Computer Methods in Engineering Exercise on pattern search

All the exercises in derivative-free optimization will be based on the same problems. The problems will be described in this exercise, and the different methods will be applied in the following exercises. This means that you will probably prefer to re-use your code from one exercise to the next. You are of course free to do so, but you can also choose to write new code for each exercise.

There will be underlying problems to solve, one regarding optimal location of a platform for CO₂ injection, and one regarding optimal location of wind turbines. The problems will be described in the following.

Problem 1

A company is looking at a reservoir geometry and has decided that the best locations to place CO₂ injectors named A, B, C, D and E are at the coordinates (10, 10), (30, 50), (16.667, 29), (0.555, 29.888) and (22.2221, 49.988), respectively, in the xy-plane. The company also needs a central platform to run the injectors from. It is anticipated that during an average week the total volume pumped through each injector pipeline will be 10, 18, 20, 14, and 25 (arbitrary units) for injectors A, B, C, D, and E, respectively. Longer pipelines require more powerful compressors in order to compensate for friction loss in the pipes. The company therefore wants to minimize the total distance-volume (distance from platform to injector multiplied by volume).

- **a)** To accomplish this, where in the xy-plane should the platform be located? Use the steepest descent method. Visually verify that your solution is correct by plotting the function.
- **b)** Use the pattern search algorithm to solve the same problem. Compare the two solutions (number of iterations, number of objective function evaluations, run time, accuracy, etc.).

Problem 2

A company have an offshore wind-farm. They currently have 5 wind turbines, all located inside their license which we assume to be inside the area [0-20,0-20] in the xy-plane, where the unit is in kilometers. Assume that the existing wind turbines have coordinates (2.5,3.5), (17.2,3.2), (4.2,16.8), (16.2,18.4) and (11.1,9.8). The company is looking at placing an additional wind turbine (a sixth wind turbine in addition to the existing ones).

The wind power is given by the function

$$\operatorname{erf}\left(\frac{1}{5}\sqrt{\sum_{i}(x_{w}-x_{i})^{2}+(y_{w}-x_{i})^{2}}\right) \tag{1}$$

where erf is the error function, i runs over all existing wind turbine locations (x_i, y_i) , and (x_w, y_w) is the location of the suggested new wind turbine location.

The company is also given a bonus for placing turbines away from the license border, given as

$$\operatorname{erf}(\operatorname{abs}(x_w - 0.0)) + \operatorname{erf}(\operatorname{abs}(x_w - 20.0)) + \operatorname{erf}(\operatorname{abs}(y_w - 0.0)) + \operatorname{erf}(\operatorname{abs}(y_w - 20.0))$$
(2)

The objective is then to maximize the sum of the two equations above.

- a) Use the pattern search method to find the optimal wind turbine location for an additional wind turbine (a sixth wind turbine in addition to the existing ones).
- b) What happens when you re-run your algorithm do you get the same result?