1. **Problem**
A farm produces 4 types of fruit: A, B, C, and D. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	68	4
B	72	13
C	78	8
D	131	14

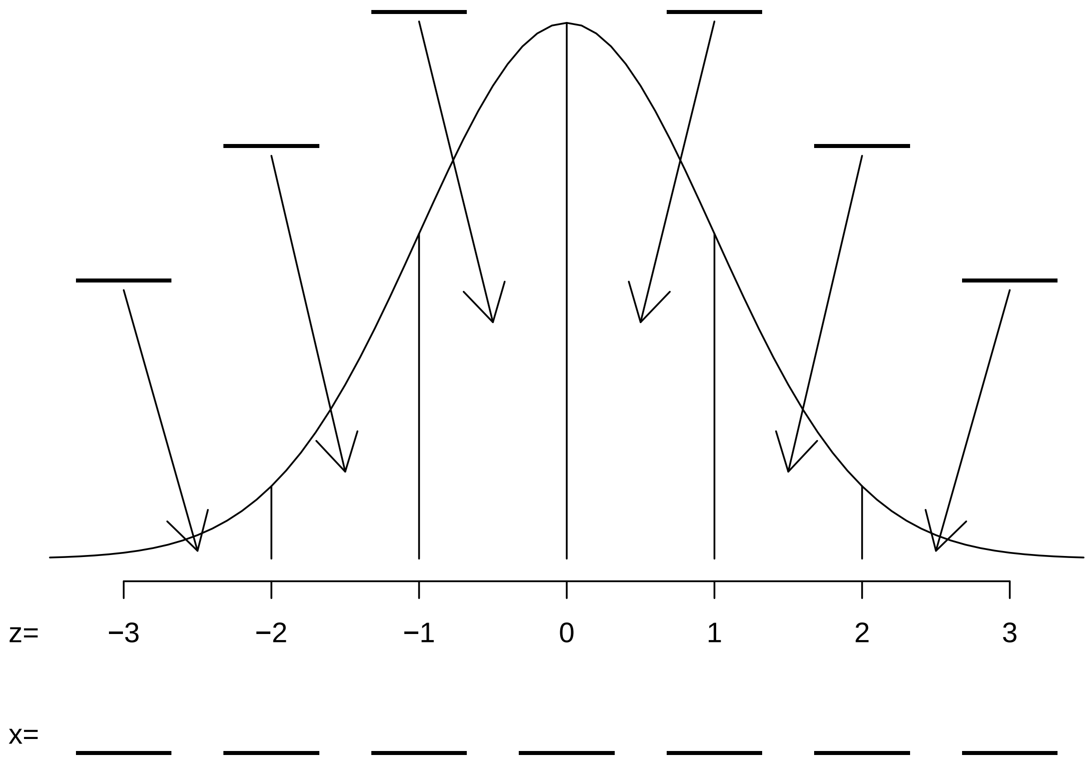
One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	75.24
B	75.25
C	65.36
D	116.9

Which specimen is the most unusually large (relative to others of its type)?

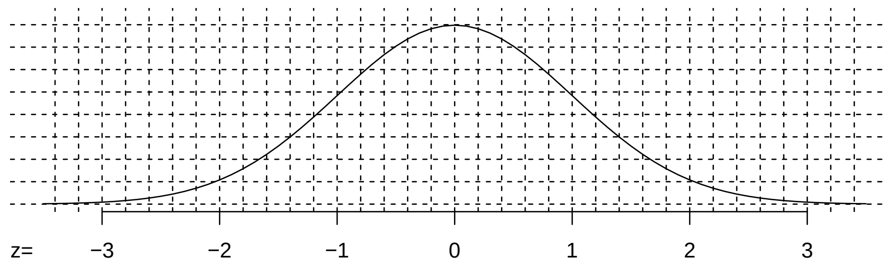
2. **Problem**
   A normal random variable X has a mean $\mu = 32.7$ and standard deviation $\sigma = 3.1$. Please label the density curve with:

   (a) The appropriate values of x.
   (b) The areas of the sections.

