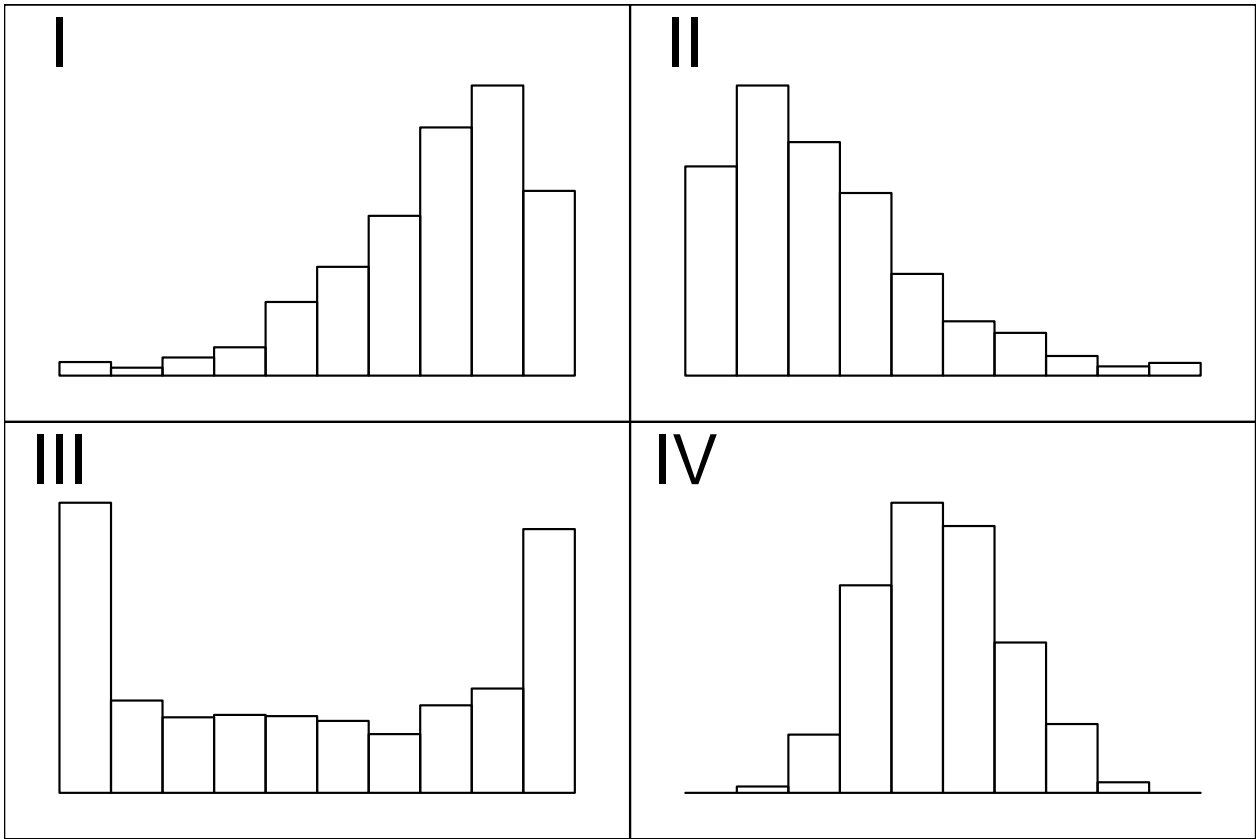


MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondence with any human. Please show work!

| question | available points | earned points |
|----------|------------------|---------------|
| 1 | 10 | |
| 2 | 15 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 15 | |
| 8 | 20 | |
| EC | 5 | |
| EC | 5 | |
| Total | 100 | |

1. (10 Points)



For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

- (a) The distribution of quiz scores on an easy quiz. Most students did very well, but a few did poorly.
- (b) The distribution of weights of newborn babies
- (c) The distribution of annual income for school employees where a high percentage of employees are entry-level teachers and only a few are high-paid administrators.
- (d) The distribution of hours that students studied for an exam when about half of students studied a lot and a similar number of students studied very little.

