1. **Problem**
   A farm produces 4 types of fruit: A, B, C, and D. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	73	9
B	111	5
C	64	7
D	66	6

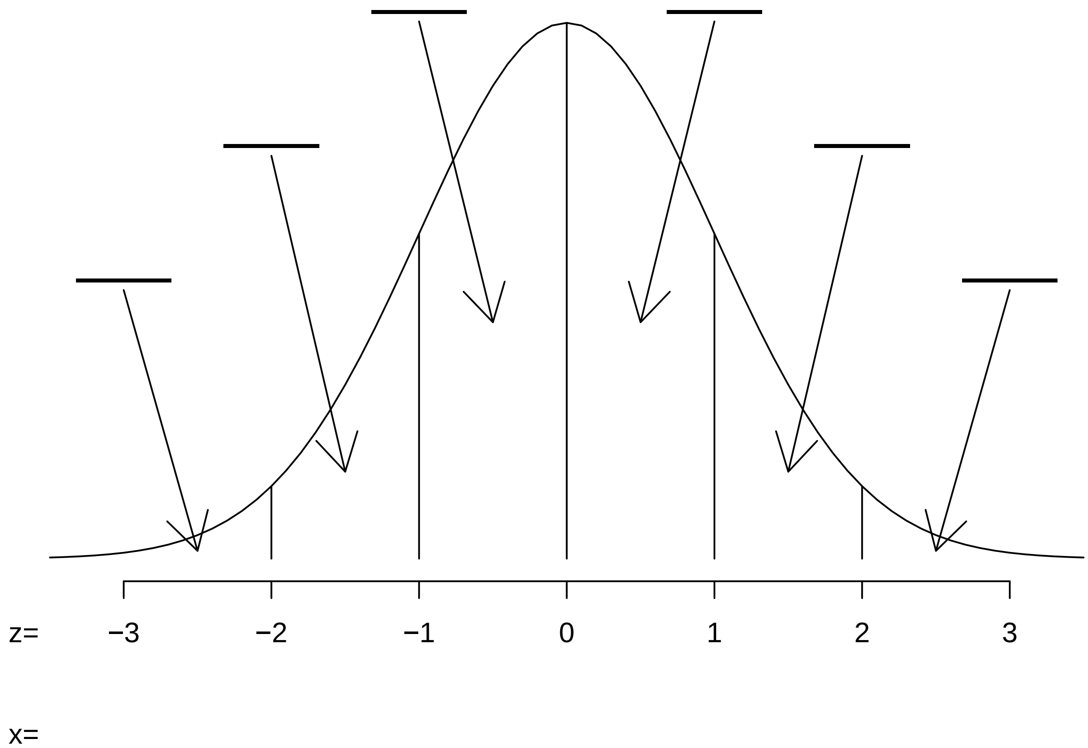
One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	87.94
B	108.6
C	68.62
D	68.46

Which specimen is the most unusually large (relative to others of its type)?

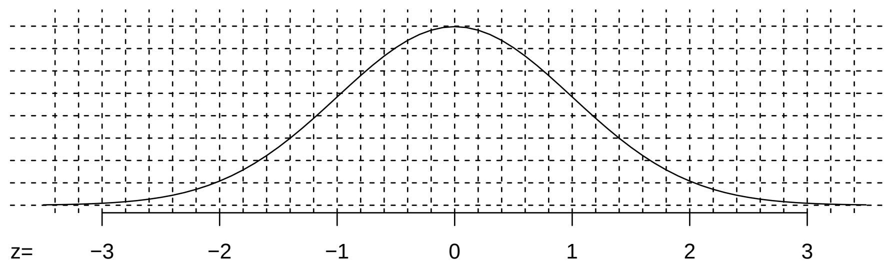
2. **Problem**
A normal random variable X has a mean $\mu = 3.9$ and standard deviation $\sigma = 0.7$. Please label the density curve with:

(a) The appropriate values of x.
(b) The areas of the sections.

