

1. Problem

A continuous random variable was measured 400 times. The resulting frequency distribution is shown below.

class	frequency
56.5–57	91
57–57.5	35
57.5–58	23
58–58.5	25
58.5–59	18
59–59.5	15
59.5–60	18
60–60.5	29
60.5–61	32
61–61.5	56
61.5–62	58

- (a) Describe the overall shape of the distribution. (symmetric mound, skew left, skew right, uniform, or bimodal)
- (b) Estimate the range of the distribution (range = max-min).
- (c) What percent of the measurements are greater than 59?
- (d) What percent of the measurements are greater than 61?
- (e) What percent of the measurements are between 59 and 61?
- (f) What percent of the measurements are within 1 of 59.5? In other words, what percent of measurements satisfy $|x - 59.5| \leq 1$?
- (g) Of the measurements greater than 59, what percent are greater than 61?
- (h) Estimate the value of the 51.75th percentile. In other words, determine a value such that 51.75% of the measurements are less than or equal to it.

2. Problem

Please make a frequency table and a dot plot from the following (unsorted) data.

59	56	57	61	60
61	60	57	58	62
59	55	61	62	60
62	58	60	56	62

3. Problem

From a very large population, a small sample of measurements was taken.

119	119	122	120	124	122
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Please calculate the (Bessel corrected) sample standard deviation using the following formula:

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

4. Problem

From a very large population, a small sample of measurements was taken.

52	46	60	50	47
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Please calculate the average absolute deviation using the following formula:

$$AAD = \frac{\sum |x - \bar{x}|}{n}$$

5. Problem

A continuous random variable X was measured 22 times. The sorted measurements are shown below.

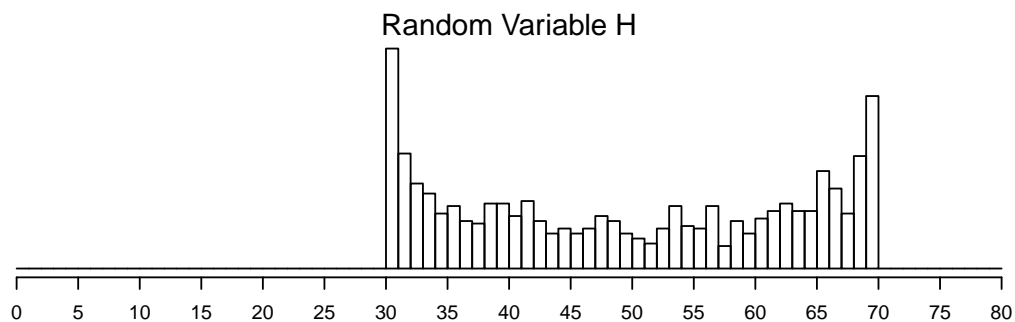
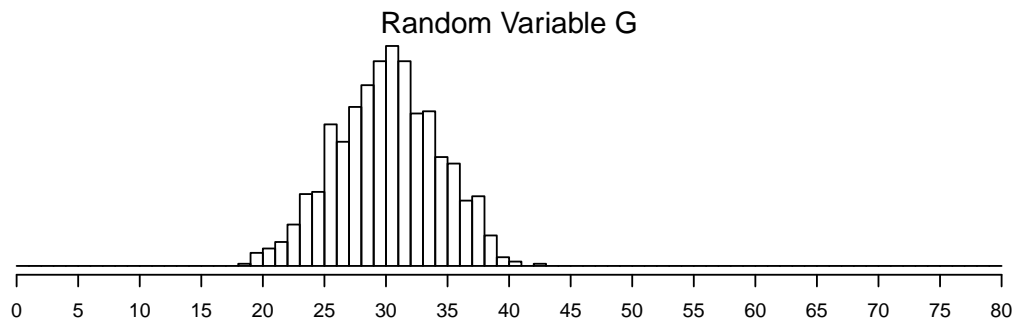
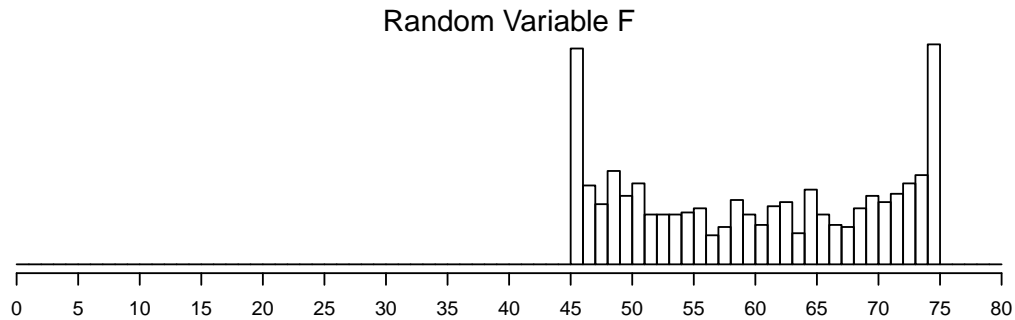
21.43	22.03	22.37	22.51	22.68	22.91	23.19	23.49	23.71	23.98
24.16	24.28	24.4	24.4	24.54	24.65	24.65	24.71	24.8	24.82
24.85	24.94								

