Normal standardization

$$z = \frac{x - \mu}{\sigma} \qquad x = \mu + z\sigma$$

Central Limit Theorem

If:

- W is "any" random variable with mean = μ_w and standard deviation = σ_w.
- Random variable X is **sum** of n instances of W.

$$X = W_1 + W_2 + W_3 + \cdots + W_n$$

- Random variable Y is **average** of n instances of W.

$$Y = \frac{W_1 + W_2 + W_3 + \cdots + W_n}{n}$$

Then:

- The following formulas are exactly true:

$$\mu_x = n\mu_w \qquad \mu_y = \mu_w$$
$$\sigma_x = \sigma_w\sqrt{n} \qquad \sigma_y = \frac{\sigma_w}{\sqrt{n}}$$

- X and Y are approximately normal (if $n \geq 30$)
- X and Y are exactly normal if W is normal

Special case of central limit theorem: Bernoulli, Binomial, and $\hat{p}$ sampling

If:

- W is a Bernoulli random variable:

w	$P(w)$
0	q
1	p

- X is sum of n instances of W (X is binomial)
- $\hat{p}$ is average of n instances of W (proportion sampling)

Then:

- The following are exactly true:

$$\mu_w = p \qquad \mu_x = np \qquad \mu_{\hat{p}} = p$$
$$\sigma_w = \sqrt{pq} \qquad \sigma_x = \sqrt{pq}\sqrt{n} \qquad \sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}}$$

- X and $\hat{p}$ are approximately normal (if $np \geq 10$ and $nq \geq 10$)

Question 1

A farm produces 4 types of fruit: A, B, C, and D. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	80	8
B	130	10
C	95	15
D	115	5

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	92
B	142
C	81.5
D	109

a. Determine a z-score for each specimen.

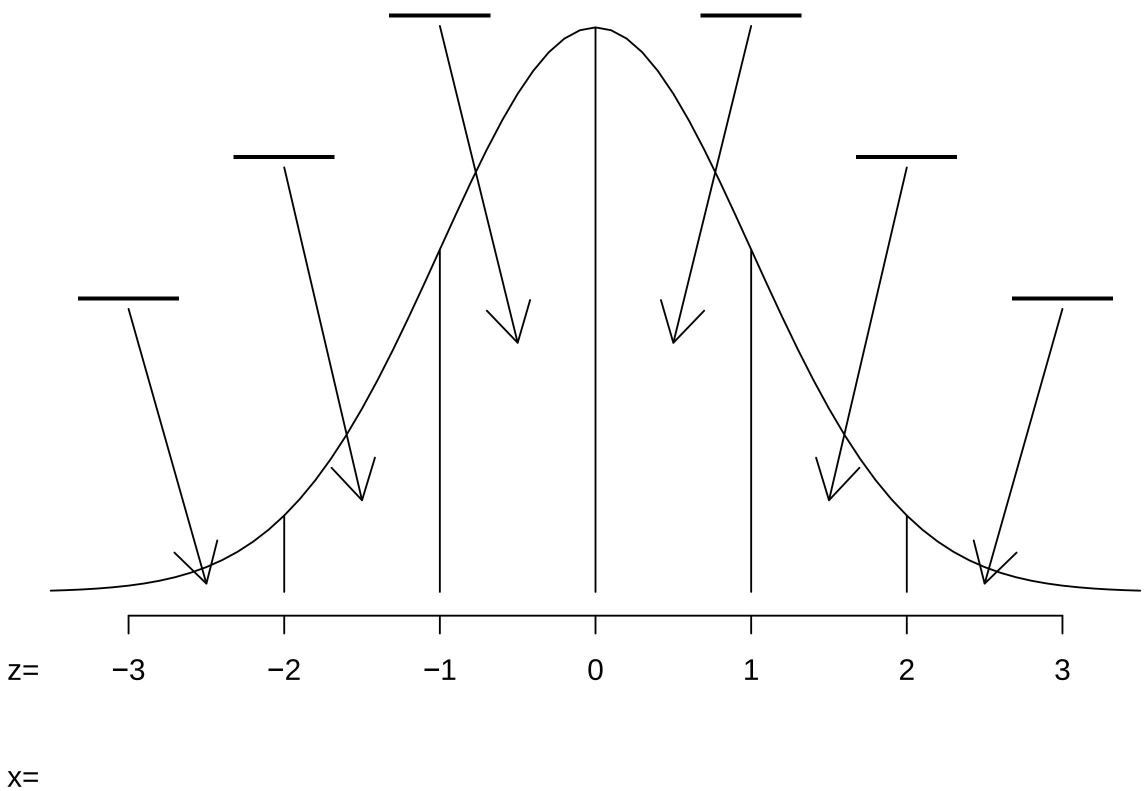
b. Which specimen was most unusually large?

c. Which specimen was most unusually small?

