2.16$$

Find the p-value.

$$p\text{-value} = P(|T| > 2.16)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 325$.)

$$P(|T| > 2.34) = 0.02$$

$$P(|T| > 2.06) = 0.04$$

Basically, because t is between 2.34 and 2.06, we know the p-value is between 0.02 and 0.04.

$$0.02 < p\text{-value} < 0.04$$

Compare the p-value and the significance level ($\alpha = 0.05$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) $0.02 < p$-value < 0.04

(b) Yes, we reject the null hypothesis.

14. **Problem:**
A null hypothesis claims a population has a mean $\mu = 240$. You decide to run two-tail test on a sample of size $n = 11$ using a significance level $\alpha = 0.05$.

You then collect the sample:

243.4	245	243	244.1	238.3
244	241.3	236.6	239.1	244.6
241				

(a) Determine the *p*-value.

(b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad \mu = 240$$
$$H_A \quad \text{claims} \quad \mu \neq 240$$

Find the mean and standard deviation of the sample.

$$\bar{x} = 241.855$$
$$s = 2.822$$

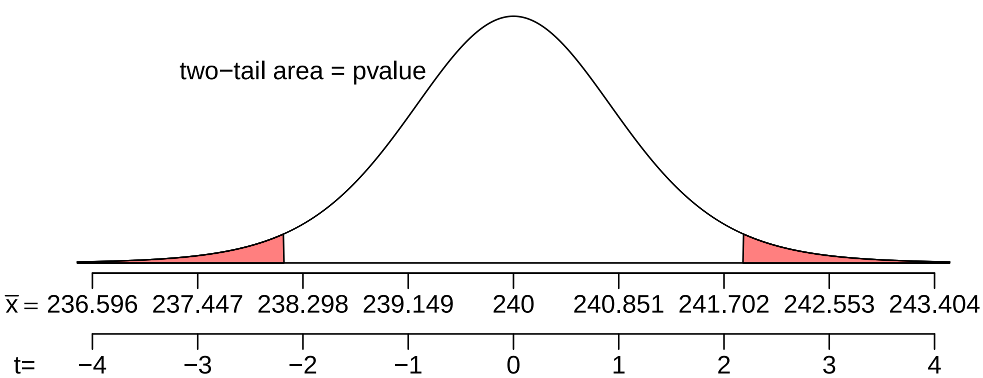
Determine the degrees of freedom.

$$df = 11 - 1 = 10$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{2.822}{\sqrt{11}} = 0.851$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{241.855 - 240}{0.851} = 2.18$$

Find the p-value.

$$p\text{-value} = P(|T| > 2.18)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 10$.)

$$P(|T| > 2.23) = 0.05$$
$$P(|T| > 1.81) = 0.1$$

Basically, because t is between 2.23 and 1.81, we know the p-value is between 0.05 and 0.1.

$$0.05 < p\text{-value} < 0.1$$

Compare the p-value and the significance level ($\alpha = 0.05$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) $0.05 < p\text{-value} < 0.1$

(b) No, we do not reject the null hypothesis.