 Name_____

Practice Exam 3: PART II

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

1) Compare the scores: a score of 75 on a test with a mean of 65 and a standard deviation of 8 and a score of 75 on a test with a mean of 70 and a standard deviation of 4.

1) same
z scores are the same

$x = 75 \quad \mu = 65 \quad \sigma = 8$

$z = \frac{75-65}{8} = 1.25$

$x = 75 \quad \mu = 70 \quad \sigma = 4$

$z = \frac{75-70}{4} = 1.25$

2) SAT scores have a mean of 1026 and a standard deviation of 209. ACT scores have a mean of 20.8 and a standard deviation of 4.8. A student takes both tests while a junior and scores 860 on the SAT and 16 on the ACT. Compare the scores.

2) student did better on SAT.

SAT

$\mu = 1026 \quad \sigma = 209$

$z = \frac{860-1026}{209} = -0.79$

ACT

$\mu = 20.8 \quad \sigma = 4.8$

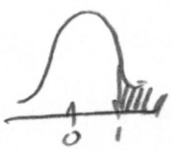
$z = \frac{16-20.8}{4.8} = -1$

3) The mean is $\mu = 15.2$ and the standard deviation is $\sigma = 0.9$. Find the probability that X is greater than 16.1.

3) 0.1587

$P(x > 16.1) = P(z \geq 1) = 1 - 0.8413 = 0.1587$

$z\text{ score} = \frac{16.1-15.2}{0.9} = 1$



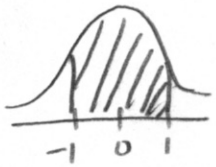
4) The mean is $\mu = 15.2$ and the standard deviation is $\sigma = 0.9$. Find the probability that X is between 14.3 and 16.1.

4) 0.6826

$P(14.3 \leq x \leq 16.1) = P(-1 \leq z \leq 1)$

z scores: $\frac{14.3-15.2}{0.9}$ $\frac{16.1-15.2}{0.9}$ $= 0.8413 - 0.1587$

$-1 \qquad 1$



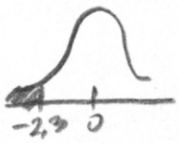
5) A physical fitness association is including the mile run in its secondary-school fitness test. The time for this event for boys in secondary school is known to possess a normal distribution with a mean of 440 seconds and a standard deviation of 40 seconds. Find the probability that a randomly selected boy in secondary school can run the mile in less than 348 seconds.

5) 0.0107

$\mu = 440 \quad \sigma = 40$

$P(x < 348) = P(z \leq -2.3) = 0.0107$

$z\text{score} = \frac{348 - 440}{40} = -2.3$



6) Suppose a brewery has a filling machine that fills 12 ounce bottles of beer. It is known that the amount of beer poured by this filling machine follows a normal distribution with a mean of 12.29 ounces and a standard deviation of 0.04 ounce. Find the probability that the bottle contains between 12.19 and 12.25 ounces.

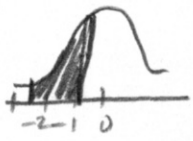
6) 0.1525

$\mu = 12.29 \quad \sigma = 0.04$

$P(12.19 < x < 12.25) = P(-2.5 < z < -1) = 0.1587 - 0.0062$

$z\text{score}: \quad \frac{12.19 - 12.29}{0.04} \quad \frac{12.25 - 12.29}{0.04}$

$-2.5 \quad -1$



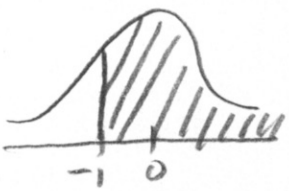
7) The tread life of a particular brand of tire is a random variable best described by a normal distribution with a mean of 60,000 miles and a standard deviation of 2000 miles. What is the probability a particular tire of this brand will last longer than 58,000 miles?

