1. **Problem**
A farm produces 4 types of fruit: A, B, C, and D. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	76	14
B	69	8
C	137	7
D	130	4

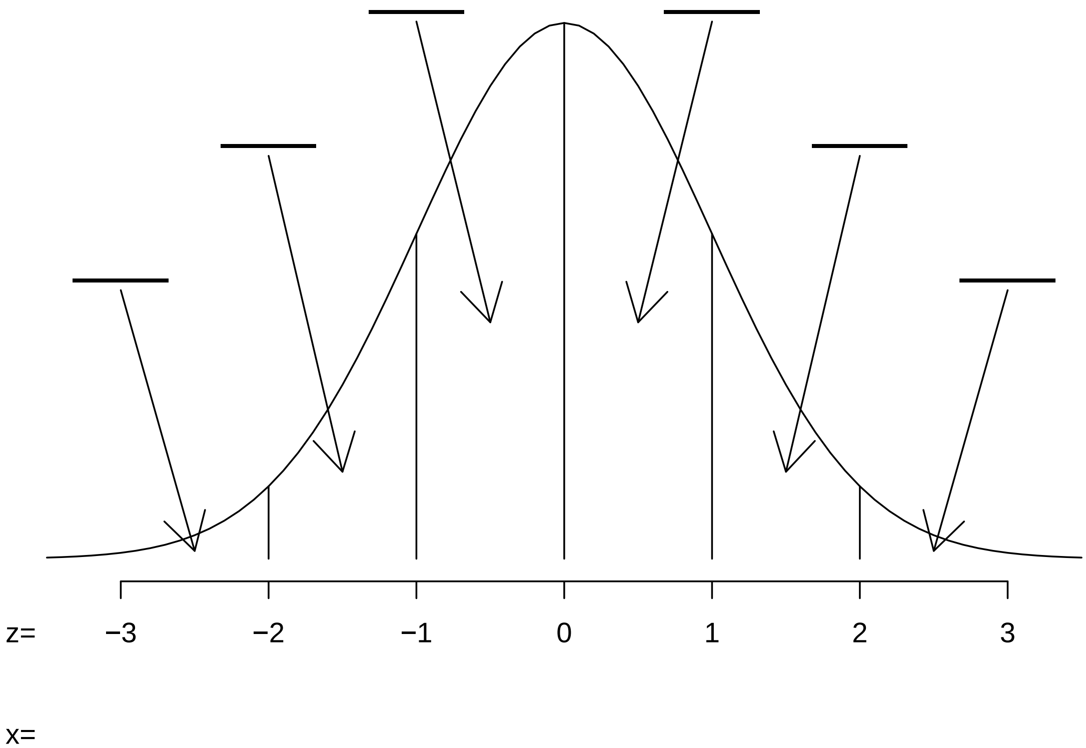
One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	84.12
B	65.32
C	125.2
D	126.6

Which specimen is the most unusually far from average (relative to others of its type)?

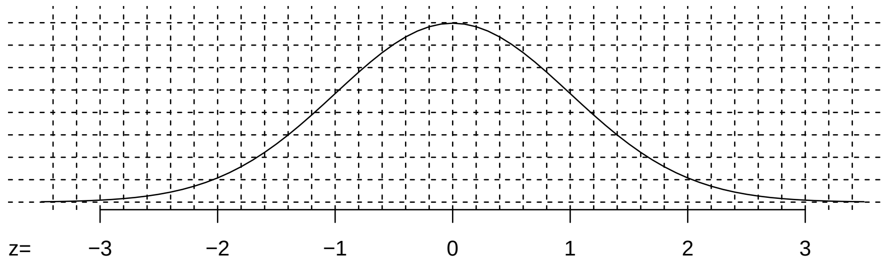
2. **Problem**
   A normal random variable X has a mean $\mu = 25.6$ and standard deviation $\sigma = 6.4$. Please label the density curve with:

   (a) The appropriate values of x.
   (b) The areas of the sections.

