Normal standardization

$$z = \frac{x - \mu}{\sigma} \qquad x = \mu + z\sigma$$

Central Limit Theorem

If:

- W is "any" random variable with mean = μ_w and standard deviation = σ_w.
- Random variable X is **sum** of n instances of W.

$$X = W_1 + W_2 + W_3 + \cdots + W_n$$

- Random variable Y is **average** of n instances of W.

$$Y = \frac{W_1 + W_2 + W_3 + \cdots + W_n}{n}$$

Then:

- The following formulas are exactly true:

$$\mu_x = n\mu_w \qquad \mu_y = \mu_w$$
$$\sigma_x = \sigma_w\sqrt{n} \qquad \sigma_y = \frac{\sigma_w}{\sqrt{n}}$$

- X and Y are approximately normal (if $n \geq 30$)
- X and Y are exactly normal if W is normal

Special case of central limit theorem: Bernoulli, Binomial, and $\hat{p}$ sampling

If:

- W is a Bernoulli random variable:

w	$P(w)$
0	q
1	p

- X is sum of n instances of W (X is binomial)
- $\hat{p}$ is average of n instances of W (proportion sampling)

Then:

- The following are exactly true:

$$\mu_w = p \qquad \mu_x = np \qquad \mu_{\hat{p}} = p$$
$$\sigma_w = \sqrt{pq} \qquad \sigma_x = \sqrt{pq}\sqrt{n} \qquad \sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}}$$

- X and $\hat{p}$ are approximately normal (if $np \geq 10$ and $nq \geq 10$)

Question 1

A farm produces 4 types of fruit: A, B, C, and D. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	80 μ	8 σ
B	130	10
C	95	15
D	115	5

One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g) x
A	92
B	142
C	81.5
D	109

a. Determine a z-score for each specimen.

$$Z = \frac{x-\mu}{\sigma}$$

type	Z
A	1.5
B	1.2
C	-0.9
D	-1.2

b. Which specimen was most unusually large?

A

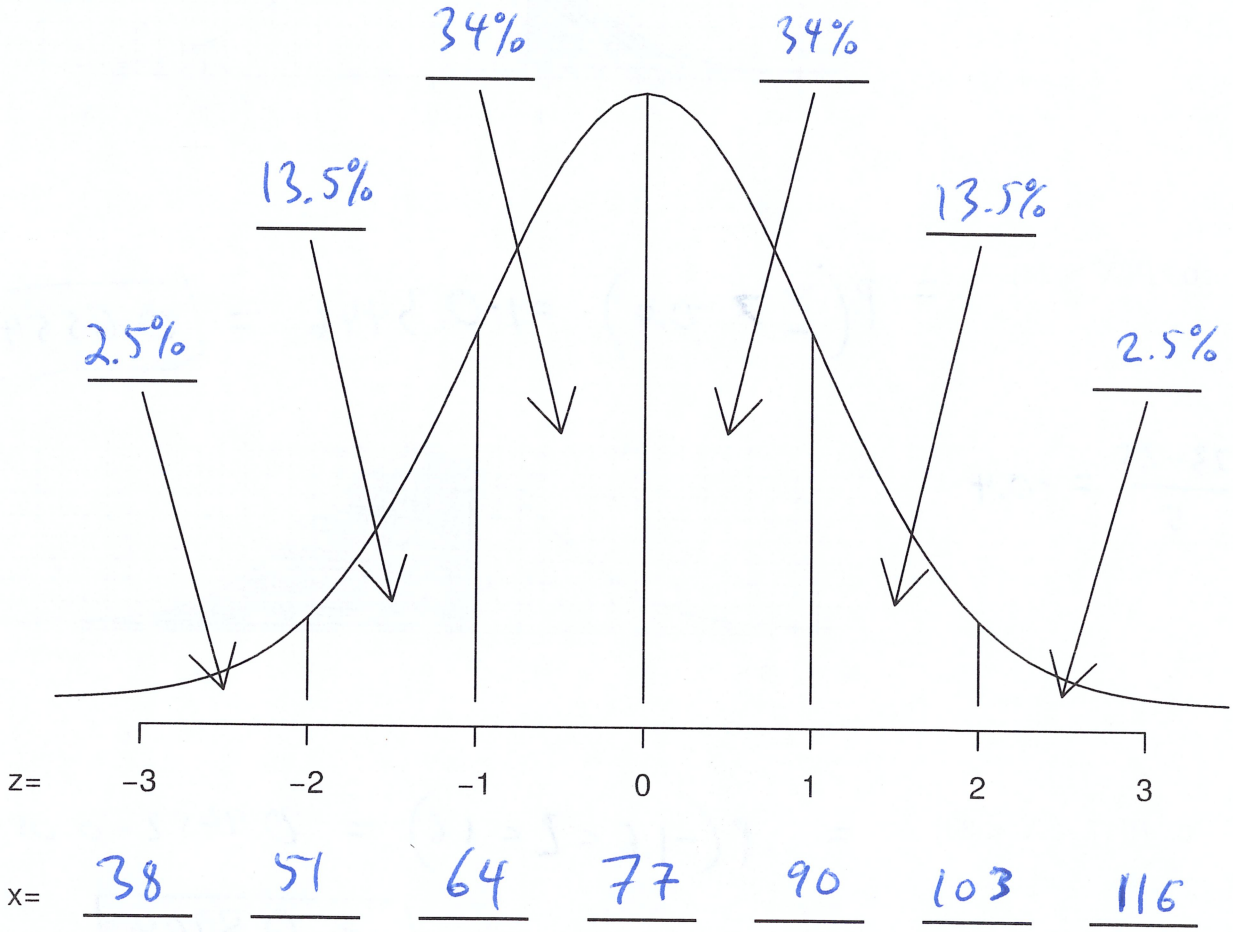
c. Which specimen was most unusually small?