7) 0.8413

$\mu = 60,000 \quad \sigma = 2000$

$P(x > 58,000) = P(z > -1) = 1 - 0.1587$

$\frac{58,000 - 60,000}{2000}$



8) A new phone system was installed last year to help reduce the expense of personal calls that were being made by employees. Before the new system was installed, the amount being spent on personal calls followed a normal distribution with an average of $500 per month and a standard deviation of $50 per month. Refer to such expenses as PCE's (personal call expenses). Using the distribution above, what is the probability that a randomly selected month had a PCE of between $375.00 and $590.00?

8) 0.9579

$\mu = 500 \quad \sigma = 50$

$P(375 < x < 590) = P(-2.5 < z < 1.8) = 0.9641 - 0.0062$

$\frac{375-500}{50} \quad \frac{590-500}{50}$

$-2.5 \quad 1.8$

9) **Please circle one. Will you use the CLT or wil you use the normal distribution as an approximation to the BINOMIAL DISTRIBUTION?**
The weights of the fish in a certain lake are normally distributed with a mean of 20 lb and a standard deviation of 9. If 9 fish are randomly selected, what is the **probability that the MEAN** weight will be between 17.6 and 23.6 lb?

9) 0.673

condition: normal dist. $\mu = 20 \quad \sigma = 9 \quad n = 9$

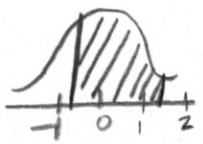
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{9}{\sqrt{9}} = \frac{9}{3} = 3$

$P(17.6 < \bar{x} < 23.6) = P(-0.8 < z < 1.2) = 0.8849 - 0.2119$

$= 0.673$

$\frac{17.6-20}{3} \quad \frac{23.6-20}{3}$

$-0.8 \quad 1.2$



10) **Please circle one. Will you use the CLT or wil you use the normal distribution as an approximation to the BINOMIAL DISTRIBUTION?**
A study of the amount of time it takes a mechanic to rebuild the transmission for a 2005 Chevrolet Cavalier shows that the mean is 8.4 hours and the standard deviation is 1.8 hours. If 40 mechanics are randomly selected, find the **probability that the MEAN** rebuild time is less than 7.6 hours.

10) 0.0025

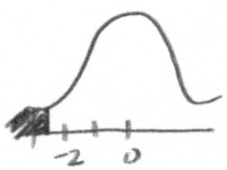
condition: $n \geq 30$ ✓ $\quad n = 40 \quad \mu = 8.4 \quad \sigma = 1.8$

$P(\bar{x} < 7.6) = P(z < -2.81) = 0.0025$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$z = \frac{(7.6 - 8.4)}{(1.8/\sqrt{40})}$

$z = -2.81$



11) **Please circle one. Will you use the CLT or wil you use the normal distribution as an approximation to the BINOMIAL DISTRIBUTION?** 11) _____

A final exam in Math 160 has a mean of 73 with standard deviation 7.8. If 24 students are randomly selected, find the **probability that the MEAN** of their test scores is greater than 71.

conditions: not met $n = 24$

12) **Please circle one. Will you use the CLT or wil you use the normal distribution as an approximation to the BINOMIAL DISTRIBUTION?** 12) _____

A final exam in Math 160 has a mean of 73 with standard deviation 7.8. If 24 students are randomly selected, find the **probability that the MEAN** of their test scores is less than 76.

conditions: not met $n = 24$

Solve the problem.

13) **Please circle one. Will you use the CLT or wil you use the normal distribution as an approximation to the BINOMIAL DISTRIBUTION?**
A study of the amount of time it takes a mechanic to rebuild the transmission for a 2005 Chevrolet Cavalier shows that the mean is 8.4 hours and the standard deviation is 1.8 hours. If 40 mechanics are randomly selected, find the **probability that their mean** rebuild time exceeds 9.1 hours.

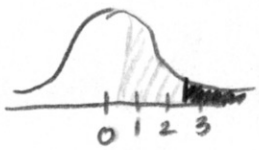
13) 0.0069

conditions: $n \geq 30$ $n=40$ $\mu=8.4$ $\sigma=1.8$

$P(\bar{x} > 9.1) = P(z > 2.46) = 1 - 0.9931$

$\downarrow$

$z = \frac{(9.1-8.4)}{(1.8/\sqrt{40})} = 2.46$



14) **Please circle one. Will you use the CLT or wil you use the normal distribution as an approximation to the BINOMIAL DISTRIBUTION?**
Assume that women's heights are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. If 90 women are randomly selected, find the **probability that they have a mean** height between 62.9 inches and 64.0 inches.

