 Name: ANSWER KEY

Instructions

- This is a practice exam. It is due **March 10**.
- The actual exam will be on **Thursday March 12**.
- The formula sheet will be provided on the actual exam.
- Enjoy!

Formulas

Discrete Probability Distributions

$$\mu = \sum x \cdot \Pr(x)$$

$$\sigma = \sqrt{(x-\mu)^2 \cdot \Pr(x)}$$

Factorial

$$0! = 1$$

$$1! = 1$$

$$2! = 2 \cdot 1$$

$$3! = 3 \cdot 2 \cdot 1$$

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Combinatorics

$${}_n\mathrm{P}_x = \frac{n!}{(n-x)!}$$

$${}_n\mathrm{C}_x = \frac{n!}{(n-x)! \cdot x!}$$

Binomial Distributions

n = number of trials
p = chance of success on each trial
q = chance of not succeeding on each trial
X = random variable representing the (unknown) number of successes
x = a specific number of successes

$$q = 1 - p$$

$$\Pr(x) = {}_n\mathrm{C}_x \cdot p^x \cdot q^{n-x}$$

$$\mu = np$$

$$\sigma = \sqrt{npq}$$

Question 1 (20 pts)

A large vessel contains marbles. Each marble has a pattern and a color. The frequencies are shown in the two-way table below.

	red	yellow	blue	violet	total
checkered	25	6	15	9	55
dotted	12	11	14	20	57
striped	30	2	27	29	88
total	67	19	56	58	200

a. What is probability a randomly selected marble is checkered?

$$\Pr(\text{checkered}) = \frac{55}{200} = 27.5\%$$

b. What is the probability a randomly selected marble is blue?

$$\Pr(\text{blue}) = \frac{56}{200} = 28\%$$

c. If you know a randomly selected marble is checkered, then what is the probability it is blue?

$$\Pr(\text{blue GIVEN checkered}) = \frac{15}{55} = 27.3\%$$

d. What is the probability a randomly selected marble is checkered and blue?

$$\Pr(\text{checkered AND blue}) = \frac{15}{200} = 7.5\%$$

e. What is the probability a randomly selected marble is checkered or blue?

$$\Pr(\text{checkered OR blue}) = \frac{55+56-15}{200} = \frac{96}{200} = 48\%$$

f. Is a red marble or a violet marble more likely to be striped?

$$\Pr(\text{striped GIVEN red}) = \frac{30}{67} = 44.8\%$$

$$\Pr(\text{striped GIVEN violet}) = \frac{29}{58} = 50\%$$

Choose one:

- ☐ red
- ☒ violet

Question 2 (20 pts)

A spinner has the probability distribution shown below.

x	$\Pr(x)$	$x \cdot \Pr(x)$	$x - \mu$	$(x-\mu)^2$	$(x-\mu)^2 \cdot \Pr(x)$
2	0.1	0.2	-16.5	272.25	27.225
17	0.47	7.99	-1.5	2.25	1.0575
18	0.08	1.44	-0.5	0.25	0.02
25	0.29	7.25	6.5	42.25	12.2525
27	0.06	1.62	8.5	72.25	4.335
		$\mu = 18.5$			$\sigma^2 = 44.89$ $\sigma = 6.7$

a. What is the probability of spinning 18?

$0.08 = 8\%$

b. What is the probability of spinning 2 or 27?

$0.1 + 0.06 = 0.16 = 16\%$

c. If spinning twice, what is the probability of first spinning 2 and then spinning 27?

$(0.1)(0.06) = 0.006 = 0.6\%$

d. What is the probability of spinning at most 18?

$0.1 + 0.47 + 0.08 = 0.65 = 65\%$

e. Determine the mean of the discrete probability distribution.

$\mu = 18.5$

f. Determine the standard deviation of the discrete probability distribution.

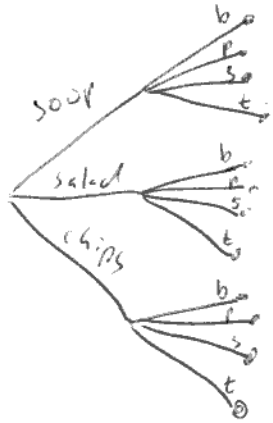
$\sigma = 6.7$

Question 3 (5 pts)

Joann will pick an appetizer and a main course. The possible appetizers and main courses are listed below.

appetizers	main courses
soup	burger
salad	pasta
chips	stirfry
	tacos

a. Draw a **tree diagram** showing the possibilities.



b. How many possibilities are there?