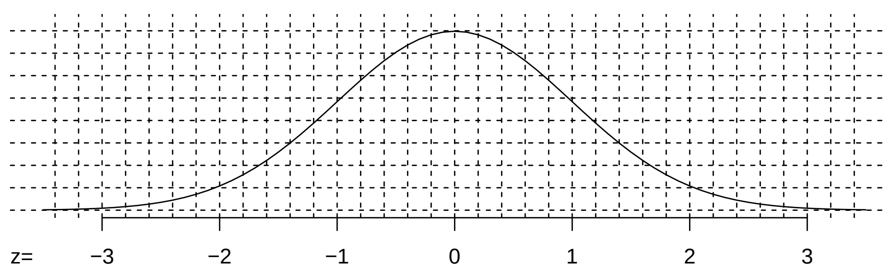


3. **Problem**
   Let X be normally distributed with mean 56 and standard deviation 14. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

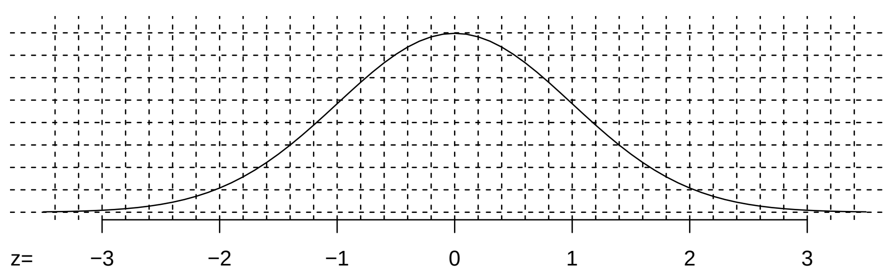
   (a) $P(X < 68.6)$



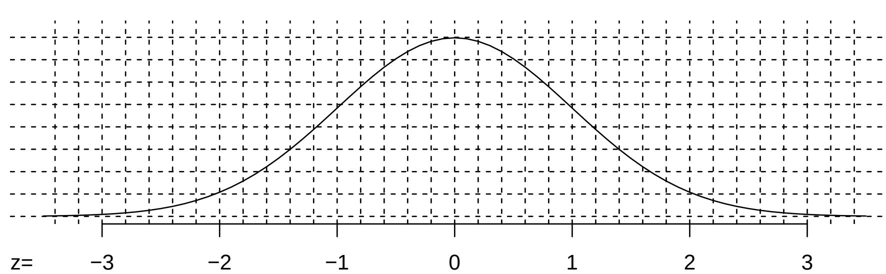
   (b) $P(X > 61.6)$



   (c) $P(|X - 56| < 7)$



   (d) $P(|X - 56| > 1.4)$



4. **Problem**
   Let X be normally distributed with mean 115 and standard deviation 3.3. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

   (a) What's the probability that X is less than 116? **Draw a sketch**.

   (b) What's the probability that X is more than 120? **Draw a sketch**.

   (c) What's the probability that X is between 116 and 120? **Draw a sketch**.

5. **Problem**
   Let random variable W have mean $\mu_W = 57$ and standard deviation $\sigma_W = 14$. Let random variable X represent the **average** of $n = 49$ instances of W.

   (a) Determine the expected value of X. $\mu_X = ?$

   (b) Determine the standard deviation of X. $\sigma_X = ?$

   (c) Using normal approximation, determine $P(X < 58.22)$.

   (d) Using normal approximation, determine $P(X > 53.82)$.

6. **Problem**
A very large population has a mean of 94.4 and a standard deviation of 9.6. When a random sample of size 36 is taken, what is the probability that the **sample mean** ($\bar{x}$) is between 91.9 and 94?

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(91.9 < \overline{X} < 94)$. **Draw a sketch**

7. **Problem**
Let random variable W have the probability distribution shown below.

w	$P(w)$
0	0.84
1	0.16

Let random variable $\hat{p}$ (sample proportion) represent the average of $n = 121$ instances of W.

(a) Determine the mean and standard deviation of the $\hat{p}$.

(b) Determine $P(\hat{p} < 0.1)$. Do NOT use a continuity correction. **Draw a sketch**

8. **Problem**
A very large population has a population proportion $p = 0.35$. When a random sample of size 36 is taken, what is the probability that the **sample proportion** ($\hat{p}$) is less than 0.42?

Do NOT use a continuity correction.

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(\hat{p} < 0.42)$. **Draw a sketch**

9. **Problem**
Let random variable W have mean $\mu_W = 39$ and standard deviation $\sigma_W = 2$. Let random variable X represent the **sum** of $n = 225$ instances of W.

