1. **Problem**
   A farm produces 4 types of fruit: A, B, C, and D. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	76	6
B	81	11
C	111	12
D	90	9

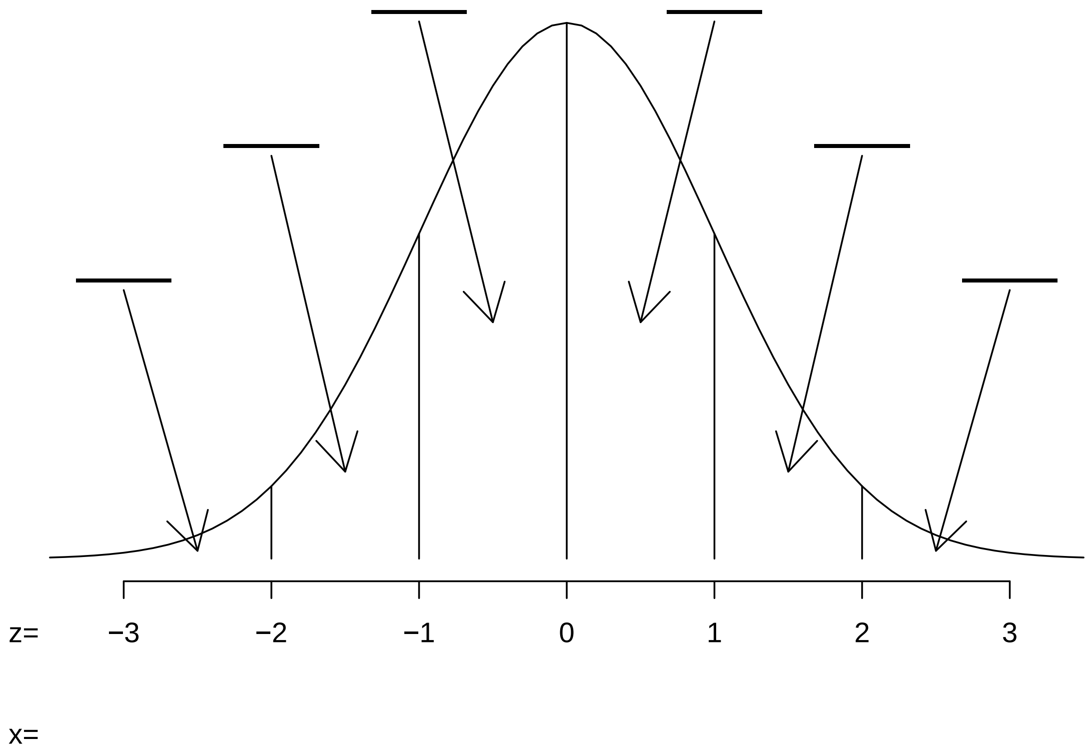
   One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	82.6
B	97.28
C	103.9
D	79.38

   Which specimen is the most unusually large (relative to others of its type)?

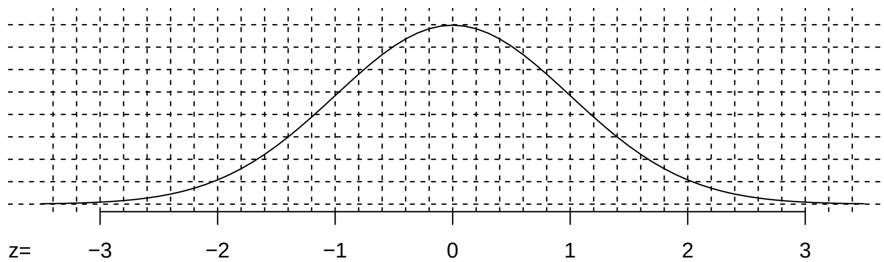
2. **Problem**
A normal random variable X has a mean $\mu = 20.1$ and standard deviation $\sigma = 0.6$. Please label the density curve with:

(a) The appropriate values of x.
(b) The areas of the sections.

