

1. Problem

In a deck of strange cards, there are 437 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	blue	gray	orange	teal	Total
dog	32	16	44	12	34	138
mop	19	24	17	30	14	104
shovel	39	40	46	21	49	195
Total	90	80	107	63	97	437

- (a) What is the probability a random card is teal given it is a dog?
- (b) What is the probability a random card is a mop?
- (c) Is a mop or a shovel more likely to be blue?
- (d) What is the probability a random card is black?
- (e) What is the probability a random card is a shovel given it is gray?
- (f) What is the probability a random card is both a mop and teal?
- (g) What is the probability a random card is either a dog or black (or both)?

2. Problem

In a deck of strange cards, there are 761 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	pink	violet	white	yellow	Total
bike	50	32	46	38	28	194
horn	34	19	16	14	36	119
lamp	37	47	11	49	20	164
quilt	39	24	17	27	31	138
rug	42	22	18	43	21	146
Total	202	144	108	171	136	761

- (a) What is the probability a random card is violet?
- (b) What is the probability a random card is both a bike and yellow?
- (c) What is the probability a random card is a bike?
- (d) What is the probability a random card is pink given it is a quilt?
- (e) Is a horn or a lamp more likely to be violet?
- (f) What is the probability a random card is either a rug or pink (or both)?
- (g) What is the probability a random card is a rug given it is pink?

3. Problem

A spinner has the probability distribution shown below.

x	$\Pr(x)$
2	0.08
4	0.24
6	0.58
24	0.1

- (a) What is the probability of spinning 6? In other words, what is $\Pr(X = 6)$?
- (b) What is the probability of spinning 4 or 6? In other words, what is $\Pr(X = 4 \text{ or } X = 6)$?
- (c) If spinning twice, what is the probability of first spinning 4 and then spinning 6? In other words, what is $\Pr(X_1 = 4 \text{ and } X_2 = 6)$?
- (d) What is the probability of spinning at most 4? In other words, what is $\Pr(X \leq 4)$?
- (e) Determine the mean of the probability distribution by using $\mu = \sum x \cdot \Pr(x)$.
- (f) Determine the standard deviation of the probability distribution by using $\sigma = \sqrt{\sum (x - \mu)^2 \cdot \Pr(x)}$.

4. Problem

A spinner has the probability distribution shown below.

x	$\Pr(x)$
1	0.14
3	0.42
27	0.08
29	0.36

- (a) What is the probability of spinning 3? In other words, what is $\Pr(X = 3)$?
- (b) What is the probability of spinning 3 or 27? In other words, what is $\Pr(X = 3 \text{ or } X = 27)$?
- (c) If spinning twice, what is the probability of first spinning 3 and then spinning 27? In other words, what is $\Pr(X_1 = 3 \text{ and } X_2 = 27)$?
- (d) What is the probability of spinning at least 3? In other words, what is $\Pr(X \geq 3)$?
- (e) Determine the mean of the probability distribution by using $\mu = \sum x \cdot \Pr(x)$.
- (f) Determine the standard deviation of the probability distribution by using $\sigma = \sqrt{\sum (x - \mu)^2 \cdot \Pr(x)}$.

5. Problem

Norma is buying a pizza. Norma still has to decide on a type of crust and a sauce. The shop carries 2 types of crust and 6 sauces. How many different combinations is Norma choosing between? Please make a **tree diagram**.

6. Problem

Wilhelmina is buying a pizza. Wilhelmina still has to decide on a topping and a type of crust. The shop carries 3 toppings and 5 types of crust. How many different combinations is Wilhelmina choosing between? Please make a **tree diagram**.

7. Problem

A company needs to select 2 members to be on a committee. The company is considering a pool of 18 applicants. How many committees are possible?

8. Problem

A committee is judging the performances of 8 different acrobats. The committee needs to assign 5 equivalent prizes. How many ways could the committee assign the prizes?

9. Problem

A company needs to select a secretary, a CFO, a vice president, a president, and a manager. Each position will be held by a different person. The company is considering the same pool of 20 applicants for each position. How many configurations are possible?

10. Problem

A basketball team has 9 players. The coach needs to pick players to fill 2 different positions. How many ways could the coach do this?

11. Problem

Each trial has 0.35 probability of success. There will be 9 trials. We will measure the number of successes (but not worry about the exact sequence).

- (a) Why is this a binomial distribution?
- (b) What is the probability of getting exactly 5 successes? In other words, determine $\Pr(X = 5)$.
- (c) What is the probability of getting exactly 2 successes? In other words, determine $\Pr(X = 2)$.
- (d) What is the probability of getting more than 2 successes? In other words, determine $\Pr(X > 2)$.
- (e) What is the probability of getting at least 2 successes? In other words, determine $\Pr(X \geq 2)$.
- (f) What is the probability of getting less than 2 successes? In other words, determine $\Pr(X < 2)$.
- (g) What is the probability of getting at most 2 successes? In other words, determine $\Pr(X \leq 2)$.
- (h) Determine the mean number of successes.
- (i) Determine the standard deviation of successes.

