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Fatigue Design

Complements of Machine Elements

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Mestrado em Engenharia Mecânica

All machine and structural designs are problems in fatigue because the forces of Nature are always at work and each object must respond in some fashion.

Carl Osgood

Recommended bibliography

- Haberhauer, Horst: Maschinenelemente. Springer Berlin Heidelberg, Berlin, Heidelberg, 2018, ISBN 978-3-662-53047-4

Hyperlink

Lecture 1

Summary

Introduction

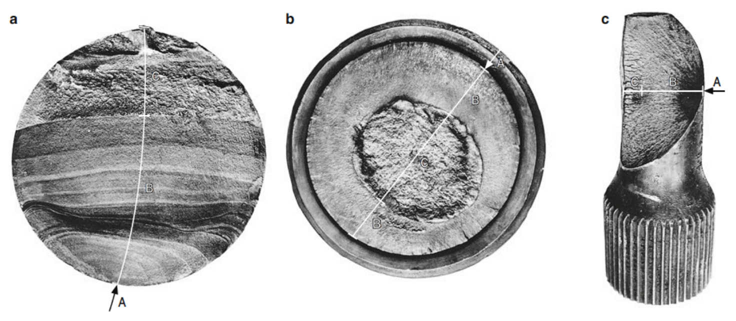


Figure 1: Typical fatigue fractures. **a.** bending fatigue fracture on the eccentric shaft of a crusher ($d = 230\,\mathrm{mm}$) at the transition from the shaft to the eccentric, **b.** Circular bending fatigue fracture on the driving axis of a tilting plow. Cause: too small fillet radius, **c.** torsional fatigue failure on a torsion bar spring [1].

A: First crack at the flaw or notch on the surface; B: Zone of progressive fatigue fracture; C: Final fracture (forced fracture)

Review on yield criteria

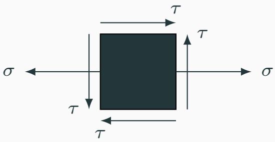


Figure 2: Element of a shaft under bending (σ) and torsion (τ).

Principal stresses:

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\sigma_2 = \frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

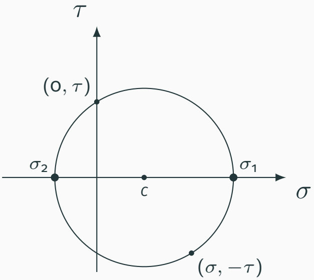


Figure 3: Mohr's Circle or circle of stress for combined stresses [2].

Maximum shear stress theory:

$$\sigma_e = \sigma_1 - \sigma_2 = \sqrt{\sigma^2 + 4\tau^2}$$

Distortion-Energy Theory [1]

$$\sigma_e = \sqrt{{\sigma_1}^2 + {\sigma_2}^2 - \sigma_1\sigma_2}$$

$$\sigma_e = \sqrt{\sigma^2 + 3\tau^2}$$

The relation between the shear strength and the tensile strength is predicted by the maximum distortion energy theory as:

$$\tau_{adm} = \frac{\sigma_{adm}}{\sqrt{3}}$$

$$\frac{\sigma_{adm}}{\sqrt{3}\tau_{adm}} = 1$$

Which can be included into the maximum distortion energy theory equation:

$$\sigma_e = \sqrt{\sigma^2 + 3\left(\frac{\sigma_{adm}}{\sqrt{3}\tau_{adm}}\tau\right)^2}$$

[1] Topic mostly transcribed from Paulo M.S.T. Castro [3] based on a *must read* review of the german literature

Bach theory

$$\varepsilon_x = \frac{\sigma}{E} \quad \text{and} \quad \varepsilon_y = \frac{-\nu\sigma}{E}$$

$$\varepsilon_{max} = \frac{1}{E}\left[\frac{1}{2}(1-\nu)\sigma + \frac{1}{2}(1+\nu)\sqrt{\sigma^2 + 4\tau^2}\right]$$

$$\sigma_e = E\varepsilon_{max}$$

For an isotropic material with $\nu = 0.3$:

$$\sigma_e = 0.35\sigma + 0.65\sqrt{\sigma^2 + 4\tau^2}$$

A correction factor (*effort ratio*) can be added to the equation:

$$\sigma_e = 0.35\sigma + 0.65\sqrt{\sigma^2 + 4(\alpha_0\tau)^2}$$

$$\alpha_0 = \frac{\sigma_{adm}}{(1+\nu)\tau_{adm}}$$

Effort ratio

According to literature [4, 1], the ratio $\frac{\tau_{adm}}{\sigma_{adm}} = \frac{1}{\sqrt{3}}$ is sometimes not verified experimentally.

So, it is current to use a modification factor α_0, called *effort ratio*[2,3]. For the maximum distortion energy theory:

$$\boxed{\sigma_e = \sqrt{\sigma^2 + 3\left(\alpha_0 \tau\right)^2}}$$

The effort ratio is [4]:

$$\boxed{\alpha_0 = \frac{\sigma_{Grenz}}{\varphi \tau_{Grenz}} = \frac{\sigma_{adm}}{\varphi \tau_{adm}}}$$

Grenz is translated as limit, permissible stress.

