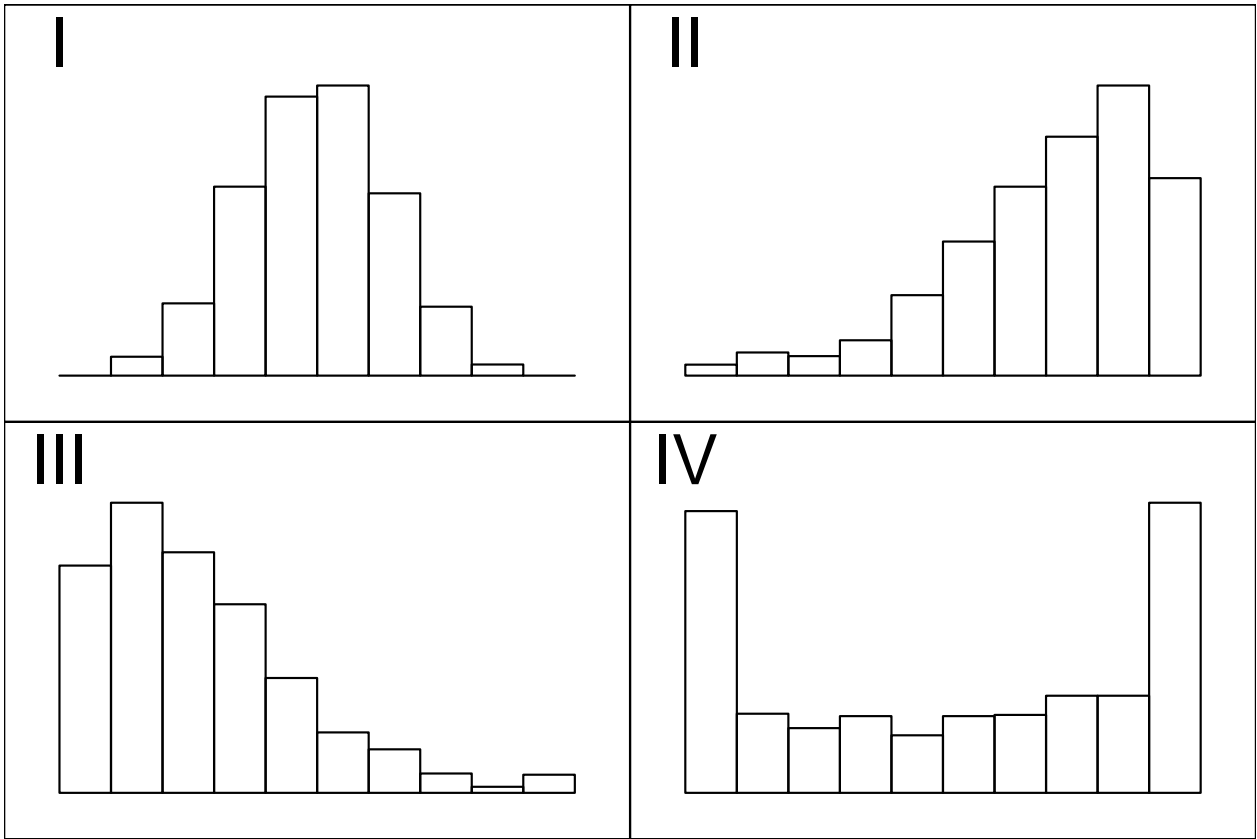


MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondence with any human. Please show work!

| question | available points | earned points |
|----------|------------------|---------------|
| 1 | 10 | |
| 2 | 15 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 15 | |
| 8 | 20 | |
| EC | 5 | |
| EC | 5 | |
| Total | 100 | |

1. (10 Points)



For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

- (a) The distribution of test scores on a very difficult exam, in which most students have poor to average scores, but a few did quite well.
- (b) The distribution of lengths of newborn babies
- (c) The distribution of hours spent per week reading by adults. In this distribution, many people do not read much, and a similar number of people read a lot.
- (d) The distribution of quiz scores on an easy quiz. Most students did very well, but a few did poorly.

