

1. Problem

Please make a frequency table and a dot plot from the following (unsorted) data.

46	51	48	48	45	51
48	49	45	51	45	47
47	47	50	45	46	47
50	49	51	47	51	46

2. Problem

From a very large population, a small sample of measurements was taken.

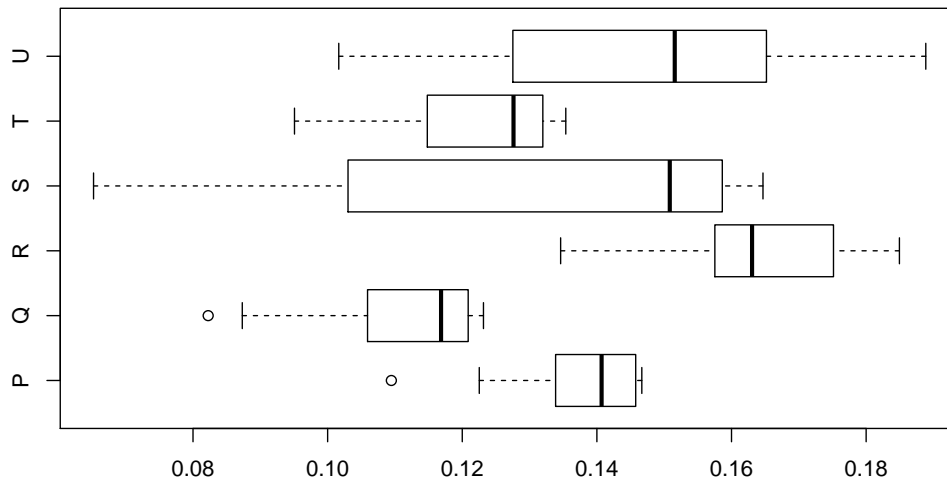
60	57	65	62	61	55
----	----	----	----	----	----

Please calculate the Average Absolute Deviation (also called the Average Distance from Mean: ADM) using the following formula:

$$AAD = \frac{\sum |x - \bar{x}|}{n}$$

3. Problem

Six random variables were each measured 25 times. The resulting boxplots are shown.



- (a) Which variable produced the largest measurement?
- (b) Which variable produced the smallest measurement?
- (c) Which distribution has the largest median?
- (d) Which distribution has the smallest median?
- (e) Which distribution has the largest 25th percentile?
- (f) Which distribution has the smallest 25th percentile?
- (g) Which distribution has the largest 75th percentile?
- (h) Which distribution has the smallest 75th percentile?
- (i) Which distribution has the largest IQR?
- (j) Which distribution has the smallest IQR?

4. Problem

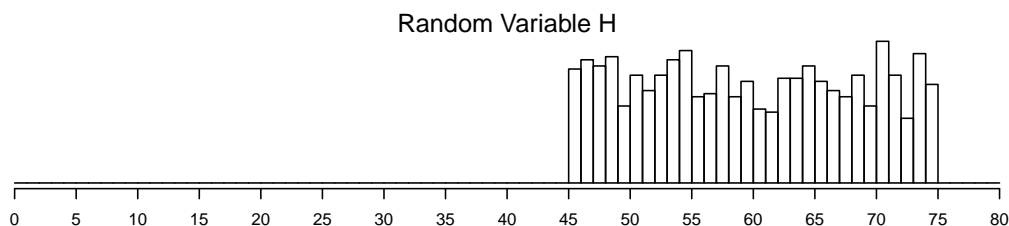
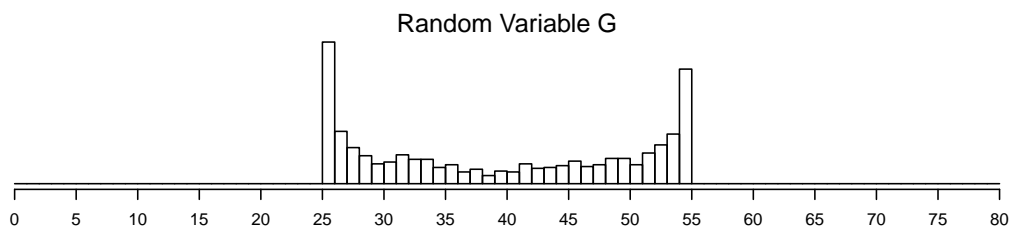
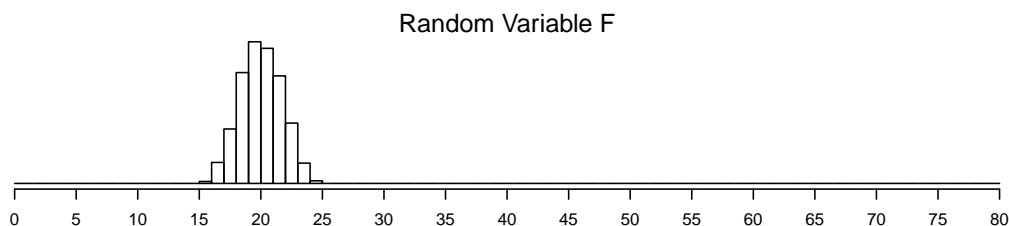
We can estimate the mean of **symmetric** distributions.

$$\bar{x} \approx \frac{\max(x) + \min(x)}{2}$$

We can **roughly** estimate the standard deviation of certain distributions.

