Name: ________________________ **Section: <u>MAT098/181 C-</u>**

MAT098/181C EXAM #3 (FORM B Key)

*A scientific calculator is permitted. **Cellphones may not be used as calculators and must be off or on vibrate during the exam**. Show all work on the test or on the work*

1. Micah is a student in a Community College and the following table summarizes his midterm exam scores for three courses he enrolled in along with the class average and standard deviation for each exam. *(15 points)*

Course Name	Midterm Score	Class Average	Standard Deviation
Statistics	78	75	2
Psychology	70	75	2
History	85	80	5

a) Assuming the exam scores are normally distributed, calculate Micah's z-score for each of the three courses.

Statistics z-score: _______________

$$z = \frac{78 - 75}{2} = 1.5$$

Psychology z-score: _______________

$$z = \frac{70 - 75}{2} = -2.5$$

History z-score: _______________

$$z = \frac{85 - 80}{5} = 1$$

b) On which course did Micah do the best on RELATIVE to the rest of his class?

Statistics.

c) EXPLAIN your reasoning your answer in part b)

The z-score is larger and above the mean.

2. The GPA's of BHCC students are normally distributed with a mean = 2.7 and a standard deviation = 0.4. **Please label the graph below with the following: *(12 points)***

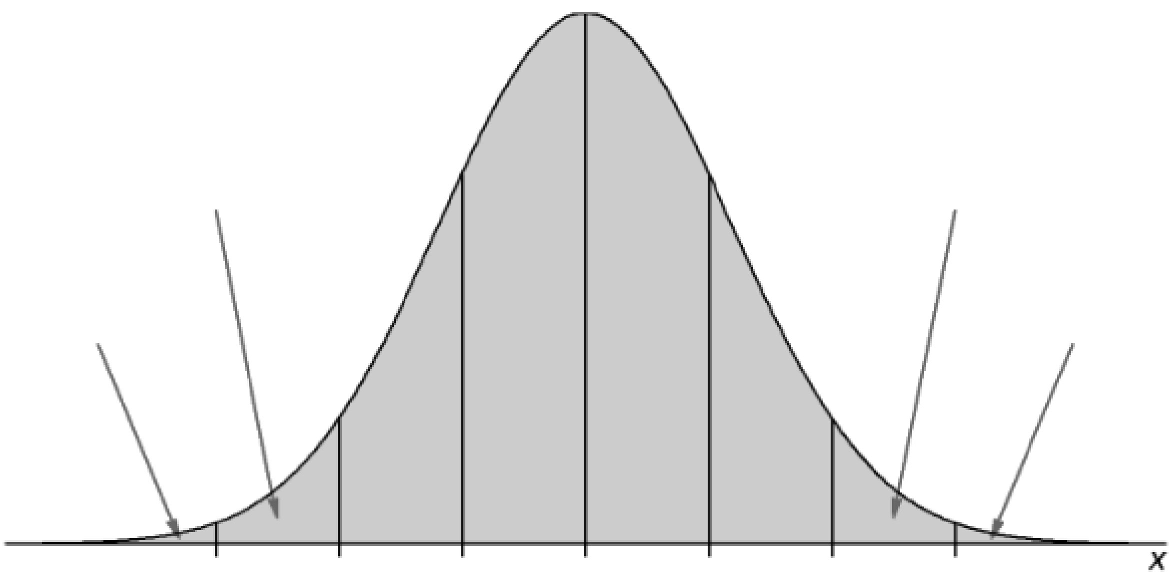
a) The tick marks on the x-axis of the graph below are one standard deviation apart. Label the axis with the ***appropriate GPA values***.

b) ***Label the Z-score*** of each value below its x-value.

c) Using the Empirical rule, label each region of the graph with the area for that region.

d) What interval will contain 95% of the GPA's around the mean?

$$(\mathbf{2.7 - 2 \times 0.4, 2.7 + 2 \times 0.4) = (1.9, 3.5})$$



3. Let x be a random variable that represents the length of time it takes a student to do statistics homework each week. After interviewing many students, it was found that x has an approximately normal distribution with mean $\mu = 6.5$ hours and standard deviation $\sigma = 0.8$ hours. *(18 points: 4,4,4,2,2,2)*

For parts a, b, c, Convert each of the following x intervals to standardized z intervals. Find the probabilities.

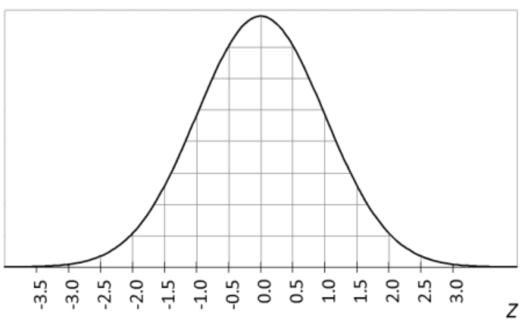
a) $P(x < 7.5) = \boldsymbol{P\left(z < \frac{7.5-6.5}{0.8}\right) = P(z < 1.25) = .8944}$

b.) $P(x > 8.5) = \boldsymbol{P(z > 2.5) = 1 - .9938 = .0062}$

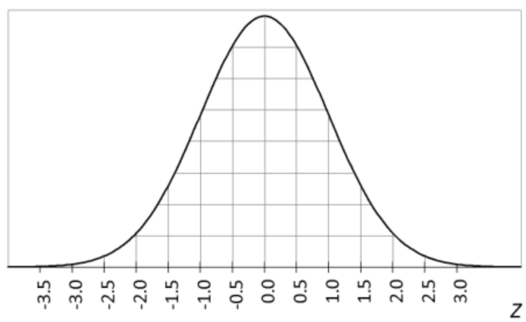
c.) $P(5.7 < x < 8) = \boldsymbol{P(-1 < z < 1.88) = .9699 - .1587 = .8112}$

For the z intervals you calculated above, shade the area under the curve that represents the associated probability.

