Normal standardization

$$z = \frac{x-\mu}{\sigma} \qquad x = \mu + z\sigma$$

Central Limit Theorem

- **If**:
  - W is a random variable with mean μ_w and SD σ_w.
  - Random variable X is **sum** of n instances of W.

$$X = W_1 + W_2 + W_3 + \cdots + W_n$$

  - Random variable Y is **average** of n instances of W.

$$Y = \frac{W_1 + W_2 + W_3 + \cdots + W_n}{n}$$

- **Then**:

$$\mu_x = n\mu_w \qquad \sigma_x = \sigma_w\sqrt{n}$$
$$\mu_y = \mu_w \qquad \sigma_y = \frac{\sigma_w}{\sqrt{n}}$$

  - X and Y are approximately normal (even if W is not normal)

Special case of central limit theorem: Bernoulli, Binomial, and $\hat{p}$ sampling

- **Given:**
  - W is a Bernoulli random variable:

w	$P(w)$
0	q
1	p

  - X is a binomial random variable (sum of n instances of W)
  - $\hat{p}$ is a sample proportion (average of n instances of W)
- **Therefore:**

$$\mu_w = p \qquad \sigma_w = \sqrt{pq}$$
$$\mu_x = np \qquad \sigma_x = \sqrt{pq}\sqrt{n}$$
$$\mu_{\hat{p}} = p \qquad \sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}}$$

Question 1

A farm produces 4 types of fruit: A, B, C, and D. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	101	11
B	64	6
C	127	13
D	67	9

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	104.7
B	71.74
C	114.1
D	63.22

Which specimen is the most unusually small (relative to others of its type)?

Question 1 solution

We compare the z-scores. The smallest z-score corresponds to the specimen that is most unusually small.

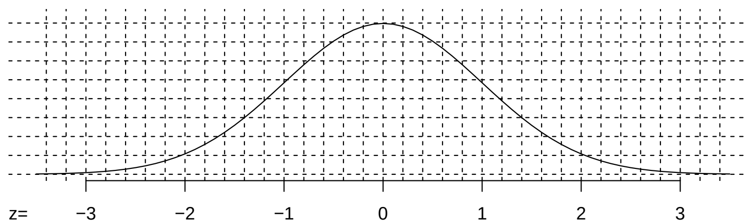
Type of fruit	formula	z-score
A	$z = \frac{104.7-101}{11}$	0.34
B	$z = \frac{71.74-64}{6}$	1.29
C	$z = \frac{114.1-127}{13}$	-0.99
D	$z = \frac{63.22-67}{9}$	-0.42

Thus, the specimen of type C is the most unusually small.

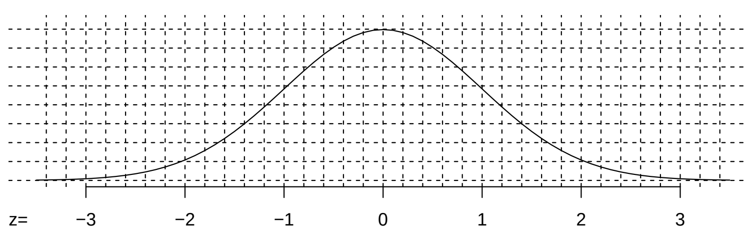
Question 2

Let X be normally distributed with mean 59 and standard deviation 14. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

a. $P(X < 61.8)$

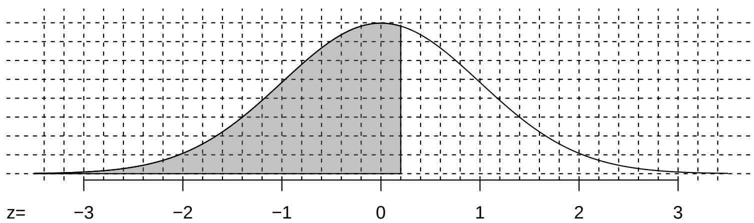


b. $P(X > 46.4)$



Question 2 solution

a. $P(X < 61.8) \; = \; P(Z < 0.2) \; = \; \boxed{0.5793}$



b. $P(X > 46.4) \; = \; P(Z > -0.9) \; = \; \boxed{0.8159}$