1. **Problem**
   A farm produces 4 types of fruit: $A$, $B$, $C$, and $D$. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

| Type of fruit | Mean mass (g) | Standard deviation of mass (g) |
|---|---|---|
| $A$ | 119 | 9 |
| $B$ | 94 | 12 |
| $C$ | 114 | 14 |
| $D$ | 76 | 13 |

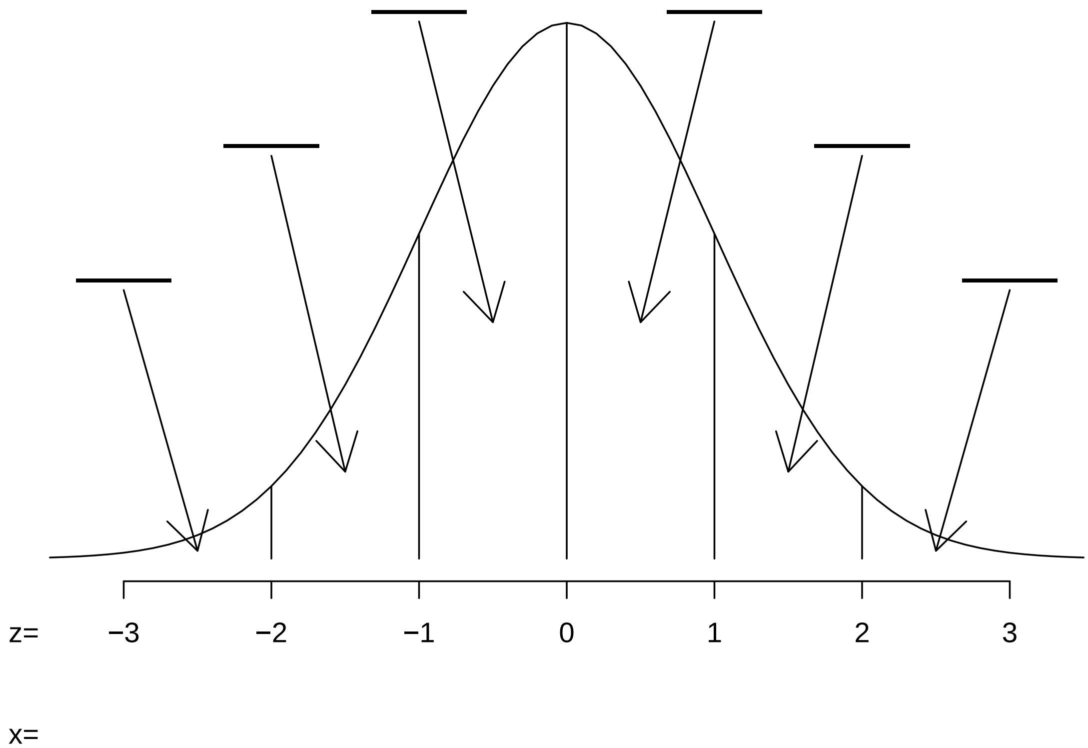
One specimen of each type is weighed. The results are shown below.

| Type of fruit | Mass of specimen (g) |
|---|---|
| $A$ | 122.4 |
| $B$ | 89.56 |
| $C$ | 105.6 |
| $D$ | 84.19 |

Which specimen is the most unusually large (relative to others of its type)?

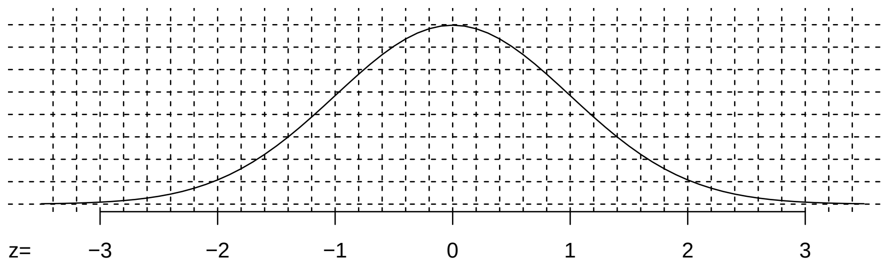
2. **Problem**
A normal random variable $X$ has a mean $\mu = 4.2$ and standard deviation $\sigma = 0.8$. Please label the density curve with:

(a) The appropriate values of $x$.
(b) The areas of the sections.

