Single-sample hypothesis testing

$$
\begin{aligned}
H_0 &= \text{null hypothesis} \\
H_A &= \text{alternative hypothesis} \\
p\text{-value} &= \text{probability of sample at least as extreme as observed, } \textbf{given } H_0 \\
\alpha &= \text{significance level}
\end{aligned}
$$

- Calculate the p-value.
  - "at least as extreme" can mean "as large or larger", "as small or smaller", or "as far from expected in either direction".
- If p-value is small enough, we reject the null hypothesis. (This logic is similar to *reductio ad absurdum*.)

$$\text{If} \quad p\text{-value} < \alpha \quad \text{then} \quad \text{reject } H_0$$

$$\text{If} \quad p\text{-value} \geq \alpha \quad \text{then} \quad \text{do not reject } H_0$$

Single-sample proportion testing

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Right tail (one tail)

- H_0 claims $p = p_0$
- H_A claims $p > p_0$
- $p\text{-value} = P(Z > z_0)$

Left tail (one tail)

- H_0 claims $p = p_0$
- H_A claims $p < p_0$
- $p\text{-value} = P(Z < z_0)$

Two tail

- H_0 claims $p = p_0$
- H_A claims $p \neq p_0$
- $p\text{-value} = P(|Z| > |z_0|)$

Single-sample mean testing, σ known

$$z_0 = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

Right tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu > \mu_0$
- $p\text{-value} = P(Z > z_0)$

Left tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu < \mu_0$
- $p\text{-value} = P(Z < z_0)$

Two tail

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu \neq \mu_0$
- $p\text{-value} = P(|Z| > |z_0|)$

Single-sample mean testing, σ unknown

$$t_0 = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Right tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu > \mu_0$
- $p\text{-value} = P(T > t_0)$

Left tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu < \mu_0$
- $p\text{-value} = P(T < t_0)$

Two tail

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu \neq \mu_0$
- $p\text{-value} = P(|T| > |t_0|)$