Name: ______________________ **Section: MAT098/181 C-**

A scientific calculator is permitted. ***Cellphones may not be used as calculators and must be off or on vibrate during the exam****. Show all work on the test or on the work paper*

MAT098/181C EXAM #3 (FORM A Key)

1. Jose is a student in a College Algebra class and the following table summarizes Jose's scores for these three tests along with the class average and standard deviation for each test. *(15 points)*

Exam Number	Jose's Score	Class Average	Standard Deviation
Exam #1	70	75	10
Exam #2	79	67	8
Exam #3	85	80	5

a) Assuming the test scores are normally distributed, calculate Jose's Z-score for each of the three exams.

Exam #1 z-score: ________________

$$z = \frac{75 - 70}{10} = 0.5$$

Exam #2 z-score: ________________

$$z = \frac{79 - 67}{8} = 1.5$$

Exam #3 z-score: ________________

$$z = \frac{85 - 80}{5} = 1$$

b) On which exam did Jose do the best on RELATIVE to the rest of his class?

Exam #2

c) EXPLAIN your reasoning your answer in part b)

The z-score is larger and above the mean.

2. The GPA's of BHCC students are normally distributed with a mean = 3.0 and a standard deviation = 0.3. **Please label the graph below with the following: *(12 points)***

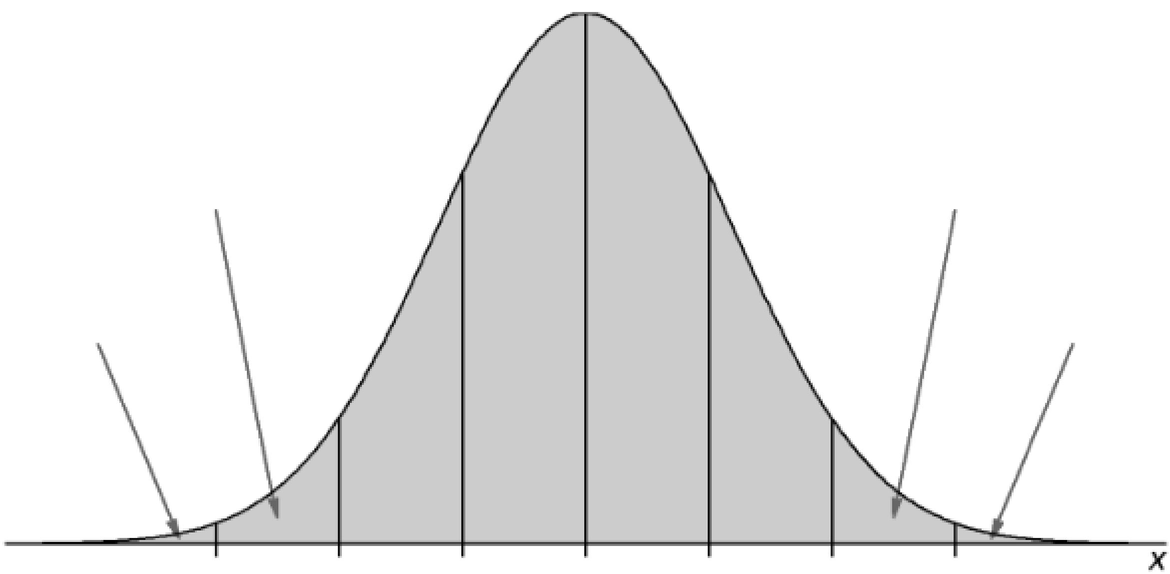
a) The tick marks on the x-axis of the graph below are one standard deviation apart. Label the axis with the ***appropriate GPA values***.

b) ***Label the Z-score*** of each value below its x-value

c) Using the Empirical rule, label each region of the graph with the area for that region

d) What interval will contain 95% of the GPA's around the mean?

$$(\mathbf{3.0 - 2 \times 0.3, 3.0 + 2 \times 0.3) = (2.4, 3.6})$$



3. Let x be a random variable that represents the length of time it takes a student to write a term paper for Dr. Adam's Sociology class. After interviewing many students, it was found that x has an approximately normal distribution with mean μ = 7.3 hours and standard deviation σ = 0.8 hours. *(18 points: 4,4,4,2,2,2)*

   For parts a, b, and c, convert each of the following x intervals to standardized z intervals. Find the probabilities.

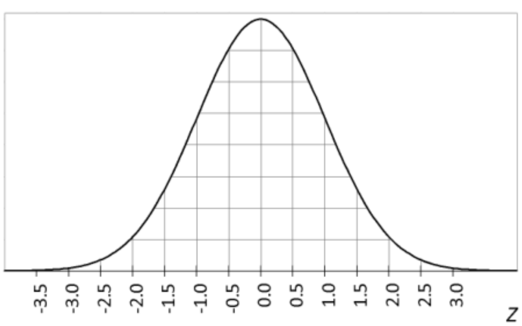
   a) P(x < 7.7) = $= \boldsymbol{P\left(z < \frac{7.7-7.3}{0.8}\right) = P(z < 0.5) = .6915}$

   b.) P(x > 9.3) = $= \boldsymbol{P\left(z > \frac{9.3-7.3}{0.8}\right) = P(z > 2.5) = 1 - .9938 = .0062}$

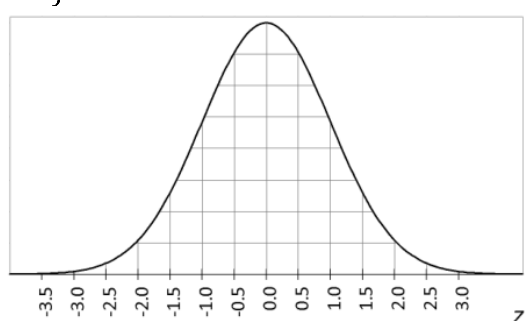
   c.) P(5.7 < x < 8) = $= \boldsymbol{P(-2 < z < 0.88) = .8106 - .0228 = .7878}$

   For the z intervals you calculated above, shade the area under the curve that represents the associated probability.