For the the maximum distortion energy theory, $\varphi = \sqrt{3} = 1.73$

[2] From german: *Anstrengungsverhältnis*

[3] In Niemman [1] it is described as *Schubfestigkeitsfaktor* which is “shear stress factor”

Equivalent stress

Criteria	Equivalent stress	Effort ratio
1. Maximum normal stress[4]	$\sigma_e = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\left(\alpha_0 \tau\right)^2}\right]$	$\alpha_0 = \frac{\sigma_{adm}}{\tau_{adm}}$
2. Maximum shear stress[5]	$\sigma_e = \sqrt{\sigma^2 + 4\left(\alpha_0 \tau\right)^2}$	$\alpha_0 = \frac{\sigma_{adm}}{2\tau_{adm}}$
3. Maximum distortion energy[6]	$\sigma_e = \sqrt{\sigma^2 + 3\left(\alpha_0 \tau\right)^2}$	$\alpha_0 = \frac{\sigma_{adm}}{1.73\tau_{adm}}$
4. Maximum strain (Bach)	$\sigma_e = 0.35\sigma + 0.65\sqrt{\sigma^2 + 4\left(\alpha_0 \tau\right)^2}$	$\alpha_0 = \frac{\sigma_{adm}}{(1+\nu)\tau_{adm}}$

1. To be used mainly for brittle materials;

2. For ductile materials in the event of failure (static) due to sliding fracture (e.g. with copper and copper alloys);

3. For ductile materials.

[4]NH *Normalspannungs-Hypothese*
[5]SH *Schubspannungs-Hypothese*
[6]GEH *Gestaltänderungsenergie-Hypothese*

Load cases

According to Bach [4, 1] there are essentially three basic load cases:

Load case I (*ruhende*): static load

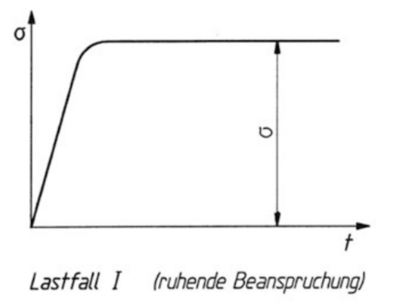


Figure 4: Lastfall I (*ruhende Beanspruchung* [4]

Load case II (*rein schwellende*): pulsating stress R=0

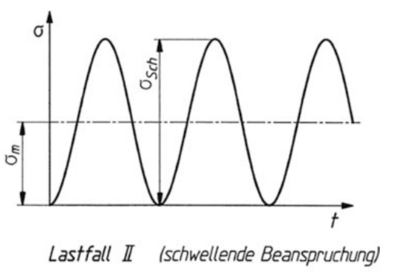


Figure 5: Lastfall II (*schwellende Beanspruchung* [4]

Load cases

Load case III (*rein wechselnde*): fully reversed stress R=-1

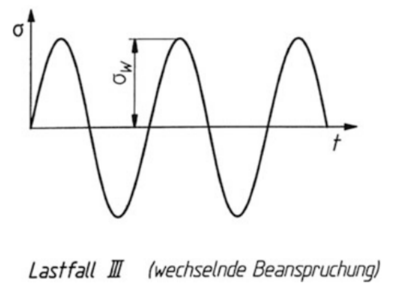


Figure 6: Lastfall III (*wechselnde Beanspruchung*) [4]

General load case (*allgemeiner*): oscilating stress – load case I plus load case III.

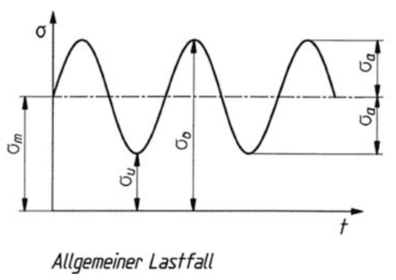


Figure 7: Allgemeiner Lastfall [4]

Effort ratio

Values for the *effort ratio* for different loading combinations:

Load case		[4]	[5]	[6]	[7]
σ_{III}	τ_{I}	0.7	0.7	0.5	0.7
σ_{II}	τ_{I}			0.7	
σ	τ	1	1	1	1
σ_{I}	τ_{III}	1.5	1.5	2	1.5
σ_{I}	τ_{II}			1.5	
σ_{III}	τ_{II}	0.7	0.7	0.75	
σ_{II}	τ_{III}		1.5	1.35	

Safety factor

$$\boxed{\sigma_e = \sqrt{\sigma^2 + 3\left(\alpha_0 \tau\right)^2}}$$

For the the maximum distortion energy theory, $\alpha_0 = \frac{1}{\sqrt{3}} \frac{\sigma_{adm}}{\tau_{adm}}$

$$\sigma_e^2 = \sigma^2 + 3\left(\frac{1}{\sqrt{3}} \frac{\sigma_{adm}}{\tau_{adm}} \tau\right)^2 = \sigma^2 + \left(\frac{\sigma_{adm}}{\tau_{adm}}\right)^2 \tau^2$$

