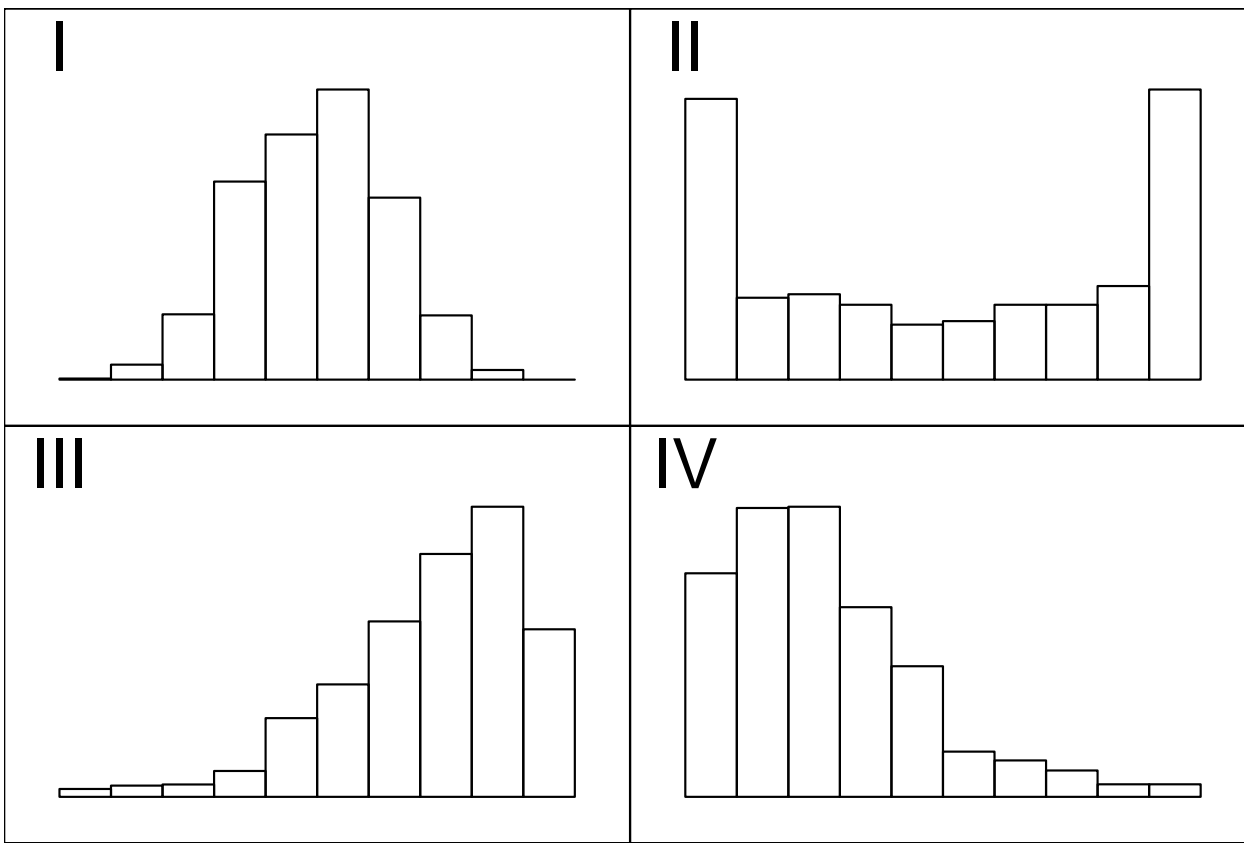


MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondence with any human. Please show work!

question	available points	earned points
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	15	
8	20	
EC	5	
EC	5	
Total	100	

1. (10 Points)



For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

- (a) The distribution of hours that students studied for an exam when about half of students studied a lot and a similar number of students studied very little.
- (b) The distribution of quiz scores on an easy quiz. Most students did very well, but a few did poorly.
- (c) The distribution of lengths of newborn babies
- (d) The distribution of annual income for NBA basketball players where only a few are high-paid superstars.

2. (15 Points)

In a deck of strange cards, there are 361 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	blue	pink	red	Total
cat	22	42	29	12	105
gem	37	19	49	34	139
quilt	26	45	33	13	117
Total	85	106	111	59	361

- (a) What is the probability a random card is a cat?
- (b) What is the probability a random card is both a cat and blue?
- (c) What is the probability a random card is pink given it is a gem?
- (d) What is the probability a random card is a quilt given it is red?
- (e) What is the probability a random card is either a quilt or pink (or both)?
- (f) Is a cat or a quilt more likely to be pink?
- (g) What is the probability a random card is pink?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
<i>A</i>	150	4
<i>B</i>	74	11
<i>C</i>	136	15
<i>D</i>	114	10

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
<i>A</i>	146.3
<i>B</i>	78.07
<i>C</i>	131.6
<i>D</i>	110.4

Which specimen is the most unusually small (relative to others of its type)?

4. (10 points)

A tree's leaves were found to be normally distributed with a mean of 31.2 millimeters and a standard deviation of 7.1 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 21.3 and 28.6 millimeters?

5. (10 points)

A species of duck is known to have a mean weight of 215.7 grams and a standard deviation of 19.8 grams. A researcher plans to measure the weights of 81 of these ducks sampled randomly. What is the probability the **sample mean** will be between 210.2 and 221.2 grams?

