



**UNIVERSITÀ  
DEGLI STUDI  
DI PADOVA**



**INFORMATION ENGINEERING DEPARTMENT  
COMPUTER ENGINEERING - AI & ROBOTICS**

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Francesco Crisci 2076739



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# Chapter 1

## Introduction

### 1.1 General Introduction

**Natural Language Processing** is a field of artificial intelligence that allows machines to read, derive meaning from text, and produce documents. It works in the background of many services, from chatbots through virtual assistants to social media tracking. Such language technologies are already showing major penetration into the information and communication industry.

NLP distinguishes itself from other AI application domains, as for instance computer vision or speech recognition. Text data is fundamentally discrete. But new words can always be created. Few words are very frequent, and there is a long tail of rare words. Out-of-Vocabulary words are always being discovered.

Languages have the following characteristics:

- it is **ambiguous**: units can have different meanings;
- it is **compositional**: meaning of a unit defined as a function of the meaning of its components.
- it is **recursive**: units can be repeatedly combined.
- they unveil **hidden structure**: local changes in a sentence might have global effects.

#### Ambiguity

word can belong to several categories, like noun, verbs or modal. The word bank, for example, have different meanings: river bank or money bank. The morphological composition, e.g., the word un-do-able is ambiguous between not doable and can be undone.

#### Compositionality

At each level, meaning of a larger unit is provided by some function of the meaning of its immediate components and the way they are combined.

#### Recursion

The rules of the grammar can iterate to generate an infinite number of structures, each with its specific meaning. Recursion is considered the main difference between human and other animals' languages.

#### Hidden structure

Local changes can disrupt the interpretation of a sentence. This suggests the existence of hidden structure.

### 1.2 Search & Learning

Many natural language processing problems can be written mathematically in the form of optimization:

$$\hat{\mathbf{y}} = \arg \max_{\mathbf{y} \in \mathcal{Y}(\mathbf{x})} \Psi(\mathbf{x}, \mathbf{y}; \Theta)$$

where  $\mathbf{x}$  is the input, which is an element of a set  $\mathcal{X}$ ;  $\mathbf{y}$  is the output, which is an element of a set  $\mathcal{Y}(\mathbf{x})$ .  $\Psi$  is a scoring function, also called the **model**, which maps from the set  $\mathcal{X} \times \mathcal{Y}$  to the real numbers;  $\Theta$  is a vector of **parameters** for  $\Psi$ .  $\hat{\mathbf{y}}$  is the predicted output, which is chosen to maximize the scoring function.

The **Search** module is responsible for finding the candidate output  $\hat{\mathbf{y}}$  with the highest score relative to the input  $\mathbf{x}$ . the **Learning** module is responsible for finding the model parameters  $\Theta$  that maximizes the predictive



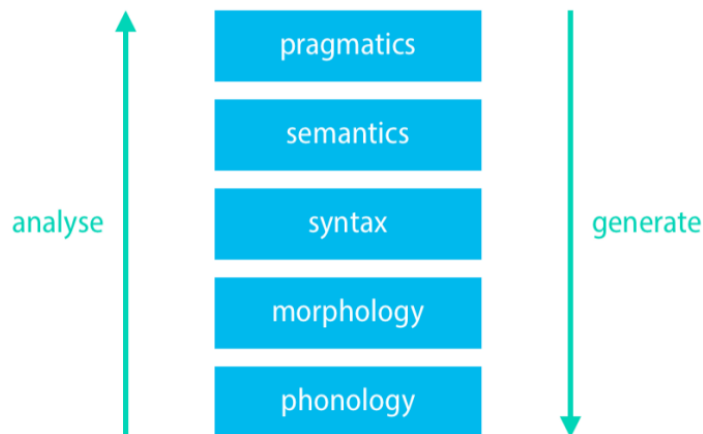
performance.

**Structured prediction** is an umbrella term for supervised machine learning techniques that involves predicting structured objects, rather than scalar discrete or real values.

## 1.3 Essentials of linguistics

**Natural language** is a structured system for communication, consisting of a vocabulary and a grammar.

Linguistics is the scientific study of language, and in particular the relationship between language form and language meaning. Besides form and meaning, another important subject of study for linguistics is how languages is used in context. **Phonology** studies the rules that organize patterns of sounds in human languages.



It is different from **phonetics**, which is concerned with the production, transmission and perception of sounds, without prior knowledge of the language being spoken.

