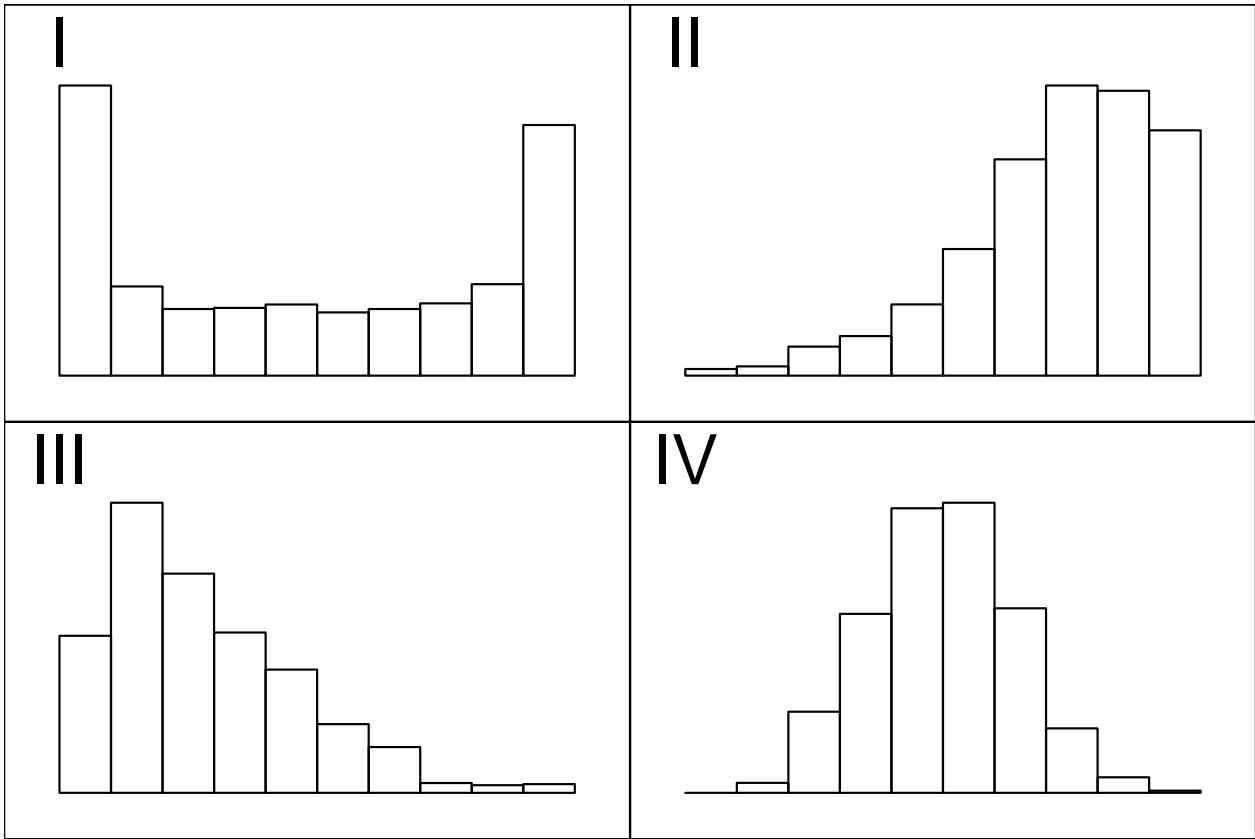


MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondence with any human. Please show work!

| question | available points | earned points |
|----------|------------------|---------------|
| 1 | 10 | |
| 2 | 15 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 15 | |
| 8 | 20 | |
| EC | 5 | |
| EC | 5 | |
| Total | 100 | |

1. (10 Points)



For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

- (a) The distribution of annual income for NBA basketball players where only a few are high-paid superstars.
- (b) The distribution of heights of adult men
- (c) The distribution of hours spent per week reading by adults. In this distribution, many people do not read much, and a similar number of people read a lot.
- (d) The distribution of ages at a skilled nursing facility, where most of the patients are elderly but a few are quite young.

