1. **Problem**
   A student is taking a multiple choice test with 600 questions. Each question has 4 choices. You want to detect whether the student does better than random guessing, so you decide to run a hypothesis test with a significance level of 0.1.

   Then, the student takes the test and gets 161 questions correct.

   (a) What kind of hypothesis test is appropriate?
   (b) State the hypotheses
   (c) Determine the p-value.
   (d) Decide whether we reject or retain the null hypothesis.
   (e) Do we think the student did better than random guessing?

2. **Problem**
   A student is taking a multiple choice test with 300 questions. Each question has 3 choices. You want to detect whether the student does better than random guessing, so you decide to run a hypothesis test with a significance level of 0.01.

   Then, the student takes the test and gets 121 questions correct.

   (a) What kind of hypothesis test is appropriate?
   (b) State the hypotheses
   (c) Determine the p-value.
   (d) Decide whether we reject or retain the null hypothesis.
   (e) Do we think the student did better than random guessing?

3. **Problem**

   A null hypothesis claims a population has a mean $\mu = 220$. You decide to run two-tail test on a sample of size $n = 451$ using a significance level $\alpha = 0.02$. You then collect the sample and find it has mean $\bar{x} = 227.36$ and standard deviation $s = 63.31$.

   (a) Determine the p-value.

   (b) Do you reject the null hypothesis?

4. **Problem**
   A null hypothesis claims a population has a mean $\mu = 150$. You decide to run two-tail test on a sample of size $n = 45$ using a significance level $\alpha = 0.02$. You then collect the sample and find it has mean $\bar{x} = 163.23$ and standard deviation $s = 37.77$.

   (a) Determine the p-value.

   (b) Do you reject the null hypothesis?

5. **Problem**
A null hypothesis claims a population has a mean $\mu = 210$. You decide to run two-tail test on a sample of size $n = 9$ using a significance level $\alpha = 0.02$.

You then collect the sample:

306.7	257.7	232.3	226.6	268.5
223.4	183	239	262.1	

(a) Determine the p-value.

(b) Do you reject the null hypothesis?

6. **Problem**
   A null hypothesis claims a population has a mean $\mu = 190$. You decide to run two-tail test on a sample of size $n = 10$ using a significance level $\alpha = 0.02$.

   You then collect the sample:

187.9	195.6	194.5	204.8	208.4
200.1	188.9	186.7	202.8	198.6

   (a) Determine the p-value.

   (b) Do you reject the null hypothesis?

1. This is a right-tail (one-tail) proportion test because we only care whether the student does better than random.

   Determine the null population proportion.

   $$p_0 = \frac{1}{4} = 0.25$$

   State the hypotheses.

   $$H_0 \quad \text{claims} \quad p = 0.25$$
   $$H_A \quad \text{claims} \quad p > 0.25$$

   Determine the standard error.

   $$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.25(1-0.25)}{600}} = 0.0177$$

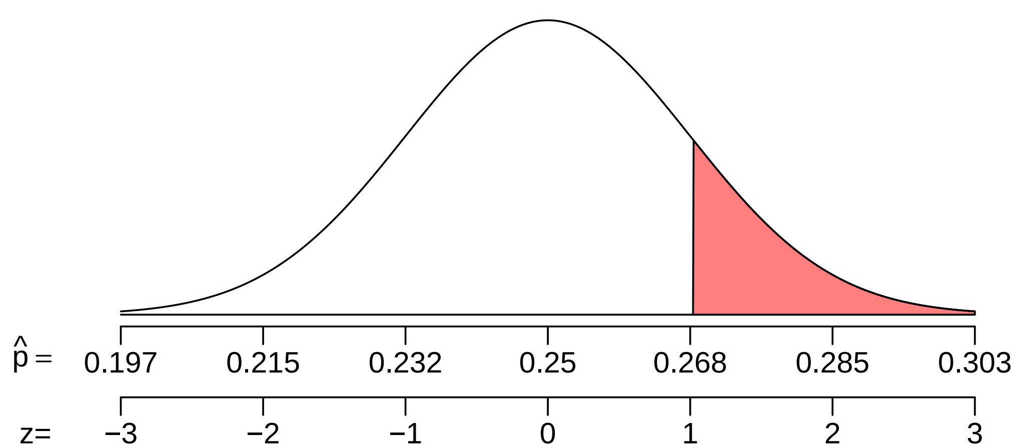
   Determine the sample proportion.

   $$\hat{p} = \frac{161}{600} = 0.268$$

   Determine a z score. For simplicity, we ignore the continuity correction.

   $$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.268 - 0.25}{0.0177} = 1.02$$

   The p-value is a right area.



