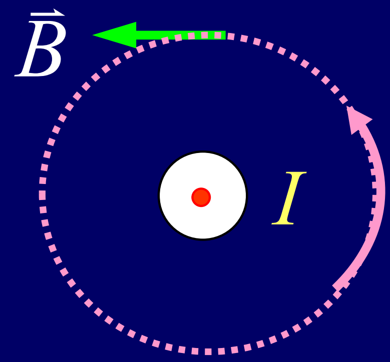
$$B=\frac{\mu_0 I}{4\pi a}(\cos\theta_1-\cos\theta_2)$$



(1) 无限长直导线

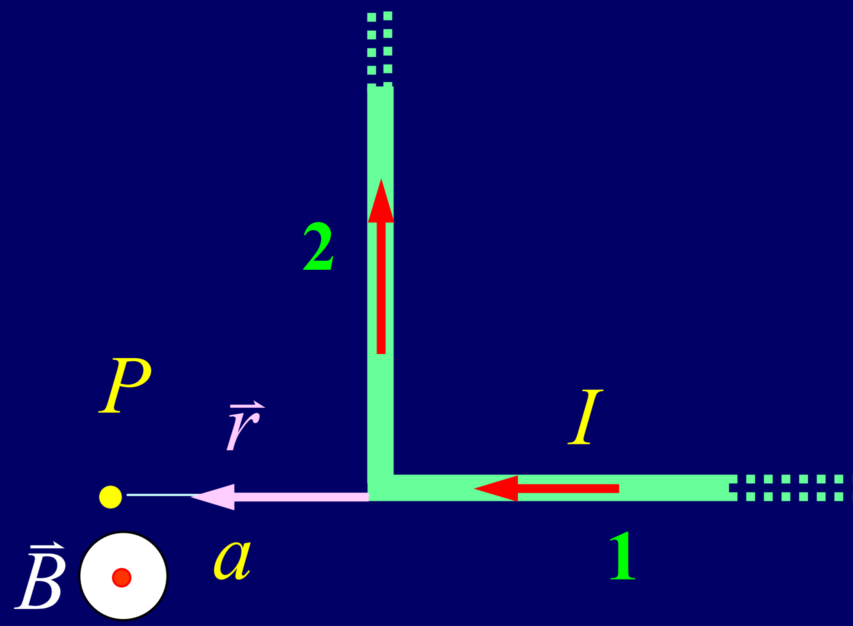
$$B=\frac{\mu_0 I}{2\pi a}$$

方向：右螺旋法则

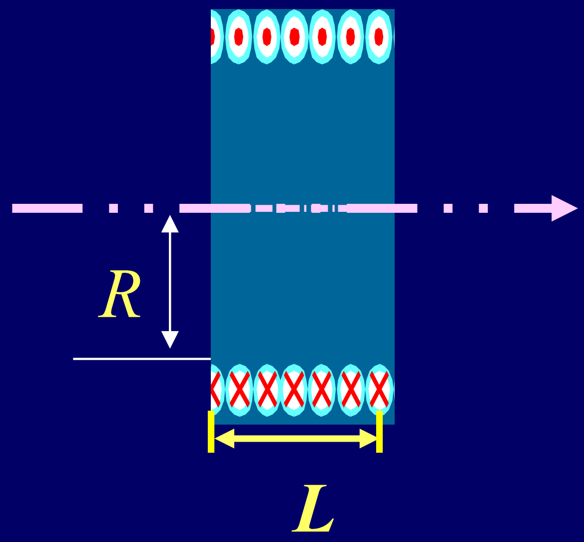
(2) 半无线长直导线

$$B_1=0$$

$$B_2=\frac{\mu_0 I}{4\pi a}$$



$$B=\frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$$

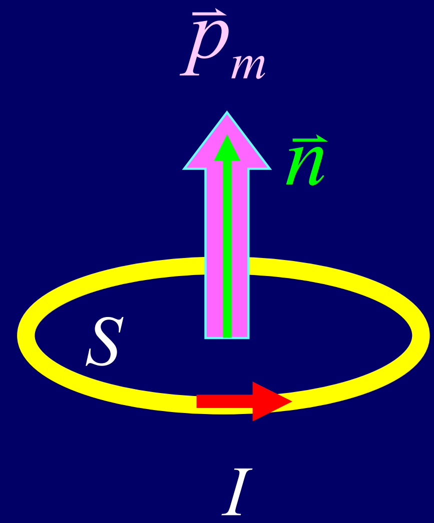


(1)如果由*N* 匝圆线圈组成 *L<<R*

$$B=\frac{\mu_0 I R^2 N}{2(R^2+x^2)^{3/2}}$$

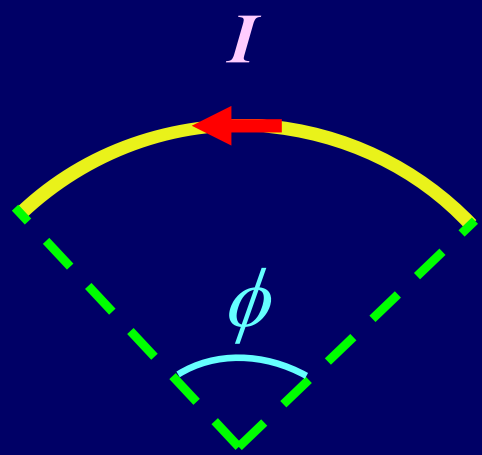
