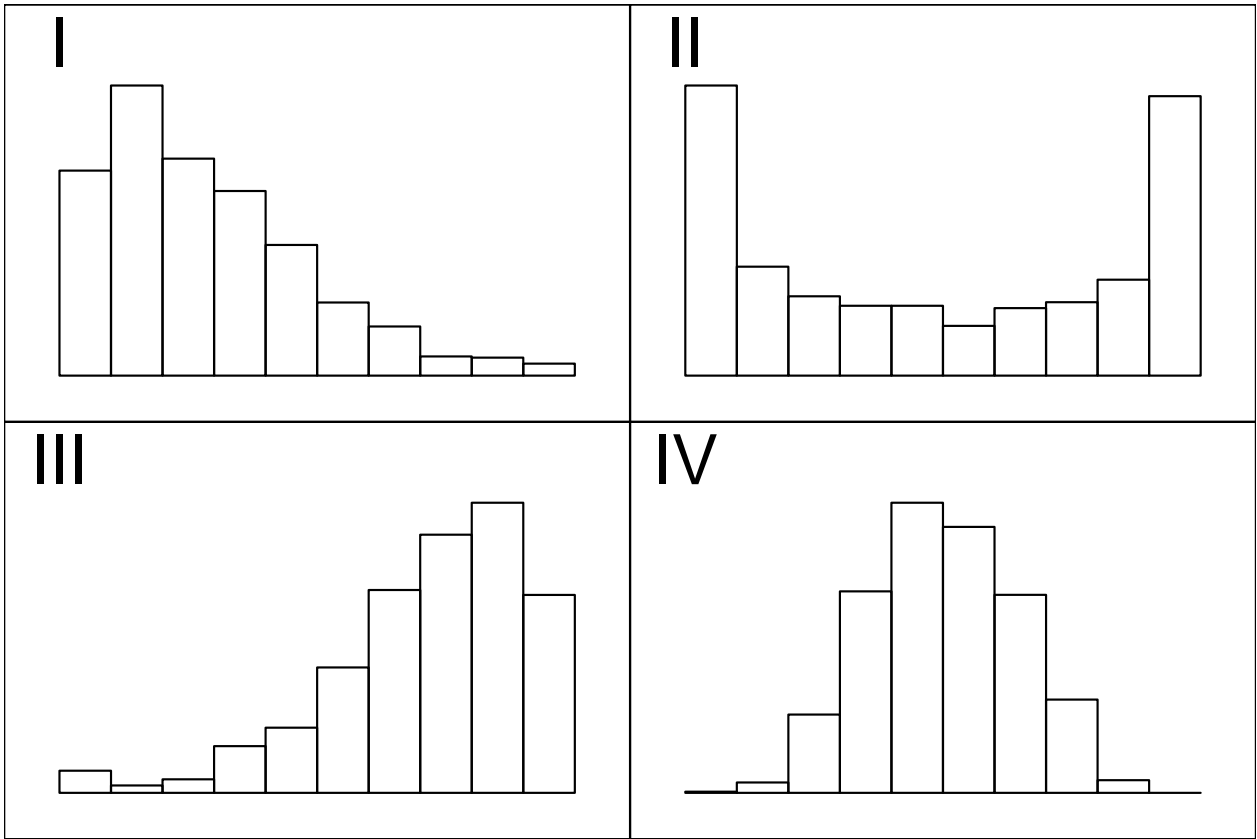


**MAT-181 FINAL TAKE-HOME EXAM**

This exam is to be taken without discussion or correspondence with any human. Please show work!

question	available points	earned points
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	15	
8	20	
EC	5	
EC	5	
Total	100	

## 1. (10 Points)



For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

- (a) The distribution of annual income for school employees where a high percentage of employees are entry-level teachers and only a few are high-paid administrators.
- (b) The distribution of quiz scores on an easy quiz. Most students did very well, but a few did poorly.
- (c) The distribution of heights of adult women
- (d) The distribution of hours that students studied for an exam when about half of students studied a lot and a similar number of students studied very little.

## 2. (15 Points)

In a deck of strange cards, there are 374 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	gray	red	Total
horn	39	20	14	73
jigsaw	11	24	30	65
needle	19	34	36	89
pig	15	43	13	71
shovel	12	26	38	76
Total	96	147	131	374

- (a) What is the probability a random card is either a jigsaw or gray (or both)?
- (b) What is the probability a random card is both a shovel and gray?
- (c) Is a horn or a shovel more likely to be red?
- (d) What is the probability a random card is blue given it is a jigsaw?
- (e) What is the probability a random card is a pig?
- (f) What is the probability a random card is a jigsaw given it is red?
- (g) What is the probability a random card is gray?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
<i>A</i>	92	9
<i>B</i>	96	11
<i>C</i>	103	6
<i>D</i>	77	4

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
<i>A</i>	95.87
<i>B</i>	105.6
<i>C</i>	107
<i>D</i>	78.6

Which specimen is the most unusually far (in either direction) from average (relative to others of its type)?

4. (10 points)

A tree's leaves were found to be normally distributed with a mean of 161.5 millimeters and a standard deviation of 3.7 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 162.9 and 166.6 millimeters?

5. (10 points)

A species of duck is known to have a mean weight of 200.7 grams and a standard deviation of 67.5 grams. A researcher plans to measure the weights of 81 of these ducks sampled randomly. What is the probability the **sample mean** will be between 205.2 and 215.7 grams?

6. (10 points)

An ornithologist wishes to characterize the average body mass of *Piranga rubra*. She randomly samples 32 adults of *Piranga rubra*, resulting in a sample mean of 36.13 grams and a sample standard deviation of 7.1 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

A student is taking a multiple choice test with 800 questions. Each question has 2 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

Then, the student takes the test and gets 426 questions correct.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses.

