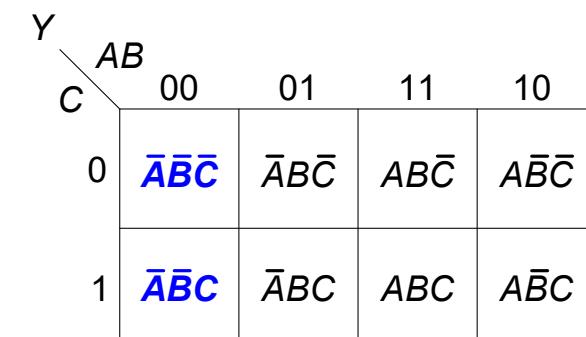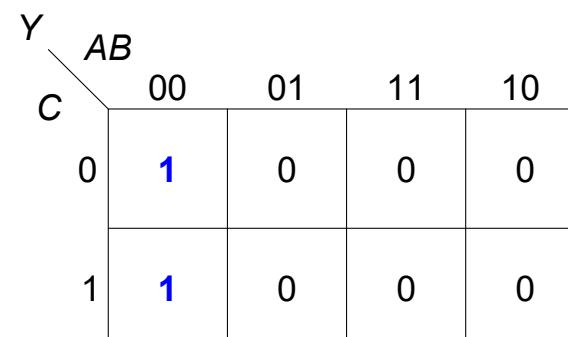# Karnaugh Maps (K-Maps)

- Boolean expressions can be minimized by combining terms

- K-maps minimize equations graphically

- $PA + P\overline{A} = P$
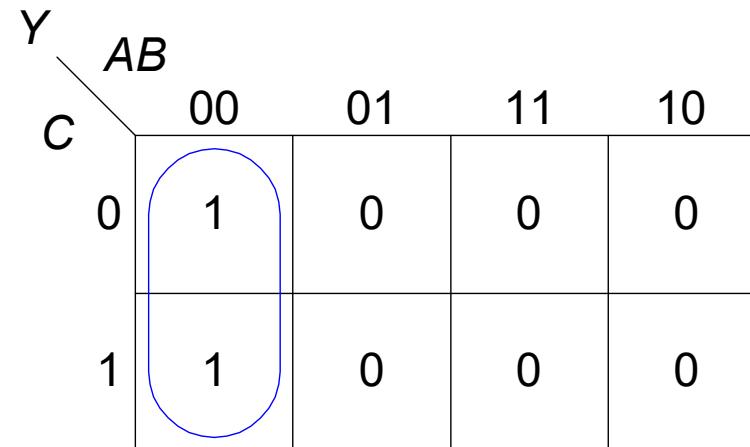
A	B	C	Y
0	0	0	**1**
0	0	1	**1**
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Y \ AB

C	00	01	11	10
0	**1**	0	0	0
1	**1**	0	0	0

Y \ AB

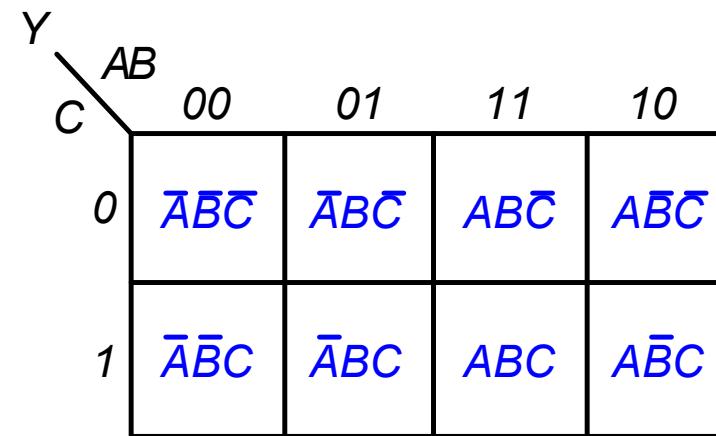| C	00	01	11	10
0	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}B\overline{C}$	$AB\overline{C}$	$A\overline{B}\overline{C}$
1	$\overline{A}\,\overline{B}C$	$\overline{A}BC$	ABC	$A\overline{B}C$