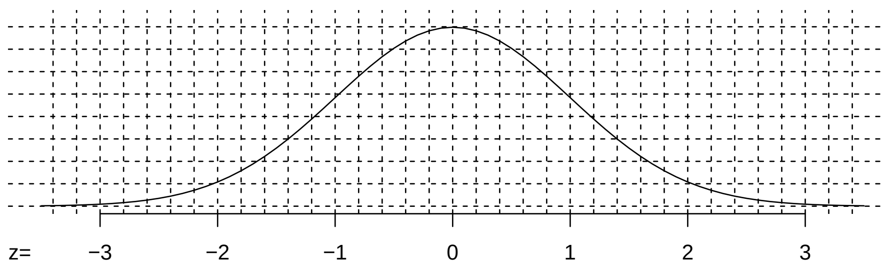


3. **Problem**
   Let X be normally distributed with mean 81 and standard deviation 13. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

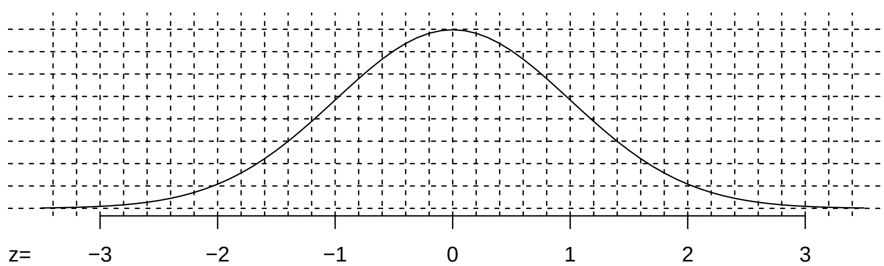
   (a) $P(X < 69.3)$

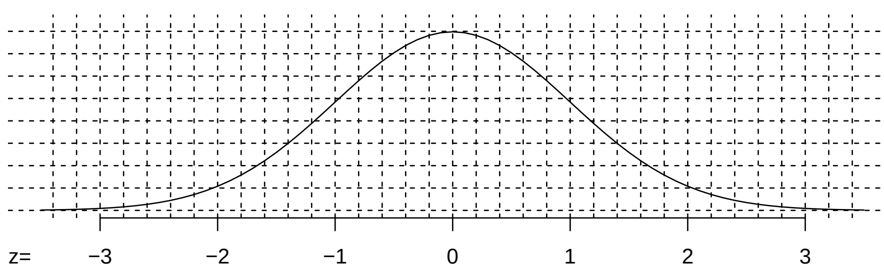
   (b) $P(X > 94)$

   (c) $P(|X - 81| < 11.7)$

   (d) $P(|X - 81| > 7.8)$

4. **Problem**
Let X be normally distributed with mean 112 and standard deviation 36.9. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

(a) What's the probability that X is less than 101? **Draw a sketch**.

(b) What's the probability that X is more than 122? **Draw a sketch**.

(c) What's the probability that X is between 101 and 122? **Draw a sketch**.

5. **Problem**
Let random variable W have mean $\mu_w = 15$ and standard deviation $\sigma_w = 2$. Let random variable X represent the **average** of $n = 36$ instances of W.

(a) Determine the expected value of X. $\mu_x = ?$

(b) Determine the standard deviation of X. $\sigma_x = ?$

(c) Using normal approximation, determine $P(X < 15.68)$.

(d) Using normal approximation, determine $P(X > 15.07)$.

6. **Problem**
A very large population has a mean of 113.9 and a standard deviation of 28.8. When a random sample of size 81 is taken, what is the probability that the **sample mean** ($\bar{x}$) is between 114 and 116?

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(114 < \overline{X} < 116)$. **Draw a sketch**

7. **Problem**
Let random variable W have the probability distribution shown below.

w	$P(w)$
0	0.58
1	0.42

Let random variable $\hat{p}$ (sample proportion) represent the average of $n = 49$ instances of W.

(a) Determine the mean and standard deviation of the $\hat{p}$.

(b) Determine $P(\hat{p} < 0.53)$. Do NOT use a continuity correction. **Draw a sketch**

8. **Problem**
A very large population has a population proportion $p = 0.28$. When a random sample of size 64 is taken, what is the probability that the **sample proportion** ($\hat{p}$) is less than 0.25?

Do NOT use a continuity correction.

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(\hat{p} < 0.25)$. **Draw a sketch**

9. **Problem**
Let random variable W have mean $\mu_w = 42$ and standard deviation $\sigma_w = 5$. Let random variable X represent the **sum** of $n = 64$ instances of W.

(a) Determine the expected value of X. $\mu_x = ?$

(b) Determine the standard deviation of X. $\sigma_x = ?$

(c) Using normal approximation, determine $P(X < 2672.4)$.