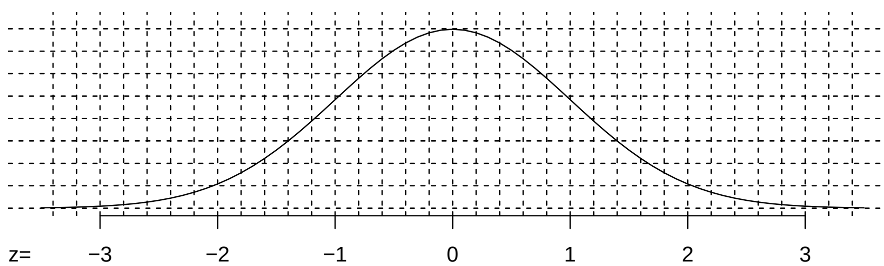


3. **Problem**
Let $X$ be normally distributed with mean 76 and standard deviation 11. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

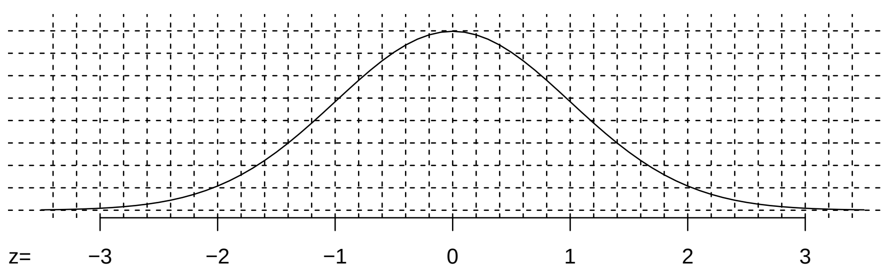
(a) $P(X < 61.7)$



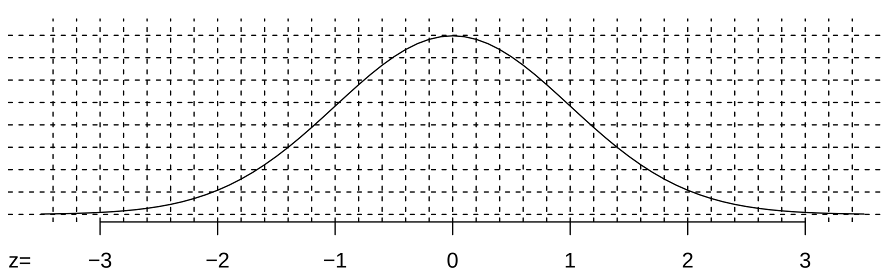

(b) $P(X > 87)$




(c) $P(|X - 76| < 19.8)$




(d) $P(|X - 76| > 8.8)$



4. **Problem**
Let $X$ be normally distributed with mean 98.8 and standard deviation 22.2. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

(a) What's the probability that $X$ is less than 84.8? **Draw a sketch**.

(b) What's the probability that $X$ is more than 98.4? **Draw a sketch**.

(c) What's the probability that $X$ is between 84.8 and 98.4? **Draw a sketch**.

5. **Problem**
Let random variable $W$ have mean $\mu_W = 19$ and standard deviation $\sigma_W = 4$. Let random variable $X$ represent the **average** of $n = 100$ instances of $W$.

(a) Determine the expected value of $X$. $\mu_X = ?$

(b) Determine the standard deviation of $X$. $\sigma_X = ?$

(c) Using normal approximation, determine $P(X < 18.72)$.

(d) Using normal approximation, determine $P(X > 18.86)$.

6. **Problem**
A very large population has a mean of 64.5 and a standard deviation of 13.2. When a random sample of size 36 is taken, what is the probability that the **sample mean** ($\bar{x}$) is between 63.4 and 66.3?

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(63.4 < \overline{X} < 66.3)$. **Draw a sketch**

7. **Problem**
Let random variable $W$ have the probability distribution shown below.

| $w$ | $P(w)$ |
|---|---|
| 0 | 0.36 |
| 1 | 0.64 |

Let random variable $\hat{p}$ (sample proportion) represent the average of $n = 81$ instances of $W$.

(a) Determine the mean and standard deviation of the $\hat{p}$.

(b) Determine $P(\hat{p} < 0.68)$. Do NOT use a continuity correction. **Draw a sketch**

8. **Problem**

   A very large population has a population proportion $p = 0.09$. When a random sample of size 289 is taken, what is the probability that the **sample proportion** ($\hat{p}$) is less than 0.12?

   *Do NOT use a continuity correction*.

   (a) Determine the mean and standard deviation of the sampling distribution.

