1. **Problem**
   A farm produces 4 types of fruit: A, B, C, and D. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	62	11
B	109	12
C	137	13
D	123	7

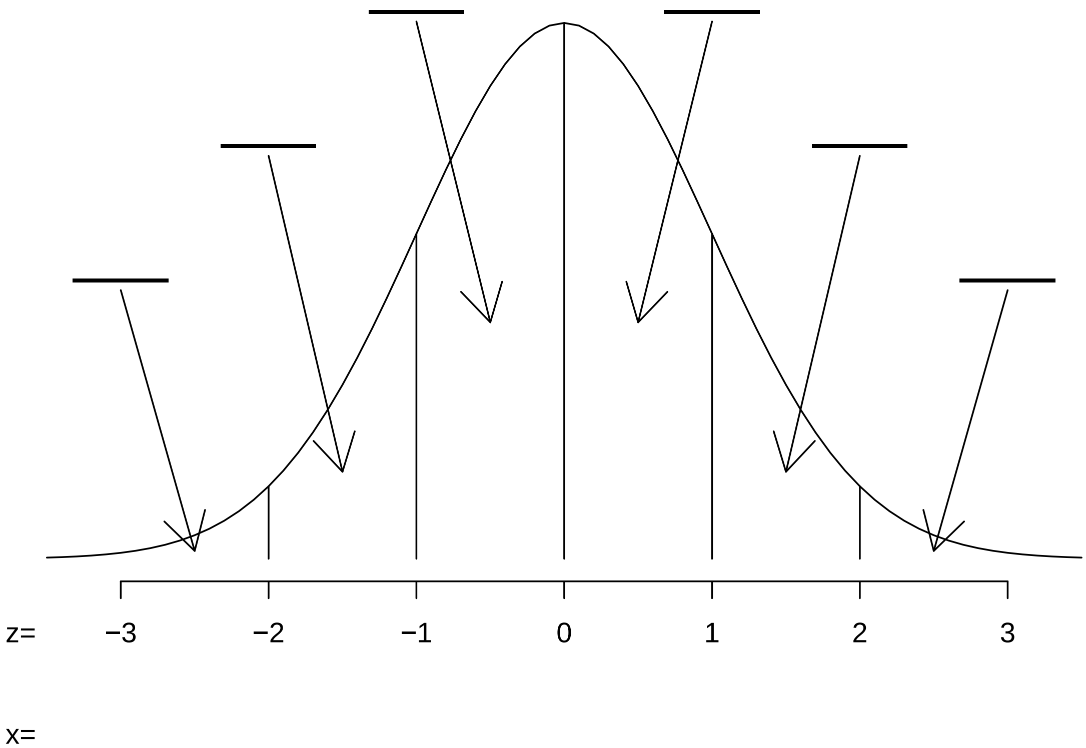
One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	82.35
B	93.16
C	130
D	117.5

Which specimen is the most unusually small (relative to others of its type)?

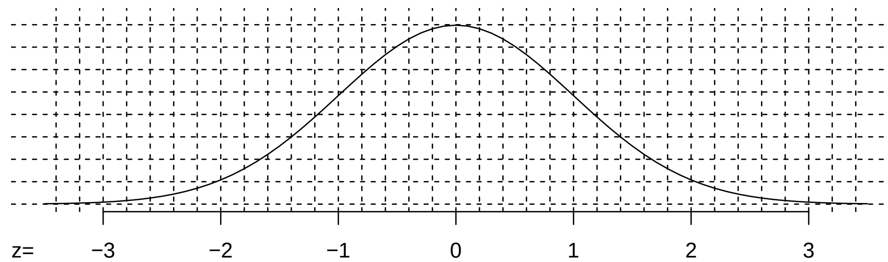
2. **Problem**
A normal random variable X has a mean $\mu = 5.3$ and standard deviation $\sigma = 0.6$. Please label the density curve with:

(a) The appropriate values of x.
(b) The areas of the sections.