**Morphology** is the study of how words are composed by **morphemes**, which are the smallest meaningful units of language. The structure of a word consists of several morphemes: one root or stem and zero or more affixes. **Inflectional morphology** means that there is no change in the grammatical category. **Derivational morphology** means that there is a change in the grammatical category.

A language can be of the following types:

- **Isolating**: a language in which each word form consists typically of a single morpheme;
- **Analytic**: no inflection to indicate grammatical relationship, may still contain derivational morphemes;
- **Synthetic**: uses inflection or agglutination to express relationships within a sentence.

The term **morphologically rich language** refers to a language in which substantial grammatical information is expressed at word level.

what we have seen in our examples so far is **concatenative morphology**: morphemes are placed one after the other. Some languages, as for instance semitic languages, are based on **template morphology**.

**Syntax** studies the rules and constraints that govern how words can be organized into sentences. Differently from formal language theory, NLP systems need to be robust to input that does not follow the rules of grammar.

### Part of speech

A **part of speech** is a category for words that play similar roles within the syntactic structure of a sentence. PoS can be defined:

- **distributionally**: Kim saw the {elephant, movie, mountain, error } before we did.
- **functionally**: verbs=predicates; nouns=arguments; adverbs=modify verbs, etc.

**Open class** tags: noun, verb, adjective, and adverb. New words in the language are usually added to these classes.

**Closed class** tags: determiners, prepositions, conjunctions, etc. Closed word classes rarely receive new members.

There are several representations for syntactic structure. Most common are Phrase structure and Dependency tree.



**Phrase structure** is a tree-like representation with leaf nodes representing sentence words and internal nodes representing word grouping called **phrases**.

**Dependency tree** is a tree-like representation where the nodes represent words and punctuation in the sentence, and arcs represent grammatical relations between a **head** and a **dependent**. Dependency trees use labels at arcs, representing grammatical relations. Arcs in a dependency tree are often called **dependencies**.

**Semantics** is the study of the meaning of linguistic expressions such as words, phrases and sentences. The focus is on what expressions conventionally/abstractly mean, rather than on what they might mean in a particular context. The linguistic study of word meaning is called **lexical semantics**. The internal semantic structure of a word refers to the similarity with other words. The external semantic structure of a word refers to the allowability to combine with other words.

**Lexical ambiguity** arises because a word can have different meanings, called word senses.

**Principle of compositionality**: the meaning of a whole expression is a function of the meanings of its parts and of the way they are syntactically combined. Syntax provides the scaffolding for semantic composition: the meaning of a sentence isn't just the amalgamation of the meaning of its component words.

**Meaning representation** Many representations for semantic structure. Most common are logical, predicate-argument or graph structure.

**Pragmatics** studies the way linguistic expressions with their semantic meanings are used for specific communicative goals. In contrast to semantics, pragmatics explicitly asks the question what an expression means in a given context. An important concept in pragmatics is the speech act, which describes an action performed through language.

## Discourse analysis

**Discourse analysis** studies written and spoken language relation to its social context. **Discourse** refers to a piece of text with multiple sub-topics and coherence relations between them, such as explanation, elaboration and contrast. There are many formalisms for representing discourse structure, as for example discourse representation theory and rhetorical structure theory. **Dialogue** is a cooperative kind of discourse, where two or more participants are involved.

## 1.4 Text Language Processing

Cleaning up a text data set requires the use of specialized **regular expressions**. Most programming languages have facilities for compiling regular expressions into efficient finite automata and running these automata on input text. Regular expressions come in many variants. We describe here the so-called **extended** regular expressions. We have different categories and operators:

