Complements of Machine Elements
Rolling Bearings – Friction torque models

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Exercise

Consider a deep groove ball bearing (6206-C in Figure 1) with normal radial clearance.

The bearing has a radial load $F_r = 2000\,\text{N}$ and an axial load $F_a = 1000\,\text{N}$. The rolling bearing rotates at $n = 3000\,\text{rpm}$ and the mineral lubricant has a kinematic viscosity at the operating temperature $\nu = 20\,\text{mm}^2\,\text{s}^{-1}$.

Determine the total friction torque of the rolling bearing using the:

1. Coloumb model;
2. Arvrid Palmgren model considering for $f_0 = 1.75$, $f_1 = 1.45$ and $\mu_1 = 0.002 \cdot \left(\frac{F}{C_0}\right)^{\frac{1}{2}}$;
3. SKF model (disregard drag losses).

1 Coulomb model

The Coulomb model is given by equation (1).

$$M_t = \mu \cdot F \cdot \frac{d}{2} \tag{1}$$

The force F is calculated with equation (2).

$$F = \sqrt{{F_r}^2 + {F_a}^2} = \sqrt{2000^2 + 1000^2} = 2236\,\text{N} \tag{2}$$

The coefficient of friction μ is found in Table 2 of Formulary. The value expected for a deep groove ball bearing is from 1×10^{-3} to 1.5×10^{-3}. Then

Main dimensions			Basic load ratings		Fatigue limit load	Limiting speed	Speed rating	Factor	Mass	Designation
d	D	B	dyn. C_r	stat. C_{0r}	C_{ur}	n_G	$n_{\vartheta r}$	f_0	m	➤225 | 1.12 ➤226 | 1.13
			N	N	N	min^{-1}	min^{-1}		≈ kg	
30	42	7	4 500	2 950	149	24 500	11 500	14,2	0,027	**61806**
	42	7	4 500	2 950	149	8 100	–	14,2	0,027	**61806-2RSR**
	42	7	4 500	2 950	149	20 800	11 500	14,2	0,027	**61806-2Z**
	47	9	7 700	5 000	310	21 700	11 900	15,7	0,051	**61906**
	47	9	7 700	5 000	310	7 600	–	15,7	0,053	**61906-2RSR**
	47	9	7 700	5 000	310	18 400	11 900	15,7	0,053	**61906-2Z**
	55	13	13 500	8 000	390	18 500	13 100	14,8	0,122	**6006**
	55	9	11 900	7 300	370	18 900	10 500	15,1	0,082	**16006**
	55	13	13 500	8 000	390	7 000	–	14,8	0,126	**6006-2RSR**
	55	13	13 500	8 000	390	15 700	13 100	14,8	0,126	**6006-2Z**
	62	16	20 800	11 300	700	17 800	13 400	13,8	0,195	**6206-C**
	62	16	20 800	11 300	700	15 100	13 400	13,8	0,201	**6206-C-2BRS**
	62	16	20 800	11 300	700	10 600	–	13,8	0,201	**6206-C-2HRS**
	62	16	20 800	11 300	700	15 100	13 400	13,8	0,201	**6206-C-2Z**
	62	20	20 700	11 300	570	6 700	–	13,8	0,243	**62206-2RSR**
	72	27	30 000	15 800	1 060	6 000	–	13	0,486	**62306-2RSR**
	72	19	32 000	16 200	1 090	15 100	11 500	13	0,328	**6306-C**
	72	19	32 000	16 200	1 090	12 800	11 500	13	0,339	**6306-C-2BRS**
	72	19	32 000	16 200	1 090	8 900	–	13	0,34	**6306-C-2HRS**
	72	19	32 000	16 200	1 090	12 800	11 500	13	0,339	**6306-C-2Z**
	90	23	45 500	25 000	1 640	10 800	8 600	13	0,74	**6406**