   To determine that right area, we use the z table.

   $$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.268) \\ &= P(Z > 1.02) \\ &= 1 - P(Z < 1.02) \\ &= 0.1539 \end{aligned}$$

Compare *p*-value to α (which is 0.1).

$$p\text{-value} > \alpha$$

Make the conclusion: we retain the null hypothesis.

We think the student might just be guessing.

(a) Right tail (one-tail) proportion test

(b) Hypotheses: H_0 claims $p = 0.25$ and H_A claims $p > 0.25$.

(c) The *p*-value is 0.1539

(d) We retain the null hypothesis.

(e) We think the student might just be guessing.

2. This is a right-tail (one-tail) proportion test because we only care whether the student does better than random.

   Determine the null population proportion.

$$p_0 = \frac{1}{3} = 0.333$$

   State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.333$$
$$H_A \quad \text{claims} \quad p > 0.333$$

   Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.333(1-0.333)}{300}} = 0.0272$$

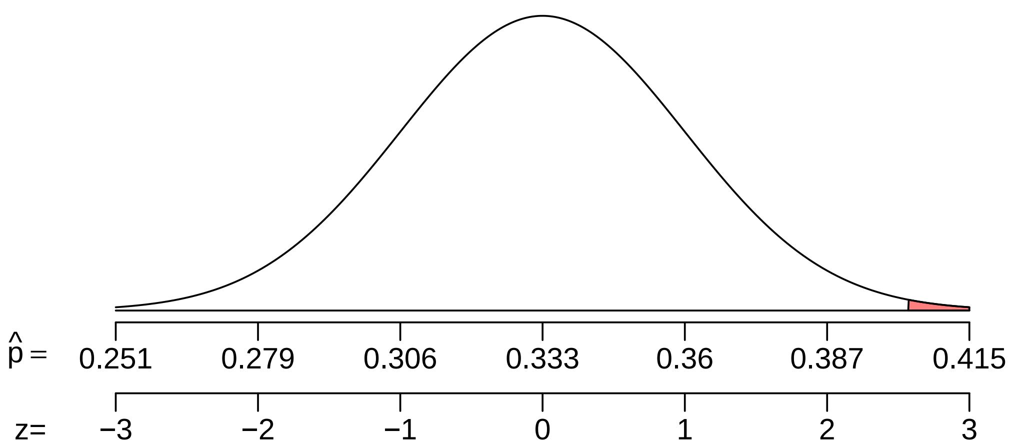
   Determine the sample proportion.

$$\hat{p} = \frac{121}{300} = 0.403$$

   Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.403 - 0.333}{0.0272} = 2.57$$

   The p-value is a right area.



   To determine that right area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.403) \\ &= P(Z > 2.57) \\ &= 1 - P(Z < 2.57) \\ &= 0.0051 \end{aligned}$$

Compare p-value to α (which is 0.01).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

We think the student did better than random guessing typically allows.

(a) Right tail (one-tail) proportion test

(b) Hypotheses: H_0 claims $p = 0.333$ and H_A claims $p > 0.333$.

(c) The p-value is 0.0051

(d) We reject the null hypothesis.

(e) We think the student did better than random guessing typically allows.

