1

QUANTITATIVE CONCEPTS

LEARNING OUTCOME

- Define the concept of interest;
- Compare simple and compound interest;
- Define present value, future value, and discount rate;
- Describe how time and discount rate affect present and future values;
- Explain the relevance of net present value in valuing financial investments;
- Describe applications of time value of money;
- Explain uses of mean, median, and mode, which are measures of frequency or central tendency;
- Explain uses of range, percentile, standard deviation, and variance, which are measures of dispersion;
- Describe and interpret the characteristics of a normal distribution;
- Describe and interpret correlation.

I. The time value of money

1. Interest

Interest

- Is the **cost of borrowing** money
- Is the amount of money lenders receives for lending out money and borrowers have to pay more for borrowing

Two main types of interest	**Simple interest**	The cost to the borrower or the rate of return to the lender, per period, on the original principal.
		Simple interest = Simple interest rate × Principal × Number of periods
	Compound interest	Is often referred to as "interest on interest".
		Future value = Original principal x (1 + simple interest rate)$^{\text{no. of period}}$

I. The time value of money

1. Interest

Interest

Two main types of interest	**Simple interest**	***Example:*** You invest \$100 (the principal) at a 5% annual rate for one year. Compute the simple interest
		Answer: $\$100 \times 5\% \times 1 = \$5$
	Compound interest	***Example:*** If an amount of \$5,000 is deposited into a savings account at an annual interest rate of 5%, **compounded monthly**, what is the value of the investment after 10 years?
		Answer: $5000 \times \left(1 + \frac{0.05}{12}\right)^{(12 \times 10)} = 8235.05$

I. The time value of money

1. Interest

Annual Percentage Rate and Effective Annual Rate

Annual percentage rate (APR)	Effective Annual Rate (EAR)
A simple interest rate that does not involve compounding	The annual rate of interest that investors actually realize as a result of compounding

$$EAR = [(1+\frac{APR}{\text{number of periods per year}})^{\text{numberof period per years}}] - 1$$

__Example:__ Compute EAR if the stated annual rate is 12%, compounded quarterly (3 months).

__Answer:__

APR = 12%; number of period = 4

Periodic rate is: $\frac{APR}{\text{number of periods per year}} = \frac{12}{4} = 3\%$.

Thus, EAR = $(1 + 0.03)^4 - 1 = 1.1255 - 1 = 0.1255 = 12.55\%$.

I. The time value of money

2. Present Value and Future Value

<u>Example: Simple and compound interest rates</u>

A credit card charges interest at an APR of 15.24%, compounded daily. A bank pays 0.2% monthly on the average amount on deposit over the month. A loan is made with a 6.0% annual rate, compounded quarterly. The following table shows what the expected annual rate is for each of these situations. The EAR is higher than the APR because of compounding.

<u>Answer:</u>

	APR	EAR
Credit card	15.24%	$16.46\% = \left[\left(1+\dfrac{0.1524}{365}\right)^{365}\right] - 1$
Bank deposit	2.4% (= 0.2% x 12)	$2.43\% = \left[\left(1+\dfrac{0.024}{12}\right)^{12}\right] - 1$
Loan	6.0%	$6.14\% = \left[\left(1+\dfrac{0.06}{4}\right)^{4}\right] - 1$

I. The time value of money

2. Present Value and Future Value

People would rather have money today than in the future.

The investor can **distinguish** between the **worth of investment** that offers different returns at a different time

Present value (PV)

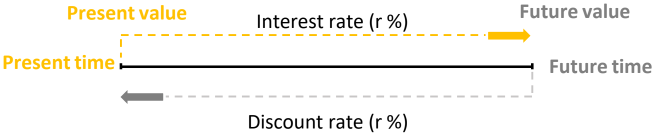
- The current value of a future sum of money or stream of cash flows given a specified rate of return.
- Determining the appropriate discount rate is the key to properly valuing future cash flows

$$PV=\frac{FV}{(1+r)^n}$$

- The value of a current asset at a future date based on an assumed rate of growth.
- Estimate how much an investment made today will be worth in the future.

$$FV = PV\times(1+r)^n$$

Future value (FV)



I. The time value of money

2. Present Value and Future Value

Example: Comparing investment

You are choosing between two investments of equal risk, given by discount rate to use is 12% and outflow for each is £500

Project A is expected to pay out £1,000 three years from now

Present value of £1,000 in three years discounted at 12%

$$= \frac{£\ 1{,}000}{(1.12)^3} = £711.78$$

Project B is expected to pay out £1,200 five years from now

Present value of £1,200 in five years discounted at 12%

$$= \frac{£\ 1{,}200}{(1.12)^5} = £680.91$$

Project A is worth more in present value terms, so it is the better investment.