- **Search**: `/d/` matches d in woodchuck.
- **Sets of characters**: `/[abc]/` matches a, b or c; `/[^abc]/` matches any character other than a, b and c; `/[a-z]/` matches any character from a to z; `/./` matches any character; also called **wildcard**.
- **Aliases**: `\d` stands for any digit; same as `[0-9]`; `\D` stands for any character other than a digit, same as `[^0-9]`; `\w` stands for any alphanumeric or underscore; `\W` is the converse of `\w`; `\s` stands for any whitespace character; `\S` converse of `\s`; `\.` stands for period; `\n` stands for newline.
- **Repetition**: `/d{2,5}/` matches between 2 and 5 digits; `/d{3,}/` matches 3 or more digits; `/d{4}\w?/` matches exactly 4 digits and one optional alphanumeric or underscore; `/[a-z]+/` matches one or more lowercase letters; `/\s+java\s+/` matches java with one or more whitespace characters before and after; `/[^\+]* /` matches zero or more characters other than + (Kleen star).
- **Anchors**: `/^/` matches beginning of input string or line; `/$/` matches end of input string or line; `/\b/` matches word boundary; `/^The` matches occurrence of The at the beginning of a string. With `|` you can have disjunctions and groups `/the|any` matches the or any; `/grupp(y|ies)/` matches gruppy and gruppies.
- **Substitute and Back-reference**: we can replace matches of a regular expression by a given pattern using the substitute operator `s/word1/word2`; we can use matches of a regular expression through the back-reference operator.

## Words

We need to distinguish between words and punctuation marks. Punctuation is critical to find sentence boundaries and for identifying some aspects of meaning. There are two ways of talking about words: **types** are the distinct words appearing in a document and **tokens** that are the individual occurrences of words in a document. the set of all types in a corpus is the **Vocabulary**  $V$ . The vocabulary size  $|V|$  is the number of types in the



corpus. The size of the corpus  $N$  is the number of tokens, if we ignore punctuation marks. In very large corpora, the relation between  $N$  and  $|V|$  can be expressed as

$$|V| = kN^\beta$$

In very large corpora, the  $r$ -th most frequent type has frequency  $f(r)$  that scales according to

$$f(r) \propto \frac{1}{(r + \beta)^\alpha}$$

The **word-form** is the full inflected or derived form of the word. Each word-form is associated with a single **lemma**, the citation form used in dictionaries. alternatively to  $|V|$ , another measure of the number of words in a corpus is the number of lemmas.

## Corpora

**Corpus:** large collection of text, in computer-readable form. Several dimensions of variation for corpora should be taken into account: language, genre, etc.

**Language:** it is important to test NLP algorithms on more than one language. languages lacking large corpora are considered low-resource languages.

**Genre:** text documents might come from newswire, fiction, scientific articles, Wikipedia, etc.

**Time:** language changes over time. for some languages we have good corpora of texts from different historical periods.

**Collection process:** how big is the data and how was it sampled? How was the data pre-processed, and what metadata is available?

**Annotation:** what are the specifics of the used annotation? How was the data annotated? How were the annotators trained?

## Text Normalization

**Text normalization** is the process of transforming a text into some predefined standard form. This consists of several tasks. There is no all-purpose normalization procedure: text normalization depends on what type of text is being normalized and what type of NLP task needs to be carried out afterwards. Text normalization is also important for applications other than NLP, such as text mining and WEB search engines.

## Language identification

**Language identification** is the task of detecting the source language for the input text. Several statistical techniques for this task: functional word frequency, N-gram language models, distance measure based on mutual information, etc.

## Spell checker

**Spell checkers** correct grammatical mistakes in text. They use approximate string matching algorithms such as **Levenshtein distance** to find correct spellings.

## Contractions

The following should be managed before further normalization: contracted forms, abbreviations and slangs. The most straightforward technique is to create a dictionary of contractions and abbreviations with their corresponding expansions.

## Punctuation

**Punctuation** marks in text need to be isolated and treated as if they were separate words. This is critical for finding sentence boundaries and for identifying some aspects of meaning.

## Tokenization

**Tokenization** is the process of segmenting text into units called tokens. Tokenization techniques can be grouped into three families: word, character and subword tokenization. Tokens are then organized into a vocabulary and, depending on the specific NLP application, may later be mapped into natural numbers.

## Word tokenization

Word tokenization is a very common approach for European languages. For English, most of the text is already tokenized after previous steps, with the following important exceptions: special compound names and city names.





## Character tokenization

Major east Asian languages write text without any spaces between words. For most Chinese NLP task, character tokenization works better than word tokenization: each character generally represents a single unit of meaning and word tokenization results in huge vocabulary, with large number of rare words.

## Subword tokenization

Many NLP systems need to deal with **unknown words**, that is, words that are not in the vocabulary of the system. To deal with the problem of unknown words, modern tokenizers automatically induce sets of tokens that include tokens smaller than words, called **subwords**. Subword tokenization reduces vocabulary size, and has become the most common tokenization method for large language modelling and neural models in general. Subword tokenization is inspired by algorithms originally developed in information theory as a simple and fast form of data compression alternative to Lempel-Ziv-Welch.