(2) $x=0$ **载流圆线圈的圆心处** $B_o=\frac{\mu_0 I}{2R}\cdot\frac{\pi R^2}{\pi R^2}$

定义 $\boxed{\vec{p}_m=IS\vec{n}}$ 磁矩



$$\vec{B}_0=\frac{\mu_0}{2\pi}\frac{\vec{p}_m}{R^3}$$

(3) 一段圆弧在圆心处产生的磁场



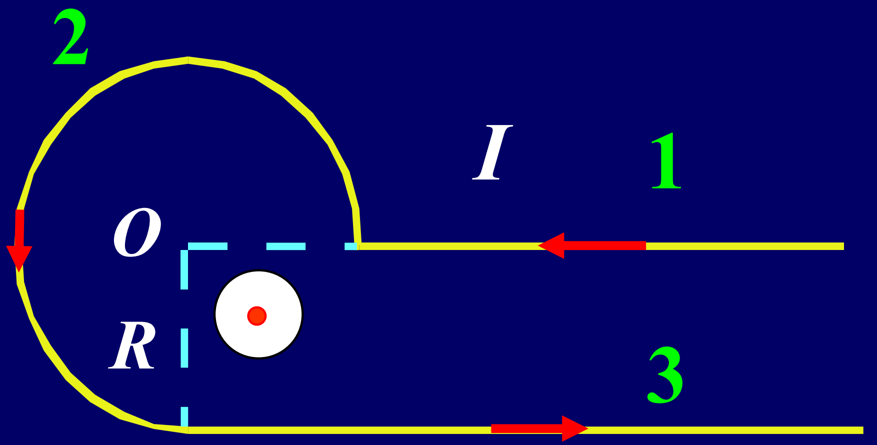
$$B = \frac{\mu_0 I}{2R} \cdot \frac{\phi}{2\pi} = \frac{\mu_0 I \phi}{4\pi R}$$

(4) $$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

$$x >> R \quad \Longrightarrow \quad B \approx \frac{\mu_0 I R^2}{2x^3} \cdot \frac{\pi}{\pi} = \frac{\mu_0 I S}{2\pi x^3}$$