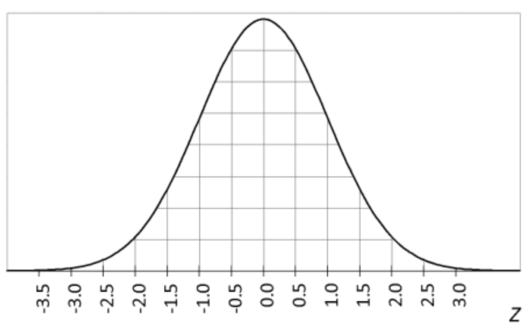
a)



b)



c)



4. ***Draw a sketch for each part.** *(15 points)* The selling price of a used car follow a normal distribution with mean μ = 6400 dollars and standard deviation σ = 300 dollars. What is the probability that the selling price of this used car is

a) less than 6000 dollars.

$$\boldsymbol{P(x < 6000) = P(z < -1.33) = .0918}$$

b) more than 7000 dollars.

$$\boldsymbol{P(x > 7000) = P(z > 2) = 1 - .9772 = .0228}$$

c) between 6000 and 7000 dollars.

$$\boldsymbol{P(6000 < x < 7000) = P(-1.33 < z < 2) = .9772 - .0918 = .8854}$$

5. Amy uses her cell phone for x minutes per day. The random variable x can be modeled by a normal distribution with mean $\mu = 28$ minutes and standard deviation $\sigma = 8$ minutes. Find the probability that on 7 randomly selected days, the average (mean) time that she spends on her cell phone is between 26 to 30 minutes. ***Draw a sketch.*** *(15 points)*

$$P(26 < \bar{x} < 30) = P\left(\frac{26-28}{8/\sqrt{7}} < z < \frac{30-28}{8/\sqrt{7}}\right) = P(-0.66 < z < 0.66)$$
$$= .7454 - .2546 = 0.4908$$

6. A clinical researcher determined that 5% of the people have flu symptoms without receiving vaccine. A random sample of 244 people is obtained from a population of 5000. The 244 people are examined for flu symptoms. If $\hat{p}$ is the sample proportion that have flu symptoms, what is the mean of the sampling distribution of $\hat{p}$? *(10 points)*

a) Will the distribution of $\hat{p}$, the sample proportion that have flu symptoms, be approximately normal? Are the conditions met?

$$np = 244 \cdot 0.05 = 12.2 \geq 10 \quad and \quad nq = 244 \cdot 0.95 = 231.8 \geq 10$$

b) What is the mean of the sampling distribution of $\hat{p}$?

$$p = 0.05$$

c) What is the standard deviation?

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.05 \cdot 0.95}{244}} = 0.01395$$

7. According to ACT.org, 57% of the students who retake the test increased their composite score. In a high school, 110 students are retaking the ACT test. Let x be a random variable that represents the number of students that increased their composite score. The high school wants a probability distribution for x. *(15 points)*

a) Write a brief description of why the normal approximation to the binomial would apply. Are the assumptions satisfied? Explain.

$$\boldsymbol{np = 110 \cdot 0.57 = 62.7 \geq 10 \quad and \quad nq = 110 \cdot 0.43 = 47.3 \geq 10}$$

b) $P(x \geq 61)$ ***Draw a sketch.**

$$\boldsymbol{P(x \geq 60.5) = P\left(z > \frac{60.5 - 62.7}{\sqrt{110 \cdot 0.57 \cdot 0.43}}\right) = P(z > -0.42) = 1 - .3372 = .6628}$$

****EXTRA CREDIT:**

8. Suppose a brewery has a filling machine that fills 12-ounce bottles of beer. It is known that the amount of beer poured by this filling machine follows a normal distribution with a mean of 12.23 ounces and a standard deviation of 0.04 ounce. The company is interested in reducing the amount of extra beer that is poured into the 12 ounce bottles. The company is seeking to identify the highest 1.5% of the fill amounts poured by this machine. For what fill amount are they searching? Round to the nearest thousandth

Left Area is $\mathbf{1 - 1.5\% = .9850}$

z-score is 2.17

$$\boldsymbol{x = \mu + z \cdot \sigma = 12.23 + 2.17 \cdot 0.04 \approx 12.317}$$

9. Find the z value such that 90% of the area under a standard normal curve lies between –z and z.

Left Area is $\mathbf{\frac{1-90\%}{2} = .0500}$

z-score is $\mathbf{-1.65}$ and 1.65

Formula sheet:

Empirical Rule

- about 68% of the x values lie within 1 standard deviation of the mean.
- about 95% of the x values lie within 2 standard deviations of the mean.
- about 99.7% of the x values lie within 3 standard deviations of the mean.

z-score

$$z = \frac{x - \mu}{\sigma}$$

Central Limit Theorem

Mean of the sample mean is $\mu_{\bar{x}} = \mu$

Standard deviation of the sample mean is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

z-score for sample mean

$$z = \frac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

Binomial Distribution

Mean: $\mu = np$

Standard Deviation: $\sigma = \sqrt{np(1-p)}$

Sampling Distribution of Sample Proportion

Mean: p

Standard Deviation: $\sigma = \sqrt{\frac{p(1-p)}{n}}$