2. (15 Points)

In a deck of strange cards, there are 263 cards. Each card has an image and a color. The amounts are shown in the table below.

| | black | indigo | red | violet | Total |
|-------|-------|--------|-----|--------|-------|
| horn | 11 | 25 | 19 | 17 | 72 |
| kite | 35 | 12 | 46 | 18 | 111 |
| lamp | 13 | 31 | 14 | 22 | 80 |
| Total | 59 | 68 | 79 | 57 | 263 |

(a) What is the probability a random card is either a kite or red (or both)?

(b) What is the probability a random card is a kite given it is red?

(c) What is the probability a random card is both a kite and violet?

(d) What is the probability a random card is violet?

(e) Is a horn or a kite more likely to be indigo?

(f) What is the probability a random card is violet given it is a horn?

(g) What is the probability a random card is a kite?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

| Type of fruit | Mean mass (g) | Standard deviation of mass (g) |
|---------------|---------------|--------------------------------|
| <i>A</i> | 130 | 6 |
| <i>B</i> | 128 | 7 |
| <i>C</i> | 121 | 8 |
| <i>D</i> | 61 | 13 |

One specimen of each type is weighed. The results are shown below.

| Type of fruit | Mass of specimen (g) |
|---------------|----------------------|
| <i>A</i> | 122.4 |
| <i>B</i> | 123.5 |
| <i>C</i> | 113.5 |
| <i>D</i> | 74 |

Which specimen is the most unusually far (in either direction) from average (relative to others of its type)?

4. (10 points)

A tree's leaves were found to be normally distributed with a mean of 114.2 millimeters and a standard deviation of 9.3 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 103.2 and 107.1 millimeters?

5. (10 points)

A species of duck is known to have a mean weight of 283.5 grams and a standard deviation of 17.5 grams. A researcher plans to measure the weights of 49 of these ducks sampled randomly. What is the probability the **sample mean** will be between 279 and 284 grams?

6. (10 points)

An ornithologist wishes to characterize the average body mass of *Dendroica pensylvanica*. She randomly samples 19 adults of *Dendroica pensylvanica*, resulting in a sample mean of 12.34 grams and a sample standard deviation of 1.3 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

A student is taking a multiple choice test with 400 questions. Each question has 5 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

Then, the student takes the test and gets 93 questions correct.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses.

(c) Determine the test statistic (z or t), draw a sketch, and determine the p -value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

You have collected the following data:

| x | y | xy |
|-------------|-------------|-------------|
| 61 | 81 | |
| 45 | 28 | |
| 89 | 99 | |
| 72 | 40 | |
| 54 | 46 | |
| 69 | 65 | |
| 24 | 64 | |
| 18 | 38 | |
| $\sum x =$ | $\sum y =$ | $\sum xy =$ |
| $\bar{x} =$ | $\bar{y} =$ | |
| $s_x =$ | $s_y =$ | |

(a) Complete the table.

(b) Calculate the correlation coefficient (r) using the formula below.

