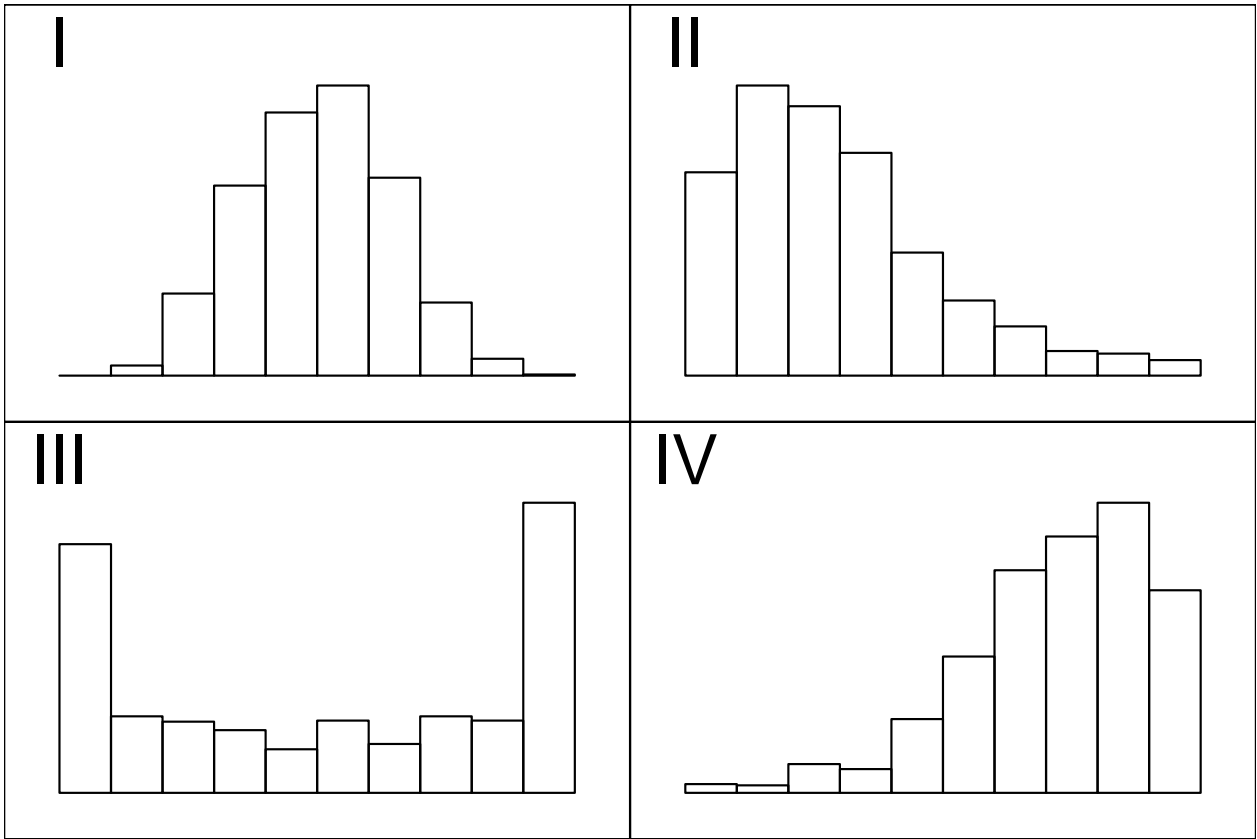## MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondance with any human. Please show work!

question	available points	earned points
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	15	
8	20	
EC	5	
EC	5	
Total	100	

1. (10 Points)

For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

(a) The distribution of weights of newborn babies

(b) The distribution of ages at a skilled nursing facility, where most of the patients are elderly but a few are quite young.

(c) The distribution of hours spent per week reading by adults. In this distribution, many people do not read much, and a similar number of people read a lot.

(d) The distribution of annual income for school employees where a high percentage of employees are entry-level teachers and only a few are high-paid administrators.

2. (15 Points)

In a deck of strange cards, there are 415 cards. Each card has an image and a color. The amounts are shown in the table below.

	blue	gray	green	pink	teal	Total
jigsaw	45	12	10	38	33	138
tree	19	29	30	35	28	141
wheel	21	48	27	24	16	136
Total	85	89	67	97	77	415

(a) What is the probability a random card is both a wheel and green?

(b) Is a jigsaw or a wheel more likely to be blue?

(c) What is the probability a random card is a wheel?

(d) What is the probability a random card is green given it is a wheel?

(e) What is the probability a random card is either a wheel or green (or both)?

(f) What is the probability a random card is a tree given it is pink?

(g) What is the probability a random card is pink?

3. (10 points)

A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	113	5
B	134	4
C	133	13
D	99	7

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	112.2
B	136.6
C	138.7
D	97.11

Which specimen is the most unusually far (in either direction) from average (relative to others of its type)?

4. (10 points)

A tree's leaves were found to be normally distributed with a mean of 69.7 millimeters and a standard deviation of 9.5 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 53.1 and 89.8 millimeters?

5. (10 points)

A species of duck is known to have a mean weight of 171.6 grams and a standard deviation of 72 grams. A researcher plans to measure the weights of 64 of these ducks sampled randomly. What is the probability the **sample mean** will be between 158.1 and 162.6 grams?

