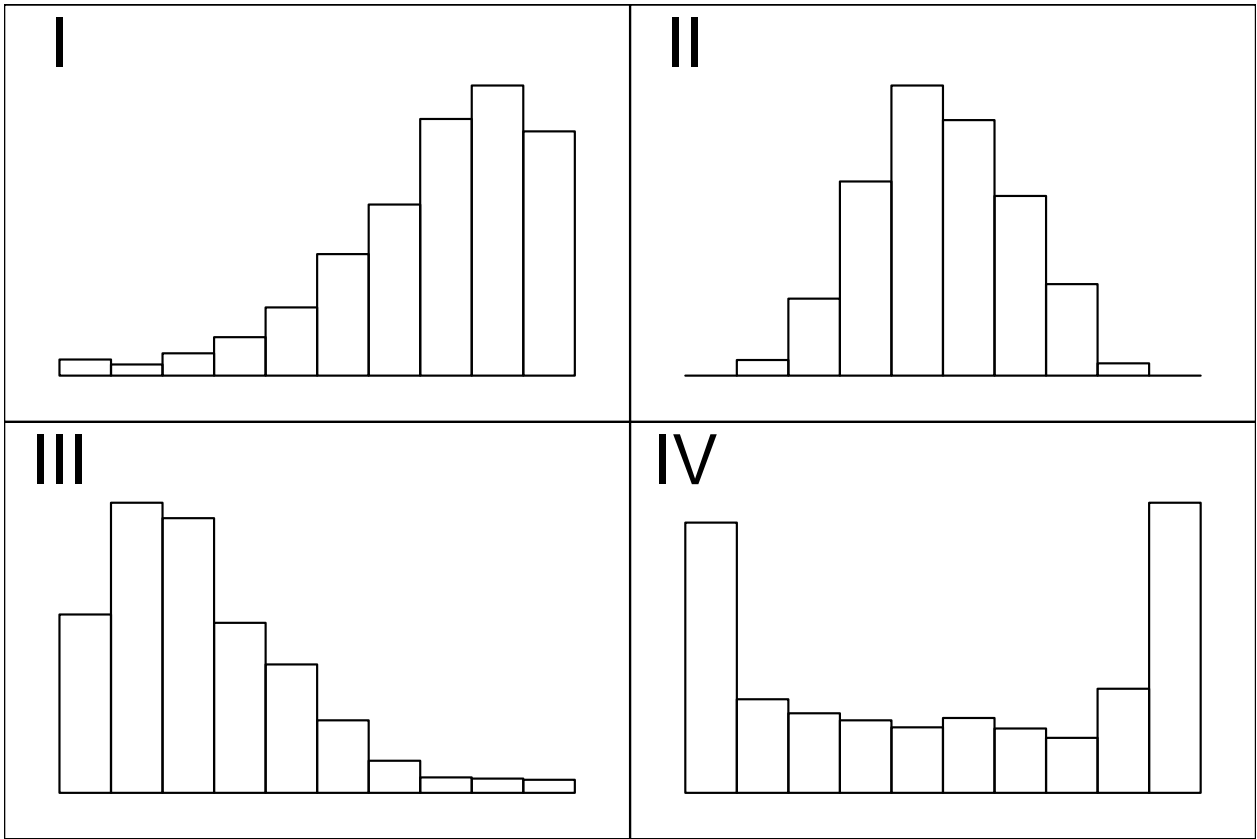# MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondance with any human. Please show work!

question	available points	earned points
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	15	
8	20	
EC	5	
EC	5	
Total	100	

1. (10 Points)

For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

(a) The distribution of heights of adult men

(b) The distribution of hours spent per week reading by adults. In this distribution, many people do not read much, and a similar number of people read a lot.

(c) The distribution of annual income for NBA basketball players where only a few are high-paid superstars.

(d) The distribution of quiz scores on an easy quiz. Most students did very well, but a few did poorly.

2. (15 Points)

In a deck of strange cards, there are 406 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	green	red	Total
dog	35	24	15	74
horn	36	26	46	108
kite	44	42	20	106
rug	49	28	41	118
Total	164	120	122	406

(a) Is a horn or a rug more likely to be red?

(b) What is the probability a random card is a rug?

(c) What is the probability a random card is both a dog and gray?

(d) What is the probability a random card is a kite given it is gray?

(e) What is the probability a random card is green?

(f) What is the probability a random card is red given it is a dog?

(g) What is the probability a random card is either a horn or red (or both)?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	70	12
B	125	15
C	69	5
D	92	10

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	58.12
B	138.4
C	64.35
D	90.2

Which specimen is the most unusually far (in either direction) from average (relative to others of its type)?

4. (10 points)

A tree's leaves were found to be normally distributed with a mean of 35.8 millimeters and a standard deviation of 7.5 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 34.3 and 48.6 millimeters?

5. (10 points)

A species of duck is known to have a mean weight of 238.4 grams and a standard deviation of 55 grams. A researcher plans to measure the weights of 121 of these ducks sampled randomly. What is the probability the **sample mean** will be between 236.9 and 243.9 grams?

6. (10 points)

   An ornithologist wishes to characterize the average body mass of *Catharus fuscescens*. She randomly samples 30 adults of *Catharus fuscescens*, resulting in a sample mean of 40.67 grams and a sample standard deviation of 4.76 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

   A student is taking a multiple choice test with 700 questions. Each question has 5 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

   Then, the student takes the test and gets 159 questions correct.

