Central Limit Theorem

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- X has mean $\mu_x = n \cdot \mu_w$ and standard deviation $\sigma_x = \sigma_w \sqrt{n}$.
- X is approximately normal, especially if n is "large".

$$X \sim \mathcal{N}(n\mu_w, \sigma_w\sqrt{n})$$

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- Y has mean $\mu_y = \mu_w$ and standard deviation $\sigma_y = \frac{\sigma_w}{\sqrt{n}}$.

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- Y has mean $\mu_y = \mu_w$ and standard deviation $\sigma_y = \frac{\sigma_w}{\sqrt{n}}$.
- Y is approximately normal, especially if n is "large".

$$Y \sim \mathcal{N}\left(\mu_w, \frac{\sigma_w}{\sqrt{n}}\right)$$

Example 1

- Let W be a random variable with the following probability distribution.

w	$P(w)$
26	0.52
27	0.43
29	0.05

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- Notice W has mean $\mu_w = 26.58$ and standard deviation $\sigma_w = 0.737$.
- Let X be the sum of 12 instances of W.

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- Let W be a random variable with the following probability distribution.

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29	0.05

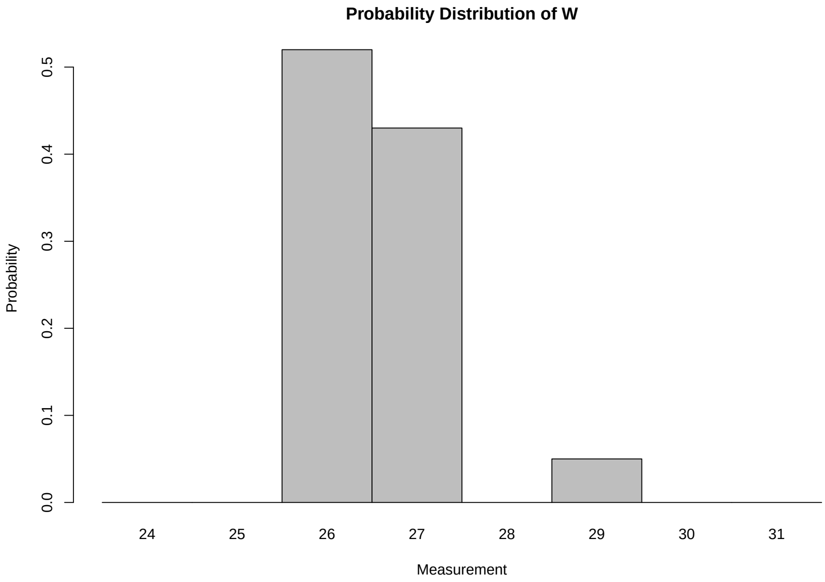
- Notice W has mean $\mu_w = 26.58$ and standard deviation $\sigma_w = 0.737$.
- Let X be the sum of 12 instances of W.
- We predict X is approximately normal, with mean and standard deviation from formulas.

$$\mu_x = n\mu_w = (12)(26.58) = 318.96$$

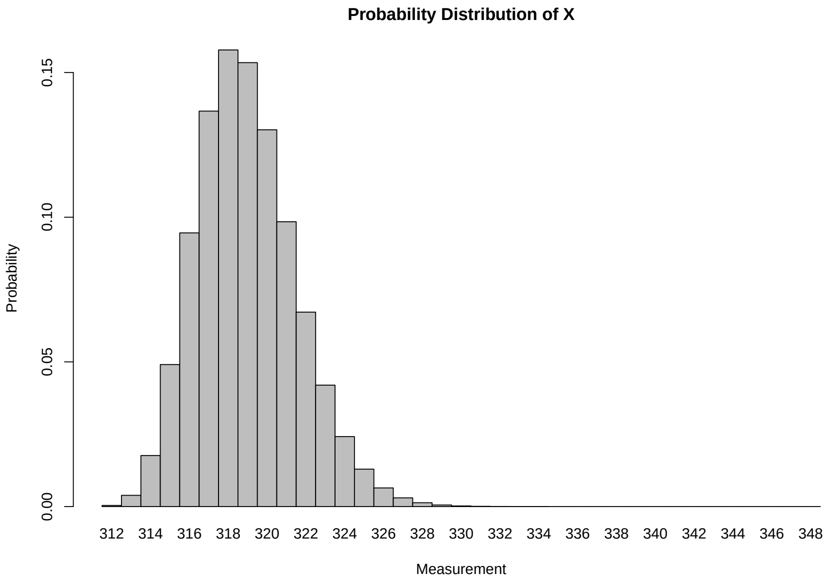
$$\sigma_w = \sigma_w\sqrt{n} = (0.737)(\sqrt{12}) = 2.55$$