Subword tokenization schemes consist off three different algorithms:

- the **token learner** takes a raw training corpus and induces a set of tokens, called **vocabulary**.
- the **token segmenter** takes a vocabulary and a raw sentence, and segments the sentence into the tokens in the vocabulary.
- the **token merger** takes a token sequence and reconstructs the original sentence.

Three algorithms are widely used for subword tokenization: byte-pair encoding, unigram and WordPiece tokenization.

### BPE: learner

The BPE **token learner** is usually run inside words, not merging across word boundaries. To this end, use a special end-of-word marker. The algorithm iterates through the following steps:

- begin with a vocabulary composed by all individual characters.
- choose the two symbols A, B that are most frequently adjacent.
- add a new merged symbol AB to the vocabulary.
- replace every adjacent A, B in the corpus with AB.

Stop when the vocabulary reaches size k, a hyperparameter.

### BPE: Encoder

Two versions of **BPE token segmenter**: apply merge rules in frequency order all over the data set, or, for each word, left-to-right, match longest token from vocabulary. Encoding is computationally expensive. Many systems use some form of caching: pre-tokenize all the words and save how a word should be tokenized in a dictionary; when an unknown word is seen: apply the encoder to tokenize the word and add the tokenization to the dictionary for future reference.

### BPE: Decoder

**BPE token merger**: to decode, we have to concatenate all the tokens together to get the whole word and use the end-of-word marker to solve possible ambiguities.

## WordPiece

**WordPiece** is a subword tokenization algorithm used by the large language model BERT. Like BPE, WordPiece starts from the initial alphabet and learns merge rules. The main difference is the way pair A, B is selected to be merged:

$$\frac{f(A, B)}{f(A) \times f(B)}$$

The algorithm prioritizes the merging of pairs where the individual parts are less frequent in the vocabulary. Text normalization also includes **sentence segmentation**: breaking up a text into individual sentences. This can be undone using cues like periods, question marks, or exclamation points.

**Lower casing** is very useful to standardize words and before stop word removal. Most programming languages have facilities for string lowercasing.

**Stop word removal** includes getting rid of common articles, pronouns, prepositions and coordinations. Stop word removal heavily depends on the task at hand, since it can wipe out relevant information.



**Stemming** refers to the process of slicing a word with the intention of removing affixes. Stemming is problematic in the linguistic perspective, since it sometimes produces words that are not in the language, or else words that have a different meaning.

**Lemmatization** has the objective of reducing a word to its base form, also called **lemma**, therefore grouping together different forms of the same word. Lemmatization and Stemming are mutually exclusive, and the former is much more resource-intensive than the latter.

## Chapter 2

# Word Embeddings

What is the meaning of a word? The linguistic study of word meaning is called **lexical semantics**. A model of word meaning should allow us to relate different words and draw inferences to address meaning-related tasks. A word-form can have multiple meanings; each meaning is called a **word sense**, or sometimes a **synet**. **Word sense disambiguation** (WSD) is the task of determining which sense of a word is being used in a particular context.

Lexical semantic relationship between words are important components of word meaning. Two words are **synonyms** if they have a common word sense. Two words are **similar** if they have similar meanings. Two words are **related** if they refer to related concepts. Two words are **antonyms** if they define a binary opposition. One word is an **hyponim** of another if the first has a more specific sense. Notions of **hypernym** or **hyperonym** are defined symmetrically. Words can have **affective** meanings implying positive or negative connotations/evaluation.

### 2.1 Distributional semantics

**Distributional semantics** develops methods for quantifying semantic similarities between words based on their distributional properties, meaning their **neighboring words**. The basic idea lies in the so-called **distributional hypothesis**: linguistic items with similar distribution have similar meanings. The basic approach is to collect distributional information in high-dimensional vectors, and to define distributional/semantic similarity in terms of vector similarity. Similar words are mapped into "close enough" vectors.

Lexical semantic relationships, represented as word vector differences, are generally preserved.



Figure 2.1: Example: Vectors from sentiment analysis application, projected into 2-dimensional space.