The total of the measurements is 523.5.

- (a) Determine the percentile rank of the measurement 24.65. In other words, determine what percent of data are less than or equal to 24.65.
- (b) Determine the measurement corresponding to a percentile rank of 0.136. In other words, determine x such that 13.6% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

6. Problem

Three random variables (F, G, and H) were measured 1000 times each. The resulting histograms show the three distributions.

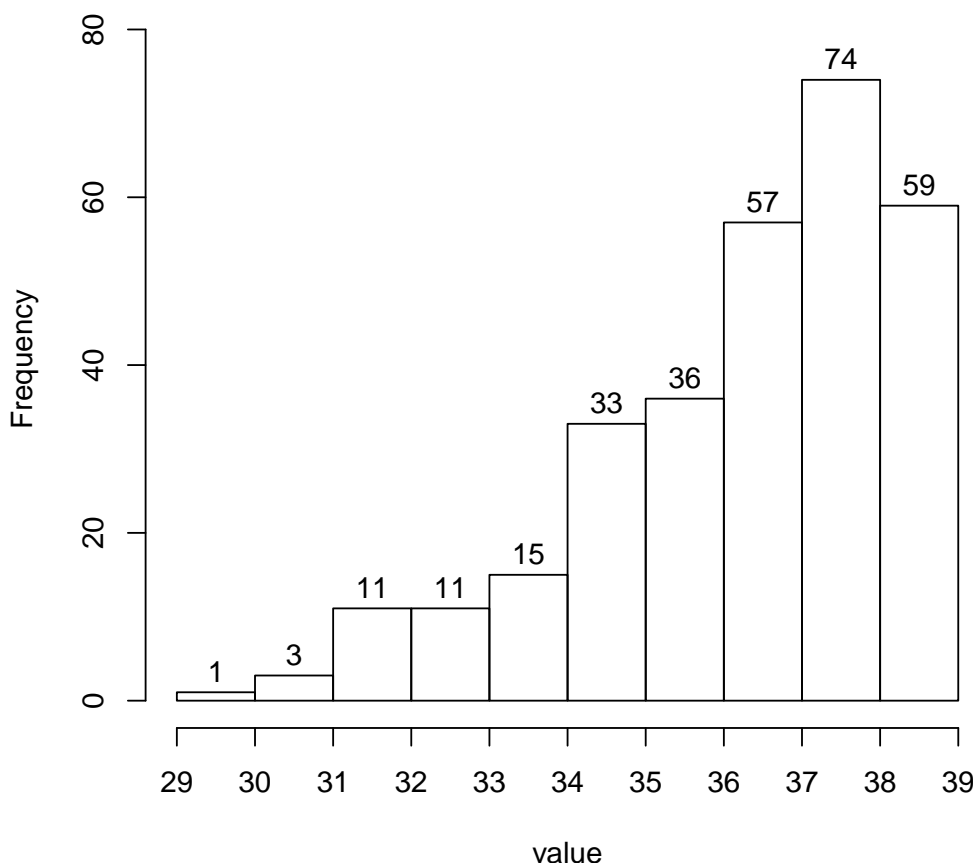


- (a) Which distribution has the highest mean? (F, G, or H)
- (b) Which distribution has the lowest mean? (F, G, or H)
- (c) Which distribution has the largest standard deviation? (F, G, or H)
- (d) Which distribution has the smallest standard deviation? (F, G, or H)

7. Problem

A continuous random variable was measured 300 times. The resulting histogram is shown below.