ELSEVIER

K-Map

- Circle 1's in adjacent squares

- In Boolean expression, include only literals whose true and complement form are ***not*** in the circle

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0
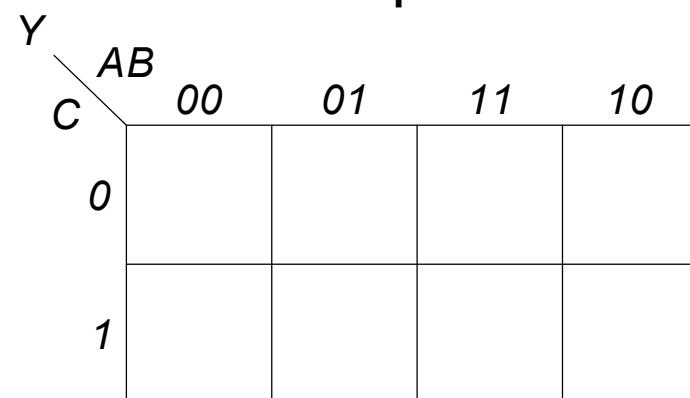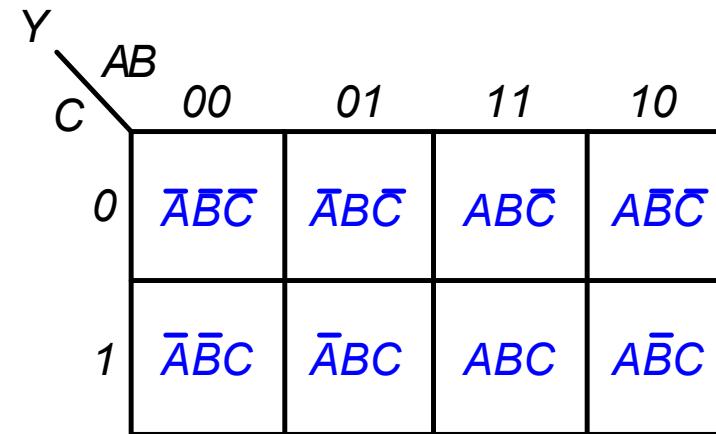
Y

C \ AB	00	01	11	10
0	1	0	0	0
1	1	0	0	0

$$Y = \overline{A}\,\overline{B}$$

ELSEVIER

3-Input K-Map

Y \ AB

	00	01	11	10
C				
0	$\overline{A}\overline{B}\overline{C}$	$\overline{A}B\overline{C}$	$AB\overline{C}$	$A\overline{B}\overline{C}$
1	$\overline{A}\overline{B}C$	$\overline{A}BC$	ABC	$A\overline{B}C$

Truth Table

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	**1**
0	1	1	**1**
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	**1**

K-Map

Y \ AB

	00	01	11	10
C				
0				
1				

ELSEVIER

3-Input K-Map

	AB 00	01	11	10
C 0	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}B\overline{C}$	$AB\overline{C}$	$A\overline{B}\,\overline{C}$
1	$\overline{A}\,\overline{B}C$	$\overline{A}BC$	ABC	$A\overline{B}C$