Shape	SD estimate
bell	range/6
uniform	range/4
bimodal	range/2

Three random variables (F, G, and H) were measured 1000 times each. The resulting histograms show the three distributions.

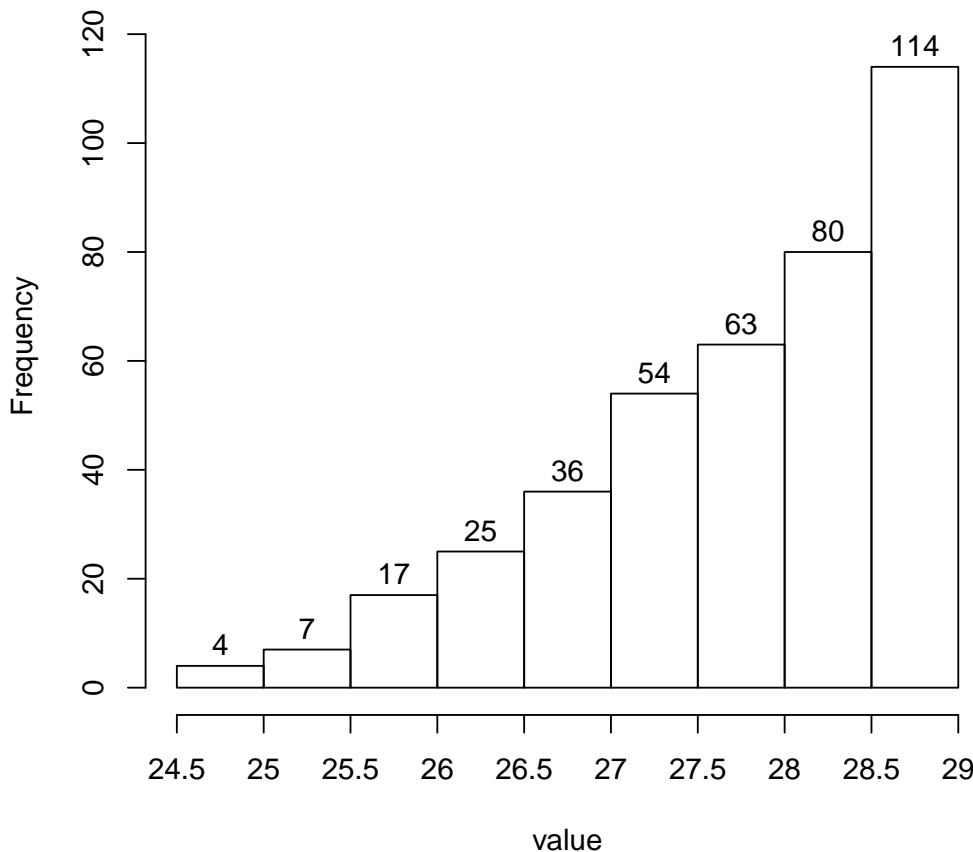


- Estimate the mean of F.
- Estimate the mean of G.
- Estimate the mean of H.
- Estimate the standard deviation of F.
- Estimate the standard deviation of G.
- Estimate the standard deviation of H.

5. Problem

A continuous random variable was measured 400 times. The resulting histogram is shown below.

Histogram of data



- (a) Describe the overall shape of the distribution. (symmetric mound, skew left, skew right, uniform, or bimodal)
- (b) Estimate the range of the distribution (range = max-min).
- (c) What percent of the measurements are less than 27.5?
- (d) What percent of the measurements are less than 26.5?
- (e) What percent of the measurements are between 26.5 and 27.5?
- (f) What percent of the measurements are within 0.25 from 27.25? In other words, what percent of measurements satisfy $|x - 27.25| \leq 0.25$?
- (g) Of the measurements less than 27.5, what percent are less than 26.5?
- (h) Estimate the value of the 7th percentile. In other words, determine a value such that 7% of the measurements are less than or equal to it.

6. Problem

From a very large population, a small sample of measurements was taken.