D

Question 2

A normal random variable X has a mean $\mu = 77$ and standard deviation $\sigma = 13$. Please label the density curve with:

a. The appropriate values of x.
b. The areas of the sections.

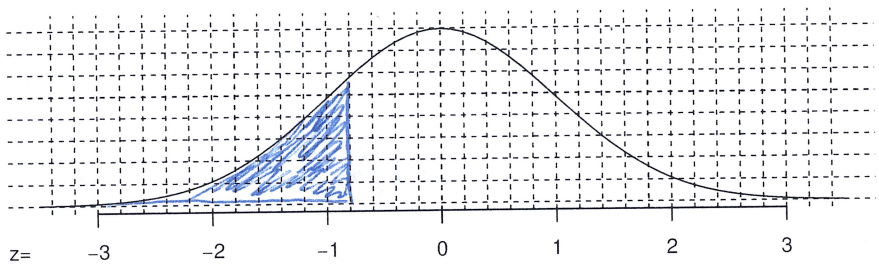


Question 3

Let X be normally distributed with mean 25 and standard deviation 5. Please calculate the probabilities shown below and also shade a corresponding region under the density curve.

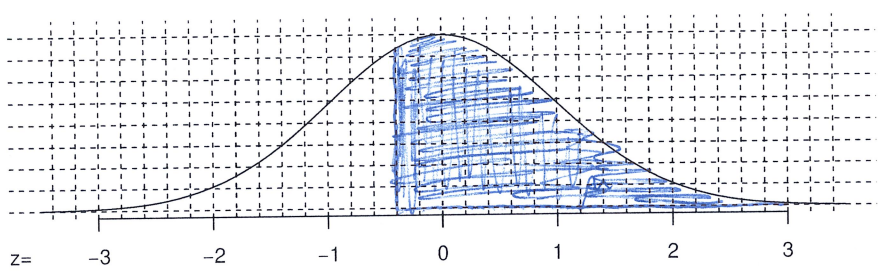
a. $P(X < 21)$ $= P(Z < -0.8) = \boxed{0.2119}$