   (b) Determine $P(\hat{p} < 0.12)$. **Draw a sketch**

9. **Problem**
Let random variable $W$ have mean $\mu_W = 49$ and standard deviation $\sigma_W = 7$. Let random variable $X$ represent the **sum** of $n = 64$ instances of $W$.

(a) Determine the expected value of $X$. $\mu_X = ?$

(b) Determine the standard deviation of $X$. $\sigma_X = ?$

(c) Using normal approximation, determine $P(X < 3082.8)$.

(d) Using normal approximation, determine $P(X > 3130.96)$.

10. **Problem**
Let random variable $W$ have the probability distribution shown below.

| $w$ | $P(w)$ |
|---|---|
| 0 | 0.4 |
| 1 | 0.6 |

Let random variable $X$ represent the sum of $n = 49$ instances of $W$. (Thus $X$ is the sample total, or number of successes.)

What is the probability that $X$ is at least 23 but at most 29? **Use a normal approximation with continuity corrections**.

11. **Problem**
    Let each trial have a chance of success $p = 0.39$. If 160 trials occur, what is the probability of getting more than 50 but less than 76 successes?

    In other words, let $X \sim \text{Bin}(n = 160,\ p = 0.39)$ and find $P(50 < X < 76)$.

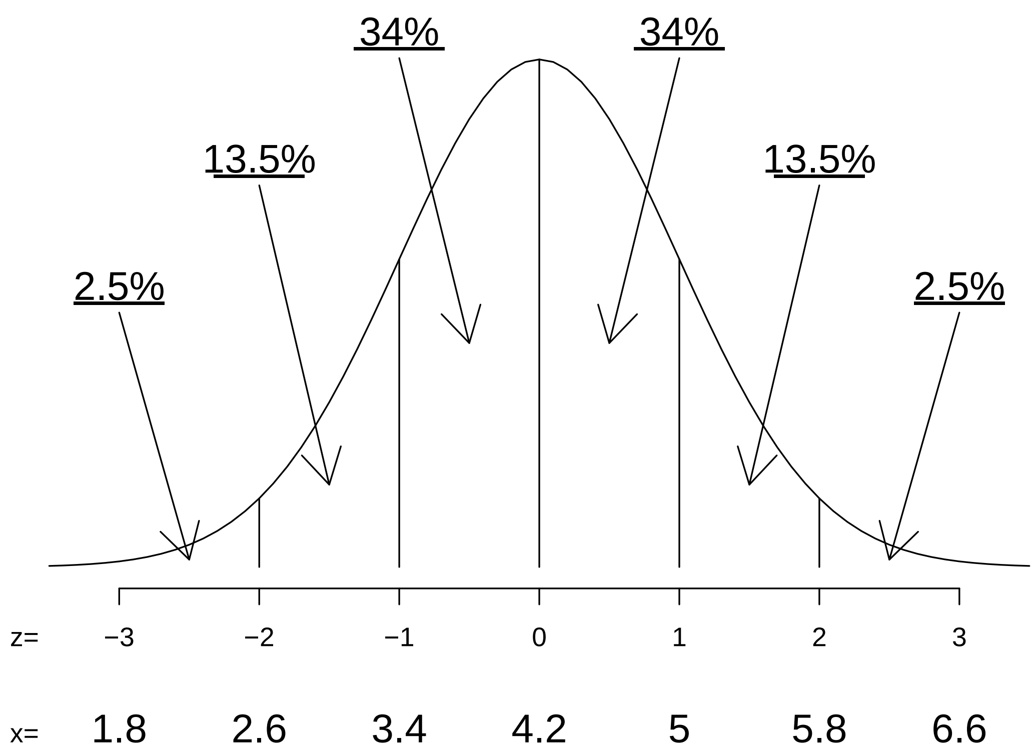
    Use a normal approximation along with the continuity correction.

1. We compare the $z$-scores. The largest $z$-score corresponds to the specimen that is most unusually large.

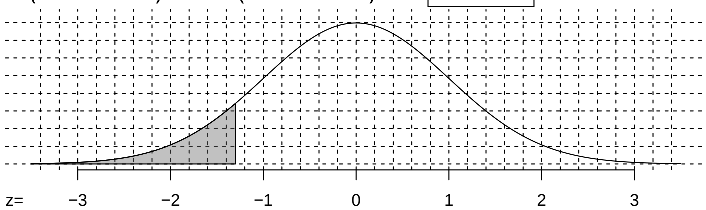
| Type of fruit | formula | $z$-score |
|---|---|---|
| $A$ | $z = \frac{122.4-119}{9}$ | 0.38 |
| $B$ | $z = \frac{89.56-94}{12}$ | -0.37 |
| $C$ | $z = \frac{105.6-114}{14}$ | -0.6 |
| $D$ | $z = \frac{84.19-76}{13}$ | 0.63 |

Thus, the specimen of type $D$ is the most unusually large.