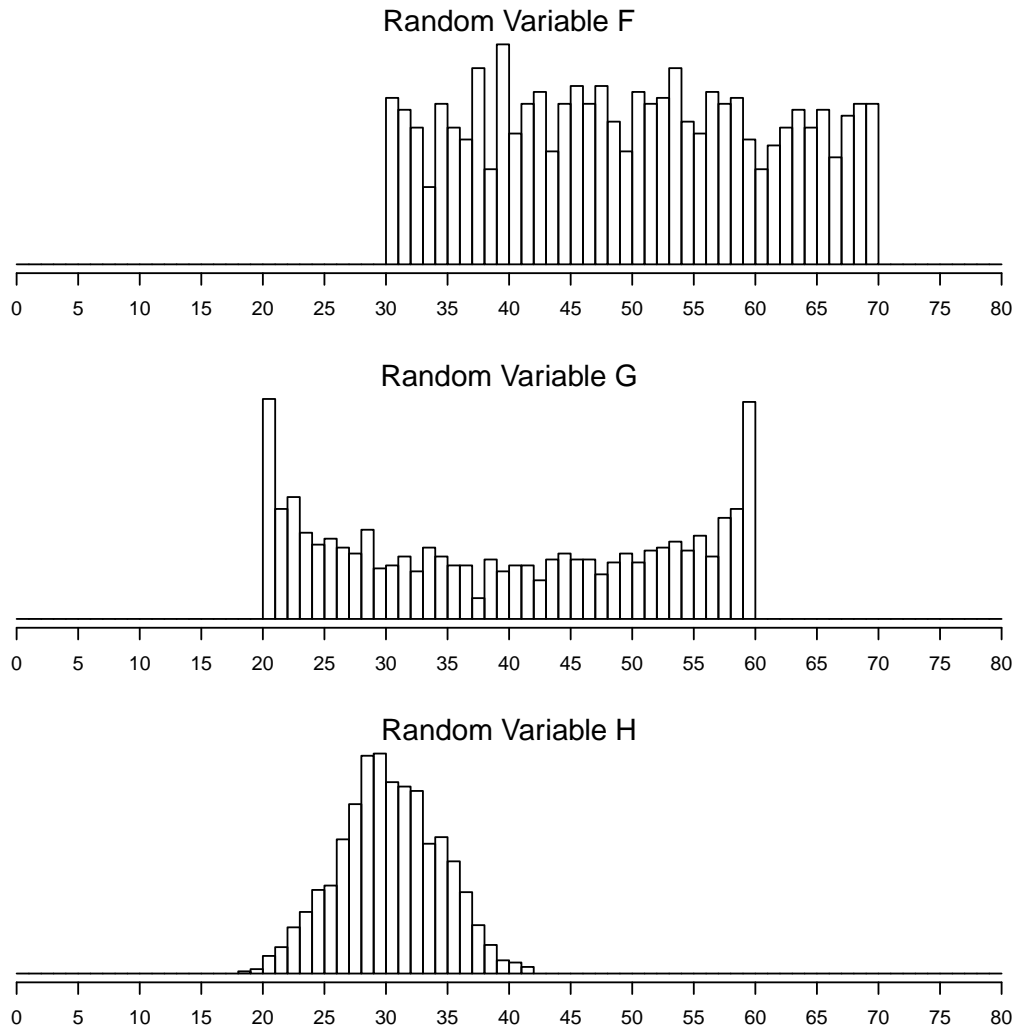
158	164	160	164
-----	-----	-----	-----

Please calculate the (Bessel corrected) sample standard deviation using the following formula:

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

7. Problem

Three random variables (F, G, and H) were measured 1000 times each. The resulting histograms show the three distributions.



.image

- (a) Which distribution has the highest mean? (F, G, or H)
- (b) Which distribution has the lowest mean? (F, G, or H)
- (c) Which distribution has the largest standard deviation? (F, G, or H)
- (d) Which distribution has the smallest standard deviation? (F, G, or H)

8. Problem

A continuous random variable was measured 100 times. The resulting frequency distribution is shown below.

class	frequency
50–60	3
60–70	3
70–80	4
80–90	8
90–100	11
100–110	16
110–120	16
120–130	22
130–140	17

- (a) Describe the overall shape of the distribution. (symmetric mound, skew left, skew right, uniform, or bimodal)
- (b) Estimate the range of the distribution (range = max-min).
- (c) What percent of the measurements are less than 80?
- (d) What percent of the measurements are less than 60?
- (e) What percent of the measurements are between 60 and 80?
- (f) What percent of the measurements are within 10 of 80? In other words, what percent of measurements satisfy $|x - 80| \leq 10$?
- (g) Of the measurements less than 80, what percent are less than 60?
- (h) Estimate the value of the 45th percentile. In other words, determine a value such that 45% of the measurements are less than or equal to it.

9. Problem

A continuous random variable X was measured 61 times. The sorted measurements are shown below.

