Central Limit Theorem: Binomial and Proportion Sampling

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Bernoulli Review

Let's use X to refer to a Bernoulli random variable. It has only two possible outcomes: 0 or 1. We often call 0 a "fail" and a 1 a "success". We allow for the probability of success to be any number between 0 and 1, and we use a lowercase p to represent that probability.

Outcome	Probability
0	$1-p$
1	p

$$
\begin{aligned}
P(X=0) &= 1-p \\
P(X=1) &= p
\end{aligned}
$$

As an example, some basketball player may have an 80% chance of making a free throw. We could describe each free throw attempt as a Bernoulli random variable where $p=0.8$.

Outcome	Probability
0	0.2
1	0.8

$$
\begin{aligned}
P(X=0) &= 0.2 \\
P(X=1) &= 0.8
\end{aligned}
$$

Binomial Review

When we sum n instances of a Bernoulli random variable, the result is an integer between 0 and n (inclusive, inclusive). We learned how to determine the probability that the sum equalled some value.

$$P\left(\sum X=a\right) = {}_{n}\mathrm{C}_{a} \cdot p^{a} \cdot (1-p)^{n-a}$$

So, for example, if the basketball player with $p=0.8$ took 5 freethrows, we could calculate the following probabilities.

Outcome	Probability
0	0.00032

Outcome	Probability
1	0.0064
2	0.0512
3	0.2048
4	0.4096
5	0.32768