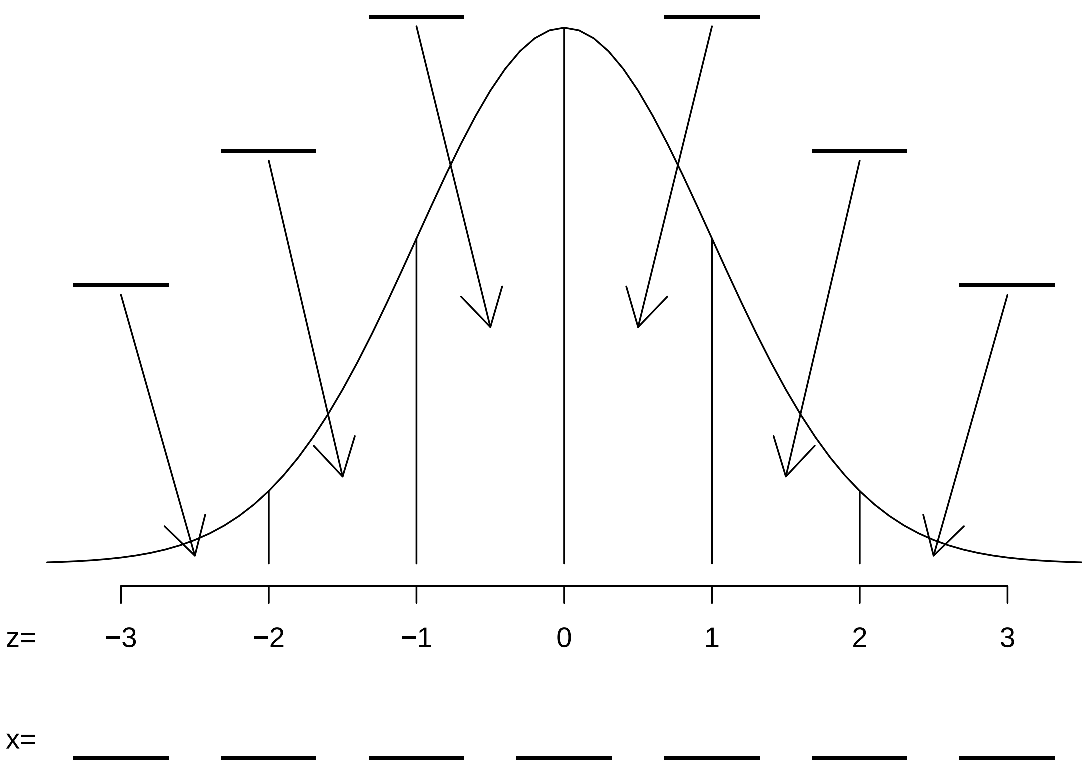
1. **Problem**
   Yasmine took 4 exams for her statistics class. Each exam yielded normally distributed scores (the class's scores were normally distributed). Yasmine's scores and the exams' means and standard deviations are shown below.

Exam	Yasmine's score	Mean of all scores	SD of all scores
1	68.57	70.1	9
2	57.48	71.8	7.7
3	85.21	69.9	8.1
4	49.93	61.9	10.5

   On which exam did Yasmine perform worst compared to other people?

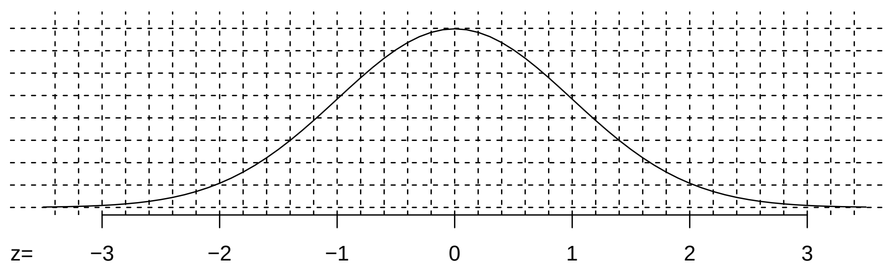
2. **Problem**
A normal random variable X has a mean μ = 7.5 and standard deviation σ = 2.5. Please label the density curve with:

(a) The appropriate values of x.
(b) The areas of the sections.

