Computer Methods in Engineering
Exercise on LU decomposition

Problem 1

Consider a one-dimensional rod of length $L = 8$, divided into N equally spaced nodes. The thermal conductivity $k(x)$ varies along the rod as follows:

- $k(x) = k_1 = 1$ for $0 \leq x < L/4$
- $k(x) = k_2 = 2$ for $L/4 \leq x < L/2$
- $k(x) = k_3 = 3$ for $L/2 \leq x < 3L/4$
- $k(x) = k_4 = 1$ for $3L/4 \leq x \leq L$

The steady-state temperature distribution $T(x)$ in the rod is governed by the equation:

$$\frac{d}{dx}\left(k(x)\frac{dT}{dx}\right) = 0$$

with boundary conditions $T(0) = T_0 = 30$ and $T(L) = T_L = 20$.

a) Discretize the equation using the finite difference method for $N = 9$, of which we have 7 interior nodes, while the first and last nodes are the boundary conditions at T_0 and T_L. Write down the resulting linear system $\mathbf{AT} = \mathbf{b}$.

b) Use LU decomposition to solve the linear system for the temperature at the interior nodes.

c) Plot the temperature distribution along the rod.