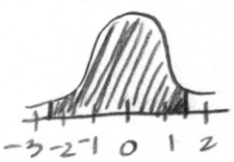
14) 0.9318

conditions: normal distribution

$\mu = 63.6$ $\sigma = 2.5$ $n = 90$

$P(62.9 < \bar{x} < 64.0) = P(-2.66 < z < 1.52) = 0.9357 - 0.0039$

$\frac{62.9-63.6}{2.5/\sqrt{90}}$ $\frac{64-63.6}{2.5/\sqrt{90}}$



-2.66 1.52

15) **Please circle one. Will you use the CLT or wil you use the normal distribution as an approximation to the BINOMIAL DISTRIBUTION?**

A product is manufactured in batches of 120 and the overall rate of defects is 5%.
Estimate the probability that a randomly selected batch contains more than 6 defects.

15) no solution

success = defect $n=120$ $p=0.05$ $q=0.95$

conditions: $np=6$ ✗ no $nq=114>10$ ✓

16) **Please circle one. Will you use the CLT or wil you use the normal distribution as an approximation to the BINOMIAL DISTRIBUTION?**

In one county, the conviction rate for speeding is 85%. Estimate the probability that of the next 100 speeding summonses issued, there will be at least 90 convictions.

16) 0.1038

success = conviction for speeding $p=0.85$ $n=100$

$q=0.15$

conditions: $np=85\geq 10$ ✓ $nq=15\geq 10$ ✓

$\mu=np=85$

$\sigma=\sqrt{npq}=\sqrt{100\times 0.85\times 0.15}=3.57$

$P(x\geq 90)$

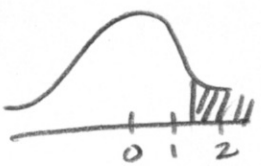
↓ cont. correction

$P(x\geq 89.5) = P(z\geq 1.26) = 1-0.8962$

$= 0.1038$

$\frac{89.5-85}{3.57}$

$z=1.26$



17) **Please circle one. Will you use the CLT or wil you use the normal distribution as an approximation to the BINOMIAL DISTRIBUTION?**

A certain question on a test is answered correctly by 22% of the respondents. Estimate the probability that among the next 150 responses there will be at most 40 correct answers.

17) 0.9306

success = correct answer $p = 0.22$ $n = 150$

$q = 0.78$

conditions: $np = 33 \geq 10$ ✓ $nq = 117 \geq 10$ ✓

$\mu = 33$ $\sigma = \sqrt{npq} = 5.07$

$P(x \leq 40)$

↓ correction

$P(x \leq 40.5) = P(z < 1.48) = 0.9306$

↓

$z = \frac{40.5 - 33}{5.07}$

$z = 1.48$

Provide an appropriate response.

18) **Please circle one. Will you use the CLT or wil you use the normal distribution as an approximation to the BINOMIAL DISTRIBUTION?**

If the probability of a newborn kitten being female is 0.5, find the probability that in 100 births, 55 or more will be female.

18) 0.1841

success = female $p = 0.5$ $n = 100$

$q = 0.5$

conditions: $np = 50 \geq 10$ ✓ $nq = 50 \geq 10$ ✓

$\mu = np = 50$ $\sigma = \sqrt{npq} = 5$

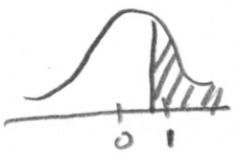
$P(x \geq 55)$

↓ correction

$P(x \geq 54.5) = P(z > 0.9) = 1 - 0.8159$

$z = \frac{54.5 - 50}{5}$

$z = 0.9$



$p = 0.36 \quad q = 0.64$

23) According to a study conducted in one city, 36% of adults in the city have credit card debts of more than \$2000. A simple random sample of n = 150 adults is obtained from the city. Describe the sampling distribution of $\hat{p}$, the sample proportion of adults who have credit card debts of more than \$2000. 23) B