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

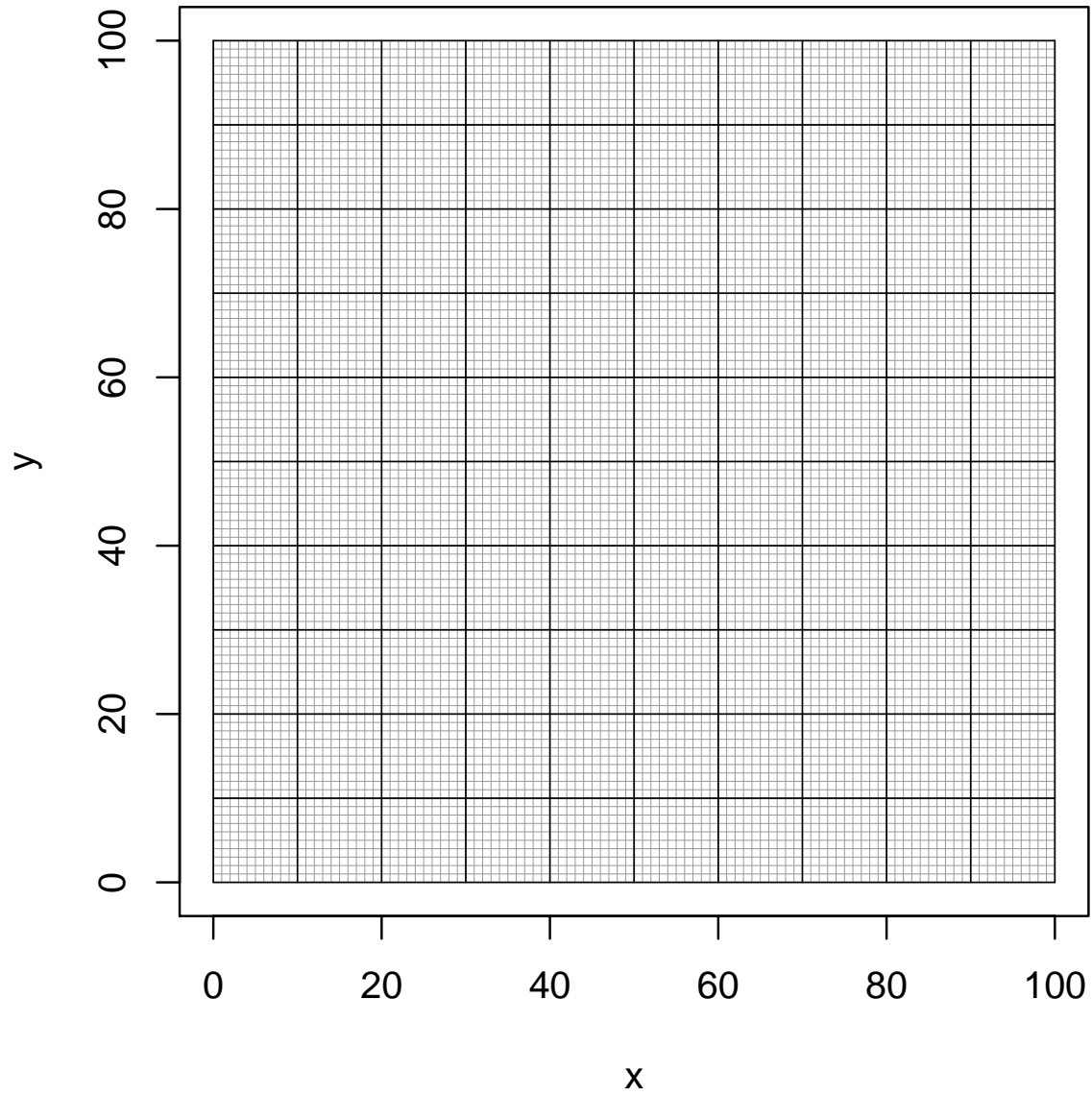
(c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r \frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

(d) Write the equation of the regression line (using the calculated values of a and b .)

(e) Please plot the data and a corresponding regression line.



9. (Extra credit: 5 points)

Let each trial have a chance of success $p = 0.24$. If 113 trials occur, what is the probability of getting more than 21 but at most 34 successes?

In other words, let $X \sim \text{Bin}(n = 113, p = 0.24)$ and find $P(21 < X \leq 34)$.

Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

A null hypothesis claims a population has a mean $\mu = 120$. You decide to run two-tail test on a sample of size $n = 12$ using a significance level $\alpha = 0.02$.

You then collect the sample:

| | | | | |
|-------|-------|-------|-------|-------|
| 121.8 | 122.7 | 141.3 | 130.4 | 129.5 |
| 110.5 | 122.7 | 115.6 | 138 | 130.6 |
| 128.3 | 129.3 | | | |

- (a) Determine the p -value.
- (b) Do you reject the null hypothesis?