Computer Methods in Engineering Exercise 6

In this exercise we will work on using the conjugate gradient method to solve an elliptic equation. More specifically, we will solve the Laplace equation we encountered in Exercise 3 . Recall that Exercise 3 was considering flow in porous media as governed by the Darcy equation:

$$\vec{q} = -\frac{k}{u} \nabla p \quad ,$$

were q is the volumetric flow rate, k is the permeability (a measure for how well the porous medium allows for transport of fluids), μ is the viscosity of the fluid, and p is the fluid pressure. At steady state this gives the Laplace equation $\nabla^2 p = 0$. We considered a two-dimensional model, thus

$$\nabla = \left(\frac{\partial}{\partial x'}, \frac{\partial}{\partial y}\right)$$

Assume a sand-body connecting two fluid reservoirs at different pressure. The left reservoir has a pressure $p_l = 1 \times 10^5\,\mathrm{Pa}$, while the right reservoir has a pressure $p_r = 2 \times 10^5\,\mathrm{Pa}$. The sand-body has a shape between the two reservoirs as outlined in Fig. 1, where the grid cell size is $100\,\mathrm{m} \times 100\,\mathrm{m}$. Further, assume a viscosity of $1 \times 10^{-3}\,\mathrm{Pa}\,\mathrm{s}$, a permeability of $1 \times 10^{-10}\,\mathrm{m}^2$, and assume a sand body thickness of $10\,\mathrm{m}$.

We saw in Exercise 3 that this gave the matrix representation for the pressure field as $A\vec{P} = \vec{b}$, where the *A* matrix was given as

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		1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	1	-3	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	ı
0	0	1	-3	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
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1	0	0	0	0	-4	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	1	0	0	0	1	-4	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
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0	0	0	0	0	0	1	0	0	0	1	-4	1	0	0	0	1	0	0	0	0	0	0	0	0	1
$A = \begin{vmatrix} 0 \end{vmatrix}$	0	0	0	0	0	0	1	0	0	0	1	-4	1	0	0	0	1	0	0	0	0	0	0	0	1
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0	0	0	0	0	0	0	0	0	1	0	0	0	1	-3	0	0	0	0	1	0	0	0	0	0	1
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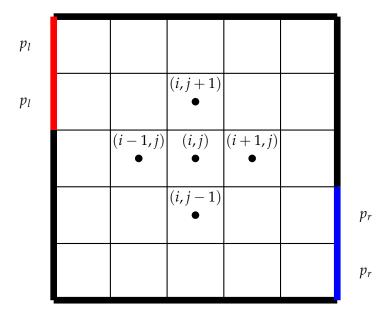


Figure 1: Grid representation of the sand-body connecting the two reservoirs. Thick black lines indicate no-flow boundaries, while red lines indicate boundaries connected to the left reservoir, and blue lines indicate boundaries connected to the right reservoir.

and the vector \vec{b} as



Problem 1

Write a Python code to solve for the pressure \vec{P} using the steepest decent method. If you want a residual smaller than 10^{-8} , how many iterations do you need.

Problem 2

Write a Python code to solve for the pressure \vec{P} using the conjugate gradient method. How does your solution compare with the solution you obtained in Exercise 3 . Compare the time used by your own python code for the conjugate gradient method to the numpy library linalg.inv.

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