Dividing by σ_{adm}^2:

$$\frac{\sigma_e^2}{\sigma_{adm}^2} = \frac{\sigma^2}{\sigma_{adm}^2} + \frac{\tau^2}{\tau_{adm}^2}$$

$$\frac{\sigma_e}{\sigma_{adm}} = \sqrt{\frac{\sigma^2}{\sigma_{adm}^2} + \frac{\tau^2}{\tau_{adm}^2}}$$

With $\sigma_e S = \sigma_{adm}$:

$$\frac{1}{S} = \sqrt{\frac{\sigma^2}{\sigma_{adm}^2} + \frac{\tau^2}{\tau_{adm}^2}}$$

$$\boxed{S = \frac{1}{\sqrt{\frac{\sigma^2}{\sigma_{adm}^2} + \frac{\tau^2}{\tau_{adm}^2}}}}$$

Smith diagram

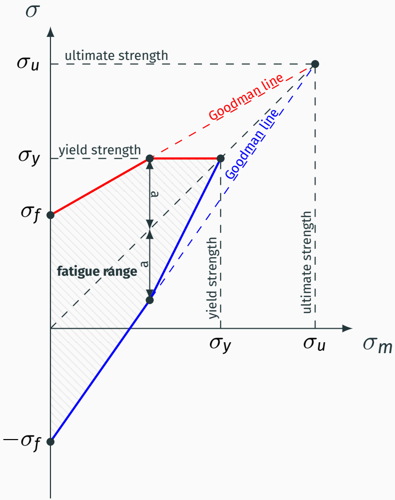


Figure 8: Smith diagram.

Smith Diagram for Traction

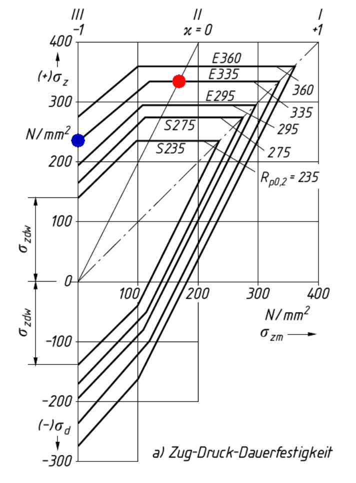


Figure 9: Smith diagram for traction for structural steels according to DIN EN 10025 [5]

- Fully reversed R=-1: $\sigma_{zdW} = 235\,\mathrm{MPa}$
- Pulsating R=0: $\sigma_{zdSch} = 335\,\mathrm{MPa}$

Yield strength: $\sigma_{zdF} = 335\,\mathrm{MPa}$

Smith Diagram for Bending

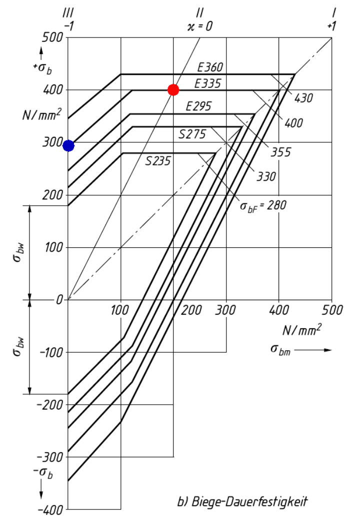


Figure 10: Smith diagram for bending for structural steels according to DIN EN 10025 [5]

- Fully reversed R=-1: $\sigma_{bW} = 290\,\mathrm{MPa}$
- Pulsating R=0: $\sigma_{bSch} = 400\,\mathrm{MPa}$

Yield strength: $\sigma_{bF} = 400\,\mathrm{MPa}$

Smith Diagram for Torsion

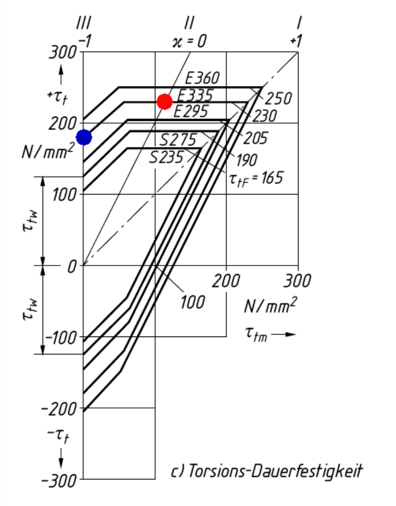


Figure 11: Smith diagram for torsion for structural steels according to DIN EN 10025 [5]

- Fully reversed R=-1: $\tau_{tW} = 180\,\mathrm{MPa}$
- Pulsating R=0: $\tau_{tSch} = 230\,\mathrm{MPa}$

Yield strength: $\tau_{tF} = 230\,\mathrm{MPa}$

Determine effort ratio from Smith diagram

Consider a shaft made of steel E335 according to DIN 10025 EN under bending and torsion[7]. Determine the *effort ratio* for:

- σ_{III} reversed bending ($\sigma_{bw} = 290\,\mathrm{MPa}$) and τ_I constant torsion ($\tau_{tF} = 230\,\mathrm{MPa}$):

$$\alpha_0 = \frac{\sigma_{bw}}{\sqrt{3}\tau_{tF}} = \frac{290}{230\sqrt{3}} = 0.728$$