$$\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{p}_m}{x^3}$$

例 右图中，求 O 点的磁感应强度



解 $B_1 = 0$

$$B_2 = \frac{\mu_0 I}{4\pi R} \cdot \frac{3\pi}{2} = \frac{3\mu_0 I}{8R}$$

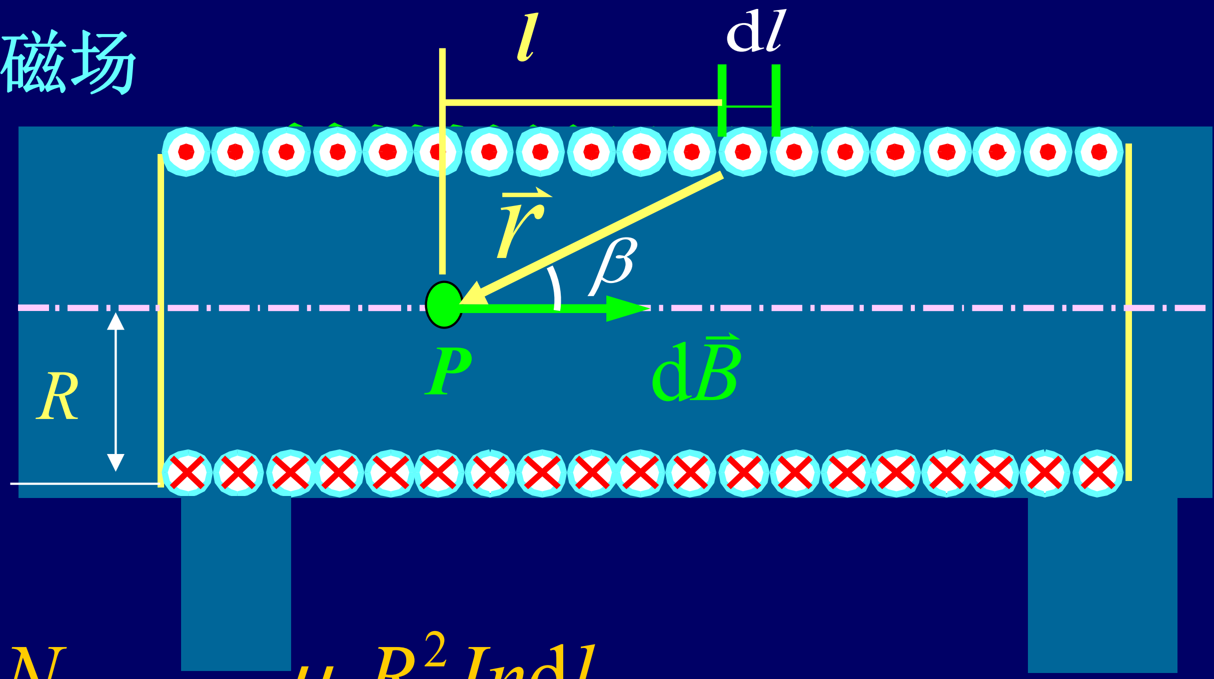
$$B_3 = \frac{\mu_0 I}{4\pi R}(\cos\theta_1 - \cos\theta_2)$$

$$\theta_1 = \pi/2 \quad \theta_2 = \pi$$

$$= \frac{\mu_0 I}{4\pi R}$$

$$B = B_1 + B_2 + B_3$$

3. 载流螺线管轴线上的磁场



已知螺线管半径为R

dl长度上有dN 匝

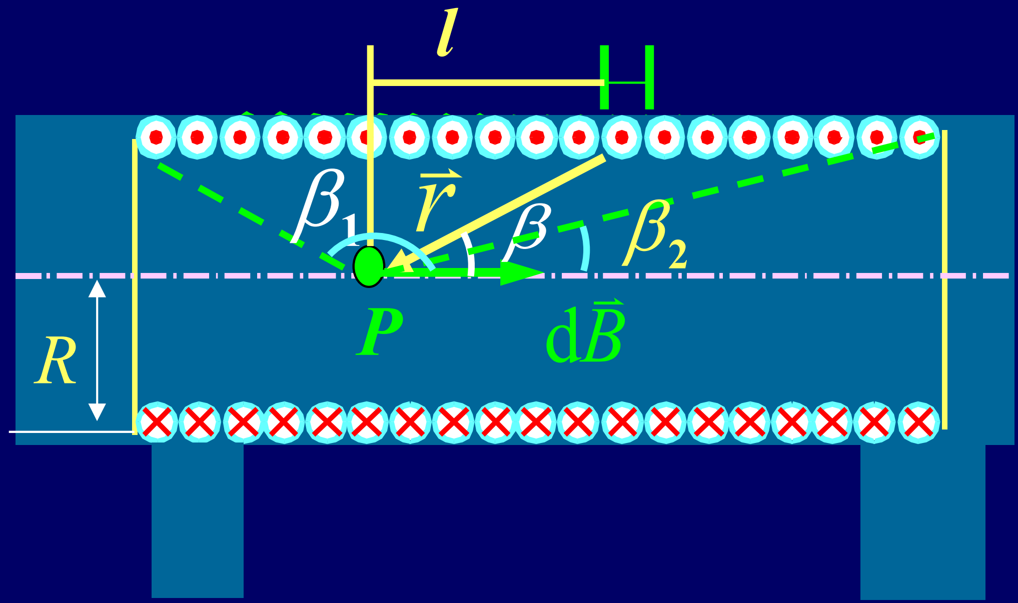
$$\mathrm{d}N = n\mathrm{d}l$$

$$\mathrm{d}B = \frac{\mu_0 R^2 I \mathrm{d}N}{2(R^2 + l^2)^{3/2}} = \frac{\mu_0 R^2 I n \mathrm{d}l}{2(R^2 + l^2)^{3/2}}$$