2. (15 Points)

In a deck of strange cards, there are 757 cards. Each card has an image and a color. The amounts are shown in the table below.

| | black | blue | indigo | orange | red | Total |
|-------|-------|------|--------|--------|-----|-------|
| cat | 25 | 11 | 38 | 29 | 18 | 121 |
| gem | 26 | 43 | 50 | 20 | 15 | 154 |
| mop | 30 | 13 | 12 | 36 | 35 | 126 |
| tree | 47 | 21 | 14 | 45 | 46 | 173 |
| wheel | 33 | 27 | 48 | 34 | 41 | 183 |
| Total | 161 | 115 | 162 | 164 | 155 | 757 |

- (a) What is the probability a random card is both a mop and blue?
- (b) What is the probability a random card is a gem given it is red?
- (c) What is the probability a random card is either a mop or red (or both)?
- (d) Is a cat or a gem more likely to be indigo?
- (e) What is the probability a random card is orange given it is a mop?
- (f) What is the probability a random card is a gem?
- (g) What is the probability a random card is blue?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

| Type of fruit | Mean mass (g) | Standard deviation of mass (g) |
|---------------|---------------|--------------------------------|
| <i>A</i> | 138 | 15 |
| <i>B</i> | 118 | 7 |
| <i>C</i> | 101 | 8 |
| <i>D</i> | 122 | 5 |

One specimen of each type is weighed. The results are shown below.

| Type of fruit | Mass of specimen (g) |
|---------------|----------------------|
| <i>A</i> | 136.5 |
| <i>B</i> | 128.8 |
| <i>C</i> | 103.4 |
| <i>D</i> | 126 |

Which specimen is the most unusually far (in either direction) from average (relative to others of its type)?

4. (10 points)

A tree's leaves were found to be normally distributed with a mean of 146.8 millimeters and a standard deviation of 6.8 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 144.2 and 154.1 millimeters?

5. (10 points)

A species of duck is known to have a mean weight of 186.3 grams and a standard deviation of 37.5 grams. A researcher plans to measure the weights of 225 of these ducks sampled randomly. What is the probability the **sample mean** will be between 183.8 and 185.8 grams?

6. (10 points)

An ornithologist wishes to characterize the average body mass of *Dendroica palmarum*. She randomly samples 31 adults of *Dendroica palmarum*, resulting in a sample mean of 10.15 grams and a sample standard deviation of 1.34 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

A student is taking a multiple choice test with 500 questions. Each question has 2 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

Then, the student takes the test and gets 270 questions correct.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses.

(c) Determine the test statistic (z or t), draw a sketch, and determine the p -value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

You have collected the following data:

| x | y | xy |
|-------------|-------------|-------------|
| 9.3 | 4 | |
| 7.4 | 2.9 | |
| 1.5 | 8.2 | |
| 3.4 | 6.3 | |
| 9.1 | 5.2 | |
| 5.5 | 3.1 | |
| 2.7 | 8.6 | |
| $\sum x =$ | $\sum y =$ | $\sum xy =$ |
| $\bar{x} =$ | $\bar{y} =$ | |
| $s_x =$ | $s_y =$ | |

(a) Complete the table.

(b) Calculate the correlation coefficient (r) using the formula below.

