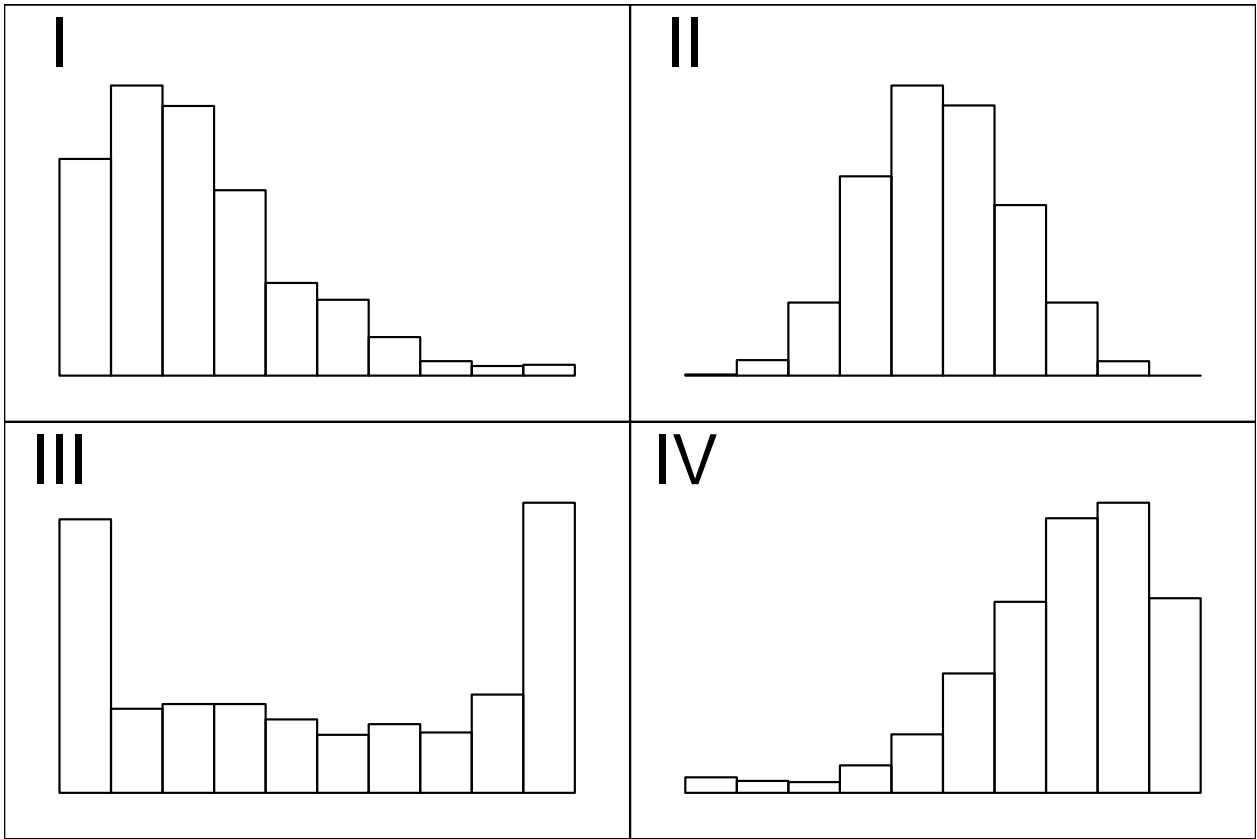


MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondence with any human. Please show work!

question	available points	earned points
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	15	
8	20	
EC	5	
EC	5	
Total	100	

1. (10 Points)



For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

- (a) The distribution of test scores on a very difficult exam, in which most students have poor to average scores, but a few did quite well.
- (b) The distribution of lengths of newborn babies
- (c) The distribution of quiz scores on an easy quiz. Most students did very well, but a few did poorly.
- (d) The distribution of hours that students studied for an exam when about half of students studied a lot and a similar number of students studied very little.

2. (15 Points)

In a deck of strange cards, there are 514 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	orange	pink	violet	yellow	Total
bike	10	49	25	29	11	124
gem	45	28	18	34	16	141
lamp	23	33	17	13	12	98
pig	14	46	26	24	41	151
Total	92	156	86	100	80	514

- (a) What is the probability a random card is a pig given it is blue?
- (b) What is the probability a random card is a lamp?
- (c) What is the probability a random card is orange?
- (d) What is the probability a random card is blue given it is a gem?
- (e) What is the probability a random card is either a bike or blue (or both)?
- (f) What is the probability a random card is both a gem and orange?
- (g) Is a bike or a gem more likely to be blue?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
<i>A</i>	101	8
<i>B</i>	111	11
<i>C</i>	123	12
<i>D</i>	69	14

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
<i>A</i>	89.08
<i>B</i>	112.1
<i>C</i>	106.4
<i>D</i>	60.88

Which specimen is the most unusually small (relative to others of its type)?

4. (10 points)

A tree's leaves were found to be normally distributed with a mean of 141.4 millimeters and a standard deviation of 2.7 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 144 and 147.2 millimeters?

5. (10 points)

A species of duck is known to have a mean weight of 111.5 grams and a standard deviation of 67.5 grams. A researcher plans to measure the weights of 81 of these ducks sampled randomly. What is the probability the **sample mean** will be between 96.5 and 128 grams?