The obtained vectors are called **word embeddings**. The approach is called **vector semantics** and is the

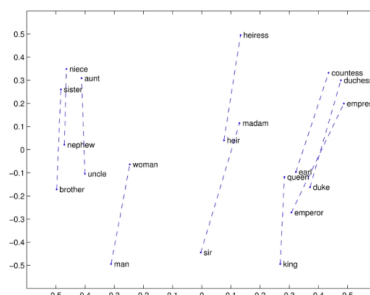


Figure 2.2: Example: Vectors projected into 2-dimensional space.

standard way to represent word meaning in NLP. All the learning algorithms we report are **unsupervised**. We discuss two families of word embeddings:

**Sparse vectors:** Vector components are computed through some function of the counts of nearby words;

**Dense vectors:** Vector components are computed through some optimization or approximation process.



There is a review of vectors and vector's operations which I'm gonna omit. You are at the fifth year of engineering. If you need a review, I'm sorry, but you're dumb :)

## 2.2 Term-context matrix

Let  $\{w_1, \dots, w_W\}$  be a set of **term** words and let  $\{c_1, \dots, c_C\}$  be a set of **context** words. For each word  $w_i$  in the corpus, we consider context words appearing inside a window of size  $L$ , at the left or at the right of  $w_i$ . A **term-context matrix**  $F$  is a matrix  $W \times C$ , where each element  $f_{i,j}$  is the number of times context word  $c_j$  appears in the context of term word  $w_i$  in the corpus.

## 2.3 Probabilities & information theory

**Pointwise mutual information** is a measure of how often two events  $x$  and  $y$  occur, compared with what we would expect if they were independent:

$$I(x, y) = \log_2 \frac{P(x, y)}{P(x)P(y)}$$

It is symmetric, it can take positive or negative values. If the random variables are independent then it is zero. The PMI between  $w_i$  and  $c_j$  is

$$PMI(w_i, c_j) = \log_2 \frac{P(w_i, c_j)}{P(w_i)P(c_j)}$$

The numerator tells us how often we observed the two words together. The denominator tells us how often we would expect the two words to co-occur assuming they each occurred independently. The ratio gives us an estimate of how much more the two words co-occur than we expected by chance. We can rewrite the PMI as:

$$PMI(w_i, c_j) = \log_2 \frac{P(w_i|c_j)}{P(w_i)}$$

PMI may then be understood as quantifying how much our confidence in the outcome  $w_i$  increases after we observe  $c_j$ .

## 2.4 Probability estimation

How do we estimate the Probabilities  $P(w_i, c_j), P(w_i), P(c_j)$ ? We can estimate:

$$P(w_i, c_j) = \frac{f_{i,j}}{\sum_{i=1}^W \sum_{j=1}^C f_{i,j}}$$

Note that the denominator is the total number of possible target/context pairs.

$$P(w_i) = \sum_{j=1}^C P(w_i, c_j) = \frac{\sum_{j=1}^C f_{i,j}}{\sum_{i=1}^W \sum_{j=1}^C f_{i,j}}$$

Similarly we can estimate:

$$P(c_j) = \sum_{i=1}^W P(w_i, c_j) = \frac{\sum_{i=1}^W f_{i,j}}{\sum_{i=1}^W \sum_{j=1}^C f_{i,j}}$$

It is difficult to estimate negative values of PMI. We can use **Positive PMI**:

$$PPMI(w_t, w_c) = \max\{\log_2 \frac{P(w_t, w_c)}{P(w_t)P(w_c)}, 0\}$$

## 2.5 Practical issues

PMI has the problem of being **biased** towards infrequent events: very rare words tend to have very high PMI values. One way to reduce this bias is to slightly change the computation for  $P(w_c)$  in the PPMI:

$$PPMI_\alpha(w_t, w_c) = \max\{\log_2 \frac{P(w_t, w_c)}{P(w_t)P_\alpha(w_c)}, 0\}$$

$$P_\alpha(w_c) = \frac{C(w_c)^\alpha}{\sum_{v \in V} (C(v))^\alpha}$$

We can use the rows of the PPMI term-context matrix as word embeddings. Notice that these vectors have the size of the vocabulary, which can be quite large, and when viewed as arrays, they are very sparse. Dot product between these word embeddings needs to use specialised software libraries, implementing vectors as dictionaries rather than arrays.

We can obtain dense word embeddings from the ePPMI term-context matrix. This is done through a matrix decomposition technique known as **truncated singular value decomposition**.

