Name: Solution **Section: MAT098/181 C-**

MAT098/181C EXAM #3 (FORM C)

A scientific calculator is permitted. ***Cellphones may not be used as calculators and must be off or on vibrate during the exam****. Show all work on the test or on the work paper*

1. Sam received a score of 85 on a history test for which the class average was 80 with standard deviation 10. He received a score of 80 on a biology test for which the class average was 74 with standard deviation 4.
   On which test did he do better RELATIVE to the rest of the class? Please JUSTIFY your answer.

   Assuming that the scores of each test follow a normal distribution, calculate Sam's z-score for each of the three exams.

   History test z-score : $z = \frac{x-\mu}{\sigma} = \frac{85-80}{10} = .5$

   Biology test z-score : $z = \frac{x-\mu}{\sigma} = \frac{80-74}{4} = 1.5$

   On which exam did Sam do the best RELATIVE to the rest of his class?

   Sam did better RELATIVE to the rest of the class on the biology test.
   Because on biology test, Sam's score is 1.5 standard deviation above the mean. On the history test, Sam's score is only 0.5 standard deviation above the mean. ■

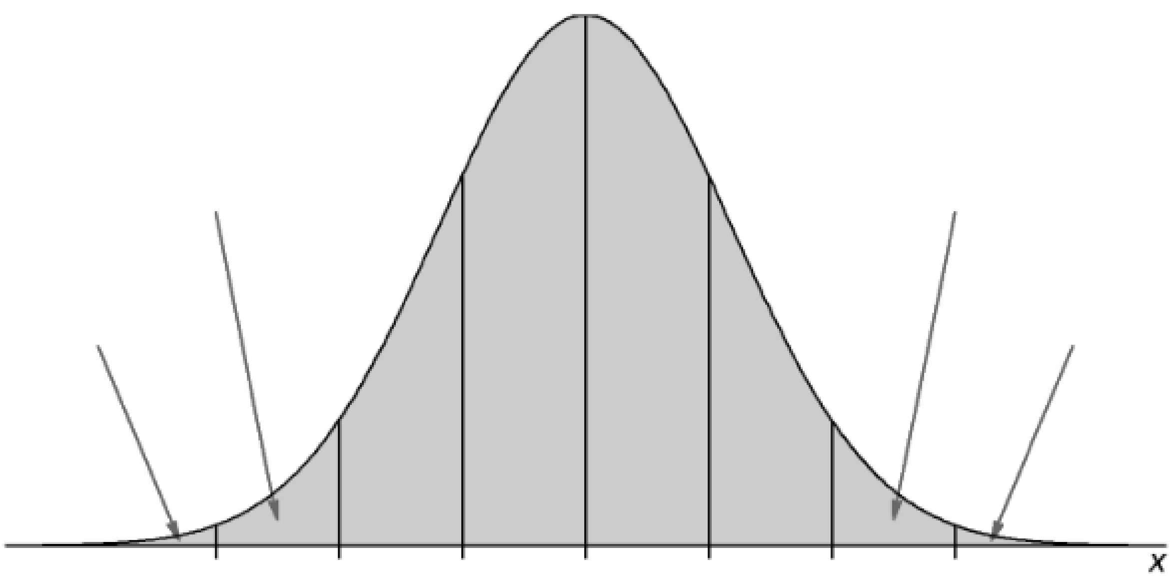
2. Suppose that students at BHCC have a normally distributed GPA with a mean of 3.0 and a standard deviation = 0.25. **Please label the graph below with the following:** ***(12 points)***

a) The tick marks on the x-axis of the graph below are one standard deviation apart. Label the axis with the ***appropriate GPA values***.

b) ***Label the Z-score*** of each value below its x-value

c) Using the Empirical rule, label each region of the graph with the area for that region

d) What interval will contain 95% of the GPA's around the mean?



3. Let x be a random variable that represents the red blood cell (RBC) count in millions per cubic of a healthy female. If x has a normal distribution with mean μ = 4.8 and standard deviation σ = 0.8. *(18 points: 4,4,4,2,2,2)*

   Find the probabilities in part a, b, and c, by first converting each of the following x intervals to standardized z intervals.

