PAMUKKALE ÜNİVERSİTESİ MÜHENDİSLİK FAKÜLTESİ BİLGİSAYAR MÜHENDİSLİĞİ BÖLÜMÜ

MAT 340 AYRIK MATEMATİK DERSİ VİZE SORULARI

SORU 1)

− (100 25)* 2 ^{25}

SORU 2)

a.) If $m < n$, find the number of one-to-one functions from A to B.

One-to-one function - iff f(x)=f(y) then x=y.

Being a function f , the domain f must equal A. Being 1-1, every x in A must have a unique image in B

ex.) A={a,b} , B={a,b,c}

Input: a b
Choices: a b
a c
b a
b c
c a
c b

There are 6 one-to-one functions

Input a b
Choices 3 2 = 3*2 *(apply the product rule)* = 6 = $\frac{3!}{1!} = \frac{3!}{(3-2)!}$.

$\frac{3!}{(3-2)!}$ = 3P2 = $\frac{n!}{(n-m)!}$: Permutation Rule: repetition not allowed and order is important.

Prove for all sets m<n
A| = m , |B| = n

a(1) a(2) a(3) ... a(m)
a(n) a(n-1) a(n-2) ... a(n-m) = a(n) * a(n-1) * a(n-2) *... * a(n-m) = nPm = $\frac{n!}{(n-m)!}$

Therefore if m<n there are $\frac{n!}{(n-m)!}$ functions from A to B.

SORU 3)

1 and 3: If B ⊆ A, and {1, 3} ⊆ B, B can be represented uniquely as B = {1, 3} ∪ C, where C is a subset of {2, 4, . . . , n}. As the set {2, 4, . . . , n} has n − 2 elements, there are 2n−2 such subsets C. Hence, there are 2n−2 subsets B of A which contain both 1 and 3.

1:

3:

|1U3|=|1|+|3|-|1n3|

SORU 4) Her katta bir daire bulunan 11 katlı bir blokta bulunan 6 daireyi böcekler basmıştır.

a) Böcek basan evlerin hepsinin yan yana olduğu kaç durum vardır?

(a) We want to count the permutations of six Ts (presence of termites) and five As (absence of termites) where all six Ts are together. Think of the six Ts as a single symbol T0 . Then count the permutations of five As and T0 . This gives a total of P (6; 5, 1) = 6! 5!1! = 6.

b) Böcek basan hiçbir evin yanyana olmadığı kaç farklı durum vardır?

(b) This time we want those permutations of six Ts and five As where no two Ts are together. There is only one possibility (alternate the Ts and As starting with a T): T AT AT AT AT AT.

SORU 4) Bir reklam kampanya mektubunun üç gruptan kişilere gönderilmesi düşünülmektedir: Fenerbahçe derneği üyeleri (F), yılda 45.000 TL'den çok kazananlar (Z), ve 5 yaşın altında çocuğu olanlar (C) . Mektup listesinde 15.000 Fenerbahçe derneği üyesi ve 15.000 Galatasaray derneği üyesi (G) toplam 30.000 kişi bulunmaktadır. Listedeki 30.000 kişiden 17.500'ü yılda 45.000 TL'den fazla kazanmaktadır ve bunların 10.001'i Fenerbahçe derneği üyeleridir. 3500 kişinin 5 yaşın altında çocuğu vardır ve bunların 1000'i Galatasaray, 2500'ü ise Fenerbahçe derneği üyesidir, 900'ü yılda 45.000 TL'den fazla kazanmaktadır. Sadece 1 Fenerbahçe derneği üyesinin hem yıl 45.000 TL'den çok kazancı hem de 5 yaşın altında çocuğu vardır.

a) Mektup listesinde kaç kişi Fenerbahçe derneği üyesi veya yılda 45.000 TL'den çok kazançlı veya 5 yaşın altında çocuk sahibidir. (Veya dilsel değil mantıksal veyadır.)

b) Galatasaray derneği üyesi olup kazancı 45.000 TL'den az olan kaç kişi vardır?

