1. **Problem:**
A random sample of 1900 rabbits from Massachusetts yielded 1349 who are sick. Find a 95% confidence level for the proportion of rabbits in Massachusetts who are sick.

   (a) Are the conditions for inference met? (The necessary conditions for inference are $\hat{p}n \geq 10$ and $(1 - \hat{p})n \geq 10$.)

   (b) Construct the interval.

Solution: Identify the givens.

$$n = 1900$$
$$\hat{p} = \frac{1349}{1900} = 0.71$$
$$\gamma = 0.95$$

Check the conditions.

$$0.71 \times 1900 = 1349 > 10$$
$$(1 - 0.71) \times 1900 = 551 > 10$$

The conditions are satisfied, so we can continue with our inference.

Determine $z^\star$ such that $P(|Z| < z^\star) = 0.95$. It is helpful to know that $\ell = \frac{\gamma+1}{2} = \frac{0.95+1}{2} = 0.975$. (Use the z-table to find $z^\star$.)

$$z^\star = 1.96$$

Use the formula (proportion) for the bounds.

$$LB = \hat{p} - z^\star\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad UB = \hat{p} + z^\star\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.71 - 1.96\sqrt{\frac{(0.71)(0.29)}{1900}} \qquad = 0.71 + 1.96\sqrt{\frac{(0.71)(0.29)}{1900}}$$

$$= 0.69 \qquad = 0.73$$

Determine the interval.

$$CI = (0.69\,,\ 0.73)$$

We are 95% confident that the true population proportion is between 69% and 73%.

(a) The conditions are met. The number of sick rabbits is more than 10, and the number of healthy rabbits is more than 10.

(b) CI = (0.69, 0.73)

2. **Problem:**
   Marcel has discovered a new species of fish. He hopes to characterize the average length of this new species, so he obtains a sample of 12 specimens, which have a sample mean of 24.6 centimeters and a sample standard deviation of 4.4 centimeters. Determine the 99% confidence interval of the new species' average length.

Solution: We are given the sample size, sample mean, sample standard deviation, and confidence level.

$$\begin{aligned} n &= 12 \\ \bar{x} &= 24.6 \\ s &= 4.4 \\ \gamma &= 0.99 \end{aligned}$$

Find the degrees of freedom.

$$\begin{aligned} df &= n - 1 \\ &= 12 - 1 \\ &= 11 \end{aligned}$$

Determine the critical t value, $t^{\star}$, such that $P(|T| < t^{\star}) = 0.99$ and $df = 11$.

$$t^{\star} = 3.11$$

Use the formula for bounds (mean, σ unknown).

$$\begin{aligned} LB &= \bar{x} - t^{\star}\frac{s}{\sqrt{n}} \\ &= 24.6 - 3.11 \times \frac{4.4}{\sqrt{12}} \\ &= 20.65 \end{aligned} \qquad \begin{aligned} UB &= \bar{x} + t^{\star}\frac{s}{\sqrt{n}} \\ &= 24.6 + 3.11 \times \frac{4.4}{\sqrt{12}} \\ &= 28.55 \end{aligned}$$

We are 99% confident that the population mean is between 20.65 and 28.55 centimeters.

$$CI = (20.65, 28.55)$$

3. **Problem:**
   A researcher hopes to characterize the average time on social media spent by BHCC students with a 94% confidence interval. Somehow the researcher knows the standard deviation is 22 minutes. How large of a sample is needed to to get the margin of error down to 2 minutes?

Solution: We are given the population standard deviation, confidence level, and margin of error.

$$\sigma = 22$$
$$\gamma = 0.94$$
$$ME = 2$$

Determine the critical z value, $z^\star$, such that $P(|Z| < z^\star) = 0.94$. Remember, $\ell = \frac{\gamma+1}{2} = \frac{0.94+1}{2} = 0.97$

$$z^\star = 1.88$$

Use the formula for sample size.

$$n = \left(\frac{z^\star \sigma}{ME}\right)^2$$

$$= \left(\frac{(1.88)(22)}{2}\right)^2$$

$$= 427.6624$$

Round up.

$$n = 428$$

4. **Problem:**
A candy maker claims 17.6% of the candies are purple. We decide to test this claim with a 0.02 significance level. We collect a random sample of 800 candies and 164 of them are purple.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses

(c) Determine the p-value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Make a conclusion.

