

# Welded Joints

## Complements of Machine Elements

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Mestrado em Engenharia Mecânica

*The simplest things are also the most extraordinary things,  
and only the wise can see them.*

Paulo Coelho, The Alchemist

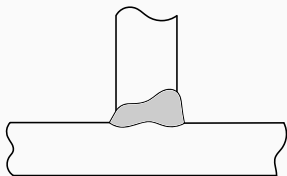
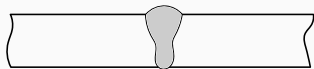
## Recommended bibliography

- Ballio, G. and Mazzolani, F.M., Theory and Design of Steel Structures, Chapman and Hall.

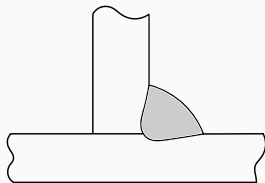
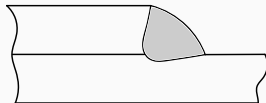
References

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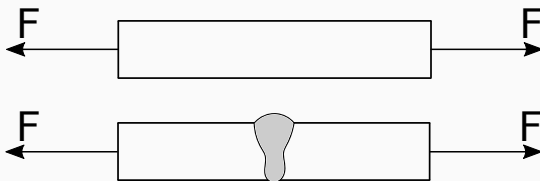
**Figure 1:** Butt welds [1]



**Figure 2:** Fillet welds [1]

- butt weld - junta de penetração completa
- fillet weld - cordão de ângulo

## Equal strength criteria



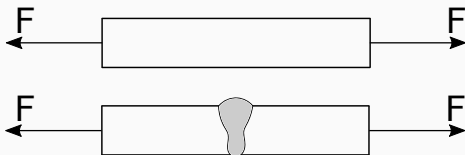
**Figure 3:** Continuous plate (top) and welded plate (bottom)

Without taking into account “joint efficiency” the stress is estimated for both plates by [1]:

$$\sigma = \frac{F}{A}$$

The dimensioning is then given by assuring that stress is lower than the permissible stress:  $\sigma < \sigma_{adm}$

## Equal strength criteria



**Figure 3:** Continuous plate (top) and welded plate (bottom)

We consider both plates with the same stress, because:

- the over-thickness of the joint should not be considered as additional strength;
- the metallurgic heterogeneity between materials is not considered, because joint material should provide at least the same strength (ASME Boiler and Pressure Vessel Code);
- the residual stresses are not considered for static loading conditions, because they are removed by plastic deformation.



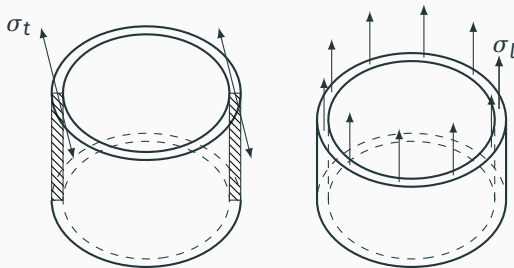
## Joint efficiency

In practical applications, a joint efficiency coefficient should be used.

From ASME Boiler and Pressure Vessel Code (BPVC), Section VIII, Division 1 or Code de Construction des Appareils a Pression Non Soumis a l'Action de la Flamme, Règles de Calcul S.N.C.T.T.I, use a joint efficiency coefficient [1, 2].

ASME code gives the following coefficients:

Degree of radiographic examination	$k$ (with <i>repris</i> )	$k$ (without <i>repris</i> )
Full	1	0.9
Spot	0.85	0.8
None	0.70	0.65



**Figure 4:** Thin wall reservoir with butt welds subjected to internal pressure  $p_i$ .

The hoop stress  $\sigma_t$  and the axial stress  $\sigma_l$  are given by:

$$2 \cdot \sigma_t \cdot l = p_i \cdot d_i \cdot l \Leftrightarrow \sigma_t = \frac{p_i \cdot d_i}{2 \cdot t}$$

$$\sigma_l \cdot \pi \cdot d_i \cdot t = \frac{p_i \cdot \pi \cdot d_i^2}{4} \Leftrightarrow \sigma_l = \frac{p_i \cdot d_i}{4 \cdot t}$$

# Joint efficiency

The radial stress is  $\sigma_r = -p_i$  on the inner wall and  $\sigma_r = 0$  for the outer wall.