Example 7. A particular political campaign mailing is expected to appeal to three groups of people: liberals, people earning more than \$45,000 a year, and people with children under five years of age. The mailing list includes 30,000 people, including 15,000 conservatives and 15,000 liberals. Of the 30,000 on the mailing list, 17,500 earn more than \$45,000 a year, including 10,001 of the liberals. In the set of people, 3500 have children under five years of age, including 1000 conservatives, 2500 liberals, and 900 of those who earn more than \$45,000 a year. Only one of the liberals earns more than \$45,000 a year and also has children under the age of five. How many people on the mailing list are liberals, or earn more than \$45,000 a year, or have children under five years of age? (As usual, by *or* we mean the *inclusive or*.)

Solution. Among people on the mailing list, let L be the set of liberals, E the set of people who earn more than \$45,000 a year, and C the set of people who have children under five years of age (see Figure 1.15).

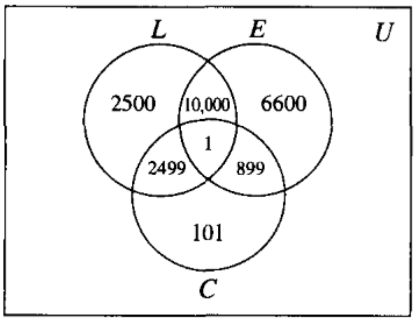

Figure 1.15 Counting liberals and children.

The Principle of Inclusion-Exclusion for Three Sets says that

$$\begin{aligned}|L \cup E \cup C| &= |L| + |E| + |C| - |L \cap E| - |L \cap C| - |E \cap C| + |L \cap E \cap C| \\ &= 15{,}000 + 17{,}500 + 3500 - 10{,}001 - 2500 - 900 + 1 \\ &= 22{,}600\end{aligned}$$ ■

Here, as often happens, there is a different way to count this collection of elements. First, note that of the 900 people who have children under five years of age and who earn more than \$45,000 a year, only one is a liberal; the other 899 are conservatives. So, among the 1000 conservatives with children under five years of age, $1000 - 899 = 101$ do not earn more than \$45,000. There are 15,000 liberals, plus $17{,}500 - 10{,}001 = 7499$ conservatives making more than \$45,000 a year, plus 101 conservatives with children under five years of age. Therefore, the answer is

$$15{,}000 + 7{,}499 + 101 = 22{,}600$$

SORU 6) S= {1, 2, 3, . . . , 26, 27} kümesinden herhangi 15 tamsayı seçilirse seçilen sayılar arasında toplamı 28 olan en az iki sayı vardır. Bunu ispatlayınız.

(a) There are 13 pairs of numbers among numbers of S whose sum is exactly 28, namely {1, 27}, {2, 26}, . . . , {13, 15}. Denote these sets by H1, H2, . . ., H13 respectively. Denote by H14 = {14}. Now we choose 15 numbers (pigeons) among numbers of S, or the same we assign to each chosen number(pigeon) the set (the pigeonhole) it belongs to. Since the number of pigeons is strictly more than the number of pigeonholes then by the Pigeonhole Principle, there are two pigeons (two numbers) that will be assign to the same pigeonhole, or the same, sum up to 28.

SORU 7) $\sum_{i=1}^{n}(2i-1) = n^2$ olduğunu ispatlayınız.

Solution:

Basis Step: It is true for $n = 1$, since 1=1.

Inductive Step: Assume that it is true for n, so $\sum_{i=1}^{n}(2i-1) = n^2$. Show that it is true for $n+1$, so $\sum_{i=1}^{n+1}(2i-1) = (n+1)^2$. Indeed,

$$\sum_{i=1}^{n+1}(2i-1) = \sum_{i=1}^{n}(2i-1) + 2(n+1) - 1 = n^2 + 2n + 1 = (n+1)^2.$$

SORU 8) 100 kişilik bir grupta 85 erkek ve 15 kadın vardır. Bu gruptan 5 erkek ve 5 kadın üye seçimi yapılacaktır. Kaç farklı üye seçimi yapılabilir?

Solution: There are (85 5) ways to pick the 5 men, and (15 5) ways to pick the 5 women. By the product rule , there are (85 5)*(15 5) ways to pick the committee of 5 men and 5 women.

SORU 9) A ile B şehrini birleştiren 3 köprü ve B ile C şehrini birleştiren dört köprü vardır. Bir satıcı A şehrinden C şehrine B şehrinden geçerek gidecektir. Satıcının A şehrinden C şehrine farklı köprülerden gidip tekrar A şehrine geri döneceği kaç farklı yolculuk planlanabilir?