- σ_{III} reversed bending ($\sigma_{bw} = 290\,\mathrm{MPa}$) and τ_{III} reversed torsion ($\tau_{tw} = 180\,\mathrm{MPa}$):

$$\alpha_0 = \frac{\sigma_{bw}}{\sqrt{3}\tau_{tW}} = \frac{290}{180\sqrt{3}} = 0.930$$

- σ_I constant bending ($\sigma_{bF} = 400\,\mathrm{MPa}$) and τ_{III} reversed torsion ($\tau_{tw} = 180\,\mathrm{MPa}$):

$$\alpha_0 = \frac{\sigma_{bF}}{\sqrt{3}\tau_{tW}} = \frac{400}{180\sqrt{3}} = 1.283$$

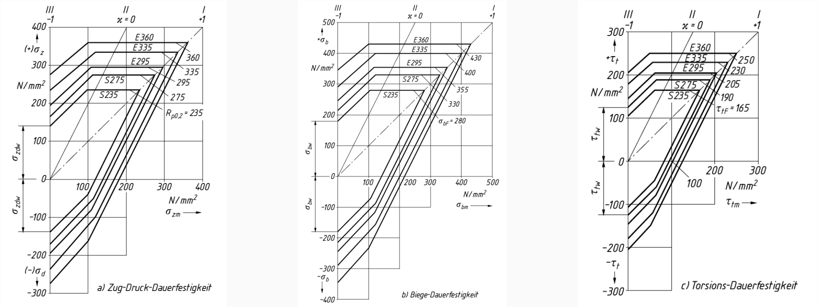
[7] Example transcribed from [8], *effort ratio* to be used with the maximum distortion energy (*Gestaltänderungsenergiehypothese – GEH*) theory with $\varphi = \sqrt{3}$

Comparison of *effort ratio* values

The following Table presents a comparison between the *effort ratio* calculated with the strength properties taken from Smith diagrams and values taken from literature.

The values taken from the literature are generic values, so, it is expected to find slight variations not only due to the loading conditions but also due to material specificity (ductility, anisotropy, manufacturing) [1].

Load case	Decker [6]	Roloff/Mattek [5]	Smith diagrams
σ_{III}, τ_{I}	0.5	0.7	**0.728**
σ_{III}, τ_{III}	1	1	**0.930**
σ_{I}, τ_{III}	2	1.5	**1.283**



(a) Traction **(b)** Bending **(c)** Torsion

Figure 12: Smith diagrams for structural steels according to DIN EN 10025 [5]

Comparing the smith diagram for bending and the smith diagram for traction, it is possible to state that the yield strength $\sigma_{bF} > \sigma_{zdF}$ is bigger for bending.

Smith diagram

$$\boxed{S = \frac{1}{\sqrt{\frac{\sigma^2}{\sigma^2_{adm}} + \frac{\tau^2}{\tau^2_{adm}}}}}$$

From the Smith diagram for each loading type, it is evident that bending stresses and normal stresses should be treated separately [8, 5, 4, 1]:

$$\boxed{S = \frac{1}{\sqrt{\left(\frac{\sigma_{zd}}{\sigma_{adm,zd}} + \frac{\sigma_{bw}}{\sigma_{adm,bw}}\right)^2 + \frac{\tau_t^2}{\tau^2_{adm,t}}}}}$$

The shear stress due to shear forces is usually disregarded on these equations [1].

Equivalent moment

The equivalent stress can be transformed into an equivalent moment:

$$\sigma_e = \sqrt{\sigma^2 + 3\left(\alpha_0 \tau\right)^2}$$

Recall that for a round solid section:

$$\sigma = \frac{32 M_f}{\pi d^3} \quad \text{and} \quad \tau = \frac{16 M_t}{\pi d^3}$$

$$\sigma_e = \sqrt{\left(\frac{32 M_f}{\pi d^3}\right)^2 + 3\left(\alpha_0 \frac{2 \times 16 M_t}{2\pi d^3}\right)^2}$$

$$\sigma_e = \frac{32}{\pi d^3}\sqrt{{M_f}^2 + \frac{3}{4}\left(\alpha_0 M_t\right)^2}$$

$$\sigma_e \frac{\pi d^3}{32} = M_e = \sqrt{{M_f}^2 + \frac{3}{4}\left(\alpha_0 M_t\right)^2}$$

Lecture 2

Summary

DIN 743

The DIN 743:2012 standard “Calculation of load capacity of shafts and axles” [8]:

- Part 1: General
- Part 2: Theoretical stress concentration factors and fatigue notch factors
- Part 3: Strength of materials
- Part 4: Fatigue limit, endurance limit - Equivalently damaging continuous stress
- Supplement 1: “Examples to part 1 to 3”
- Supplement 2: “Examples to part 4”

[8] In german: *Tragfähigkeitsberechnung von Wellen und Achsen*

DIN 743 - fatigue verification[9]

$$\boxed{S = \frac{1}{\sqrt{\left(\frac{\sigma_{zda}}{\sigma_{zdADK}} + \frac{\sigma_{ba}}{\sigma_{bADK}}\right)^2 + \left(\frac{\tau_{ta}}{\tau_{tADK}}\right)^2}}}$$

The safety factor should be:

$$S \geq S_{min} \geq 1.2$$

If the load quantification is uncertain, higher safety factors should be used.

