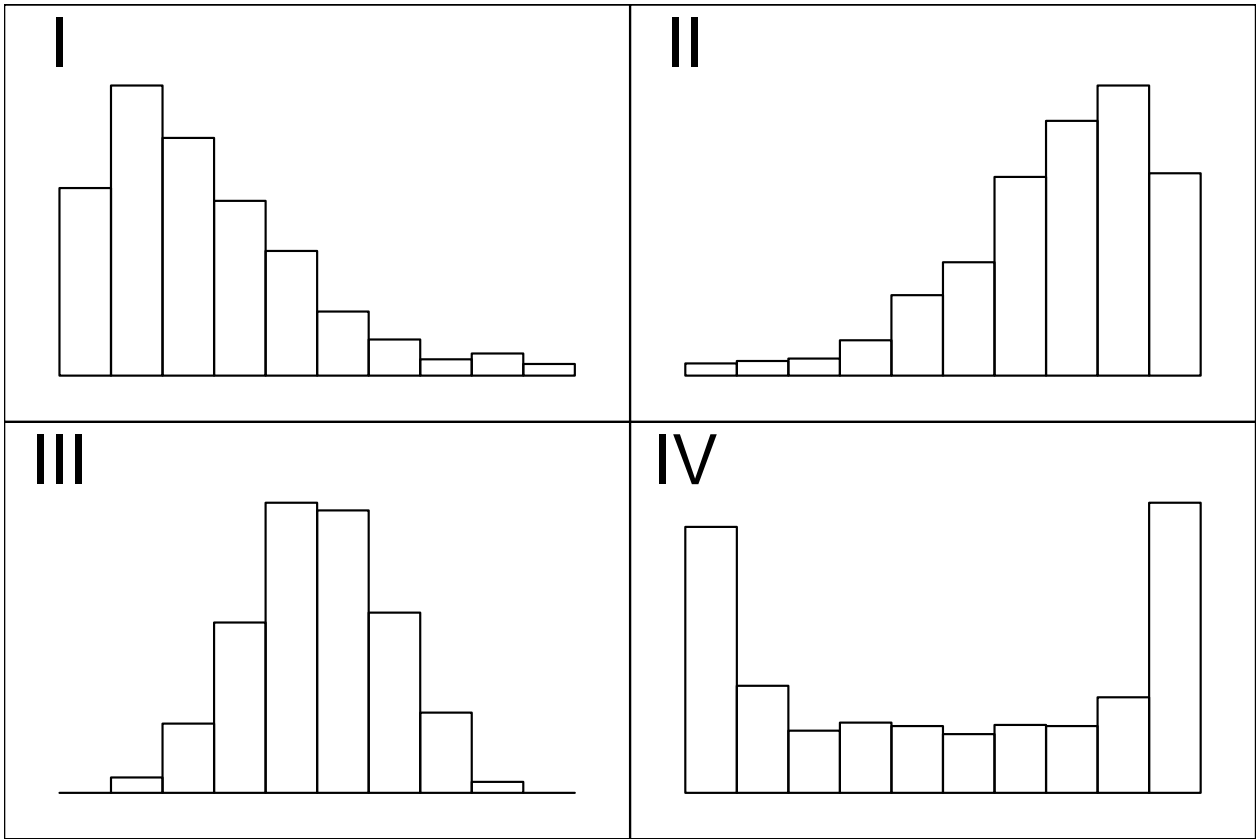# MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondance with any human. Please show work!

question	available points	earned points
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	15	
8	20	
EC	5	
EC	5	
Total	100	

1. (10 Points)

For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

(a) The distribution of hours that students studied for an exam when about half of students studied a lot and a similar number of students studied very little.

(b) The distribution of quiz scores on an easy quiz. Most students did very well, but a few did poorly.

(c) The distribution of annual income for school employees where a high percentage of employees are entry-level teachers and only a few are high-paid administrators.

(d) The distribution of weights of newborn babies

2. (15 Points)

In a deck of strange cards, there are 486 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	orange	pink	red	Total
bike	29	22	34	21	106
cat	35	18	48	45	146
dog	19	10	30	49	108
quilt	16	40	42	28	126
Total	99	90	154	143	486

(a) What is the probability a random card is a cat?

(b) What is the probability a random card is a dog given it is orange?

(c) Is a bike or a cat more likely to be red?

(d) What is the probability a random card is either a cat or red (or both)?

(e) What is the probability a random card is orange?

(f) What is the probability a random card is both a cat and red?

(g) What is the probability a random card is red given it is a bike?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	73	13
B	60	7
C	114	6
D	94	12

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	61.43
B	56.01
C	112.6
D	104.1

Which specimen is the most unusually far (in either direction) from average (relative to others of its type)?

4. (10 points)

A tree's leaves were found to be normally distributed with a mean of 122.8 millimeters and a standard deviation of 5.7 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 125.4 and 127.2 millimeters?

5. (10 points)

A species of duck is known to have a mean weight of 215.3 grams and a standard deviation of 20 grams. A researcher plans to measure the weights of 100 of these ducks sampled randomly. What is the probability the **sample mean** will be between 215.3 and 219.3 grams?

6. (10 points)

An ornithologist wishes to characterize the average body mass of *Vireo philadelphicus*. She randomly samples 20 adults of *Vireo philadelphicus*, resulting in a sample mean of 13.5 grams and a sample standard deviation of 1.52 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

A student is taking a multiple choice test with 300 questions. Each question has 4 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

Then, the student takes the test and gets 89 questions correct.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses.