12. Problem

Each trial has 0.58 probability of success. There will be 7 trials. We will measure the number of successes (but not worry about the exact sequence).

- (a) Why is this a binomial distribution?
- (b) What is the probability of getting exactly 4 successes? In other words, determine $\Pr(X = 4)$.
- (c) What is the probability of getting exactly 5 successes? In other words, determine $\Pr(X = 5)$.
- (d) What is the probability of getting more than 5 successes? In other words, determine $\Pr(X > 5)$.
- (e) What is the probability of getting at least 5 successes? In other words, determine $\Pr(X \geq 5)$.
- (f) What is the probability of getting less than 5 successes? In other words, determine $\Pr(X < 5)$.
- (g) What is the probability of getting at most 5 successes? In other words, determine $\Pr(X \leq 5)$.
- (h) Determine the mean number of successes.
- (i) Determine the standard deviation of successes.

1. (a) $P(\text{teal given dog}) = 0.246$
 (b) $P(\text{mop}) = 0.238$
 (c) $P(\text{blue given mop}) = 0.231$ and $P(\text{blue given shovel}) = 0.205$, so a mop is more likely to be blue than a shovel is.
 (d) $P(\text{black}) = 0.206$
 (e) $P(\text{shovel given gray}) = 0.43$
 (f) $P(\text{mop and teal}) = 0.032$
 (g) $P(\text{dog or black}) = 0.449$
2. (a) $P(\text{violet}) = 0.142$
 (b) $P(\text{bike and yellow}) = 0.0368$
 (c) $P(\text{bike}) = 0.255$
 (d) $P(\text{pink given quilt}) = 0.174$
 (e) $P(\text{violet given horn}) = 0.134$ and $P(\text{violet given lamp}) = 0.0671$, so a horn is more likely to be violet than a lamp is.
 (f) $P(\text{rug or pink}) = 0.352$
 (g) $P(\text{rug given pink}) = 0.153$
3. Make a table (for parts d and e).

x	$\text{Pr}(x)$	$x \cdot \text{Pr}(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 \cdot \text{Pr}(x)$
2	0.08	0.16	-5	25	2
4	0.24	0.96	-3	9	2.16
6	0.58	3.48	-1	1	0.58
24	0.1	2.4	17	289	28.9
		$\sum x \cdot \text{Pr}(x) = 7$			$\sigma^2 = 33.64$
		$\mu = 7$			$\sigma = 5.8$

- (a) 0.58
- (b) 0.82
- (c) 0.1392
- (d) 0.32
- (e) $\mu = 7$
- (f) $\sigma = 5.8$

4. Make a table (for parts d and e).

x	$\Pr(x)$	$x \cdot \Pr(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 \cdot \Pr(x)$
1	0.14	0.14	-13	169	23.66
3	0.42	1.26	-11	121	50.82
27	0.08	2.16	13	169	13.52
29	0.36	10.44	15	225	81
		$\sum x \cdot \Pr(x) = 14$			$\sigma^2 = 169$
		$\mu = 14$			$\sigma = 13$