(d) Using normal approximation, determine $P(X > 2638.8)$.

10. **Problem**
Let random variable W have the probability distribution shown below.

w	$P(w)$
0	0.45
1	0.55

Let random variable X represent the sum of $n = 46$ instances of W. (Thus X is the sample total, or number of successes.)

What is the probability that X is at least 18 but at most 32? **Use a normal approximation with continuity corrections**.

11. **Problem**

Let each trial have a chance of success $p = 0.72$. If 41 trials occur, what is the probability of getting more than 22 but at most 33 successes?

In other words, let $X \sim \text{Bin}(n = 41,\ p = 0.72)$ and find $P(22 < X \leq 33)$.

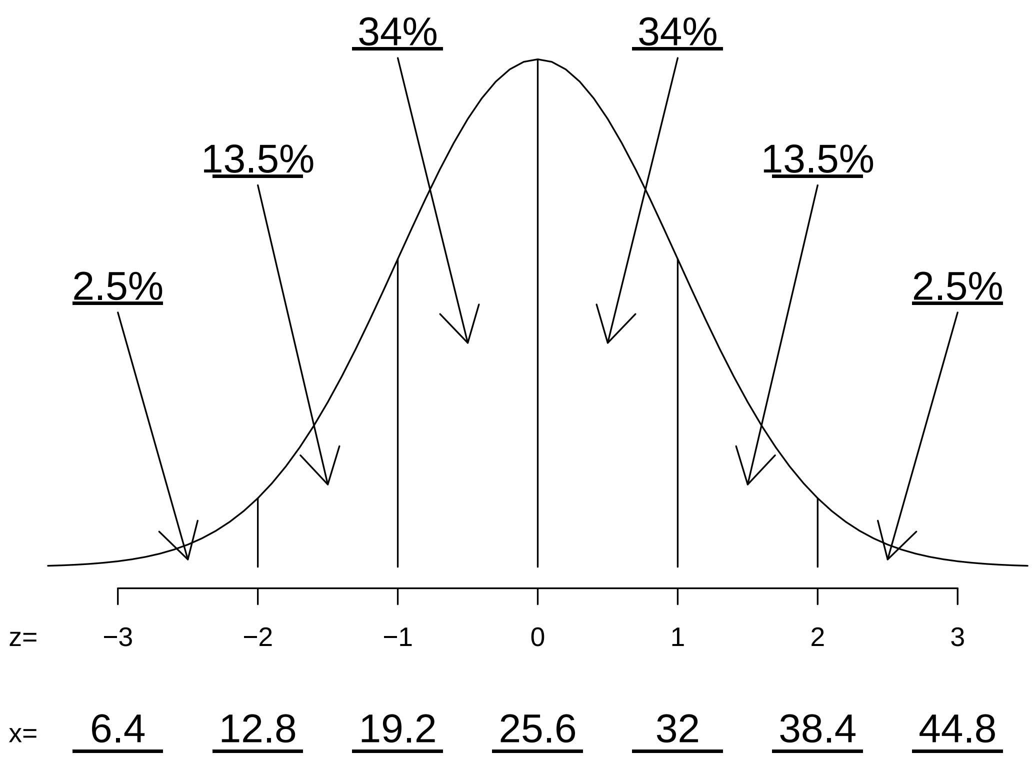
Use a normal approximation along with the continuity correction.

1. We compare the absolute z-scores. The largest absolute z-score corresponds to the specimen that is most unusually far from average.

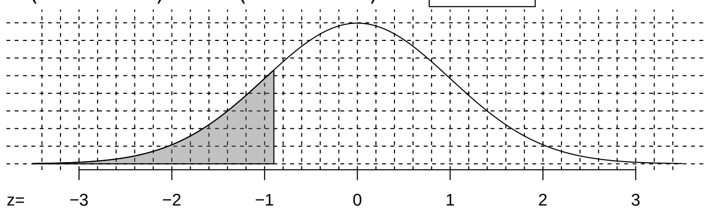
Type of fruit	formula	absolute z-score
A	$\lvert z\rvert = \frac{\lvert 84.12-76\rvert}{14}$	0.58
B	$\lvert z\rvert = \frac{\lvert 65.32-69\rvert}{8}$	0.46
C	$\lvert z\rvert = \frac{\lvert 125.2-137\rvert}{7}$	1.68
D	$\lvert z\rvert = \frac{\lvert 126.6-130\rvert}{4}$	0.86

Thus, the specimen of type C is the most unusually far from average.