$$l = R\cot\beta \qquad \left.\begin{aligned} \mathrm{d}l &= -R\csc^2\beta\,\mathrm{d}\beta \\ R^2 + l^2 &= R^2\csc^2\beta \end{aligned}\right\}$$

$$\mathrm{d}B = \frac{-\mu_0 R^2 I n \cdot R\csc^2\beta\mathrm{d}\beta}{2(R^2\csc^2\beta)^{3/2}} = -\frac{\mu_0}{2} nI\sin\beta\mathrm{d}\beta$$

$$\mathrm{d}B = -\frac{\mu_0}{2} nI \sin\beta \mathrm{d}\beta$$



$$B = \int_{\beta_1}^{\beta_2} -\frac{\mu_0}{2} nI \sin\beta \mathrm{d}\beta$$

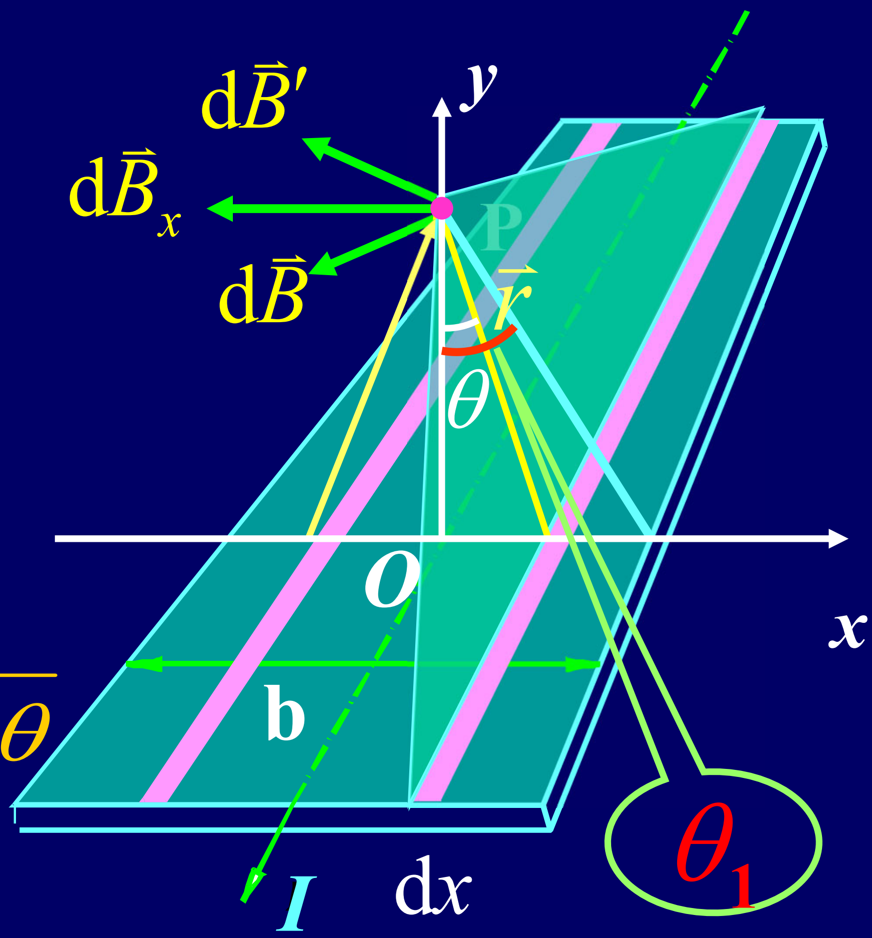
$$= \frac{\mu_0 nI}{2}(\cos\beta_2 - \cos\beta_1)$$

讨 论

(1) 无限长载流螺线管 $\beta_1 \to \pi,\ \beta_2 \to 0 \quad \Rightarrow \quad B = \mu_0 nI$

(2) 半无限长载流螺线管 $\beta_1 \to \pi/2,\ \beta_2 \to 0 \quad \Rightarrow \quad B = \mu_0 n \frac{I}{2}$

