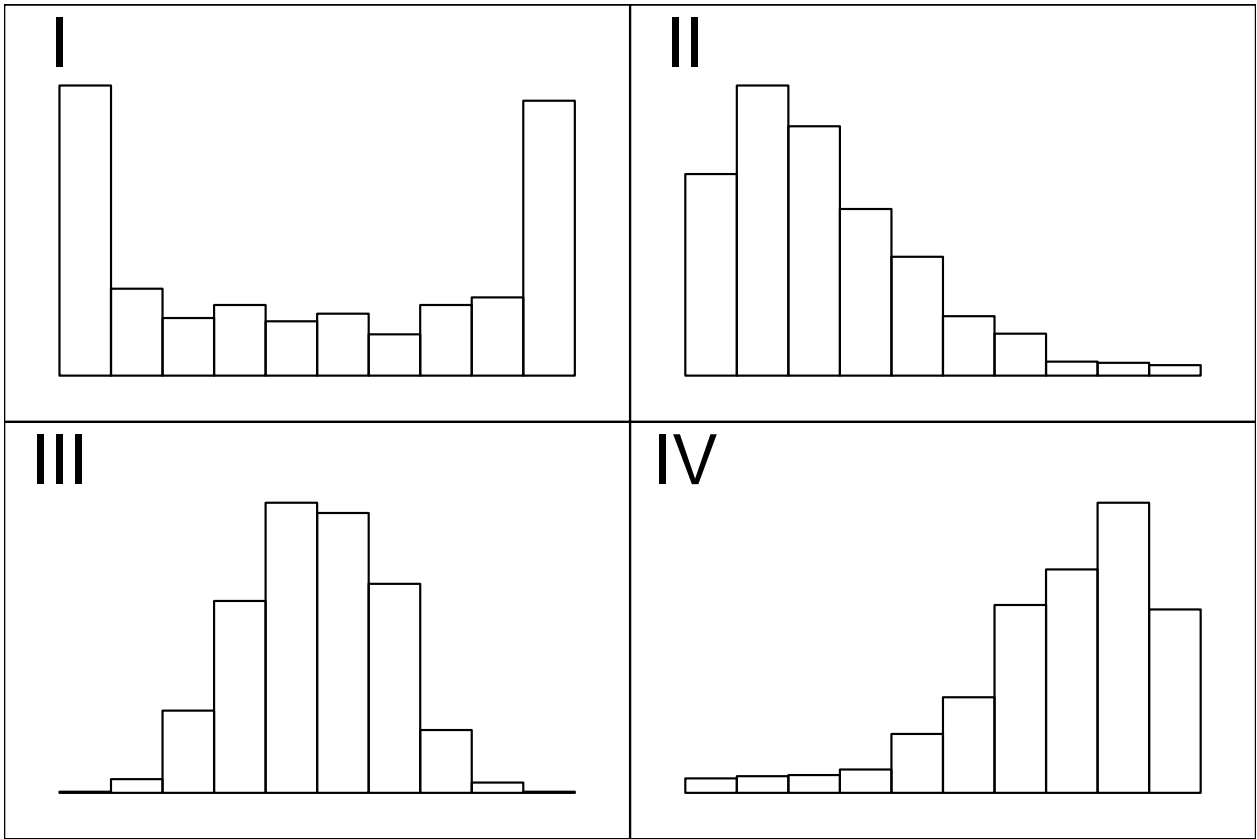# MAT-181 FINAL TAKE-HOME EXAM

This exam is to be taken without discussion or correspondance with any human. Please show work!

question	available points	earned points
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	15	
8	20	
EC	5	
EC	5	
Total	100	

1. (10 Points)

For each description below, choose which histogram best fits (I, II, III, or IV). Each histogram should be used once.

(a) The distribution of annual income for NBA basketball players where only a few are high-paid superstars.

(b) The distribution of heights of adult women

(c) The distribution of hours spent per week reading by adults. In this distribution, many people do not read much, and a similar number of people read a lot.

(d) The distribution of ages at a skilled nursing facility, where most of the patients are elderly but a few are quite young.

