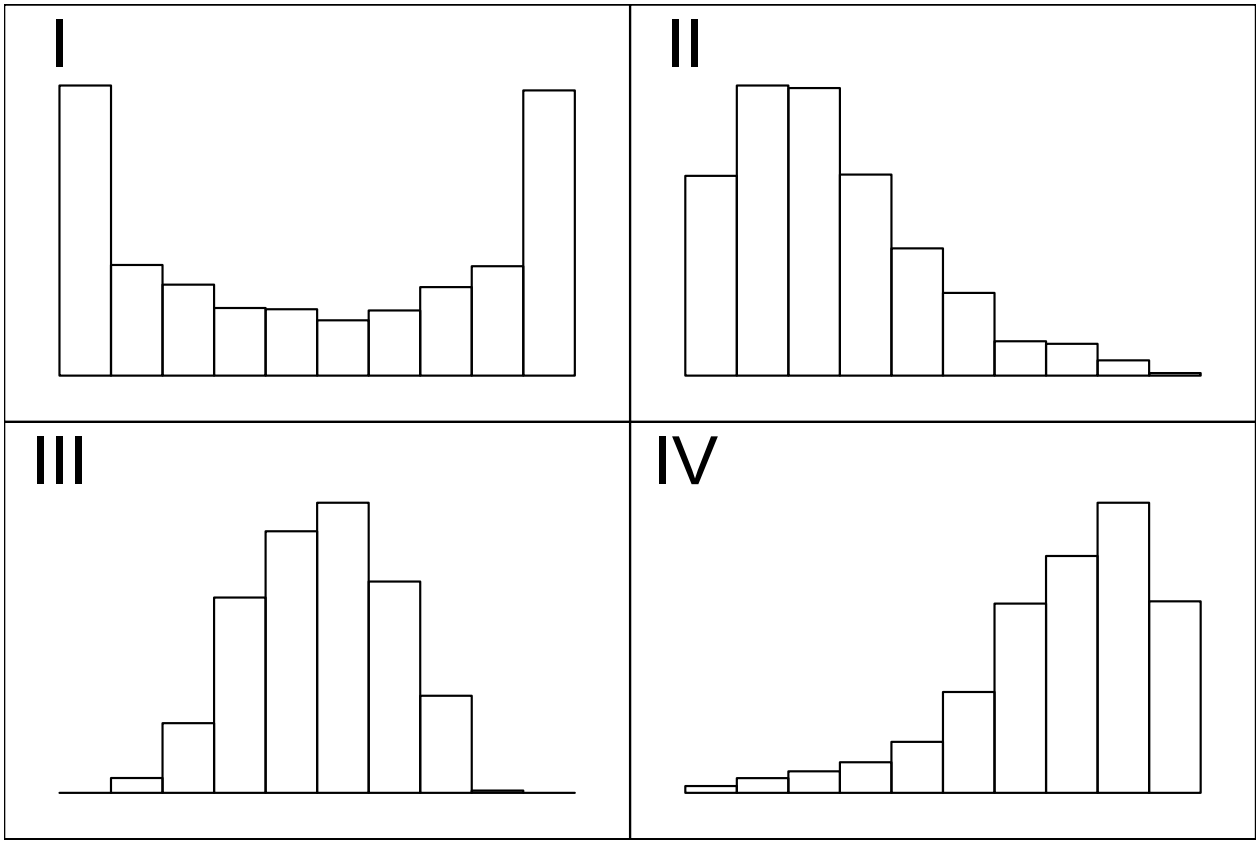
MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondance with any human. Please show work!

question	available points	earned points
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	15	
8	20	
EC	5	
EC	5	
Total	100	

1. (10 Points)



For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

(a) The distribution of heights of adult men

(b) The distribution of ages at a skilled nursing facility, where most of the patients are elderly but a few are quite young.

(c) The distribution of annual income for school employees where a high percentage of employees are entry-level teachers and only a few are high-paid administrators.

(d) The distribution of hours that students studied for an exam when about half of students studied a lot and a similar number of students studied very little.

2. (15 Points)

In a deck of strange cards, there are 309 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	red	teal	violet	Total
dog	22	38	37	21	118
jigsaw	29	45	15	14	103
quilt	28	24	25	11	88
Total	79	107	77	46	309

(a) What is the probability a random card is both a jigsaw and violet?

(b) What is the probability a random card is a quilt given it is teal?

(c) Is a dog or a jigsaw more likely to be red?

(d) What is the probability a random card is gray given it is a jigsaw?

(e) What is the probability a random card is a quilt?

(f) What is the probability a random card is either a quilt or teal (or both)?

(g) What is the probability a random card is violet?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	123	14
B	138	8
C	103	10
D	146	6

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	101.3
B	136.1
C	106
D	136.3

Which specimen is the most unusually small (relative to others of its type)?

4. (10 points)

   A tree's leaves were found to be normally distributed with a mean of 129 millimeters and a standard deviation of 6.2 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 114.4 and 125.8 millimeters?

5. (10 points)

   A species of duck is known to have a mean weight of 164.3 grams and a standard deviation of 130 grams. A researcher plans to measure the weights of 169 of these ducks sampled randomly. What is the probability the **sample mean** will be between 141.3 and 180.8 grams?

6. (10 points)

   An ornithologist wishes to characterize the average body mass of *Denrdoica magnolia*. She randomly samples 16 adults of *Denrdoica magnolia*, resulting in a sample mean of 9.33 grams and a sample standard deviation of 1.67 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

   A student is taking a multiple choice test with 700 questions. Each question has 5 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

   Then, the student takes the test and gets 159 questions correct.