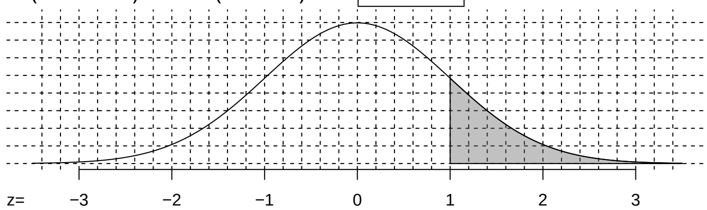
2. The filled in areas and x values are shown below.



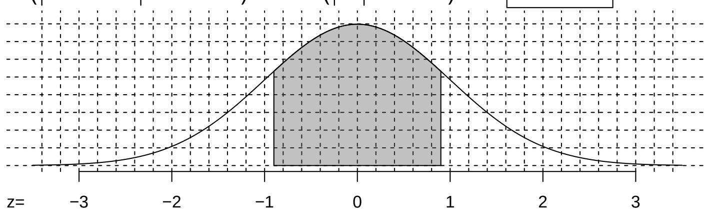
3. (a) $P(X < 69.3) = P(Z < -0.9) = \boxed{0.1841}$



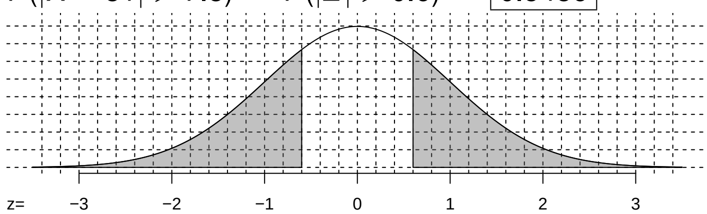
(b) $P(X > 94) = P(Z > 1) = \boxed{0.1587}$



(c) $P(|X - 81| < 11.7) = P(|Z| < 0.9) = \boxed{0.6318}$

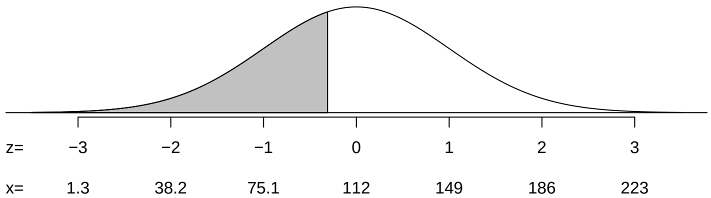


(d) $P(|X - 81| > 7.8) = P(|Z| > 0.6) = \boxed{0.5486}$

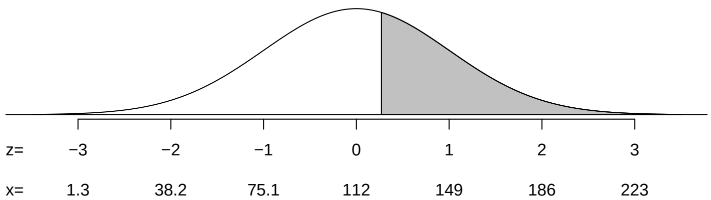


4. Notice the three probabilities will add up to 1.

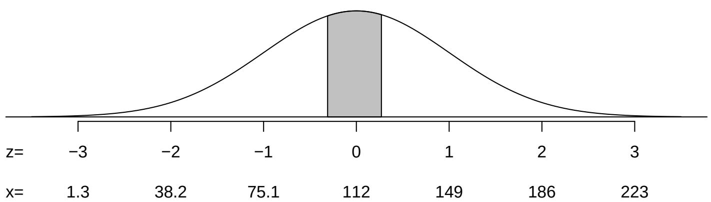
   (a) $P(X < 101) = P(Z < -0.31) = \boxed{0.3783}$



   (b) $P(X > 122) = P(Z > 0.27) = \boxed{0.3936}$



   (c) $P(101 < X < 122) = P(-0.31 < Z < 0.27) = \boxed{0.2281}$



5. We use the Central Limit Theorem for **sample average** sampling ($\bar{x}$ sampling). We recognize that in this problem X is an AVERAGE of 36 instances of W.

