Name: ________________________ **Section: <u>MAT098/181C-</u>**

MAT098/181C EXAM #4 (FORM B Key)

A scientific calculator is permitted. ***Cellphones may not be used as calculators and must be off or on vibrate during the exam****. Show all work on the test or on the work*

1. A random sample of 225 fathers from Cambridge yielded 128 who see parenting as central to their identity. Find a 95% confidence interval for the proportion of fathers in Cambridge who see parenting as central to their identity. (Round to three decimal places) *20 pts*

   a) Determine whether the conditions are met.

$$np, nq \geq 10$$

   b) Construct the 95% confidence interval.

   1. $\hat{p} = 128/225 = 0.569$ $\quad n = 225 \quad Z_c = 1.96$
   2. $np, nq \geq 10$
   3.

$$\hat{p} \pm Z_c \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.569 \pm 0.065 = (0.504, 0.634)$$

   4. We are 95% confident that the interval from 50.4% and 63.4% actually does contain the true value of the population proportion of fathers in Cambridge who see parenting as central to their identity.

2. A researcher wants to estimate the average spending for Mother's Day gifts of Americans this year. The researcher chooses a random sample of 51 people and finds the average spending \$170 with a sample standard deviation of \$25. Construct a 95% confidence interval for the average spending for Mother's Day gifts of Americans this year. (Round final answer to two decimal places) *20 pts*

$$\bar{x} \pm T_c \cdot \frac{s}{\sqrt{n}} = 170 \pm 2.01 \cdot \frac{25}{\sqrt{51}} = 170 \pm 7.04 = (162.96, 177.04)$$

3. How many BHCC students must be randomly selected to estimate the average time spent on their cell phones during class? We want 95% confidence that the sample mean is within 0.5 minutes of the population mean, and the population standard deviation is known to be 6.2 minutes. (1*2 pts)*

$$n = \left(\frac{Z_c \cdot \sigma}{E}\right)^2 = \left(\frac{1.96 \cdot 6.2}{0.5}\right)^2 = 591$$

For the next three problems, state:

1) Null, Alternate Hypothesis, type of test & level of significance
2) Check the conditions.
3) Compute the sample test statistic, draw a picture and find the P-value.
4) State the conclusion about the Null Hypothesis.
5) Interpret the conclusion.

4. Parents' experiences with education strongly influence what their children do after high school. A study shows that 80% of those raised by graduates said their parents encouraged them to attend college. In a random sample of 125 children with graduate parents, 81.6% of them say their parents encouraged them to attend college. Use a 0.05 significant level to test the claim that 80% of graduate parents encourage their children to attend college. (*24 pts)*

$p = 0.8 \qquad \hat{p} = .816 \qquad n = 125 \qquad \alpha = 0.05$

1) H_0: $p = 0.8$
   H_a: $p \neq 0.8$

2) $np \geq 10, nq \geq 10$
3) $$z = \frac{0.816 - 0.8}{\sqrt{\frac{0.8 * 0.2}{125}}} \approx 0.45$$
4) P-value is 0.3264*2 > 0.05. Fail to reject null hypothesis.
5) At a 5% significant level, there is not enough evidence to support the alternative hypothesis that the proportion of graduate parents encourage their children to attend college is not 80%.

5. A study states that Latinos enjoy the 2nd longest life expectancy of any ethnic groups in the U.S. today. They live to 83.5 years on average. A randomly selected sample of 46 Latino Americans reported that their grandparents live to 84.2 years on average. Assume that we know the population standard deviation of life expectancy is 15 years. Use a 0.05 significance level to test the claim that the life expectancy of Latinos is more than 83.5 years. (*24 pts)*

$\mu = 83.5 \quad \bar{x} = 84.2 \quad n = 46 \quad \sigma = 15 \quad \alpha = 0.05$

1) H_0: $\mu = 83.5$
   H_a: $\mu > 83.5$

2) $n > 30$
3) $$z = \frac{84.2 - 83.5}{\frac{15}{\sqrt{46}}} \approx 0.32$$
4) P-value is 0.3745 > 0.05. Fail to reject the null hypothesis.
5) At a 5% significant level, there is not enough evidence to support the alternative hypothesis that the life expectancy of Latinos is more than 83.5 years.

(EXTRA CREDIT) The mean number of absences a student has per semester is believed to be about 4 days. Faculty in a university do not believe this figure. They randomly survey 9 students. The number of absences they took for the last semester are as follows:

0, 0, 1, 3, 4, 4, 5, 5, 6

Let $x =$ the number of absences a student had for the last semester. Assume that x follows a normal distribution. Should the faculty team believe that the mean number is 4 days? (round to one decimal place) *5 pts*

$\mu = 4 \quad \bar{x} = 3.1 \quad n = 9 \quad s = 2.3 \quad \alpha = 0.05$

1) H_0: $\mu = 4$
   H_a: $\mu \neq 4$

2) normal distribution
3) $$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{3.1 - 4}{\frac{2.3}{\sqrt{9}}} \approx -1.17$$
4) P-value is more than 0.2 > 0.05. Fail to reject hypothesis.
5) At a 5% significant level, there is enough not evidence to support the alternative hypothesis that mean number of absence a student has per semester is not 4 days.

Confidence Interval for Population Parameters

<table>
<tr><th>Concept</th><th>Population Proportion p</th><th colspan="2">Population Mean μ</th></tr>
<tr><td>confidence interval formula</td><td>$\hat{p} \pm Z_c \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$</td><td>$\sigma$ known
$\bar{x} \pm Z_c \cdot \frac{\sigma}{\sqrt{n}}$</td><td>$\sigma$ unknown
$\text{df} = n - 1$
$\bar{x} \pm T_c \cdot \frac{s}{\sqrt{n}}$</td></tr>
<tr><td rowspan="2">sample size formula</td><td>$\hat{p} = \frac{x}{n}$ known
$n = \hat{p} \cdot \hat{q} \cdot \left(\frac{Z_c}{E}\right)^2$</td><td colspan="2" rowspan="2">$n = \left(\frac{Z_c \cdot \sigma}{E}\right)^2$</td></tr>
<tr><td>$\hat{p}$ unknown
$n = \frac{1}{4} \cdot \left(\frac{Z_c}{E}\right)^2$</td></tr>
</table>

- 90% confidence interval: $Z_c \approx 1.645$
- 95% confidence interval: $Z_c \approx 1.960$
- 99% confidence interval: $Z_c \approx 2.576$

Hypothesis Testing

<table>
<tr><th>Concept</th><th>Population Proportion p</th><th colspan="2">Population Mean μ</th></tr>
<tr><td>test statistics</td><td>$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$</td><td>$\sigma$ known
$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$</td><td>$\sigma$ unknown
$\text{df} = n - 1$
$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$</td></tr>
</table>

- If the P-value $< \alpha$, we reject the null hypothesis.
- If the P-value $\geq \alpha$, we fail to reject the null hypothesis.