   (a) What kind of hypothesis test is appropriate?

   (b) State the hypotheses.

   (c) Determine the test statistic (z or t), draw a sketch, and determine the p-value.

   (d) Decide whether we reject or retain the null hypothesis.

   (e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

   You have collected the following data:

x	y	xy
640	25	
900	52	
550	42	
600	35	
610	53	
400	44	
280	28	
570	19	
120	25	
$\sum x =$	$\sum y =$	$\sum xy =$
$\bar{x} =$	$\bar{y} =$	
$s_x =$	$s_y =$	

   (a) Complete the table.

   (b) Calculate the correlation coefficient (r) using the formula below.

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

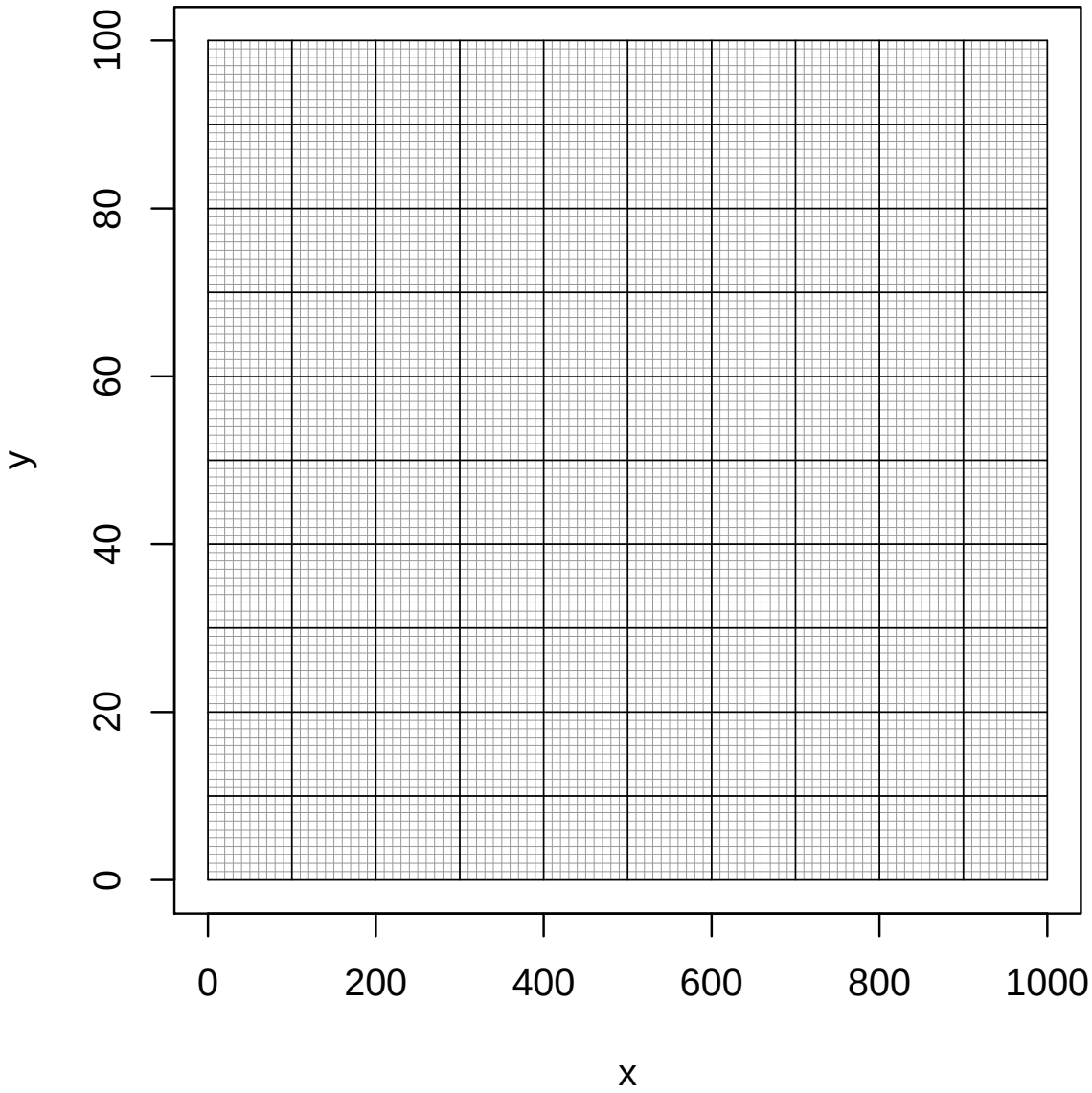
   (c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r\frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

   (d) Write the equation of the regression line (using the calculated values of a and b.)

(e) Please plot the data and a corresponding regression line.



9. (Extra credit: 5 points)

   Let each trial have a chance of success $p = 0.49$. If 129 trials occur, what is the probability of getting at least 51 but at most 77 successes?

   In other words, let $X \sim \text{Bin}(n = 129,\ p = 0.49)$ and find $P(51 \leq X \leq 77)$.

   Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

    A null hypothesis claims a population has a mean μ = 160. You decide to run two-tail test on a sample of size n = 8 using a significance level α = 0.1.

    You then collect the sample:

176	169.1	160.3	162.8	165.3
151.6	166.2	169.1		

    (a) Determine the *p*-value.

    (b) Do you reject the null hypothesis?