1. **Problem:**
   It is generally accepted that a population's proportion is 0.523. However, you think that maybe the population proportion is less than 0.523, so you decide to run a one-tail hypothesis test with a significance level of 0.025 with a sample size of 600.

   Then, when you collect the random sample, you find its proportion is 0.482. Do you reject or retain the null hypothesis?

   (a) Determine the *p*-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.523$$

$$H_A \quad \text{claims} \quad p < 0.523$$

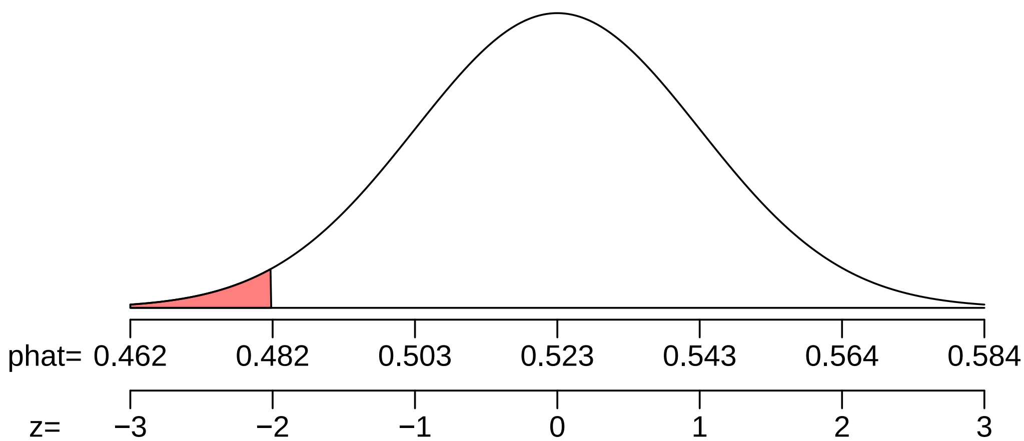
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.523(1-0.523)}{600}} = 0.0204$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.482 - 0.523}{0.0204} = -2.01$$

The p-value is a left area.



To determine that left area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} < 0.482) \\ &= P(Z < -2.01) \\ &= 0.0222 \end{aligned}$$

Compare p-value to α (which is 0.025).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) The p-value is 0.0222

(b) We reject the null hypothesis.

2. **Problem:**
   It is generally accepted that a population's proportion is 0.828. However, you think that maybe the population proportion is less than 0.828, so you decide to run a one-tail hypothesis test with a significance level of 0.05 with a sample size of 500.

   Then, when you collect the random sample, you find its proportion is 0.796. Do you reject or retain the null hypothesis?

   (a) Determine the p-value.
   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.828$$

$$H_A \quad \text{claims} \quad p < 0.828$$

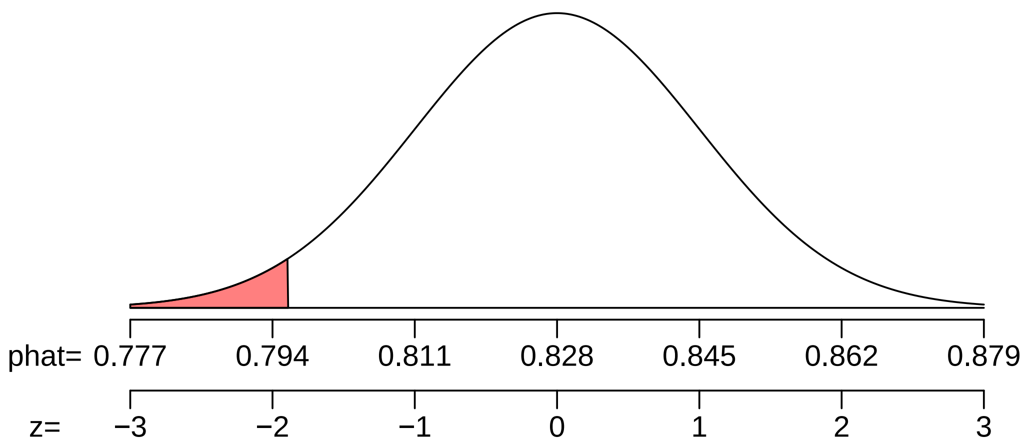
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.828(1-0.828)}{500}} = 0.0169$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p}-p_0}{\sigma_{\hat{p}}} = \frac{0.796-0.828}{0.0169} = -1.89$$

The p-value is a left area.



To determine that left area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} < 0.796) \\ &= P(Z < -1.89) \\ &= 0.0294 \end{aligned}$$

Compare p-value to α (which is 0.05).

$$p\text{-value} < \alpha$$

Make the conclusion: we reject the null hypothesis.

(a) The p-value is 0.0294

(b) We reject the null hypothesis.