   (a) Determine the expected value of X. $\mu_X = ?$
   (b) Determine the standard deviation of X. $\sigma_X = ?$
   (c) Using normal approximation, determine $P(X < 8789.7)$.
   (d) Using normal approximation, determine $P(X > 8770.8)$.

10. **Problem**
    Let random variable W have the probability distribution shown below.

w	$P(w)$
0	0.35
1	0.65

Let random variable X represent the sum of $n = 211$ instances of W. (Thus X is the sample total, or number of successes.)

What is the probability that X is at least 123 but at most 152? **Use a normal approximation with continuity corrections**.

11. **Problem**

Let each trial have a chance of success $p = 0.35$. If 80 trials occur, what is the probability of getting more than 24 but less than 30 successes?

In other words, let $X \sim \text{Bin}(n = 80,\ p = 0.35)$ and find $P(24 < X < 30)$.

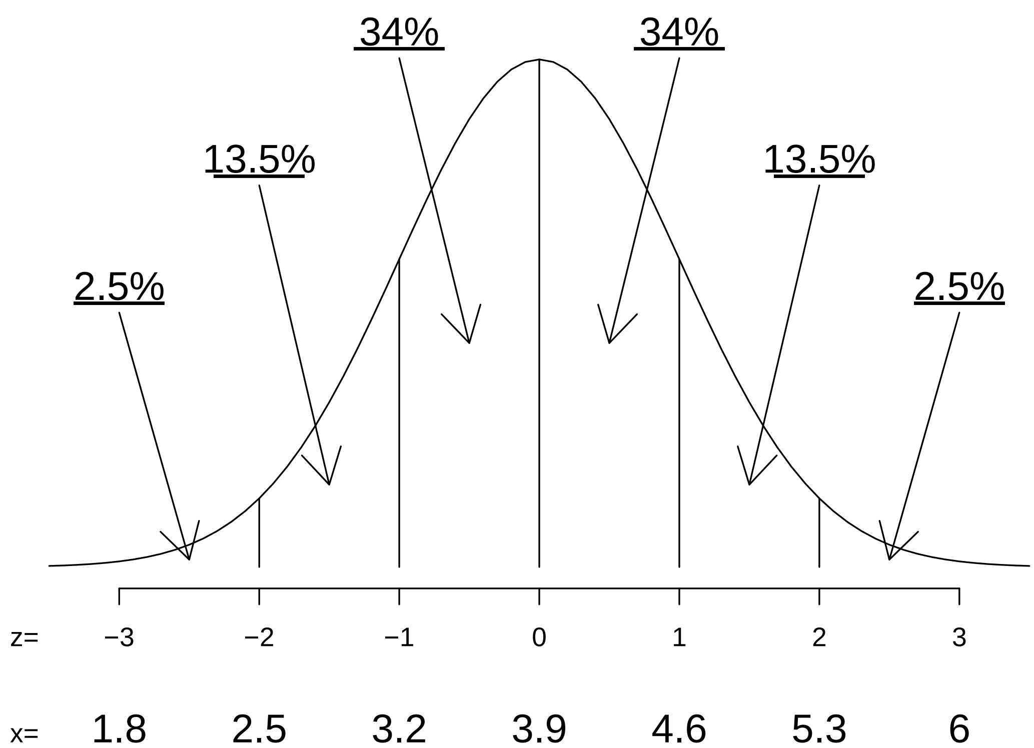
Use a normal approximation along with the continuity correction.

1. We compare the z-scores. The largest z-score corresponds to the specimen that is most unusually large.

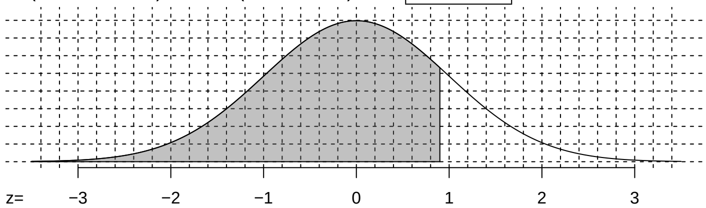
Type of fruit	formula	z-score
A	$z = \frac{87.94-73}{9}$	1.66
B	$z = \frac{108.6-111}{5}$	-0.49
C	$z = \frac{68.62-64}{7}$	0.66
D	$z = \frac{68.46-66}{6}$	0.41

Thus, the specimen of type A is the most unusually large.

