

1. Problem:

A continuous random variable X was measured 12 times. The sorted data are shown below, along with each datum's index (ordinal rank).

i	x
1	71.169
2	77.925
3	80.589
4	82.451
5	85.374
6	98.129
7	98.353
8	103.815
9	104.102
10	105.836
11	110.491
12	122.708

The total of the measurements is 1140.942.

- (a) Determine the percentile of the value 110.491. In other words, determine what percent of data are less than or equal to 110.491.
- (b) Determine the 83.3th percentile. In other words, determine x such that 83.3% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 110.491$. This means $i = 11$. We know $n = 12$. Determine the percentile ℓ .

$$\ell = \frac{11}{12}$$

$$\ell = 0.917$$

So, the percentile is 0.917, or 91.7th percentile.

- (b) We are given $\ell = 0.833$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n}\right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (12)(0.833)$$

$$i = 10$$

Determine the x associated with $i = 10$.

$$x = \text{105.836}$$

- (c) The mean: $\bar{x} = \frac{1140.942}{12} = \text{95.078}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 12$ and so n is even.

$$\text{median} = \frac{x_6 + x_7}{2} = \frac{98.129 + 98.353}{2}$$

So, median = 98.241.

2. Problem:

A continuous random variable X was measured 6 times. The sorted data are shown below, along with each datum's index (ordinal rank).

i	x
1	80.253
2	81.075
3	81.089
4	84.161
5	84.825
6	88.305

The total of the measurements is 499.708.

- (a) Determine the percentile of the value 81.089. In other words, determine what percent of data are less than or equal to 81.089.
- (b) Determine the 33.3th percentile. In other words, determine x such that 33.3% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 81.089$. This means $i = 3$. We know $n = 6$. Determine the percentile ℓ .

$$\ell = \frac{3}{6}$$

$$\ell = 0.5$$

So, the percentile is 0.5, or 50th percentile.

- (b) We are given $\ell = 0.333$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (6)(0.333)$$

$$i = 2$$

Determine the x associated with $i = 2$.

$$x = \text{81.075}$$

- (c) The mean: $\bar{x} = \frac{499.708}{6} = \text{83.285}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 6$ and so n is even.

$$\text{median} = \frac{x_3 + x_4}{2} = \frac{81.089 + 84.161}{2}$$

So, median = 82.625.

3. Problem:

A continuous random variable X was measured 50 times. The sorted data are shown below.

71.834	72.768	73.442	74.507	74.924	76.367	77.012	77.109	77.259	77.277
78.293	78.770	79.865	79.904	82.711	83.252	83.859	84.336	84.789	86.212
86.858	86.902	86.959	87.882	87.944	88.315	88.749	90.611	93.660	93.694
93.737	94.342	94.577	95.016	95.327	100.167	102.098	103.201	103.386	104.547
104.901	105.637	107.647	109.338	110.962	112.341	112.733	124.337	128.635	128.889

The total of the measurements is 4597.882.

- (a) Determine the percentile of the value 112.733. In other words, determine what percent of data are less than or equal to 112.733.
- (b) Determine the 18th percentile. In other words, determine x such that 18% of the data are less than or equal to x .
- (c) Determine the mean of the measurements.
- (d) Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 112.733$. This means $i = 47$. We know $n = 50$. Determine the percentile ℓ .

$$\ell = \frac{47}{50}$$

$$\ell = 0.94$$

So, the percentile is 0.94, or 94th percentile.

- (b) We are given $\ell = 0.18$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (50)(0.18)$$

$$i = 9$$

Determine the x associated with $i = 9$.

$$x = \text{77.259}$$

- (c) The mean: $\bar{x} = \frac{4597.882}{50} = \text{91.958}$

- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 50$ and so n is even.

$$\text{median} = \frac{x_{25} + x_{26}}{2} = \frac{87.944 + 88.315}{2}$$

So, median = 88.1295.

4. Problem:

A continuous random variable X was measured 90 times. The sorted data are shown below.