例： 无限长载流平板



解 $$\mathrm{d}I = \frac{I\mathrm{d}x}{b}$$

$$\mathrm{d}B = \frac{\mu_0 \mathrm{d}I}{2\pi r} = \frac{\mu_0 I \mathrm{d}x}{2\pi r b}$$

$$B_P = B_x = \int \mathrm{d}B_x = \int \mathrm{d}B\cos\theta = \int \frac{\mu_0 I \mathrm{d}x}{2\pi r b \sec\theta}$$

$$r = y\sec\theta$$

$$x = y\tan\theta \qquad \mathrm{d}x = y\sec^2\theta\,\mathrm{d}\theta$$

$$\theta_1 = \arctan\frac{b}{2y}$$

$$B_P = \frac{\mu_0 I}{\pi b}\int_0^{\theta_1}\mathrm{d}\theta = \frac{\mu_0 I}{\pi b}\arctan\frac{b}{2y}$$

分析： $$B_p = \frac{\mu_0 I}{\pi b} \arctan\frac{b}{2y}$$

(1) $y >> b$ $\Rightarrow$ $$\arctan\frac{b}{2y} \approx \frac{b}{2y}$$

$$B_P \approx \frac{\mu_0 I b}{2y\pi b} = \frac{\mu_0 I}{2\pi y}$$

无限长载流直导线

(2) $y << b$ $\Rightarrow$ $$\arctan\frac{b}{2y} \approx \frac{\pi}{2}$$

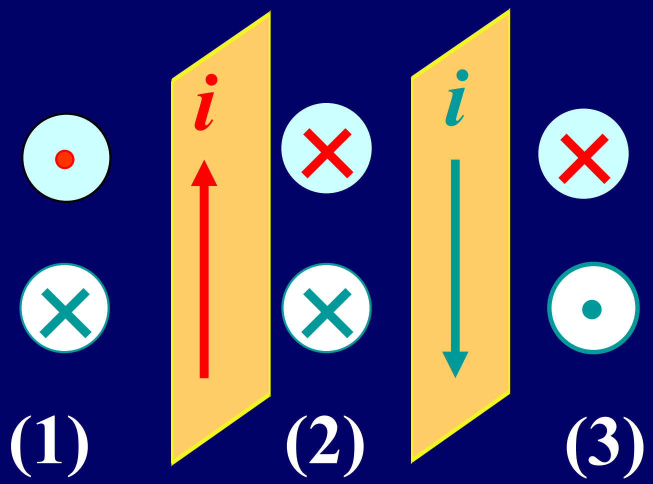
无限大薄板

均匀磁场

$$B_P \approx \frac{\mu_0 I \pi}{2\pi b} = \frac{\mu_0 I}{2b} = \frac{1}{2}\mu_0 i$$

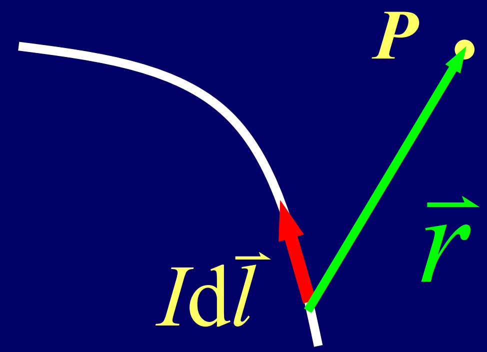
$$B_1 = B_3 = 0 \qquad B_2 = \mu_0 i$$

磁屏蔽



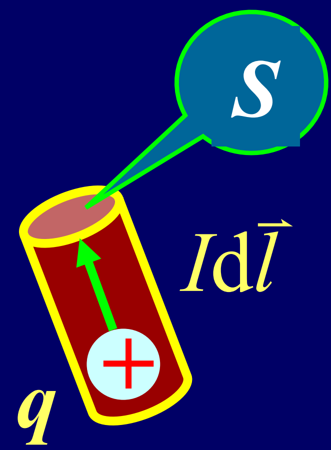
三. 运动电荷的磁场

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{l} \times \vec{r}^{\,0}}{r^2}$$



电荷密度

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{n \cdot s\mathrm{d}l \cdot q}{\mathrm{d}t} = nsq\upsilon$$



$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{(nsq\upsilon)\mathrm{d}\vec{l} \times \vec{r}^{\,0}}{r^2}$$