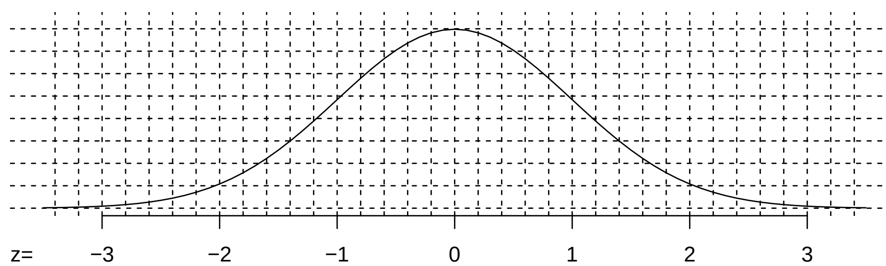


3. **Problem**
Let X be normally distributed with mean 49 and standard deviation 11. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

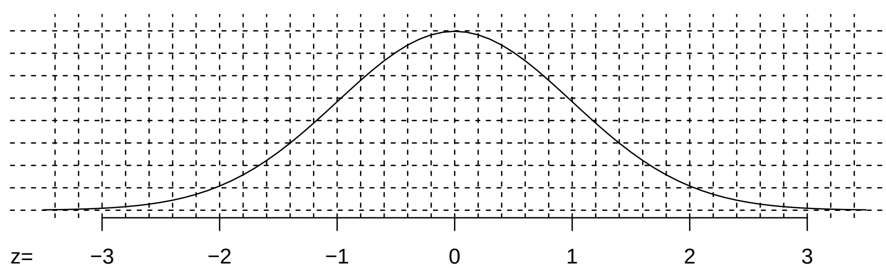
(a) $P(X < 50.1)$



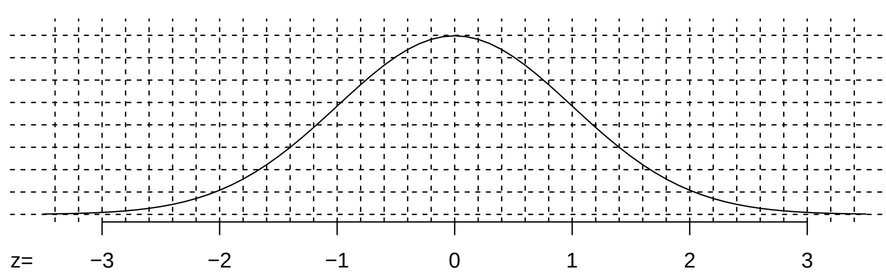
(b) $P(X > 60)$



(c) $P(|X - 49| < 6.6)$



(d) $P(|X - 49| > 5.5)$



4. **Problem**
   Let X be normally distributed with mean 105.6 and standard deviation 19.3. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

   (a) What's the probability that X is less than 94.8? **Draw a sketch**.

   (b) What's the probability that X is more than 130? **Draw a sketch**.

   (c) What's the probability that X is between 94.8 and 130? **Draw a sketch**.

5. **Problem**
Let random variable W have mean $\mu_W = 49$ and standard deviation $\sigma_W = 10$. Let random variable X represent the **average** of $n = 64$ instances of W.

(a) Determine the expected value of X. $\mu_X = ?$

(b) Determine the standard deviation of X. $\sigma_X = ?$

(c) Using normal approximation, determine $P(X < 49.09)$.

(d) Using normal approximation, determine $P(X > 47.61)$.

6. **Problem**
   A very large population has a mean of 100.7 and a standard deviation of 21. When a random sample of size 100 is taken, what is the probability that the **sample mean** ($\bar{x}$) is between 101 and 105?

   (a) Determine the mean and standard deviation of the sampling distribution.

   (b) Determine $P(101 < \overline{X} < 105)$. **Draw a sketch**

7. **Problem**
Let random variable W have the probability distribution shown below.

w	$P(w)$
0	0.28
1	0.72

Let random variable $\hat{p}$ (sample proportion) represent the average of $n = 49$ instances of W.

(a) Determine the mean and standard deviation of the $\hat{p}$.

(b) Determine $P(\hat{p} < 0.66)$. Do NOT use a continuity correction. **Draw a sketch**

8. **Problem**
A very large population has a population proportion $p = 0.78$. When a random sample of size 225 is taken, what is the probability that the **sample proportion** ($\hat{p}$) is less than 0.75?

Do NOT use a continuity correction.

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(\hat{p} < 0.75)$. **Draw a sketch**

9. **Problem**
Let random variable W have mean $\mu_W = 56$ and standard deviation $\sigma_W = 6$. Let random variable X represent the **sum** of $n = 64$ instances of W.