   (a) $\mu_X = \mu_W = 15$

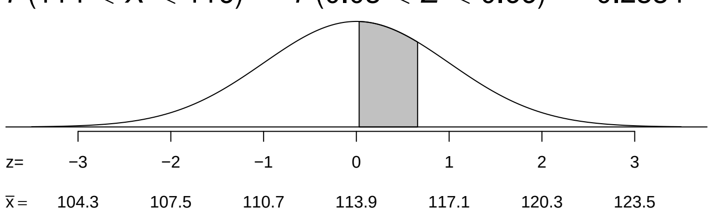
   (b) $\sigma_X = \frac{\sigma_W}{\sqrt{n}} = 0.3333333$

   (c) 0.9793

   (d) 0.4168

6. (a) Central limit of average formulas: $\mu_{\bar{x}} = 113.9$ and $\sigma_{\bar{x}} = \frac{28.8}{\sqrt{81}} = 3.2$.

   (b) $P(114 < \overline{X} < 116) = P(0.03 < Z < 0.66) = 0.2334$



7. (a) We can recognize W is a Bernoulli variable with $p = 0.42$ and $q = 0.58$. Thus,

$$\mu_W = p = 0.42$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.42)(0.58)} = 0.4935585$$

Then, we use the central limit of average formulas:

$$\mu_{\hat{p}} = \mu_W = 0.42$$

and

$$\sigma_{\hat{p}} = \frac{\sigma_W}{\sqrt{n}} = \frac{0.4935585}{\sqrt{49}} = 0.0705084$$

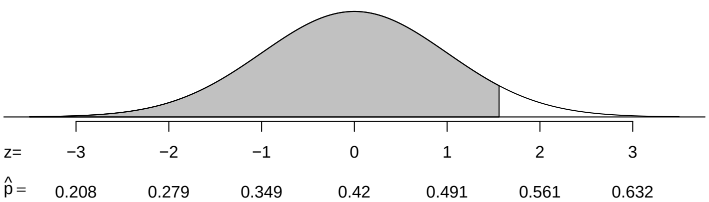
But, if we recognized $\hat{p}$ follows the formulas of a $\hat{p}$ **sampling distribution**:

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}} = \sqrt{\frac{pq}{n}}$$

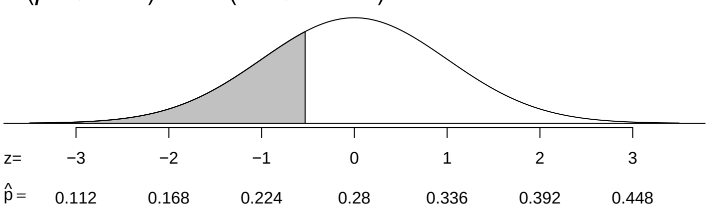
we could have just used those instead.

(b) $P(\hat{p} < 0.53) \;=\; P(Z < 1.56) \;=\; 0.9406$



8. (a) Use $\hat{p}$ sampling formulas: $\mu_{\hat{p}} = 0.28$ and $\sigma_{\hat{p}} = \frac{\sqrt{(0.28)(0.72)}}{\sqrt{64}} = 0.0561249$.

(b) $P(\hat{p} < 0.25) \;=\; P(Z < -0.53) \;=\; 0.2981$



9. (a) 2688

(b) 40

(c) 0.3483

(d) 0.8907

10. We recognize W is a Bernoulli variable with $p = 0.55$ and $q = 0.45$. Thus,

$$\mu_W = p = 0.55$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.55)(0.45)} = 0.4974937$$

.

We use the Central Limit Formulas (for a sum).

$$\mu_X = n\mu_W = (46)(0.55) = 25.3$$

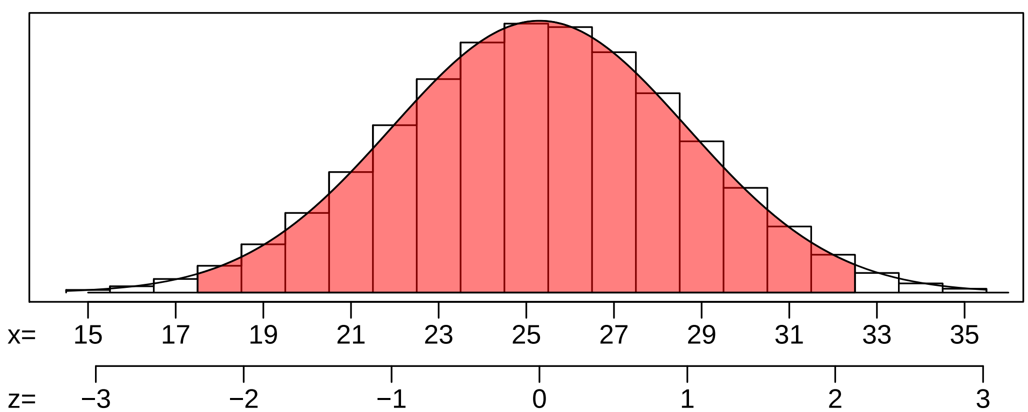
$$\sigma_X = \sqrt{n}\sigma_W = \sqrt{46}(0.4974937) = 3.3742$$