2. (15 Points)

In a deck of strange cards, there are 420 cards. Each card has an image and a color. The amounts are shown in the table below.

| | blue | gray | indigo | white | yellow | Total |
|-------|------|------|--------|-------|--------|-------|
| horn | 46 | 10 | 13 | 44 | 29 | 142 |
| lamp | 31 | 18 | 21 | 14 | 39 | 123 |
| mop | 11 | 47 | 15 | 45 | 37 | 155 |
| Total | 88 | 75 | 49 | 103 | 105 | 420 |

- (a) What is the probability a random card is blue?
- (b) What is the probability a random card is both a mop and yellow?
- (c) What is the probability a random card is a mop given it is blue?
- (d) What is the probability a random card is a mop?
- (e) What is the probability a random card is either a horn or gray (or both)?
- (f) What is the probability a random card is blue given it is a horn?
- (g) Is a lamp or a mop more likely to be white?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

| Type of fruit | Mean mass (g) | Standard deviation of mass (g) |
|---------------|---------------|--------------------------------|
| <i>A</i> | 71 | 7 |
| <i>B</i> | 73 | 4 |
| <i>C</i> | 60 | 14 |
| <i>D</i> | 110 | 6 |

One specimen of each type is weighed. The results are shown below.

| Type of fruit | Mass of specimen (g) |
|---------------|----------------------|
| <i>A</i> | 77.51 |
| <i>B</i> | 70.92 |
| <i>C</i> | 64.76 |
| <i>D</i> | 106.3 |

Which specimen is the most unusually small (relative to others of its type)?

4. (10 points)

A tree's leaves were found to be normally distributed with a mean of 30.5 millimeters and a standard deviation of 2.6 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 24.2 and 29.6 millimeters?

5. (10 points)

A species of duck is known to have a mean weight of 217.1 grams and a standard deviation of 25 grams. A researcher plans to measure the weights of 100 of these ducks sampled randomly. What is the probability the **sample mean** will be between 212.6 and 216.6 grams?

6. (10 points)

An ornithologist wishes to characterize the average body mass of *Catharus fuscescens*. She randomly samples 16 adults of *Catharus fuscescens*, resulting in a sample mean of 42.97 grams and a sample standard deviation of 3.95 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

A student is taking a multiple choice test with 500 questions. Each question has 5 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

Then, the student takes the test and gets 115 questions correct.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses.

(c) Determine the test statistic (z or t), draw a sketch, and determine the p -value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

You have collected the following data:

| x | y | xy |
|-------------|-------------|-------------|
| 860 | 74 | |
| 120 | 28 | |
| 820 | 68 | |
| 340 | 22 | |
| 470 | 48 | |
| 960 | 67 | |
| 260 | 30 | |
| 120 | 25 | |
| $\sum x =$ | $\sum y =$ | $\sum xy =$ |
| $\bar{x} =$ | $\bar{y} =$ | |
| $s_x =$ | $s_y =$ | |

(a) Complete the table.

(b) Calculate the correlation coefficient (r) using the formula below.