(a) Determine the expected value of X. $\mu_X = ?$

(b) Determine the standard deviation of X. $\sigma_X = ?$

(c) Using normal approximation, determine $P(X < 3644)$.

(d) Using normal approximation, determine $P(X > 3594.56)$.

10. **Problem**
Let random variable W have the probability distribution shown below.

w	$P(w)$
0	0.66
1	0.34

Let random variable X represent the sum of $n = 177$ instances of W. (Thus X is the sample total, or number of successes.)

What is the probability that X is at least 47 but at most 72? **Use a normal approximation with continuity corrections**.

11. **Problem**

Let each trial have a chance of success $p = 0.77$. If 156 trials occur, what is the probability of getting more than 108 but less than 133 successes?

In other words, let $X \sim \text{Bin}(n = 156,\ p = 0.77)$ and find $P(108 < X < 133)$.

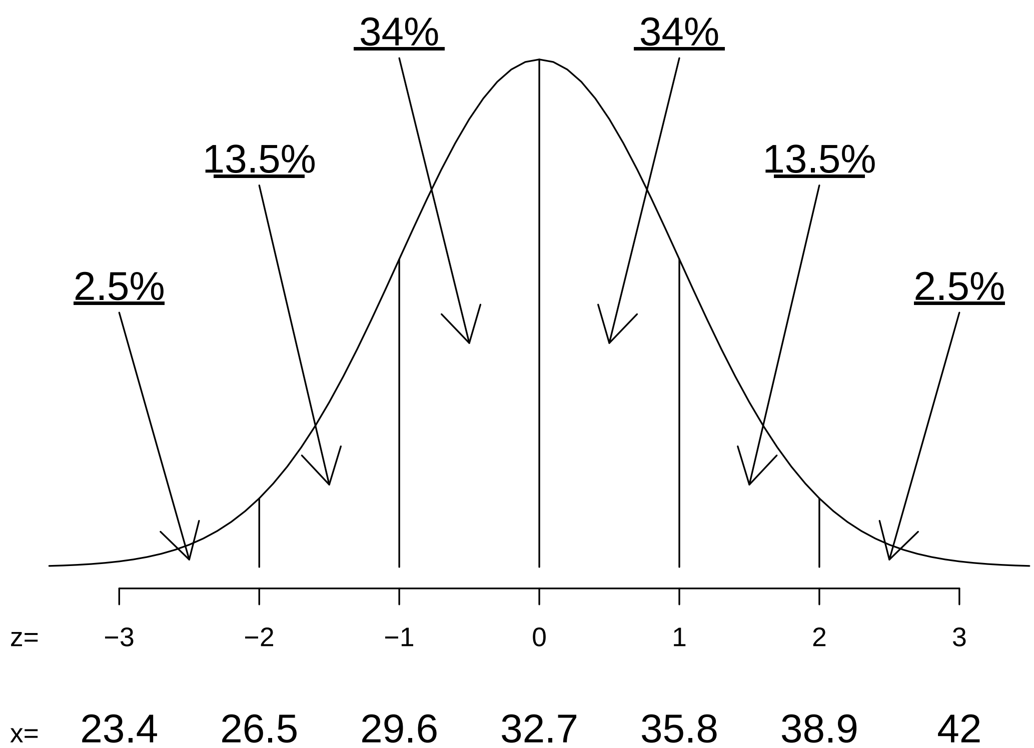
Use a normal approximation along with the continuity correction.

1. We compare the z-scores. The largest z-score corresponds to the specimen that is most unusually large.

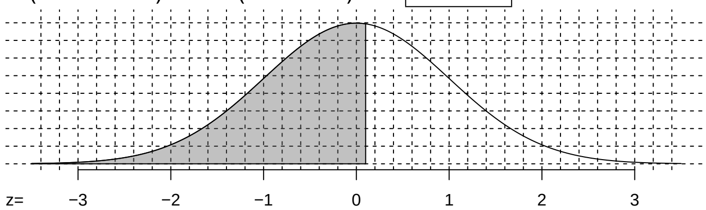
Type of fruit	formula	z-score
A	$z = \frac{75.24-68}{4}$	1.81
B	$z = \frac{75.25-72}{13}$	0.25
C	$z = \frac{65.36-78}{8}$	-1.58
D	$z = \frac{116.9-131}{14}$	-1.01

Thus, the specimen of type A is the most unusually large.