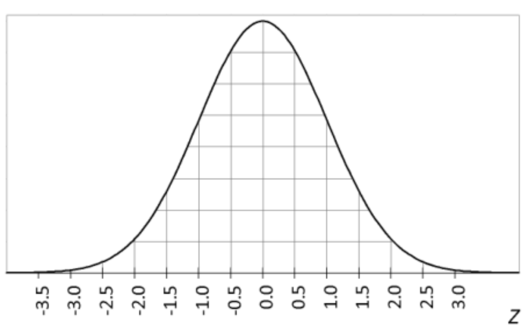
   a) P(x ≤ 4.2) = p($z \leq -.75$) =.227 Use Standard Normal (z) Distribution Table or use technology. Recall that $z = \frac{x-\mu}{\sigma}$

   b) b.) P(x ≥ 4.5) = p(z≥-.375)=1-p(z≤-.375)=.646 Use Standard Normal (z) Distribution Table or use technology. Recall that $z = \frac{x-\mu}{\sigma}$

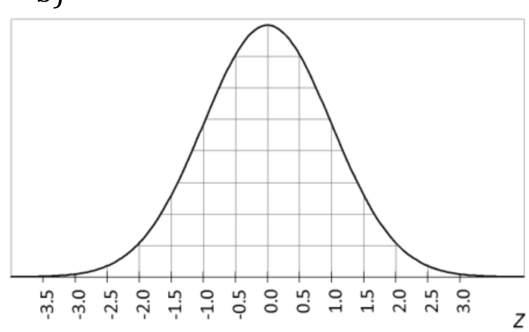
   c) c.) P(4.0 ≤ x ≤5.5) = p(-1≤z≤.875)=.651 Use Standard Normal (z) Distribution Table or use technology.

   For the z intervals you calculated above, shade the area under the curve that represents the associated probability.

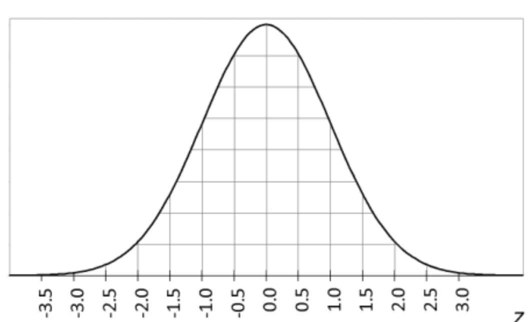
   a)



b)



c)



4. ***Draw a sketch for each part.*** *(15 points)* Suppose that after 12-hour fast, a person's blood glucose level follows a normal distribution with mean μ =85 milligrams of glucose per deciliter of blood and standard deviation σ = 25 milligrams of glucose per deciliter of blood. What is the probability that after a 12-hour fast,

a) the blood glucose level is less than 110 milligrams of glucose per deciliter of blood?

P(x<110): $z = \frac{x-\mu}{\sigma} = \frac{110-85}{20} = 1.25$

P(x<110)=p(z<1.25)=.894 Use Standard Normal (z) Distribution Table or use technology.

b) the blood glucose level is greater than 60 milligrams of glucose per deciliter of blood?

P(x>60): $z = \frac{x-\mu}{\sigma} = \frac{60-85}{20} = -1.25$

P(x>60)= p(z>-1.25)=1-p(z<-1.25)=.933 Use Standard Normal (z) Distribution Table or use technology.

c) the blood glucose level is between 60 and 110 milligrams of glucose per deciliter of blood?

P(60<x<110)= p(-1.25<z<1.25)=.789

5. Suppose that the average diameter of a certain type of roller bearing is mound-shaped and symmetric with mean μ = 6mm and standard deviation σ = 1.1mm. If 35 roller bearings are tested, describe the $\bar{x}$ distribution and find the probability that the ***average (mean)*** $\bar{x}$ is between 5.5 and 6.5 mm. *(15 points)*

   Since we have a normal distribution, then from the central limit theorem, we expect the $\bar{x}$ distribution to be approximately normal, with $\mu_{\bar{x}} = \mu = 6mm$ and standard deviation

