Confidence Interval

- The confidence level, γ, represents how confident we are the interval will contain the population parameter (population proportion or population mean).
- To get $z^\star$, find $z^\star$ such that $P(|Z| < z^\star) = \gamma$. To do that, first get percentile, ℓ, from confidence level (γ):

$$\ell = \frac{\gamma + 1}{2}$$

  then, use the z-table to find $z^\star$ such that $P(Z < z^\star) = \ell$.

Proportion

The population proportion, p, is estimated with an interval (to indicate uncertainty) based on a sample proportion, $\hat{p}$.

- Bounds:

$$\hat{p} \pm z^\star \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- Necessary sample size for a given margin of error:
  - If $\hat{p}$ is known:

$$n = \hat{p}(1-\hat{p})\left(\frac{z^\star}{ME}\right)^2$$

  - If $\hat{p}$ is unknown, assume it is 0.5 to be conservative

$$n = \frac{1}{4}\left(\frac{z^\star}{ME}\right)^2$$

Mean

The population mean, μ, is estimated with an interval (to indicate uncertainty) based on a sample mean, $\bar{x}$.

- Bounds:
  - If σ is known:

$$\bar{x} \pm z^\star \cdot \frac{\sigma}{\sqrt{n}}$$

  - If σ is unknown, use the sample standard deviation (and $t^\star$). Remember, $df = n - 1$. To get $t^\star$, find $t^\star$ such that $P(|T| < t^\star) = \gamma$ and $df = n - 1$.

$$\bar{x} \pm t^\star \cdot \frac{s}{\sqrt{n}}$$

- Necessary sample size for a given margin of error:

$$n = \left(\frac{z^\star \sigma}{ME}\right)^2$$

Hypothesis Testing (Single-Sample)

$H_0 =$ null hypothesis
$H_A =$ alternative hypothesis
p-value $=$ probability of getting sample at least as extreme as observed sample, **given** H_0
$\alpha =$ significance level = chance of type II error given H_0

- Calculate the p-value.
  - "at least as extreme" can mean "as large or larger", "as small or smaller", or "as far from expected in either direction".
- If p-value is small enough, we reject the null hypothesis. (This logic is similar to *reductio ad absurdum* or proof by contradiction.)

$$\text{If} \quad p\text{-value} < \alpha \quad \text{then} \quad \text{reject } H_0$$

$$\text{If} \quad p\text{-value} \geq \alpha \quad \text{then} \quad \text{do not reject } H_0$$

Single-sample proportion testing

Necessary conditions: $\hat{p}n \geq 10$ and $(1-\hat{p})n \geq 10$.

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$$

Right tail (one tail)

- H_0 claims $p = p_0$
- H_A claims $p > p_0$
- p-value $= P(Z > z_0)$

Left tail (one tail)

- H_0 claims $p = p_0$
- H_A claims $p < p_0$
- p-value $= P(Z < z_0)$

Two tail

- H_0 claims $p = p_0$
- H_A claims $p \neq p_0$
- p-value $= P(|Z| > |z_0|)$

Single-sample mean testing, σ known

$$z_0 = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

Right tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu > \mu_0$
- p-value $= P(Z > z_0)$

Left tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu < \mu_0$
- p-value $= P(Z < z_0)$

Two tail

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu \neq \mu_0$
- p-value $= P(|Z| > |z_0|)$

Single-sample mean testing, σ unknown

$$df = n - 1 \qquad\qquad t_0 = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Right tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu > \mu_0$
- p-value $= P(T > t_0)$

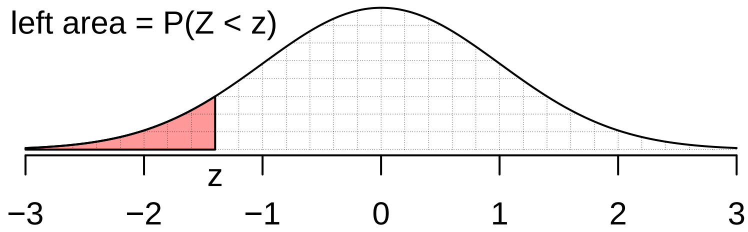
Left tail (one tail)

