İstanbul Bilgi University

Department of Computer Engineering

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CMPE 100: Introduction to Computing

Worksheet08

1. (50 points) A number is prime if no number divides it except 1 and itself. To test if a number p is prime we can take its modulo of all numbers starting with 2 up to p-1, and none of these modulos are 0 then we can say that the number is prime.

   Let us define a helper function which tells if a number is divisible by numbers up to some other number, d(p,k), its value is true if any number from 2 to k divides p. We can write this function in a recursive manner as follows: Now, design the program to compute it.

$$d(p,k) = \begin{cases} false & \text{if } k <= 1 \\ true & \text{if } k >= 2 \quad and \quad (p \;\; mod \;\; k) = 0 \\ d(p, k-1) & \text{otherwise} \end{cases}$$

   Therefore primality test is converted into the following:

   isPrime(p)=not (d(p,p-1))

2. (50 points) Design a Racket function named power to find an integer power of a number, x^n seems to require n-1 multiplications at first sight. However, this computation can be simplified greatly. Think about 2^8, which would require 7 multiplications to compute. Since the exponent 8 is an even number, this can be written as $2^8 = 2^{4^2}$. Therefore we need 3 multiplications to compute 2^4 and another multiplication to compute its square. It can be further simplified as $2^8 = 2^{4^2} = 2^{2^{2^2}}$, which now requires only 3 multiplications. In case of an odd exponent, one can rewrite only for the even part: $2^9 = 2^8 * 2$.

   A recursive formulation is:

$$x^n = \begin{cases} 1/(x^n) & \text{if } n < 0 \\ 1 & \text{if } n = 0 \\ x & \text{if } n = 1 \\ \left(x^{n/2}\right)^2 & \text{if } n \;\; mod \;\; 2 \;\; = \;\; 0 \\ \left(x^{(n-1)/2}\right)^2 * x & \text{if } n \;\; mod \;\; 2 \;\; \neq \;\; 0 \end{cases}$$

Note: Ensure that your programs are fully documented, using comments.