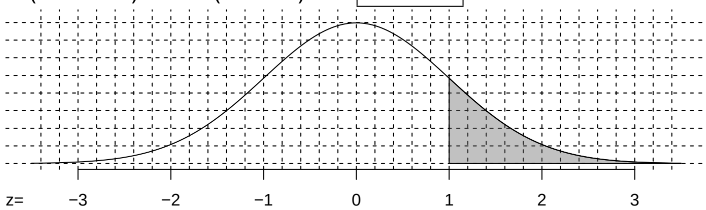
2. The filled in areas and $x$ values are shown below.



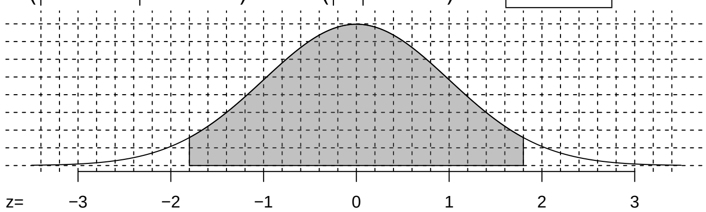
3. (a) $P(X < 61.7) = P(Z < -1.3) = \boxed{0.0968}$



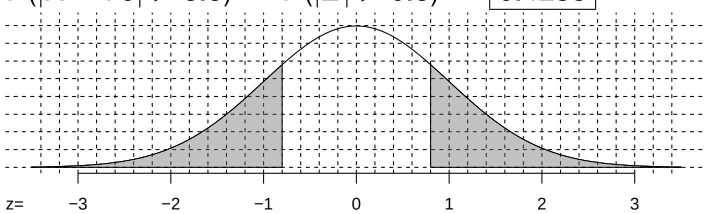

(b) $P(X > 87) = P(Z > 1) = \boxed{0.1587}$




(c) $P(|X - 76| < 19.8) = P(|Z| < 1.8) = \boxed{0.9282}$

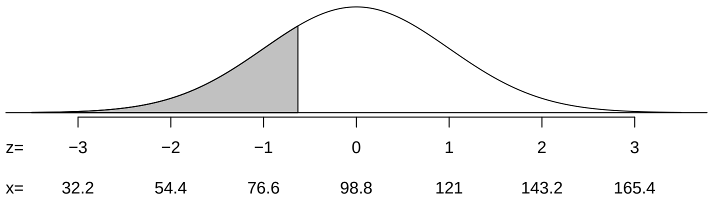



(d) $P(|X - 76| > 8.8) = P(|Z| > 0.8) = \boxed{0.4238}$

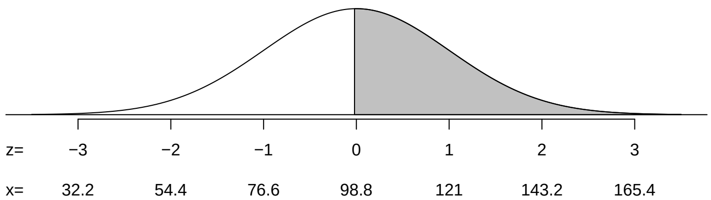


4. Notice the three probabilities will add up to 1.

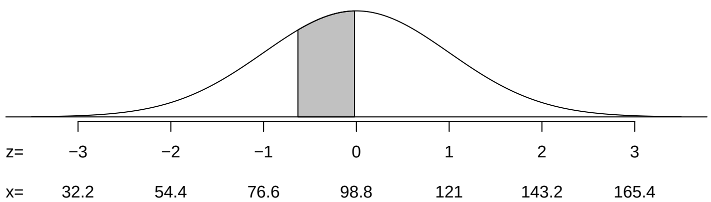
   (a) $P(X < 84.8) = P(Z < -0.63) = \boxed{0.2643}$




