MAT098/181 Name________________________________

Practice Exam 3: PART I

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) The SAT is an exam used by colleges and universities to evaluate undergraduate applicants. The test scores are normally distributed. In a recent year, the mean test score was 1484 and the standard deviation was 298. The test scores of four students selected at random are 1930, 1340, 2150, and 1450. Find the z–scores that correspond to each value. 1) ____________

$\mu = 1484 \qquad \sigma = 298$

$$z = \frac{1930-1484}{298} = 1.50 \qquad z = \frac{1340-1484}{298} = -0.48$$

$$z = \frac{2150-1484}{298} = 2.23 \qquad z = \frac{1450-1484}{298} = -0.11$$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

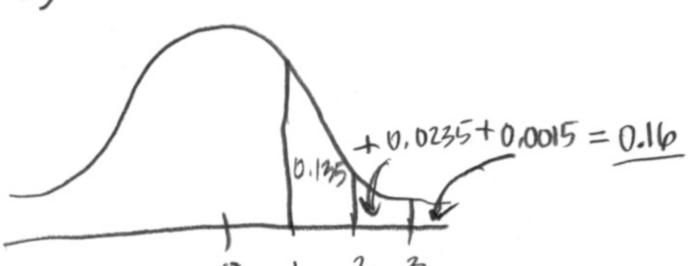
For each question #2–5, draw a sketch of the standard normal curve. Use the Empirical Rule to find the probability that a z–score randomly selected from the normal distribution meets the given condition.

2) The z–score is greater than 1. 2) B
A) 0.8413 B) 0.1587 C) 0.1397 D) 0.5398

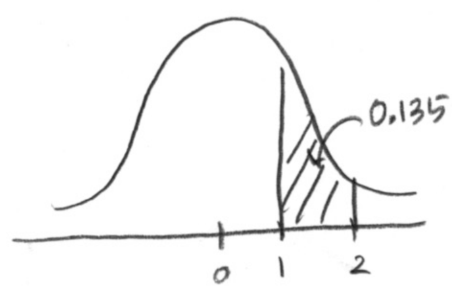
3) The z–score is between 1 and 2. 3) D
A) 0.5398 B) 0.8413 C) 0.2139 D) 0.1359

4) The ~~z–score is~~ between –1.5 and 2.5. 4) A
A) 0.9270 B) 0.9831 C) 0.6312 D) 0.7182

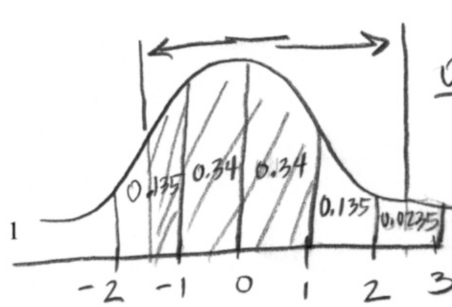
2)



3)



4)



$$\frac{0.135}{2} + 0.68 + 0.135 + \frac{0.0235}{2} = 0.89425$$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

5) Suppose a brewery has a filling machine that fills 12 ounce bottles of beer. It is known that the amount of beer poured by this filling machine follows a normal distribution with a mean of 10.14 onces and a standard deviation of 0.04 ounce. Find the probability that the bottle contains more than 10.14 ounces of beer. 5) 0.5

$\mu = 10.14 \qquad \sigma = 0.04$

$P(x > 10.14) = P(z > 0) = 0.5$

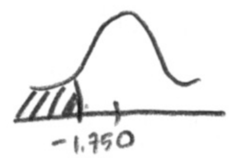
$\hookrightarrow z \text{ score} = \frac{10.14 - 10.14}{0.04} = 0$

Assume that X has a normal distribution, and find the indicated probability.

6) The mean is $\mu = 60.0$ and the standard deviation is $\sigma = 4.0$. Find the probability that X is less than 53.0. 6) 0.0401

$P(x < 53) = P(z < -1.75) = 0.0401$

$\hookrightarrow z \text{ score} = \frac{53 - 60}{4} = -1.75$

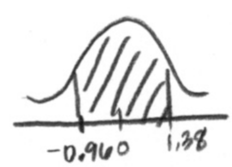


7) The mean is $\mu = 22.0$ and the standard deviation is $\sigma = 2.4$. Find the probability that X is between 19.7 and 25.3. 7) 0.7477

$P(19.7 < x < 25.3) = P(-0.96 < z < 1.38) = 0.9162 - 0.1685$

z scores: $\frac{19.7 - 22}{2.4}$ $\quad \frac{25.3 - 22}{2.4}$

$-0.96 \qquad 1.38$



Find the indicated probability.

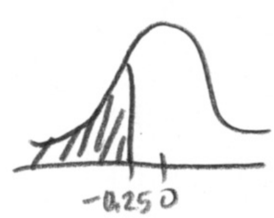
8) The volumes of soda in quart soda bottles are normally distributed with a mean of 32.3 oz and a standard deviation of 1.2 oz. What is the probability that the volume of soda in a randomly selected bottle will be less than 32 oz? 8) 0.4013

$\mu = 32.3 \qquad \sigma = 1.2$

$P(x < 32) = P(z < -0.25) = 0.4013$

$\downarrow$

$z \text{ score} = \frac{32 - 32.3}{1.2} = -0.25$



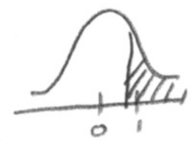
9) The weekly salaries of teachers in one state are normally distributed with a mean of \$490 and a standard deviation of \$45. What is the probability that a randomly selected teacher earns more than \$525 a week?

