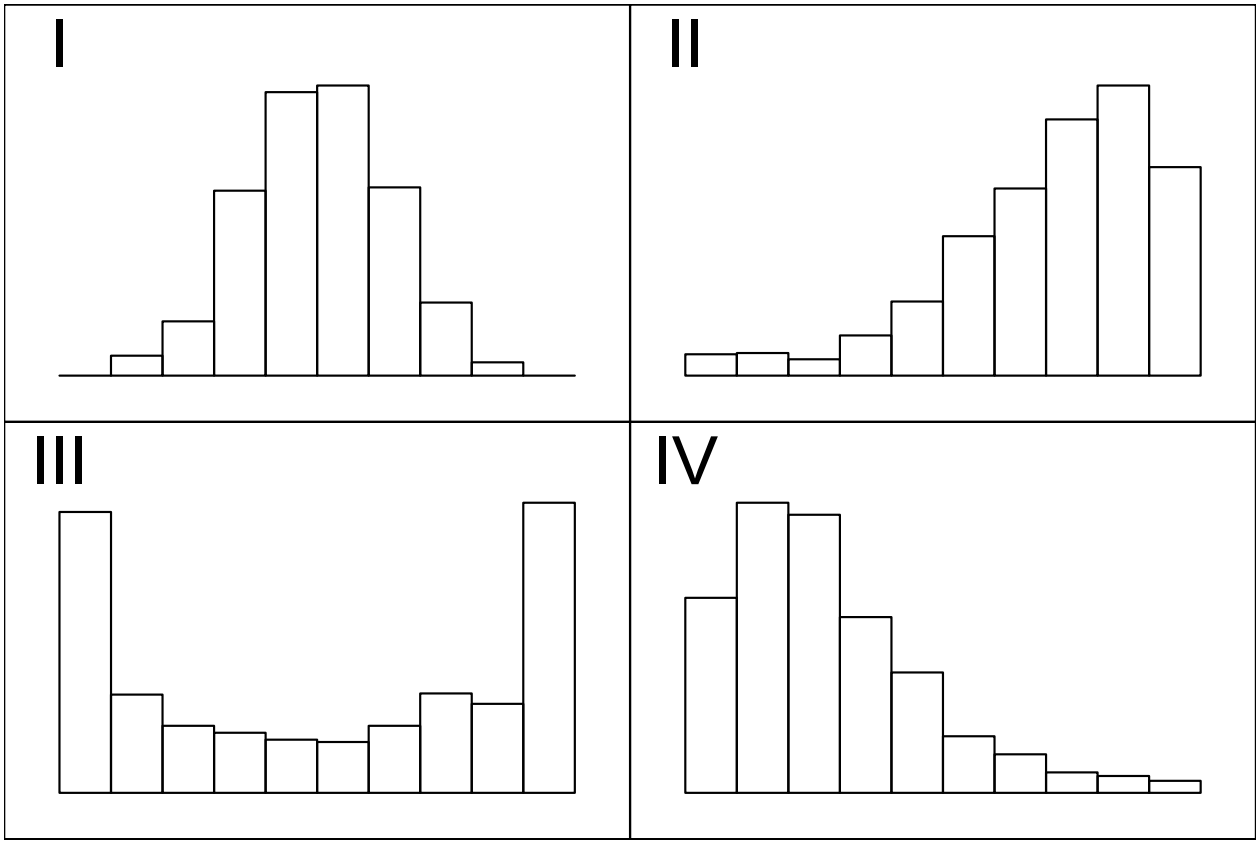
MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondance with any human. Please show work!

question	available points	earned points
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	15	
8	20	
EC	5	
EC	5	
Total	100	

1. (10 Points)



For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

(a) The distribution of hours that students studied for an exam when about half of students studied a lot and a similar number of students studied very little.

(b) The distribution of quiz scores on an easy quiz. Most students did very well, but a few did poorly.

(c) The distribution of heights of adult women

(d) The distribution of test scores on a very difficult exam, in which most students have poor to average scores, but a few did quite well.

2. (15 Points)

In a deck of strange cards, there are 752 cards. Each card has an image and a color. The amounts are shown in the table below.

	gray	indigo	orange	teal	yellow	Total
cat	14	26	35	25	33	133
lamp	24	37	48	18	42	169
needle	15	41	10	22	13	101
shovel	28	27	40	50	36	181
wheel	31	45	19	34	39	168
Total	112	176	152	149	163	752

(a) What is the probability a random card is either a wheel or orange (or both)?

(b) What is the probability a random card is orange?

(c) What is the probability a random card is both a lamp and gray?

(d) What is the probability a random card is gray given it is a shovel?

(e) What is the probability a random card is a lamp given it is indigo?

(f) Is a needle or a wheel more likely to be indigo?

(g) What is the probability a random card is a lamp?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	89	7
B	115	5
C	90	14
D	133	9

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	88.16
B	119.9
C	101.6
D	127.3

Which specimen is the most unusually small (relative to others of its type)?

4. (10 points)

   A tree's leaves were found to be normally distributed with a mean of 151.4 millimeters and a standard deviation of 9.3 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 129.2 and 145.3 millimeters?

5. (10 points)

   A species of duck is known to have a mean weight of 133.9 grams and a standard deviation of 48 grams. A researcher plans to measure the weights of 144 of these ducks sampled randomly. What is the probability the **sample mean** will be between 132.9 and 134.9 grams?