50.93	50.97	50.99	51.15	51.19	51.53	51.57	51.66	51.67	51.68
51.69	51.69	51.71	51.75	51.86	51.86	51.9	51.92	51.97	51.99
51.99	52.02	52.03	52.05	52.09	52.1	52.14	52.17	52.18	52.23
52.26	52.26	52.27	52.28	52.36	52.39	52.42	52.47	52.53	52.53
52.55	52.56	52.59	52.61	52.66	52.69	52.7	52.74	52.8	52.82
52.83	52.85	52.88	52.9	52.91	52.93	52.93	52.94	52.96	52.96
52.96									

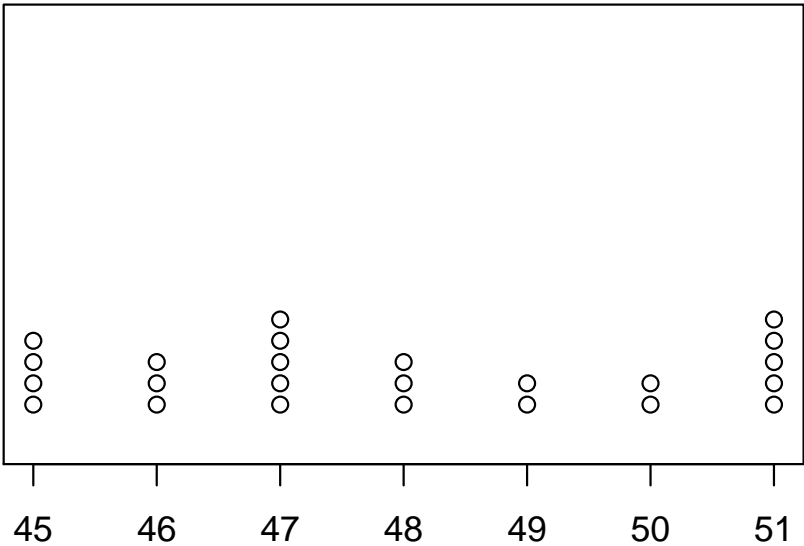
The total of the measurements is 3185.22.

- (a) Determine the percentile rank of the measurement 52.83. In other words, determine what percent of data are less than or equal to 52.83.
- (b) Determine the measurement corresponding to a percentile rank of 0.984. In other words, determine x such that 98.4% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

1. Make a frequency table.

value	frequency
45	4
46	3
47	5
48	3
49	2
50	2
51	5

Make the dot plot.



2. We fill out the table column by column.

x	$x - \bar{x}$	$ x - \bar{x} $
60	0	0
57	-3	3
65	5	5
62	2	2
61	1	1
55	-5	5

x	$x - \bar{x}$	$ x - \bar{x} $
=====	=====	=====
$\sum x = 360$		$\sum x - \bar{x} = 16$
$\bar{x} = 60$		

We are ready for the formula.

$$s = \frac{\sum |x - \bar{x}|}{n}$$

$$= \frac{16}{6}$$

$$= \boxed{2.6666667}$$

3. (a) U
(b) S
(c) R
(d) Q
(e) R
(f) S
(g) S
(h) P
4. (a) 20
(b) 40
(c) 60
(d) 1.6666667
(e) 15
(f) 7.5
5. (a) skew left
(b) 4.5
(c) 35.75%
(d) 13.25%
(e) 22.5%
(f) 13.5%
(g) 37.06%
(h) 26

6. We fill out the table column by column.

x	$x - \bar{x}$	$(x - \bar{x})^2$
158	-3.5	12.25
164	2.5	6.25
160	-1.5	2.25
164	2.5	6.25
=====	=====	=====
$\sum x = 646$		$\sum (x - \bar{x})^2 = 27$
$\bar{x} = 161.5$		

We are ready for the formula.

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$= \sqrt{\frac{27}{4 - 1}}$$

$$= \sqrt{9}$$

$$= \boxed{3}$$

7. (a) F
 (b) H
 (c) G
 (d) H
8. (a) skew left
 (b) 90
 (c) 10%
 (d) 3%
 (e) 7%
 (f) 12%
 (g) 30%
 (h) 110

9. Let x represent a measurement of interest. Let i represent that measurement's index. Let ℓ represent that measurement's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 52.83$. This means $i = 51$. We know $n = 61$. Determine the percentile ℓ .

$$\ell = \frac{51}{61}$$

$$\ell = 0.836$$

So, the percentile rank is $\boxed{0.836}$, or 83.6th percentile.

- (b) We are given $\ell = 0.984$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (61)(0.984)$$

$$i = 60$$

Determine the x associated with $i = 60$.

$$x = \boxed{52.96}$$

- (c) The mean: $\bar{x} = \frac{3185.22}{61} = \boxed{52.217}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 61$ and so n is odd.

$$\text{median} = x_{(61+1)/2} = x_{31}$$

So, median = $\boxed{52.26}$.