### 2.5.1 Truncated singular value decomposition

For a matrix  $A \in \mathbb{R}^{n \times m}$ , the **Frobenius norm** is

$$\|A\|_F = \sum_{i=1}^n \sum_{j=1}^m a_{i,j}^2$$

where  $a_{i,j}$  are the elements of the matrix. Let  $|V|$  be the vocabulary size, and let  $K \ll |V|$  be the size of the desired embedding. Let  $P \in \mathbb{R}^{|V| \times |V|}$  be the PPMI term-context matrix. Let  $U, V \in \mathbb{R}^{|V| \times K}$  be learnable parameters. The basic idea is to approximate  $P$  by means of a matrix  $\tilde{P}(U, V) \in \mathbb{R}^{|V| \times |V|}$  such that

$$\min_{U, V} \|P - \tilde{P}(U, V)\|_F$$

**Truncated singular value decomposition** solves the following constrained optimization problem:

$$\min_{U \in \mathbb{R}^{|V| \times K}, S \in \mathbb{R}^{K \times K}, V \in \mathbb{R}^{|V| \times K}} \|P - USV^T\|_F$$

subject to conditions  $U^T U = \mathbb{I}$ ,  $V^T V = \mathbb{I}$  and  $S$  is diagonal with its elements in decreasing order.  $U$  are the target embeddings,  $SV^T$  are the context embeddings.

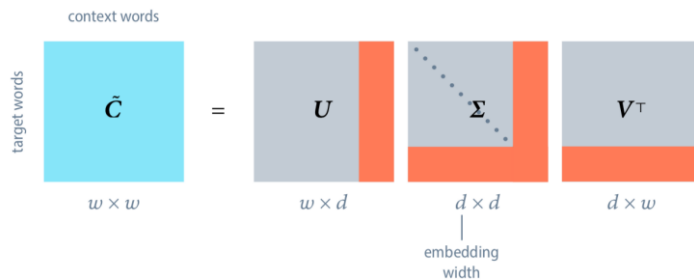
Let  $U_k, V_k \in |V| \times K$  be the  $k$ -th columns of matrices  $U$  and  $V$ , respectively. In truncated SVD,  $P$  is approximated by the sum of  $K$  rank-1 matrices:

$$P \approx \sum_{k=1}^K s_k U_k V_k^T$$

Equivalently, each element  $p_{i,j}$  of  $P$  is approximated through the dot product between the embedding of the  $i$ -th target word and the embedding of the  $j$ -th context word, weighted by the singular values

$$p_{i,j} \approx \sum_{k=1}^K s_k u_{i,k} v_{j,k}$$

We can alternatively view the previous approximation approach as a factorization problem for matrix  $P$ ,  $P = USV^T$ , where  $U, S, V \in |V| \times |V|$ ,  $U^T U = \mathbb{I}$ ,  $V^T V = \mathbb{I}$  and  $S$  is a diagonal with **singular values**  $s_1 > s_2 > \dots > s_V$  in its diagonal. We then choose  $K \ll |V|$  on the basis of mass distribution at the singular values, and truncate matrices  $U, S, V$  accordingly.



## 2.6 Neural word embeddings

Word embeddings can also be computed using neural networks. Generally speaking, neural word embeddings have the following properties: small number of dimension, vectors are dense, the dimensions don't have a clear interpretation, components can be negative. Neural word embeddings work better than previous embeddings in every NLP task.

**Word2vec** is a software package including two different algorithms for learning word embeddings: skip-gram with negative sampling (SGNS) and continuous bag-of-words (CBOW). The idea is that instead of counting how often a context word appears near a target word, we train a classifier on the following binary prediction task: *Is a given context word likely to appear near a given target word?* We don't really care about this prediction task: instead, we use the learned parameters as the word embeddings.



### 2.6.1 Skip-Gram

More specifically, the basic steps of the **skip-gram** algorithm are as follows:

For each target word  $w_t \in V$ :

I treat  $w_t$  and any neighboring context word  $w_c$  as positive examples;

II randomly sample other words  $w_n \in V$ , called **noise words**, to produce negative examples for  $w_t$

Using **logistic regression** to train a classifier to distinguish positive and negative examples. Use the learned weights as the embeddings.