30.002	30.012	30.021	30.023	30.053	30.055	30.108	30.139	30.176	30.244
30.283	30.285	30.302	30.318	30.504	30.542	30.562	30.569	30.719	30.729
30.736	30.768	30.867	31.013	31.069	31.145	31.300	31.304	31.360	31.367
31.558	31.626	31.656	31.711	31.728	31.928	31.937	31.973	32.064	32.095
32.176	32.209	32.334	32.413	32.423	32.569	32.669	32.682	32.748	32.778
32.793	32.901	33.136	33.389	33.420	33.519	33.619	33.719	33.894	33.936
33.940	33.994	34.002	34.005	34.087	34.101	34.134	34.141	34.141	34.208
34.323	34.332	34.384	34.486	34.624	34.721	34.776	34.807	34.854	34.867
34.878	34.880	34.893	34.900	34.900	34.918	34.930	34.942	34.945	34.950

The total of the measurements is 2930.241.

- Determine the percentile of the value 30.569. In other words, determine what percent of data are less than or equal to 30.569.
- Determine the 25.6th percentile. In other words, determine x such that 25.6% of the data are less than or equal to x .
- Determine the mean of the measurements.
- Determine the median of the measurements.

Solution: Let x represent a datum of interest. Let i represent that datum's index. Let ℓ represent that datum's percentile. Let n represent the sample size (number of measurements). In general,

$$\ell = \frac{i}{n}$$

- (a) We are given $x = 30.569$. This means $i = 18$. We know $n = 90$. Determine the percentile ℓ .

$$\ell = \frac{18}{90}$$

$$\ell = 0.2$$

So, the percentile is 0.2, or 20th percentile.

- (b) We are given $\ell = 0.256$. We can use algebra to solve for i .

$$\ell = \frac{i}{n}$$

Multiply both sides by n .

$$n \cdot (\ell) = n \cdot \left(\frac{i}{n} \right)$$

Simplify both sides.

$$n\ell = i$$

To make me happy, switch the sides.

$$i = n\ell$$

Now, we can evaluate i .

$$i = (90)(0.256)$$

$$i = 23$$

Determine the x associated with $i = 23$.

$$x = \text{30.867}$$

- (c) The mean: $\bar{x} = \frac{2930.241}{90} = \text{32.558}$

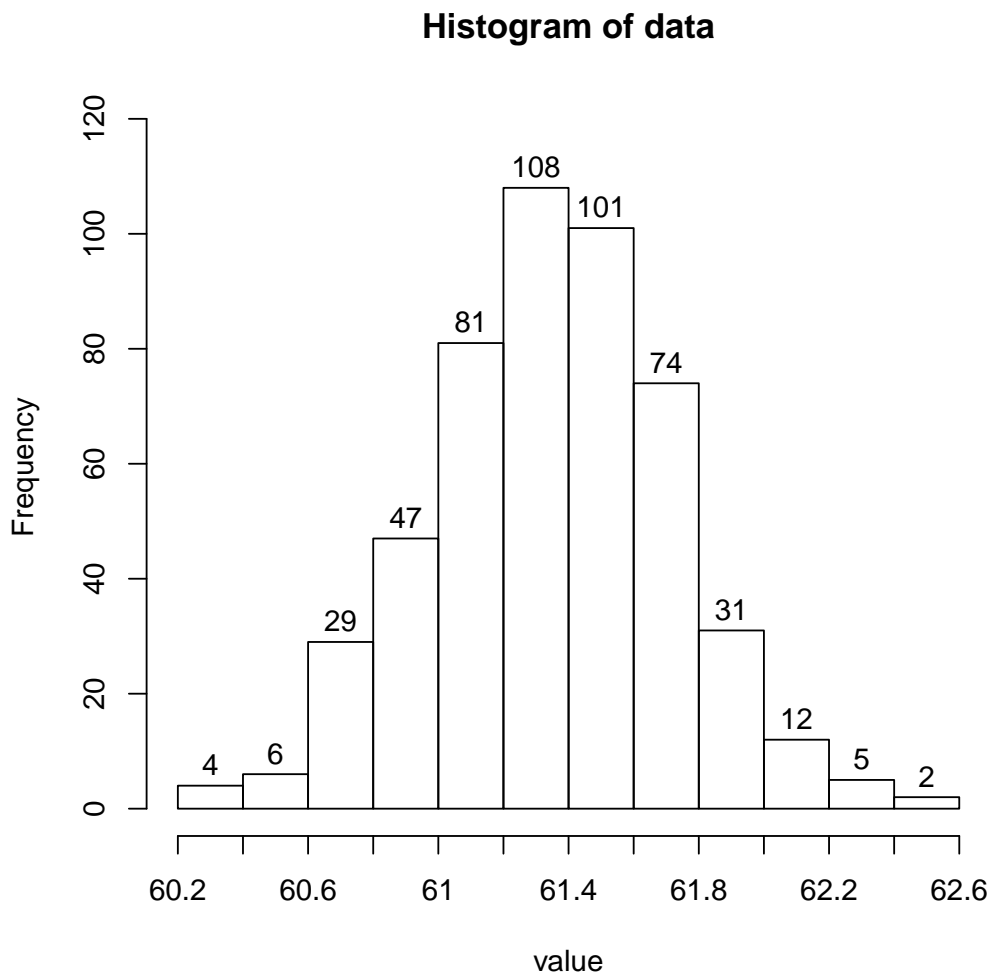
- (d) If n is odd, then median is $x_{i=\frac{n+1}{2}}$, the value of x when $i = \frac{n+1}{2}$. Otherwise, if n is even, the median is mean of $x_{i=\frac{n}{2}}$ and $x_{i=\frac{n}{2}+1}$. In this case, $n = 90$ and so n is even.