$Z = \frac{21-25}{5} = -0.8$



b. $P(X > 23)$ $= P(Z > -0.4) = 1 - 0.3446 = \boxed{0.6554}$

$Z = \frac{23-25}{5} = -0.4$

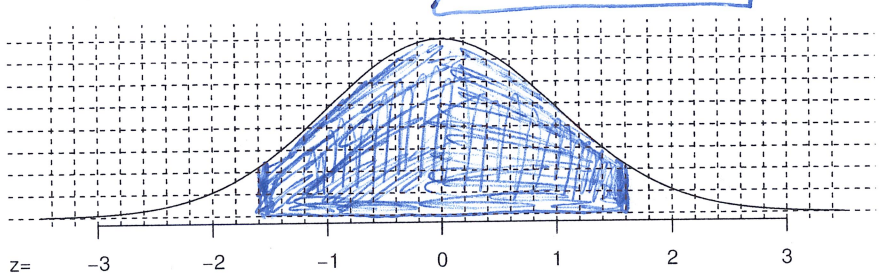


c. $P(17 < X < 33)$ $= P(-1.6 < Z < 1.6) = 0.9452 - 0.0548$

$\boxed{= 0.8904}$

$Z_1 = \frac{17-25}{5} = -1.6$

$Z_2 = \frac{33-25}{5} = 1.6$



$l_1 = 0.0548$

$l_2 = 0.9452$

Question 4

Let random variable W have a **mean of** $\mu_w = 10.50$ and a **standard deviation of** $\sigma_w = 5.77$.

Let X represent the sum of $n = 81$ instances of W, and let Y represent the average of $n = 81$ instances of W.

a. Determine μ_x

$\mu_x = 10.5 \cdot 81 = \boxed{850.5}$

b. Determine σ_x

$\sigma_x = 5.77 \times \sqrt{81} = \boxed{51.93}$

c. What is the probability that X is between 773 and 928? *Do NOT use a continuity correction.*

$Z_{low} = \frac{773 - 850.5}{51.93} = -1.49$

$Z_{high} = \frac{928 - 850.5}{51.93} = 1.49$

$\ell_{low} = 0.0681$

$\ell_{high} = 0.9319$

$P(-1.49 < Z < 1.49) = 0.9319 - 0.0681$

$\boxed{= 0.8638}$

d. Determine μ_y

$\mu_y = \boxed{10.5}$

e. Determine σ_y

$\sigma_y = \frac{5.77}{\sqrt{81}} = \boxed{0.6411}$

f. What is the probability that Y is between 10.2 and 10.8? *Do NOT use a continuity correction.*

$Z_1 = \frac{10.2 - 10.5}{0.6411} = -0.47$

$Z_2 = \frac{10.8 - 10.5}{0.6411} = 0.47$

$\ell_1 = 0.3192$

$\ell_2 = 0.6808$

$P(10.2 < Y < 10.8) = P(-0.47 < Z < 0.47)$

$= 0.6808 - 0.3192$

$\boxed{= 0.3616}$

Question 5

An unfair coin has a $p = 0.63$ chance of landing tails. When $n = 100$ of these unfair coins are flipped, what is the probability of getting at least 55 but at most 71 tails? Please use a **normal approximation** with a **continuity correction**.

upper bound inclusive +0.5

Lower bound inclusive −0.5

Binomial

$$\mu = np = (0.63)(100) = 63$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(0.63)(0.37)} = 4.83$$

$$Z_{low} = \frac{54.5 - 63}{4.83} = -1.76 \qquad l_{low} = 0.0392$$

$$Z_{high} = \frac{71.5 - 63}{4.83} = 1.76 \qquad l_{high} = 0.9608$$

Let X represent # of tails.

$$P(55 \le X \le 71) = P(-1.76 < Z < 1.76)$$

$$= 0.9608 - 0.0392$$

$$\boxed{= 0.9216}$$

notice you could also use

$$P(|Z| < 1.76) = 2 \times 0.9608 - 1 = 0.9216$$

Question 6

About 8% of men are color blind ($p = 0.08$). If you gather a simple random sample of $n = 121$ ~~humans~~ men, what is the probability that the sample proportion ($\hat{p}$) is between 0.03 and 0.07? Please use a **normal approximation**, but *do NOT use a continuity correction.*

proportion sampling

$$\mu_{\hat{p}} = 0.08$$

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.08)(0.92)}{121}} = 0.0247$$

$$Z_{low} = \frac{0.03 - 0.08}{0.0247} = -2.02 \qquad l_{low} = 0.0217$$

$$Z_{high} = \frac{0.07 - 0.08}{0.0247} = -0.40 \qquad l_{high} = 0.3446$$

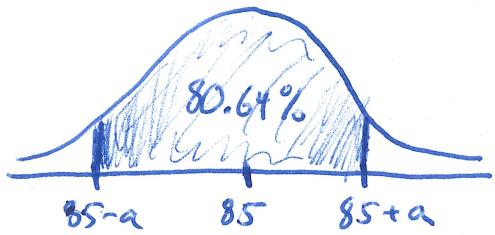
$$P(0.03 < \hat{p} < 0.07) = P(-2.02 < Z < -0.40)$$

