Quantum computing 1st laboratory

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1 13 variantas

1.1 calculate

$$\frac{10}{1-2i}+(1-i)^3=\frac{10}{1-2i}\cdot\frac{1+2i}{1+2i}+(1-i)(1-2i-1)=\frac{10+20i}{5}+(1-i)(-2i)$$
$$=2+4i+-2-2i=2i$$

1.2 $u_1 = 2+3i$, $w = 1-i$, $z = 1+i$

$$|w|+\bar{u}+\frac{z^2}{|z|}=\sqrt{1^2+(-1)^2}+2-3i+\frac{(1+i)^2}{\sqrt{1^2+1^2}}=\sqrt{2}+2-3i+\frac{1+2i-1}{\sqrt{2}}=$$
$$=2+\sqrt{2}-3i+\sqrt{2}i=2+\sqrt{2}+(\sqrt{2}-3)i$$

1.3 Write the complex number in polar representation ($\rho e^{i\theta}$)

$$z=-10+10i$$
$$\rho=|z|=\sqrt{(-10)^2+(10)^2}=10\sqrt{2}$$
$$\theta=\arctan(\frac{10}{-10})=\arctan(-1)=\frac{3\pi}{4}$$
$$z(\text{in polar})=\rho e^{\theta i}=10\sqrt{2}e^{\frac{3\pi}{4}i}$$

1.4 Solve equation

$$3x^2-3x+3=0$$
$$x^2-x+1=0$$
$$D=b^2-4ac=1-4=-3$$
$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$$
$$x_{1,2}=\frac{1\pm\sqrt{3}i}{2}=0.5\pm\frac{\sqrt{3}}{2}i$$

1.5 Are these 3 vectors are linearly independent?

$$\vec{v_1}=\begin{bmatrix}1+4i\\-4-i\\3+4i\\1-2i\end{bmatrix},\vec{v_2}=\begin{bmatrix}3-5i\\-5+3i\\2-3i\\-4-i\end{bmatrix},\vec{v_3}=\begin{bmatrix}-3-3i\\-3+4i\\-4-5i\\3+3i\end{bmatrix}$$

To check if they are linearly independent - we have to solve $Ax=0$ equation (We can calculate the determinant, and check whether its not zero = linear independent)

$$\left[\begin{array}{ccc|c}1+4i&3-5i&-3-3i&0\\-4-i&-5+3i&-3+4i&0\\3+4i&2-3i&-4-3i&0\\1-2i&-4-i&3+3i&0\end{array}\right]\rightsquigarrow\left[\begin{array}{ccc|c}1+4i&3-5i&-3-3i&0\\0&-34i&-28-23i&0\\3+4i&2-3i&-4-3i&0\\1-2i&-4-i&3+3i&0\end{array}\right]\rightsquigarrow$$

$$\left[\begin{array}{ccc|c}1+4i&3-5i&-3-3i&0\\0&-34i&-28-23i&0\\0&-15+8i&5+2i&0\\1-2i&-4-i&3+3i&0\end{array}\right]\rightsquigarrow\left[\begin{array}{ccc|c}1+4i&3-5i&-3-3i&0\\0&-34i&-28-23i&0\\0&-15+8i&5+2i&0\\0&7-6i&12i&0\end{array}\right]\rightsquigarrow$$

$$\left[\begin{array}{ccc|c}1+4i&3-5i&-3-3i&0\\0&-34i&-28-23i&0\\0&0&-536-291i&0\\0&7-6i&12i&0\end{array}\right]\rightsquigarrow\left[\begin{array}{ccc|c}1+4i&3-5i&-3-3i&0\\0&-34i&-28-23i&0\\0&0&-536-291i&0\\0&0&742-7i&0\end{array}\right]$$

We see that $742 - 7iz = 0 \rightarrow z = 0$ Simiraly $-34iy - (28 - 23i)z = 0 (z = 0) \rightarrow y = 0$ and $(1 + 4i)z + (3 - 5i)y + (-3 - 3i)z = 0 \rightarrow x = 0$

Which means that vectors $\vec{v_1}, \vec{v_2}, \vec{v_3}$ are linearly independent.