3. State the hypotheses.

$$H_0 \text{ claims } \mu = 220$$
$$H_A \text{ claims } \mu \neq 220$$

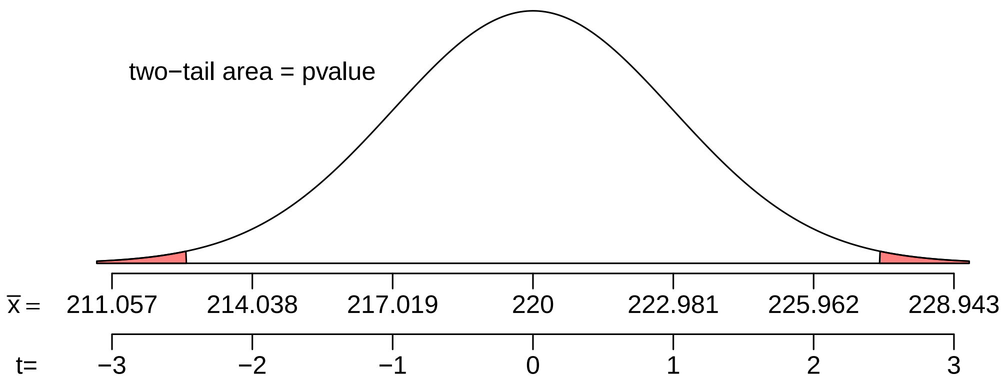
Determine the degrees of freedom.

$$df = 451 - 1 = 450$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{63.31}{\sqrt{451}} = 2.981$$

Make a sketch.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{227.36 - 220}{2.981} = 2.47$$

Find the p-value.

$$p\text{-value} = P(|T| > 2.47)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with df = 450.)

$$P(|T| > 2.59) = 0.01$$

$$P(|T| > 2.33) = 0.02$$

Basically, because t is between 2.59 and 2.33, we know the p-value is between 0.01 and 0.02.

$$0.01 < p\text{-value} < 0.02$$

Compare the p-value and the significance level ($\alpha = 0.02$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) $0.01 < p$-value < 0.02

(b) Yes, we reject the null hypothesis.

4. State the hypotheses.

$$H_0 \text{ claims } \mu = 150$$
$$H_A \text{ claims } \mu \neq 150$$

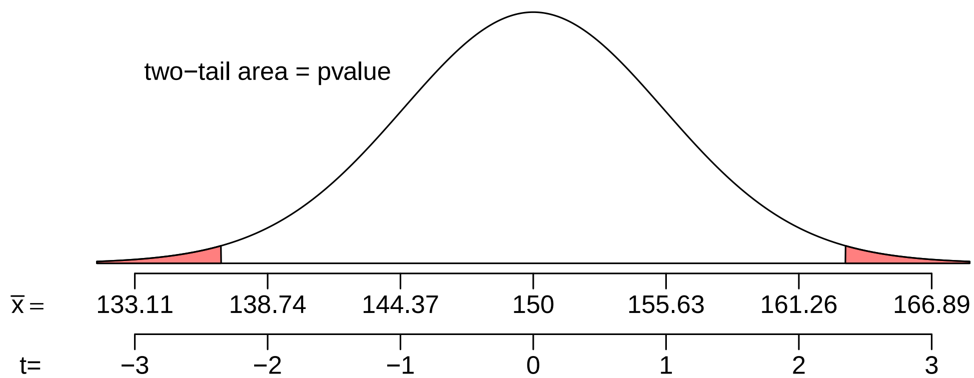
Determine the degrees of freedom.

$$df = 45 - 1 = 44$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{37.77}{\sqrt{45}} = 5.63$$

Make a sketch.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{163.23 - 150}{5.63} = 2.35$$

Find the p-value.

$$p\text{-value} = P(|T| > 2.35)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 44$.)

$$P(|T| > 2.41) = 0.02$$

$$P(|T| > 2.12) = 0.04$$

Basically, because t is between 2.41 and 2.12, we know the p-value is between 0.02 and 0.04.

$$0.02 < p\text{-value} < 0.04$$

Compare the p-value and the significance level ($\alpha = 0.02$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) $0.02 < p$-value < 0.04

(b) No, we do not reject the null hypothesis.