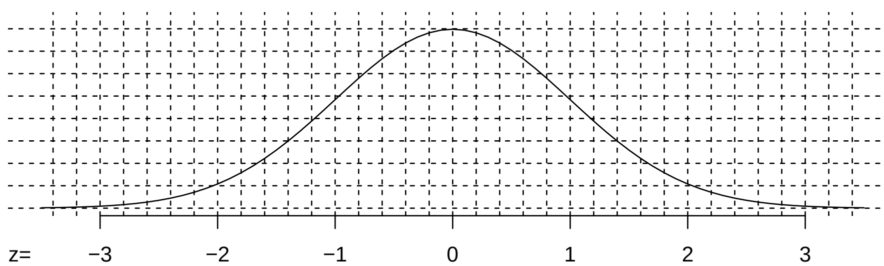


3. **Problem**
Let X be normally distributed with mean 103 and standard deviation 24. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

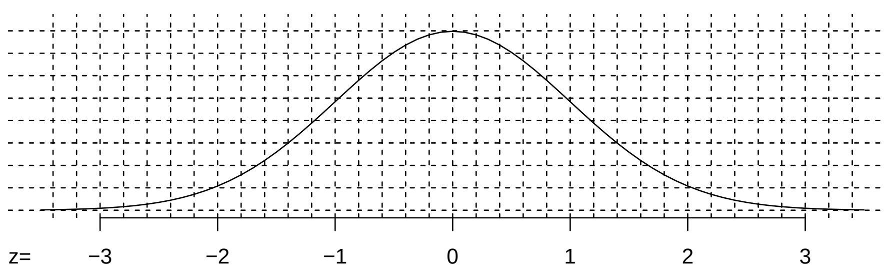
(a) $P(X < 139)$



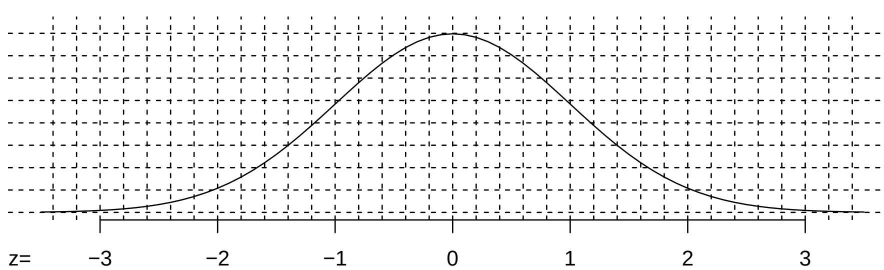
(b) $P(X > 79)$



(c) $P(|X - 103| < 19)$



(d) $P(|X - 103| > 17)$



4. **Problem**
   Let X be normally distributed with mean 103 and standard deviation 8.9. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

   (a) What's the probability that X is less than 95.3? **Draw a sketch**.

   (b) What's the probability that X is more than 119? **Draw a sketch**.

   (c) What's the probability that X is between 95.3 and 119? **Draw a sketch**.

5. **Problem**
   Let random variable W have mean $\mu_w = 8$ and standard deviation $\sigma_w = 1$. Let random variable X represent the **average** of $n = 81$ instances of W.

   (a) Determine the expected value of X. $\mu_x = ?$

   (b) Determine the standard deviation of X. $\sigma_x = ?$

   (c) Using normal approximation, determine $P(X < 7.88)$.

   (d) Using normal approximation, determine $P(X > 7.98)$.

6. **Problem**
A very large population has a mean of 107.4 and a standard deviation of 30. When a random sample of size 144 is taken, what is the probability that the **sample mean** ($\bar{x}$) is between 106 and 109?

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(106 < \overline{X} < 109)$. **Draw a sketch**

7. **Problem**
   Let random variable W have the probability distribution shown below.

w	$P(w)$
0	0.47
1	0.53

Let random variable $\hat{p}$ (sample proportion) represent the average of $n = 100$ instances of W.

(a) Determine the mean and standard deviation of the $\hat{p}$.

(b) Determine $P(\hat{p} < 0.57)$. Do NOT use a continuity correction. **Draw a sketch**

8. **Problem**
A very large population has a population proportion $p = 0.26$. When a random sample of size 100 is taken, what is the probability that the **sample proportion** ($\hat{p}$) is less than 0.21?

Do NOT use a continuity correction.

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(\hat{p} < 0.21)$. **Draw a sketch**

9. **Problem**
Let random variable W have mean $\mu_W = 33$ and standard deviation $\sigma_W = 2$. Let random variable X represent the **sum** of $n = 225$ instances of W.