电流元内总电荷数 $\mathrm{d}N = ns\mathrm{d}l$

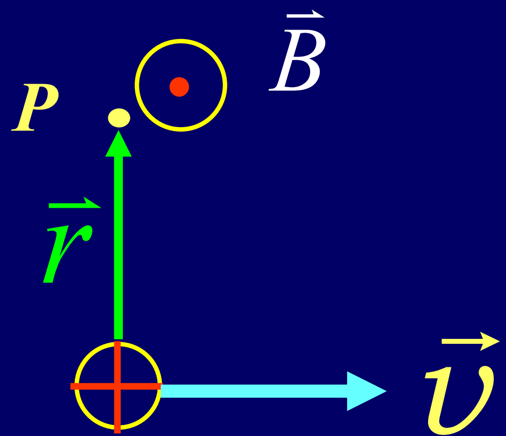
$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{\mathrm{d}N \cdot q\vec{\upsilon} \times \vec{r}^{\,0}}{r^2}$$

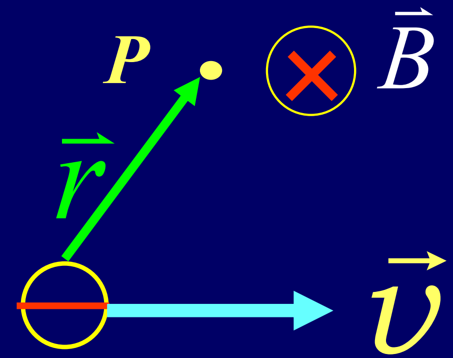
一个电荷产生的磁场

$$\boxed{\vec{B} = \frac{\mathrm{d}\vec{B}}{\mathrm{d}N} = \frac{\mu_0}{4\pi}\frac{q\vec{\upsilon} \times \vec{r}^{\,0}}{r^2}}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}^{\,0}}{r^2}$$

例如：





§11.3　磁场的高斯定理

静电场：$\Phi_e = \oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \sum q_i / \varepsilon_0$　　静电场是有源场

磁　场：$\oint \vec{B} \cdot \mathrm{d}\vec{S} = ?$

一. 磁感应线

1. 规定

(1) 方向：磁力线切线方向为磁感应强度 $\vec{B}$ 的方向

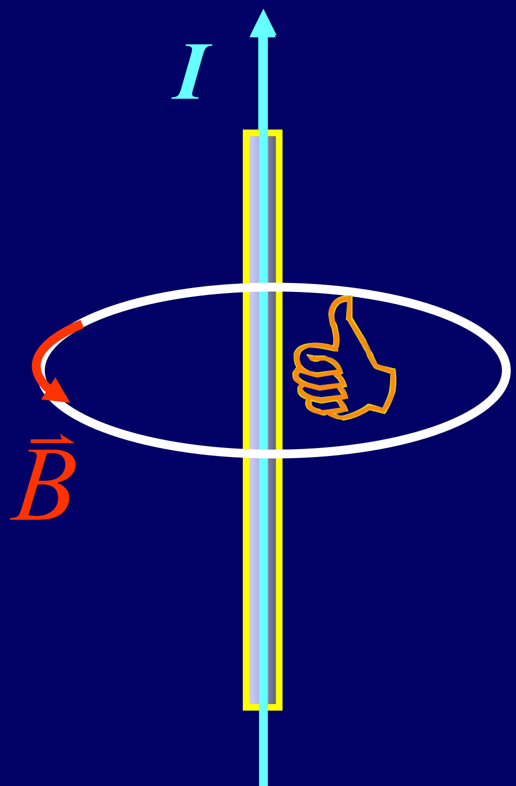
(2) 大小：垂直 $\vec{B}$ 的单位面积上穿过的磁力线条数为磁感应强度 $\vec{B}$ 的大小

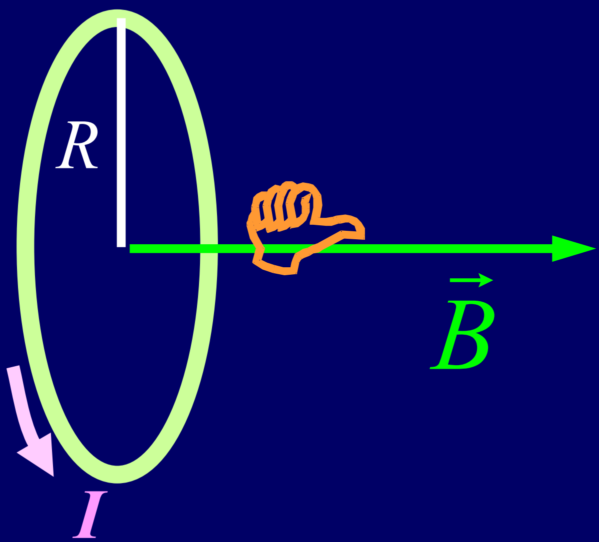
$$B = \frac{\mathrm{d}\Phi_\mathrm{m}}{\mathrm{d}S_\perp}$$

