Name: Solution **Section: MAT098/181 C-**

MAT098/181C EXAM #3 (FORM D)

A scientific calculator is permitted. ***Cellphones may not be used as calculators and must be off or on vibrate during the exam****. Show all work on the test or on the work paper*

1. John received a score of 79 on a sociology test for which the class average was 67 with standard deviation 8. He received a score of 80 on a psychology test for which the class average was 70 with standard deviation 2.5. He scored 85 on a history test for which the class average was 80 with standard deviation 5.
   On which test did he do best RELATIVE to the rest of the class? Please JUSTIFY your answer.
   If the scores of each test follow a normal distribution, let calculate John's z-score for each of the three tests.

   Test #1 z-score : $z = \frac{\mathrm{x}-\mu}{\sigma} = \frac{79-67}{8} = 1.5$

   Test #2 z-score : $z = \frac{\mathrm{x}-\mu}{\sigma} = \frac{80-70}{2.5} = 4$

   John did better RELATIVE to the rest of the class on Test#2 because on Test#2, John's score is 4 standard deviation above the mean. On Test#1, John's score is 1.5 standard deviation above the mean. ■

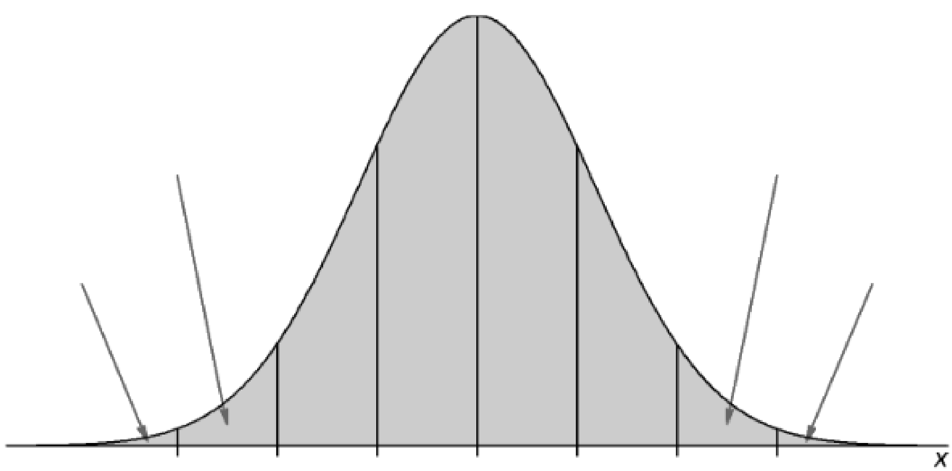
2. Suppose that students at BHCC have a normally distributed GPA with a mean of 3.0 and a standard deviation = 0.33. **Please label the graph below with the following:** ***(12 points)***

a) The tick marks on the x-axis of the graph below are one standard deviation apart. Label the axis with the ***appropriate GPA values***.

b) ***Label the Z-score*** of each value below its x-value

c) Using the Empirical rule, label each region of the graph with the area for that region

d) What interval will contain 95% of the GPA's around the mean?



3. Between one month and five months old, young deer have a body weight that is normally distributed with mean $\mu = 27.2$ kilograms and standard deviation $\sigma = 4.3$ kilograms. Let x be the weight of a young deer in kilograms.
   Find the probabilities in part a, b, and c, by first converting each of the following x intervals to standardized z intervals. *(18 points: 4,4,4,2,2,2)*

   Use Standard Normal (z) Distribution Table or use technology. Recall that $z = \frac{x-\mu}{\sigma}$

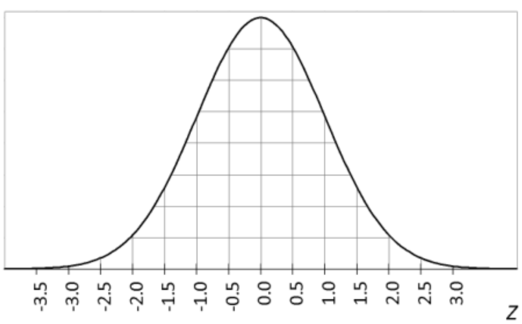
a) a) $P(x \leq 30) = p(z \leq .65) = .742$

b.) $P(x \geq 19) = p(z \geq -1.91) = .972$

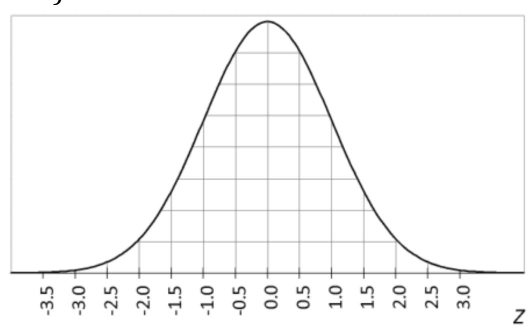
c.) $P(32 \leq x \leq 35) = p(1.12 \leq z \leq 1.81) = 0.096$

For the z intervals you calculated above, shade the area under the curve that represents the associated probability.