(a) Determine the expected value of X. $\mu_X = ?$

(b) Determine the standard deviation of X. $\sigma_X = ?$

(c) Using normal approximation, determine $P(X < 7456.2)$.

(d) Using normal approximation, determine $P(X > 7415.1)$.

10. **Problem**

Let random variable W have the probability distribution shown below.

w	$P(w)$
0	0.13
1	0.87

Let random variable X represent the sum of $n = 106$ instances of W. (Thus X is the sample total, or number of successes.)

What is the probability that X is at least 85 but at most 95? **Use a normal approximation with continuity corrections**.

11. **Problem**

    Let each trial have a chance of success $p = 0.16$. If 199 trials occur, what is the probability of getting more than 22 but less than 27 successes?

    In other words, let $X \sim \text{Bin}(n = 199,\ p = 0.16)$ and find $P(22 < X < 27)$.

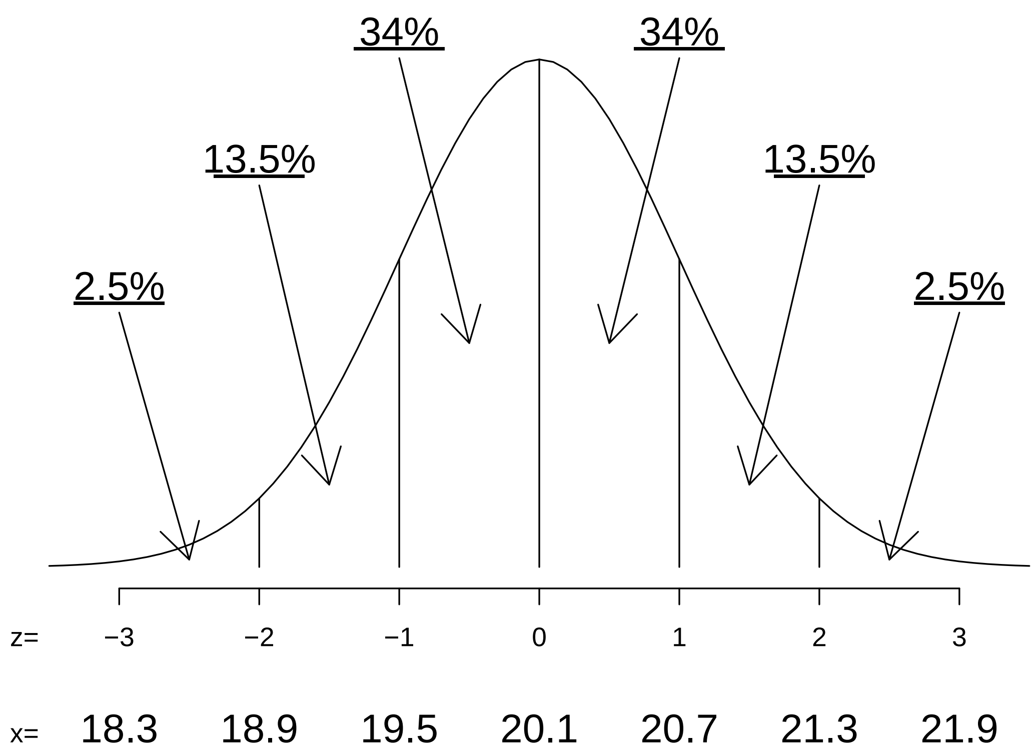
    Use a normal approximation along with the continuity correction.

1. We compare the z-scores. The largest z-score corresponds to the specimen that is most unusually large.

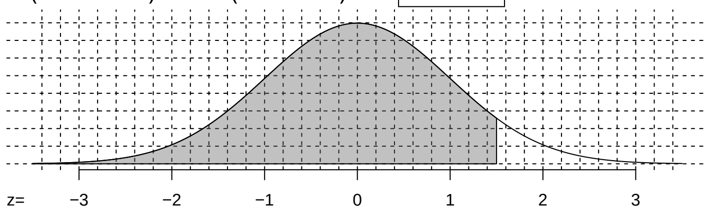
Type of fruit	formula	z-score
A	$z = \frac{82.6-76}{6}$	1.1
B	$z = \frac{97.28-81}{11}$	1.48
C	$z = \frac{103.9-111}{12}$	-0.59
D	$z = \frac{79.38-90}{9}$	-1.18

Thus, the specimen of type B is the most unusually large.