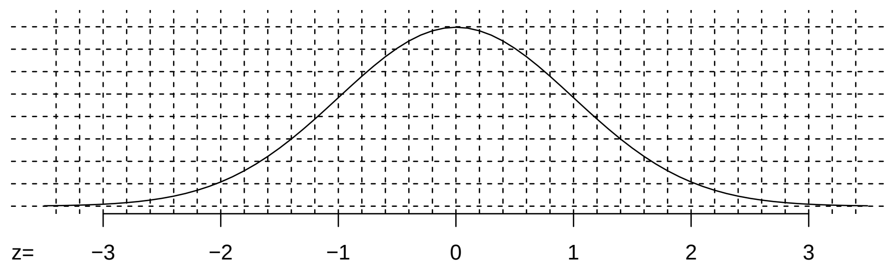


3. **Problem**
Let X be normally distributed with mean 55 and standard deviation 12. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

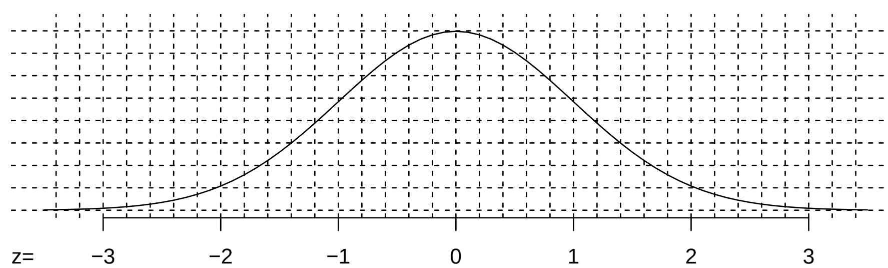
(a) $P(X < 64.6)$



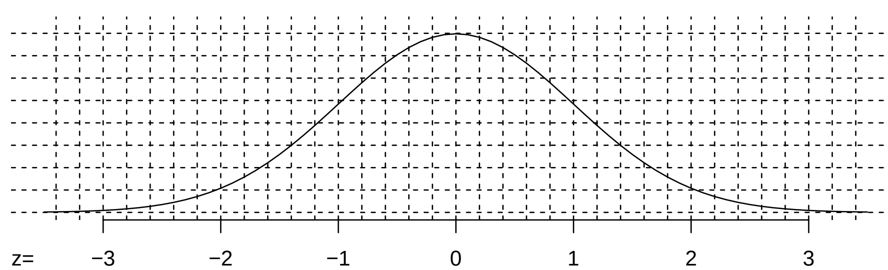
(b) $P(X > 57.4)$



(c) $P(|X - 55| < 10.8)$



(d) $P(|X - 55| > 15.6)$



4. **Problem**
   Let X be normally distributed with mean 115.9 and standard deviation 27.4. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

   (a) What's the probability that X is less than 104? **Draw a sketch**.

   (b) What's the probability that X is more than 138? **Draw a sketch**.

   (c) What's the probability that X is between 104 and 138? **Draw a sketch**.

5. **Problem**
   Let random variable W have mean $\mu_W = 35$ and standard deviation $\sigma_W = 7$. Let random variable X represent the **average** of $n = 121$ instances of W.

   (a) Determine the expected value of X. $\mu_X = ?$

   (b) Determine the standard deviation of X. $\sigma_X = ?$

   (c) Using normal approximation, determine $P(X < 35.21)$.

   (d) Using normal approximation, determine $P(X > 34.93)$.

6. **Problem**
A very large population has a mean of 95.4 and a standard deviation of 10.4. When a random sample of size 64 is taken, what is the probability that the **sample mean** ($\bar{x}$) is between 93.7 and 94.9?

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(93.7 < \overline{X} < 94.9)$. **Draw a sketch**

7. **Problem**
   Let random variable W have the probability distribution shown below.

w	$P(w)$
0	0.4
1	0.6

Let random variable $\hat{p}$ (sample proportion) represent the average of $n = 81$ instances of W.

(a) Determine the mean and standard deviation of the $\hat{p}$.

(b) Determine $P(\hat{p} < 0.58)$. Do NOT use a continuity correction. **Draw a sketch**

8. **Problem**
A very large population has a population proportion $p = 0.37$. When a random sample of size 144 is taken, what is the probability that the **sample proportion** ($\hat{p}$) is less than 0.35?

Do NOT use a continuity correction.

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(\hat{p} < 0.35)$. **Draw a sketch**

9. **Problem**
Let random variable W have mean $\mu_W = 59$ and standard deviation $\sigma_W = 17$. Let random variable X represent the **sum** of $n = 144$ instances of W.

(a) Determine the expected value of X. $\mu_X = ?$

(b) Determine the standard deviation of X. $\sigma_X = ?$

(c) Using normal approximation, determine $P(X < 8585.76)$.