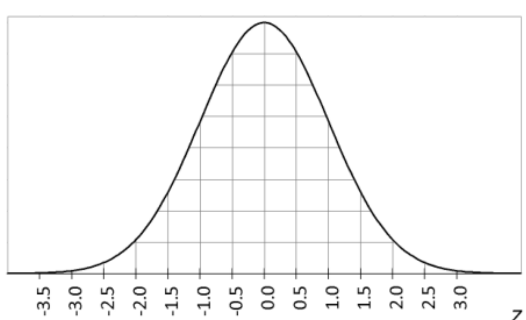
a)



b)



c)



4. *(15 points*) Suppose that a laptop has a battery with a life span that is normally distributed with a mean of 2.3 years and a standard deviation of 0.4 year. Let x be the battery life span. Keeping in mind that the life span of a rechargeable battery is the time before the battery must be replaced because it no longer holds a charge, answer the following questions:

a) What is the probability that the battery will still function for a period over 3 years? **Draw a sketch.**

P(x>3): $z = \frac{x-\mu}{\sigma} = \frac{3-2.3}{0.4} = 1.75$ ⇒P(x>3) =p(z>1.75) =.040

b) What is the probability that the battery will fail before the 2-year guarantee period? **Draw a sketch.**

P(x<2): $z = \frac{x-\mu}{\sigma} = \frac{2-2.3}{0.4} = -.75$ ⇒P(x<2) =p(z<-.75) =.227

c) What is the probability that the battery will be replaced between the 2 and 3-year period? **Draw a sketch.**

P(2<x<3)=p(-.75<z<1.75)=.733

5. Let x be the heights of 18-year-old men. If the heights are mound-shaped and symmetric, with mean 68 inches and standard deviation 3 inches, and if a random sample nine 18-year-men is selected, what is the probability **that the average height (mean) is between 67 and 69 inches?**

   Since we have a normal distribution, then from the central limit theorem, we expect the $\bar{x}$ distribution to be approximately normal, with $\mu_{\bar{x}} = \mu = 68$ inches and standard deviation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{9}} = 1$$

   Let now convert the interval $67<\bar{x}<69$ to a standard z interval and use the Standard Normal (z) Distribution Table or technology to find $p(67<\bar{x}<69)$.

$$z = \frac{x-\mu}{\sigma} = \frac{67-68}{1} = -1$$

$$z = \frac{x-\mu}{\sigma} = \frac{69-68}{1} = 1$$

   Therefore, $p(67<\bar{x}<69)=p(-1<z<1)=.683$
   <u>Interpretation:</u> About 68.3% of all such samples have average height between 67 and 69 inches. ■

6. Few years ago, one of the students at BHCC conducted a poll which showed that 65% of the students at BHCC often purchased used text books instead of new ones. A random sample of 75 students who purchased text books at BHCC Bookstore were surveyed. (*10 points)*

   a) Can we approximate the *p* distribution with a normal distribution? Explain.
   Notice that $np = 75(.65) = 48.45$ and $n(1-p) = 75(1-.65) = 26.25$ are both greater than 5. Therefore, the p distribution will be a good approximation with a normal distribution

   b) What is the mean of the sampling distribution of *p*?
   The mean is $p = .65$

   c) What is the standard deviation of the sampling distribution of *p*?
   The standard deviation of the sampling distribution of *p* is

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.65(1-.65)}{75}} = .055 \approx .06. ■$$

7. It is found that the probability that a new vaccine will reduce the risk of flu illness is about 60% among the overall population during seasons when most circulating flu viruses are like the vaccine viruses.

   (a) If a random of 500 people were administered a new vaccine during the flu seasons, use the normal approximation to the binomial to find the probability that more than 280 of these randomly selected people will be protected from the flu.

   Let r be the number of people that were administered the new vaccine. To find p(r≥280), we use the normal curve with
   $\mu = np = 500(.6) = 300$ and $\sigma = \sqrt{np(1-p)}$=$\sqrt{500(.6)(1-.6)}$=10.95
   Since r≥280, we subtract .5. Consequently, p(r≥280)= p(r≥280-.5)=p(x≥279.5). Now, let convert the interval x≥279.5 to a standard z.
   $z = \frac{279.5-300}{10.95} = -1.87$. Then p(r≥280)= p(x≥279.5)= p(z≥-1.87)
   =1- p(z≤-1.87)=96.93%.■

   (b) Will the normal distribution make a good approximation to the binomial for this problem? Explain your answer.

   Since $np = 500(.60) = 300$ and $n(1-p) = 500(1-.60) = 200$ are both greater than 5, the normal distribution will make a good approximation to the binomial. ■

8 **EXTRA CREDIT:

a) A coffee machine automatically pours coffee into cups. The amount of coffee dispensed into a cup is normally distributed with a mean of 6.7 ounces and standard deviation of 0.3 ounce. Suppose that we are interested in reducing the amount of extra coffee that is poured into the 6.7 once cup. We are seeking to identify the highest 2.5% of the fill amounts poured by this machine. For what fill amount are we searching? Round to the nearest hundredth.

Left Area is 1-2.5%=.975

z-score corresponding to .975 is 1.96

$x = \mu + z\sigma = 6.7 + 1.96(.3) = 7.29$ ■

b) The machine has just been loaded with 956 cups. How many of these do you expect will not overflow when served?

What is 84% of 956?

$x = .(84) * 956$

$x =$ 803.04. Thefore, we expect that 803 cups will not overflow when served. ■

Formula sheet:

Empirical Rule

- about 68% of the *x* values lie within 1 standard deviation of the mean.
- about 95% of the *x* values lie within 2 standard deviations of the mean.
- about 99.7% of the *x* values lie within 3 standard deviations of the mean.

z-score

$$z = \frac{x - \mu}{\sigma}$$

Central Limit Theorem

Mean of the sample mean is $\mu_{\bar{x}} = \mu$

Standard deviation of the sample mean is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

z-score for sample mean

$$z = \frac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

Binomial Distribution

Mean: $\mu = np$

Standard Deviation: $\sigma = \sqrt{np(1-p)}$

Sampling Distribution of Sample Proportion

Mean: p

Standard Deviation: $\sigma = \sqrt{\frac{p(1-p)}{n}}$