3. **Problem:**
   It is generally accepted that a population's proportion is 0.83. However, you think that maybe the population proportion is more than 0.83, so you decide to run a one-tail hypothesis test with a significance level of 0.01 with a sample size of 3000.

   Then, when you collect the random sample, you find its proportion is 0.815. Do you reject or retain the null hypothesis?

   (a) Determine the *p*-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.83$$

$$H_A \quad \text{claims} \quad p > 0.83$$

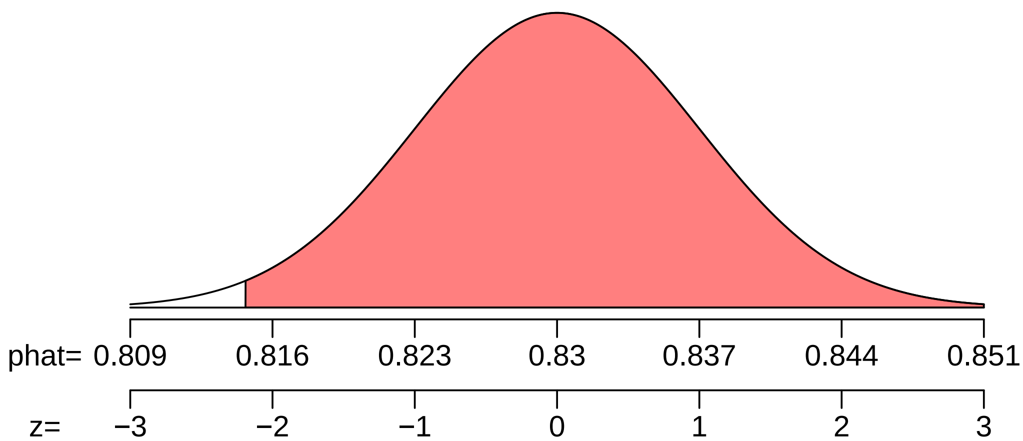
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.83(1-0.83)}{3000}} = 0.00686$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.815 - 0.83}{0.00686} = -2.19$$

The p-value is a right area.



To determine that right area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.815) \\ &= P(Z > -2.19) \\ &= 1 - P(Z < -2.19) \\ &= 0.9857 \end{aligned}$$

Compare p-value to α (which is 0.01).

$$p\text{-value} > \alpha$$

Make the conclusion: we retain the null hypothesis.

(a) The p-value is 0.9857

(b) We retain the null hypothesis.

4. **Problem:**
   It is generally accepted that a population's proportion is 0.472. However, you think that maybe the population proportion is more than 0.472, so you decide to run a one-tail hypothesis test with a significance level of 0.02 with a sample size of 700.

   Then, when you collect the random sample, you find its proportion is 0.508. Do you reject or retain the null hypothesis?

   (a) Determine the *p*-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.472$$

$$H_A \quad \text{claims} \quad p > 0.472$$

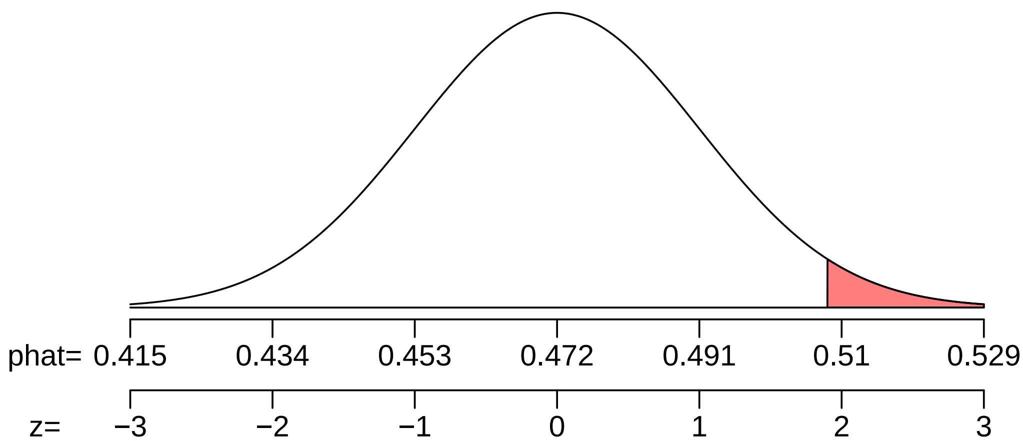
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.472(1-0.472)}{700}} = 0.0189$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.508 - 0.472}{0.0189} = 1.9$$

The p-value is a right area.



To determine that right area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(\hat{p} > 0.508) \\ &= P(Z > 1.9) \\ &= 1 - P(Z < 1.9) \\ &= 0.0287 \end{aligned}$$

Compare p-value to α (which is 0.02).

$$p\text{-value} > \alpha$$

Make the conclusion: we retain the null hypothesis.