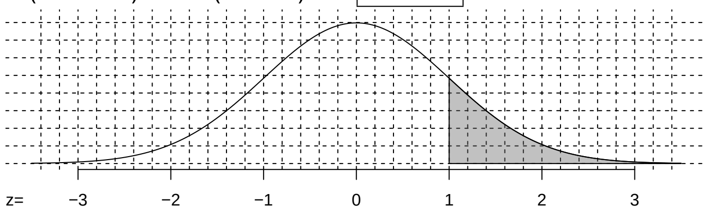
2. The filled in areas and x values are shown below.



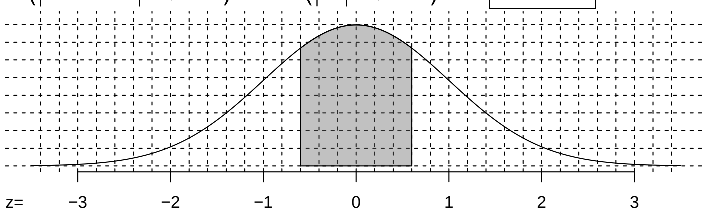
3. (a) $P(X < 50.1) = P(Z < 0.1) = \boxed{0.5398}$



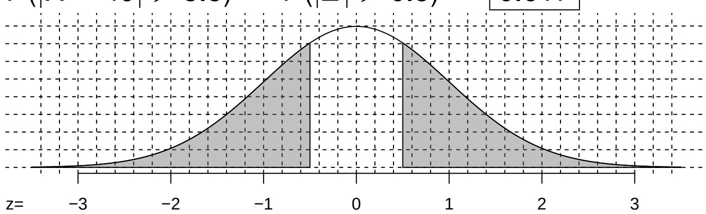
(b) $P(X > 60) = P(Z > 1) = \boxed{0.1587}$



(c) $P(|X - 49| < 6.6) = P(|Z| < 0.6) = \boxed{0.4514}$

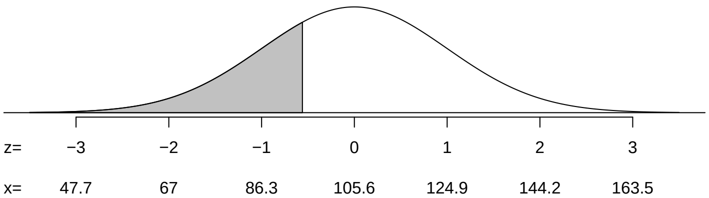


(d) $P(|X - 49| > 5.5) = P(|Z| > 0.5) = \boxed{0.617}$

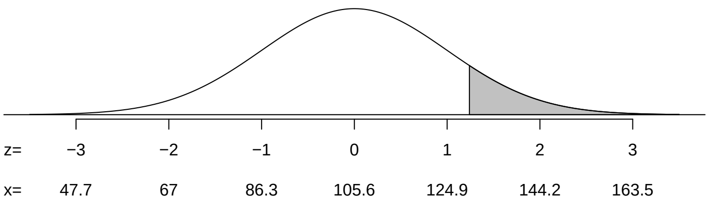


4. Notice the three probabilities will add up to 1.

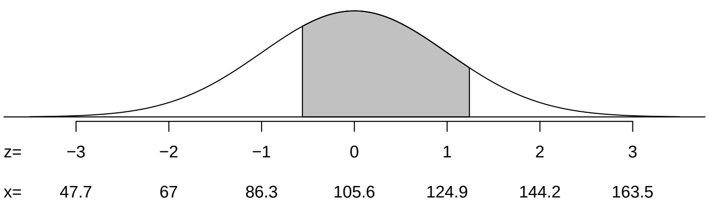
   (a) $P(X < 94.8) = P(Z < -0.56) = \boxed{0.2877}$



   (b) $P(X > 130) = P(Z > 1.24) = \boxed{0.1075}$



   (c) $P(94.8 < X < 130) = P(-0.56 < Z < 1.24) = \boxed{0.6048}$



5. (a) 49

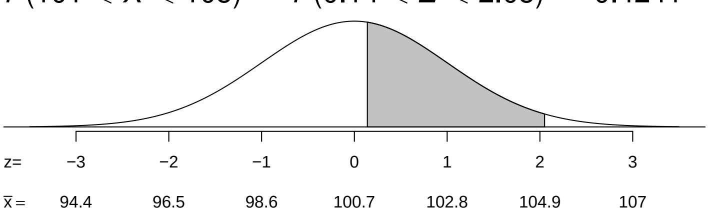
   (b) 1.25

   (c) 0.5279

   (d) 0.8665

6. (a) Central limit of average formulas: $\mu_{\bar{x}} = 100.7$ and $\sigma_{\bar{x}} = \frac{21}{\sqrt{100}} = 2.1$.