Typically the radial stress is the average of the previous two values, i.e.  $\sigma_r = -\frac{p_i}{2}$

Using the Tresca criterium:

$$\sigma_t - \sigma_r = \sigma_{adm}$$

$$\frac{p_i \cdot d_i}{2 \cdot t} + \frac{p_i}{2} = \sigma_{adm}$$

With the joint efficiency  $k$ :

$$\frac{p_i \cdot d_i}{2 \cdot t} + \frac{p_i}{2} = \sigma_{adm} \cdot k$$

The following expression is found on 'Recommandation ISO R831' [1]:

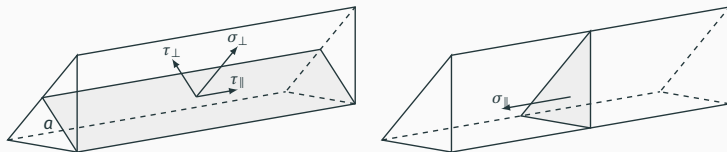
$$t = \frac{p_i \cdot r_i}{\sigma_{adm} \cdot k - 0.5 \cdot p_i}$$

The ASME code [3] gives:

$$t = \frac{p_i \cdot r_i}{\sigma_{adm} \cdot k - 0.6 \cdot p_i}$$

Valid for  $t < 0.5 \cdot r_i$  and  
 $p_i < 0.385 \cdot \sigma_{adm} \cdot k$

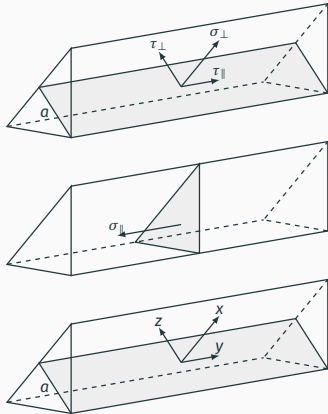
## Criteria based on “Recommendation ISO R 617”



**Figure 5:** Considered stresses on the throat [1, 4]

- $a$  is the throat length;
- $\sigma_{\perp}$  is the normal stress;
- $\tau_{\perp}$  is perpendicular to the weld axis;
- $\tau_{\parallel}$  is the shear stress parallel to the weld axis;
- $\sigma_{\parallel}$  is the normal to weld cross section and parallel to the weld axis.

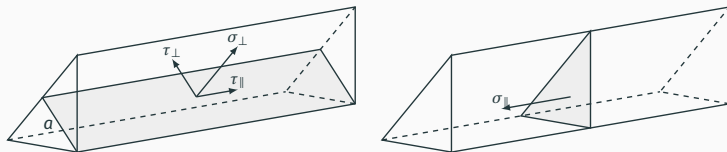
# Criteria based on “Recommendation ISO R 617”



**Figure 6:** Considered stresses on the throat and equivalence to a Cartesian coordinate system [1, 4]

$$\left\{ \begin{array}{l} \sigma_x = \sigma_{\perp} \\ \sigma_y = \sigma_{\parallel} \\ \sigma_z = 0 \\ \tau_{xz} = \tau_{\perp} \\ \tau_{xy} = \tau_{\parallel} \\ \tau_{yz} = 0 \end{array} \right.$$

# Criteria based on “Recommendation ISO R 617”



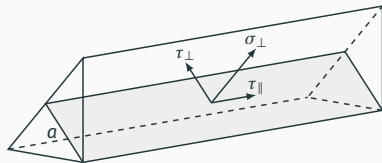
**Figure 5:** Considered stresses on the throat [1, 4]

Using the von-Mises criteria:

$$\sigma_e^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 - (\sigma_x \cdot \sigma_y + \sigma_y \cdot \sigma_z + \sigma_x \cdot \sigma_z) + 3 \cdot (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)$$

$$\sigma_e^2 = \sigma_{\perp}^2 + \sigma_{\parallel}^2 - \sigma_{\perp} \cdot \sigma_{\parallel} + 3 \cdot (\tau_{\perp}^2 + \tau_{\parallel}^2)$$

# Criteria based on “Recommendation ISO R 617”



**Figure 7:** Actual stresses considered for the criteria [1, 4]

Based on experiments, the equation is typically modified to

[1]:

$$\sigma_e^2 = \sigma_{\perp}^2 + \sigma_{\parallel}^2 - \sigma_{\perp} \cdot \sigma_{\parallel} + \lambda \cdot (\tau_{\perp}^2 + \tau_{\parallel}^2)$$

With  $\lambda = 1.8$ .

Under static loads, experiments allowed to assume that  $\sigma_{\parallel}$  has a negligible effect on the strength of the weld.

$$\sigma_e^2 = \sigma_{\perp}^2 + \lambda \cdot (\tau_{\perp}^2 + \tau_{\parallel}^2)$$

# Stress ellipsoid

A tool for geometrical representation of the three-dimensional stress state at a given point.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\frac{\sigma_{\perp}^2}{\sigma_e^2} + \lambda \cdot \left( \frac{\tau_{\perp}^2 + \tau_{\parallel}^2}{\sigma_e^2} \right) = 1$$

Assuming  $\sigma_{\parallel} = 0$ :

