Skorokhod Spaces

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1 Space C

Definition 1.1 (Modulus of continuity)
The **modulus of continuity** for $x \in \mathcal{C}[0,1]$ is defined as

$$w_x(\delta) = w(x,\delta) = \sup_{|s-t|<\delta} |x(s) - x(t)|, \quad \delta \in [0,1]$$

Definition 1.2
The **modulus of continuity** for $x \in \mathcal{D}[0,1]$ is defined as

$$w'_x(\delta) = w'(x,\delta) = \inf_{t_i} \max_{1 \le i \le v} w_x[t_{i-1}, t_i),$$

where infimum is taken over δ-spase sets $\{t_i\}$, i.e. $\min_{1\le i\le v}(t_i - t_{i-1}) > \delta$.

Proposition 1.3

- $w'_x(\delta) \le w_x(2\delta)$
- $w_x(\delta) \le 2w'_x(\delta) + j(x)$.

2 Compactness

Recall Arzela-Ascoli Theorem,

Theorem 2.1
A set $A \subset \mathcal{D}$ is relatively compact in the J_1 Skorokhod topology iff

1. $\sup_{x\in A} \|x\|^* < \infty$.
2. $\lim_{\delta\to 0} \sup_{x\in A} w'_x(\delta) = 0$.