(a) The p-value is 0.0287

(b) We retain the null hypothesis.

5. **Problem:**
   It is generally accepted that a population's proportion is 0.824. However, you think that maybe the population proportion is not 0.824, so you decide to run a two-tail hypothesis test with a significance level of 0.05 with a sample size of 5000.

   Then, when you collect the random sample, you find its proportion is 0.814. Do you reject or retain the null hypothesis?

   (a) Determine the *p*-value.

   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.824$$

$$H_A \quad \text{claims} \quad p \neq 0.824$$

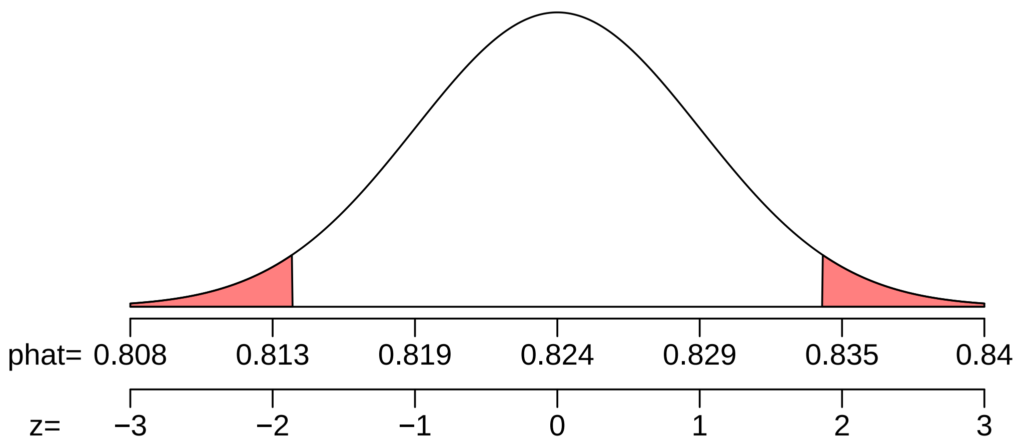
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.824(1-0.824)}{5000}} = 0.00539$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.814 - 0.824}{0.00539} = -1.86$$

The p-value is a two-tail area.



To determine that two-tail area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(|Z| > 1.86) \\ &= 2 \cdot P(Z < -1.86) \\ &= 0.0628 \end{aligned}$$

Compare p-value to α (which is 0.05).

$$p\text{-value} > \alpha$$

Make the conclusion: we don't reject the null hypothesis.

(a) The p-value is 0.0628

(b) We don't reject the null hypothesis.

6. **Problem:**
   It is generally accepted that a population's proportion is 0.754. However, you think that maybe the population proportion is not 0.754, so you decide to run a two-tail hypothesis test with a significance level of 0.1 with a sample size of 900.

   Then, when you collect the random sample, you find its proportion is 0.777. Do you reject or retain the null hypothesis?

   (a) Determine the *p*-value.
   (b) Decide whether we reject or retain the null hypothesis.

Solution: State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.754$$

$$H_A \quad \text{claims} \quad p \neq 0.754$$

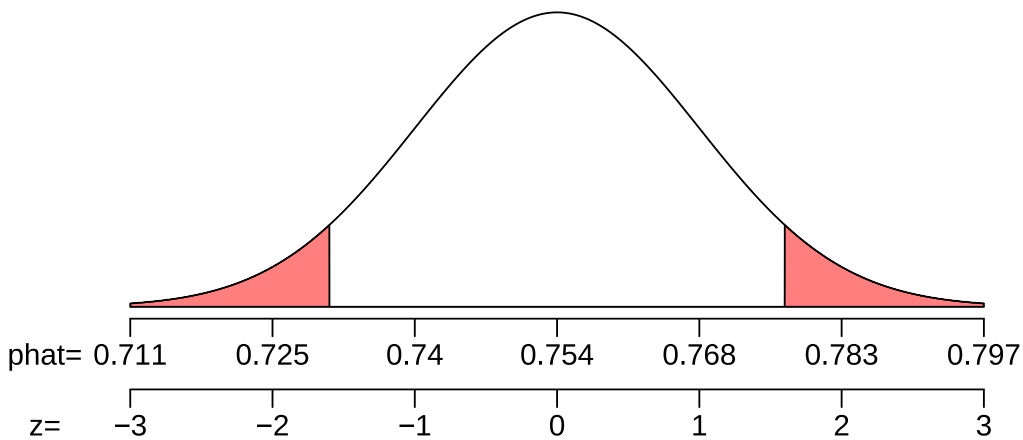
Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.754(1-0.754)}{900}} = 0.0144$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.777 - 0.754}{0.0144} = 1.6$$

The p-value is a two-tail area.