I. The time value of money

2. Present Value and Future Value

Net present value (NPV)

Definition	Formula
NPV is **the present value** of future cash flows **minus the value** of the cost of the initial investment	$= \frac{\textbf{Cash flow}}{\mathbf{(1+i)^t}}$ **– initial investment** *where:* i: required return (discount rate) t: number of time periods

Example:

Imagine a project that costs $1,000 and will provide three cash flows of $500, $300, and $800 over the next three years. Assume there is no salvage value at the end of the project and the required rate of return is 8%. Calculate the NPV of the project.

$$NPV = \frac{\$500}{(1+0.08)^1} + \frac{\$300}{(1+0.08)^2} + \frac{\$800}{(1+0.08)^3} - \$1000$$

= $355.23

I. The time value of money

2. Present Value and Future Value

Application of the time value of money

Valuation of Financial Instruments

The current value (price) of a financial product should **equal the present value of its expected future cash flows (PV)**.

↓

Investors **can assess the price** of a financial instrument and evaluate its price.

Price < PV	**Price = PV**	**Prive > PV**
↓	↓	↓
Underpriced	Priced fairly	Overpriced

I. The time value of money

2. Present Value and Future Value

Application of the time value of money

Evaluation of Annuity		
Involves the initial payment of an amount, in exchange for a fixed number of future payments of a certain amount		
Annuity Due	Cash flows (A) occur **at the beginning** of each period	
	$FV = A \times \frac{(1+i)^n - 1}{i} \times (1+i)$	$PV = A \times [\frac{1}{i} - \frac{1}{i(1+i)^n}] \times (1+i)$
Ordinary Annuity	Cash flows (A) occur **at the end** of each period.	
	$FV = A \times \frac{(1+i)^n - 1}{i}$	$PV = A \times [\frac{1}{i} - \frac{1}{i(1+i)^n}]$
Perpetuities	Annuities with **infinite lives**	
	Perpetuities has no FV	$PV = \frac{A}{i}$

A: Cash flow each period
i: rate per period
n: number of period

I. The time value of money

2. Present Value and Future Value

Application of the time value of money

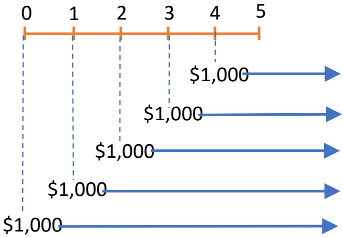
Example: Let's assume that you invest $1,000 at the beginning of the year for the next five years, at 5% interest. How much would you have at the end of the five-year period?

This is an Annuity Due.

The total amount you get after 5 years is:

$$FV = A \times \frac{(1+i)^n - 1}{i} \times (1+i) = 1{,}000 \times \frac{(1+5\%)^5 - 1}{5\%} \times (1+5\%) = \mathbf{5{,}801.9}$$

It can also be explained in this way



$$\mathbf{FV = PV \times (1 + r)^n}$$

$FV_5 = 1{,}000 \times (1.05)^1 = \$1{,}050.0$

$FV_4 = 1{,}000 \times (1.05)^2 = \$1{,}102.5$

$FV_3 = 1{,}000 \times (1.05)^3 = \$1{,}157.6$

$FV_2 = 1{,}000 \times (1.05)^4 = \$1{,}215.5$

$FV_1 = 1{,}000 \times (1.05)^5 = \$1{,}276.2$

FV of an annuity due: $**5,801.9**

I. The time value of money

Vocabulary

Vocabulary	Meaning
Interest	Lãi vay
Principal	Gốc
Lender	Người cho vay
Borrower	Người đi vay
Simple/ Compound interest	Lãi đơn/ kép
Effective annual rate	Lãi suất thực hưởng
Saving	Tiết kiệm
Deposit	Tiền gửi
Quarterly	Hàng quý
Net present value	Giá trị hiện tại thuần
Ordinary Annuity	Dòng tiền đều cuối kì
Annuity due	Dòng tiền đều đầu kì
Perpetuities	Dòng tiền đều vô hạn