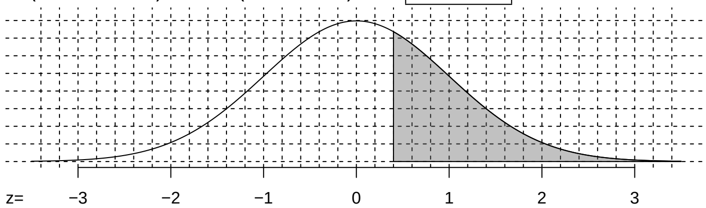
2. The filled in areas and x values are shown below.



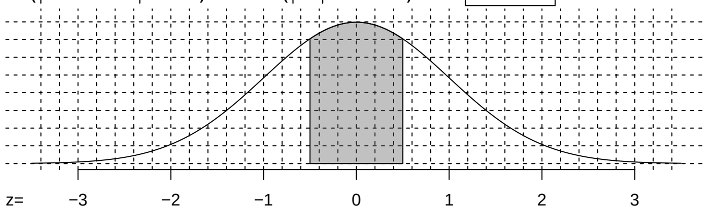
3. (a) $P(X < 68.6) = P(Z < 0.9) = \boxed{0.8159}$



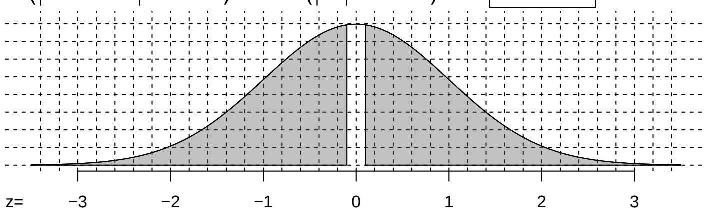
(b) $P(X > 61.6) = P(Z > 0.4) = \boxed{0.3446}$



(c) $P(|X - 56| < 7) = P(|Z| < 0.5) = \boxed{0.383}$

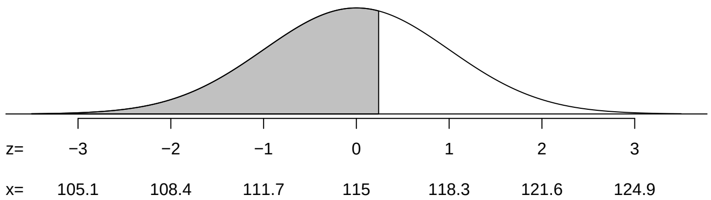


(d) $P(|X - 56| > 1.4) = P(|Z| > 0.1) = \boxed{0.9204}$

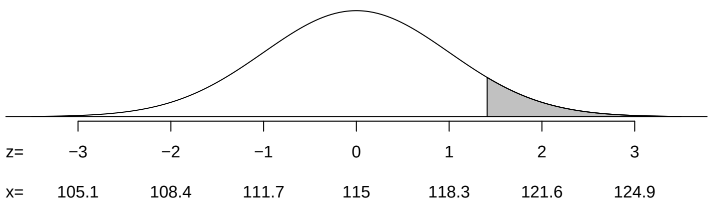


4. Notice the three probabilities will add up to 1.

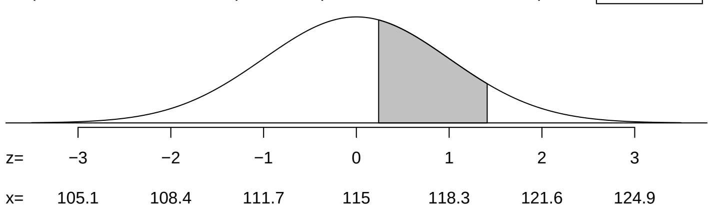
   (a) $P(X < 116) \;=\; P(Z < 0.24) \;=\; \boxed{0.5948}$



   (b) $P(X > 120) \;=\; P(Z > 1.41) \;=\; \boxed{0.0793}$



   (c) $P(116 < X < 120) \;=\; P(0.24 < Z < 1.41) \;=\; \boxed{0.3259}$



5. (a) 57

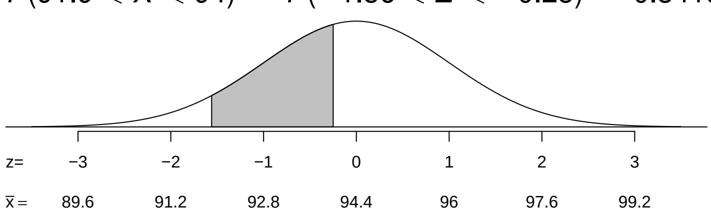
   (b) 2

   (c) 0.7291

   (d) 0.9441

6. (a) Central limit of average formulas: $\mu_{\bar{x}} = 94.4$ and $\sigma_{\bar{x}} = \frac{9.6}{\sqrt{36}} = 1.6$.