   (b) $P(X > 98.4) = P(Z > -0.02) = \boxed{0.508}$




   (c) $P(84.8 < X < 98.4) = P(-0.63 < Z < -0.02) = \boxed{0.2277}$




5. (a) 19

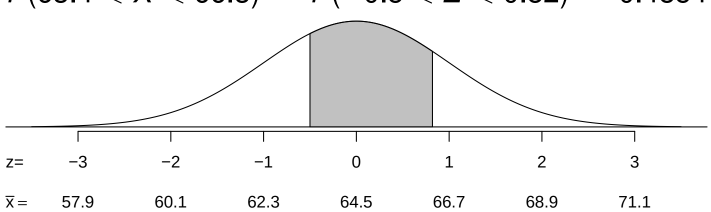
   (b) 0.4

   (c) 0.242

   (d) 0.6368

6. (a) Central limit of average formulas: $\mu_{\bar{x}} = 64.5$ and $\sigma_{\bar{x}} = \frac{13.2}{\sqrt{36}} = 2.2$.

   (b) $P(63.4 < \overline{X} < 66.3) = P(-0.5 < Z < 0.82) = 0.4854$



7. (a) We can recognize $W$ is a Bernoulli variable with $p = 0.64$ and $q = 0.36$. Thus,

$$\mu_W = p = 0.64$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.64)(0.36)} = 0.48$$

Then, we use the central limit of average formulas:

$$\mu_{\hat{p}} = \mu_W = 0.64$$

and

$$\sigma_{\hat{p}} = \frac{\sigma_W}{\sqrt{n}} = \frac{0.48}{\sqrt{81}} = 0.0533$$

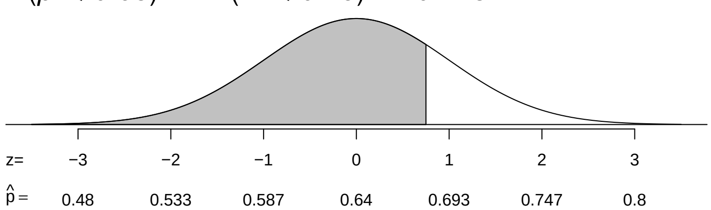
But, if we recognized $\hat{p}$ follows the formulas of a $\hat{p}$ **sampling distribution**:

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}} = \sqrt{\frac{pq}{n}}$$

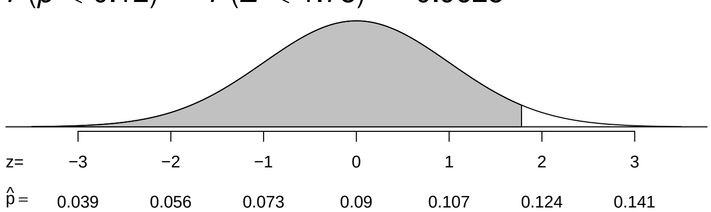
we could have just used those instead.

(b) $P(\hat{p} < 0.68) \ = \ P(Z < 0.75) \ = \ 0.7734$




8. (a) Use $\hat{p}$ sampling formulas: $\mu_{\hat{p}} = 0.09$ and $\sigma_{\hat{p}} = \frac{\sqrt{(0.09)(0.91)}}{\sqrt{289}} = 0.0168$.

(b) $P(\hat{p} < 0.12) \ = \ P(Z < 1.78) \ = \ 0.9625$




9. (a) 3136

(b) 56

(c) 0.1711

(d) 0.5359

10. We recognize $W$ is a Bernoulli variable with $p = 0.6$ and $q = 0.4$. Thus,

$$\mu_W = p = 0.6$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.6)(0.4)} = 0.4899$$

.

We use the Central Limit Formulas (for a sum).

$$\mu_X = n\mu_W = (49)(0.6) = 29.4$$

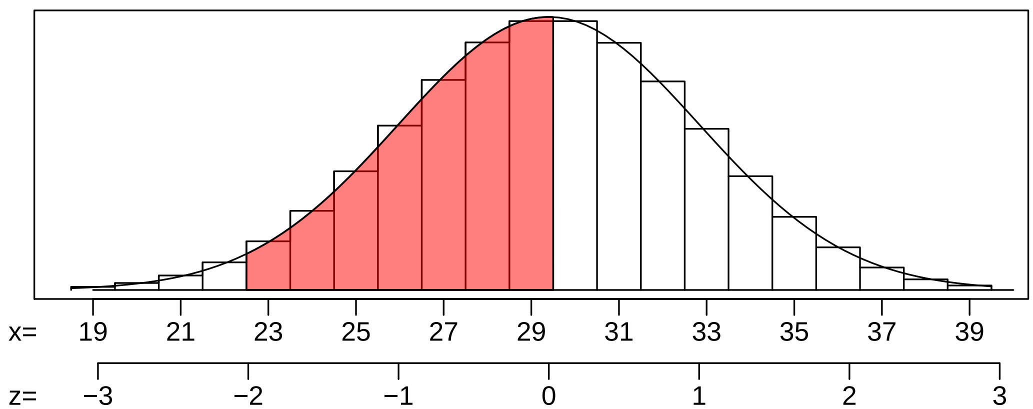
$$\sigma_X = \sqrt{n}\sigma_W = \sqrt{49}(0.4899) = 3.4293$$

It should be mentioned that you could have also just recognized $X$ is binomial:

$$\mu = np = (49)(0.6) = 29.4$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{(49)(0.6)(1-0.6)} = 3.4293$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.