   $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.1}{\sqrt{35}} \approx .2$$

   Let now convert the interval 5.5<$\bar{x}$ <6.5 to a standard z interval and use the Standard Normal (z) Distribution Table or technology to find p(5.5<$\bar{x}$ <6.5).

   $$z = \frac{x - \mu}{\sigma} = \frac{5.5 - 6}{.2} = -2.5$$

   $$z = \frac{x - \mu}{\sigma} = \frac{6.5 - 6}{.2} = 2.5$$

   Therefore, p(5.5<$\bar{x}$ <6.5)=p(-2.5<z<2.5)=.988.
   Interpretation: About 98.8% of all such samples have average diameter between 5.5 and 6.5mm.

6. A random sample of 200 Math text books purchased at BHCC Bookstore showed that 70% of the text books purchased were College Algebra text books. If p is the probability of the College Algebra text books purchased, (*10 points)*

   a) Can we approximate the p distribution with a normal distribution? Explain.
      Notice that $np = 200(.7) = 140$ and $n(1-p) = 200(1-.7) = 60$ are both greater than 5. Therefore, the p distribution will be a good approximation with a normal distribution.

   b) What is the mean of the sampling distribution of p?

      The mean is $p = .7$

   c) What is the standard deviation of the sampling distribution of p?
      The standard deviation of the sampling distribution of p is

      $\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.7(1-.7)}{200}}$=.03■

7. It is found that 70% of the aluminum cans sold in one of the communities in California were recycled. *(15 points)*
   (a) If a random of 400 cans are sold today, use the normal approximation to the binomial to find the probability that 300 or more will be recycled?

Let r be the number of recycled cans. To find p(r≥300), we use the normal curve with

$\mu = np = 400(.7) = 280$ and $\sigma = \sqrt{np(1-p)}$=$\sqrt{400(.7)(1-.7)}$=9.17.

Since r≥300 indicates the left endpoint, we subtract .5. Consequently,

p(r≥300)= p(300≤r)=p(300-.5≤x)=p(299.5≤x).

Now, let convert the interval 299.5≤x to a standard z.

$z = \frac{299.5-280}{9.17} = 2.13$. The p(r≥300)= p(299.5≤x)=p(2.13≤z)=0.0166

(b) Will the normal distribution make a good approximation to the binomial for this problem? Explain your answer.

Since $np = 400(.70) = 280$ and $n(1-p) = 400(1-.70) = 120$ are both greater than 5, the normal distribution will make a good approximation to the binomial. ■

8. ****EXTRA CREDIT:**

a) A coffee machine automatically pours coffee into cups. The amount of coffee dispensed into a cup is normally distributed with a mean of 6.7 ounces and standard deviation of 0.3 ounce. Suppose that we are interested in reducing the amount of extra coffee that is poured into the 6.7 once cup. We are seeking to identify the highest 2.5% of the fill amounts poured by this machine. For what fill amount are we searching? Round to the nearest hundredth.

Left Area is 1-2.5%=.975

z-score corresponding to .975 is 1.96

$x = \mu + z\sigma = 6.7 + 1.96(.3) = 7.29$ ■

b) The machine has just been loaded with 956 cups. How many of these do you expect will not overflow when served?

What is 84% of 956?

$x \quad = \; .(84) * 956$

$x \quad =$ 803.04. Thefore, we expect that 803 cups will not overflow when served. ■

Formula sheet:

Empirical Rule

- about 68% of the x values lie within 1 standard deviation of the mean.
- about 95% of the x values lie within 2 standard deviations of the mean.
- about 99.7% of the x values lie within 3 standard deviations of the mean.

z-score

$$z = \frac{x - \mu}{\sigma}$$

Central Limit Theorem

Mean of the sample mean is $\mu_{\bar{x}} = \mu$

Standard deviation of the sample mean is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

z-score for sample mean

$$z = \frac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

Binomial Distribution

Mean: $\mu = np$

Standard Deviation: $\sigma = \sqrt{np(1-p)}$

Sampling Distribution of Sample Proportion

Mean: p

Standard Deviation: $\sigma = \sqrt{\frac{p(1-p)}{n}}$