Figura 1: Rolling bearing properties taken from manufacture's catalog.

we can compute a minimum, maximum and a mean coefficient of friction and then predict the total friction torque.

$$M_{t_{min}} = 1 \times 10^{-3} \times 2236 \times \frac{30}{2} = 33.54\,\mathrm{N\,mm} \tag{3}$$

$$M_{t_{max}} = 1.5 \times 10^{-3} \times 2236 \times \frac{30}{2} = 50.31\,\mathrm{N\,mm} \tag{4}$$

$$M_{t_{mean}} = 1.25 \times 10^{-3} \times 2236 \times \frac{30}{2} = 41.93\,\mathrm{N\,mm} \tag{5}$$

2 Arvrid Palmgren model

The Arvrid Palmgren model is given by equation (6).

$$M_t = M_0 + M_1 \tag{6}$$

The no-load friction torque is:

$$M_0 = f_0 \cdot 10^{-7} \cdot (\nu \cdot n)^{2/3} \cdot d_m^3 \tag{7}$$

The load dependent friction torque is:

$$M_1 = \mu_1 \cdot f_1 \cdot F \cdot \frac{d_m}{2} \tag{8}$$

The force F is calculated according to the equation (2) and the mean diameter d_m is given by:

$$d_m = \frac{D + d}{2} \tag{9}$$

The necessary coefficients are provided on the exercise, i.e. $f_0 = 1.75$, $f_1 = 1.45$ and $\mu_1 = 0.002\left(\frac{F}{C_0}\right)^{\frac{1}{2}}$.

The no-load friction torque is then calculated, for the present case as:

$$M_0 = 1.75 \times 10^{-7} \times (20 \times 3000)^{2/3} \times 46^3 = 26.11\,\mathrm{Nmm} \tag{10}$$

The coefficient of friction μ_1 required to calculate the load dependent losses M_1 is:

$$\mu_1 = 0.002 \times \left(\frac{2236}{11300}\right)^{\frac{1}{2}} = 0.000889 \tag{11}$$

$$M_1 = 1.45 \times 0.000889 \times 2236 \times \frac{46}{2} = 66.34\,\mathrm{Nmm} \tag{12}$$

The total friction torque using the Arvrid Palmgren model is:

$$M_t = 26.11 + 66.34 = 92.45\,\mathrm{Nmm} \tag{13}$$

3 SKF friction torque model

If we disregard the drag losses, the total friction torque is given by the equation (14). Please recall that the present bearing doesn't have seals.

$$M_t = M'_{rr} + M_{sl} \tag{14}$$

For the current exercise the total friction torque is evaluated as:

$$M_t = 66.12 + 40.09 = 106.21\,\mathrm{Nmm} \tag{15}$$

Now, let's see how to calculate each component of the SKF friction torque model.

3.1 Rolling friction torque M'_{rr}

$$M'_{rr} = \phi_{ish} \cdot \phi_{rs} \cdot \left[G_{rr} \cdot (n \cdot \nu)^{0.6}\right] \tag{16}$$

The value of G_{rr} is predicted for the present bearing as (see Table 1a of SKF model document):

$$G_{rr} = R_1 \cdot d_m^{1.96} \cdot \left(F_r + \frac{R_2}{\sin \alpha_F} F_a\right)^{0.54} \tag{17}$$

Which for the present case evaluates as:

$$G_{rr} = 3.9 \times 10^{-7} \times 46^{1.96} \times \left(2000 + \frac{1.7}{\sin 13.747^\circ} \times 1000\right)^{0.54} = 0.097575 \tag{18}$$

With α_F calculated by:

$$\alpha_F = 24.6 \cdot \left(\frac{F_a}{C_0}\right)^{0.24} = 13.747^\circ \tag{19}$$

So, we can calculate the nominal rolling friction torque as:

$$M_{rr} = G_{rr} \cdot (n \cdot \nu)^{0.6} = 0.097575 \times (3000 \times 20)^{0.6} = 71.82\,\text{Nmm} \tag{20}$$