The standard calculates the safety factor against fatigue failure for infinite life.

[9] A comprehensive step by step explanation about the calculation guideline of DIN 743 is given in Course Notes [3]

DIN 743 - plastic deformation verification

Verification for plastic deformation:

$$\boxed{S = \frac{1}{\sqrt{\left(\frac{\sigma_{zdmax}}{\sigma_{zdFK}} + \frac{\sigma_{bmax}}{\sigma_{bFK}}\right)^2 + \left(\frac{\tau_{tmax}}{\tau_{tFK}}\right)^2}}}$$

For the plastic deformation verification, the stress concentration factors is not need (already discussed in “Machine Elements”).

Strength of the part is calculated for any diameter d from the values for the specimen with d_B [10].

[10]To be discussed during KISSSoft class both for fatigue (two cases) and static verification

Soderberg criterium

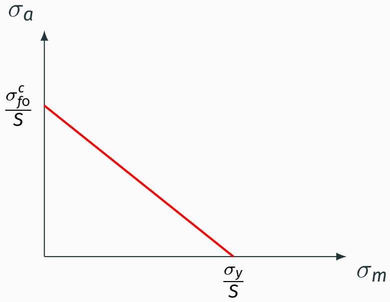


Figure 13: Soderberg criterium with a safety factor.

The equation should be used after:

- correct the fatigue limit strength $\sigma_{fo}^c = C_1 \cdot C_2 \cdot C_3 \cdot C_4 \cdot \sigma_{fo}$;
- add an adequate stress concentration factor for fatigue;
- consider a safety factor.

$$\boxed{\frac{K_f \cdot \sigma_a}{\sigma_{fo}^c} + \frac{\sigma_m}{\sigma_y} = \frac{1}{S}}$$

Review on Soderberg criterium

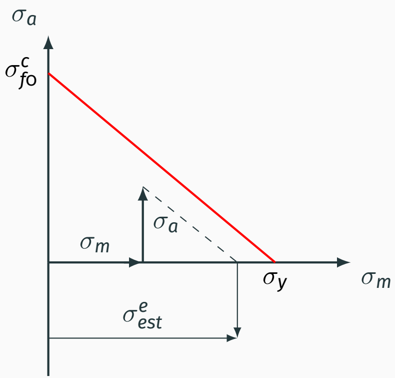


Figure 14: Determination of an equivalent static stress.

$$\frac{\sigma_a}{\sigma_{fo}^c} + \frac{\sigma_m}{\sigma_y} = 1$$

The Soderberg line slope is $\frac{\sigma_{fo}^c}{\sigma_y}$

An equivalent static stress is then computed as:

$$\sigma_{est}^{eq} = \sigma_m + \frac{\sigma_y}{\sigma_{fo}^c} \cdot \sigma_a$$

And for practical applications:

$$\boxed{\sigma_{est}^{eq} = \sigma_m + \frac{\sigma_y}{\sigma_{fo}^c} \cdot K_f \cdot \sigma_a}$$

Comparison of different fatigue design criteria

For a solid shaft with circular cross section:

$$\sigma_a = K_f \frac{32 M_{f_a}}{\pi d^3}$$

$$\sigma_m = \frac{32 M_{f_m}}{\pi d^3}$$

$$\tau_a = K_f \frac{16 M_{t_a}}{\pi d^3}$$

$$\tau_m = \frac{16 M_{t_m}}{\pi d^3}$$

An equivalent static stress is calculated using the Soderberg criterium for normal stresses:

$$\sigma_{est}^{eq} = \sigma_m + K_f \frac{\sigma_y}{\sigma_{fo}^c} \sigma_a$$

And also for shear stresses:

$$\tau_{est}^{eq} = \tau_m + K_f \frac{\sigma_y}{\sigma_{fo}^c} \tau_a$$

ASME-elliptic criterium

Using Soderberg and maximum shear stress theory for a solid circular section:

$$\frac{\sigma_y}{S} = \sqrt{\left(\sigma_m + K_f \frac{\sigma_y}{\sigma_{fo}^c} \sigma_a\right)^2 + 4\left(\tau_m + K_f \frac{\sigma_y}{\sigma_{fo}^c} \tau_a\right)^2}$$

The section diameter can be estimated as:

$$d^3 = \frac{32S}{\pi} \sqrt{\left(\frac{M_{f_m}}{\sigma_y} + K_f \frac{M_{f_a}}{\sigma_{fo}^c}\right)^2 + \left(\frac{M_{t_m}}{\sigma_y} + K_f \frac{M_{t_a}}{\sigma_{fo}^c}\right)^2}$$

Also known as ASME-elliptic criterium [9, 10, 11]

ASME criterium

Using Soderberg and maximum distortion energy theory for a solid circular section:

$$\frac{\sigma_y}{S} = \sqrt{\left(\sigma_m + K_f \frac{\sigma_y}{\sigma_{fo}^c}\sigma_a\right)^2 + 3\left(\tau_m + K_f \frac{\sigma_y}{\sigma_{fo}^c}\tau_a\right)^2}$$

The section diameter can be estimated as:

$$d^3 = \frac{32S}{\pi}\sqrt{\left(\frac{M_{f_m}}{\sigma_y} + K_f \frac{M_{f_a}}{\sigma_{fo}^c}\right)^2 + \frac{3}{4}\left(\frac{M_{t_m}}{\sigma_y} + K_f \frac{M_{t_a}}{\sigma_{fo}^c}\right)^2}$$

Also known as ASME criterium [9, 10, 11]

Comparison of different fatigue design criteria

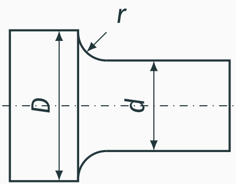


Figure 15: Shaft under alternated bending and alternating torsion.