A) Approximately normal; $\mu_p = 0.36$, $\sigma_p = 0.0015$
B) Approximately normal; $\mu_p = 0.36$, $\sigma_p = 0.039$
C) Binomial; $\mu_p = 54$, $\sigma_p = 5.88$
D) Exactly normal; $\mu_p = 0.36$, $\sigma_p = 0.039$

$np = 54 \geq 10$ ✓ $nq = 96 \geq 10$ ✓

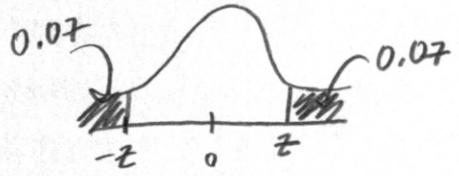
$\mu_p = p = 0.36 \quad \sigma_p = \sqrt{\frac{0.36 \times 0.64}{150}}$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

EXTRA CREDIT

Find the indicated z-score.

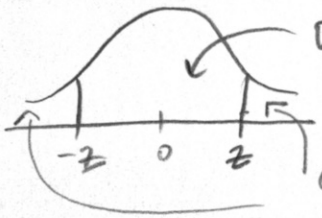
24) Find the z-score for which the area under the standard normal curve to its right is 0.07. 24) 1.48



From the table

$0.07 \approx 0.0694 \rightarrow z = -1.48$

25) Find the z-scores for which 90% of the distribution's area lies between -z and z. 25) -1.65, 1.65



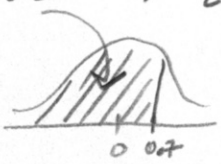
From the table

0.10 → divide by 2 = $0.05 \approx 0.495 \rightarrow z = -1.65$

26) The length of time it takes college students to find a parking spot in the library parking lot follows a normal distribution with a mean of 7.0 minutes and a standard deviation of 1 minute. Find the cut-off time which 75.8% of the college students exceed when trying to find a parking spot in the library parking lot. 26) 7.7 min

$\mu = 7.0 \quad \sigma = 1$

0.758 → $z = 0.7$



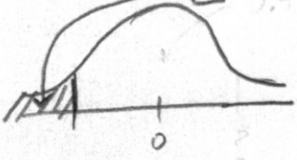
$z = \frac{x - \mu}{\sigma}$

$0.7 = \frac{x - 7.0}{1}$

$0.7 = x - 7.0 \rightarrow x = 7.7$

27) A brewery has a beer dispensing machine that dispenses beer into the company's 12 ounce bottles. The distribution for the amount of beer dispensed by the machine follows a normal distribution with a standard deviation of 0.11 ounce. The company can control the mean amount of beer dispensed by the machine. What value of the mean should the company use if it wants to guarantee that 98.5% of the bottles contain at least 12 ounces (the amount on the label)? Round to the nearest thousandth. 27) 12.239

$1 - 0.985 = 0.015$



$z = -2.17$ $\quad z = 2.17$

$\sigma = 0.11$

$z = \frac{x - \mu}{\sigma}$

$-2.17 = \frac{12 - \mu}{0.11}$

$-0.2387 = 12 - \mu$

$\mu = 12.2387$

Answer Key
Testname: MAT098-181 PRACTICE EXAM 3 PART II

1) The two scores are statistically the same.
2) A score of 860 on the SAT test was better.
3) 0.1587
4) 0.6826
5) 0.0107
6) 0.1525
7) 0.8413
8) 0.9579
9)
10) 0.0025
11) 0.8962
12) 0.9699
13) 0.0069
14) 0.9318
15) 0.4168
16) 0.1038
17) 0.9306
18) 0.1841
19) 0.0896
20) 0.6644
21) D
22) D
23) B
24) 1.48
25) (-1.645, 1.645)
26) 7.7 min
27) 12.239 oz