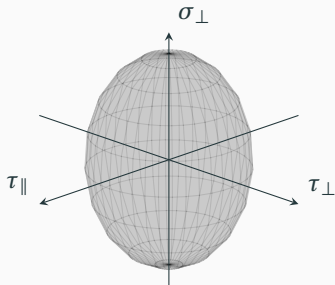
$$\sigma_e^2 = \sigma_{\perp}^2 + \lambda \cdot (\tau_{\perp}^2 + \tau_{\parallel}^2)$$

$$\frac{\sigma_{\perp}^2}{\sigma_e^2} + \frac{\tau_{\perp}^2}{\left(\frac{\sigma_e}{\sqrt{\lambda}}\right)^2} + \frac{\tau_{\parallel}^2}{\left(\frac{\sigma_e}{\sqrt{\lambda}}\right)^2} = 1$$



# Stress ellipsoid

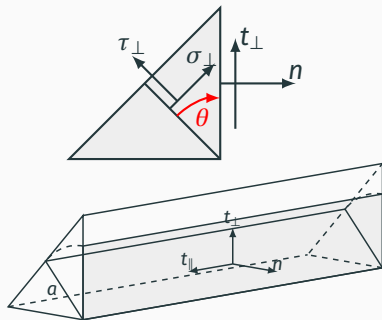
$$\frac{\sigma_{\perp}^2}{\sigma_e^2} + \frac{\tau_{\perp}^2}{\left(\frac{\sigma_e}{\sqrt{\lambda}}\right)^2} + \frac{\tau_{\parallel}^2}{\left(\frac{\sigma_e}{\sqrt{\lambda}}\right)^2} = 1$$



**Figure 8:** Stress Ellipsoid

# Simplified method

Reclining the the throat section into the plane of connection of the parts.



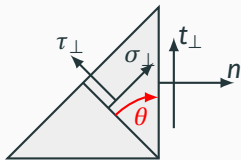
**Figure 9:** Reclining throat mechanism [1, 4]

$$\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \sigma_{\perp} \\ \tau_{\perp} \\ \tau_{\parallel} \end{bmatrix} = \begin{bmatrix} n \\ t_{\perp} \\ t_{\parallel} \end{bmatrix}$$

For  $\theta = \frac{\pi}{4}$ :

$$\begin{cases} \sigma_{\perp} \cdot \frac{\sqrt{2}}{2} - \tau_{\perp} \cdot \frac{\sqrt{2}}{2} = n \\ \sigma_{\perp} \cdot \frac{\sqrt{2}}{2} + \tau_{\perp} \cdot \frac{\sqrt{2}}{2} = t_{\perp} \\ \tau_{\parallel} = t_{\parallel} \end{cases}$$

# Simplified method



**Figure 10:** Stress components transformation [1, 4]

$$\begin{cases} \sigma_{\perp} = \frac{\sqrt{2}}{2} (n + t_{\perp}) \\ \tau_{\perp} = \frac{\sqrt{2}}{2} (t_{\perp} - n) \\ \tau_{\parallel} = t_{\parallel} \end{cases}$$

Substituting into:

$$\begin{cases} \sigma_{\perp} \cdot \frac{\sqrt{2}}{2} - \tau_{\perp} \cdot \frac{\sqrt{2}}{2} = n \\ \sigma_{\perp} \cdot \frac{\sqrt{2}}{2} + \tau_{\perp} \cdot \frac{\sqrt{2}}{2} = t_{\perp} \\ \tau_{\parallel} = t_{\parallel} \end{cases}$$

$$\sigma_e^2 = \sigma_{\perp}^2 + \lambda \cdot (\tau_{\perp}^2 + \tau_{\parallel}^2)$$

$$\sigma_e^2 = \left( \frac{\sqrt{2}}{2} (n + t_{\perp}) \right)^2 + \lambda \cdot \left( \left( \frac{\sqrt{2}}{2} (t_{\perp} - n) \right)^2 + t_{\parallel}^2 \right)$$

## Simplified method

$$\sigma_e^2 = \left( \frac{\sqrt{2}}{2} (n + t_{\perp}) \right)^2 + \lambda \cdot \left( \left( \frac{\sqrt{2}}{2} (t_{\perp} - n) \right)^2 + t_{\parallel}^2 \right)$$

$$2 \cdot \sigma_e^2 = n^2 + 2 \cdot n \cdot t_{\perp} + t_{\perp}^2 + \lambda \cdot t_{\perp}^2 - 2 \cdot \lambda \cdot t_{\perp} \cdot n + \lambda \cdot n^2 + 2 \cdot \lambda \cdot t_{\parallel}^2$$

$$2 \cdot \sigma_e^2 = (1 + \lambda) \cdot (n^2 + t_{\perp}^2) + 2 \cdot (1 - \lambda) \cdot n \cdot t_{\perp} + 2 \cdot \lambda \cdot t_{\parallel}^2$$

With  $\lambda = 1.8$ .