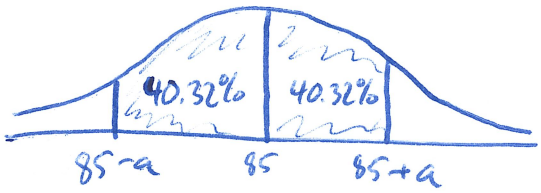
$$= 0.3446 - 0.0217$$

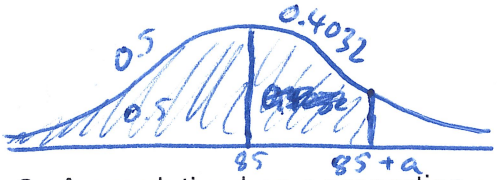
$$= \boxed{0.3229}$$

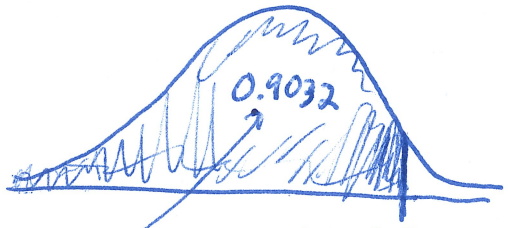
Extra Credit

1. Let random variable X be normally distributed with mean $\mu = 85$ and standard deviation $\sigma = 10$. Determine a such that $P(|X - 85| < a) = 0.8064$.









$x = 85 + a$

use table!

$z = 1.3$

$x = \mu + z\sigma$

$85 + a = 85 + 1.3 \cdot 10$

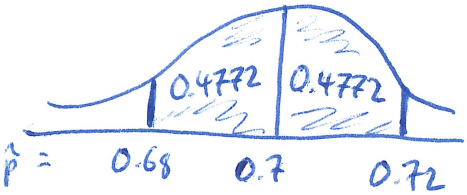
$\boxed{a = 13}$

2. A population has a proportion $p = 0.7$. The sample proportion is $\hat{p}$. Determine n such that $P(0.68 < \hat{p} < 0.72) = 0.9544$. In other words, determine the necessary sample size such that the sample proportion is between 0.68 and 0.72 in 95.44% of random samples of that size.

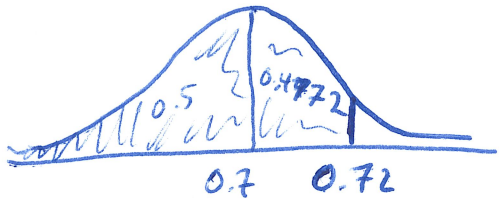
$\hat{p}$ is a random variable

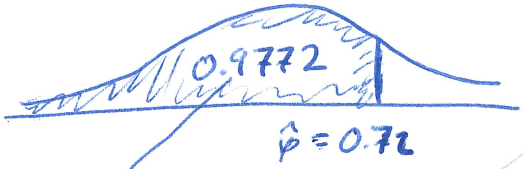
$\hat{p} =$ 0.68 0.7 0.72

$z =$? 0 ?



$\hat{p} =$ 0.68 0.7 0.72





$\hat{p} = 0.72$

use table

$z = 2$

$\hat{p} = \mu + z\sigma$

$0.72 = 0.7 + 2\sigma$

$0.02 = 2\sigma$

$0.01 = \sigma$

$\sqrt{\frac{0.7 \times 0.3}{n}} = 0.01$

$\frac{0.7 \times 0.3}{n} = 0.01^2 = 0.0001$

$\boxed{n = \frac{0.7 \times 0.3}{0.0001} = 2100}$