$D = 50\,\mathrm{mm}$, $d = 42\,\mathrm{mm}$, $r = 5\,\mathrm{mm}$, Surface roughness $R_z = 4\,\mathrm{\mu m}$ [11]

Load at cross section with diameter d:

$\sigma_b = 500\,\mathrm{MPa} \pm 50\,\mathrm{MPa}$ and $\tau_t = 100\,\mathrm{MPa} \pm 30\,\mathrm{MPa}$

Material: 34CrMo4 (strength values according to DIN 743-3, $d_B \leq 16\,\mathrm{mm}$):

$\sigma_B = \sigma_R = 1000\,\mathrm{MPa}, \sigma_S = \sigma_y = 800\,\mathrm{MPa}, \sigma_{zdW} = 400\,\mathrm{MPa}, \sigma_{bW} = 500\,\mathrm{MPa}, \tau_{tW} = 300\,\mathrm{MPa}$

[11]Example and result taken from Supplement 1 of DIN 743

Comparison of different fatigue design criteria

The ASME and ASME elliptic follow the same procedure already discussed during “Machine Elements”:

- stress concentration factor K_t from Peterson [10, 12] assuming the notch sensitivity factor as $q = 1$:

$$K_f = 1 + q \cdot (K_t - 1)$$

- *Marine* factors (C_1, C_2 and C_3) as described on [13] or [9];
- Fatigue strength calculated as $\sigma_{fo}^c = 0.5 \cdot \sigma_R \cdot C_1 \cdot C_2 \cdot C_3$

Criteria	Safety factor
ASME	1.11
ASME Elliptic	1.08
DIN 743	2.61

Critical speed of a shaft

The shaft should be verified for:

- static loading;
- fatigue;
- critical speed.

The critical speed should be always accessed. Why?

Because the rotating shafts deflect during operation. The critical speed (natural frequency) is the point at which the shaft becomes dynamically unstable and large deflections associated with vibration are likely to develop [14].

Potential energy:

$$U = \frac{1}{2}k\delta^2$$

Kinetic energy:

$$T = \frac{1}{2}m\dot{\delta}^2$$

The potential energy is zero when the mass passes through the static equilibrium position and the kinetic energy is maximum. When the mass is at the position of maximum displacement, velocity is zero and the potential energy is maximum.

Critical speed of a shaft

The total mechanical energy is the sum of the potential and kinetic energies, which is constant over time:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(U+T\right)=0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}k\delta^2+\frac{1}{2}m\dot{\delta}^2\right]=0$$

$$k\delta\dot{\delta}+m\dot{\delta}\ddot{\delta}=0\longleftrightarrow\dot{\delta}\left(m\ddot{\delta}+k\delta\right)=0$$

$$\ddot{\delta}+\omega^2\delta=0$$

ω is the first natural frequency, usually used as critical speed:

$$\boxed{\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{g}{\delta}}}$$

This model is valid only for a shaft that can be represented by a single mass and spring.

For a multiple-mass system, *Rayleigh method* or *Dunkerley equation* should be used.

KISSSoft Shaft Design

Summary

KISSSoft

KISSSoft presents several capabilities:

- Gears
- Shafts and Bearings
- Shaft-Hub Connections
- Bolts
- Springs
- Belts and Chain Drives

There are other comercial software with similar capabilities, namely: Masta; Symscape; Romax; GWJ Technology GmbH and Mesys (both includes DIN 743:2012 calculation).

The focus should be the methods, not the software tools.

KISSSoft main window

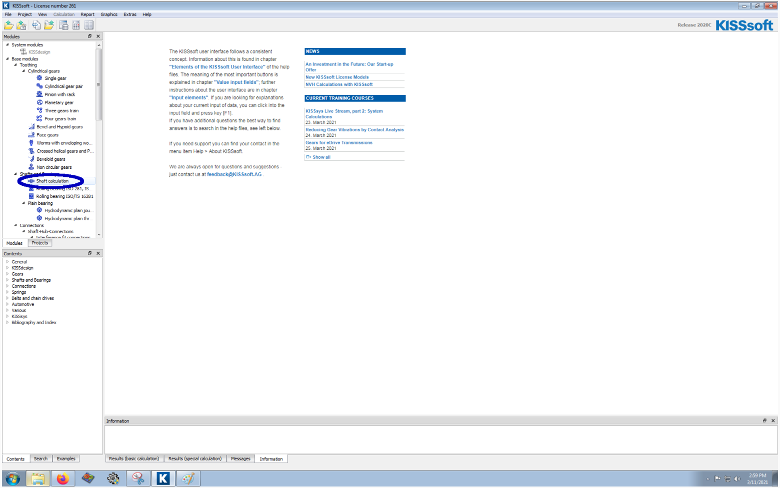


Figure 16: KISSSoft main window, base modules tree and shaft calculation module.