(d) Using normal approximation, determine $P(X > 8728.56)$.

10. **Problem**

Let random variable W have the probability distribution shown below.

w	$P(w)$
0	0.63
1	0.37

Let random variable X represent the sum of $n = 97$ instances of W. (Thus X is the sample total, or number of successes.)

What is the probability that X is at least 35 but at most 40? **Use a normal approximation with continuity corrections**.

11. **Problem**

Let each trial have a chance of success $p = 0.33$. If 157 trials occur, what is the probability of getting at least 40 but less than 49 successes?

In other words, let $X \sim \text{Bin}(n = 157,\ p = 0.33)$ and find $P(40 \leq X < 49)$.

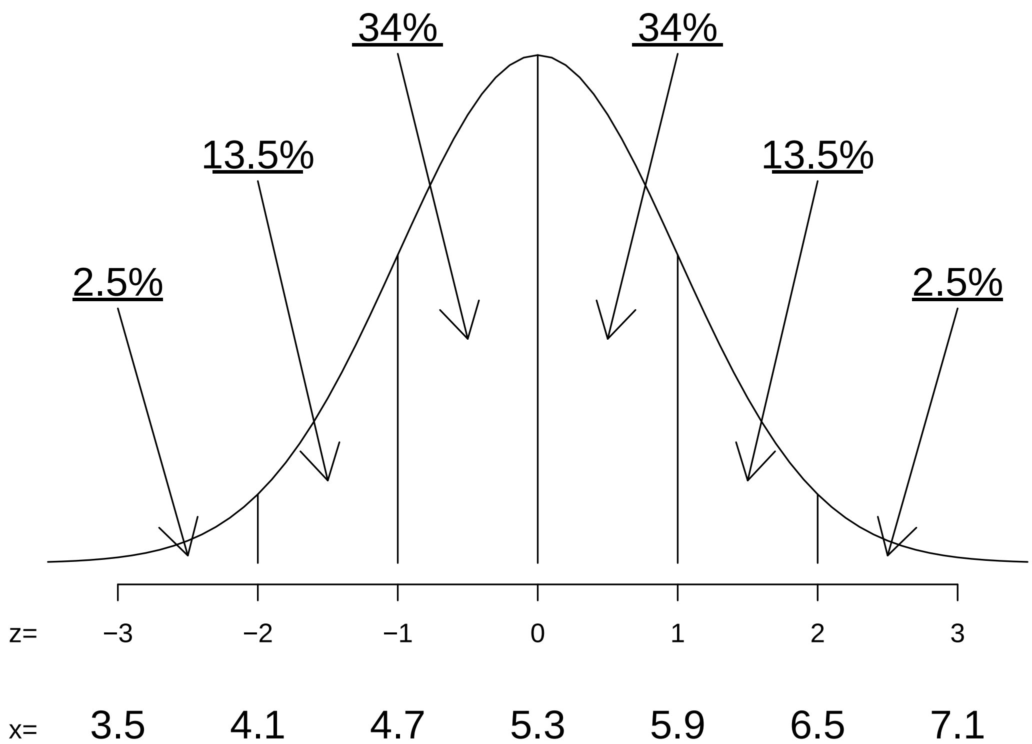
Use a normal approximation along with the continuity correction.

1. We compare the z-scores. The smallest z-score corresponds to the specimen that is most unusually small.

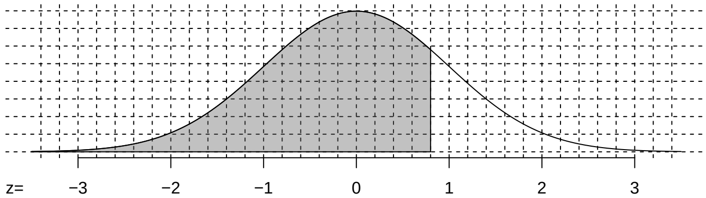
Type of fruit	formula	z-score
A	$z = \frac{82.35-62}{11}$	1.85
B	$z = \frac{93.16-109}{12}$	-1.32
C	$z = \frac{130-137}{13}$	-0.54
D	$z = \frac{117.5-123}{7}$	-0.78

Thus, the specimen of type B is the most unusually small.