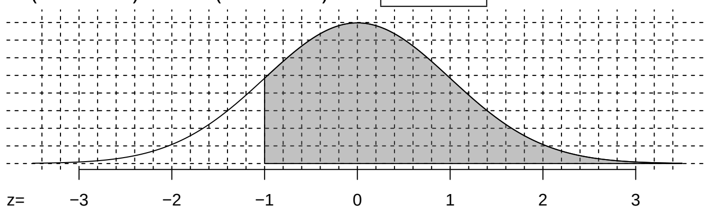
2. The filled in areas and x values are shown below.



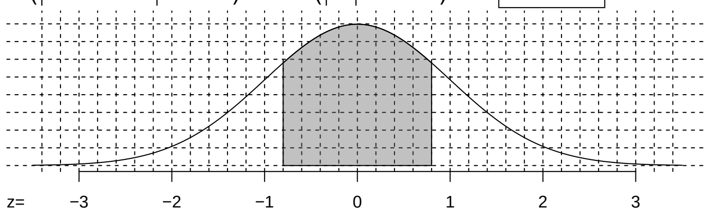
3\. (a) $P(X < 139) = P(Z < 1.5) = \boxed{0.9332}$



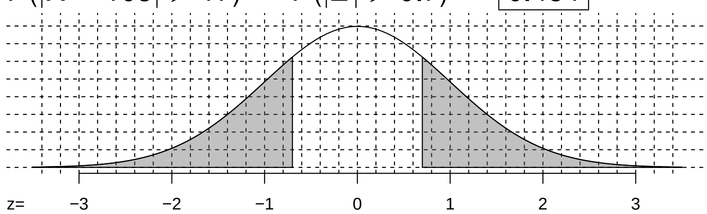
(b) $P(X > 79) = P(Z > -1) = \boxed{0.8413}$



(c) $P(|X - 103| < 19) = P(|Z| < 0.8) = \boxed{0.5762}$

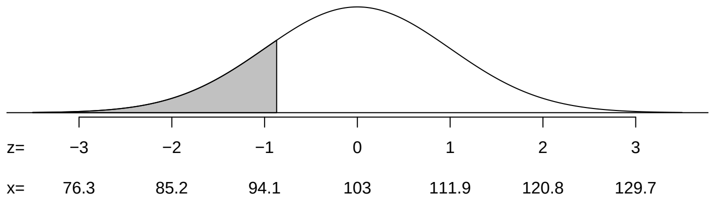


(d) $P(|X - 103| > 17) = P(|Z| > 0.7) = \boxed{0.484}$

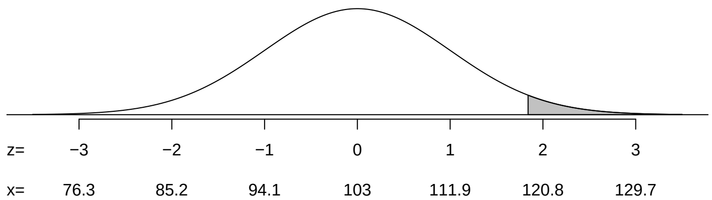


4. Notice the three probabilities will add up to 1.

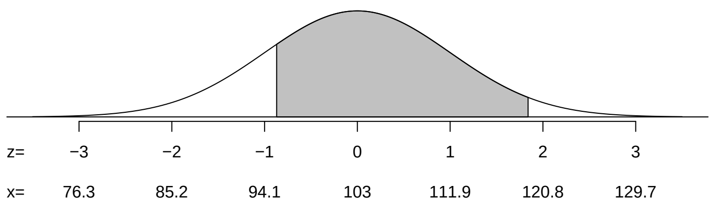
   (a) $P(X < 95.3) = P(Z < -0.87) = \boxed{0.1922}$



   (b) $P(X > 119) = P(Z > 1.84) = \boxed{0.0329}$



   (c) $P(95.3 < X < 119) = P(-0.87 < Z < 1.84) = \boxed{0.7749}$



5. (a) 8

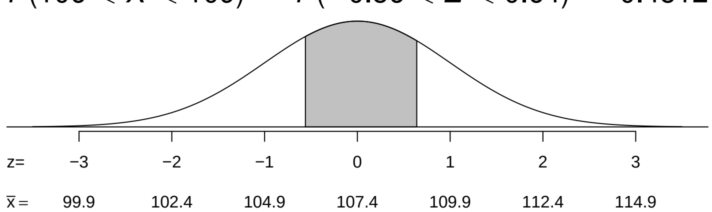
   (b) 0.1111

   (c) 0.1401

   (d) 0.5714

6. (a) Central limit of average formulas: $\mu_{\bar{x}} = 107.4$ and $\sigma_{\bar{x}} = \frac{30}{\sqrt{144}} = 2.5$.