   (b) $P(91.9 < \overline{X} < 94) \;=\; P(-1.56 < Z < -0.25) \;=\; 0.3419$



7. (a) We can recognize W is a Bernoulli variable with $p = 0.16$ and $q = 0.84$. Thus,

$$\mu_W = p = 0.16$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.16)(0.84)} = 0.3666$$

Then, we use the central limit of average formulas:

$$\mu_{\hat{p}} = \mu_W = 0.16$$

and

$$\sigma_{\hat{p}} = \frac{\sigma_W}{\sqrt{n}} = \frac{0.3666}{\sqrt{121}} = 0.0333$$

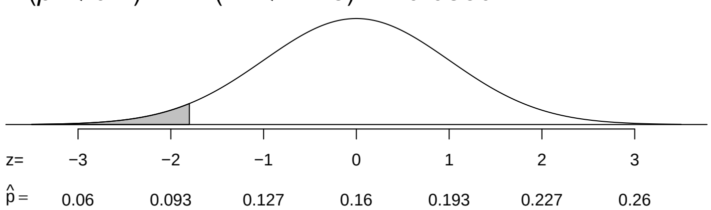
But, if we recognized $\hat{p}$ follows the formulas of a $\hat{p}$ **sampling distribution**:

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}} = \sqrt{\frac{pq}{n}}$$

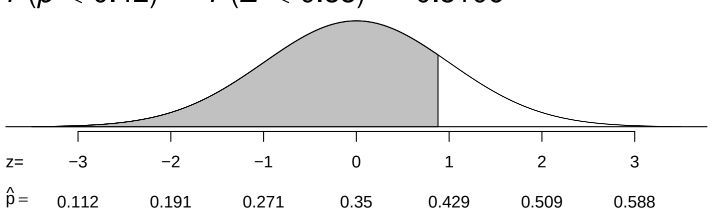
we could have just used those instead.

(b) $P(\hat{p} < 0.1) \;=\; P(Z < -1.8) \;=\; 0.0359$



8. (a) Use $\hat{p}$ sampling formulas: $\mu_{\hat{p}} = 0.35$ and $\sigma_{\hat{p}} = \frac{\sqrt{(0.35)(0.65)}}{\sqrt{36}} = 0.0795$.

(b) $P(\hat{p} < 0.42) \;=\; P(Z < 0.88) \;=\; 0.8106$



9. (a) 8775

(b) 30

(c) 0.6879

(d) 0.5557

10. We recognize W is a Bernoulli variable with $p = 0.65$ and $q = 0.35$. Thus,

$$\mu_W = p = 0.65$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.65)(0.35)} = 0.477$$

.

We use the Central Limit Formulas (for a sum).

$$\mu_X = n\mu_W = (211)(0.65) = 137.15$$

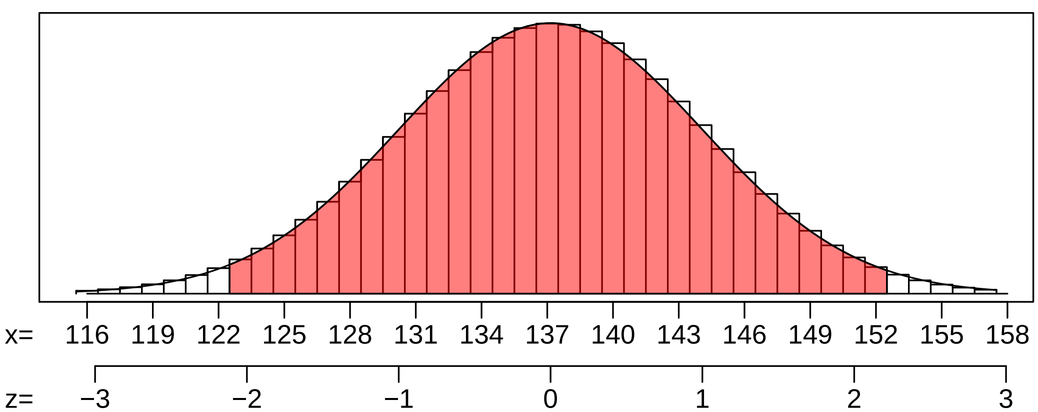
$$\sigma_X = \sqrt{n}\sigma_W = \sqrt{211}(0.477) = 6.9284$$