2. 磁感应线的特征

(1) 无头无尾的闭合曲线

(2) 与电流相互套连，服从右手螺旋定则

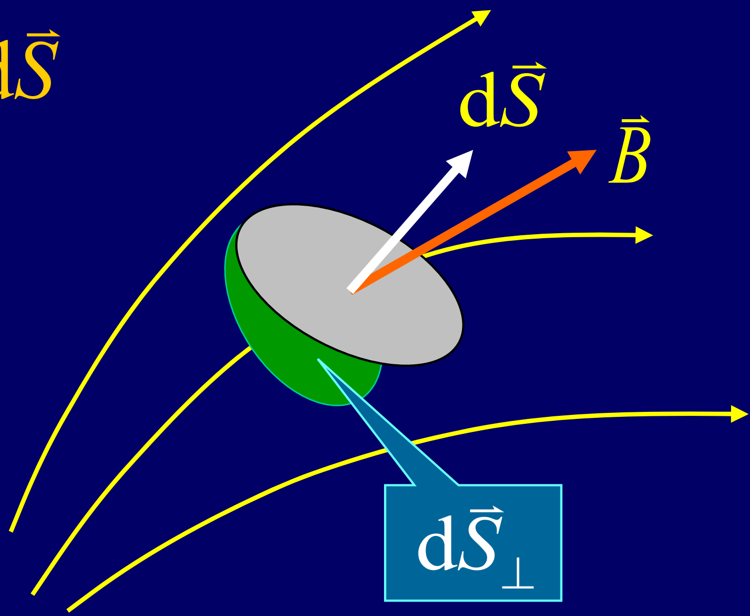




二. 磁通量

通过面元dS的磁力线条数 —— 通过该面元的磁通量$d\Phi_m$

$$B = \frac{d\Phi_m}{dS_\perp} \Rightarrow d\Phi_m = \vec{B} \cdot d\vec{S}$$



对于有限曲面 $\Phi_m = \int \vec{B} \cdot d\vec{S}$

对于闭合曲面 $\Phi_m = \oint_S \vec{B} \cdot d\vec{S}$

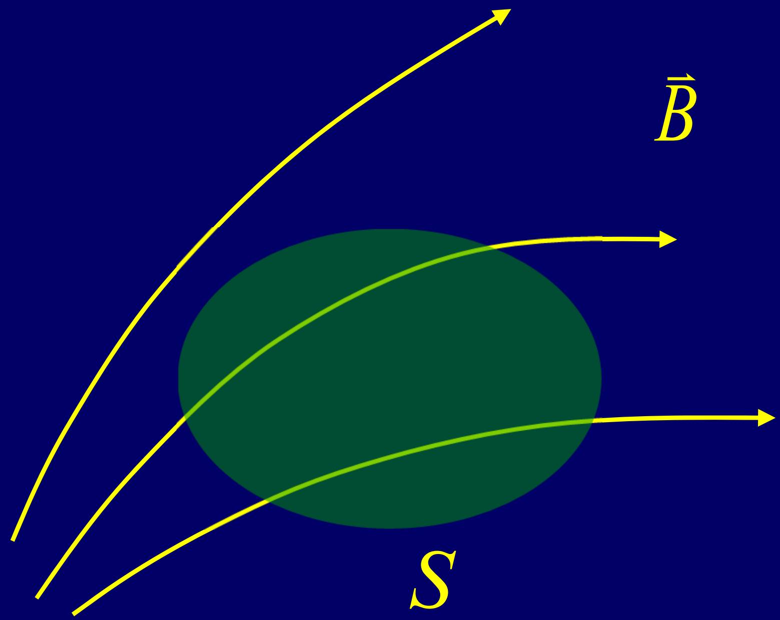
规定：外法线为正

磁力线穿入 $\Phi_m < 0$

磁力线穿出 $\Phi_m > 0$

单位： $1W_b = 1T\,m^2$

三. 磁场的高斯定理



磁场线都是闭合曲线

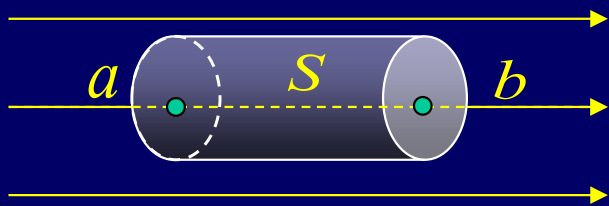
$$\boxed{\Phi_m = \oint_S \vec{B} \cdot \mathrm{d}\vec{S} = 0}$$ **(磁高斯定理)**

