Stability of ODE

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When we talk about stability, it is always in reference to something. For example, fixed points stable, periodic solution stable, set stable, etc.

Definition 0.1 (Three types of fixed point stability)**.** Suppose $f'(x_0) = 0$ so that x_0 is a fixed point. We say

1. x_0 is **stable** or (Lyapunov stable) if $\forall \epsilon > 0, \exists \delta > 0$ such that if $\|y - x_0\| < \delta$, then $\|\varphi_t(y) - x_0\| < \epsilon$. In other words:"start close, stay close".
2. x_0 is **asymptotically stable** if x_0 is stable and $\lim_{t \to \infty} \varphi_t(y) = x_0$. (collapse to fixed point)
3. x_0 is **spectrally stable** (linear stability) if all eigenvalues of $Df(x_0)$ have negative real parts.