6. (10 points)

An ornithologist wishes to characterize the average body mass of *Seiurus aurocapillus*. She randomly samples 29 adults of *Seiurus aurocapillus*, resulting in a sample mean of 22.62 grams and a sample standard deviation of 1.44 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

A student is taking a multiple choice test with 800 questions. Each question has 4 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

Then, the student takes the test and gets 221 questions correct.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses.

(c) Determine the test statistic (z or t), draw a sketch, and determine the p -value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

You have collected the following data:

x	y	xy
24	37	
62	40	
51	18	
61	44	
75	29	
26	21	
21	15	
$\sum x =$	$\sum y =$	$\sum xy =$
$\bar{x} =$	$\bar{y} =$	
$s_x =$	$s_y =$	

(a) Complete the table.

(b) Calculate the correlation coefficient (r) using the formula below.

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

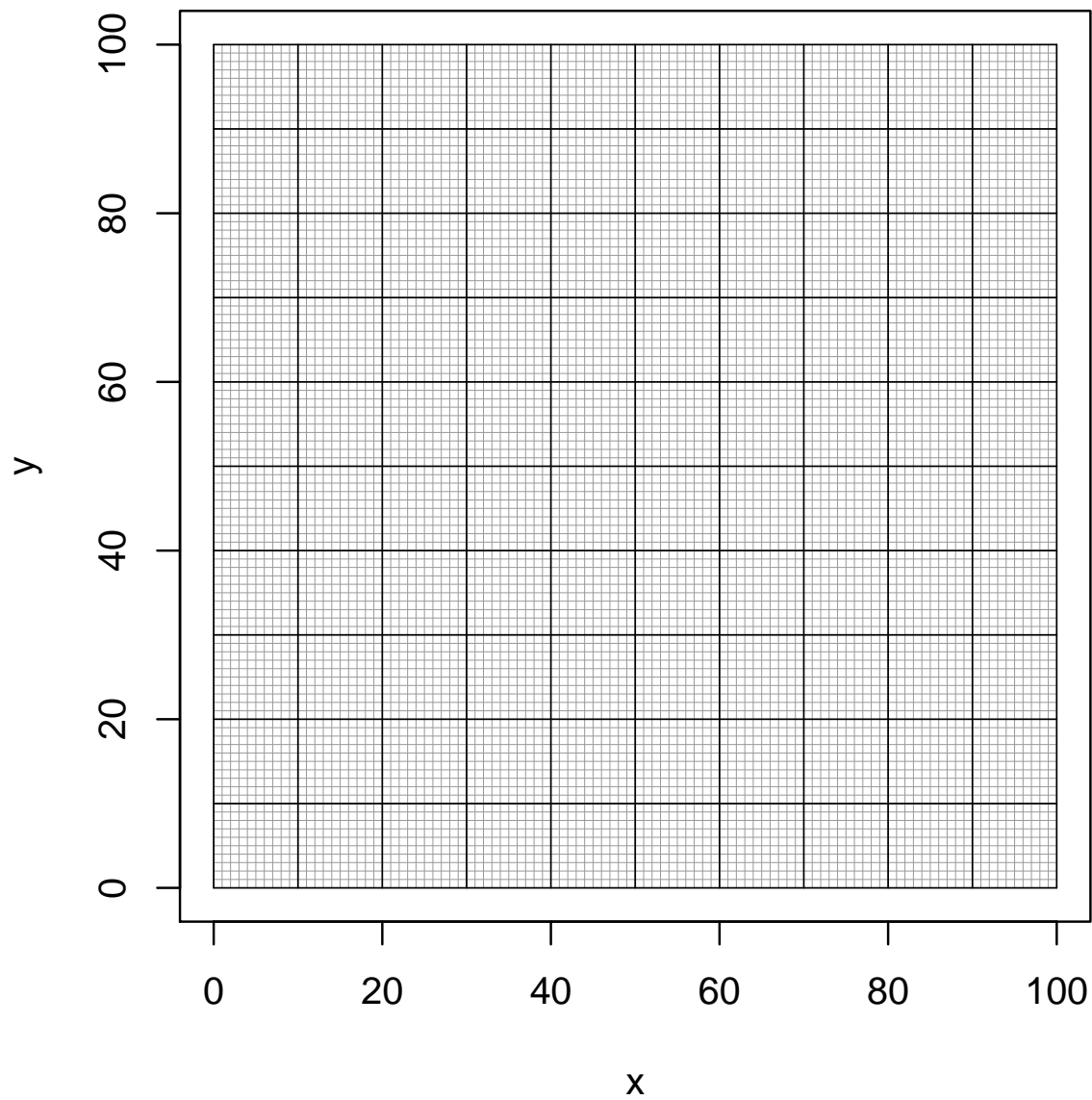
(c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r \frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

(d) Write the equation of the regression line (using the calculated values of a and b .)

(e) Please plot the data and a corresponding regression line.



9. (Extra credit: 5 points)

Let each trial have a chance of success $p = 0.87$. If 118 trials occur, what is the probability of getting more than 100 but less than 107 successes?

In other words, let $X \sim \text{Bin}(n = 118, p = 0.87)$ and find $P(100 < X < 107)$.

Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

A null hypothesis claims a population has a mean $\mu = 250$. You decide to run two-tail test on a sample of size $n = 11$ using a significance level $\alpha = 0.02$.

You then collect the sample:

319.8	350.5	234.2	305	229.3
269.4	292	245.2	290	259.5
322.6				

- (a) Determine the p -value.
- (b) Do you reject the null hypothesis?