电流产生的磁感应线既没有起始点，也没有终止点。

——磁场是无源场

例 证明在 磁力线 为平行直线的空间中，同一根磁力线 上各点的磁感应强度值相等。

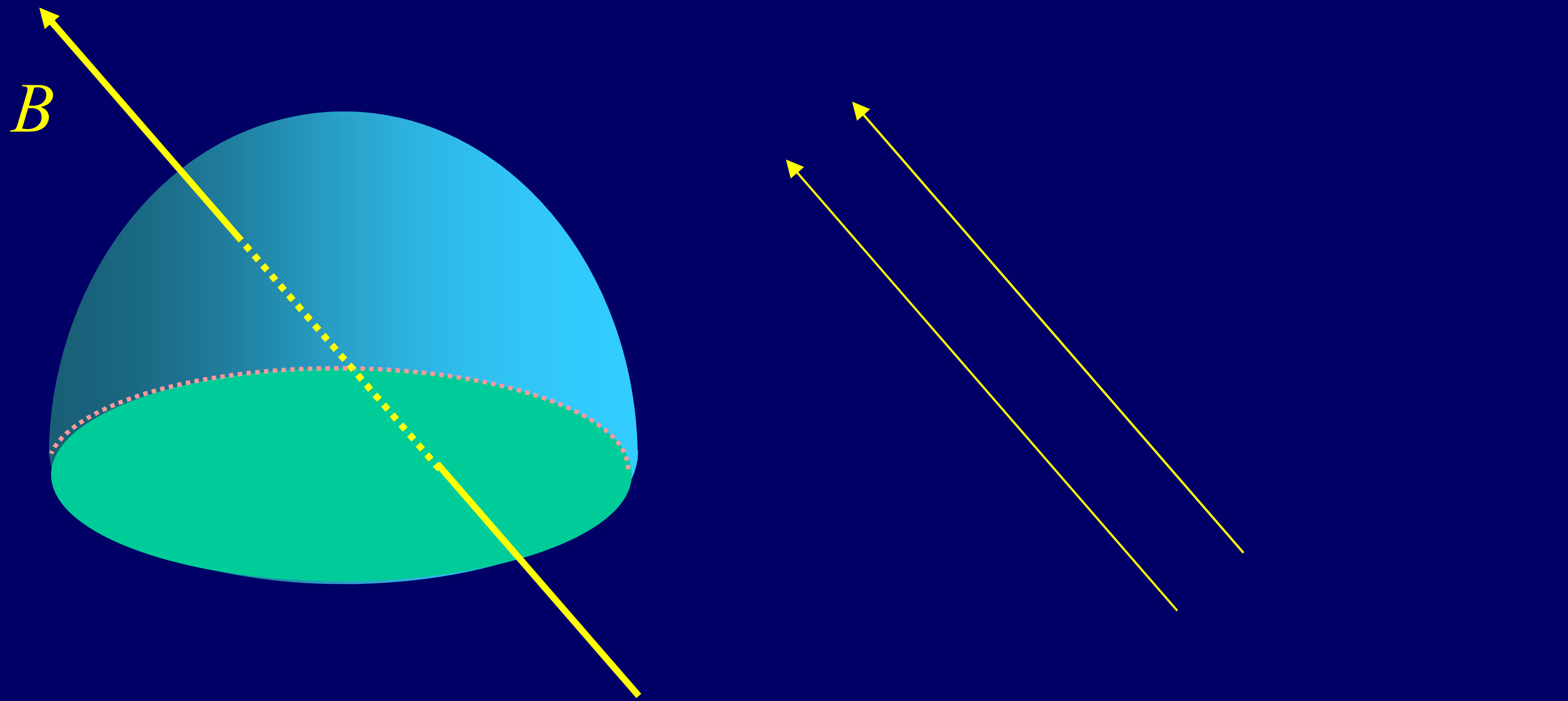
解



$$\Phi_m = \oint_S \vec{B} \cdot \mathrm{d}\vec{S}$$

$$= -B_a \Delta S + B_b \Delta S = 0$$

$$B_a = B_b$$


B