6. (10 points)

An ornithologist wishes to characterize the average body mass of *Hylocichla mustelina*. She randomly samples 20 adults of *Hylocichla mustelina*, resulting in a sample mean of 57.29 grams and a sample standard deviation of 7.67 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

A student is taking a multiple choice test with 200 questions. Each question has 2 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

Then, the student takes the test and gets 110 questions correct.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses.

(c) Determine the test statistic (z or t), draw a sketch, and determine the p -value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

You have collected the following data:

x	y	xy
58	760	
54	620	
93	470	
41	650	
90	480	
41	570	
34	740	
64	680	
25	800	
$\sum x =$	$\sum y =$	$\sum xy =$
$\bar{x} =$	$\bar{y} =$	
$s_x =$	$s_y =$	

(a) Complete the table.

(b) Calculate the correlation coefficient (r) using the formula below.

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

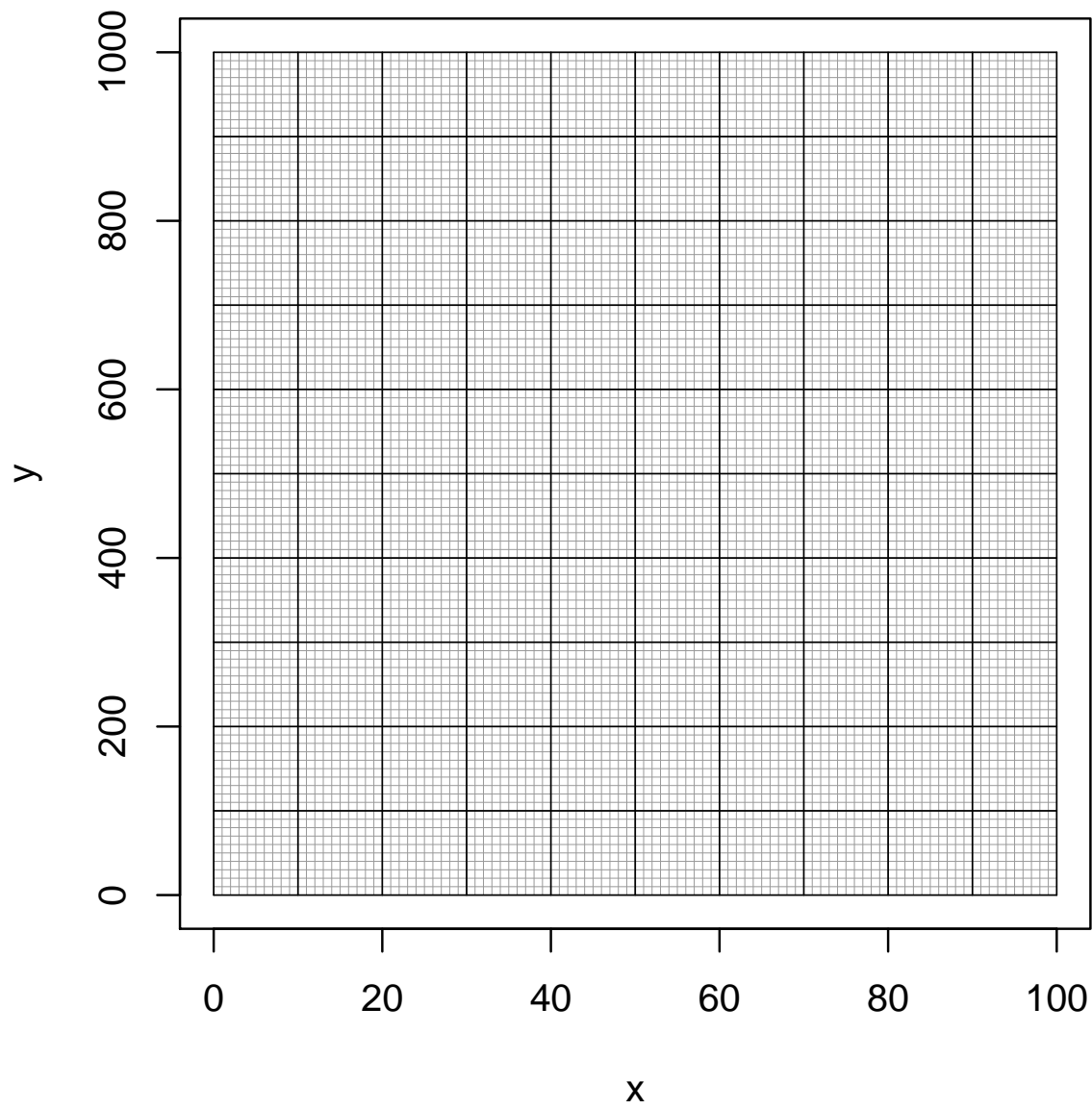
(c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r \frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

(d) Write the equation of the regression line (using the calculated values of a and b .)

(e) Please plot the data and a corresponding regression line.



9. (Extra credit: 5 points)

Let each trial have a chance of success $p = 0.46$. If 174 trials occur, what is the probability of getting more than 64 but less than 87 successes?

In other words, let $X \sim \text{Bin}(n = 174, p = 0.46)$ and find $P(64 < X < 87)$.

Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

A null hypothesis claims a population has a mean $\mu = 60$. You decide to run two-tail test on a sample of size $n = 9$ using a significance level $\alpha = 0.01$.

You then collect the sample:

57.1	74.5	65	61.3	69.1
76.7	62.8	70.2	78.7	

- (a) Determine the p -value.
- (b) Do you reject the null hypothesis?