It should be mentioned that you could have also just recognized X is binomial:

$$\mu = np = (46)(0.55) = 25.3$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{(46)(0.55)(1-0.55)} = 3.3742$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{17.5 - 25.3}{3.3742} = -2.31$$

$$z_2 = \frac{32.5 - 25.3}{3.3742} = 2.13$$

Find the percentiles (from z-table).

$$\ell_1 = 0.0104$$

$$\ell_2 = 0.9834$$

Calculate the probability.

$$P(18 \le X \le 32) = 0.9834 - 0.0104 = 0.973$$

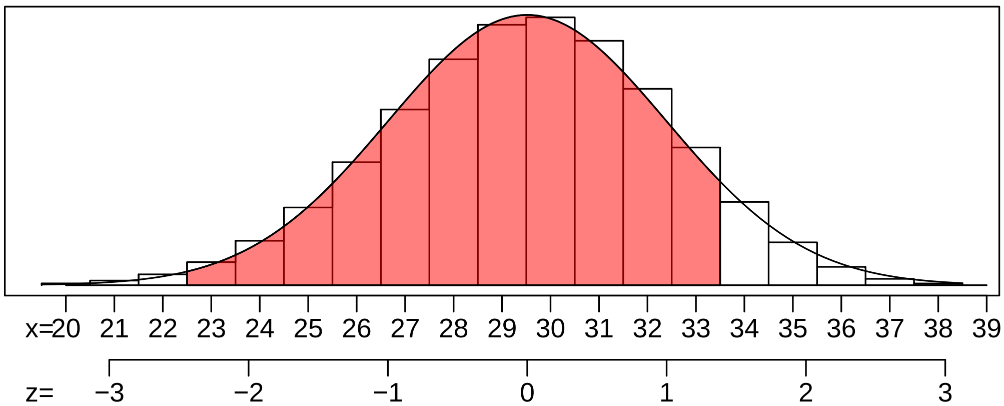
11. Find the mean.

$$\mu = np = (41)(0.72) = 29.52$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(41)(0.72)(1-0.72)} = 2.875$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{22.5 - 29.52}{2.875} = -2.44$$

$$z_2 = \frac{33.5 - 29.52}{2.875} = 1.38$$

Find the percentiles (from z-table).

$$\ell_1 = 0.0073$$

$$\ell_2 = 0.9162$$

Calculate the probability.

$$P(22 < X \leq 33) = 0.9162 - 0.0073 = 0.909$$

Normal Distributions

$z = \frac{x-\mu}{\sigma}$
$x = \mu + z\sigma$

Central Limit Theorem

Let random variable W have mean μ_w and standard deviation σ_w.
Let random variable X represent the sum of n instances of W.
Let random variable Y represent the average of n instances of W.
Then:

$$\mu_x = (n)(\mu_w) \qquad \mu_y = \mu_w$$

$$\sigma_x = (\sigma_w)(\sqrt{n}) \qquad \sigma_y = \frac{\sigma_w}{\sqrt{n}}$$

and X and Y are both approximately normal.

Bernoulli Random Variable

$\mu = p$
$\sigma = \sqrt{pq}$

Binomial Random Variable (sum of Bernoullis)

$\mu = np$
$\sigma = \sqrt{npq}$

Proportion Sampling Random Variable (average of Bernoullis)

$\mu = p$
$\sigma = \sqrt{\frac{pq}{n}}$

Continuity Correction

- If:
  - X is a discrete variable with a support of consecutive integers
  - we are approximating X with a normal distribution
- Then we can apply a continuity correction:

$$P(X \leq x_0) = P\left(Z < \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X < x_0) = P\left(Z < \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right)$$

$$P(X \geq x_0) = P\left(Z > \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X > x_0) = P\left(Z > \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right)$$