   (b) $P(106 < \overline{X} < 109) = P(-0.56 < Z < 0.64) = 0.4512$



7. (a) We can recognize W is a Bernoulli variable with $p = 0.53$ and $q = 0.47$. Thus,

$$\mu_W = p = 0.53$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.53)(0.47)} = 0.4991$$

Then, we use the central limit of average formulas:

$$\mu_{\hat{p}} = \mu_W = 0.53$$

and

$$\sigma_{\hat{p}} = \frac{\sigma_W}{\sqrt{n}} = \frac{0.4991}{\sqrt{100}} = 0.0499$$

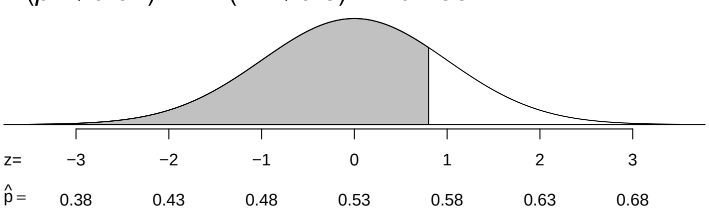
But, if we recognized $\hat{p}$ follows the formulas of a $\hat{p}$ **sampling distribution**:

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}} = \sqrt{\frac{pq}{n}}$$

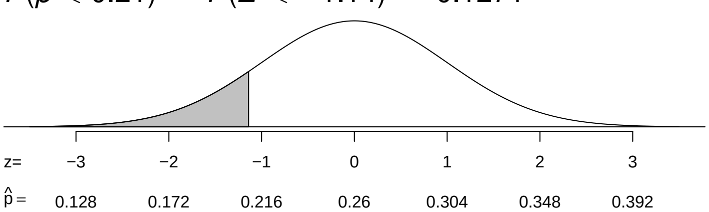
we could have just used those instead.

(b) $P(\hat{p} < 0.57) = P(Z < 0.8) = 0.7881$



8. (a) Use $\hat{p}$ sampling formulas: $\mu_{\hat{p}} = 0.26$ and $\sigma_{\hat{p}} = \frac{\sqrt{(0.26)(0.74)}}{\sqrt{100}} = 0.0439$.

(b) $P(\hat{p} < 0.21) = P(Z < -1.14) = 0.1271$



9. (a) 7425

(b) 30

(c) 0.8508

(d) 0.6293

10. We recognize W is a Bernoulli variable with $p = 0.87$ and $q = 0.13$. Thus,

$$\mu_W = p = 0.87$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.87)(0.13)} = 0.3363$$

.

We use the Central Limit Formulas (for a sum).

$$\mu_X = n\mu_W = (106)(0.87) = 92.22$$

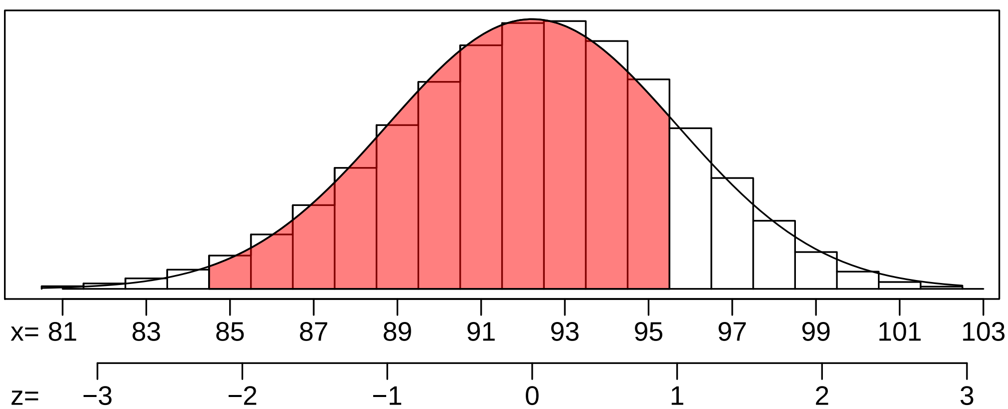
$$\sigma_X = \sqrt{n}\sigma_W = \sqrt{106}(0.3363) = 3.4625$$

