1. **Problem:**
   As an ornithologist, you wish to determine the average body mass of *Vireo griseus*. You randomly sample 27 adults of *Vireo griseus*, resulting in a sample mean of 10.11 grams and a sample standard deviation of 0.838 grams. Determine a 95% confidence interval of the true population mean.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 27 \\ \bar{x} &= 10.11 \\ s &= 0.838 \\ \gamma &= 0.95 \end{aligned}$$

Find the degrees of freedom.

$$\begin{aligned} df &= n - 1 \\ &= 27 - 1 \\ &= 26 \end{aligned}$$

Determine the critical t value, $t^\star$, such that $P(|T| < t^\star) = 0.95$ and $df = 26$.

$$t^\star = 2.06$$

Use the formula for bounds (mean, σ unknown).

$$\begin{aligned} LB &= \bar{x} - t^\star \frac{s}{\sqrt{n}} \\ &= 10.11 - 2.06 \times \frac{0.838}{\sqrt{27}} \\ &= 9.78 \end{aligned} \qquad \begin{aligned} UB &= \bar{x} + t^\star \frac{s}{\sqrt{n}} \\ &= 10.11 + 2.06 \times \frac{0.838}{\sqrt{27}} \\ &= 10.4 \end{aligned}$$

We are 95% confident that the population mean is between 9.78 and 10.4.

$$CI = (9.78,\ 10.4)$$

2. **Problem:**
   You are tasked with estimating the proportion of widgets that are defective. In a sample of 340 widgets, you determine that 45% were defective. Determine a 80% confidence interval of the population proportion.

Solution: Identify the givens.

$$n = 340$$
$$\hat{p} = 0.45$$
$$\gamma = 0.8$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.8$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.8+1}{2} = 0.9$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.28$$

Use the formula (proportion) for the bounds.

$$LB = \hat{p} - z^\star\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad UB = \hat{p} + z^\star\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.45 - 1.28\sqrt{\frac{(0.45)(0.55)}{340}} \qquad = 0.45 + 1.28\sqrt{\frac{(0.45)(0.55)}{340}}$$

$$= 0.415 \qquad = 0.485$$

Determine the interval.

$$CI = (0.415\,,\ 0.485)$$

We are 80% confident that the true population proportion is between 41.5% and 48.5%.

3. **Problem:**
   Some snails have clockwise shells, and others have counter-clockwise shells. You plan to estimate the proportion with clockwise shells by sampling. You want to be 98% confident that the sample proportion is within 0.007 of the population proportion. How many snails do you need?

Solution: We are given the confidence level and the margin of error.

$$\gamma = 0.98$$
$$ME = 0.007$$

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.98$. It is helpful to get the percentile of $z^\star$ by using $\ell = \frac{1+\gamma}{2} = \frac{1+0.98}{2} = 0.99$. This lets you find $z^\star$ such that $P(Z < z^\star) = 0.99$.

$$z^\star = 2.33$$

Use the appropriate formula.

$$n = \frac{1}{4}\left(\frac{z^\star}{ME}\right)^2$$

$$= \frac{1}{4}\left(\frac{2.33}{0.007}\right)^2$$

$$= 27698.4693878$$

When determining a necessary sample size, always round up (ceiling).

$$n = 27699$$

4. **Problem:**
A fair 10-sided die has a discrete uniform distribution with an expected value of $\mu = 5.5$ and a standard deviation $\sigma = 2.87$.

You are told to check if a 10-sided die has an expected value different than 5.5. You are told to roll the die 95 times and do a significance test with a significance level of 0.05.

You then roll the die 95 times and get a mean of 6.009. Should we conclude the die is unfair?

(a) What kind of hypothesis test is appropriate?
(b) State the hypotheses
(c) Determine the p-value.
(d) Decide whether we reject or retain the null hypothesis.
(e) Do we conclude the die is unfair?

Solution: We should use a two-tail test of population mean.