5. State the hypotheses.

$$H_0 \quad \text{claims} \quad \mu = 210$$
$$H_A \quad \text{claims} \quad \mu \neq 210$$

Find the mean and standard deviation of the sample.

$$\bar{x} = 244.367$$
$$s = 34.741$$

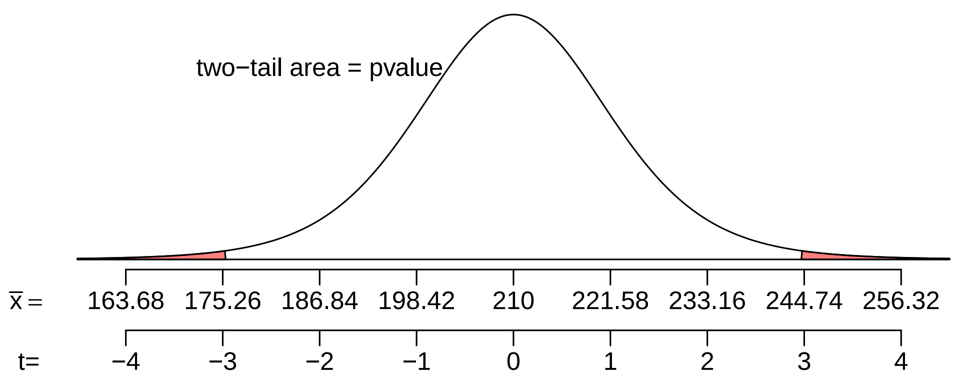
Determine the degrees of freedom.

$$df = 9 - 1 = 8$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{34.741}{\sqrt{9}} = 11.58$$

Make a sketch.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{244.367 - 210}{11.58} = 2.97$$

Find the p-value.

$$p\text{-value} = P(|T| > 2.97)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 8$.)

$$P(|T| > 3.36) = 0.01$$
$$P(|T| > 2.9) = 0.02$$

Basically, because t is between 3.36 and 2.9, we know the p-value is between 0.01 and 0.02.

$$0.01 < p\text{-value} < 0.02$$

Compare the p-value and the significance level ($\alpha = 0.02$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) $0.01 < p\text{-value} < 0.02$

(b) Yes, we reject the null hypothesis.

6. State the hypotheses.

$$H_0 \text{ claims } \mu = 190$$
$$H_A \text{ claims } \mu \neq 190$$

Find the mean and standard deviation of the sample.

$$\bar{x} = 196.83$$
$$s = 7.446$$

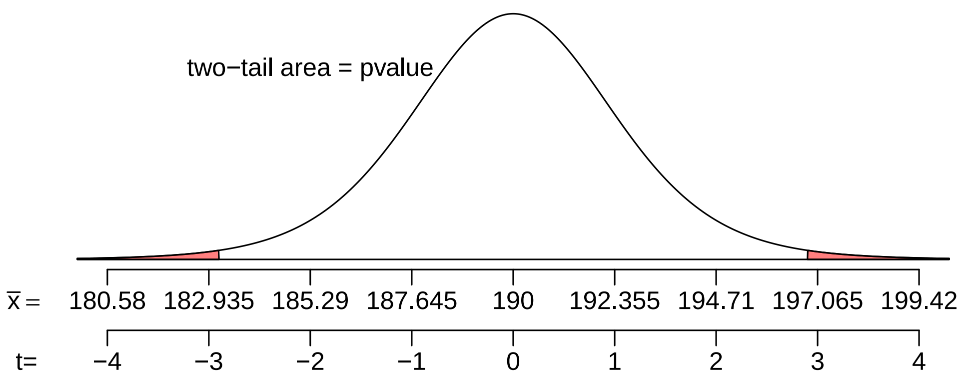
Determine the degrees of freedom.

$$df = 10 - 1 = 9$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{7.446}{\sqrt{10}} = 2.355$$

Make a sketch.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{196.83 - 190}{2.355} = 2.9$$

Find the p-value.

$$p\text{-value} = P(|T| > 2.9)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 9$.)

$$P(|T| > 3.25) = 0.01$$
$$P(|T| > 2.82) = 0.02$$

Basically, because t is between 3.25 and 2.82, we know the p-value is between 0.01 and 0.02.

$$0.01 < p\text{-value} < 0.02$$

Compare the p-value and the significance level ($\alpha = 0.02$).

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) $0.01 < p\text{-value} < 0.02$

(b) Yes, we reject the null hypothesis.