### 2.6.2 Logistic Regression

For any pair of words  $w_t, u \in V$ , we need to define the probability that  $u$  is/is not a context word for target word  $w_t$

$$P(+|w_t, u) \quad P(-|w_t, u) = 1 - P(+|w_t, u)$$

For each word  $w \in V$ , we construct two complementary embeddings: a target embedding  $e_t(w) \in \mathbb{R}^d$  and a context embedding  $e_c(w) \in \mathbb{R}^d$ . Integer  $d$  is the size of the embedding. Operator  $e_t(w)$  is always assigned to target words, operator  $e_c(w)$  is always assigned to context or noise words. We use the **dot product**:

$$e_t(w) \cdot e_c(w) = |e_t(w)| |e_c(w)| \cos \theta$$

where  $\theta$  is the angle between  $e_t(w)$  and  $e_c(w)$ . We also use the **logistic sigmoid function**  $\sigma$ , which maps real values to probabilities. We can now define:

$$P(+|w, u) = \sigma(e_t(w) \cdot e_c(u)) = \frac{1}{1 + \exp(-e_t(w) \cdot e_c(u))}$$

We then have vectors  $e_t(w), e_c(w)$  with small angle: positive cosine similarity,  $P(+|w, u)$  close to one, and vectors  $e_t(w), e_c(w)$  with large angle: negative cosine similarity,  $P(+|w, u)$  close to zero.

## 2.7 Training

For simplicity, let us consider a dataset with only one target/context pair  $(w, u)$ , along with  $k$  noise words  $v_1, \dots, v_k$  (negative examples). Skip-gram makes the simplifying assumption that all context words are independent. Then the log-likelihood of the data is:

$$LL_w = \log \left( P(+|w, u) \prod_{i=1}^k P(-|w, v_k) \right) = \log P(+|w, u) + \sum_{i=1}^k \log P(-|w, v_k)$$

Which we can rewrite as:

$$LL_w = \log \sigma(e_t(w) \cdot e_c(u)) + \sum_{i=1}^k \log(1 - \sigma(e_t(w) \cdot e_c(v_k)))$$

We can alternatively minimize the inverse of  $LL_w$ , which in case of the logistic regression is the **cross-entropy** loss function:

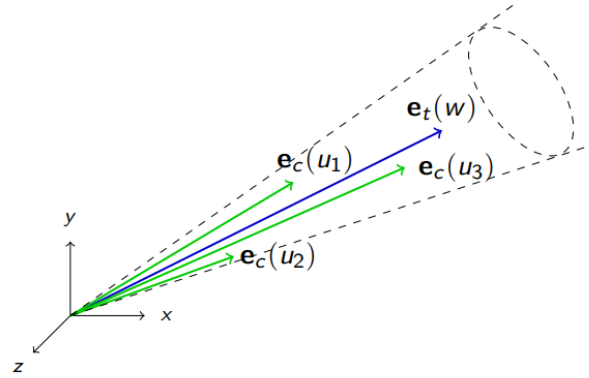
$$L_{CE} = -\log \sigma(e_t(w) \cdot e_c(u)) - \sum_{i=1}^k \log(1 - \sigma(e_t(w) \cdot e_c(v_k)))$$

The model parameters are the two  $\mathbb{R}^{|V| \times d}$  matrices with the target and context embeddings  $e_t(w), e_c(w), w \in V$ , which are randomly initialized. By making an update to minimize  $L_{CE}$  we force an increase in similarity between  $e_t(w)$  and  $e_c(w)$ , and we force a decrease in similarity between  $e_t(w)$  and  $e_c(v_k)$ , for all of the noise words  $v_k$ . We train the model with **stochastic gradient descent**, as usual for logistic regression. In the end, we retain the target embeddings  $e_t(w)$  and ignore the context embeddings  $e_c(w)$ .

### 2.7.1 Geometric interpretation

Target vector  $e_t(w)$  is surrounded by its context vectors  $e_c(u_j)$  which form a thin hypercone:

Let  $w$  be a target word with context words  $u_j$ 's. Let also  $w'$  be a target word similar to  $w$ . Then we have that  $e_c(u_j)$  form a thin cone surrounding vector  $e_t(w)$ ; vector  $e_t(w')$  must be placed within that cone, because  $w'$  and  $w$  share their context word;  $e_t(w)$  and  $e_t(w')$  are forced to be at a small angle. As a result, similar words cluster together. Within a similar reasoning, we can argue that different words repel each other.