Find the $z$ scores.

$$z_1 = \frac{22.5 - 29.4}{3.4293} = -2.01$$

$$z_2 = \frac{29.5 - 29.4}{3.4293} = 0.03$$

Find the percentiles (from $z$-table).

$$\ell_1 = 0.0222$$

$$\ell_2 = 0.512$$

Calculate the probability.

$$P(23 \le X \le 29) = 0.512 - 0.0222 = 0.4898$$

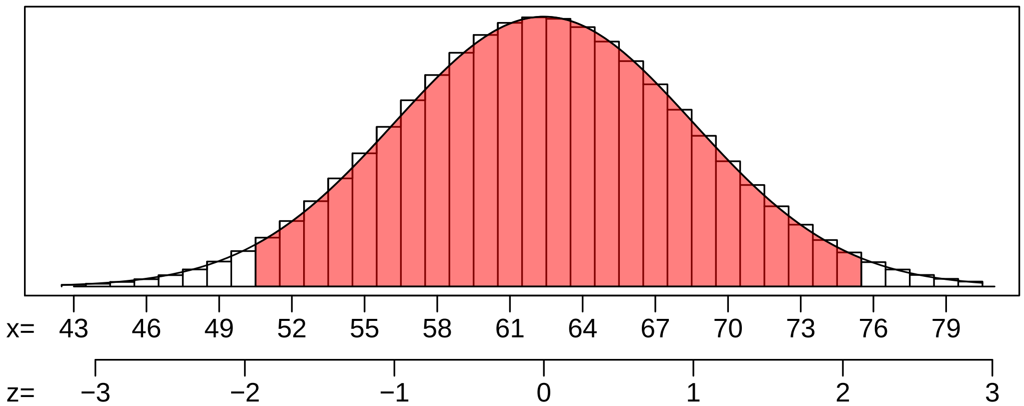
11. Find the mean.

$$\mu = np = (160)(0.39) = 62.4$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(160)(0.39)(1-0.39)} = 6.1696$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.




Find the $z$ scores.

$$z_1 = \frac{50.5 - 62.4}{6.1696} = -1.93$$

$$z_2 = \frac{75.5 - 62.4}{6.1696} = 2.12$$

Find the percentiles (from $z$-table).

$$\ell_1 = 0.0268$$

$$\ell_2 = 0.983$$

Calculate the probability.

$$P(50 < X < 76) = 0.983 - 0.0268 = 0.956$$

## Normal Distributions

$z = \frac{x-\mu}{\sigma}$
$x = \mu + z\sigma$

## Central Limit Theorem

Let random variable $W$ have mean $\mu_w$ and standard deviation $\sigma_w$.
Let random variable $X$ represent the sum of $n$ instances of $W$.
Let random variable $Y$ represent the average of $n$ instances of $W$.
Then:

$$\mu_x = (n)(\mu_w) \qquad \mu_y = \mu_w$$
$$\sigma_x = (\sigma_w)(\sqrt{n}) \qquad \sigma_y = \frac{\sigma_w}{\sqrt{n}}$$

and $X$ and $Y$ are both approximately normal.

## Bernoulli Random Variable

$\mu = p$
$\sigma = \sqrt{pq}$

## Binomial Random Variable (sum of Bernoullis)

$\mu = np$
$\sigma = \sqrt{npq}$

## Proportion Sampling Random Variable (average of Bernoullis)

$\mu = p$
$\sigma = \sqrt{\frac{pq}{n}}$

## Continuity Correction

- If:
  - $X$ is a discrete variable with a domain of consecutive integers
  - we are approximating $X$ with a normal distribution
- Then:

$$P(X \leq x_0) = P\left(Z < \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X < x_0) = P\left(Z < \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right)$$
$$P(X \geq x_0) = P\left(Z > \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X > x_0) = P\left(Z > \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right)$$