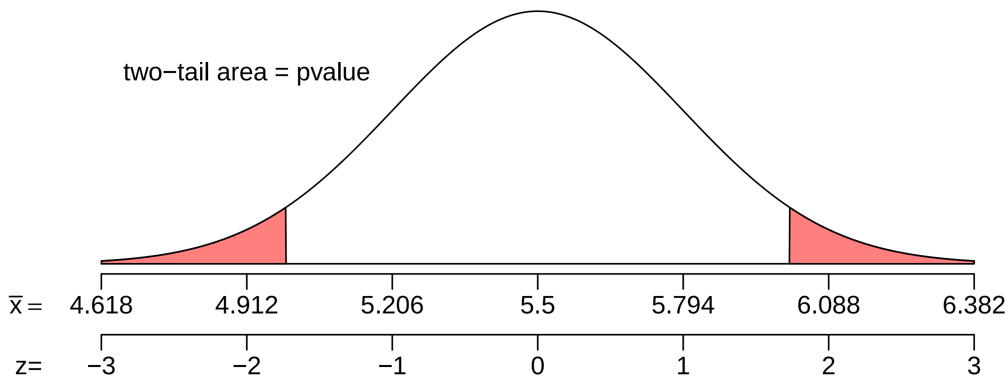
State the hypotheses:

$$H_0 \text{ claims } \mu = 5.5$$
$$H_A \text{ claims } \mu \neq 5.5$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.87}{\sqrt{95}} = 0.294$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{6.009 - 5.5}{0.294} = 1.73$$

Find the p-value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(|Z| > 1.73) \\ &= 2 \cdot P(Z < -1.73) \\ &= \boxed{0.0836} \end{aligned}$$

Compare the p-value and the significance level ($\alpha = 0.05$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

We conclude the die might be fair.

(a) Two-tail single mean test
(b) Hypotheses: H_0 claims $\mu = 5.5$ and H_A claims $\mu \neq 5.5$.
(c) p-value = 0.0836
(d) No, we do not reject the null hypothesis.
(e) We conclude the die might be fair.

5. **Problem:**
A student is taking a multiple choice test with 700 questions. Each question has 5 choices. You want to detect whether the student does better than random guessing, so you decide to run a hypothesis test with a significance level of 0.025.

Then, the student takes the test and gets 162 questions correct.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses

(c) Determine the p-value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Do we think the student did significantly better than random guessing?

Solution: This is a right-tail (one-tail) proportion test because we only care whether the student does better than random.

Determine the null population proportion.

$$p_0 = \frac{1}{5} = 0.2$$

State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.2$$
$$H_A \quad \text{claims} \quad p > 0.2$$

Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.2(1-0.2)}{700}} = 0.0151$$

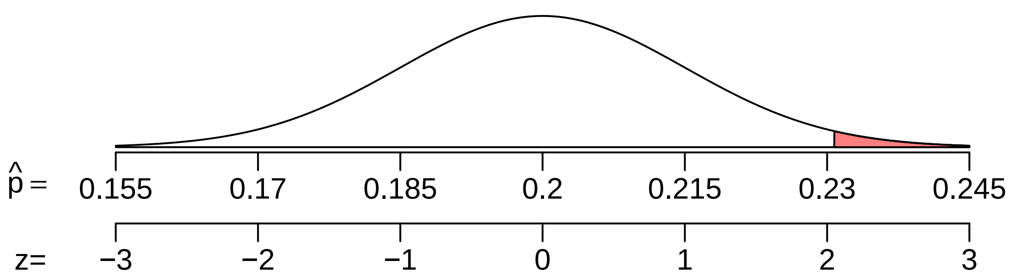
Determine the sample proportion.

$$\hat{p} = \frac{162}{700} = 0.231$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.231 - 0.2}{0.0151} = 2.05$$

Make a sketch of the null's sampling distribution. The p-value is a right area.