II. DESCRIPTIVE STATISTICS

1. Basic knowledge of statistics and probability

Probability

Definition
Describe how likely an event could occur. The probability of an event is a number between 0 and 1 (0% - 100%) *E.g: When flipping a coin, 2 events may occur: getting head side or getting number side. The probability of getting head side is: 1 : 2 = ½ = 50%*

Rules			
Addition rules		**Multiplication rules**	
Event A and B are mutually exclusive	Event A and B are non-mutually exclusive	Event A and B are independent	Event A and B are dependent
If event A occurs, then event B cannot occur	A & B still can occur at the same time	The outcome of event A does not affect the outcome of B	The outcome of event A affects the outcome of B

II. DESCRIPTIVE STATISTICS

1. Basic knowledge of statistics and probability

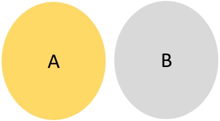
Probability Rules

Addition rules

Mutually exclusive events

E.g: flipping a coin, get head side or number side

The probability that at least A or B occur:
P(A or B) = P(A) + P(B)

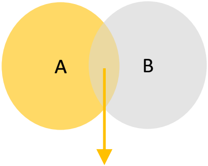


P (both A and B) = 0

Non-mutually exclusive events

E.g: Choose 2 cards from 3 cards J, Q, K.
Calculate the probability that at least 1 J or 1 Q is chosen.
These are non-mutually exclusive events because there is a chance that both J and Q are chosen

The probability that A or B will occur:
P(A or B) = P(A) + P(B) – P(A and B)
in which A: 1J is chosen
B: 1Q is chosen



Probability that both A & B happen

II. DESCRIPTIVE STATISTICS

1. Basic knowledge of statistics and probability

Probability Rules

Multiplication rules

Dependent events

E.g: Event A: Number of people who smoke increases
Event B: Number of people who have lung cancer increases

If event A happens, the probability of event B increases

Probability that number of people who smoke and number of people who have lung cancer increase : **P(A&B)= P(B) x P(A|B)**

Independent events

E.g: There are 2 bags, each bag has 5 red balls and 5 blue balls. Take 1 ball in each bag.
Event A: Take the red ball in the first bag
Event B: Take the red ball in the second bag

The probability of these two events does not affect each other.

Probability that the red balls are taken in both first and second bag:
P(A&B) = P(A) x P(B)

II. DESCRIPTIVE STATISTICS

1. Basic knowledge of statistics and probability

	Factorial	Permutation	Combination
Formula	$P_n = n!$ $= n \times (n-1) \times \ldots \times 1$	$P_n^k = \dfrac{n!}{(n-k)!}$	$C_n^k = \dfrac{n!}{(n-k)!k!}$
Example	How many ways to arrange 3 students into 3 chairs?	From the given digits: 2, 3, 5, how many numbers have 2 different digits that can be created?	There is a group of 3 students, 2 students have to sweep the floor. How many ways to choose those 2 students?
Solution	$P_3 = 3!$ $= 3 \times 2 \times 1$ $= 6$ ways	$P_3^2 = \dfrac{3!}{(3-2)!}$ $= 6$ numbers	$C_3^2 = \dfrac{3!}{(3-2)!2!}$ $= 3$ ways

II. DESCRIPTIVE STATISTICS

2. Measures of Frequency and Average

Arithmetic mean

Definition

The arithmetic mean is the sum of the observation values divided by the number of observations

Example

Let's say that a stock's returns over the last five years are 20%, 6%, -10%, -1%, and 6%. Calculate the arithmetic mean.

Answer:

$$\text{The arithmetic mean} = \frac{20 + 6 - 10 - 1 + 6}{5} = 4.2$$

II. DESCRIPTIVE STATISTICS

2. Measures of Frequency and Average

Geometric mean

Definition

Geometric mean return is the average return assuming that returns are compounding.