(c) Determine the test statistic (z or t), draw a sketch, and determine the p-value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

   You have collected the following data:

x	y	xy
7.9	60	
5	57	
2	91	
7.2	54	
5.5	41	
3.5	72	
4.5	52	
$\sum x =$	$\sum y =$	$\sum xy =$
$\bar{x} =$	$\bar{y} =$	
$s_x =$	$s_y =$	

(a) Complete the table.

(b) Calculate the correlation coefficient (r) using the formula below.

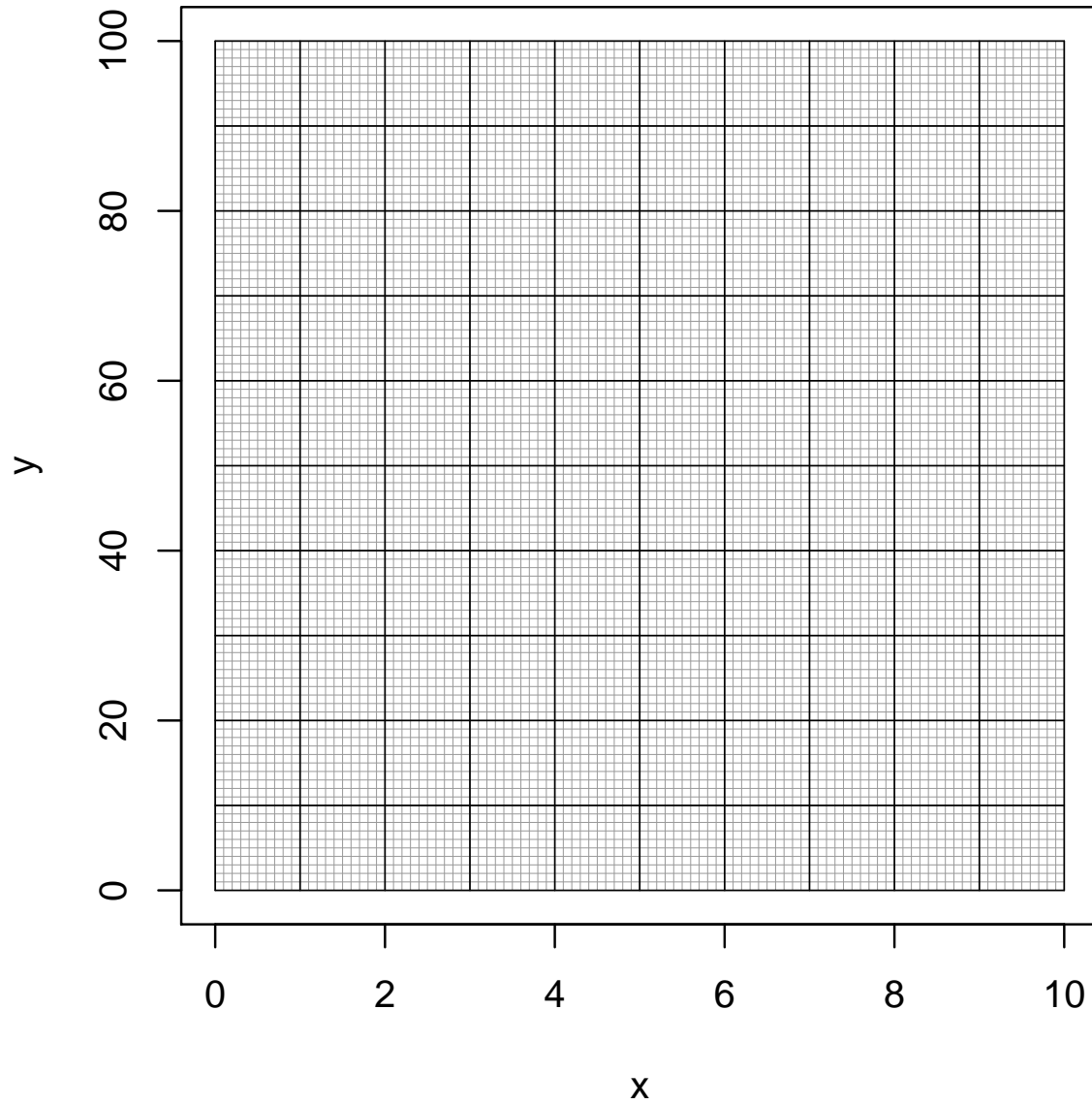$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

(c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r\frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

(d) Write the equation of the regression line (using the calculated values of a and b.)

(e) Please plot the data and a corresponding regression line.

9. (Extra credit: 5 points)

Let each trial have a chance of success $p = 0.81$. If 195 trials occur, what is the probability of getting more than 145 but at most 166 successes?

In other words, let $X \sim \text{Bin}(n = 195, p = 0.81)$ and find $P(145 < X \le 166)$.

Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

A null hypothesis claims a population has a mean μ = 120. You decide to run two-tail test on a sample of size n = 8 using a significance level α = 0.02.

You then collect the sample:

121.1	127.3	123.1	118.2	124.1
123	121.4	123.3		

(a) Determine the p-value.

(b) Do you reject the null hypothesis?