1.6 Raskite $A^2 + 3B^\dagger A^{-2} + C^{-1}$ (pateikdami detalius skaičiavimus), jeigu:

$$A = \begin{bmatrix} 4+i & -4-6i \\ 3+5i & 6+7i \end{bmatrix}, B = \begin{bmatrix} -9+7i & 7-8i \\ -4+4i & 5-2i \end{bmatrix}, C = \begin{bmatrix} 6-7i & -3+6i \\ 8-4i & -5+6i \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4+i & -4-6i \\ 3+5i & 6+7i \end{bmatrix} \cdot \begin{bmatrix} 4+i & -4-6i \\ 3+5i & 6+7i \end{bmatrix} =$$

$$= \begin{bmatrix} (4+i)^2 + (-4-6i)(3+5i) & (4+i)(3+5i) + (3+5i)(6+7i) \\ (4+i)(-4-6i) + (-4-6i)^2 & (3+5i)(-4-6i) + (6+7i)^2 \end{bmatrix} =$$

$$= \begin{bmatrix} (15+8i) + (38i+18) & (7+23i) + (51i-17) \\ (20i-10) + (48i-20) & (-12-18i-20i+30) + (84i-13) \end{bmatrix} =$$

$$= \begin{bmatrix} 33+46i & -10+74i \\ 69i-30 & 46i+5 \end{bmatrix}$$

$$B^\dagger = \begin{bmatrix} -9-7i & -4-4i \\ 7+8i & 5+2i \end{bmatrix}$$

$$3B^\dagger = 3\begin{bmatrix} -9-7i & -4-4i \\ 7+8i & 5+2i \end{bmatrix} = \begin{bmatrix} -27-21i & -12-12i \\ 21+24i & 15+6i \end{bmatrix}$$

Since

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{(4+i)(6+7i) - (-4-6i)(3+5i)}\begin{bmatrix} 6+7i & 4+6i \\ -3-5i & 4+i \end{bmatrix} = \frac{1}{-1+72i}\begin{bmatrix} 6+7i & 4+6i \\ -3-5i & 4+i \end{bmatrix}$$

$$A^{-2} = \frac{1}{-1+72i}\begin{bmatrix} 6+7i & 4+6i \\ -3-5i & 4+i \end{bmatrix} \cdot \frac{1}{-1+72i}\begin{bmatrix} 6+7i & 4+6i \\ -3-5i & 4+i \end{bmatrix} = \frac{1}{-5183-144i}\begin{bmatrix} 5+46i & -8+92i \\ 10-74i & 33-30i \end{bmatrix}$$

$$C^{-1} = \frac{1}{12+11i}\begin{bmatrix} -5+6i & 3-6i \\ -8+4i & 6-7i \end{bmatrix}$$

$$A^2 + 3B^\dagger A^{-2} + C^{-1} =$$

$$= \begin{bmatrix} 33+46i & -10+74i \\ 69i-30 & 46i+5 \end{bmatrix} + \begin{bmatrix} -27-21i & -12-12i \\ 21+24i & 15+6i \end{bmatrix} \cdot \frac{1}{-5183-144i}\begin{bmatrix} 5+46i & -8+92i \\ 10-74i & 33-30i \end{bmatrix} + \frac{1}{12+11i}\begin{bmatrix} -5+6i & 3-6i \\ -8+4i & 6-7i \end{bmatrix}$$

$$= \begin{bmatrix} 33+46i & -10+74i \\ 69i-30 & 46i+5 \end{bmatrix} + \frac{1}{-5183-144i}\begin{bmatrix} -177-579i & 1392-2352i \\ -405+36i & -1701+1488i \end{bmatrix} + \frac{1}{12+11i}\begin{bmatrix} -5+6i & 3-6i \\ -8+4i & 6-7i \end{bmatrix}$$

using calculator(because these are enormous numbers) =

$$\frac{1}{1424863925} \cdot \begin{bmatrix} 47105811246 + 41905358578i & 10873175506 + 130995334033i \\ -14417256847 + 106158201246i & 7553341983 + 64315479656i \end{bmatrix}$$

1.7 Write vector $\vec{v_4}$ in the basis of $\vec{v_1}, \vec{v_2}, \vec{v_3}$

gauso budu galim bandyti

$$\vec{v_1} = \begin{bmatrix} 1-4i \\ -1+4i \\ -1-3i \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 2+i \\ -1+3i \\ -1-2i \end{bmatrix}, \vec{v_3} = \begin{bmatrix} -4+2i \\ -4+2i \\ 4-2i \end{bmatrix}, \vec{v_4} = \begin{bmatrix} 3+i \\ 1-4i \\ -2-2i \end{bmatrix}$$