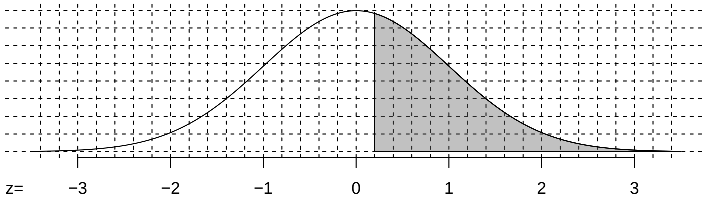
2. The filled in areas and x values are shown below.



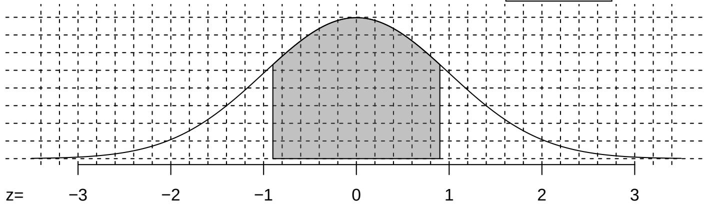
3. (a) $P(X < 64.6) = P(Z < 0.8) = \boxed{0.7881}$



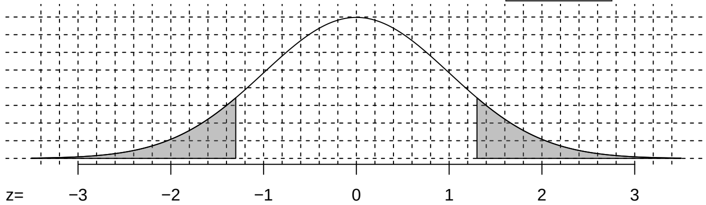
(b) $P(X > 57.4) = P(Z > 0.2) = \boxed{0.4207}$



(c) $P(|X - 55| < 10.8) = P(|Z| < 0.9) = \boxed{0.6318}$

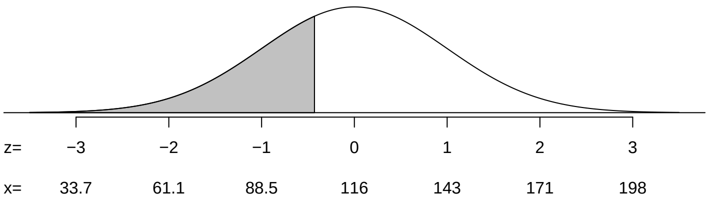


(d) $P(|X - 55| > 15.6) = P(|Z| > 1.3) = \boxed{0.1936}$

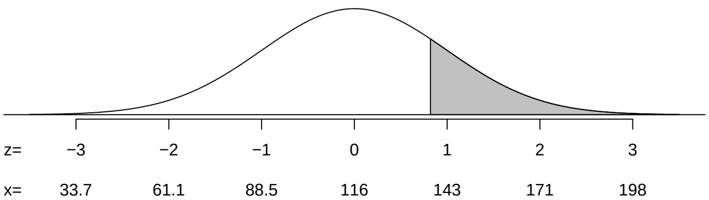


4. Notice the three probabilities will add up to 1.

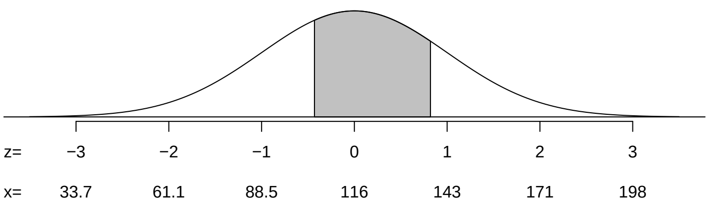
   (a) $P(X < 104) = P(Z < -0.43) = \boxed{0.3336}$



   (b) $P(X > 138) = P(Z > 0.82) = \boxed{0.2061}$



   (c) $P(104 < X < 138) = P(-0.43 < Z < 0.82) = \boxed{0.4603}$



5. We use the Central Limit Theorem for **sample average** sampling ($\bar{x}$ sampling). We recognize that in this problem X is an AVERAGE of 121 instances of W.