2. (15 Points)

In a deck of strange cards, there are 462 cards. Each card has an image and a color. The amounts are shown in the table below.

	black	indigo	orange	teal	Total
gem	23	39	41	12	115
kite	14	15	31	16	76
mop	25	46	19	48	138
quilt	37	17	29	50	133
Total	99	117	120	126	462

(a) Is a kite or a quilt more likely to be teal?

(b) What is the probability a random card is a quilt?

(c) What is the probability a random card is teal given it is a mop?

(d) What is the probability a random card is teal?

(e) What is the probability a random card is both a quilt and orange?

(f) What is the probability a random card is a kite given it is black?

(g) What is the probability a random card is either a kite or orange (or both)?

3. (10 points)

   A farm produces 4 types of fruit: *A*, *B*, *C*, and *D*. The fruits' masses follow normal distributions, with parameters dependent on the type of fruit.

Type of fruit	Mean mass (g)	Standard deviation of mass (g)
A	110	15
B	126	5
C	80	12
D	83	6

   One specimen of each type is weighed. The results are shown below.

Type of fruit	Mass of specimen (g)
A	83
B	126.4
C	66.08
D	94.1

   Which specimen is the most unusually far (in either direction) from average (relative to others of its type)?

4. (10 points)

A tree's leaves were found to be normally distributed with a mean of 53.3 millimeters and a standard deviation of 6.4 millimeters. If you pick a random leaf from that tree, what is the probability the length is between 44.8 and 53.7 millimeters?

5. (10 points)

A species of duck is known to have a mean weight of 178.7 grams and a standard deviation of 36 grams. A researcher plans to measure the weights of 225 of these ducks sampled randomly. What is the probability the **sample mean** will be between 175.7 and 184.7 grams?

6. (10 points)

   An ornithologist wishes to characterize the average body mass of *Melospiza georgiana*. She randomly samples 32 adults of *Melospiza georgiana*, resulting in a sample mean of 13.6 grams and a sample standard deviation of 1.14 grams. Determine a 95% confidence interval of the true population mean.

7. (15 points)

A student is taking a multiple choice test with 500 questions. Each question has 2 choices. You want to detect whether the student does significantly better than random guessing, so you decide to run a hypothesis test with a significance level of 0.05.

Then, the student takes the test and gets 268 questions correct.

(a) What kind of hypothesis test is appropriate?

(b) State the hypotheses.

(c) Determine the test statistic (z or t), draw a sketch, and determine the p-value.

(d) Decide whether we reject or retain the null hypothesis.

(e) Did the student do significantly better than random guessing?

8. (20 points) [Note: this question uses 2 pages.]

   You have collected the following data:

x	y	xy
62	2.2	
46	3.1	
73	3.4	
85	3.7	
65	4.8	
99	4.4	
71	6.5	
$\sum x =$	$\sum y =$	$\sum xy =$
$\bar{x} =$	$\bar{y} =$	
$s_x =$	$s_y =$	

(a) Complete the table.

(b) Calculate the correlation coefficient (r) using the formula below.

