Large Deviations

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I am working on a problem related to functional central limit theorem. I obtained some results that are strange or hard to interpret. Consider the sequence of random processes X_n. I have obtained the functional law of large numbers limit of the family of processes $\bar{X}^{(n)}(t) = n^{-1}X_n(t)$. Let's call the FLLN limit $\bar{X}$. Now I only consider the cases when $\bar{X}$ has an equilibrium point $\bar{X}^*$. For FCLT, I consider the process $\hat{X}^n(t) = \sqrt{n}(\bar{X}^n)(t) - \bar{X}^*$. I think I showed that $\hat{X}^n \Rightarrow$ to $\hat{X}$, which is the solution to the SDE $d\hat{X}(t) = (a - b\hat{X}(t))dt + \sigma dB(t)$. However, the constants a, b don't have to be positive. So suppose $b < 0$ the limiting process could go to infinity and don't have a stationary distribution. When I center around fluid limit's equilibrium point, isn't the diffusion limit supposed to be centered around 0? Am I wrong to analyze FCLT by centering around the equilibrium point?

the fluid limit $\bar{X}$ solves the ODE

$$d\bar{X}(t) = adt - bd\bar{X}(t)$$

where $a < 0, b < 0$. So $\frac{a}{b}$ is an equilibrium point but it is not locally stable. Does this mean I should not center around $\frac{a}{b}$ in this case?