6. (10 points)

An ornithologist wishes to characterize the average body mass of *Archilochus colubris*. She randomly samples 14 adults of *Archilochus colubris*, resulting in a sample mean of 3.35 grams and a sample standard deviation of 0.686 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

A student is taking a multiple choice test with 600 questions. Each question has 4 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

Then, the student takes the test and gets 170 questions correct.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses.

(c) Determine the test statistic (z or t), draw a sketch, and determine the p-value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

   You have collected the following data:

x	y	xy
280	68	
890	78	
110	39	
770	77	
480	61	
360	62	
$\sum x =$	$\sum y =$	$\sum xy =$
$\bar{x} =$	$\bar{y} =$	
$s_x =$	$s_y =$	

(a) Complete the table.

(b) Calculate the correlation coefficient (r) using the formula below.
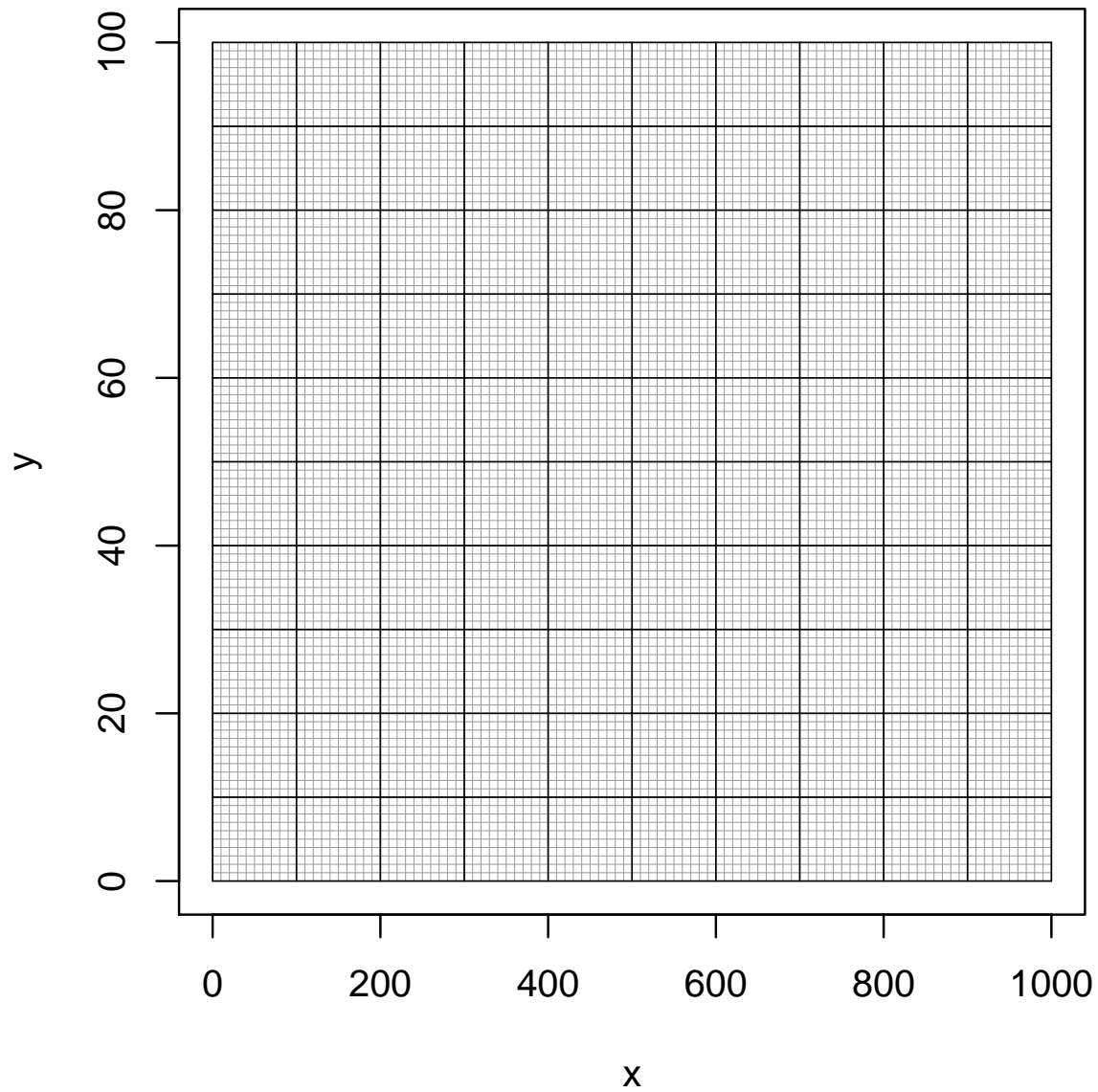
$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

(c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r\frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

(d) Write the equation of the regression line (using the calculated values of a and b.)

(e) Please plot the data and a corresponding regression line.

9. (Extra credit: 5 points)

Let each trial have a chance of success $p = 0.39$. If 75 trials occur, what is the probability of getting more than 21 but less than 36 successes?

In other words, let $X \sim \text{Bin}(n = 75, p = 0.39)$ and find $P(21 < X < 36)$.

Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

   A null hypothesis claims a population has a mean $\mu = 150$. You decide to run two-tail test on a sample of size $n = 9$ using a significance level $\alpha = 0.05$.

   You then collect the sample:

134.7	204.4	157.5	138.7	154.9
216.2	217.3	225.1	162.3	

   (a) Determine the p-value.

   (b) Do you reject the null hypothesis?