12

Question 4 (5 pts)

Joshua will pick an appetizer, a main course, and a dessert. The options are listed below.

appetizers	main courses	desserts
soup	burger	cake
salad	pasta	cookies
hummus	stirfry	pie
	tacos	
	sushi	

How many possibilities are there for Joshua? (Do not make a tree!)

$3 \times 5 \times 3 = \boxed{45}$

Question 5 (10 pts)

Jan will paint 4 different rooms different colors. There are 9 colors to choose from. How many options does Jan have for how the apartment will be painted?

$$_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 9 \cdot 8 \cdot 7 \cdot 6 = 3024$$

Question 6 (10 pts)

Karl will select 4 toppings for his pizza. There are 12 different toppings available. How many different pizzas are possible?

$$_{12}C_4 = \frac{12!}{(12-4)!\,4!} = \frac{12!}{8!\,4!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2} = 11 \cdot 5 \cdot 9$$

$$= \boxed{495}$$

Question 7 (20 pts)

When Emmaline is shooting archery, each arrow has an 70% chance of hitting the bullseye. Emmaline will shoot 12 arrows. We are interested in how many bullseyes Emmaline will hit.

a. Identify the parameters.

$n =$ 12 $\quad p =$ 0.7 $\quad q =$ 0.3

b. What is the probability that Emmaline hits the bullseye 9 times?

$\Pr(9) =$ 23.97%

$_{12}C_9 \cdot 0.7^9 \cdot 0.3^3 = 0.2397$

c. What is the probability that Emmaline hits the bullseye at least 10 times?

$\Pr(X \geq 10) =$ 0.1678+0.0712+0.0138 = 0.2528

$\Pr(10) = {}_{12}C_{10} \cdot 0.7^{10} \cdot 0.3^2 = 0.1678$

$\Pr(11) = {}_{12}C_{11} \cdot 0.7^{11} \cdot 0.3^1 = 0.0712$

$\Pr(12) = {}_{12}C_{12} \cdot 0.7^{12} \cdot 0.3^0 = 0.0138$

25.28%

d. What is the probability that Emmaline hits the bullseye fewer than 9 times?

$\Pr(X < 9) =$ $1 - (0.2397 + 0.2528)$

$= 0.5075 =$ 50.75%

e. What is the expected value of this binomial distribution?

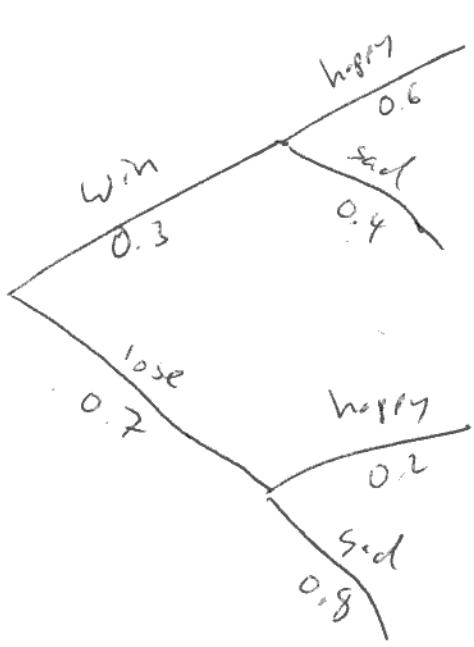
$\mu = (12)(0.7) =$ 8.4

f. What is the standard deviation of this binomial distribution?

$\sigma = \sqrt{(12)(0.7)(0.3)} =$ 1.587

Question 8 (10 pts)

In a game, there is a 30% chance Bob will win. If Bob wins, there is an 60% chance he is happy. If Bob does **not** win, there is a 20% chance he is happy. After playing the game, Bob is **not** happy; what is the probability that Bob won the game?



$0.18 = P(W \text{ and } H)$

$0.32 = P(W \text{ and } S)$

$0.14 = P(L \text{ and } H)$

$0.56 = P(L \text{ and } S)$

$$P(W \mid S) = \frac{P(W \text{ and } S)}{P(S)} = \frac{P(W \text{ and } S)}{P(W \text{ and } S) + P(L \text{ and } S)}$$

$$= \frac{0.32}{0.32 + 0.56} = 0.3636$$

$= 36.36\%$

Extra Credit (10 pts)

Candice has two upcoming tennis matches. She has a 75% chance of winning the first match and a 20% chance of winning the second match.

a. What is the probability that Candice wins both matches?

$0.75 \times 0.2 = 15\%$

b. What is the probability that Candice loses both matches? (Assume ties are impossible such that Candice either wins or loses each match.)

$0.25 \times 0.8 = 20\%$

c. What is the probability that Candice wins once and loses once (in either order)?

$100\% - 15\% - 20\% = 65\%$