Shaft Editor

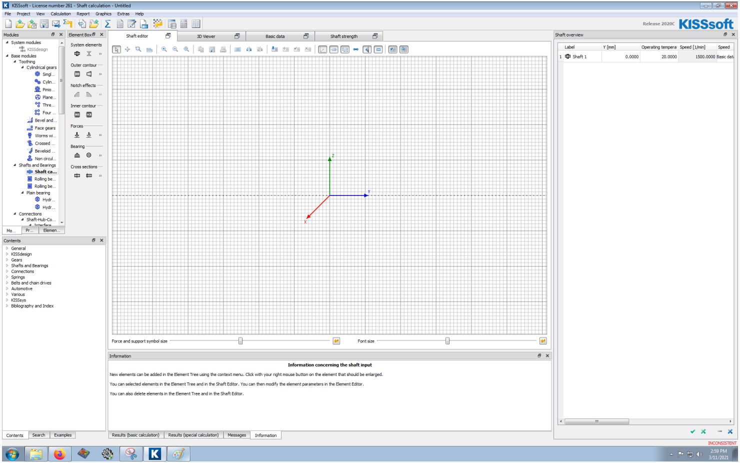


Figure 17: KISSSoft shaft editor.

KISSSoft examples

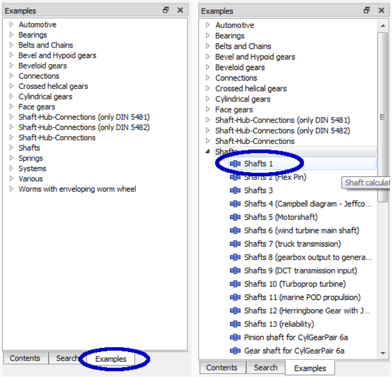


Figure 18: KISSSoft built-in examples.

- Select “Examples” in bottom left of KISSSoft main window
- Select “Shafts” tree
- Select “Shafts 1” example

The example is a shaft, two rolling element bearings, a helical gear pinion with a key and an interference fit.

Shaft 2D view

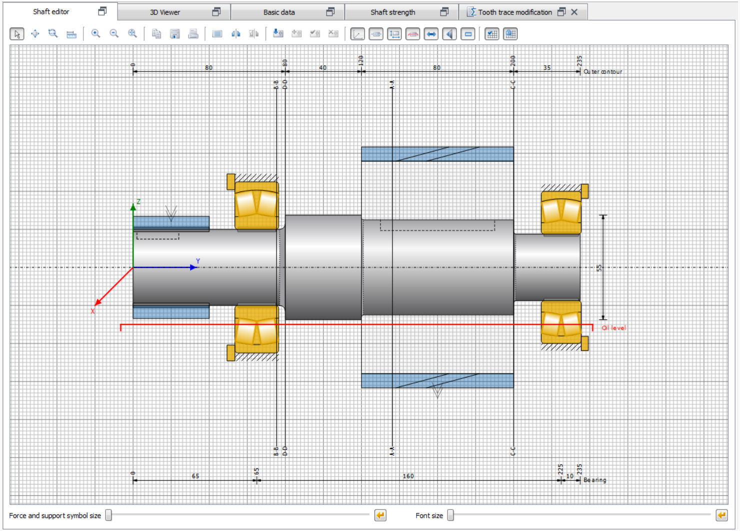


Figure 19: 2D view of the Shaft 1 example.

Shaft 3D view

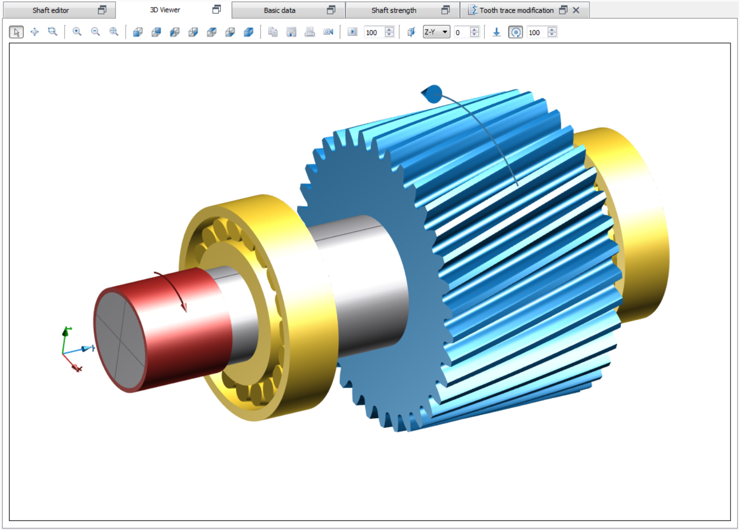


Figure 20: 3D view of the Shaft 1 example.