It should be mentioned that you could have also just recognized X is binomial:

$$\mu = np = (106)(0.87) = 92.22$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{(106)(0.87)(1-0.87)} = 3.4625$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{84.5 - 92.22}{3.4625} = -2.23$$

$$z_2 = \frac{95.5 - 92.22}{3.4625} = 0.95$$

Find the percentiles (from z-table).

$$\ell_1 = 0.0129$$

$$\ell_2 = 0.8289$$

Calculate the probability.

$$P(85 \leq X \leq 95) = 0.8289 - 0.0129 = 0.816$$

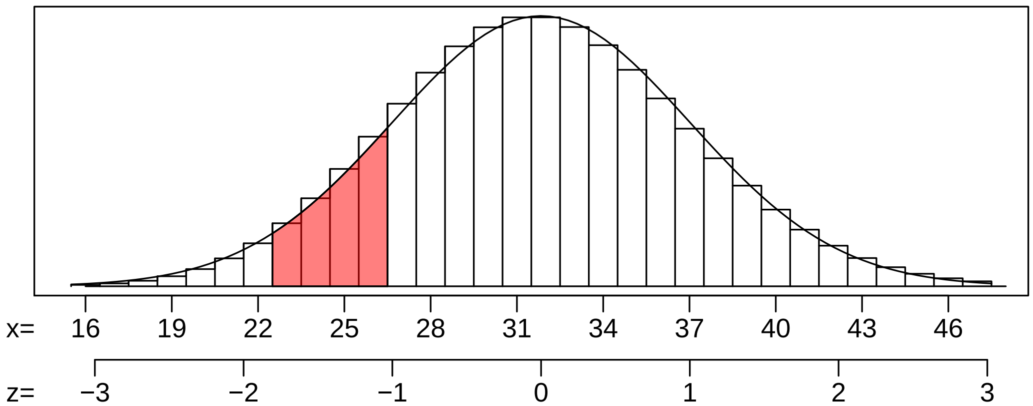
11. Find the mean.

$$\mu = np = (199)(0.16) = 31.84$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(199)(0.16)(1-0.16)} = 5.1716$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{22.5 - 31.84}{5.1716} = -1.81$$

$$z_2 = \frac{26.5 - 31.84}{5.1716} = -1.03$$

Find the percentiles (from z-table).

$$\ell_1 = 0.0351$$

$$\ell_2 = 0.1515$$

Calculate the probability.

$$P(22 < X < 27) = 0.1515 - 0.0351 = 0.117$$

Normal Distributions

$z = \frac{x-\mu}{\sigma}$
$x = \mu + z\sigma$

Central Limit Theorem

Let random variable W have mean μ_w and standard deviation σ_w.
Let random variable X represent the sum of n instances of W.
Let random variable Y represent the average of n instances of W.
Then:

$$\mu_x = (n)(\mu_w) \qquad \mu_y = \mu_w$$

$$\sigma_x = (\sigma_w)(\sqrt{n}) \qquad \sigma_y = \frac{\sigma_w}{\sqrt{n}}$$

and X and Y are both approximately normal.

Bernoulli Random Variable

$\mu = p$
$\sigma = \sqrt{pq}$

Binomial Random Variable (sum of Bernoullis)

$\mu = np$
$\sigma = \sqrt{npq}$

Proportion Sampling Random Variable (average of Bernoullis)

$\mu = p$
$\sigma = \sqrt{\frac{pq}{n}}$

Continuity Correction

- If:
  - X is a discrete variable with a support of consecutive integers
  - we are approximating X with a normal distribution
- Then we can apply a continuity correction:

$$P(X \leq x_0) = P\left(Z < \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X < x_0) = P\left(Z < \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right)$$

$$P(X \geq x_0) = P\left(Z > \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X > x_0) = P\left(Z > \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right)$$