9) 0.2177

$\mu = 490 \quad \sigma = 45$

$P(x > 525) = P(z > 0.78) = 1 - 0.7823$

↳ $z\text{ score} = \dfrac{525-490}{45} = 0.78$



10) The length of time it takes college students to find a parking spot in the library parking lot follows a normal distribution with a mean of 6.0 minutes and a standard deviation of 1 minute. Find the probability that a randomly selected college student will take between 4.5 and 7.0 minutes to find a parking spot in the library lot.

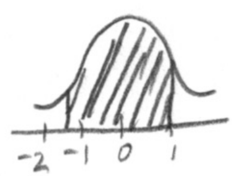
10) 0.7745

$\mu = 6.0 \quad \sigma = 1$

$P(4.5 < x < 7.0) = P(-1.5 < z < 1) = 0.8413 - 0.0668$

z scores: $\dfrac{4.5-6.0}{1} \qquad \dfrac{7.0-6.0}{1}$

$-1.5 \qquad 1$



11) A new phone system was installed last year to help reduce the expense of personal calls that were being made by employees. Before the new system was installed, the amount being spent on personal calls followed a normal distribution with an average of \$600 per month and a standard deviation of \$50 per month. Refer to such expenses as PCE's (personal call expenses). Using the distribution above, what is the probability that a randomly selected month had a PCE of between \$475.00 and \$690.00?

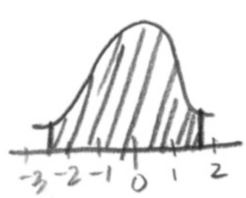
11) 0.9579

$\mu = 600 \quad \sigma = 50$

$P(475 < x < 690) = P(-2.5 < z < 1.8) = 0.9641 - 0.0062$

$\dfrac{475-600}{50} \qquad \dfrac{690-600}{50}$

$-2.5 \qquad 1.8$



12) **USE THE CENTRAL LIMIT THEOREM.** 12) 0.9192

The amount of snowfall falling in a certain mountain range is normally distributed with a mean of 70 inches, and a standard deviation of 10 inches. What is the **probability that the MEAN** annual snowfall during 25 randomly picked years will exceed 72.8 inches?

$\mu = 70 \quad \sigma = 10$ conditions: normal distribution

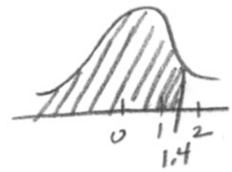
$n = 25$

$P(\bar{x} > 72.8) = P(z < 1.4) = 0.9192$

$\hookrightarrow$ $z\text{score} = \dfrac{72.8 - 70}{10/\sqrt{25}}$

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

$= \dfrac{2.8}{10/5} = \dfrac{2.8}{2} = 1.4$



13) **USE THE CENTRAL LIMIT THEOREM.** 13) 0.9192

The annual precipitation amounts in a certain mountain range are normally distributed with a mean of 109 inches, and a standard deviation of 10 inches. What is the **probability that the MEAN** annual precipitation during 25 randomly picked years will be less than 111.8 inches?

$\mu = 109 \quad \sigma = 10$ conditions: normal distribution

$n = 25$

$P(\bar{x} < 111.8) = P(z < 1.4) = 0.9192$

$\downarrow$

$z\text{score} = \dfrac{111.8 - 109}{10/\sqrt{25}}$

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

$= \dfrac{2.8}{10/5} = \dfrac{2.8}{2} = 1.4$

USE THE CENTRAL LIMIT THEOREM.

14) A study of the amount of time it takes a mechanic to rebuild the transmission for a 2005 Chevrolet Cavalier shows that the mean is 8.4 hours and the standard deviation is 1.8 hours. If 40 mechanics are randomly selected, find the **probability that the MEAN** rebuild time exceeds 8.7 hours. 14) 0.1469

$n = 40$

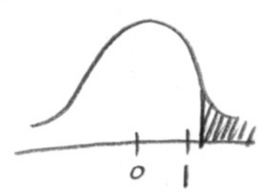
$\mu = 8.4 \quad \sigma = 1.8$ condition: $n \geq 30$ ✓

$P(\bar{x} > 8.7) = P(z > 1.05) = 1 - 0.8531$

$\downarrow$

$z\text{score} = \dfrac{(8.7 - 8.4)}{(1.8/\sqrt{40})} = 1.05$

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$



Estimate the indicated probability by using the normal distribution as an approximation to the binomial distribution.

15) With n = 20 and p = 0.60, estimate P(fewer than 8). 15) 0.0202

$q = 0.40$

$np = 12$ ✓ $nq = 8$ ✓

$\mu = np = 12$

$P(x < 8) = P(x \leq 7)$

$\sigma = \sqrt{npq}$

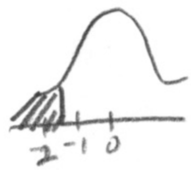
↓ cont. correction

$= \sqrt{(20)(0.6)(0.4)}$

$P(x < 7.5) = P(z < -2.05) = 0.0202$

$= 2.19$

$z = \dfrac{7.5 - 12}{2.19}$



16) Two percent of hair dryers produced in a certain plant are defective. Estimate the probability that of 10,000 randomly selected hair dryers, at least 219 are defective. 16) 0.0934

success → defective hair dryer

$p = 0.02$ $q = 0.98$ $n = 10{,}000$

$np = 200 > 10$ ✓ $nq = 9800 > 10$ ✓

$\mu = np = 200$

$P(x \geq 219)$

$\sigma = \sqrt{npq} = 14$

↓ correction

$P(x \geq 218.5)$

↓

$z = \dfrac{218.5 - 200}{14}$

$P(z \geq 1.32) = 1 - 0.9066 = 0.0934$