Formula

$$\textbf{Geometric mean} = [(1+r_1)\times\ldots(1+r_t)]^{1/t} - 1$$

Where:

- r_t: the return in period t expressed using decimals
- t : the number of periods

Example

Let's say that a stock's returns over the last five years are 20%, 6%, -10%, -1%, and 6%. Calculate the geometric mean.

Answer:

$$\sqrt[5]{(1+20\%)\times(1+6\%)\times(1-10\%)\times(1-1\%)\times(1+6\%)} - 1 = 3.74\%$$

II. DESCRIPTIVE STATISTICS

2. Measures of Frequency and Average

Median	is the midpoint of a data set when the data is arranged in ascending or descending order. Half the observations lie above the median while the other half lie below.
	<u>Example:</u> The median of 4, 1 and 7 is 4 because when the numbers are put in order (1,4,7) the number 4 is in the middle.

The value that occurs most frequently in a data set ▪ **Unimodal:** When a distribution has one value that appears most frequently ▪ **Bimodal or trimodal:** When a set of data has two or three values that occur most frequently	**Mode**
<u>Example:</u> The mode of (4, 4, 2, 3, 2, 2) is 2 because it occurs three times, which is more than any other number.	

II. DESCRIPTIVE STATISTICS

3. Measures of Dispersion

Range	The distance between the largest and the smallest value in the data set range
	Range = Maximum value – Minimum value

Example: What is the range for the 5-year annualized total returns for five investment managers if the managers' individual returns were 30%, 12%, 25%, 20%, and 23%?

Answer:

Range = 30% − 12% = 18%

II. DESCRIPTIVE STATISTICS

3. Measures of Dispersion

Standard deviation (σ)

It measures the **variability** or volatility of a data set **around the arithmetic mean** of that data set

Formula:

$$= \sqrt{\frac{[X_1-E(X)]^2+[X_2-E(X)]^2+\cdots+[X_i-E(X)]^2}{n-1}}$$

X_i= observation i (one of n possible outcomes)
n = number of observations
E(X) = the arithmetic mean

Example: What is the standard deviation for the 5-year annualized total returns for 5 investment managers if the individual returns were 30%, 12%, 25%, 20%, 23%?

Answer:

$$\bar{X} = \frac{30+12+25+20+23}{5} = 22$$

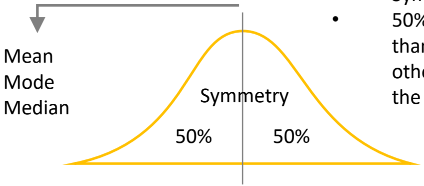
$$\sigma = \sqrt{\frac{(30-22)^2+(12-22)^2+(25-22)^2+(20-22)^2+(23-22)^2}{5-1}} = 6.67\%$$

6.67% is considered as small standard deviation, means that the values in the data set are close to the **mean ($\overline{X}$)** of the data set.

II. DESCRIPTIVE STATISTICS

3. Measures of Dispersion

Normal distribution	**Definition**
	A probability distribution that is symmetric about the mean, showing that data near the mean occurs more frequently than data far from the mean.
	Characteristics
	Being represented in a graph by a bell curve
	Having special importance in statistics because many probability distributions have a similar shape to normal distribution.

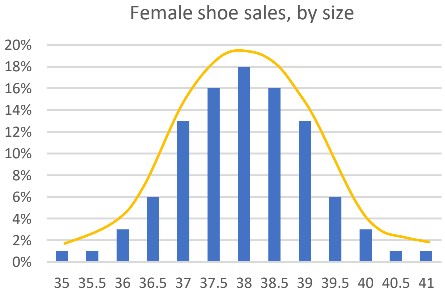


- Mean = Median = Mode
- Symmetry about the center
- 50% of the values are less than the mean while the other 50% are greater than the mean.

II. DESCRIPTIVE STATISTICS

3. Measures of Dispersion

Normal distribution example



- Mean = Median = Mode = 38
- Most of the women have a shoe size around 38.
- A very small number of women wears either extremely small or extremely big shoes.