To determine that right area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.231) \\ &= P(Z > 2.05) \\ &= 1 - P(Z < 2.05) \\ &= 0.0202 \end{aligned}$$

Compare p-value to α (which is 0.025).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

We think the student did better than random guessing typically allows.

(a) Right tail (one-tail) proportion test
(b) Hypotheses: H_0 claims $p = 0.2$ and H_A claims $p > 0.2$.
(c) The p-value is 0.0202
(d) We reject the null hypothesis.
(e) We think the student did better than random guessing typically allows.

6. **Problem:**
A null hypothesis claims a population has a mean $\mu = 53.0$. You decide to run right-tail test on a sample of size $n = 9$ using a significance level $\alpha = 0.05$.

You then collect the sample:

55.6	51.2	53.8	53.9	54.5
64.1	51.5	54.1	60.9	

(a) State the hypotheses.

(b) Determine the p-value.

(c) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \text{ claims } \mu = 53$$
$$H_A \text{ claims } \mu > 53$$

Find the mean and standard deviation of the sample (use calculator function).

$$\bar{x} = 55.511$$
$$s = 4.272$$

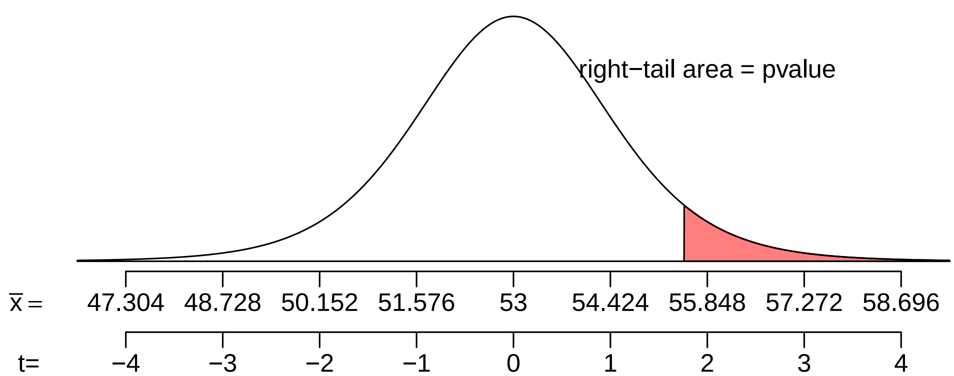
Determine the degrees of freedom.

$$df = 9 - 1 = 8$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{4.272}{\sqrt{9}} = 1.424$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{55.511 - 53}{1.424} = 1.76$$

Find the p-value.

$$p\text{-value} = P(T > 1.76)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 8$.)

$$P(T > 1.86) = 0.05$$
$$P(T > 1.4) = 0.1$$

Basically, because t is between 1.86 and 1.4, we know the p-value is between 0.05 and 0.1.

$$0.05 < p\text{-value} < 0.1$$

Compare the p-value and the significance level ($\alpha = 0.05$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) The hypotheses: H_0 claims $\mu = 53$ and H_A claims $\mu > 53$.

(b) $0.05 < p$-value < 0.1

(c) No, we do not reject the null hypothesis.

7. **Problem:**
A null hypothesis claims a population has a mean $\mu = 54.0$. You decide to run left-tail test on a sample of size $n = 11$ using a significance level $\alpha = 0.01$.

You then collect the sample:

63.4	30.6	48.6	45.5	40.5
42.4	56.7	52.7	50.8	28.9
39.7				

(a) State the hypotheses.

(b) Determine the p-value.