To calculate the actual rolling friction torque, first we need to calculate the inlet shear heating and the replenishment/starvation factors, given by the equations (21) and (22).

$$\phi_{ish} = \frac{1}{1 + 1.84 \times 10^{-9} \cdot (n \cdot d_m)^{1.28} \cdot \nu^{0.64}} \tag{21}$$

$$\phi_{rs} = \frac{1}{e^{K_{rs} \cdot \nu \cdot n \cdot (d+D) \cdot \sqrt{\frac{K_z}{2 \cdot (D-d)}}}} \tag{22}$$

The value of the inlet shear heating is evaluated as:

$$\phi_{ish} = \frac{1}{1 + 1.84 \times 10^{-9} \times (3000 \cdot 46)^{1.28} \times 20^{0.64}} = 0.955 \tag{23}$$

To calculate the value of the replenishment/starvation factor, The value of $K_z = 3.1$ is taken from Table 4 (SKF model document available on Course Contents). The value of K_{rs} is 3×10^{-8} for low level oil bath and oil jet lubrication:

$$\phi_{rs} = \frac{1}{3\times10^{-8}\times20\times3000\times(30+62)\times\sqrt{\frac{3.1}{2\times(62-30)}}} = 0.964 \tag{24}$$

But 6×10^{-8} for grease and oil-air lubrication:

$$\phi_{rs} = \frac{1}{6\times10^{-8}\times20\times3000\times(30+62)\times\sqrt{\frac{3.1}{2\times(62-30)}}} = 0.9297 \tag{25}$$

Since no details are given for the lubrication method used, lets use the biggest $\phi_{rs} = 0.964$ value that corresponds to a less efficient bearing.

$$M'_{rr} = \phi_{ish}\cdot\phi_{rs}\cdot M_{rr} = 0.955\times0.964\times = 71.82 = 66.12\,\mathrm{N\,mm} \tag{26}$$

3.2 Sliding friction torque M_{sl}

$$M_{sl} = G_{sl}\cdot\mu_{sl} \tag{27}$$

The value of G_{sl} is predicted for the present bearing as (see Table 1a of SKF model document):

$$G_{sl} = S_1\cdot d_m^{-0.145}\left(F_r^5 + \frac{S_2\cdot d_m^{1.5}}{\sin\alpha_F}F_a^4\right)^{\frac{1}{3}} \tag{28}$$

Which for the present case evaluates as:

$$G_{sl} = 3.23\times10^{-3}\times46^{-0.145}\left(2000^5 + \frac{36.5\times46^{1.5}}{\sin 13.747°}\times1000^4\right)^{\frac{1}{3}} = 798.59 \tag{29}$$

The coefficient of friction μ_{sl} is given by equation (30).

$$\mu_{sl} = \phi_{bl}\cdot\mu_{bl} + (1-\phi_{bl})\cdot\mu_{EHL} \tag{30}$$

The weighting lubrication factor ϕ_{bl} is:

$$\phi_{bl} = \frac{1}{e^{2.6\times10^{-8}(n\cdot\nu)^{1.4}\cdot d_m}} = \frac{1}{e^{2.6\times10^{-8}(3000\times20)^{1.4}\times46}} = 0.00288 \tag{31}$$

The reference coefficient of friction values are $\mu_{bl} = 0.12$ (see SKF model document) and $\mu_{EHL} = 0.05$ for lubrication with mineral oils which is the case.

Finally the coefficient of friction μ_{sl} is evaluated as:

$$\mu_{sl} = 0.00288 \times 0.12 + (1 - 0.00288) \times 0.05 = 0.0502 \tag{32}$$

The sliding friction torque is then:

$$M_{sl} = 798.59 \times 0.0502 = 40.09\,\mathrm{N\,mm} \tag{33}$$