$$X \sim \mathcal{N}(318.96,\, 2.55)$$

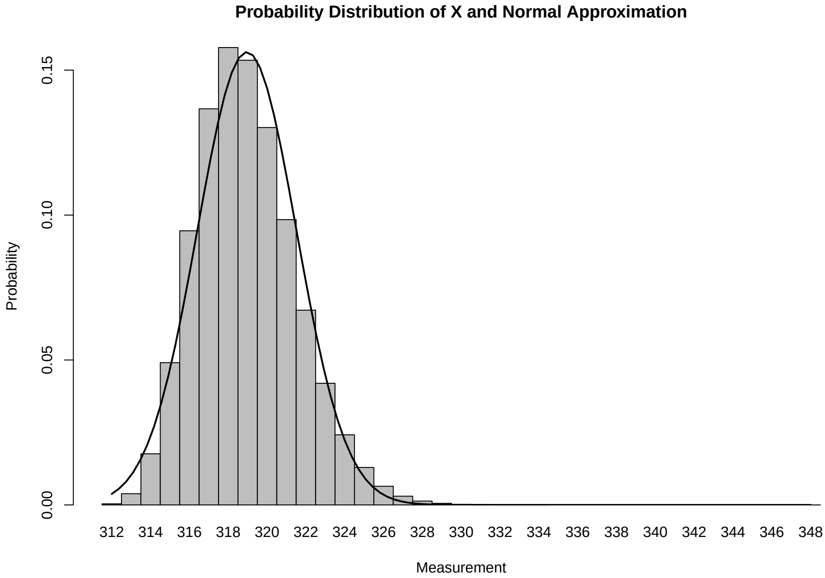
Example 1 continued...



Example 1 continued. . .



Example 1 continued...



Example 2: How to roll 100 dice

- Let random variable W represent a 6-sided die.

w	$P(w)$
1	0.1667
2	0.1667
3	0.1667
4	0.1667
5	0.1667
6	0.1667

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- Notice W has mean $\mu_w = 3.5$ and standard deviation $\sigma_w = 1.708$.

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- Let X be the sum of 100 instances of W.

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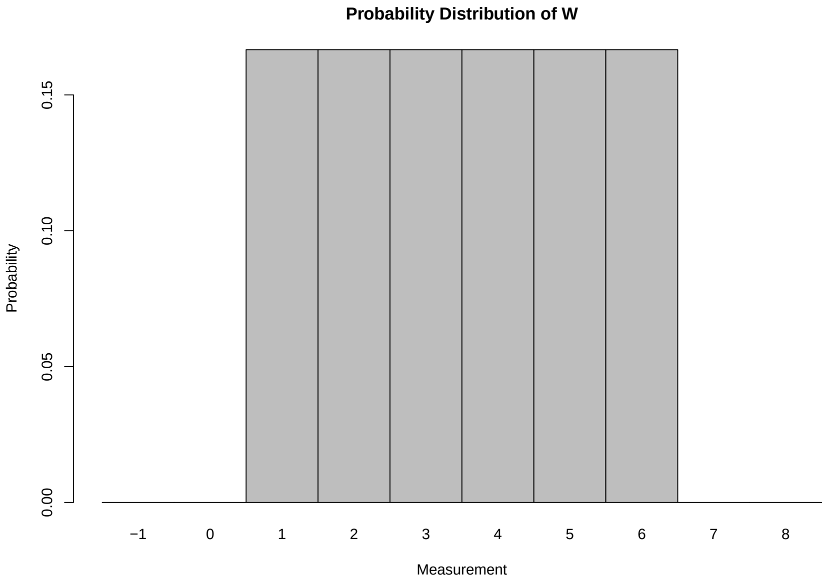
- Notice W has mean $\mu_w = 3.5$ and standard deviation $\sigma_w = 1.708$.
- Let X be the sum of 100 instances of W.
- We predict X is approximately normal, with mean and standard deviation from formulas.

$$\mu_x = n\mu_w = (100)(3.5) = 350$$

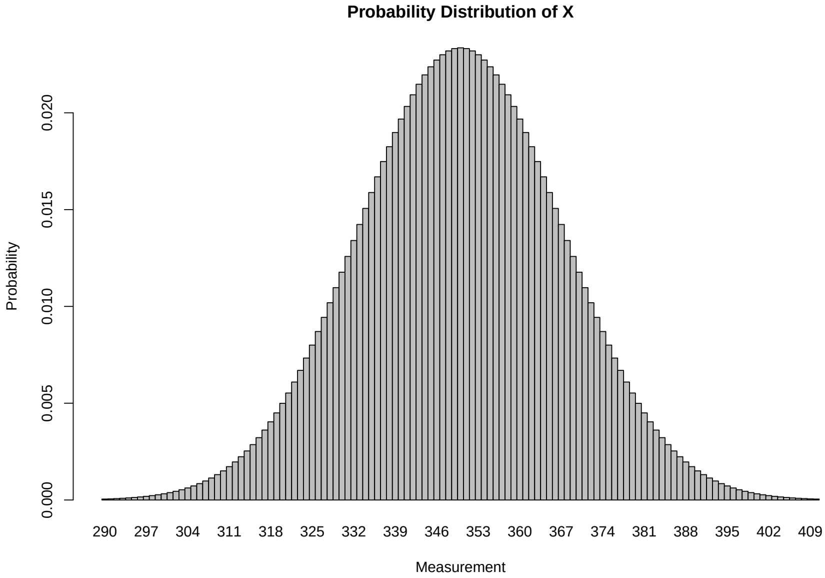
$$\sigma_w = \sigma_w\sqrt{n} = (1.708)(\sqrt{100}) = 17.08$$

$$X \sim \mathcal{N}(350, 17.08)$$

Probability distribution of standard 6-sided die



Probability distribution of **sum** of 100 6-sided dice



How to roll 1000 dice: spin this once