To determine that two-tail area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(|Z| > 1.6) \\ &= 2 \cdot P(Z < -1.6) \\ &= 0.1096 \end{aligned}$$

Compare p-value to α (which is 0.1).

$$p\text{-value} > \alpha$$

Make the conclusion: we don't reject the null hypothesis.

(a) The p-value is 0.1096
(b) We don't reject the null hypothesis.

7. **Problem:**
A null hypothesis claims a roughly symmetric population has a mean $\mu = 190$ and a standard deviation $\sigma = 27$. You decide to run two-tail test on a sample of size $n = 139$ using a significance level $\alpha = 0.1$. You then collect a sample and find it has mean $\bar{x} = 193.92$.

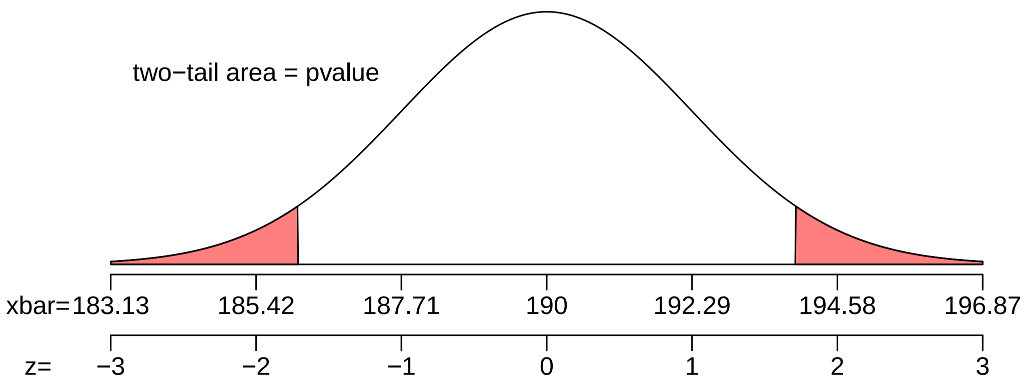
   (a) Determine the p-value.

   (b) Do you reject the null hypothesis?

Solution: Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{27}{\sqrt{139}} = 2.29$$

Make a sketch.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{193.92 - 190}{2.29} = 1.71$$

Find the p-value.

$$\begin{aligned} p\text{-value} &= P(|Z| > 1.71) \\ &= 2 \cdot P(Z < -1.71) \\ &= \boxed{0.0872} \end{aligned}$$

Compare the p-value and the significance level.

$$p\text{-value} < \alpha$$

Yes, we reject the null hypothesis.

(a) p-value = 0.0872

(b) Yes, we reject the null hypothesis.

8. **Problem:**
   A null hypothesis claims a roughly symmetric population has a mean $\mu = 100$ and a standard deviation $\sigma = 12$. You decide to run two-tail test on a sample of size $n = 279$ using a significance level $\alpha = 0.02$. You then collect a sample and find it has mean $\bar{x} = 101.5$.

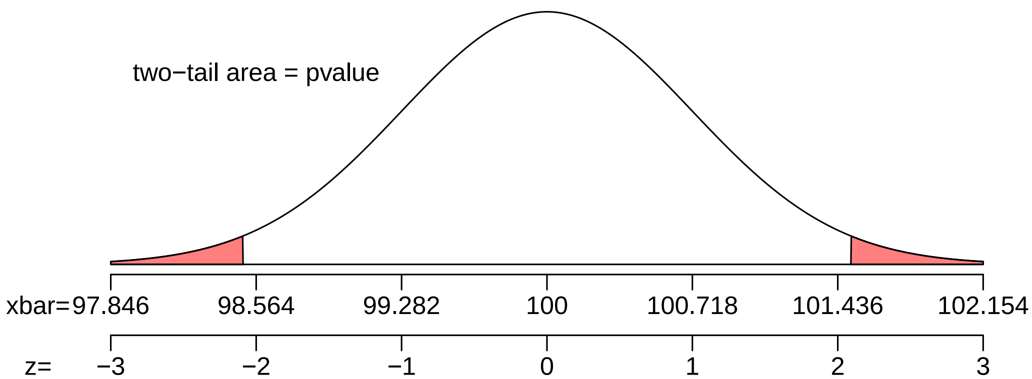
   (a) Determine the *p*-value.

   (b) Do you reject the null hypothesis?

Solution: Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{279}} = 0.718$$

Make a sketch.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{101.5 - 100}{0.718} = 2.09$$

Find the p-value.

$$\begin{aligned} p\text{-value} &= P(|Z| > 2.09) \\ &= 2 \cdot P(Z < -2.09) \\ &= \boxed{0.0366} \end{aligned}$$

Compare the p-value and the significance level.

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) p-value = 0.0366

(b) No, we do not reject the null hypothesis.