

## Instructions

This test is to be taken as an individual without outside assistance or notes. If you are suspected of cheating, you will fail this exam and your transgression will be reported.

You can either use a scientific calculator or (with a smartphone) GeoGebra Scientific Calculator.

To use GeoGebra, you must first put your smartphone on **Airplane Mode**. Then, in GeoGebra, turn on **Exam Mode**. You must leave exam mode on for the entire exam. You won't be able to use your smartphone for anything else. After you are done, you will show me how long exam mode has been running, and if that time is not how long you've been sitting, you will fail this exam.

Read each question carefully and show your work.

## Grade Table

(do not write in this table)

question	available points	earned points
1	20	
2	20	
3	5	
4	5	
5	10	
6	10	
7	20	
8	10	
EC1	10	
Total	100	

## Formulas

### Discrete Probability Distributions

$$\mu = \sum x \cdot \Pr(x)$$
$$\sigma = \sqrt{(x - \mu)^2 \cdot \Pr(x)}$$

### Factorial

$$0! = 1$$
$$1! = 1$$
$$2! = 2 \cdot 1$$
$$3! = 3 \cdot 2 \cdot 1$$
$$n! = n \cdot (n - 1) \cdot (n - 2) \cdots 3 \cdot 2 \cdot 1$$

### Combinatorics

$${}_nP_x = \frac{n!}{(n - x)!}$$
$${}_nC_x = \frac{n!}{(n - x)! \cdot x!}$$

### Binomial Distributions

$n$  = number of trials

$p$  = chance of success on each trial

$q$  = chance of not succeeding on each trial

$X$  = random variable representing the (unknown) number of successes

$x$  = a specific number of successes

$$q = 1 - p$$

$$\Pr(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

$$\mu = np$$
$$\sigma = \sqrt{npq}$$

**Question 1 (20 pts)**

A large vessel contains marbles. Each marble has a pattern and a color. The frequencies are shown in the two-way table below.

	red	yellow	blue	violet	total
checkered	22	3	23	28	76
dotted	24	25	12	19	80
striped	20	5	2	17	44
total	66	33	37	64	200

- a. What is probability a randomly selected marble is yellow?

$$\Pr(\text{yellow}) =$$

- b. What is the probability a randomly selected marble is dotted?

$$\Pr(\text{dotted}) =$$

- c. If you know a randomly selected marble is dotted, then what is the probability it is violet?

$$\Pr(\text{violet GIVEN dotted}) =$$

- d. What is the probability a randomly selected marble is checkered and blue?

$$\Pr(\text{checkered AND blue}) =$$

- e. What is the probability a randomly selected marble is checkered or blue?

$$\Pr(\text{checkered OR blue}) =$$

- f. Is a red marble or a violet marble more likely to be striped?

$$\Pr(\text{striped GIVEN red}) =$$

$$\Pr(\text{striped GIVEN violet}) =$$

Choose one:

☐ red

☐ violet

**Question 2 (20 pts)**

A spinner has the probability distribution shown below.

$x$	$\Pr(x)$	$x \cdot \Pr(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 \cdot \Pr(x)$
14	0.14				
18	0.47				
21	0.1				
22	0.19				
26	0.1				
		$\mu =$			$\sigma^2 =$ $\sigma =$

- What is the probability of spinning 26?
- What is the probability of spinning 21 or 22?
- If spinning twice, what is the probability of first spinning 21 and then spinning 22?
- What is the probability of spinning at most 22?
- Determine the mean of the discrete probability distribution.
- Determine the standard deviation of the discrete probability distribution.

**Question 3 (5 pts)**

Joann will pick an appetizer and a main course. The possible appetizers and main courses are listed below.

appetizers	main courses
soup	burger
salad	pasta
	stirfry

a. Draw a **tree diagram** showing the possibilities.

b. How many possibilities are there?

**Question 4 (5 pts)**

Joshua will pick an appetizer, a main course, and a dessert. The options are listed below.

appetizers	main courses	desserts
soup	burger	cake
salad	pasta	cookies
hummus	stirfry	pie
pickles	tacos	
	sushi	

How many possibilities are there for Joshua? (Do not make a tree!)

**Question 5 (10 pts)**

A committee is planning a conference. There are 20 potential speakers. The committee must decide which 3 speakers will speak and in which order. How many possibilities are there?

**Question 6 (10 pts)**

A committee is planning a gallery. There are 15 potential artists to exhibit, but the committee will only choose 4 of them for this gallery. How many possibilities are there?

**Question 7 (20 pts)**

When Emmaline is shooting archery, each arrow has an 80% chance of hitting the bullseye. Emmaline will shoot 10 arrows. We are interested in how many bullseyes Emmaline will hit.

- a. Identify the parameters.

$$n = \qquad p = \qquad q =$$

- b. What is the probability that Emmaline hits the bullseye 9 times?

$$\Pr(9) =$$

- c. What is the probability that Emmaline hits the bullseye at least 9 times?

$$\Pr(X \geq 9) =$$

- d. What is the probability that Emmaline hits the bullseye fewer than 9 times?

$$\Pr(X < 9) =$$

- e. What is the expected value of this binomial distribution?

$$\mu =$$

- f. What is the standard deviation of this binomial distribution?

$$\sigma =$$

### Question 8 (10 pts)

In a game, there is a 40% chance Bob will win. If Bob wins, there is an 80% chance he is happy. If Bob does **not** win, there is a 30% chance he is happy. After playing the game, Bob is happy; what is the probability that Bob won the game?



### Extra Credit (10 pts)

Candice has two upcoming tennis matches. She has a 60% chance of winning the first match and a 25% chance of winning the second match.

- What is the probability that Candice wins both matches?
- What is the probability that Candice loses both matches? (Assume ties are impossible such that Candice either wins or loses each match.)
- What is the probability that Candice wins once and loses once (in either order)?