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

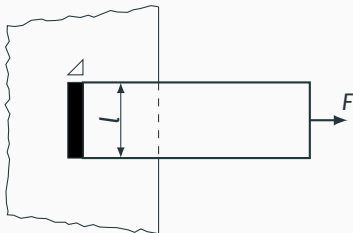
The dimensioning is then given by:

$$\sigma_e = \alpha \cdot \sigma_{adm}$$

Where  $\sigma_{adm}$  is the permissible stress of the base material.

$\alpha$  depends on the throat dimension and is estimated by:

$$\alpha = 0.8 \cdot \left(1 + \frac{1}{a}\right)$$



**Figure 11:** Case 1 [1]

General equation:

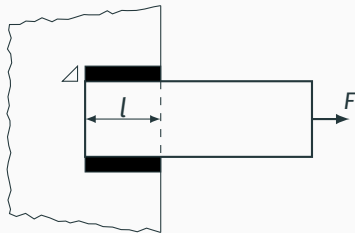
$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

Only  $n$  stress component exists:

$$n = \frac{F}{a \cdot l}$$

$$\sigma_e = \sqrt{1.4 \cdot n^2} = 1.18 \cdot \frac{F}{a \cdot l}$$

$$\frac{F}{a \cdot l} = \frac{\sigma_e}{1.18} = \frac{\alpha \cdot \sigma_{adm}}{1.18}$$



**Figure 12:** Case 2 [1]

Only  $t_{\parallel}$  stress component exists:

$$t_{\parallel} = \frac{F}{2 \cdot a \cdot l}$$

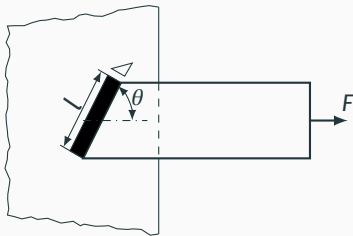
$$\sigma_e = \sqrt{1.8 \cdot \left( \frac{F}{2 \cdot a \cdot l} \right)^2} = 1.34 \cdot \frac{F}{2 \cdot a \cdot l}$$

General equation:

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

$$\frac{F}{2 \cdot a \cdot l} = \frac{\alpha \cdot \sigma_{adm}}{1.34}$$

## REApE case 3



**Figure 13:** Case 3 [1]

General equation:

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

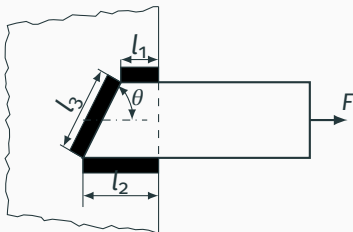
Exists both  $n$  and  $t_{\parallel}$  stress components:

$$n = \frac{F}{a \cdot l} \cdot \sin \theta \qquad t_{\parallel} = \frac{F}{a \cdot l} \cdot \cos \theta$$

$$\sigma_e = \sqrt{1.4 \cdot \left( \frac{F}{a \cdot l} \cdot \sin \theta \right)^2 + 1.8 \cdot \left( \frac{F}{a \cdot l} \cdot \cos \theta \right)^2}$$

$$\frac{F}{a \cdot l} = \frac{\alpha \cdot \sigma_{adm}}{\sqrt{1.4 \cdot \sin^2 \theta + 1.8 \cdot \cos^2 \theta}}$$





**Figure 14:** Case 4 [1]

The force is reparted by the horizontal welds  $F_{1,2}$  and the oblique weld  $F_3$ :

$$F = F_{1,2} + F_3$$

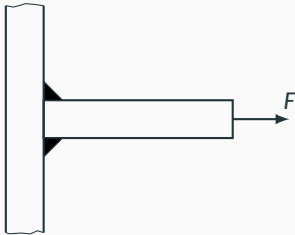
$$\frac{F_{1,2}}{l_1 \cdot a_1 + l_2 \cdot a_2} = \frac{\alpha \cdot \sigma_{adm}}{1.34}$$

General equation:

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

$$\frac{F_3}{a_3 \cdot l_3} = \frac{\alpha \cdot \sigma_{adm}}{\sqrt{1.4 \cdot \sin^2 \theta + 1.8 \cdot \cos^2 \theta}}$$

## REApE case 5



**Figure 15:** Case 5 [1]

General equation:

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

Only  $n$  stress component exists:

$$n = \frac{F}{2 \cdot a \cdot l}$$

$$\sigma_e = \sqrt{1.4 \cdot n^2} = \sqrt{1.4 \cdot \left( \frac{F}{2 \cdot a \cdot l} \right)^2}$$

$$\frac{F}{2 \cdot a \cdot l} = \frac{\alpha \cdot \sigma_{adm}}{1.18}$$



**Figure 16:** Case 6 [1]

General equation:

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

Only  $t_{\parallel}$  stress component is

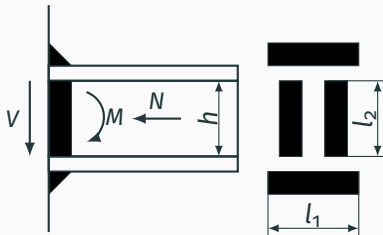
considered:

$$t_{\parallel} = \frac{V \cdot S}{2 \cdot a \cdot I}$$

$V$  is the transverse force,  $S$  is the static moment of area on the bending axis and  $I$  the second moment of area of the cross section on the bending axis.

$$\alpha \cdot \sigma_{adm} = \sqrt{1.8 \cdot t_{\parallel}^2}$$

## REApE case 7



**Figure 17:** Case 7 [1]

General equation:

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

For the beam flange to column:

$$n = \frac{N}{2 \cdot a_1 \cdot l_1 + 2 \cdot a_2 \cdot l_2} + \frac{M}{h \cdot a_1 \cdot l_1}$$

$$\alpha \cdot \sigma_{adm} = \sqrt{1.4 \cdot n^2}$$

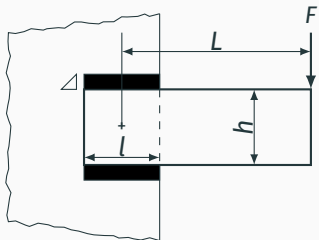
For the beam web to column:

$$n = \frac{N}{2 \cdot a_1 \cdot l_1 + 2 \cdot a_2 \cdot l_2}$$

$$t_{\parallel} = \frac{V}{2 \cdot a_2 \cdot l_2}$$

$$\alpha \cdot \sigma_{adm} = \sqrt{1.4 \cdot n^2 + 1.8 \cdot t_{\parallel}^2}$$

## REApE case 8



**Figure 18:** Case 8 [1]

$t_{\perp}$  and  $t_{\parallel}$  stress components exist:

$$t_{\perp} = \frac{F}{2 \cdot a \cdot l}$$

$$t_{\parallel} = \frac{F \cdot L}{h \cdot a \cdot l}$$

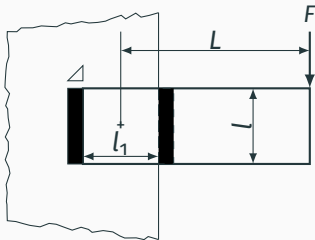
General equation:

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

$$\sigma_e = \sqrt{1.4 \cdot t_{\perp}^2 + 1.8 \cdot t_{\parallel}^2}$$

$$\alpha \cdot \sigma_{adm} = \sqrt{1.4 \cdot \left( \frac{F}{2 \cdot a \cdot l} \right)^2 + 1.8 \cdot \left( \frac{F \cdot L}{h \cdot a \cdot l} \right)^2}$$

## REApE case 9



**Figure 19:** Case 9 [1]

General equation:

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

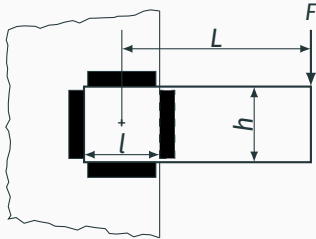
Only  $t_{\parallel}$  stress component exist:

$$t_{\parallel} = \frac{F}{2 \cdot a \cdot l} + \frac{F \cdot L}{l_1 \cdot a \cdot l}$$

$$\sigma_e = \sqrt{1.8 \cdot t_{\parallel}^2}$$

$$\alpha \cdot \sigma_{adm} = \sqrt{1.8 \cdot \left( \frac{F}{2 \cdot a \cdot l} + \frac{F \cdot L}{l_1 \cdot a \cdot l} \right)^2}$$

## REApE case 10



**Figure 20:** Case 10 [1]

General equation:

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

The moment  $M$  is reported by the vertical welds  $M_1$  and the

horizontal welds  $M_2$ :

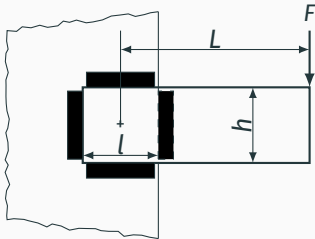
$$M = M_1 + M_2$$

$M_1$  causes  $t_{\parallel}$ :

$$t_{\parallel} = \frac{M_1}{l \cdot a_1 \cdot l_1}$$

$$\sigma_e = \alpha \cdot \sigma_{adm} = \sqrt{1.8 \cdot t_{\parallel}^2}$$

$$M_1 = \frac{\alpha \cdot \sigma_{adm}}{\sqrt{1.8}} \cdot l \cdot a_1 \cdot l_1$$