   (b) $P(101 < \overline{X} < 105) = P(0.14 < Z < 2.05) = 0.4241$



7. (a) We can recognize W is a Bernoulli variable with $p = 0.72$ and $q = 0.28$. Thus,

$$\mu_W = p = 0.72$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.72)(0.28)} = 0.449$$

Then, we use the central limit of average formulas:

$$\mu_{\hat{p}} = \mu_W = 0.72$$

and

$$\sigma_{\hat{p}} = \frac{\sigma_W}{\sqrt{n}} = \frac{0.449}{\sqrt{49}} = 0.0641$$

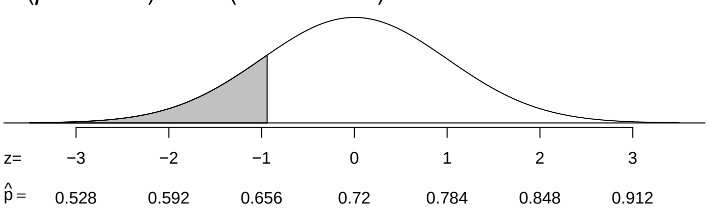
But, if we recognized $\hat{p}$ follows the formulas of a $\hat{p}$ **sampling distribution**:

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}} = \sqrt{\frac{pq}{n}}$$

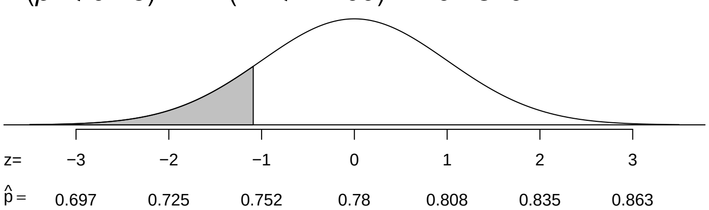
we could have just used those instead.

(b) $P(\hat{p} < 0.66) \;=\; P(Z < -0.94) \;=\; 0.1736$



8. (a) Use $\hat{p}$ sampling formulas: $\mu_{\hat{p}} = 0.78$ and $\sigma_{\hat{p}} = \frac{\sqrt{(0.78)(0.22)}}{\sqrt{225}} = 0.0276$.

(b) $P(\hat{p} < 0.75) \;=\; P(Z < -1.09) \;=\; 0.1379$



9. (a) 3584

(b) 48

(c) 0.8944

(d) 0.4129

10. We recognize W is a Bernoulli variable with $p = 0.34$ and $q = 0.66$. Thus,

$$\mu_W = p = 0.34$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.34)(0.66)} = 0.4737$$

.

We use the Central Limit Formulas (for a sum).

$$\mu_X = n\mu_W = (177)(0.34) = 60.18$$

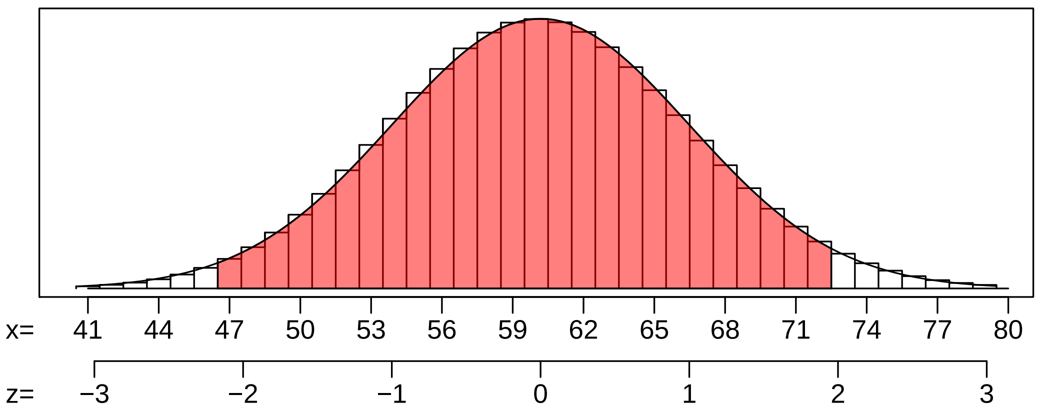
$$\sigma_X = \sqrt{n}\sigma_W = \sqrt{177}(0.4737) = 6.3023$$