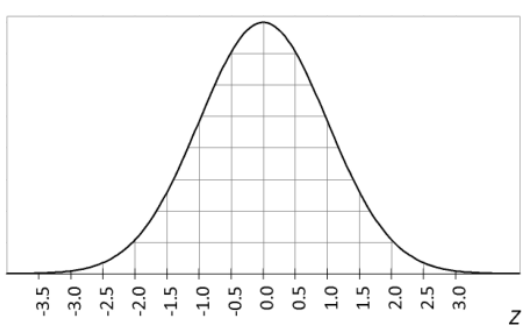
   a)



   b)



   c)



4. ***Draw a sketch for each part.** (*15 points*) Weights of a certain model of fully loaded gravel truck follow a normal distribution with mean μ = 6.4 tons and standard deviation σ = 0.3 ton. What is the probability that a fully loaded truck of this model is

a) less than 6 tons.

$$P(x < 6) = P(z < -1.33) = .0918$$

b) more than 7 tons.

$$P(x > 7) = P(z > 2) = 1 - .9772 = .0228$$

c) between 6 and 7 tons.

$$P(6 < x < 7) = P(-1.33 < z < 2) = .9772 - .0918 = .8854$$

5. The manufacturer of a new compact car claims that the miles per gallon (mpg) for the gasoline consumption is mound-shaped and symmetric with mean $\mu = 25.9$ mpg and standard deviation $\sigma = 9.5$ mpg. If 30 such cars are tested, what is the probability the ***average (mean)*** mpg $\bar{x}$ is between 23 and 28 mpg? ***Draw a sketch.** *(15 points)*

$$P(23 < \bar{x} < 28) = P\left(\frac{23 - 25.9}{9.5/\sqrt{30}} z < \frac{28 - 25.9}{9.5/\sqrt{30}}\right)$$

$$= P(-1.67 < z < 1.21) = 0.8869 - 0.0475 = 0.8394$$

6. A national caterer determined that 87% of the people who sampled their food said that it was delicious. A random sample of 144 people is obtained from a population of 5000. The 144 people are asked to sample the caterer's food. If $\hat{p}$ is the sample proportion saying that the food is delicious, what is the mean of the sampling distribution of $\hat{p}$? *(10 points)*

   a) Will the distribution of $\hat{p}$, the sample proportion saying that the food is delicious, be approximately normal? Are the conditions met?

$$np = 144 \cdot 0.87 = 125.28 \geq 10 \quad and \quad nq = 144 \cdot 0.13 = 18.72 \geq 10$$

   b) What is the mean of the sampling distribution of $\hat{p}$?

$$p = 0.87$$

   c) What is the standard deviation?

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.87 \cdot 0.13}{144}} = 0.0280$$

7. Psychology-231 can be taken as an online course on a pass/fail basis. Long experience with this course shows that about 71% of the students pass. This semester, 88 students are taking Psychology-231 online. Let x be a random variable that represents the number that will pass. The psychology department wants a probability distribution for x. *(15 points)*

(a) Write a brief description of why the normal approximation to the binomial would apply. Are the assumptions satisfied? Explain.

$$\boldsymbol{np = 88 \cdot 0.71 = 62.48 \geq 10 \quad and \quad nq = 88 \cdot 0.29 = 25.52 \geq 10}$$

(b) $P(x \geq 60)$ ***Draw a sketch.**

$$\boldsymbol{P(x \geq 59.5) = P\left(z \geq \frac{59.5 - 62.48}{\sqrt{88 \cdot 0.71 \cdot 0.29}}\right) = P(z \geq -0.70) = 1 - .2420 = .7580}$$

****EXTRA CREDIT:**

1. Suppose a brewery has a filling machine that fills 12-ounce bottles of beer. It is known that the amount of beer poured by this filling machine follows a normal distribution with a mean of 12.23 ounces and a standard deviation of 0.04 ounce. The company is interested in reducing the amount of extra beer that is poured into the 12 ounce bottles. The company is seeking to identify the highest 1.5% of the fill amounts poured by this machine. For what fill amount are they searching? Round to the nearest thousandth

   Left Area is $1 - 1.5\% = .9850$

   z-score is 2.17

$$x = \mu + z \cdot \sigma = 12.23 + 2.17 \cdot 0.04 \approx 12.317$$

2. Find the z value such that 95% of the area under a standard normal curve lies between –z and z.

   Left Area is $\frac{1-95\%}{2} = .0250$

   z-score is -1.96 and 1.96

Formula sheet:

Empirical Rule

- about 68% of the *x* values lie within 1 standard deviation of the mean.
- about 95% of the *x* values lie within 2 standard deviations of the mean.
- about 99.7% of the *x* values lie within 3 standard deviations of the mean.

z-score

$$z = \frac{x - \mu}{\sigma}$$

Central Limit Theorem

Mean of the sample mean is $\mu_{\bar{x}} = \mu$

Standard deviation of the sample mean is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

z-score for sample mean

$$z = \frac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

Binomial Distribution

Mean: $\mu = np$

Standard Deviation: $\sigma = \sqrt{np(1 - p)}$

Sampling Distribution of Sample Proportion

Mean: p

Standard Deviation: $\sigma = \sqrt{\frac{p(1-p)}{n}}$