- H_0 claims $\mu = \mu_0$
- H_A claims $\mu < \mu_0$
- p-value $= P(T < t_0)$

Two tail

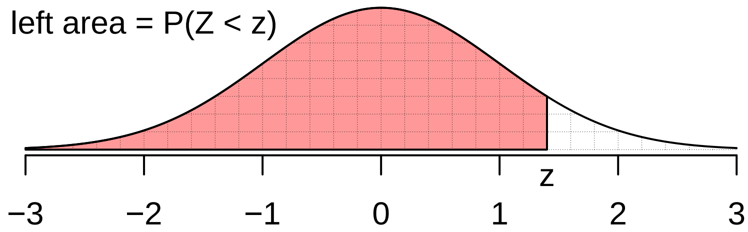
- H_0 claims $\mu = \mu_0$
- H_A claims $\mu \neq \mu_0$
- p-value $= P(|T| > |t_0|)$

z-table



z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$
-3.00	0.0013	-2.50	0.0062	-2.00	0.0228	-1.50	0.0668	-1.00	0.1587	-0.50	0.3085
-2.99	0.0014	-2.49	0.0064	-1.99	0.0233	-1.49	0.0681	-0.99	0.1611	-0.49	0.3121
-2.98	0.0014	-2.48	0.0066	-1.98	0.0239	-1.48	0.0694	-0.98	0.1635	-0.48	0.3156
-2.97	0.0015	-2.47	0.0068	-1.97	0.0244	-1.47	0.0708	-0.97	0.1660	-0.47	0.3192
-2.96	0.0015	-2.46	0.0069	-1.96	0.0250	-1.46	0.0721	-0.96	0.1685	-0.46	0.3228
-2.95	0.0016	-2.45	0.0071	-1.95	0.0256	-1.45	0.0735	-0.95	0.1711	-0.45	0.3264
-2.94	0.0016	-2.44	0.0073	-1.94	0.0262	-1.44	0.0749	-0.94	0.1736	-0.44	0.3300
-2.93	0.0017	-2.43	0.0075	-1.93	0.0268	-1.43	0.0764	-0.93	0.1762	-0.43	0.3336
-2.92	0.0018	-2.42	0.0078	-1.92	0.0274	-1.42	0.0778	-0.92	0.1788	-0.42	0.3372
-2.91	0.0018	-2.41	0.0080	-1.91	0.0281	-1.41	0.0793	-0.91	0.1814	-0.41	0.3409
-2.90	0.0019	-2.40	0.0082	-1.90	0.0287	-1.40	0.0808	-0.90	0.1841	-0.40	0.3446
-2.89	0.0019	-2.39	0.0084	-1.89	0.0294	-1.39	0.0823	-0.89	0.1867	-0.39	0.3483
-2.88	0.0020	-2.38	0.0087	-1.88	0.0301	-1.38	0.0838	-0.88	0.1894	-0.38	0.3520
-2.87	0.0021	-2.37	0.0089	-1.87	0.0307	-1.37	0.0853	-0.87	0.1922	-0.37	0.3557
-2.86	0.0021	-2.36	0.0091	-1.86	0.0314	-1.36	0.0869	-0.86	0.1949	-0.36	0.3594
-2.85	0.0022	-2.35	0.0094	-1.85	0.0322	-1.35	0.0885	-0.85	0.1977	-0.35	0.3632
-2.84	0.0023	-2.34	0.0096	-1.84	0.0329	-1.34	0.0901	-0.84	0.2005	-0.34	0.3669
-2.83	0.0023	-2.33	0.0099	-1.83	0.0336	-1.33	0.0918	-0.83	0.2033	-0.33	0.3707
-2.82	0.0024	-2.32	0.0102	-1.82	0.0344	-1.32	0.0934	-0.82	0.2061	-0.32	0.3745
-2.81	0.0025	-2.31	0.0104	-1.81	0.0351	-1.31	0.0951	-0.81	0.2090	-0.31	0.3783
-2.80	0.0026	-2.30	0.0107	-1.80	0.0359	-1.30	0.0968	-0.80	0.2119	-0.30	0.3821
-2.79	0.0026	-2.29	0.0110	-1.79	0.0367	-1.29	0.0985	-0.79	0.2148	-0.29	0.3859
-2.78	0.0027	-2.28	0.0113	-1.78	0.0375	-1.28	0.1003	-0.78	0.2177	-0.28	0.3897
-2.77	0.0028	-2.27	0.0116	-1.77	0.0384	-1.27	0.1020	-0.77	0.2206	-0.27	0.3936