## 2.7.2 Practical issues

Clean up text: convert any punctuation into token  $\langle PERIOD \rangle$ . Remove all words  $w$  such that  $f(w) \leq D$ : greatly reduces issues due to noise in data and improves the quality of the vector representation:

**Indexing**: convert words in  $V$  to non-negative integers and back again, this makes your code simpler.

Words that show up very often don't provide much context and are often regarded as noise. The process of discarding occurrences of these words by means of some probability distribution is called **subsampling**. We discard occurrence  $w_i$  with probability given by

$$P(w_i) = 1 - \sqrt{\frac{t}{f(w_i)}}$$

where  $t$  is a threshold parameter depending on dataset size.

The ratio  $k$  between the positive and the negative examples is a hyperparameter and must be adjusted on the development set. The noise words are chosen according to the relative frequency estimator and a  $\alpha$  corrector:

$$P_\alpha(v) = \frac{C(v)^\alpha}{\sum_{w \in V} (C(w))^\alpha}$$

Once hyperparameter  $L$  is fixed, context window size can be chosen randomly in the range  $[1, \dots, L]$ , with uniform distribution. This results in linear decay for context words, depending on distances from target word: words at distance 1 from target will be chosen with  $P=1$ , words at distance  $L$  from target will be chosen with  $P0\frac{1}{L}$ .

Should we estimate one embedding per word form or else estimate distinct embeddings for each **word sense**? Intuitively, if word representations are to capture the meaning of individual words, then words with multiple meanings should have multiple embeddings.

## 2.8 Miscellanea

### 2.8.1 FastText

**FastText** is an embedding that deals with unknown words and sparsity in languages with rich morphology, by using subword models. Each word in FastText is represented by itself plus a bag of character  $N$ -grams, for bounded values of  $N$ .

### 2.8.2 GloVe

**GloVe** (Global Vectors), is an embedding that accounts for global corpus statistics, which was somehow disregarded by word2vec. GloVe combines the intuitions of count-based models like PPMI, while also capturing the linear structures used by methods like word2vec.

### 2.8.3 Semantic properties

Both sparse and dense vectors are sensitive to the **context window size** used to collect counts, controlled by hyperparameter  $L$ . The choice of  $L$  depends on the goals of the representation:

- small values of  $L$  provides semantically similar words with the same parts of speech.
- large values of  $L$  provides words that are typically related but not similar.

another semantic property of embeddings is their ability to capture **relational meanings**. The **parallelogram model** can be used to solve analogy problems.



## 2.9 Evaluation

**Extrinsic evaluation:** use the model to be evaluated in some end-to-end application and measure performance.

**Intrinsic evaluation:** look at performance of model in isolation, w.r.t a given evaluation measure.

The most common evaluation metric for embedding models is extrinsic evaluation on end-to-end tasks: machine translation, sentiment analysis, etc. Three types of intrinsic evaluation:

- word **similarity**: compute a numerical score for the semantic similarity between two words;
- word **relatedness**: compute the degree of how much one word has to do with another words;
- word **analogy**: comparison of two things to show their similarities.

## 2.10 Cross-lingual word embedding

**Cross-lingual word embeddings** encode words from two or more languages in a shared high dimensional space. Vectors representing words with similar meaning are closely located, regardless of language.

Cross-lingual word embeddings are very useful in comparing the meaning of word across languages; are key to applications such as machine translation, multilingual lexicon induction, and cross-lingual information retrieval; enable model transfer between resource-rich and low-resource languages, by providing a common representation space.

The class of **objective functions** minimized by most cross-lingual word embedding methods can be formulated as:

$$J = \mathcal{L}^1 + \dots + \mathcal{L}^\ell + \Omega$$

where  $\mathcal{L}^i$  is the monolingual loss of the  $i$ -th language and  $\Omega$  is a regularization term. Joint optimization of multiple non-convex losses is difficult: most approaches optimize one loss at a time, while keeping certain variables fixed; this step-wise approach is approximate and does not guarantee to reach even a local optimum. The choice of multilingual parallel data sources is more important for the model performance than the actual underlying architecture. Methods differ along the following two dimensions: type of text **alignment**: at the level of words, sentences, or documents, which introduce stronger or weaker supervision; Type of text **comparability**: extract translations, weak translations, or similar meaning.

**Variation in ambiguity**: ambiguity for word  $w$  in one language does not transfer to the translation of  $w$  in another language.