**Figure 20:** Case 10 [1]

$M_2$  causes  $t_{\parallel}$ :

$$t_{\parallel} = \frac{M_2}{h \cdot a_2 \cdot l_2}$$

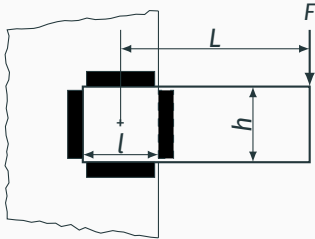
$$\sigma_e = \alpha \cdot \sigma_{adm} = \sqrt{1.8 \cdot t_{\parallel}^2}$$

$$M_2 = \frac{\alpha \cdot \sigma_{adm}}{\sqrt{1.8}} \cdot h \cdot a_2 \cdot l_2$$

$$\begin{cases} M = M_1 + M_2 \\ \frac{M_1}{M_2} = \frac{\frac{\alpha_1 \cdot \sigma_{adm}}{\sqrt{1.8}} \cdot l \cdot a_1 \cdot l_1}{\frac{\alpha_2 \cdot \sigma_{adm}}{\sqrt{1.8}} \cdot h \cdot a_2 \cdot l_2} \end{cases}$$



## REApE case 10



**Figure 20:** Case 10 [1]

General equation:

$$\sigma_e^2 = 1.4 \cdot (n^2 + t_{\perp}^2) - 0.8 \cdot n \cdot t_{\perp} + 1.8 \cdot t_{\parallel}^2$$

The force  $F$  is reported by the vertical welds  $F_1$  and the

horizontal welds  $F_2$ :

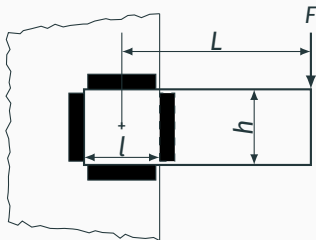
$$F = F_1 + F_2$$

$F_1$  causes  $t_{\parallel}$ :

$$t_{\parallel} = \frac{F_1}{2 \cdot a_1 \cdot l_1}$$

$$\sigma_e = \alpha \cdot \sigma_{adm} = \sqrt{1.8 \cdot t_{\parallel}^2}$$

$$F_1 = \frac{\alpha \cdot \sigma_{adm}}{\sqrt{1.8}} \cdot 2 \cdot a_1 \cdot l_1$$



**Figure 20:** Case 10 [1]

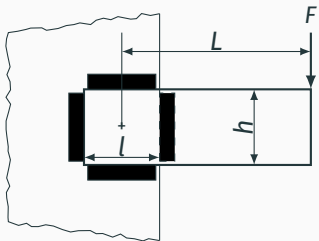
$F_2$  causes  $t_{\perp}$ :

$$t_{\perp} = \frac{F_2}{2 \cdot a_2 \cdot l_2}$$

$$\sigma_e = \alpha \cdot \sigma_{adm} = \sqrt{1.4 \cdot t_{\parallel}^2}$$

$$F_2 = \frac{\alpha \cdot \sigma_{adm}}{\sqrt{1.4}} \cdot 2 \cdot a_2 \cdot l_2$$

$$\begin{cases} F = F_1 + F_2 \\ \frac{F_1}{F_2} = \frac{\frac{\alpha_1 \cdot \sigma_{adm}}{\sqrt{1.8}} \cdot 2 \cdot a_1 \cdot l_1}{\frac{\alpha_2 \cdot \sigma_{adm}}{\sqrt{1.4}} \cdot 2 \cdot a_2 \cdot l_2} \end{cases}$$



**Figure 20:** Case 10 [1]

$t_{\parallel 2}$ . So:

$$\sigma_e = \alpha \cdot \sigma_{adm} = \sqrt{1.8 \cdot (t_{\parallel 1} + t_{\parallel 2})^2}$$

For horizontal welds:

$M_2$  causes  $t_{\parallel}$  and  $F_2$  also causes  $t_{\perp}$ . So:

For vertical welds:

$M_1$  causes  $t_{\parallel 1}$  and  $F_1$  also causes

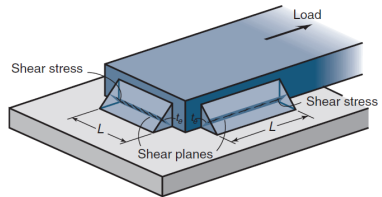
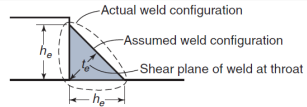
$$\sigma_e = \alpha \cdot \sigma_{adm} = \sqrt{1.4 \cdot t_{\perp}^2 + 1.8 \cdot t_{\parallel}^2}$$

# Welds treated as lines

The governing stress in fillet welds is shear on the throat of the weld as shown in Figure 21 [5].

On the fillet weld aligned parallel to the load, the shear stress occurs along the throat.

In a fillet weld aligned transverse to the load, the shear stress occurs at  $45^\circ$  to the load, acting transverse to the axis of the fillet.