Question 2

A normal random variable X has a mean $\mu = 77$ and standard deviation $\sigma = 13$. Please label the density curve with:

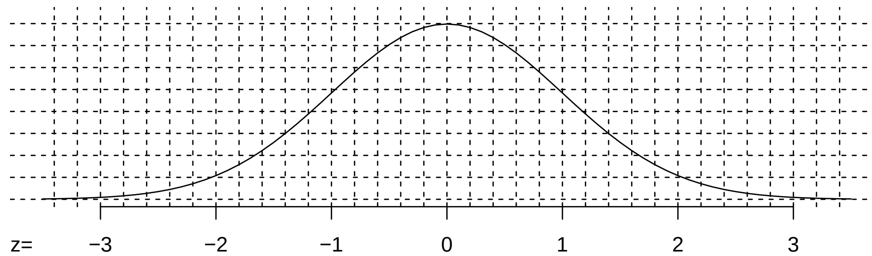
a. The appropriate values of x.
b. The areas of the sections.



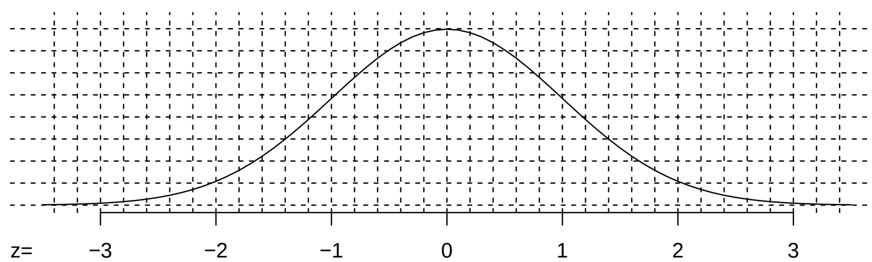
Question 3

Let X be normally distributed with mean 25 and standard deviation 5. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

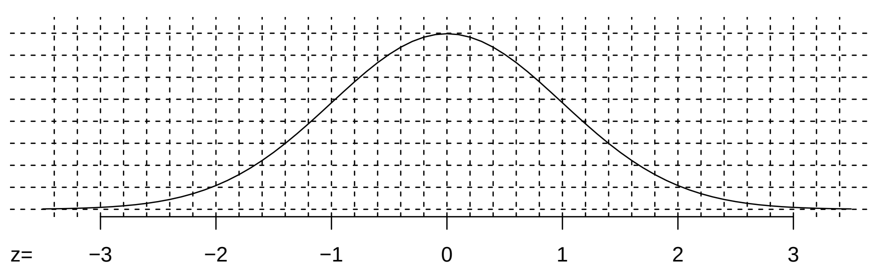
a. $P(X < 21)$



b. $P(X > 23)$



c. $P(17 < X < 33)$



Question 4

Let random variable W have a **mean of** $\mu_w = 10.50$ and a **standard deviation of** $\sigma_w = 5.77$.

Let X represent the sum of $n = 81$ instances of W, and let Y represent the average of $n = 81$ instances of W.

a. Determine μ_x

b. Determine σ_x

c. What is the probability that X is between 773 and 928? *Do NOT use a continuity correction.*

d. Determine μ_y

e. Determine σ_y

f. What is the probability that Y is between 10.2 and 10.8? *Do NOT use a continuity correction.*

Question 5

An unfair coin has a $p = 0.63$ chance of landing tails. When $n = 100$ of these unfair coins are flipped, what is the probability of getting at least 55 but at most 71 tails? Please use a **normal approximation** with a **continuity correction**.

Question 6

About 8% of men are color blind ($p = 0.08$). If you gather a simple random sample of $n = 121$ men, what is the probability that the sample proportion ($\hat{p}$) is between 0.03 and 0.07? Please use a **normal approximation**, but *do NOT use a continuity correction*.

Extra Credit

1. Let random variable X be normally distributed with mean $\mu = 85$ and standard deviation $\sigma = 10$. Determine a such that $P(|X - 85| < a) = 0.8064$.

2. A population has a proportion $p = 0.7$. The sample proportion is $\hat{p}$. Determine n such that $P(0.68 < \hat{p} < 0.72) = 0.9544$. In other words, determine the necessary sample size such that the sample proportion is between 0.68 and 0.72 in 95.44% of random samples of that size.