-2.76	0.0029	-2.26	0.0119	-1.76	0.0392	-1.26	0.1038	-0.76	0.2236	-0.26	0.3974
-2.75	0.0030	-2.25	0.0122	-1.75	0.0401	-1.25	0.1056	-0.75	0.2266	-0.25	0.4013
-2.74	0.0031	-2.24	0.0125	-1.74	0.0409	-1.24	0.1075	-0.74	0.2296	-0.24	0.4052
-2.73	0.0032	-2.23	0.0129	-1.73	0.0418	-1.23	0.1093	-0.73	0.2327	-0.23	0.4090
-2.72	0.0033	-2.22	0.0132	-1.72	0.0427	-1.22	0.1112	-0.72	0.2358	-0.22	0.4129
-2.71	0.0034	-2.21	0.0136	-1.71	0.0436	-1.21	0.1131	-0.71	0.2389	-0.21	0.4168
-2.70	0.0035	-2.20	0.0139	-1.70	0.0446	-1.20	0.1151	-0.70	0.2420	-0.20	0.4207
-2.69	0.0036	-2.19	0.0143	-1.69	0.0455	-1.19	0.1170	-0.69	0.2451	-0.19	0.4247
-2.68	0.0037	-2.18	0.0146	-1.68	0.0465	-1.18	0.1190	-0.68	0.2483	-0.18	0.4286
-2.67	0.0038	-2.17	0.0150	-1.67	0.0475	-1.17	0.1210	-0.67	0.2514	-0.17	0.4325
-2.66	0.0039	-2.16	0.0154	-1.66	0.0485	-1.16	0.1230	-0.66	0.2546	-0.16	0.4364
-2.65	0.0040	-2.15	0.0158	-1.65	0.0495	-1.15	0.1251	-0.65	0.2578	-0.15	0.4404
-2.64	0.0041	-2.14	0.0162	-1.64	0.0505	-1.14	0.1271	-0.64	0.2611	-0.14	0.4443
-2.63	0.0043	-2.13	0.0166	-1.63	0.0516	-1.13	0.1292	-0.63	0.2643	-0.13	0.4483
-2.62	0.0044	-2.12	0.0170	-1.62	0.0526	-1.12	0.1314	-0.62	0.2676	-0.12	0.4522
-2.61	0.0045	-2.11	0.0174	-1.61	0.0537	-1.11	0.1335	-0.61	0.2709	-0.11	0.4562
-2.60	0.0047	-2.10	0.0179	-1.60	0.0548	-1.10	0.1357	-0.60	0.2743	-0.10	0.4602
-2.59	0.0048	-2.09	0.0183	-1.59	0.0559	-1.09	0.1379	-0.59	0.2776	-0.09	0.4641
-2.58	0.0049	-2.08	0.0188	-1.58	0.0571	-1.08	0.1401	-0.58	0.2810	-0.08	0.4681
-2.57	0.0051	-2.07	0.0192	-1.57	0.0582	-1.07	0.1423	-0.57	0.2843	-0.07	0.4721
-2.56	0.0052	-2.06	0.0197	-1.56	0.0594	-1.06	0.1446	-0.56	0.2877	-0.06	0.4761
-2.55	0.0054	-2.05	0.0202	-1.55	0.0606	-1.05	0.1469	-0.55	0.2912	-0.05	0.4801
-2.54	0.0055	-2.04	0.0207	-1.54	0.0618	-1.04	0.1492	-0.54	0.2946	-0.04	0.4840
-2.53	0.0057	-2.03	0.0212	-1.53	0.0630	-1.03	0.1515	-0.53	0.2981	-0.03	0.4880
-2.52	0.0059	-2.02	0.0217	-1.52	0.0643	-1.02	0.1539	-0.52	0.3015	-0.02	0.4920
-2.51	0.0060	-2.01	0.0222	-1.51	0.0655	-1.01	0.1562	-0.51	0.3050	-0.01	0.4960
-2.50	0.0062	-2.00	0.0228	-1.50	0.0668	-1.00	0.1587	-0.50	0.3085	0.00	0.5000