Kadangi $v_4 = axv_1 + yv_2 + zv_3$ naudokime gaudo metoda kad gauti lygties sprendini

$$\left[\begin{array}{ccc|c} 1-4i & 2+i & -4+2i & 3+i \\ -1+4i & -1+3i & -4+2i & 1-4i \\ -1-3i & -1-2i & 4-2i & -2-2i \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1-4i & 2+i & -4+2i & 3+i \\ 0 & 17 & 8+36i & -8-19i \\ -1-3i & -1-2i & 4-2i & -2-2i \end{array}\right] \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 1-4i & 2+i & -4+2i & 3+i \\ 0 & 17 & 8+36i & -8-19i \\ 0 & -10+9i & -14-28i & -10+16i \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1-4i & 2+i & -4+2i & 3+i \\ 0 & 17 & 8+36i & -8-19i \\ 0 & 0 & 166-188i & -421+154i \end{array}\right] \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 1-4i & 2+i & -4+2i & 3+i \\ 0 & 17 & 8+36i & -8-19i \\ 0 & 0 & 1 & -\frac{2907}{1850}-\frac{788}{925}i \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1-4i & 2+i & 0 & -\frac{923}{185}+\frac{136}{185}i \\ 0 & 17 & 8+36i & -8-19i \\ 0 & 0 & 1 & -\frac{2907}{1850}-\frac{788}{925}i \end{array}\right] \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 1-4i & 2+i & 0 & -\frac{923}{185}+\frac{136}{185}i \\ 0 & 17 & 0 & -\frac{4828}{185}+\frac{8211}{185}i \\ 0 & 0 & 1 & -\frac{2907}{1850}-\frac{788}{925}i \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1-4i & 2+i & 0 & -\frac{923}{185}+\frac{136}{185}i \\ 0 & 1 & 0 & -\frac{284}{185}+\frac{483}{185}i \\ 0 & 0 & 1 & -\frac{2907}{1850}-\frac{788}{925}i \end{array}\right] \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 1-4i & 0 & 0 & \frac{128}{185}-\frac{546}{185}i \\ 0 & 1 & 0 & -\frac{284}{185}+\frac{483}{185}i \\ 0 & 0 & 1 & -\frac{2907}{1850}-\frac{788}{925}i \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{136}{185}-\frac{2}{185}i \\ 0 & 1 & 0 & -\frac{284}{185}+\frac{483}{185}i \\ 0 & 0 & 1 & -\frac{2907}{1850}-\frac{788}{925}i \end{array}\right]$$

Taigi $\vec{v_4} = (\frac{136}{185} - \frac{2}{185}i)\vec{v_1} + (-\frac{284}{185} + \frac{483}{185}i)\vec{v_2} + (\frac{-2907}{1850} - \frac{788}{925}i)\vec{v_3}$

1.8 8

using the vector basis from the 7th task, get orthonormal basis using Gram-Schmidt process (https://en.wikipedia.org/wiki/

$$\vec{v_1} = \begin{bmatrix} 1-4i \\ -1+4i \\ -1-3i \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 2+i \\ -1+3i \\ -1-2i \end{bmatrix}, \vec{v_3} = \begin{bmatrix} -4+2i \\ -4+2i \\ 4-2i \end{bmatrix}, \vec{v_4} = \begin{bmatrix} 3+i \\ 1-4i \\ -2-2i \end{bmatrix}$$

$$proj_u(v) = \frac{\langle v,u\rangle}{\langle u,u\rangle}u \quad u_1 = v_1 \quad u_k = v_k - \sum_{j=1}^{k-1} proj_{u_j}(v_k)$$

$$u_1 = \begin{bmatrix} 1-4i \\ -1+4i \\ -1-3i \end{bmatrix}$$

$$u_2 = \begin{bmatrix} 2+i \\ -1+3i \\ -1-2i \end{bmatrix} - proj_{u_1}\left(\begin{bmatrix} 2+i \\ -1+3i \\ -1-2i \end{bmatrix}\right) = \begin{bmatrix} 2+i \\ -1+3i \\ -1-2i \end{bmatrix} - \frac{\left\langle \begin{bmatrix} 2+i \\ -1+3i \\ -1-2i \end{bmatrix}, \begin{bmatrix} 1-4i \\ -1+4i \\ -1-3i \end{bmatrix} \right\rangle}{\left\langle \begin{bmatrix} 1-4i \\ -1+4i \\ -1-3i \end{bmatrix}, \begin{bmatrix} 1-4i \\ -1+4i \\ -1-3i \end{bmatrix} \right\rangle} \cdot \begin{bmatrix} 1-4i \\ -1+4i \\ -1-3i \end{bmatrix} =$$