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

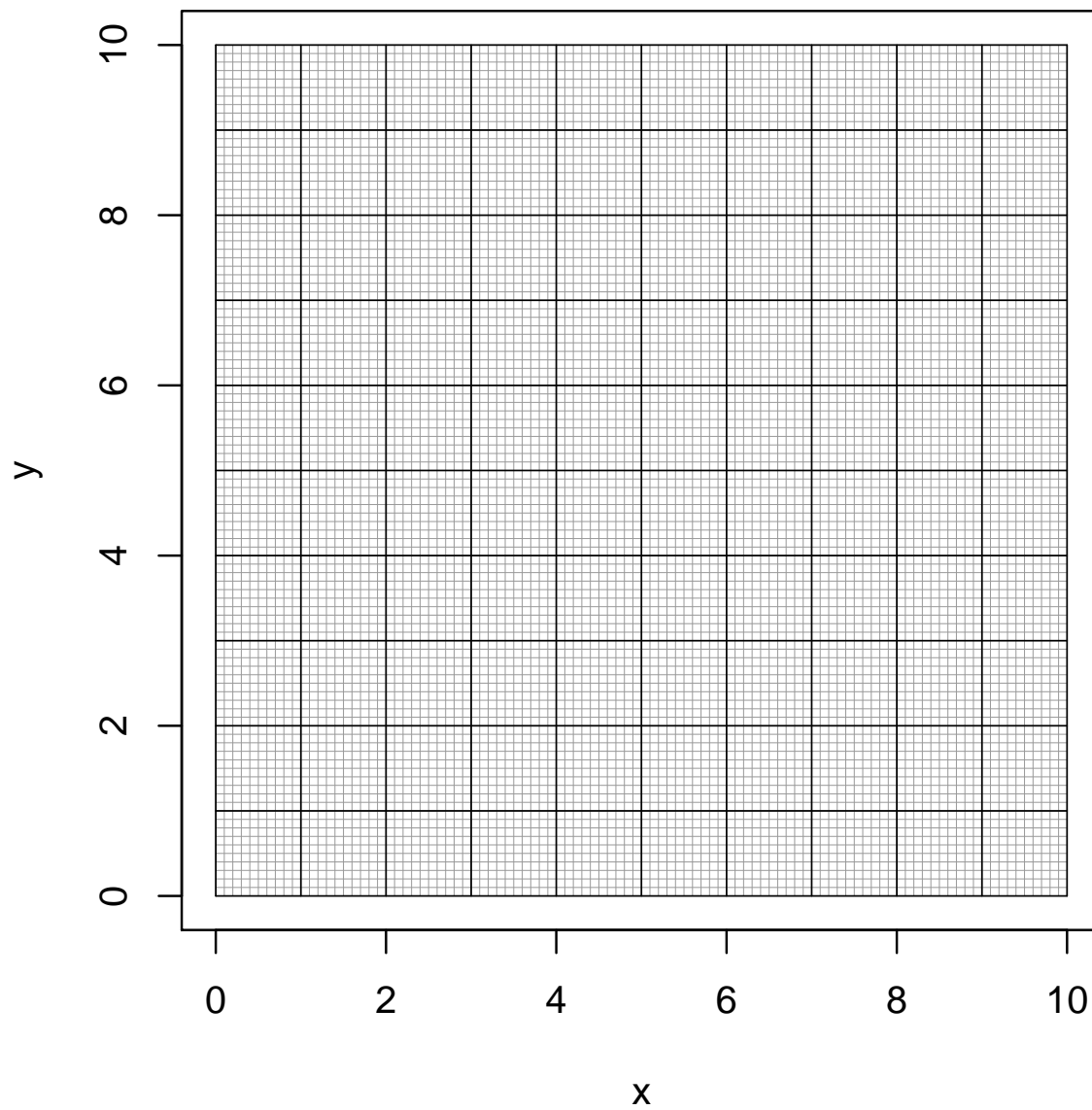
(c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r \frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

(d) Write the equation of the regression line (using the calculated values of a and b .)

(e) Please plot the data and a corresponding regression line.



9. (Extra credit: 5 points)

Let each trial have a chance of success $p = 0.83$. If 73 trials occur, what is the probability of getting at least 53 but less than 62 successes?

In other words, let $X \sim \text{Bin}(n = 73, p = 0.83)$ and find $P(53 \leq X < 62)$.

Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

A null hypothesis claims a population has a mean $\mu = 210$. You decide to run two-tail test on a sample of size $n = 9$ using a significance level $\alpha = 0.1$.

You then collect the sample:

| | | | | |
|-------|-------|-------|-------|-------|
| 202 | 247.4 | 277 | 253.7 | 279.4 |
| 280.8 | 217.1 | 190.7 | 168 | |

- (a) Determine the p -value.
- (b) Do you reject the null hypothesis?