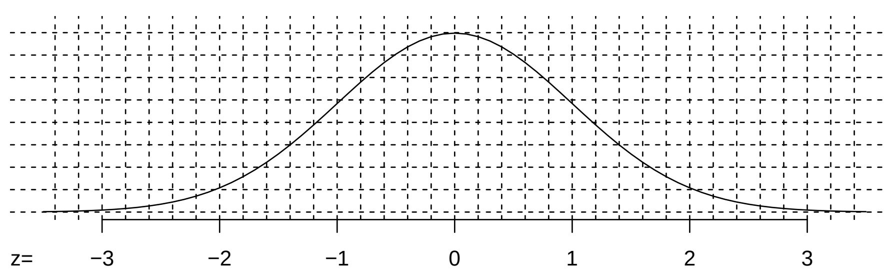


3. **Problem**
Let X be normally distributed with mean 117 and standard deviation 32. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

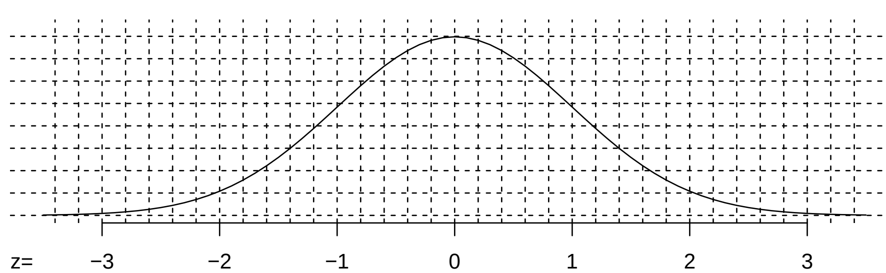
(a) $P(X < 133)$



(b) $P(X > 152.2)$



(c) $P(75.4 < X < 158.6)$



4. **Problem**
A very large population has a mean of 87.6 and a standard deviation of 16.2. When a random sample of size 36 is taken, what is the probability that the **sample mean** ($\bar{x}$) is between 86.4 and 88.7?

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(86.4 < \overline{X} < 88.7)$. **Draw a sketch**

5. **Problem**
A very large population has a population proportion $p = 0.12$. When a random sample of size 256 is taken, what is the probability that the **sample proportion** ($\hat{p}$) is more than 0.09?

Do NOT use a continuity correction.

(a) Determine the mean and standard deviation of the sampling distribution.

(b) Determine $P(\hat{p} > 0.09)$. **Draw a sketch**

6. **Problem**
Let random variable W have mean $\mu_W = 25$ and standard deviation $\sigma_W = 4$. Let random variable X represent the **sum** of $n = 225$ instances of W.

(a) Determine the expected value of X. $\mu_X = ?$

(b) Determine the standard deviation of X. $\sigma_X = ?$

(c) Using normal approximation, determine $P(X < 5636.4)$.

(d) Using normal approximation, determine $P(X > 5545.2)$.

7. **Problem**
   Let each trial have a chance of success $p = 0.06$. If 237 trials occur, what is the probability of getting at least 11 but at most 16 successes?

   In other words, let $X \sim \text{Bin}(n = 237,\ p = 0.06)$ and find $P(11 \leq X \leq 16)$.

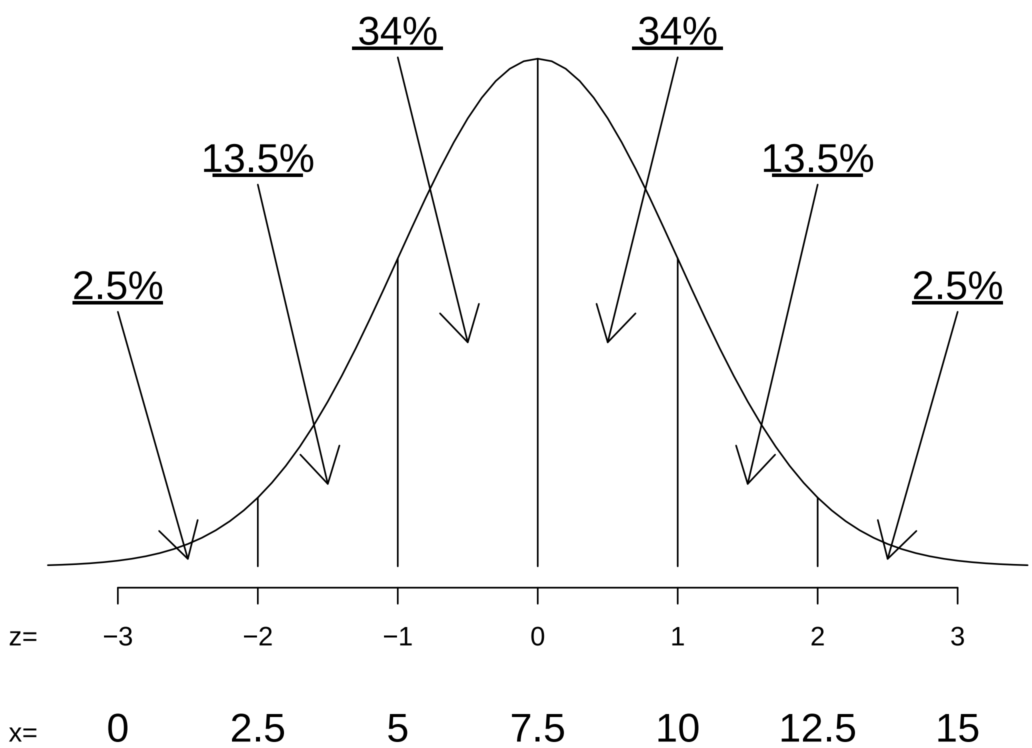
   Use a normal approximation along with the continuity correction.