   (a) $\mu_X = \mu_W = 35$

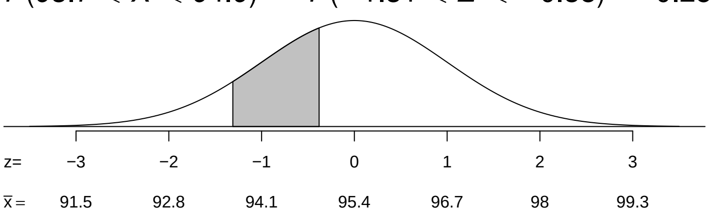
   (b) $\sigma_X = \frac{\sigma_W}{\sqrt{n}} = 0.6363636$

   (c) 0.6293

   (d) 0.5438

6. (a) Central limit of average formulas: $\mu_{\bar{x}} = 95.4$ and $\sigma_{\bar{x}} = \frac{10.4}{\sqrt{64}} = 1.3$.

   (b) $P(93.7 < \overline{X} < 94.9) = P(-1.31 < Z < -0.38) = 0.2569$



7. (a) We can recognize W is a Bernoulli variable with $p = 0.6$ and $q = 0.4$. Thus,

$$\mu_W = p = 0.6$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.6)(0.4)} = 0.4898979$$

Then, we use the central limit of average formulas:

$$\mu_{\hat{p}} = \mu_W = 0.6$$

and

$$\sigma_{\hat{p}} = \frac{\sigma_W}{\sqrt{n}} = \frac{0.4898979}{\sqrt{81}} = 0.0544331$$

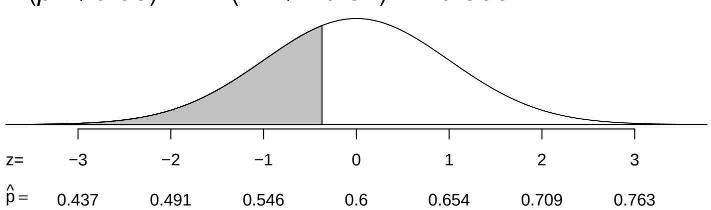
But, if we recognized $\hat{p}$ follows the formulas of a $\hat{p}$ **sampling distribution**:

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}} = \sqrt{\frac{pq}{n}}$$

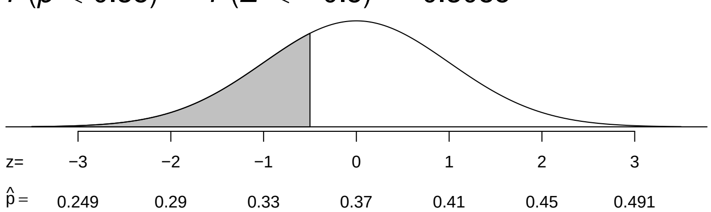
we could have just used those instead.

(b) $P(\hat{p} < 0.58) = P(Z < -0.37) = 0.3557$



8. (a) Use $\hat{p}$ sampling formulas: $\mu_{\hat{p}} = 0.37$ and $\sigma_{\hat{p}} = \frac{\sqrt{(0.37)(0.63)}}{\sqrt{144}} = 0.0402337$.

(b) $P(\hat{p} < 0.35) = P(Z < -0.5) = 0.3085$



9. (a) 8496

(b) 204

(c) 0.67

(d) 0.1271

10. We recognize W is a Bernoulli variable with $p = 0.37$ and $q = 0.63$. Thus,

$$\mu_W = p = 0.37$$

and

$$\sigma_W = \sqrt{pq} = \sqrt{(0.37)(0.63)} = 0.4828043$$

.

We use the Central Limit Formulas (for a sum).

$$\mu_X = n\mu_W = (97)(0.37) = 35.89$$

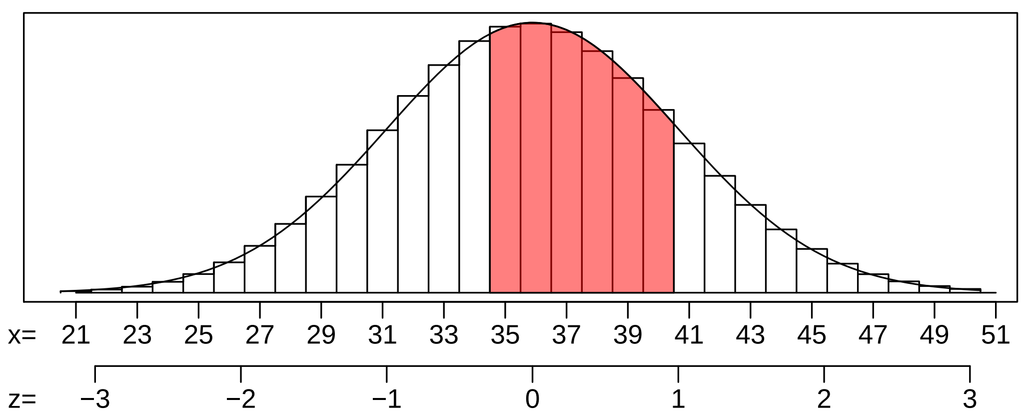
$$\sigma_X = \sqrt{n}\sigma_W = \sqrt{97}(0.4828043) = 4.7551$$