$$\text{median} = \frac{x_{45} + x_{46}}{2} = \frac{32.423 + 32.569}{2}$$

So, median = 32.496.

5. Problem:

A continuous random variable was measured 500 times. The resulting histogram is shown below.



- (a) Describe the overall shape of the distribution. (bell, skew left, skew right, uniform, or bimodal)
- (b) Estimate the range of the distribution (range = max-min).
- (c) What percent of the measurements are less than 61.8?
- (d) What percent of the measurements are less than 60.8?
- (e) Of the measurements less than 61.8, what percent are less than 60.8?
- (f) Estimate the value of the 55th percentile.

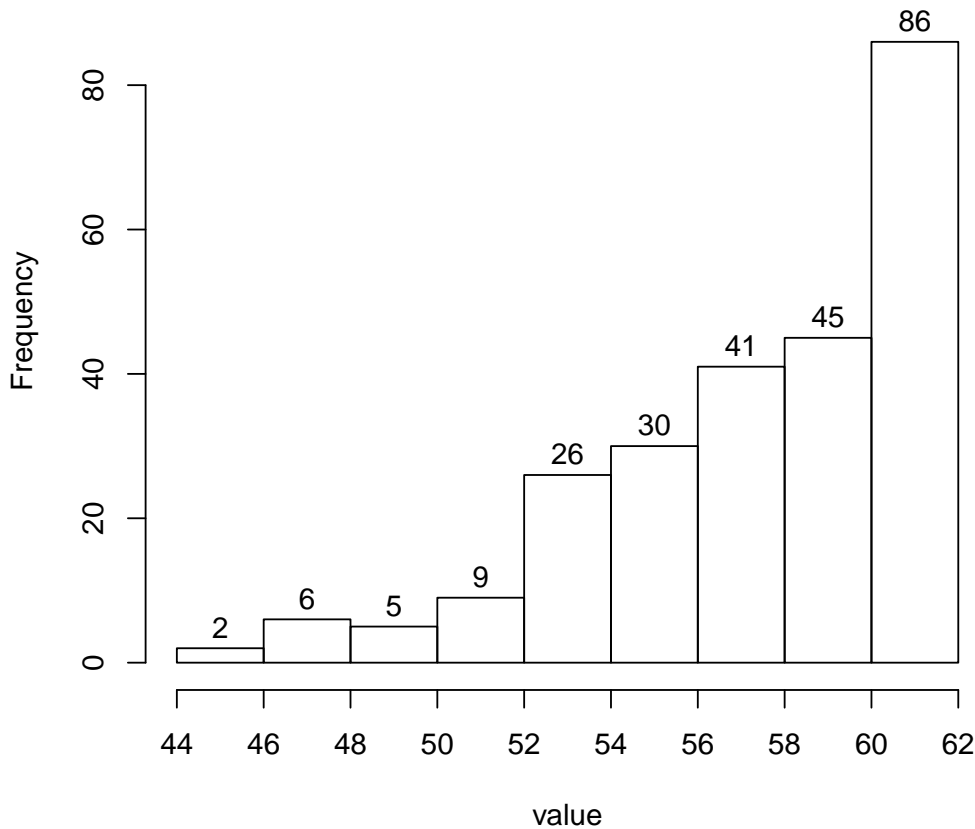
Solution:

- (a) bell
- (b) 2.4
- (c) 90%
- (d) 7.8%
- (e) 8.667%
- (f) 61.4

6. Problem:

A continuous random variable was measured 250 times. The resulting histogram is shown below.

Histogram of data



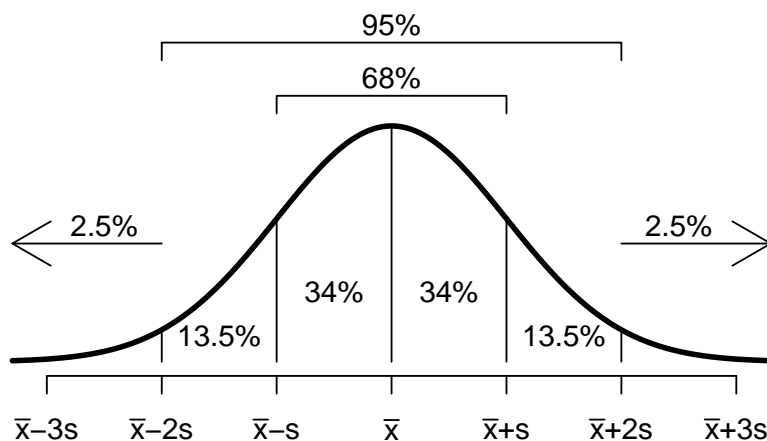
- (a) Describe the overall shape of the distribution. (bell, skew left, skew right, uniform, or bimodal)
- (b) Estimate the range of the distribution (range = max-min).
- (c) What percent of the measurements are less than 50?
- (d) What percent of the measurements are greater than 46?
- (e) Of the measurements less than 50, what percent are greater than 46?
- (f) Estimate the value of the 19.2th percentile.

Solution:

- (a) skew left
- (b) 18
- (c) 5.2%
- (d) 99.2%
- (e) 84.62%
- (f) 54

7. Problem:

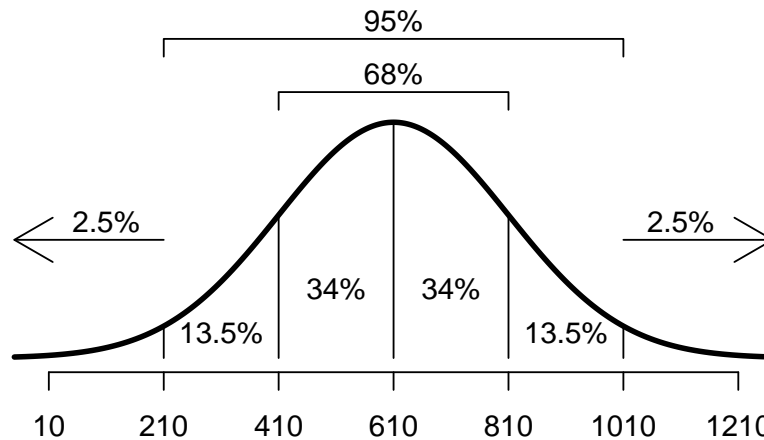
The figure below summarizes the *standard deviation rule* for normal distributions. In the figure, \bar{x} is the mean and s is the standard deviation. The percentages show the fraction of measurements that fall within various intervals.



A specific distribution is approximately normal with mean $\bar{x} = 610$ and standard deviation $s = 200$.