It should be mentioned that you could have also just recognized X is binomial:

$$\mu = np = (177)(0.34) = 60.18$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{(177)(0.34)(1-0.34)} = 6.3023$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{46.5 - 60.18}{6.3023} = -2.17$$

$$z_2 = \frac{72.5 - 60.18}{6.3023} = 1.95$$

Find the percentiles (from z-table).

$$\ell_1 = 0.015$$

$$\ell_2 = 0.9744$$

Calculate the probability.

$$P(47 \leq X \leq 72) = 0.9744 - 0.015 = 0.9594$$

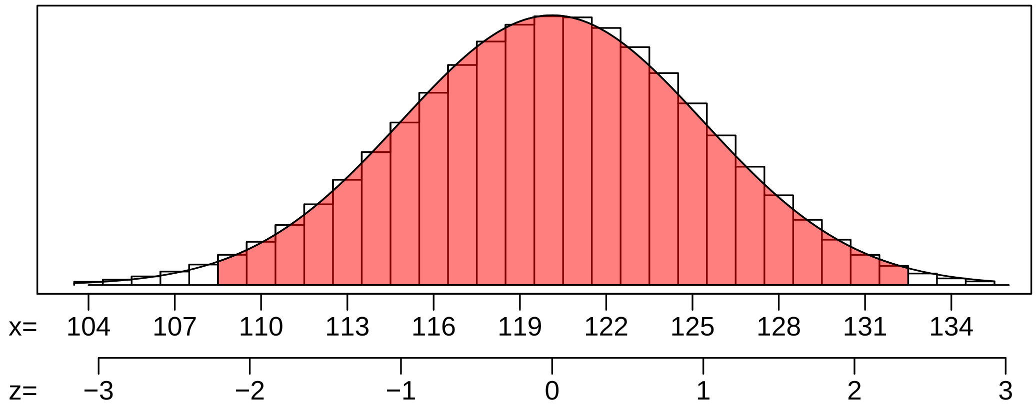
11. Find the mean.

$$\mu = np = (156)(0.77) = 120.12$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(156)(0.77)(1-0.77)} = 5.2562$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{108.5 - 120.12}{5.2562} = -2.21$$

$$z_2 = \frac{132.5 - 120.12}{5.2562} = 2.36$$

Find the percentiles (from z-table).

$$\ell_1 = 0.0136$$

$$\ell_2 = 0.9909$$

Calculate the probability.

$$P(108 < X < 133) = 0.9909 - 0.0136 = 0.977$$

Normal Distributions

$z = \frac{x-\mu}{\sigma}$
$x = \mu + z\sigma$

Central Limit Theorem

Let random variable W have mean μ_w and standard deviation σ_w.
Let random variable X represent the sum of n instances of W.
Let random variable Y represent the average of n instances of W.
Then:

$$\mu_x = (n)(\mu_w) \qquad \mu_y = \mu_w$$
$$\sigma_x = (\sigma_w)(\sqrt{n}) \qquad \sigma_y = \frac{\sigma_w}{\sqrt{n}}$$

and X and Y are both approximately normal.

Bernoulli Random Variable

$\mu = p$
$\sigma = \sqrt{pq}$

Binomial Random Variable (sum of Bernoullis)

$\mu = np$
$\sigma = \sqrt{npq}$

Proportion Sampling Random Variable (average of Bernoullis)

$\mu = p$
$\sigma = \sqrt{\frac{pq}{n}}$

Continuity Correction

- If:
  - X is a discrete variable with a support of consecutive integers
  - we are approximating X with a normal distribution
- Then we can apply a continuity correction:

$$P(X \leq x_0) = P\left(Z < \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X < x_0) = P\left(Z < \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right)$$
$$P(X \geq x_0) = P\left(Z > \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X > x_0) = P\left(Z > \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right)$$