

## Question

A continuous random variable  $X$  was measured 80 times. The sorted measurements are shown below.

72.346	73.394	73.645	74.609	74.668	75.151	75.174	75.205	75.476	75.507
75.539	75.617	75.871	75.895	76.111	76.173	76.382	76.389	76.391	76.450
76.503	76.664	76.687	76.689	76.852	76.942	77.055	77.161	77.219	77.319
77.336	77.423	77.604	77.687	77.722	77.731	77.879	77.891	78.017	78.072
78.099	78.101	78.133	78.139	78.248	78.285	78.325	78.367	78.421	78.424
78.427	78.451	78.457	78.532	78.668	78.687	78.710	78.724	78.724	78.791
78.816	78.855	78.880	79.053	79.096	79.187	79.198	79.282	79.314	79.364
79.382	79.473	79.551	79.640	79.704	79.731	79.792	79.833	79.853	79.967

Create a boxplot representing these measurements.

## Solution

We first determine the median of all measurements. We note that  $n = 80$ , so  $n$  is even. We use the appropriate method.

$$\begin{aligned}\text{median} &= \frac{x_{[\frac{n}{2}]} + x_{[\frac{n}{2}+1]}}{2} \\ &= \frac{x_{40} + x_{41}}{2} \\ &= \frac{78.072 + 78.099}{2} \\ &= 78.0855\end{aligned}$$

We now split the data into the lower group (all measurements with  $i \leq 40$ ) and the upper group (all measurements with  $i \geq 41$ ). We determine the medians of each group, and these medians represent the first quartile Q1 and the third quartile Q3.

$$Q1 = \frac{x_{20} + x_{21}}{2} = \frac{76.45 + 76.503}{2} = 76.4765$$

$$Q3 = \frac{x_{60} + x_{61}}{2} = \frac{78.791 + 78.816}{2} = 78.8035$$

We determine the interquartile range.

$$IQR = Q3 - Q1 = 78.8035 - 76.4765 = 2.327$$

We determine the lowest and highest possible reach of the whiskers.

$$\text{lowest reach} = Q1 - 1.5 \times IQR = 76.4765 - 1.5 \times 2.327 = 72.986$$

$$\text{highest reach} = Q3 + 1.5 \times IQR = 78.8035 + 1.5 \times 2.327 = 82.294$$

Any measurements outside the interval  $(72.986, 82.294)$  will be considered outliers and marked as individual dots. In this case, our outliers:

$$\text{outliers} = \{72.346\}$$

The whiskers then extend to the smallest and largest measurements within the interval  $(72.986, 82.294)$ . So, the whiskers extend from 73.394 to 79.967.