6. (10 points)

   An ornithologist wishes to characterize the average body mass of *Melospiza georgiana*. She randomly samples 28 adults of *Melospiza georgiana*, resulting in a sample mean of 13.66 grams and a sample standard deviation of 1.07 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

   A student is taking a multiple choice test with 1000 questions. Each question has 5 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

   Then, the student takes the test and gets 224 questions correct.

   (a) What kind of hypothesis test is appropriate?

   (b) State the hypotheses.

   (c) Determine the test statistic (z or t), draw a sketch, and determine the p-value.

   (d) Decide whether we reject or retain the null hypothesis.

   (e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

   You have collected the following data:

x	y	xy
7.7	45	
6.3	23	
2	58	
4.1	65	
2.7	97	
8.2	44	
8.5	36	
$\sum x =$	$\sum y =$	$\sum xy =$
$\bar{x} =$	$\bar{y} =$	
$s_x =$	$s_y =$	

   (a) Complete the table.

   (b) Calculate the correlation coefficient (r) using the formula below.

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

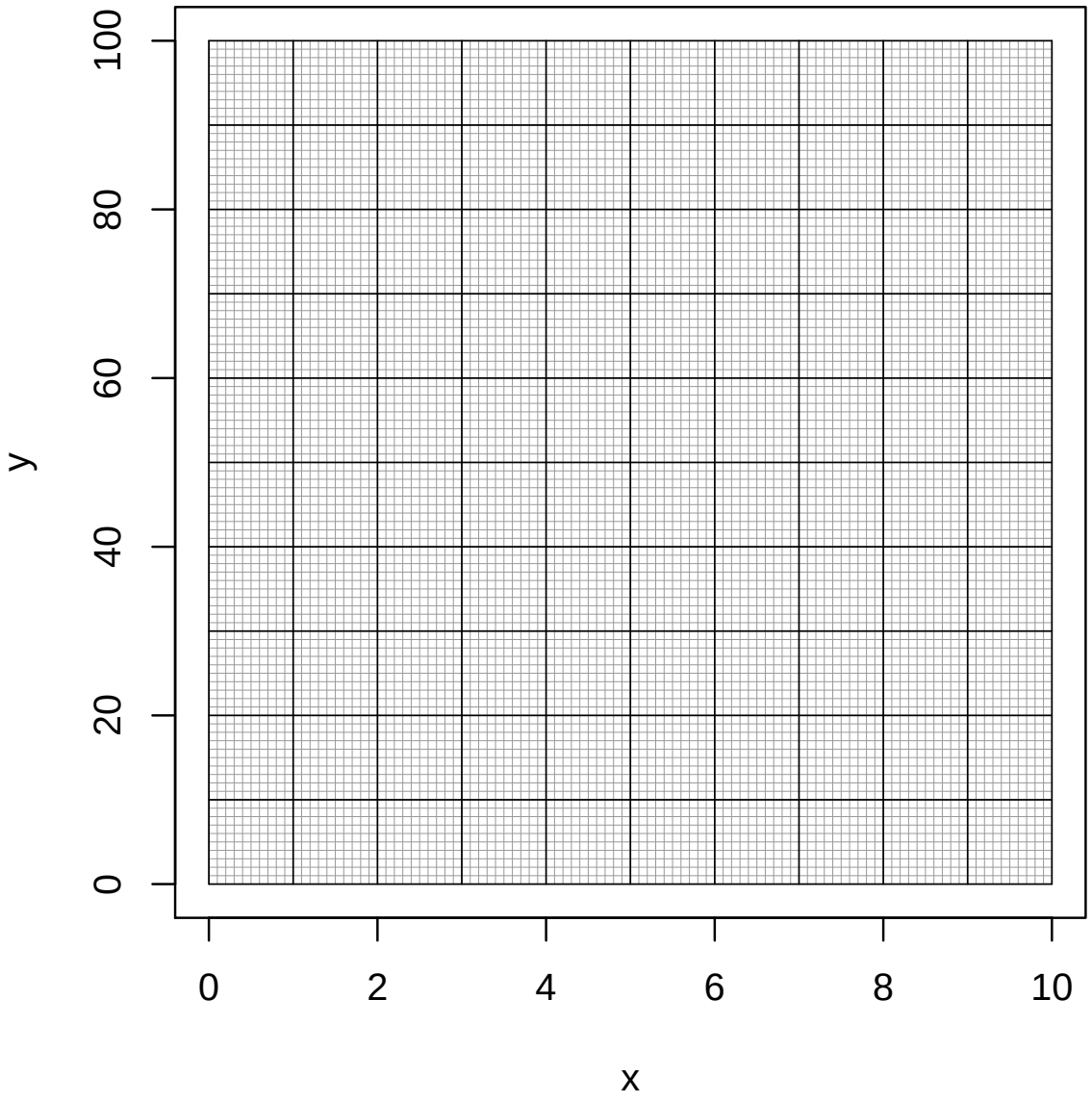
   (c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r\frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

   (d) Write the equation of the regression line (using the calculated values of a and b.)

(e) Please plot the data and a corresponding regression line.



9. (Extra credit: 5 points)

   Let each trial have a chance of success $p = 0.92$. If 164 trials occur, what is the probability of getting at least 147 but at most 155 successes?

   In other words, let $X \sim \text{Bin}(n = 164,\ p = 0.92)$ and find $P(147 \leq X \leq 155)$.

   Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

    A null hypothesis claims a population has a mean μ = 90. You decide to run two-tail test on a sample of size n = 9 using a significance level α = 0.02.

    You then collect the sample:

92.9	88.3	91.2	92.2	100.4
98.8	102.9	91.7	93.5	

    (a) Determine the *p*-value.

    (b) Do you reject the null hypothesis?