Name: ______________________ **Section: <u>MAT098/181C-</u>**

MAT098/181C EXAM #4 (FORM A Key)

A scientific calculator is permitted. ***<u>Cellphones may not be used as calculators and must be off or on vibrate during the exam</u>****. Show all work on the test or on the work*

1. Of the first 10,000 votes cast in the 2016 election, 5,180 were for candidate A. Find a 95% confidence interval for the proportion of votes that candidate A will receive. (Round to three decimal places) *20 pts*

1) $\hat{p} = 5180/10000 = 0.518$ $\quad n = 10000$ $\quad Z_c = 1.96$
2) $np, nq \geq 10$
3)

$$\hat{p} \pm Z_c \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.518 \pm 0.010 = (0.508, 0.528)$$

4) We are 95% confident that the interval from 0.508 and 0.528 actually does contain the true value of the population proportion of votes that candidate A will receive.

2. A researcher wants to estimate the average time a person spends waiting every day in Boston. The researcher chooses a random sample of 51 people and finds the average wait time is 53 minutes per day with a sample standard deviation of 29 minutes. Construct a 95% confidence interval for the average time a person spends waiting every day in Boston. (Round final answer to one decimal place) *20 pts*

$$\bar{x} \pm T_c \cdot \frac{s}{\sqrt{n}} = 53 \pm \ 2.01 \cdot \frac{29}{\sqrt{51}} = 53 \pm 8.2 = (44.8, 61.2)$$

3. How many BHCC students must be randomly selected to estimate the mean annual salary after their graduation? We want 95% confident that the sample mean is within \$1,200 of the population mean, and the population standard deviation is known to be \$10,580. (*12 pts)*

$$n = \left(\frac{Z_c \cdot \sigma}{E}\right)^2 = \left(\frac{1.96 \cdot 10580}{1200}\right)^2 = 299$$

4. A new study has found that more than 90% of college students admit to using their devices for non-class activities during class times. A professor surveyed a sample of 425 randomly selected college students and found that 92% of them use their devices for non-class activities in class. Use a 0.05 significant level to test the claim that more than 90% of college students use their devices for non-class activities in class. (*24 pts)*

$p = 0.9 \quad \hat{p} = .92 \quad n = 425 \quad \alpha = 0.05$

1) $H_0\colon\ p = 0.9$
   $H_a\colon\ p > 0.9$

2) $np \geq 10, nq \geq 10$
3) $$z = \frac{0.92 - 0.9}{\sqrt{\frac{0.9 * 0.1}{425}}} \approx 1.37$$
4) P-value is 0.0853 > 0.05. Fail to reject null hypothesis.
5) At a 5% significant level, there is not enough evidence to support the alternative hypothesis that more than 90% of college students use their devices for non-class activities in class.

5. According to a study, average smartphone life expectancy now reaches 4.7 years. A randomly selected sample of 46 individuals reported that their smartphone stop working after an average of 4.2 years. Assume that we know the population standard deviation of smartphone life expectancy is 1.2 years. Use a 0.05 significance level to test the claim that the average smartphone life expectancy is not 4.7 years. (*24 pts)*

$\mu = 4.7 \quad \bar{x} = 4.2 \quad n = 46 \quad \sigma = 1.2 \quad \alpha = 0.05$

1) H_0: $\mu = 4.7$
   H_a: $\mu \neq 4.7$

2) $n > 30$
3) $$z = \frac{4.2 - 4.7}{\frac{1.2}{\sqrt{46}}} \approx -2.83$$
4) P-value is 0.0023 < 0.05. Reject null hypothesis.
5) At a 5% significant level, there is enough evidence to support the alternative hypothesis that the average smartphone life expectancy is not 4.7 years.

(EXTRA CREDIT) The mean number of absences a student has per semester is believed to be about 4 days. Faculty in a university do not believe this figure. They randomly survey 9 students. The number of absences they took for the last semester are as follows:

0, 0, 1, 3, 4, 4, 5, 5, 6

Let $x =$ the number of absences a student had for the last semester. Assume that x follows a normal distribution. Should the faculty team believe that the mean number is 4 days? (round to one decimal place) *5 pts*

$\mu = 4 \quad \bar{x} = 3.1 \quad n = 9 \quad s = 2.3 \quad \alpha = 0.05$

1) $H_0\colon\ \mu = 4$
   $H_a\colon\ \mu \neq 4$

2) normal distribution
3) $$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{3.1 - 4}{\frac{2.3}{\sqrt{9}}} \approx -1.17$$
4) P-value is more than 0.2 > 0.05. Fail to reject hypothesis.
5) At a 5% significant level, there is enough not evidence to support the alternative hypothesis that mean number of absence a student has per semester is not 4 days.

Confidence Interval for Population Parameters

Concept	Population Proportion p	Population Mean μ	
conditions	both expected successes np and failures $n(1-p)$are at least 10	σ known variable is normally distributed in the population OR sample size is more than 30	σ unknown variable is normally distributed in the population OR sample size is more than 30
confidence interval formula	$\hat{p} \pm Z_c \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$	$\bar{x} \pm Z_c \cdot \dfrac{\sigma}{\sqrt{n}}$	$\text{df} = n - 1$ $\bar{x} \pm T_c \cdot \dfrac{s}{\sqrt{n}}$
sample size formula	$\hat{p} = \frac{x}{n}$ known $n = \hat{p} \cdot \hat{q} \cdot \left(\dfrac{Z_c}{E}\right)^2$ $\hat{p}$ unknown $n = \dfrac{1}{4} \cdot \left(\dfrac{Z_c}{E}\right)^2$	$n = \left(\dfrac{Z_c \cdot \sigma}{E}\right)^2$	

- 90% confidence interval: $Z_c \approx 1.645$
- 95% confidence interval: $Z_c \approx 1.960$
- 99% confidence interval: $Z_c \approx 2.576$

Hypothesis Testing

Concept	Population Proportion p	Population Mean μ	
conditions	both expected successes np and failures $n(1-p)$are at least 10	σ known variable is normally distributed in the population OR sample size is more than 30	σ unknown variable is normally distributed in the population OR sample size is more than 30
test statistics	$z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$	$z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$	$\text{df} = n - 1$ $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$

- If the P-value $< \alpha$, we reject the null hypothesis and there is enough evidence to support the alternative hypothesis.
- If the P-value $\geq \alpha$, we fail to reject the null hypothesis and there is not enough evidence to support the alternative hypothesis.