 **Name:** ______________________

## Instructions

This test is to be taken as an individual without outside assistance or notes. If you are suspected of cheating, you will fail this exam and your transgression will be reported.

You can either use a scientific calculator or (with a smartphone) GeoGebra Scientific Calculator.

To use GeoGebra, you must first put your smartphone on **Airplane Mode**. Then, in GeoGebra, turn on **Exam Mode**. You must leave exam mode on for the entire exam. You won't be able to use your smartphone for anything else. After you are done, you will show me how long exam mode has been running, and if that time is not how long you've been sitting, you will fail this exam.

Read each question carefully and show your work.

## Grade Table

(do not write in this table)

| question | available points | earned points |
|---|---|---|
| 1 | 15 | |
| 2 | 13 | |
| 3 | 21 | |
| 4 | 12 | |
| 5 | 12 | |
| 6 | 15 | |
| 7 | 12 | |
| EC1 | 5 | |
| EC2 | 5 | |
| Total | 100 | |

## Question 1 (15 pts)

A farm produces 4 types of fruit: $A$, $B$, $C$, and $D$. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

| Type of fruit | Mean mass (g) | Standard deviation of mass (g) |
|---|---|---|
| $A$ | 115 | 5 |
| $B$ | 95 | 20 |
| $C$ | 100 | 15 |
| $D$ | 140 | 10 |

One specimen of each type is weighed. The results are shown below.

| Type of fruit | Mass of specimen (g) |
|---|---|
| $A$ | 109 |
| $B$ | 117 |
| $C$ | 86.5 |
| $D$ | 143 |

a. Determine a $z$-score for each specimen.

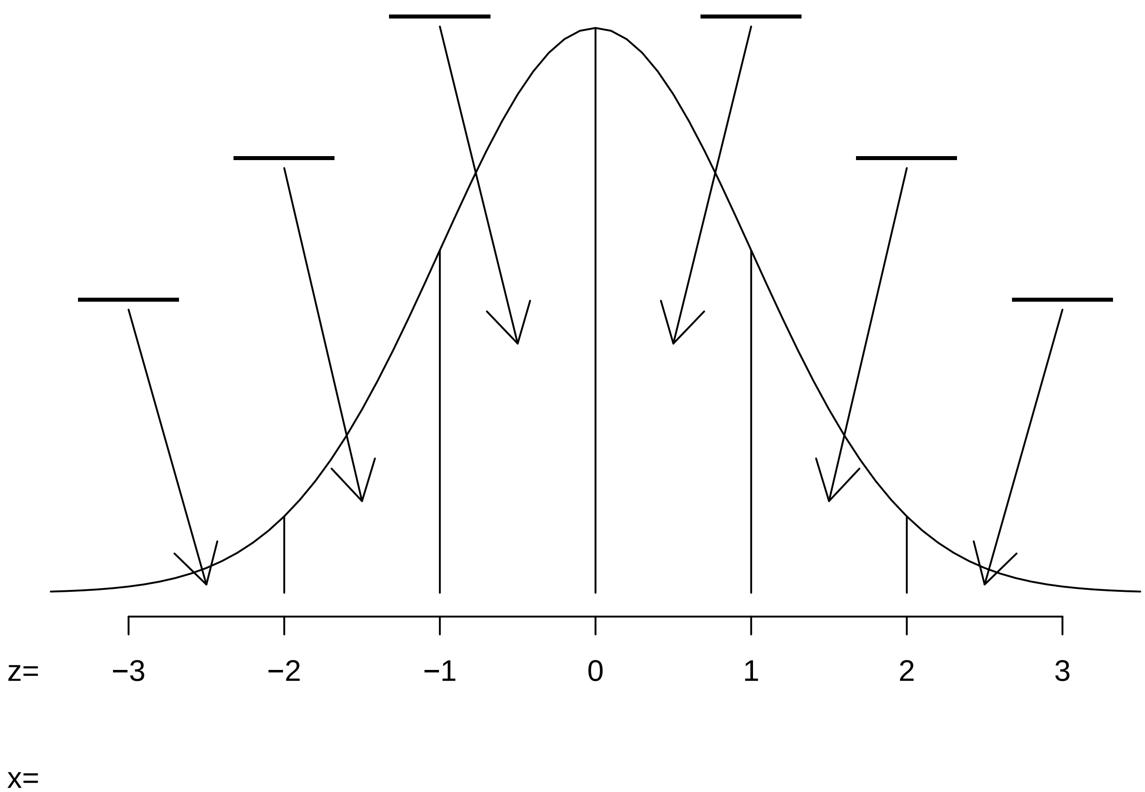
b. Which specimen was most unusually large?

c. Which specimen was most unusually small?

## Question 2 (13 pts)

A normal random variable $X$ has a mean $\mu = 15$ and standard deviation $\sigma = 3$. Please label the density curve with:

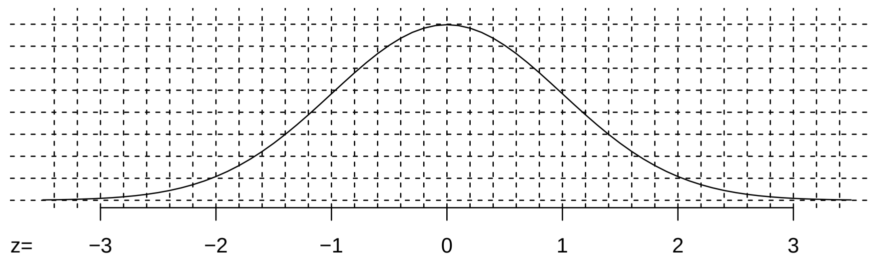
a. The appropriate values of $x$.
b. The areas of the sections.