It should be mentioned that you could have also just recognized X is binomial:

$$\mu = np = (211)(0.65) = 137.15$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{(211)(0.65)(1-0.65)} = 6.9284$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{122.5 - 137.15}{6.9284} = -2.11$$

$$z_2 = \frac{152.5 - 137.15}{6.9284} = 2.22$$

Find the percentiles (from z-table).

$$\ell_1 = 0.0174$$

$$\ell_2 = 0.9868$$

Calculate the probability.

$$P(123 \leq X \leq 152) = 0.9868 - 0.0174 = 0.9694$$

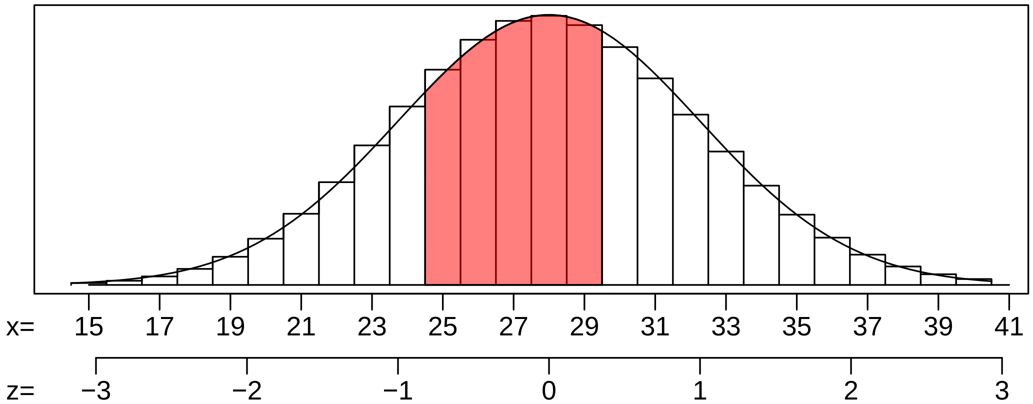
11. Find the mean.

$$\mu = np = (80)(0.35) = 28$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(80)(0.35)(1-0.35)} = 4.2661$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{24.5 - 28}{4.2661} = -0.82$$

$$z_2 = \frac{29.5 - 28}{4.2661} = 0.35$$

Find the percentiles (from z-table).

$$\ell_1 = 0.2061$$

$$\ell_2 = 0.6368$$

Calculate the probability.

$$P(24 < X < 30) = 0.6368 - 0.2061 = 0.431$$

Normal Distributions

$z = \frac{x-\mu}{\sigma}$
$x = \mu + z\sigma$

Central Limit Theorem

Let random variable W have mean μ_w and standard deviation σ_w.
Let random variable X represent the sum of n instances of W.
Let random variable Y represent the average of n instances of W.
Then:

$$\mu_x = (n)(\mu_w) \qquad \mu_y = \mu_w$$

$$\sigma_x = (\sigma_w)(\sqrt{n}) \qquad \sigma_y = \frac{\sigma_w}{\sqrt{n}}$$

and X and Y are both approximately normal.

Bernoulli Random Variable

$\mu = p$
$\sigma = \sqrt{pq}$

Binomial Random Variable (sum of Bernoullis)

$\mu = np$
$\sigma = \sqrt{npq}$

Proportion Sampling Random Variable (average of Bernoullis)

$\mu = p$
$\sigma = \sqrt{\frac{pq}{n}}$

Continuity Correction

- If:
  - X is a discrete variable with a support of consecutive integers
  - we are approximating X with a normal distribution
- Then we can apply a continuity correction:

$$P(X \leq x_0) = P\left(Z < \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X < x_0) = P\left(Z < \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right)$$

$$P(X \geq x_0) = P\left(Z > \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X > x_0) = P\left(Z > \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right)$$