  → Female shoe sizes follow normal distribution.

II. DESCRIPTIVE STATISTICS

4. Covariance

Definition	
measures the directional relationship between the returns on two variances	
Positive covarriance	**Negative Covariance**
The 2 variables move in the same direction e.i. if X increases, Y increases	The 2 variables move inversely e.i. if X increases, then Y decreases, vice versa

Formula

$$\boldsymbol{Cov(X,Y) = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}}$$

Where:

X_i: the ith value of X variable
Y_i: the ith value of Y variable
$\bar{X}$: the mean of X
$\bar{Y}$: the mean of Y
n: number of observations

II. DESCRIPTIVE STATISTICS

4. Covariance

Formula

$$\boldsymbol{Cov(X,Y) = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}}$$

Example

Assume an analyst in a company has a 3-quarter data set that shows quarterly GDP growth (x) and the company's product growth rate (y). The data set may look like:

Q1: x = 2%, y = 10%
Q2: x = 4%, y = 14%
Q3: x = 3%, y = 12%

$$\bar{X} = (2 + 4 + 3)/3 = 3; \quad \bar{Y} = (10 + 14 + 12)/3 = 12$$

$$Cov(X,Y) = \frac{(2-3) \text{ x } (10-12) + (4-3) \text{ x } (14-12) + (3-3) \text{ x } (12-12)}{3-1} = 2 > 0$$

→ The GDP growth rate and product growth rate have positive covariance.

II. DESCRIPTIVE STATISTICS

5. Correlation

Correlation	Measures the strength of the relationship between two variables
	A scale form of covariance
	Range: from -1 to +1

Equation	$Cor(X,Y) = \rho_{X,Y} = \dfrac{Cov(X,Y)}{\sigma_X \sigma_Y}$
	If $\rho_{X,Y} = 1.0$, the random variables have perfect positive correlation.
	If $\rho_{X,Y} = -1.0$, the random variables have perfect negative correlation.
	If $\rho_{X,Y} = 0$, there is no linear relationship between variables.

II. DESCRIPTIVE STATISTICS

5. Correlation

Example

Assume an analyst in a company has a 3-quarter data set that shows quarterly GDP growth (x) and a company's product growth rate (y). The data set may look like:

Q1: x = 2%, y = 10%
Q2: x = 4%, y = 14%
Q3: x = 3%, y = 12%

Calculate the correlation between the GDP growth rate and product growth rate ?

$\bar{X} = (2 + 4 + 3)/3 = 3$; $\bar{Y} = (10 + 14 + 12)/3 = 12$

$$\sigma_X = \sqrt{\frac{(2-3)^2+(4-3)^2+(3-3)^2}{3-1}} = 1$$

$$\sigma_Y = \sqrt{\frac{(10-12)^2+(14-12)^2+(12-12)^2}{3-1}} = 2$$

$$\rho_{X,Y} = \frac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{2}{1 \times 2} = 1$$

→ The GDP growth rate and the product growth rate have positive correlation and a strong relationship.

II. DESCRIPTIVE STATISTICS

Vocabulary

Vocabulary	Meaning
Statistics	Thống kê
Probability	Xác suất
Addition rule	Quy tắc cộng
Mutually exclusive events	Biến cố xung khắc
Multiplication rule	Quy tắc nhân
Independent events	Biến cố độc lập
Dependent events	Biến cố phụ thuộc
Factorial	Hoán vị
Permutation	Chỉnh hợp
Combination	Tổ hợp
Arithmetic mean	Trung bình cộng giản đơn
Geometric mean	Trung bình nhân

II. DESCRIPTIVE STATISTICS

Vocabulary

Vocabulary	Meaning
Median	Trung vị
Ascending or descending order	Thứ tự tăng hoặc giảm dần
Mode	Số yếu vị
Most frequently	Thường xuyên nhất
Normal distribution	Phân phối chuẩn
Range	Khoảng biến thiên
Standard deviation	Độ lệch chuẩn
Variability	Biến thiên
Normal distribution	Phân phối chuẩn
Covariance	Hiệp phương sai
Variable	Biến
Correlation	Hệ số tương quan