

# Computer Methods in Engineering

## Exercise 6

In this exercise we will work on using the conjugate gradient method to solve an elliptic equation. More specifically, we will solve the Laplace equation we encountered in Exercise 3 . Recall that Exercise 3 was considering flow in porous media as governed by the Darcy equation:

$$\vec{q} = -\frac{k}{\mu} \nabla p \quad ,$$

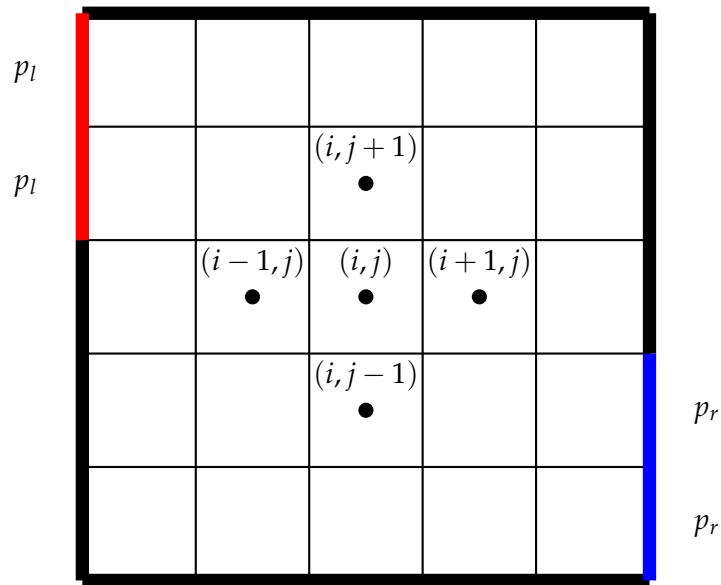
where  $q$  is the volumetric flow rate,  $k$  is the permeability (a measure for how well the porous medium allows for transport of fluids),  $\mu$  is the viscosity of the fluid, and  $p$  is the fluid pressure. At steady state this gives the Laplace equation  $\nabla^2 p = 0$ . We considered a two-dimensional model, thus

$$\nabla = \left( \frac{\partial}{\partial x'}, \frac{\partial}{\partial y} \right)$$

Assume a sand-body connecting two fluid reservoirs at different pressure. The left reservoir has a pressure  $p_l = 1 \times 10^5$  Pa, while the right reservoir has a pressure  $p_r = 2 \times 10^5$  Pa. The sand-body has a shape between the two reservoirs as outlined in Fig. 1, where the grid cell size is  $100 \text{ m} \times 100 \text{ m}$ . Further, assume a viscosity of  $1 \times 10^{-3}$  Pa s, a permeability of  $1 \times 10^{-10} \text{ m}^2$ , and assume a sand body thickness of 10 m.

We saw in Exercise 3 that this gave the matrix representation for the pressure field as  $A\vec{P} = \vec{b}$ , where the  $A$  matrix was given as

[illegible]



$$\vec{b} = \begin{bmatrix} -100000 \\ 0 \\ 0 \\ 0 \\ -100000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -200000 \\ 0 \\ 0 \\ 0 \\ 0 \\ -200000 \end{bmatrix}$$

## Write a Pyth

## Write a Python