Histogram of data



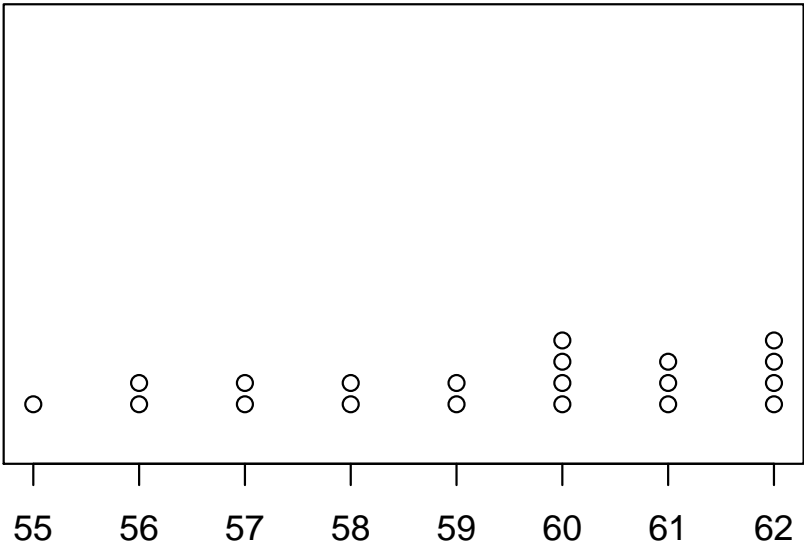
- (a) Describe the overall shape of the distribution. (symmetric mound, skew left, skew right, uniform, or bimodal)
- (b) Estimate the range of the distribution (range = max-min).
- (c) What percent of the measurements are greater than 31?
- (d) What percent of the measurements are greater than 34?
- (e) What percent of the measurements are between 31 and 34?
- (f) What percent of the measurements are within 1 from 34? In other words, what percent of measurements satisfy $|x - 34| \leq 1$?
- (g) Of the measurements greater than 31, what percent are greater than 34?
- (h) Estimate the value of the 5th percentile. In other words, determine a value such that 5% of the measurements are less than or equal to it.

1. (a) bimodal
(b) 5.5
(c) 52%
(d) 28.5%
(e) 23.5%
(f) 20%
(g) 54.81%
(h) 59.5

2. Make a frequency table.

value	frequency
55	1
56	2
57	2
58	2
59	2
60	4
61	3
62	4

Make the dot plot.



3. We fill out the table column by column.

x	$x - \bar{x}$	$(x - \bar{x})^2$
119	-2	4
119	-2	4
122	1	1
120	-1	1
124	3	9
122	1	1
=====	=====	=====
$\sum x = 726$		$\sum (x - \bar{x})^2 = 20$
$\bar{x} = 121$		

We are ready for the formula.

$$\begin{aligned}
 s &= \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} \\
 &= \sqrt{\frac{20}{6 - 1}} \\
 &= \sqrt{4} \\
 &= \boxed{2}
 \end{aligned}$$

4. We fill out the table column by column.

x	$x - \bar{x}$	$ x - \bar{x} $
52	1	1
46	-5	5
60	9	9
50	-1	1
47	-4	4
<hr/>		
=====	=====	=====
$\sum x = 255$		$\sum x - \bar{x} = 20$
$\bar{x} = 51$		

We are ready for the formula.

$$\begin{aligned}
 s &= \frac{\sum |x - \bar{x}|}{n} \\
 &= \frac{20}{5} \\
 &= \boxed{4}
 \end{aligned}$$

5. Let x represent a measurement of interest. Let i represent that measurement's index. Let ℓ represent that measurement's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

(a) We are given $x = 24.65$. This means $i = 17$. We know $n = 22$. Determine the percentile ℓ .

$$\ell = \frac{17}{22}$$

$$\ell = 0.773$$

So, the percentile rank is $\boxed{0.773}$, or 77.3th percentile.

(b) We are given $\ell = 0.136$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (22)(0.136)$$

$$i = 3$$

Determine the x associated with $i = 3$.

$$x = \boxed{22.37}$$

(c) The mean: $\bar{x} = \frac{523.5}{22} = \boxed{23.795}$

(d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 22$ and so n is even.

$$\text{median} = \frac{x_{11} + x_{12}}{2} = \frac{24.16 + 24.28}{2}$$

So, median = $\boxed{24.22}$.

6. (a) F
(b) G
(c) H
(d) G
7. (a) skew left
(b) 10
(c) 98.67%
(d) 86.33%
(e) 12.33%
(f) 16%
(g) 87.5%
(h) 32