**Figure 21:** Fillet weld [5]

# Welds treated as lines

## Parallel and transverse loading

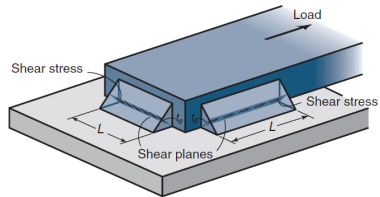
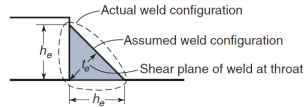
Fillet welds fail by shearing at the minimum section, which is at the throat of the weld.

This is true whether the weld has parallel (on the side) or transverse (at the end) loading. The shear stress from these types of loading is:

$$\tau = \frac{F}{t_e \cdot L} = \frac{F}{\frac{\sqrt{2}}{2} \cdot h_e \cdot L} = \frac{1.414 \cdot F}{h_e \cdot L}$$

To avoid failure:

$$\frac{\tau_{adm}}{n} = \frac{F}{t_e \cdot L}$$



**Figure 22:** Fillet weld [5]

# Welds treated as lines

## Torsional loading

For torsional loading, the resultant shear stress acting on the weld group is the vectorial sum of the direct and torsional shear stresses. The direct (or transverse) shear stress in the weld is:

$$\tau = \frac{V}{A}$$

The torsional shear stress is:

$$\tau = \frac{T \cdot r}{J}$$

$T$  is the applied torque,  $J$  is the polar moment of inertia and  $r$  is the distance from the centroid of the weld to the farthest point in the weld.

The critical section is the throat and the analysis can be simplified for line welds using the concept of unit polar moment of inertia:

$$J = t_e \cdot J_u$$

# Welds treated as lines

## Bending loading

In bending, the welded joint experiences a transverse shear stress (already discussed) as well as a normal stress. The moment  $M$  produces a normal bending stress  $\sigma$  in the welds. It is customary to assume that the stress acts normal to the throat area. The area moment of inertia is calculated from the unit area moment of inertia:

$$I = t_e \cdot I_u$$

The normal stress is:

$$\sigma = \frac{M \cdot c}{I}$$

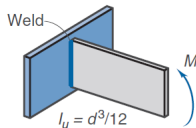
$c$  is the distance from the neutral axis to the outer fiber.

**After calculate the shear stress and the normal stress, the principal stresses can be determined and the maximum-shear stress theory or the distortion-energy theory can be applied.**

# Area and Polar moment of inertia [5]

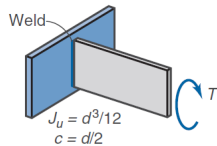


$L_w = d$



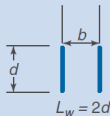
Weld

$I_u = d^3/12$

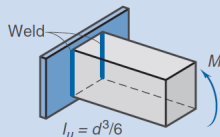


Weld

$J_u = d^3/12$   
 $c = d/2$

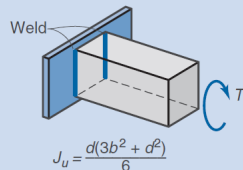


$L_w = 2d$



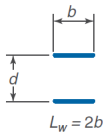
Weld

$I_u = d^3/6$

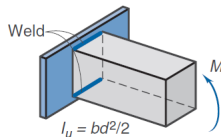


Weld

$J_u = \frac{d(3b^2 + d^2)}{6}$

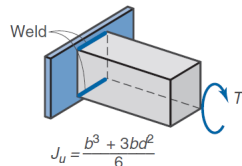


$L_w = 2b$



Weld

$I_u = bd^2/2$

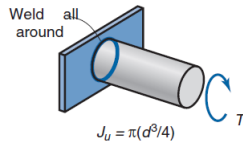
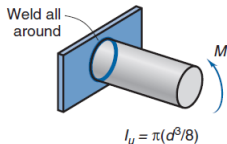
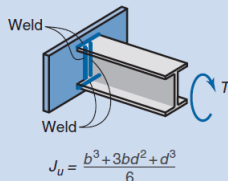
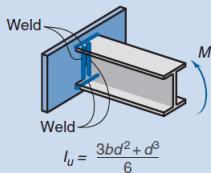
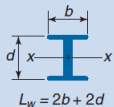
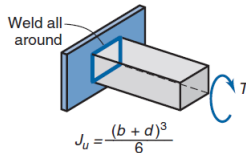
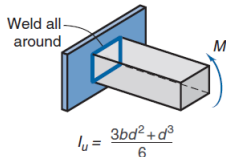
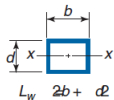