Solution: We should use a two-tail proportion test.

State the hypotheses.

$$H_0 \quad \text{claims} \quad p = 0.176$$
$$H_A \quad \text{claims} \quad p \neq 0.176$$

Determine the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.176(1-0.176)}{800}} = 0.0135$$

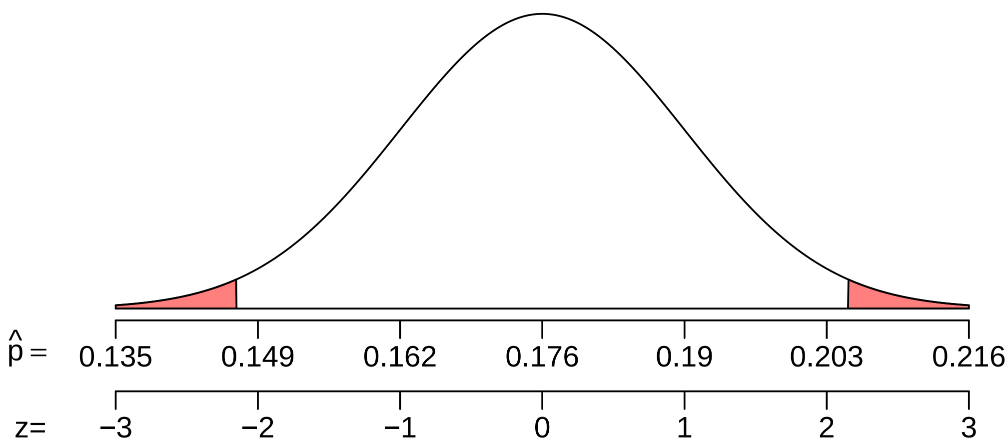
Determine the sample proportion.

$$\hat{p} = 0.205$$

Determine a z score. For simplicity, we ignore the continuity correction.

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.205 - 0.176}{0.0135} = 2.15$$

Make a sketch of the null's sampling distribution. The p-value is a two-tail area.



To determine that two-tail area, we use the z table.

$$\begin{aligned} p\text{-value} &= P(|Z| > 2.15) \\ &= 2 \cdot P(Z < -2.15) \\ &= 0.0316 \end{aligned}$$

Compare p-value to α (which is 0.02).

$$p\text{-value} > \alpha$$

Make the conclusion: we don't reject the null hypothesis.

We conclude the candy maker could be correct.

(a) Two-tail proportion test
(b) Hypotheses: H_0 claims $p = 0.176$ and H_A claims $p \neq 0.176$.
(c) The p-value is 0.0316
(d) We don't reject the null hypothesis.
(e) We conclude the candy maker could be correct.

5. **Problem:**
You work at a lightbulb company. The basic bulbs currently have an average brightness of 7860 lumens with a standard deviation of 1300 lumens. You are trying to engineer a brighter lightbulb.

Your newest model seems promising, so you decide to test, with a significance level of 0.1, whether your new bulbs have higher average brightness. A sample of 83 of these bulbs has an average brightness of 8037 lumens.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses

(c) Determine the p-value.

(d) Decide whether we reject or retain the null hypothesis.

(e) What can you conclude about your new model of lightbulb?

Solution: We should use a right-tail test of population mean.

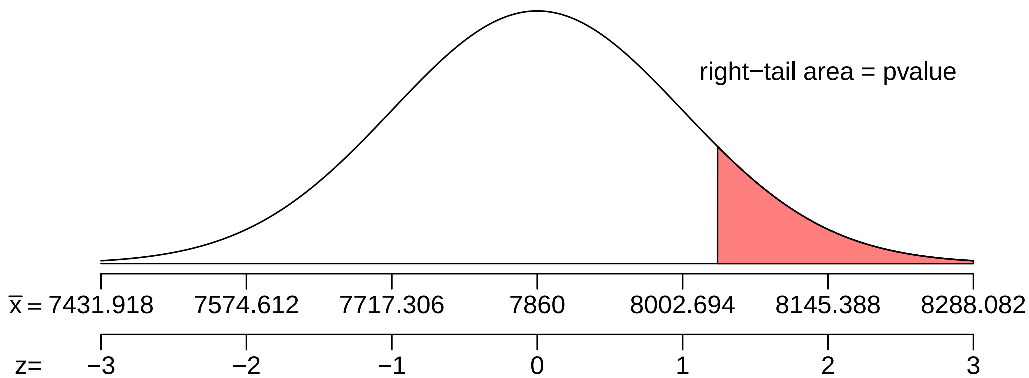
State the hypotheses:

$$H_0 \text{ claims } \mu = 7860$$
$$H_A \text{ claims } \mu > 7860$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1300}{\sqrt{83}} = 142.694$$

Make a sketch of the null's sampling distribution.