   (a) What kind of hypothesis test is appropriate?

   (b) State the hypotheses.

   (c) Determine the test statistic (z or t), draw a sketch, and determine the p-value.

   (d) Decide whether we reject or retain the null hypothesis.

   (e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

    You have collected the following data:

x	y	xy
26	18	
22	52	
15	30	
93	80	
44	55	
32	42	
81	63	
21	39	
13	19	
$\sum x =$	$\sum y =$	$\sum xy =$
$\bar{x} =$	$\bar{y} =$	
$s_x =$	$s_y =$	

(a) Complete the table.

(b) Calculate the correlation coefficient (r) using the formula below.

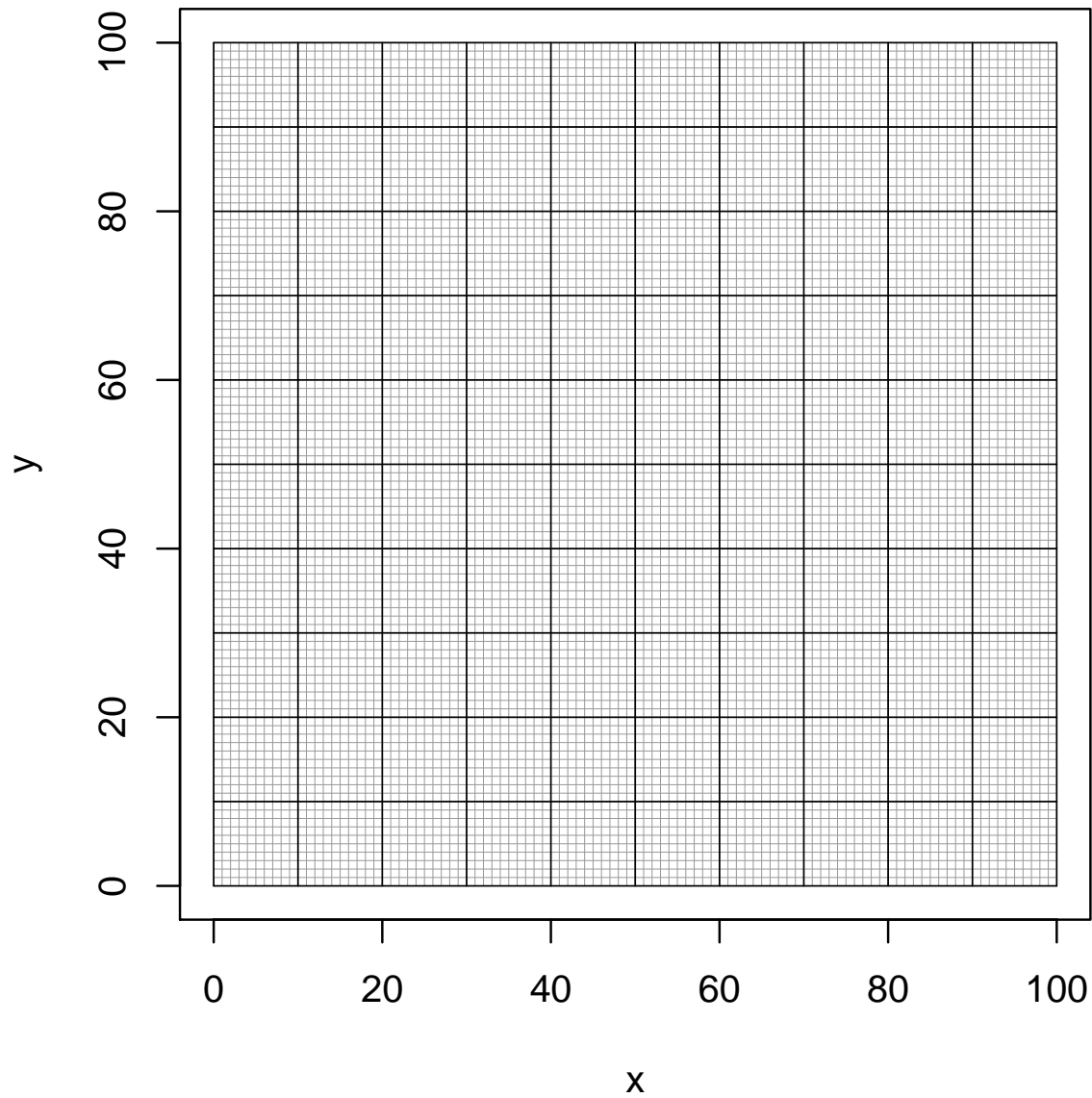$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

(c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r\frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

(d) Write the equation of the regression line (using the calculated values of a and b.)

(e) Please plot the data and a corresponding regression line.

9. (Extra credit: 5 points)

    Let each trial have a chance of success $p = 0.67$. If 168 trials occur, what is the probability of getting at least 104 but at most 128 successes?

    In other words, let $X \sim \text{Bin}(n = 168, p = 0.67)$ and find $P(104 \leq X \leq 128)$.

    Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

A null hypothesis claims a population has a mean $\mu = 60$. You decide to run two-tail test on a sample of size $n = 12$ using a significance level $\alpha = 0.1$.

You then collect the sample:

81.3	70.6	55.6	64.9	69.8
78.3	50.3	66	69	68.5
46.1	65.1			

(a) Determine the p-value.

(b) Do you reject the null hypothesis?