It should be mentioned that you could have also just recognized X is binomial:

$$\mu = np = (97)(0.37) = 35.89$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{(97)(0.37)(1-0.37)} = 4.7551$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{34.5 - 35.89}{4.7551} = -0.29$$

$$z_2 = \frac{40.5 - 35.89}{4.7551} = 0.97$$

Find the percentiles (from z-table).

$$\ell_1 = 0.3859$$

$$\ell_2 = 0.834$$

Calculate the probability.

$$P(35 \leq X \leq 40) = 0.834 - 0.3859 = 0.4481$$

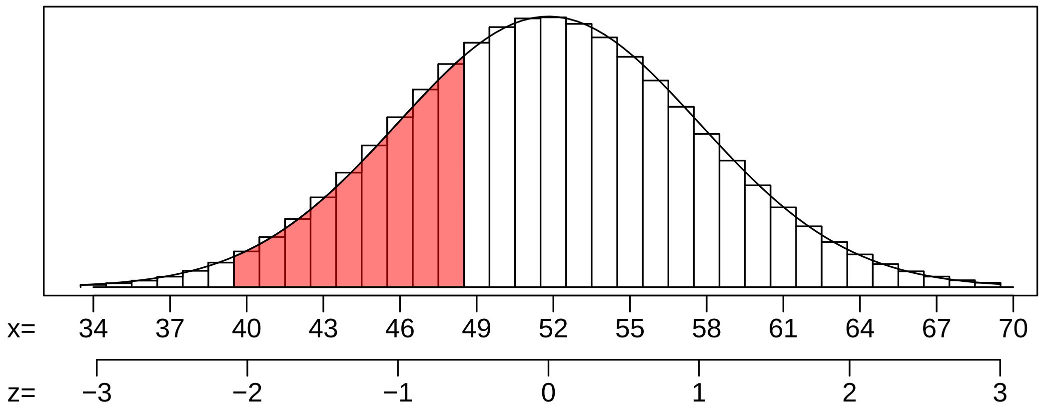
11. Find the mean.

$$\mu = np = (157)(0.33) = 51.81$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(157)(0.33)(1-0.33)} = 5.8917$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{39.5 - 51.81}{5.8917} = -2.09$$

$$z_2 = \frac{48.5 - 51.81}{5.8917} = -0.56$$

Find the percentiles (from z-table).

$$\ell_1 = 0.0183$$

$$\ell_2 = 0.2877$$

Calculate the probability.

$$P(40 \leq X < 49) = 0.2877 - 0.0183 = 0.27$$

Normal Distributions

$z = \frac{x-\mu}{\sigma}$
$x = \mu + z\sigma$

Central Limit Theorem

Let random variable W have mean μ_w and standard deviation σ_w.
Let random variable X represent the sum of n instances of W.
Let random variable Y represent the average of n instances of W.
Then:

$$\mu_x = (n)(\mu_w) \qquad \mu_y = \mu_w$$
$$\sigma_x = (\sigma_w)(\sqrt{n}) \qquad \sigma_y = \frac{\sigma_w}{\sqrt{n}}$$

and X and Y are both approximately normal.

Bernoulli Random Variable

$\mu = p$
$\sigma = \sqrt{pq}$

Binomial Random Variable (sum of Bernoullis)

$\mu = np$
$\sigma = \sqrt{npq}$

Proportion Sampling Random Variable (average of Bernoullis)

$\mu = p$
$\sigma = \sqrt{\frac{pq}{n}}$

Continuity Correction

- If:
  - X is a discrete variable with a support of consecutive integers
  - we are approximating X with a normal distribution
- Then we can apply a continuity correction:

$$P(X \le x_0) = P\left(Z < \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X < x_0) = P\left(Z < \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right)$$
$$P(X \ge x_0) = P\left(Z > \frac{x_0 - 0.5 - \mu_x}{\sigma_x}\right) \qquad P(X > x_0) = P\left(Z > \frac{x_0 + 0.5 - \mu_x}{\sigma_x}\right)$$