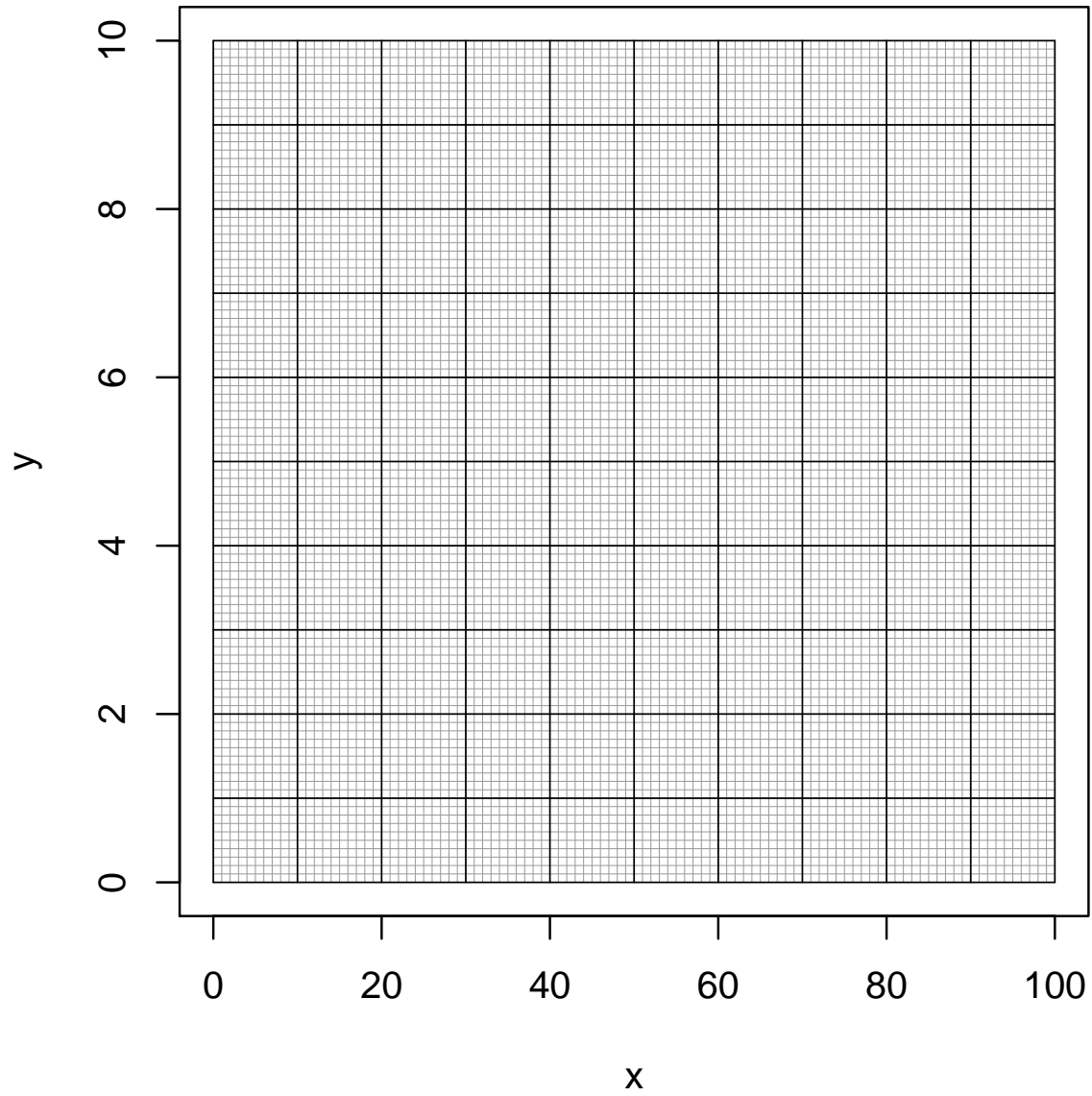$$r = \frac{\sum xy - n\bar{x}\bar{y}}{(n-1)s_x s_y}$$

(c) The least-squares regression line will be represented as $y = a + bx$. Determine the parameters (b and a) using the formulas below.

$$b = r\frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

(d) Write the equation of the regression line (using the calculated values of a and b.)

(e) Please plot the data and a corresponding regression line.

9. (Extra credit: 5 points)

   Let each trial have a chance of success $p = 0.92$. If 238 trials occur, what is the probability of getting at least 212 but less than 225 successes?

   In other words, let $X \sim \text{Bin}(n = 238, p = 0.92)$ and find $P(212 \leq X < 225)$.

   Use a normal approximation along with the continuity correction.

10. (Extra credit: 5 points)

    A null hypothesis claims a population has a mean $\mu = 150$. You decide to run two-tail test on a sample of size $n = 9$ using a significance level $\alpha = 0.05$.

    You then collect the sample:

224.8	167.6	122	201.3	171.3
186.1	183.4	146.1	156.6	

    (a) Determine the p-value.

    (b) Do you reject the null hypothesis?