1. We compare the z-scores. The smallest z-score corresponds to the worst score.

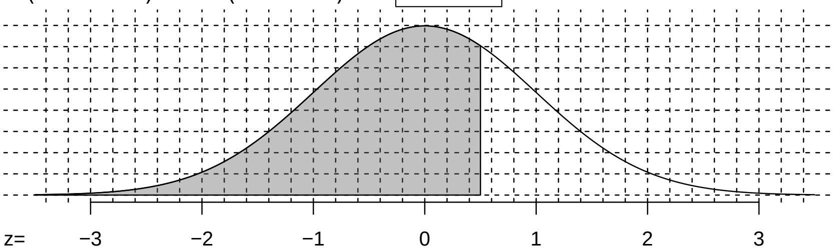
Exam	formula	z-score
1	$z = \frac{68.57-70.1}{9}$	-0.17
2	$z = \frac{57.48-71.8}{7.7}$	-1.86
3	$z = \frac{85.21-69.9}{8.1}$	1.89
4	$z = \frac{49.93-61.9}{10.5}$	-1.14

Thus, Yasmine did worst on exam 2.

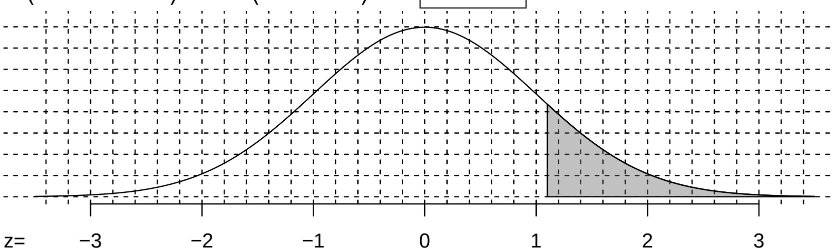
2. The filled in areas and x values are shown below.



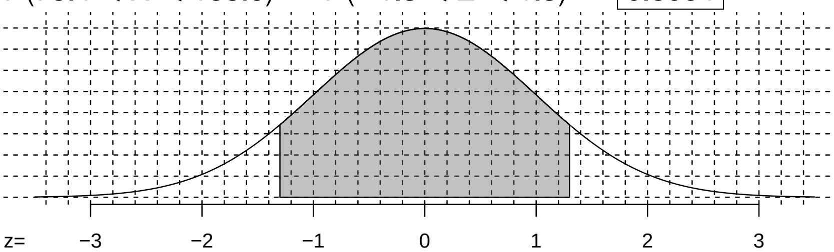
3. (a) $P(X < 133) = P(Z < 0.5) = \boxed{0.6915}$



(b) $P(X > 152.2) = P(Z > 1.1) = \boxed{0.1357}$

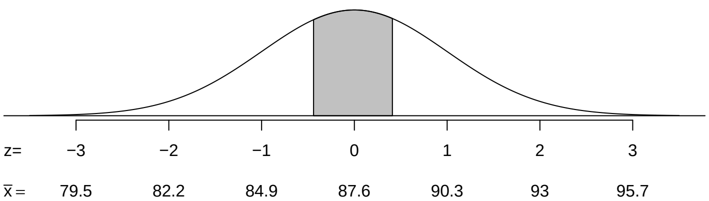


(c) $P(75.4 < X < 158.6) = P(-1.3 < Z < 1.3) = \boxed{0.8064}$



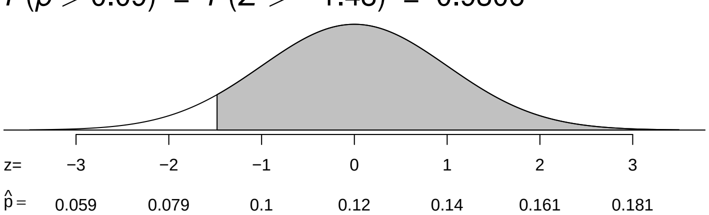
4. (a) Central limit of average formulas: $\mu_{\bar{x}} = 87.6$ and $\sigma_{\bar{x}} = \frac{16.2}{\sqrt{36}} = 2.7$.