- (a) What percent of the measurements are greater than 210?
- (b) What percent of the measurements are less than 1010?
- (c) What measurement is greater than 16% of the measurements?
- (d) What measurement is less than 50% of the measurements?
- (e) What percent of the measurements are between 410 and 810?

Solution: It is probably best to start by redrawing (relabeling) the normal distribution with the specific values.



- (a) Because we are asked for the percent of measurements *greater* than 210, we add the areas to the right of 210.

97.5%

- (b) Because we are asked for the percent of measurements *less* than 1010, we add the areas to the left of 1010.

97.5%

- (c) We determine which leftward area has a total of 16%. This occurs at 410.

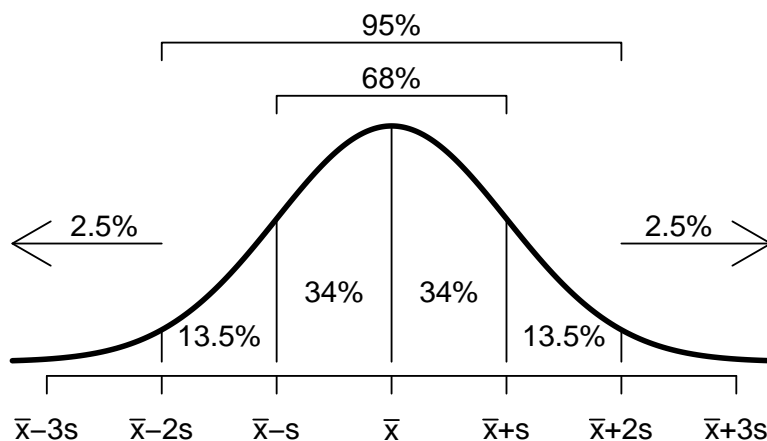
- (d) We determine which rightward area has a total of 50%. This occurs at 610.

- (e) We add the areas from 410 to 810.

68%

8. Problem:

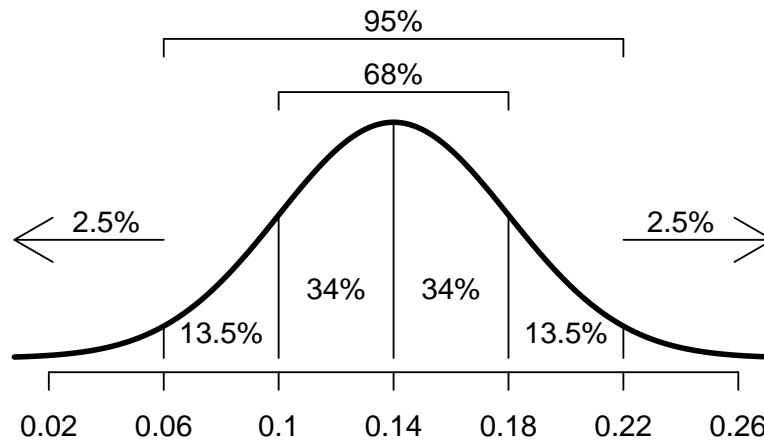
The figure below summarizes the *standard deviation rule* for normal distributions. In the figure, \bar{x} is the mean and s is the standard deviation. The percentages show the fraction of measurements that fall within various intervals.



A specific distribution is approximately normal with mean $\bar{x} = 0.14$ and standard deviation $s = 0.04$.

- (a) What percent of the measurements are greater than 0.06?
- (b) What percent of the measurements are less than 0.1?
- (c) What measurement is greater than 50% of the measurements?
- (d) What measurement is less than 2.5% of the measurements?
- (e) What percent of the measurements are between 0.1 and 0.18?

Solution: It is probably best to start by redrawing (relabeling) the normal distribution with the specific values.



- (a) Because we are asked for the percent of measurements *greater* than 0.06, we add the areas to the right of 0.06.

97.5%

- (b) Because we are asked for the percent of measurements *less* than 0.1, we add the areas to the left of 0.1.

16%

- (c) We determine which leftward area has a total of 50%. This occurs at 0.14.

- (d) We determine which rightward area has a total of 2.5%. This occurs at 0.22.

- (e) We add the areas from 0.1 to 0.18.

68%