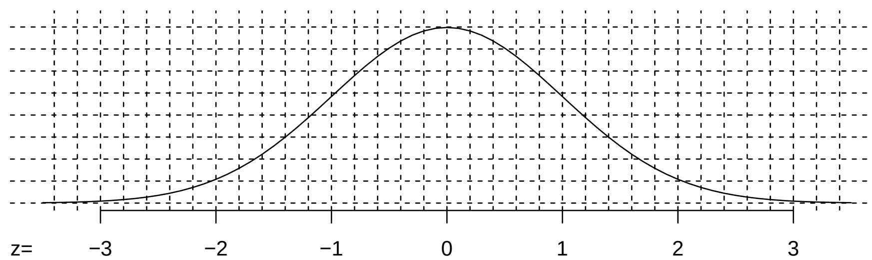
## Question 3 (21 pts)

Let $X$ be normally distributed with mean 80 and standard deviation 15. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

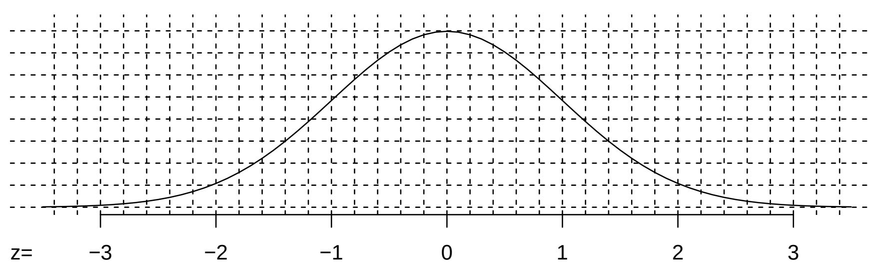
a. $P(X < 83)$




b. $P(X > 104)$




c. $P(60 < X < 100)$



## Question 4 (12 pts)

Let $W$ have the probability distribution of a standard six-sided die, giving $W$ a **mean of** $\mu_{\mathsf{w}} = 3.50$ and a **standard deviation of** $\sigma_{\mathsf{w}} = 1.71$.

Let $X$ represent the sum of $n = 100$ rolls. ($X$ is sum of 100 instances of $W$)

a. Determine $\mu_{\mathsf{x}}$

b. Determine $\sigma_{\mathsf{x}}$

c. If 100 dice are thrown, what is the probability of getting a **sample total** between 325 and 375? *Do NOT use a continuity correction.*

## Question 5 (12 pts)

Let $W$ have the probability distribution of a standard six-sided die, giving $W$ a **mean of** $\mu_{\text{W}} = 3.50$ and a **standard deviation of** $\sigma_{\text{W}} = 1.71$.

Let $Y$ represent the average of $n = 100$ rolls. ($Y$ is average of 100 instances of $W$)

a. Determine $\mu_{\text{Y}}$

b. Determine $\sigma_{\text{Y}}$

c. If 100 dice are thrown, what is the probability of getting a **sample mean** between 3.25 and 3.75? *Do NOT use a continuity correction.*

## Question 6 (15 pts)

A fair coin has a $p = 0.5$ chance of landing tails. When $n = 100$ fair coins are flipped, what is the probability of getting at least 45 but at most 55 tails? Please use a **normal approximation** with a **continuity correction**.

## Question 7 (12 pts)

About 10% of humans are left handed ($p = 0.10$). If you gather a simple random sample of $n = 144$ humans, what is the probability that the sample proportion ($\hat{p}$) is between 0.12 and 0.15? Please use a **normal approximation**, but *do NOT use a continuity correction*.

## Extra Credit

1. (5 pts) Let random variable $X$ be normally distributed with mean $\mu = 22.2$ and standard deviation $\sigma = 3.3$. Determine $x$ such that $P(X > x) = 0.8665$.

2. (5 pts) Find $z$ such that $P(|Z| > z) = 0.0574$

**Normal standardization**

$$z = \frac{x - \mu}{\sigma} \qquad x = \mu + z\sigma$$

**Central Limit Theorem**

if:

- $W$ is "any" random variable with mean = $\mu_W$ and standard deviation = $\sigma_W$.
- Random variable $X$ is **sum** of $n$ instances of $W$.

$$X = W_1 + W_2 + W_3 + \cdots + W_n$$

- Random variable $Y$ is **average** of $n$ instances of $W$.

$$Y = \frac{W_1 + W_2 + W_3 + \cdots + W_n}{n}$$

then:

- The following formulas are exactly true:

$$\mu_X = n\mu_W \qquad \mu_Y = \mu_W$$

$$\sigma_X = \sigma_W\sqrt{n} \qquad \sigma_Y = \frac{\sigma_W}{\sqrt{n}}$$

- $X$ and $Y$ are approximately normal (if $n \geq 30$).
- $X$ and $Y$ are exactly normal if $W$ is normal.

**Special case of central limit theorem: Bernoulli, Binomial, and $\hat{p}$ sampling**

if:

- $W$ is a Bernoulli random variable:

| $w$ | $P(w)$ |
|---|---|
| 0 | $q$ |
| 1 | $p$ |

- $X$ is sum of $n$ instances of $W$ (binomial).
- $\hat{p}$ is average of $n$ instances of $W$ (proportion sampling).

then:

- The following are exactly true:

$$\mu_W = p \qquad \mu_X = np \qquad \mu_{\hat{p}} = p$$

$$\sigma_W = \sqrt{pq} \qquad \sigma_X = \sqrt{pq}\sqrt{n} \qquad \sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}}$$

- $X$ and $\hat{p}$ are approximately normal (if $np \geq 10$ and $nq \geq 10$)