(b) $P(86.4 < \overline{X} < 88.7) = P(-0.44 < Z < 0.41) = 0.3291$



5. (a) Use $\hat{p}$ sampling formulas: $\mu_{\hat{p}} = 0.12$ and $\sigma_{\hat{p}} = \frac{\sqrt{(0.12)(0.88)}}{\sqrt{256}} = 0.0203101$.

(b) $P(\hat{p} > 0.09) = P(Z > -1.48) = 0.9306$



6. (a) $\mu_X = n\mu_W = (225)(25) = \boxed{5625}$

   (b) $\sigma_X = \sigma_W\sqrt{n} = (4)(\sqrt{225}) = \boxed{60}$

   (c) $P(X < 5636.4) = P(Z < 0.19) = \boxed{0.5753}$

   (d) $P(X > 5545.2) = P(Z > -1.33) = 1 - 0.0918 = \boxed{0.9082}$

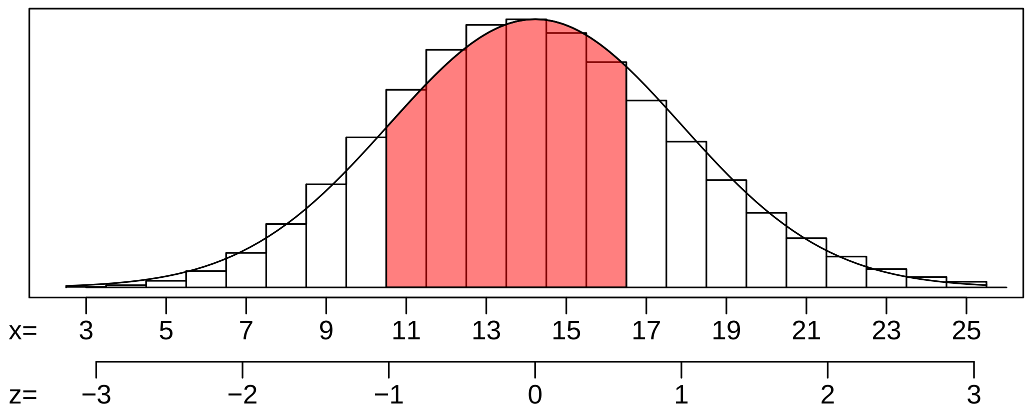
7. Find the mean.

$$\mu = np = (237)(0.06) = 14.22$$

Find the standard deviation.

$$\sigma = \sqrt{np(1-p)} = \sqrt{(237)(0.06)(1-0.06)} = 3.6561$$

Make a sketch, specifically try to picture whether you need to add or subtract 0.5 for the continuity correction.



Find the z scores.

$$z_1 = \frac{10.5 - 14.22}{3.6561} = -1.02$$

$$z_2 = \frac{16.5 - 14.22}{3.6561} = 0.62$$

Find the percentiles (from z-table).

$$\ell_1 = 0.1539$$

$$\ell_2 = 0.7324$$

Calculate the probability.

$$P(11 \le X \le 16) = 0.7324 - 0.1539 = 0.5785$$

Normal Distributions

$$z = \frac{x - \mu}{\sigma}$$

$$x = \mu + z\sigma$$

Central Limit Theorem

- **If:**
  - Random variable W has mean μ_w and standard deviation σ_w.
  - Random variable X represents the **sum** of n instances of W.

$$X = W_1 + W_2 + W_3 + \cdots + W_n$$

  - Random variable Y represents the **mean** of n instances of W.

$$Y = \frac{W_1 + W_2 + W_3 + \cdots + W_n}{n}$$

- **Then**:
  - The following formulas are exactly true:

$$\mu_x = n\mu_w \qquad \mu_y = \mu_w$$

$$\sigma_x = \sigma_w\sqrt{n} \qquad \sigma_y = \frac{\sigma_w}{\sqrt{n}}$$

  - X and Y are both approximately normal (if $n \geq 30$).
  - X and Y are exactly normal if W is normal.

Special case of CLT: Bernoulli, Binomial, and Proportion Sampling

- **If:**
  - Random variable W is Bernoulli:

w	$P(w)$
0	q
1	p

  - Random variable X represents the sum of n instances of W. (Binomial)
  - Random variable $\hat{p}$ represents the mean of n instances of W. (Proportion sampling)

- **Then**:
  - The following formulas are exactly true:

$$\mu_w = p \qquad \mu_x = np \qquad \mu_{\hat{p}} = p$$

$$\sigma_w = \sqrt{pq} \qquad \sigma_x = \sqrt{pq}\sqrt{n} \qquad \sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}}$$

  - X and $\hat{p}$ are both approximately normal (if $np \geq 10$ and $nq \geq 10$).