Truth Table

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

K-Map

Y AB
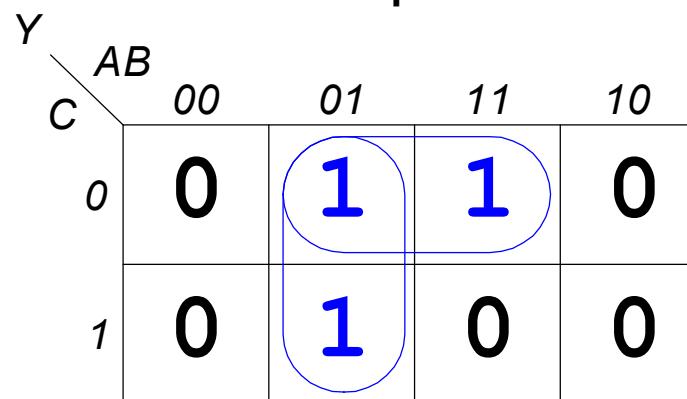
	00	01	11	10
C 0	0	1	1	0
1	0	1	0	0

$$Y = \overline{A}B + B\overline{C}$$
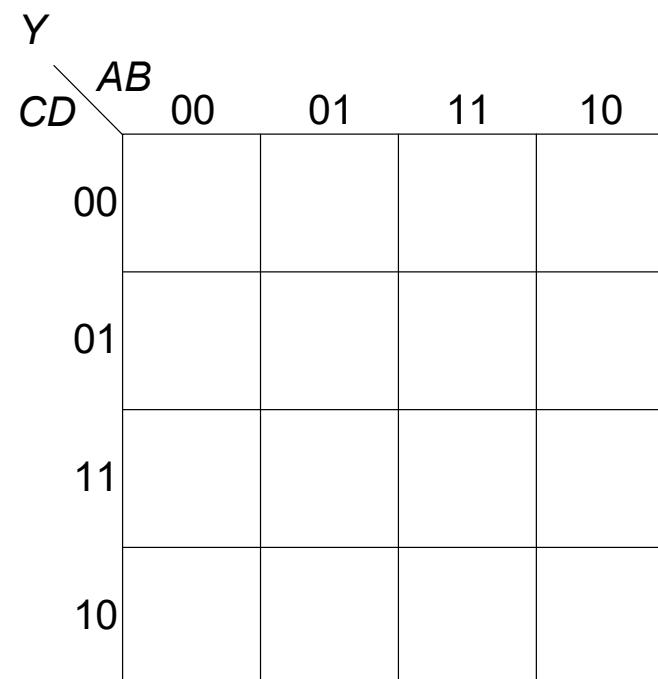
K-Map Definitions

- **Complement:** variable with a bar over it

  $\bar{A}, \bar{B}, \bar{C}$

- **Literal:** variable or its complement

  $\bar{A}, A, \bar{B}, B, C, \bar{C}$

- **Implicant:** product of literals

  $AB C, \bar{A}\bar{C}, BC$

- **Prime implicant:** implicant corresponding to the largest circle in a K-map

K-Map Rules

- Every 1 must be circled at least once

- Each circle must span a power of 2 (i.e. 1, 2, 4) squares in each direction

- Each circle must be as large as possible

- A circle may wrap around the edges

- A "don't care" (X) is circled only if it helps minimize the equation
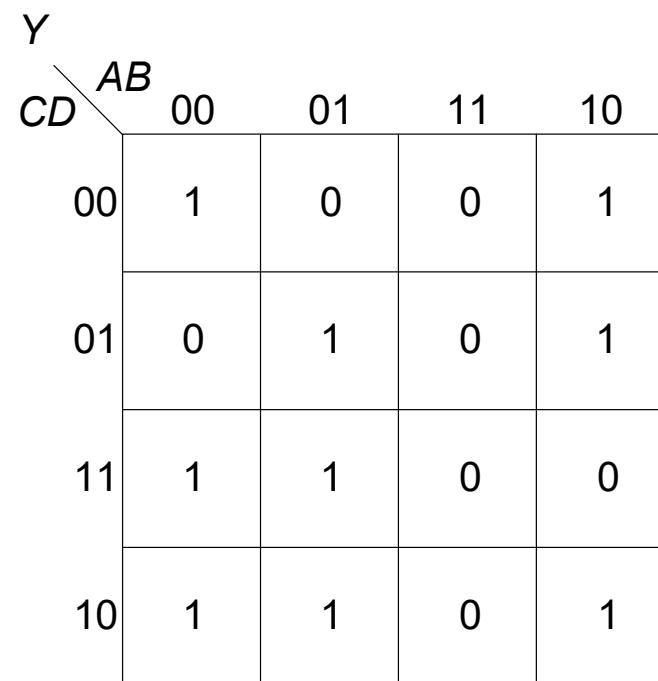
4-Input K-Map