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

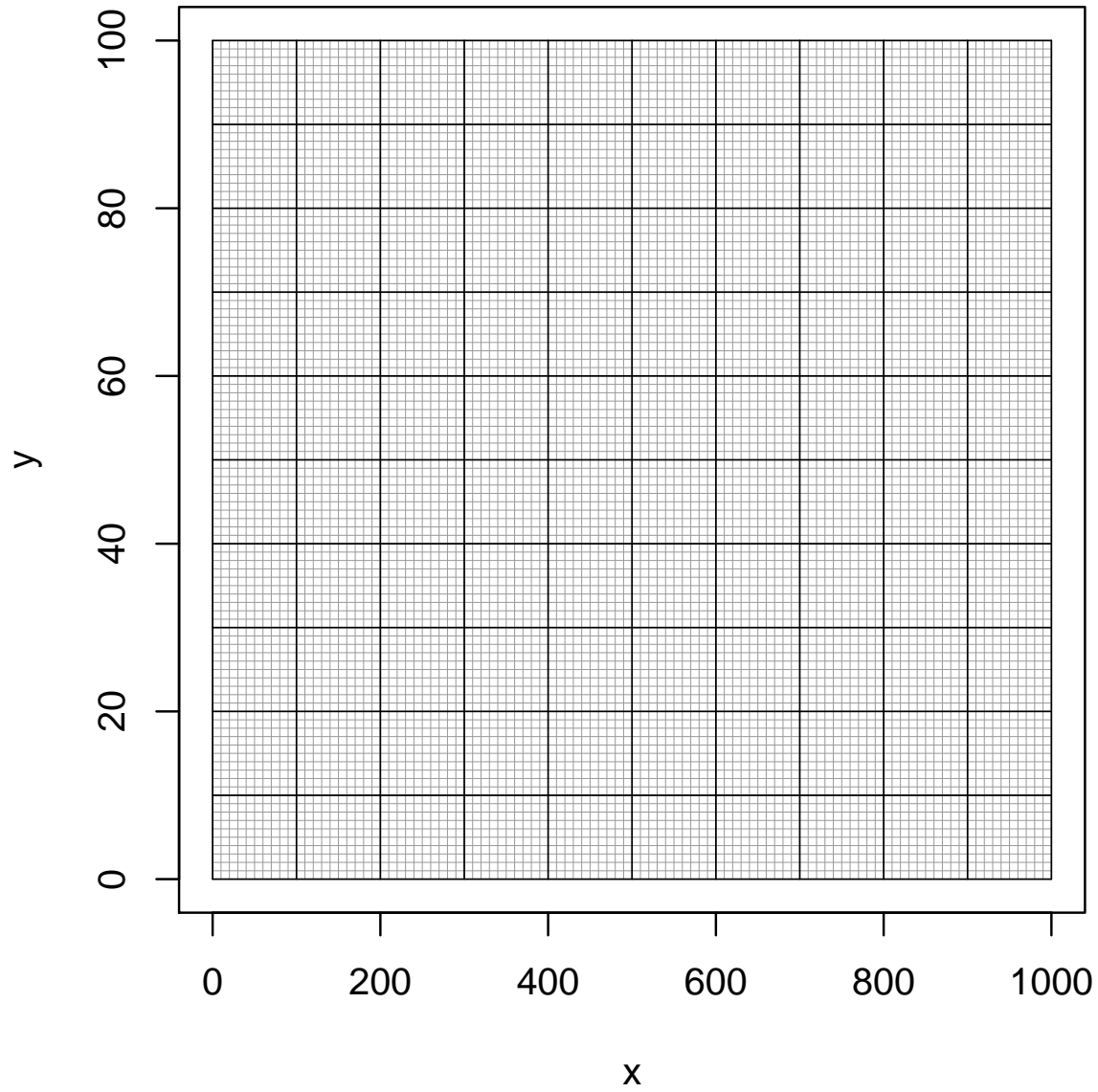
(c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r \frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

(d) Write the equation of the regression line (using the calculated values of a and b .)

(e) Please plot the data and a corresponding regression line.



9. (Extra credit: 5 points)

Let each trial have a chance of success $p = 0.81$. If 238 trials occur, what is the probability of getting at least 188 but at most 199 successes?

In other words, let $X \sim \text{Bin}(n = 238, p = 0.81)$ and find $P(188 \leq X \leq 199)$.

Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

A null hypothesis claims a population has a mean $\mu = 90$. You decide to run two-tail test on a sample of size $n = 12$ using a significance level $\alpha = 0.05$.

You then collect the sample:

| | | | | |
|------|-------|-------|------|------|
| 99.1 | 98.1 | 90.5 | 99.8 | 88.7 |
| 85 | 108.8 | 115.6 | 97.8 | 99.9 |
| 79.6 | 92.1 | | | |

- (a) Determine the p -value.
- (b) Do you reject the null hypothesis?