z-table



z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$	z	$P(Z < z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772	2.50	0.9938
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.01	0.9778	2.51	0.9940
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.02	0.9783	2.52	0.9941
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.03	0.9788	2.53	0.9943
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.04	0.9793	2.54	0.9945
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.05	0.9798	2.55	0.9946
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.06	0.9803	2.56	0.9948
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.07	0.9808	2.57	0.9949
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.08	0.9812	2.58	0.9951
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.09	0.9817	2.59	0.9952
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.10	0.9821	2.60	0.9953
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463	2.11	0.9826	2.61	0.9955
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474	2.12	0.9830	2.62	0.9956
0.13	0.5517	0.63	0.7357	1.13	0.8708	1.63	0.9484	2.13	0.9834	2.63	0.9957
0.14	0.5557	0.64	0.7389	1.14	0.8729	1.64	0.9495	2.14	0.9838	2.64	0.9959
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.15	0.9842	2.65	0.9960
0.16	0.5636	0.66	0.7454	1.16	0.8770	1.66	0.9515	2.16	0.9846	2.66	0.9961
0.17	0.5675	0.67	0.7486	1.17	0.8790	1.67	0.9525	2.17	0.9850	2.67	0.9962
0.18	0.5714	0.68	0.7517	1.18	0.8810	1.68	0.9535	2.18	0.9854	2.68	0.9963
0.19	0.5753	0.69	0.7549	1.19	0.8830	1.69	0.9545	2.19	0.9857	2.69	0.9964
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.20	0.9861	2.70	0.9965
0.21	0.5832	0.71	0.7611	1.21	0.8869	1.71	0.9564	2.21	0.9864	2.71	0.9966
0.22	0.5871	0.72	0.7642	1.22	0.8888	1.72	0.9573	2.22	0.9868	2.72	0.9967
0.23	0.5910	0.73	0.7673	1.23	0.8907	1.73	0.9582	2.23	0.9871	2.73	0.9968
0.24	0.5948	0.74	0.7704	1.24	0.8925	1.74	0.9591	2.24	0.9875	2.74	0.9969
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.25	0.9878	2.75	0.9970
0.26	0.6026	0.76	0.7764	1.26	0.8962	1.76	0.9608	2.26	0.9881	2.76	0.9971
0.27	0.6064	0.77	0.7794	1.27	0.8980	1.77	0.9616	2.27	0.9884	2.77	0.9972
0.28	0.6103	0.78	0.7823	1.28	0.8997	1.78	0.9625	2.28	0.9887	2.78	0.9973
0.29	0.6141	0.79	0.7852	1.29	0.9015	1.79	0.9633	2.29	0.9890	2.79	0.9974
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.30	0.9893	2.80	0.9974
0.31	0.6217	0.81	0.7910	1.31	0.9049	1.81	0.9649	2.31	0.9896	2.81	0.9975
0.32	0.6255	0.82	0.7939	1.32	0.9066	1.82	0.9656	2.32	0.9898	2.82	0.9976
0.33	0.6293	0.83	0.7967	1.33	0.9082	1.83	0.9664	2.33	0.9901	2.83	0.9977
0.34	0.6331	0.84	0.7995	1.34	0.9099	1.84	0.9671	2.34	0.9904	2.84	0.9977
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678	2.35	0.9906	2.85	0.9978
0.36	0.6406	0.86	0.8051	1.36	0.9131	1.86	0.9686	2.36	0.9909	2.86	0.9979
0.37	0.6443	0.87	0.8078	1.37	0.9147	1.87	0.9693	2.37	0.9911	2.87	0.9979
0.38	0.6480	0.88	0.8106	1.38	0.9162	1.88	0.9699	2.38	0.9913	2.88	0.9980
0.39	0.6517	0.89	0.8133	1.39	0.9177	1.89	0.9706	2.39	0.9916	2.89	0.9981
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713	2.40	0.9918	2.90	0.9981
0.41	0.6591	0.91	0.8186	1.41	0.9207	1.91	0.9719	2.41	0.9920	2.91	0.9982
0.42	0.6628	0.92	0.8212	1.42	0.9222	1.92	0.9726	2.42	0.9922	2.92	0.9982
0.43	0.6664	0.93	0.8238	1.43	0.9236	1.93	0.9732	2.43	0.9925	2.93	0.9983
0.44	0.6700	0.94	0.8264	1.44	0.9251	1.94	0.9738	2.44	0.9927	2.94	0.9984
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744	2.45	0.9929	2.95	0.9984
0.46	0.6772	0.96	0.8315	1.46	0.9279	1.96	0.9750	2.46	0.9931	2.96	0.9985
0.47	0.6808	0.97	0.8340	1.47	0.9292	1.97	0.9756	2.47	0.9932	2.97	0.9985
0.48	0.6844	0.98	0.8365	1.48	0.9306	1.98	0.9761	2.48	0.9934	2.98	0.9986
0.49	0.6879	0.99	0.8389	1.49	0.9319	1.99	0.9767	2.49	0.9936	2.99	0.9986
0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772	2.50	0.9938	3.00	0.9987