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Y
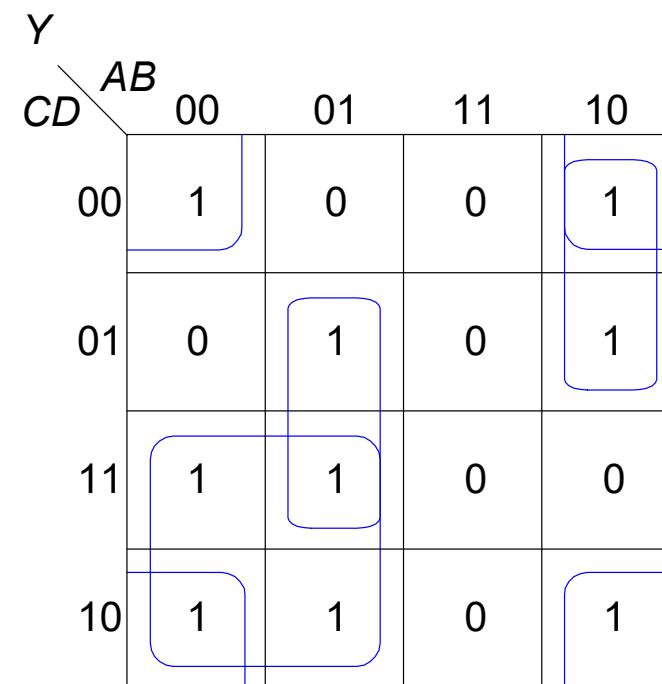
CD \ AB	00	01	11	10
00				
01				
11				
10				

4-Input K-Map

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Y

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

ELSEVIER

4-Input K-Map

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Y

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1
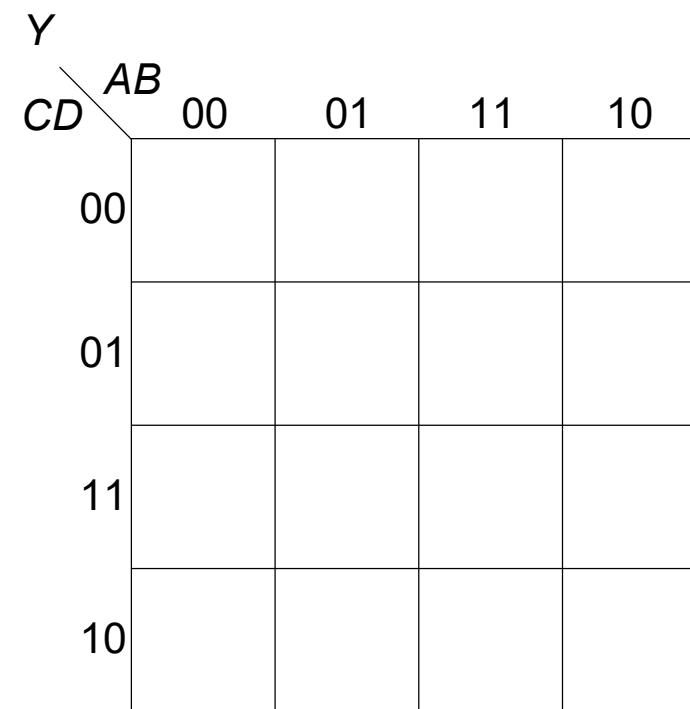
$$Y = \overline{A}\,\overline{C} + \overline{A}BD + A\overline{B}\,\overline{C} + \overline{B}\,\overline{D}$$

K-Maps with Don't Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y

CD \ AB

	00	01	11	10
00				
01				
11				
10				

ELSEVIER

K-Maps with Don't Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y

CD \ AB	00	01	11	10
00	1	0	X	1
01	0	X	X	1
11	1	1	X	X
10	1	1	X	X

ELSEVIER

K-Maps with Don't Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y

CD\AB	00	01	11	10
00	1	0	X	1
01	0	X	X	1
11	1	1	X	X
10	1	1	X	X

$$Y = A + \overline{B}\overline{D} + C$$