Weld

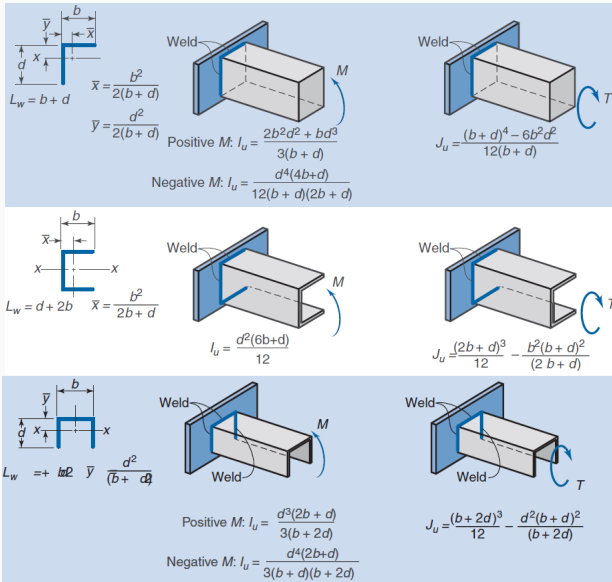
$J_u = \frac{b^3 + 3bd^2}{6}$



# Area and Polar moment of inertia [5]



# Area and Polar moment of inertia [5]



# American Welding Society

The American Welding Society specifies a minimum dimension for the welded joint based on the thicker plate of the connection:

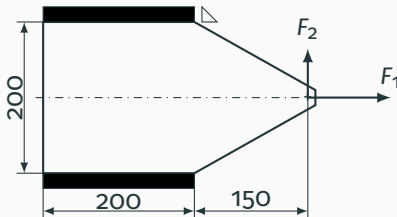
Thicker plate to connect	Minimum $h_e$
up to $1/2$	$3/16$
from $1/2$ up to $3/4$	$1/4$
from $3/4$ up to 1.5	$5/16$
from 1.5 up to 2.25	$3/8$
from 2.25 up to 6	$1/2$
above 6	$5/8$

## Minimum strength properties of electrode classes

Electrode number	$S_u$ / MPa	$S_y$ / MPa	Elongation / %
E60XX	427	345	17-25
E70XX	482	393	22
E80XX	552	462	19
E90XX	620	531	14-17
E100XX	689	600	13-16
E120XX	827	738	14

## Exercise 1: Eyebars

An eyebar subjected to an axial load  $F_1 = 37.5\text{ kN}$ , a vertical load  $F_2 = 21.65\text{ kN}$  and a load  $F_3 = 25\text{ kN}$  perpendicular [1].

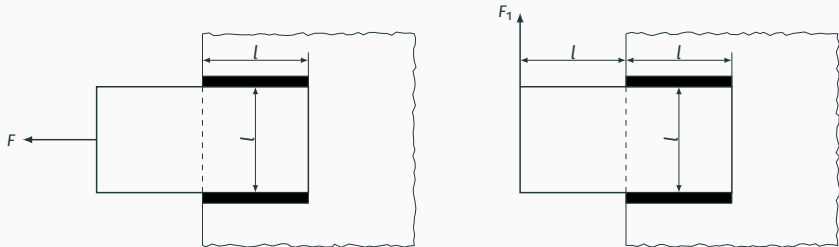


**Figure 23:** Eyebars.

Determine the weld throat dimensions if  $\sigma_{adm} = 240\text{ MPa}$ .

## Exercise 2: REApE cases

Calculate the ratio between the maximum allowed load  $F$  and  $F_1$  that can be applied for each case of Figure 24. Consider that the throat dimension  $a$  is similar in both cases.

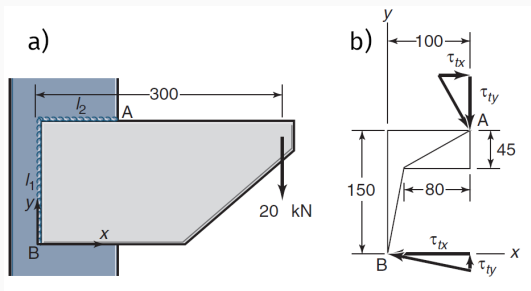


**Figure 24:** Welded joints.

### Exercise 3: Welds treated as lines

A bracket is welded to a beam as shown in Figure 25. Assume a steady loading of 20 kN and weld lengths  $l_1 = 150\text{ mm}$  and  $l_2 = 100\text{ mm}$ . Assume an electrode number of E60XX  $\sigma_{adm} = 345\text{ MPa}$  and a fillet weld.

Determine the minimum weld leg length for the eccentric loading based on torsion and a safety factor of 5.



**Figure 25:** Welded bracket: (a) dimensions, load, and coordinates; (b) torsional shear stress components at points A and B.

## References

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- [1] Castro, Paulo M.S.T. de: *Notas sobre o Dimensionamento de Ligações Soldadas*, 2015.
- [2] Ballio, G. and F.M. Mazzolani: *Theory and Design of Steel Structures*.  
**Chapman and Hall, 1983.**
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- [4] Picón, Rafael and José Cañas: *On strength criteria of fillet welds*.  
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- [5]** Osgood, Carl and Fatigue Design: *Fundamentals of Machine Elements*.  
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