Find the z score.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{8037 - 7860}{142.694} = 1.24$$

Find the p-value (using formula for left-tail test of mean).

$$\begin{aligned} p\text{-value} &= P(Z > 1.24) \\ &= 1 - P(Z < 1.24) \\ &= 1 - 0.8925 \\ &= \boxed{0.1075} \end{aligned}$$

Compare the p-value and the significance level ($\alpha = 0.1$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

We conclude your new bulbs could be just as bright on average as the basic bulbs.

(a) Right-tail single mean test
(b) Hypotheses: H_0 claims $\mu = 7860$ and H_A claims $\mu < 7860$.
(c) p-value = 0.1075
(d) No, we do not reject the null hypothesis.
(e) We conclude your new bulbs could be just as bright on average as the basic bulbs.

6. **Problem:**
A null hypothesis claims a population has a mean $\mu = 9.0$. You decide to run two-tail test on a sample of size $n = 11$ using a significance level $\alpha = 0.1$.

You then collect the sample:

9.6	9.1	9.7	9.3	10.1
10.4	8	7.4	10.6	11.1
10.2				

(a) Determine the p-value.

(b) Do you reject the null hypothesis?

Solution: State the hypotheses.

$$H_0 \text{ claims } \mu = 9$$
$$H_A \text{ claims } \mu \neq 9$$

Find the mean and standard deviation of the sample.

$$\bar{x} = 9.591$$
$$s = 1.107$$

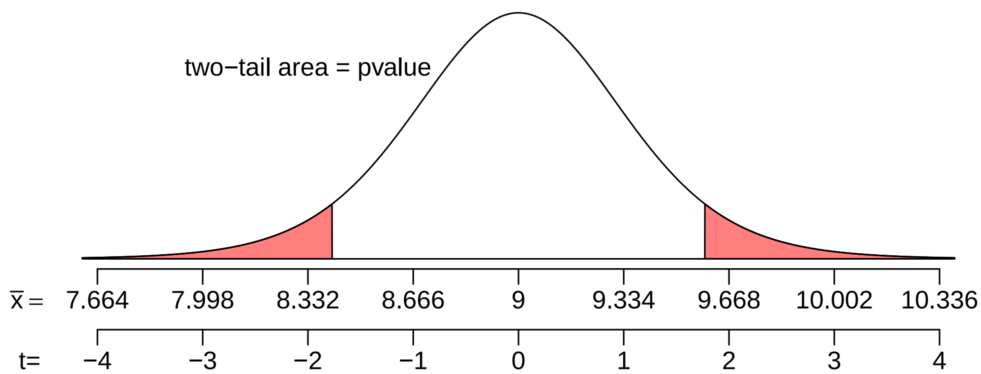
Determine the degrees of freedom.

$$df = 11 - 1 = 10$$

Find the standard error.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{1.107}{\sqrt{11}} = 0.334$$

Make a sketch of the null's sampling distribution.



Find the t score.

$$t = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{9.591 - 9}{0.334} = 1.77$$

Find the p-value.

$$p\text{-value} = P(|T| > 1.77)$$

We can't get an exact value with our table, but we can determine an interval that contains the p-value. (Look at row with $df = 10$.)

$$P(|T| > 1.81) = 0.1$$
$$P(|T| > 1.37) = 0.2$$

Basically, because t is between 1.81 and 1.37, we know the p-value is between 0.1 and 0.2.

$$0.1 < p\text{-value} < 0.2$$

Compare the p-value and the significance level ($\alpha = 0.1$).

$$p\text{-value} > \alpha$$

No, we do not reject the null hypothesis.

(a) $0.1 < p$-value < 0.2
(b) No, we do not reject the null hypothesis.