$$= \begin{bmatrix} 2+i \\ -1+3i \\ -1-2i \end{bmatrix} - \frac{18+9i}{44} \cdot \begin{bmatrix} 1-4i \\ -1+4i \\ -1-3i \end{bmatrix} = \begin{bmatrix} 2+i \\ -1+3i \\ -1-2i \end{bmatrix} - \frac{1}{44} \cdot \begin{bmatrix} 54-63i \\ -54+63i \\ 9-63i \end{bmatrix} = \frac{1}{44}\begin{bmatrix} 34+107i \\ 10+69i \\ -53-25i \end{bmatrix}$$

$$u_3 = \begin{bmatrix}-4+2i\\-4+2i\\4-2i\end{bmatrix} - proj_{u_1}(\begin{bmatrix}-4+2i\\-4+2i\\4-2i\end{bmatrix}) - proj_{u_2}(\begin{bmatrix}-4+2i\\-4+2i\\4-2i\end{bmatrix})$$

$$\begin{bmatrix}-4+2i\\-4+2i\\4-2i\end{bmatrix} - \frac{\left\langle \begin{bmatrix}-4+2i\\-4+2i\\4-2i\end{bmatrix}, \begin{bmatrix}1-4i\\-1+4i\\-1-3i\end{bmatrix} \right\rangle}{\left\langle \begin{bmatrix}1-4i\\-1+4i\\-1-3i\end{bmatrix}, \begin{bmatrix}1-4i\\-1+4i\\-1-3i\end{bmatrix} \right\rangle} \cdot \begin{bmatrix}1-4i\\-1+4i\\-1-3i\end{bmatrix} - \frac{\left\langle \begin{bmatrix}-4+2i\\-4+2i\\4-2i\end{bmatrix}, \frac{1}{44}\begin{bmatrix}34+107i\\10+69i\\-53-25i\end{bmatrix} \right\rangle}{\left\langle \frac{1}{44}\begin{bmatrix}34+107i\\10+69i\\-53-25i\end{bmatrix}, \frac{1}{44}\begin{bmatrix}34+107i\\10+69i\\-53-25i\end{bmatrix} \right\rangle} \cdot \frac{1}{44}\begin{bmatrix}34+107i\\10+69i\\-53-25i\end{bmatrix} =$$

$$= \begin{bmatrix}-4+2i\\-4+2i\\4-2i\end{bmatrix} - \frac{1+7i}{22} \cdot \begin{bmatrix}1-4i\\-1+4i\\-1-3i\end{bmatrix} - \frac{7+499i}{475} \cdot \frac{1}{22}\begin{bmatrix}34+107i\\10+69i\\-53-25i\end{bmatrix} =$$

$$= \frac{1}{10450}\left(\begin{bmatrix}-41800+20900i\\-41800+20900i\\41800-20900i\end{bmatrix} - (1+7i) \cdot \begin{bmatrix}475-1900i\\-475+1900i\\-475-1425i\end{bmatrix} - (7+499i)\begin{bmatrix}34+107i\\10+69i\\-53-25i\end{bmatrix}\right) =$$

$$= \frac{1}{10450}\left(\begin{bmatrix}-41800+20900i\\-41800+20900i\\41800-20900i\end{bmatrix} - \begin{bmatrix}13775+1425i\\-13775+-1425i\\9500-4750i\end{bmatrix} - \begin{bmatrix}-53155+17715i\\-34361+5473i\\12104-26622i\end{bmatrix}\right) =$$

$$= \frac{1}{10450}\begin{bmatrix}-2420+1760i\\6336+16852i\\20196+10472i\end{bmatrix}$$

$$u_1 = \begin{bmatrix}1-4i\\-1+4i\\-1-3i\end{bmatrix}, \quad u_2 = \frac{1}{44}\begin{bmatrix}34+107i\\10+69i\\-53-25i\end{bmatrix} \quad u_3 = \frac{1}{10450}\begin{bmatrix}-2420+1760i\\6336+16852i\\20196+10472i\end{bmatrix}$$