There are three bridges connecting two towns, A and B. Between towns B and C there are four bridges. A salesperson has to travel from A to C via B. Find (1) the number of possible choices for bridges from A to C, (2) the number of choices for round-trip travel from A to C, (3) the number of choices for round-trip travel from A to C if no bridge is repeated.

Solution. We use the rule for sequential counting in each case. There are 3·4 = 12 routes from A to C, 3·4·4·3 = 144 round-trips, and 3·4·3·2 = 72 round-trips without repeat bridges.

SORU 10) Tüm 4 I harfinin "veya" tüm 4 S harfinin birlikte olması koşulu ile MISSISSIPPI kelimesindeki harfler kaç farklı şekilde sıralanabilir.

Buradaki "veya" bilgisayar bilimleri ve matematikteki "veya" kapısı gibidir.

If I treat IIII as a single letter there are 8!/4!2! permutations. The factorials in the denominator take into account the fact that there are four Ss and two Ps. There are the same number of permutations containing SSSS. I can add those two answers to find the number of permutations containing one or the other – as long as I correct because I've double counted the permutations that contain both (inclusionexclusion principle). There are 5!/2! of those, so my answer is

2 × 8! /(4!2!) − 5!/ 2! = 1680 − 60 = 1620.

I gave partial credit for knowing how to calculate the number of permutations when some letters are repeated. I was surprised at how many students didn't see the need to worry about double counting the strings in which both blocks appeared. I'd hoped my hint would point you in that direction

SORU 11) Her biri dört şıklı ve tek şıkkın işaretlendiği bir test

a) kaç farklı şekilde işaretlenebilir?

4^{20} possible tests.

b) bulduğunuz sayı yaklaşık olarak kaç basamaklıdır?

$4^{20} = 2^{40} = (2^{10})^4 \approx (10^3)^4 = 10^{12}$. Öyle ise yaklaşım 13 basamaklıdır.

SORU 12) A park alanında tek bir çizgi halinde n adetlik boş park yeri vardır. Arabalar geldikçe rasgele park etmeye başlamıştır.

a) Sadece iki adet boş park yeri kaldıysa ve acemi bir şoför ancak iki park yeri yanyana olduğunda park edebiliyorsa, acemi şoförü park edebilme olasılığı nedir?

(a) If exactly two spaces are free, what is the probability that she is able to park?

Solution

I need to compute

$$\frac{\text{number of configurations with adjacent empty spaces}}{\text{number of ways to pick two empty spaces}}.$$

To find the numerator, treat the two empty spaces as one item among $n-1$. There are $n-1$ places to put that pair. The denominator is just $\binom{n}{2}$ so the answer is

$$\frac{n-1}{\frac{n(n-1)}{2}} = \frac{2}{n}.$$

(b) Eğer f adet boş park yeri varsa (bir önceki soruda f=2'ydi), acemi şoförün kesin park edebilmesi için f'nin ne olması gerektiğini n çift olduğu durum için n cinsinden veriniz. (Cevabınızı açıklayınız.)

(b) Suppose f spaces are free. (In the previous question $f = 2$.) What is the minimum value of f that guarantees that Auntie Em can park? (Explain how you know you've answered the question correctly.)

Solution

Think of the parking lot as a bit string, with 0 for empty spaces.

If Auntie Em can't park, then every 0 must be followed by a 1, except possibly at the end.

If we want as many 0's as possible, we don't want two 1's together. That says that the bitstrings $0101 \cdots 01$ and $101 \cdots 10$ (for n even) and $0101 \cdots 010$ (for n odd) have the fewest 1s that can prevent Auntie Em. Therefore $f = 1 + n/2$ for n even and $f = 1 + (n+1)/2$ for n odd.

(c) (Optional, hard, take home). Answer the first question if there are f empty spaces. (You've just done $f = 2$, and found values of f that make the probability 1.) Check that your answer agrees with your answer to the first question. Evaluate your expression when $n = 16$ and $f = 4$.

Solution

To answer this question I counted the configurations where she couldn't park. I considered two cases.

- If the bitstring doesn't end with a 0 then every 0 must be followed by a 1. So think of building the string from f copies of (01) and the remaining $n - 2f$ copies of 1. That can be done $\binom{n-f}{f}$ ways.
- For the bitstrings ending with 0 I need $f - 1$ copies of (01) and $n - 2f + 1$ more