(c) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \text{ claims } \mu = 54$$
$$H_A \text{ claims } \mu < 54$$

Find the mean and standard deviation of the sample (use calculator function).

$$\bar{x} = 45.436$$

$$s = 10.494$$

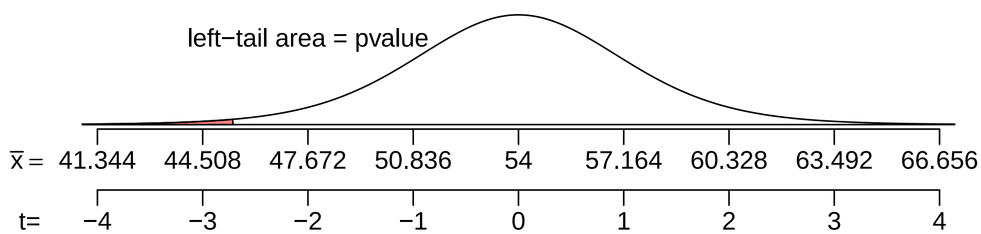
Determine the degrees of freedom.

$$df = 11 - 1 = 10$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{10.494}{\sqrt{11}} = 3.164$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{45.436 - 54}{3.164} = -2.71$$

Find the p-value.

$$p\text{-value} = P(T < -2.71)$$

The T distribution is symmetric.

$$p\text{-value} = P(T > 2.71)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 10$.)

$$P(T > 2.76) = 0.01$$

$$P(T > 2.36) = 0.02$$

Basically, because $|t|$ is between 2.76 and 2.36, we know the p-value is between 0.01 and 0.02.

$$0.01 < p\text{-value} < 0.02$$

Compare the p-value and the significance level ($\alpha = 0.01$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) The hypotheses: H_0 claims $\mu = 54$ and H_A claims $\mu < 54$.
(b) $0.01 < p$-value < 0.02
(c) No, we do not reject the null hypothesis.

8. **Problem:**
A null hypothesis claims a population has a mean $\mu = 140$. You decide to run two-tail test on a sample of size $n = 9$ using a significance level $\alpha = 0.1$.

You then collect the sample:

136.3	156.1	156.6	142.4	137.1
138	146	152.1	139	

(a) Determine the p-value.

(b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad \mu = 140$$
$$H_A \quad \text{claims} \quad \mu \neq 140$$

Find the mean and standard deviation of the sample.

$$\bar{x} = 144.844$$
$$s = 8.207$$

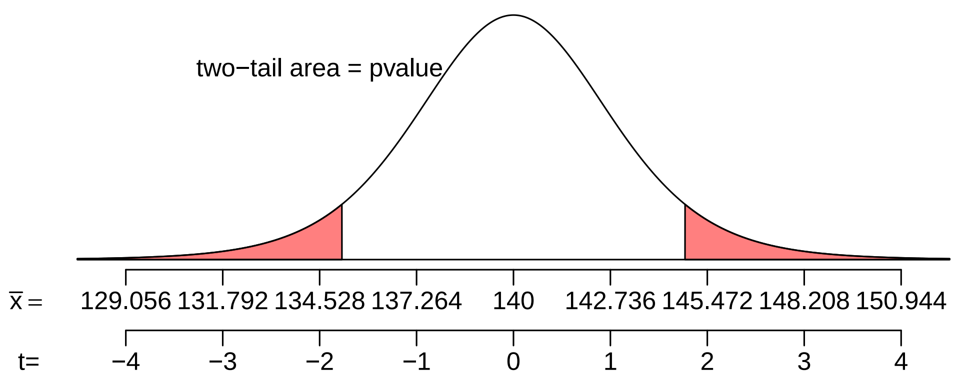
Determine the degrees of freedom.

$$df = 9 - 1 = 8$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{8.207}{\sqrt{9}} = 2.736$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{144.844 - 140}{2.736} = 1.77$$

Find the p-value.

$$p\text{-value} = P(|T| > 1.77)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 8$.)

$$P(|T| > 1.86) = 0.1$$
$$P(|T| > 1.4) = 0.2$$

Basically, because t is between 1.86 and 1.4, we know the p-value is between 0.1 and 0.2.

$$0.1 < p\text{-value} < 0.2$$

Compare the p-value and the significance level ($\alpha = 0.1$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) $0.1 < p$-value < 0.2

(b) No, we do not reject the null hypothesis.