1.9 Find eigenvectors and eigenvalues

$A = \begin{bmatrix}2 & i & 3\\0 & i & 0\\1-i & 2 & -1\end{bmatrix}$ To find eigenvalues we have to solve this equation:

$$|A-\lambda I| = 0$$

$$\left|\begin{bmatrix}2 & i & 3\\0 & i & 0\\1-i & 2 & -1\end{bmatrix} - \begin{bmatrix}\lambda & 0 & 0\\0 & \lambda & 0\\0 & 0 & \lambda\end{bmatrix}\right| = 0$$

$$\left|\begin{bmatrix}2-\lambda & i & 3\\0 & i-\lambda & 0\\1-i & 2 & -1-\lambda\end{bmatrix}\right| = 0$$

$$(2-\lambda)(i-\lambda)(-1-\lambda) - 3(i-\lambda)(1-i) = 0$$

$$(i-\lambda)((2-\lambda)(-1-\lambda) - 3(1-i)) = 0$$

$$(i-\lambda)(-2-2\lambda+\lambda+\lambda^2-3+3i) = 0$$

$$(i-\lambda)(\lambda^2-\lambda-5+3i) = 0$$

$$\lambda = i \text{ or } \lambda^2-\lambda-5+3i = 0$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{21-12i}}{2}$$

Eigenvalues: $\lambda_0 = i, \lambda_1 = \frac{1+\sqrt{21-12i}}{2}, \lambda_2 = \frac{1-\sqrt{21-12i}}{2}$
Eigenvectors:

$$(\lambda I - A)\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left(\begin{bmatrix}\lambda & 0 & 0\\0 & \lambda & 0\\0 & 0 & \lambda\end{bmatrix} - \begin{bmatrix}2 & i & 3\\0 & i & 0\\1-i & 2 & -1\end{bmatrix}\right)\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left(\begin{bmatrix}\lambda-2 & -i & -3\\0 & \lambda-i & 0\\-1+i & -2 & \lambda+1\end{bmatrix}\right)\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\lambda = i \to y \in \mathbb{C}$$

$$\begin{cases}(i-2)x - 3z = yi\\(-1+i)x + (i+1)z = 2y\end{cases}$$

$$\begin{cases}z = \frac{(i-2)x-yi}{3}\\(-1+i)x + (i+1)\cdot\frac{(i-2)x-yi}{3} = 2y \to x = \frac{-7-4i}{10}y\end{cases}$$

$$z = \frac{(i-2)x-yi}{3} = \frac{(i-2)\frac{-7-4i}{10}y - yi}{3} = ... = \frac{6-3i}{10}y$$

Eigenvector is:

$$\begin{bmatrix}\frac{-7-4i}{10}y\\y\\\frac{6-3i}{10}y\end{bmatrix}, \quad y \in \mathbb{C}$$

$$\lambda = \frac{1 \pm \sqrt{21-12i}}{2}$$

$$\begin{cases}(\frac{1\pm\sqrt{21-12i}}{2} - 2)x - yi - 3z = 0\\(\frac{1\pm\sqrt{21-12i}}{2} - i)y = 0 \to y = 0\\(-1+i)x - 2y + (\frac{1\pm\sqrt{21-12i}}{2} + 1)z = 0\end{cases}$$

$$\begin{cases}(\frac{1\pm\sqrt{21-12i}}{2} - 2)x - 3z = 0 \to z = \frac{-3\pm\sqrt{21-12i}}{6}x\\(-1+i)x + (\frac{1\pm\sqrt{21-12i}}{2}+1)z = 0 \to z = x\frac{1-i}{\frac{3\pm\sqrt{21-12i}}{2}} = x\frac{2(1-i)}{3\pm\sqrt{21-12i}} = x\frac{(1-i)(3\mp\sqrt{21-12i})}{-6+6i} = x\frac{(i-1)(-6-6i)(-3\pm\sqrt{21-12i})}{72} =\\= x\frac{(-3\pm\sqrt{21-12i})}{6}\end{cases}$$

These equations are equivalent, so the eigenvectors:

$$\begin{bmatrix}x\\0\\x\frac{(-3\pm\sqrt{21-12i})}{6}\end{bmatrix}, \quad x \in \mathbb{C}$$