T-table

	$P(T < t) =$	0.9	0.95	0.975	0.98	0.99	0.995	0.9975	0.998	0.999
	$P(T > t) =$	0.1	0.05	0.025	0.02	0.01	0.005	0.0025	0.002	0.001
	$P(\|T\| < t) =$	0.8	0.9	0.95	0.96	0.98	0.99	0.995	0.996	0.998
	$P(\|T\| > t) =$	0.2	0.1	0.05	0.04	0.02	0.01	0.005	0.004	0.002
df=	$t =$									
1		3.08	6.31	12.71	15.89	31.82	63.66	127.32	159.15	318.31
2		1.89	2.92	4.30	4.85	6.96	9.92	14.09	15.76	22.33
3		1.64	2.35	3.18	3.48	4.54	5.84	7.45	8.05	10.21
4		1.53	2.13	2.78	3.00	3.75	4.60	5.60	5.95	7.17
5		1.48	2.02	2.57	2.76	3.36	4.03	4.77	5.03	5.89
6		1.44	1.94	2.45	2.61	3.14	3.71	4.32	4.52	5.21
7		1.41	1.89	2.36	2.52	3.00	3.50	4.03	4.21	4.79
8		1.40	1.86	2.31	2.45	2.90	3.36	3.83	3.99	4.50
9		1.38	1.83	2.26	2.40	2.82	3.25	3.69	3.83	4.30
10		1.37	1.81	2.23	2.36	2.76	3.17	3.58	3.72	4.14
11		1.36	1.80	2.20	2.33	2.72	3.11	3.50	3.62	4.02
12		1.36	1.78	2.18	2.30	2.68	3.05	3.43	3.55	3.93
13		1.35	1.77	2.16	2.28	2.65	3.01	3.37	3.49	3.85
14		1.35	1.76	2.14	2.26	2.62	2.98	3.33	3.44	3.79
15		1.34	1.75	2.13	2.25	2.60	2.95	3.29	3.39	3.73
16		1.34	1.75	2.12	2.24	2.58	2.92	3.25	3.36	3.69
17		1.33	1.74	2.11	2.22	2.57	2.90	3.22	3.33	3.65
18		1.33	1.73	2.10	2.21	2.55	2.88	3.20	3.30	3.61
19		1.33	1.73	2.09	2.20	2.54	2.86	3.17	3.27	3.58
20		1.33	1.72	2.09	2.20	2.53	2.85	3.15	3.25	3.55
21		1.32	1.72	2.08	2.19	2.52	2.83	3.14	3.23	3.53
22		1.32	1.72	2.07	2.18	2.51	2.82	3.12	3.21	3.50
23		1.32	1.71	2.07	2.18	2.50	2.81	3.10	3.20	3.48
24		1.32	1.71	2.06	2.17	2.49	2.80	3.09	3.18	3.47
25		1.32	1.71	2.06	2.17	2.49	2.79	3.08	3.17	3.45
26		1.31	1.71	2.06	2.16	2.48	2.78	3.07	3.16	3.43
27		1.31	1.70	2.05	2.16	2.47	2.77	3.06	3.15	3.42
28		1.31	1.70	2.05	2.15	2.47	2.76	3.05	3.14	3.41
29		1.31	1.70	2.05	2.15	2.46	2.76	3.04	3.13	3.40
30		1.31	1.70	2.04	2.15	2.46	2.75	3.03	3.12	3.39
31		1.31	1.70	2.04	2.14	2.45	2.74	3.02	3.11	3.37
32		1.31	1.69	2.04	2.14	2.45	2.74	3.01	3.10	3.37
33		1.31	1.69	2.03	2.14	2.44	2.73	3.01	3.09	3.36
34		1.31	1.69	2.03	2.14	2.44	2.73	3.00	3.09	3.35
35		1.31	1.69	2.03	2.13	2.44	2.72	3.00	3.08	3.34
36		1.31	1.69	2.03	2.13	2.43	2.72	2.99	3.08	3.33
37		1.30	1.69	2.03	2.13	2.43	2.72	2.99	3.07	3.33
38		1.30	1.69	2.02	2.13	2.43	2.71	2.98	3.06	3.32
39		1.30	1.68	2.02	2.12	2.43	2.71	2.98	3.06	3.31
40		1.30	1.68	2.02	2.12	2.42	2.70	2.97	3.05	3.31