(a) 0.42

(b) 0.5

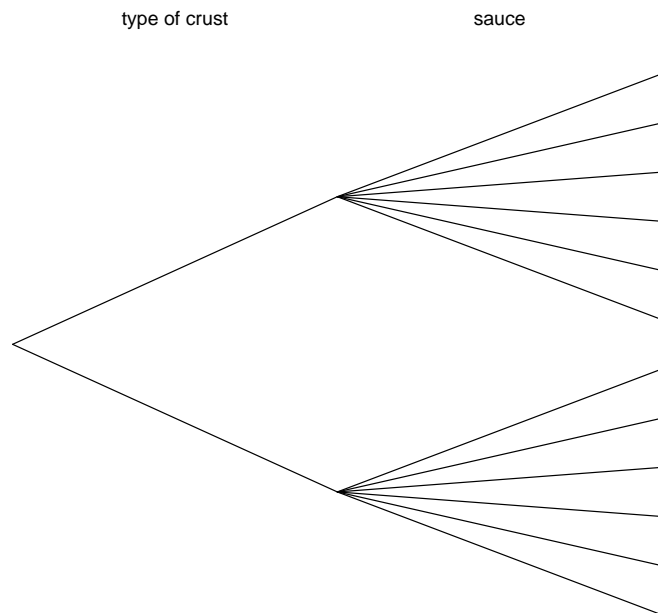
(c) 0.0336

(d) 0.86

(e) $\mu = 14$

(f) $\sigma = 13$

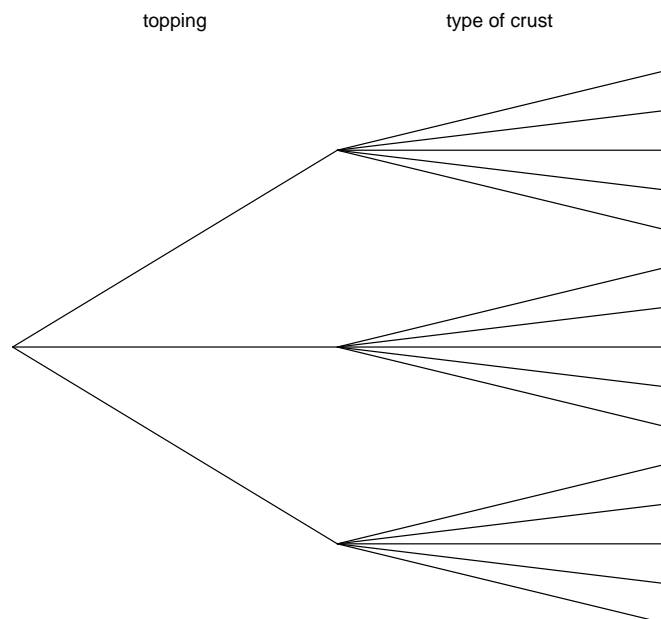
5. Make a tree.



Count the leaves (the nodes at the far right). In this case there are 12 leaves.

There are 12 combinations possible.

6. Make a tree.



Count the leaves (the nodes at the far right). In this case there are 15 leaves.

There are 15 combinations possible.

7. This scenario describes a combinations problem (order does not matter). We are considering the subsets of size 2 from a set of size 18.

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

$$n = 18$$

$$r = 2$$

$${}_{18}C_2 = \frac{18!}{(18-2)! \cdot 2!}$$

$$= \frac{18!}{16! \cdot 2!}$$

$$= \frac{18 \cdot 17}{2 \cdot 1}$$

$$= \boxed{153}$$

8. This scenario describes a combinations problem (order does not matter). We are considering the subsets of size 5 from a set of size 8.

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

$$n = 8$$

$$r = 5$$

$${}_8C_5 = \frac{8!}{(8-5)! \cdot 5!}$$

$$= \frac{8!}{3! \cdot 5!}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= \boxed{56}$$

9. This scenario describes a permutations problem (order matters). We are considering the nonrepeating sequences of size 5 from a set of size 20.

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$n = 20$$

$$r = 5$$

$${}_{20}P_5 = \frac{20!}{(20-5)!}$$

$$= \frac{20!}{15!}$$

$$= 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16$$

$$= \boxed{1860480}$$

10. This scenario describes a permutations problem (order matters). We are considering the nonrepeating sequences of size 2 from a set of size 9.

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$n = 9$$

$$r = 2$$

$${}_9P_2 = \frac{9!}{(9-2)!}$$

$$= \frac{9!}{7!}$$

$$= 9 \cdot 8$$

$$= \boxed{72}$$

11. (a) Each trial has TWO possible outcomes (which are mutually exclusive and exhaustive). Each trial has the same probability of success. We are interested in the total number of successes in a fixed number of trials.
- (b) $\Pr(X = 5) = {}_9C_5 \cdot 0.35^5 0.65^4 = 0.1181311$
- (c) $\Pr(X = 2) = {}_9C_2 \cdot 0.35^2 0.65^7 = 0.2161882$
- (d) $\Pr(X > 2) = 0.6627267$
- (e) $\Pr(X \geq 2) = 0.878915$
- (f) $\Pr(X < 2) = 0.121085$
- (g) $\Pr(X \leq 2) = 0.3372733$
- (h) Because this is a binomial distribution, $\mu = np$, so $\mu = 3.15$
- (i) Because this is a binomial distribution, $\sigma = \sqrt{npq}$, so $\sigma = 1.4309088$

12. (a) Each trial has TWO possible outcomes (which are mutually exclusive and exhaustive). Each trial has the same probability of success. We are interested in the total number of successes in a fixed number of trials.
- (b) $\Pr(X = 4) = {}_7C_4 \cdot 0.58^4 0.42^3 = 0.2934458$
- (c) $\Pr(X = 5) = {}_7C_5 \cdot 0.58^5 0.42^2 = 0.2431408$
- (d) $\Pr(X > 5) = 0.1340018$
- (e) $\Pr(X \geq 5) = 0.3771426$
- (f) $\Pr(X < 5) = 0.6228574$
- (g) $\Pr(X \leq 5) = 0.8659982$
- (h) Because this is a binomial distribution, $\mu = np$, so $\mu = 4.06$
- (i) Because this is a binomial distribution, $\sigma = \sqrt{npq}$, so $\sigma = 1.3058331$