3

1.10 Matricai A iš ankstesnės užduoties raskite

$$A = \begin{bmatrix}2 & i & 3\\0 & i & 0\\1-i & 2 & -1\end{bmatrix} \to A^{-1} = \begin{bmatrix}\frac{5+3i}{34} & \frac{-23+27i}{34} & \frac{15+9i}{34}\\0 & -i & 0\\\frac{4-1}{17} & \frac{2-9i}{17} & \frac{-5-3i}{17}\end{bmatrix}$$

$$Z = AX \to \text{ hermitian if } Z = Z^\dagger$$

Let't take Z as:

$$Z = \begin{bmatrix}5 & 4+5i & 6-16i\\4-5i & 13 & 7\\6+16i & 7 & 2\end{bmatrix}$$

$$A^{-1}AX = A^{-1}Z$$

a) kokia nors matrica X, tokia, kad AX butu nediagonaline Ermito matrica;

$$X = A^{-1}Z$$

$$X = \begin{bmatrix} \frac{5+3i}{34} & \frac{-23+27i}{34} & \frac{15+9i}{34} \\ 0 & -i & 0 \\ \frac{4-1}{17} & \frac{2-9i}{17} & \frac{-5-3i}{17} \end{bmatrix} \cdot \begin{bmatrix} 5 & 4+5i & 6-6i \\ 4-5i & 13 & 7 \\ 6+16i & 7 & 2 \end{bmatrix} = \begin{bmatrix} \frac{7+266i}{17} & \frac{-189+451i}{34} & \frac{-53+145i}{34} \\ -5-4i & -13i & -7i \\ \frac{1-149i}{17} & \frac{12-122i}{17} & \frac{12-139}{17} \end{bmatrix}$$

b) kokia nors matrica Y , tokia, kad AY butu nediagonaline unitarine matrica; Unitary matrix:

$$U \cdot U^{\dagger} = U^{\dagger} \cdot U = I$$

$$U = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

$$Y = A^{-1}AU$$

$$Y = \begin{bmatrix} \frac{5+3i}{34} & \frac{-23+27i}{34} & \frac{15+9i}{34} \\ 0 & -i & 0 \\ \frac{4-1}{17} & \frac{2-9i}{17} & \frac{-5-3i}{17} \end{bmatrix} \cdot \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix} = \begin{bmatrix} \frac{-3+5i}{34} & \frac{-9+15i}{34} & \frac{-27-23i}{34} \\ 0 & 0 & 1 \\ \frac{1+4i}{17} & \frac{3-5i}{17} & \frac{9+2i}{17} \end{bmatrix}$$

c) tenzorines sandaugas $Z \otimes A$ ir $A \otimes Z$, kur matrica Z yra:

$$Z = \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix}$$

$$Z \otimes A = \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} \otimes \begin{bmatrix} 2 & i & 3 \\ 0 & i & 0 \\ 1-i & 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 \cdot \begin{bmatrix} 2 & i & 3 \\ 0 & i & 0 \\ 1-i & 2 & -1 \end{bmatrix} & 2i \cdot \begin{bmatrix} 2 & i & 3 \\ 0 & i & 0 \\ 1-i & 2 & -1 \end{bmatrix} \\ -2i \cdot \begin{bmatrix} 2 & i & 3 \\ 0 & i & 0 \\ 1-i & 2 & -1 \end{bmatrix} & 1 \cdot \begin{bmatrix} 2 & i & 3 \\ 0 & i & 0 \\ 1-i & 2 & -1 \end{bmatrix} \end{bmatrix} =$$

$$= \begin{bmatrix} -2 & -i & -3 & 4i & -2 & 6i \\ 0 & -i & 0 & 0 & -2 & 0 \\ -1+i & -2 & 1 & 2i+2 & 4i & -2i \\ -4i & 2 & -6i & 2 & i & 3 \\ 0 & 2 & 0 & 0 & i & 0 \\ -2i-2 & -4i & 2i & 1-i & 2 & -1 \end{bmatrix}$$

$$A \otimes Z = \begin{bmatrix} 2 & i & 3 \\ 0 & i & 0 \\ 1-i & 2 & -1 \end{bmatrix} \otimes \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} = \begin{bmatrix} 2 \cdot \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} & i \cdot \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} & 3 \cdot \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} \\ 0 & i \cdot \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} & 0 \\ (1-i) \cdot \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} & 2 \cdot \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} & -1 \cdot \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} \end{bmatrix} =$$

$$= \begin{bmatrix} -2 & 4i & -i & -2 & -3 & 6i \\ -4i & 2 & 2 & i & -6i & 3 \\ 0 & 0 & -i & -2 & 0 & 0 \\ 0 & 0 & 2 & i & 0 & 0 \\ -1+i & 2i+2 & -2 & 4i & 1 & -2i \\ -2i-2 & 1-i & -4i & 2 & 2i & -1 \end{bmatrix}$$