T-table

	$P(T < t) =$	0.9	0.95	0.975	0.98	0.99	0.995	0.9975	0.998	0.999
	$P(T > t) =$	0.1	0.05	0.025	0.02	0.01	0.005	0.0025	0.002	0.001
	$P(\|T\| < t) =$	0.8	0.9	0.95	0.96	0.98	0.99	0.995	0.996	0.998
	$P(\|T\| > t) =$	0.2	0.1	0.05	0.04	0.02	0.01	0.005	0.004	0.002
df=	$t =$									
41		1.30	1.68	2.02	2.12	2.42	2.70	2.97	3.05	3.30
42		1.30	1.68	2.02	2.12	2.42	2.70	2.96	3.05	3.30
43		1.30	1.68	2.02	2.12	2.42	2.70	2.96	3.04	3.29
44		1.30	1.68	2.02	2.12	2.41	2.69	2.96	3.04	3.29
45		1.30	1.68	2.01	2.12	2.41	2.69	2.95	3.03	3.28
46		1.30	1.68	2.01	2.11	2.41	2.69	2.95	3.03	3.28
47		1.30	1.68	2.01	2.11	2.41	2.68	2.95	3.03	3.27
48		1.30	1.68	2.01	2.11	2.41	2.68	2.94	3.02	3.27
49		1.30	1.68	2.01	2.11	2.40	2.68	2.94	3.02	3.27
50		1.30	1.68	2.01	2.11	2.40	2.68	2.94	3.02	3.26
55		1.30	1.67	2.00	2.10	2.40	2.67	2.92	3.00	3.25
60		1.30	1.67	2.00	2.10	2.39	2.66	2.91	2.99	3.23
65		1.29	1.67	2.00	2.10	2.39	2.65	2.91	2.98	3.22
70		1.29	1.67	1.99	2.09	2.38	2.65	2.90	2.98	3.21
75		1.29	1.67	1.99	2.09	2.38	2.64	2.89	2.97	3.20
80		1.29	1.66	1.99	2.09	2.37	2.64	2.89	2.96	3.20
85		1.29	1.66	1.99	2.09	2.37	2.63	2.88	2.96	3.19
90		1.29	1.66	1.99	2.08	2.37	2.63	2.88	2.95	3.18
95		1.29	1.66	1.99	2.08	2.37	2.63	2.87	2.95	3.18
100		1.29	1.66	1.98	2.08	2.36	2.63	2.87	2.95	3.17
125		1.29	1.66	1.98	2.08	2.36	2.62	2.86	2.93	3.16
150		1.29	1.66	1.98	2.07	2.35	2.61	2.85	2.92	3.15
175		1.29	1.65	1.97	2.07	2.35	2.60	2.84	2.92	3.14
200		1.29	1.65	1.97	2.07	2.35	2.60	2.84	2.91	3.13
225		1.29	1.65	1.97	2.07	2.34	2.60	2.83	2.91	3.13
250		1.28	1.65	1.97	2.06	2.34	2.60	2.83	2.91	3.12
275		1.28	1.65	1.97	2.06	2.34	2.59	2.83	2.90	3.12
300		1.28	1.65	1.97	2.06	2.34	2.59	2.83	2.90	3.12
325		1.28	1.65	1.97	2.06	2.34	2.59	2.83	2.90	3.12
350		1.28	1.65	1.97	2.06	2.34	2.59	2.82	2.90	3.11
375		1.28	1.65	1.97	2.06	2.34	2.59	2.82	2.90	3.11
400		1.28	1.65	1.97	2.06	2.34	2.59	2.82	2.89	3.11
425		1.28	1.65	1.97	2.06	2.34	2.59	2.82	2.89	3.11
450		1.28	1.65	1.97	2.06	2.33	2.59	2.82	2.89	3.11
475		1.28	1.65	1.96	2.06	2.33	2.59	2.82	2.89	3.11
500		1.28	1.65	1.96	2.06	2.33	2.59	2.82	2.89	3.11
∞		1.28	1.64	1.96	2.05	2.33	2.58	2.81	2.88	3.09