(c) Determine the test statistic ( $z$  or  $t$ ), draw a sketch, and determine the  $p$ -value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Did the student do significantly better than random guessing?



8. (20 points) [Note: this question uses 2 pages.]

You have collected the following data:

$x$	$y$	$xy$
2	71	
6.7	16	
8.7	9.1	
9.6	5.5	
9.3	26	
4.2	22	
7	11	
$\sum x =$	$\sum y =$	$\sum xy =$
$\bar{x} =$	$\bar{y} =$	
$s_x =$	$s_y =$	

(a) Complete the table.

(b) Calculate the correlation coefficient ( $r$ ) using the formula below.

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

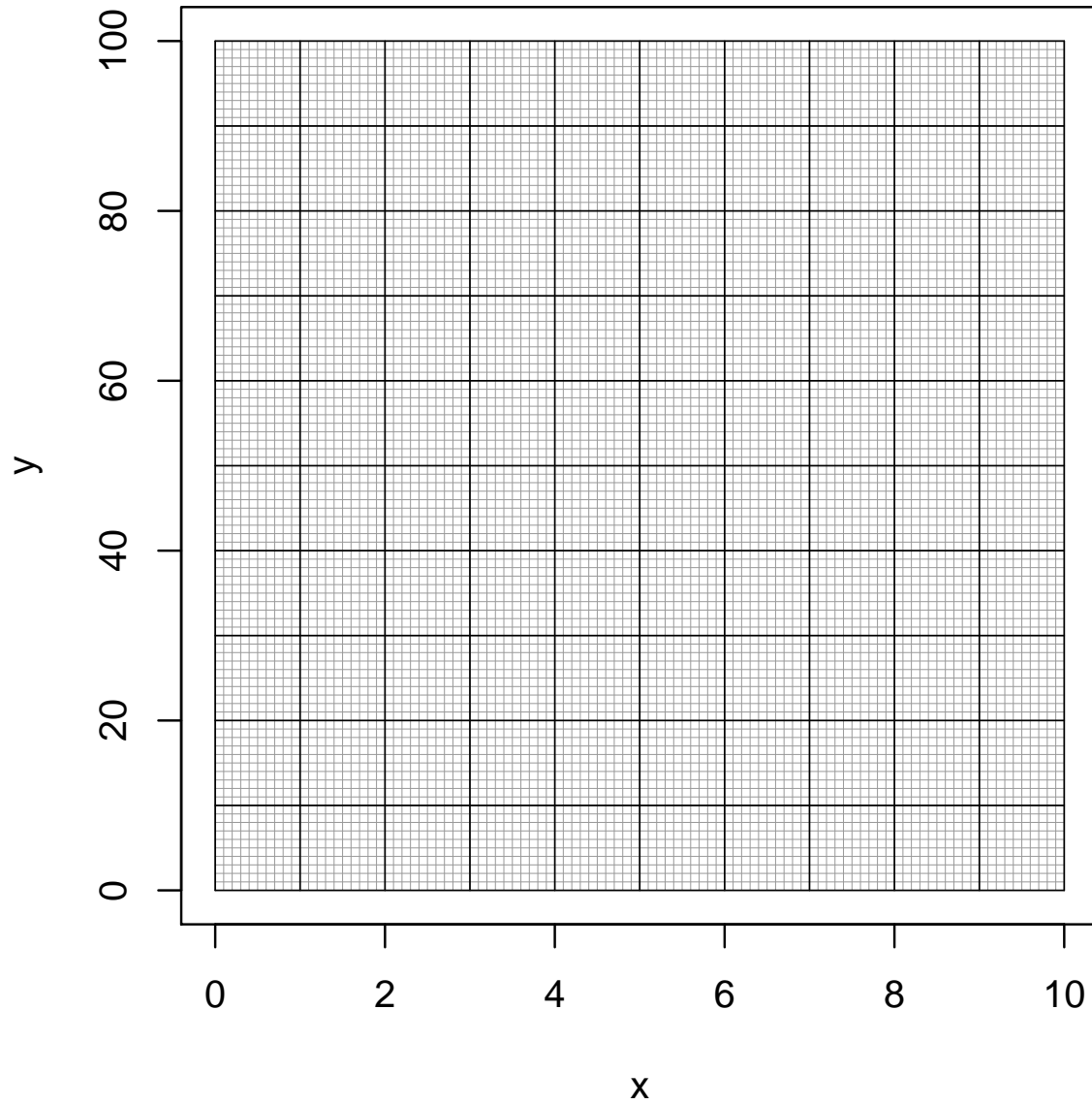
(c) The least-squares regression line will be represented as  $y = a + bx$ . Determine the parameters ( $b$  and  $a$ ) using the formulas below.

$$b = r \frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

(d) Write the equation of the regression line (using the calculated values of  $a$  and  $b$ .)

(e) Please plot the data and a corresponding regression line.



9. (Extra credit: 5 points)

Let each trial have a chance of success  $p = 0.19$ . If 185 trials occur, what is the probability of getting at least 37 but at most 43 successes?

In other words, let  $X \sim \text{Bin}(n = 185, p = 0.19)$  and find  $P(37 \leq X \leq 43)$ .

Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

A null hypothesis claims a population has a mean  $\mu = 80$ . You decide to run two-tail test on a sample of size  $n = 9$  using a significance level  $\alpha = 0.02$ .

You then collect the sample:

79.6	80.9	82.7	79.4	83
85.3	81	82.5	81.5	

- (a) Determine the  $p$ -value.
- (b) Do you reject the null hypothesis?