Change gear type

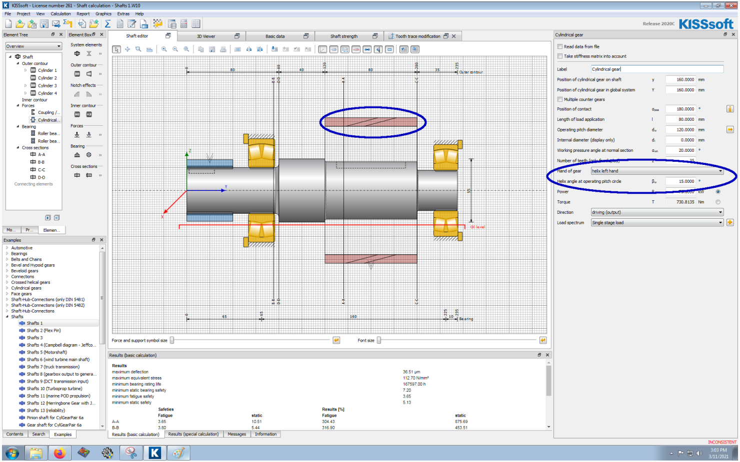


Figure 21: Change the gear type from helical to spur ($m = 3.5\,\mathrm{mm}$, $z_1 = 35$, $z_2 = 65$, $\alpha = 20^\circ$, $a = 175\,\mathrm{mm}$) and define the input Power ($75\,\mathrm{kW}$). Save the changes as a new model.

Shaft geometry and loads

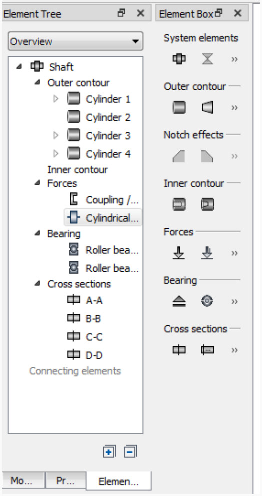


Figure 22: Shaft design: cross sections and lengths, notches, rolling bearings, forces (coupling/gears)

In order to identify or modify the shaft notches the “Outer contour” section should be explored.

The “Cross sections” are important to make the strength calculations[12].

[12] Homework 5 is based on an example to use the shaft editor

Basic data

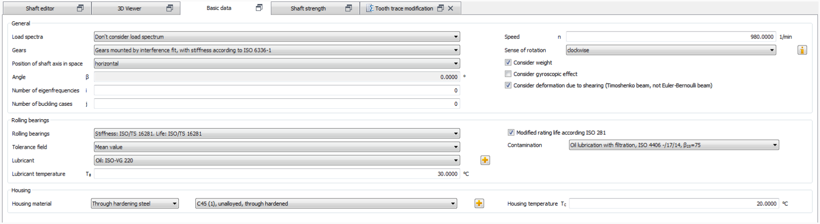


Figure 23: Basic data tab.

In this section, the user defines:

- a load spectrum, if available;
- position (**horizontal**) and the speed of the shaft (1500 rpm);
- the material of the shaft: (**C45 steel**);
- Number of eigenfrequencies (“critical speed” in KISSSoft is the second eigenfrequency);
- The behavior of the gear in the shaft (load, mass and stiffness, interference fit)

Gear load

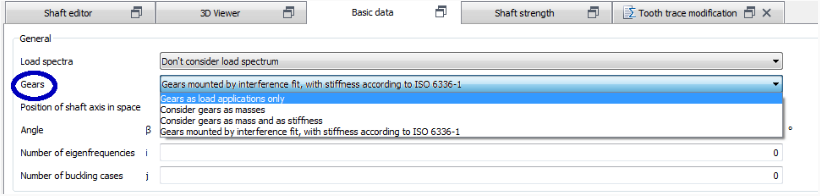


Figure 24: Select "Gears as load applications only".

This is the typical behavior that we follow during the engineering course. However, other options apply:

- Gears as masses
- Gears as mass and as stiffness
- Interference fit, with stiffness according to ISO 6336-1, to be discussed later on.

Despite those options could be more realistic, they are out of the scope of the course and the difference on the shaft strength calculation is negligible (as the user may verify).

Run basic calculation

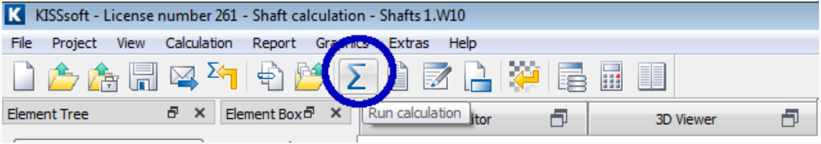


Figure 25: Select “Gears as load applications only”.

Please observe that before running the calculation, the software shows a message in bottom right: “INCONSISTENT” that after a successful calculation should become “CONSISTENT”

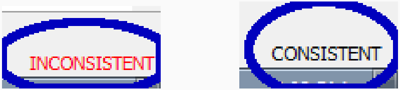


Figure 26: Check calculation consistency.

Verify forces calculated

Gear forces:

$$d_w = z \cdot m = 122.5\,\mathrm{mm}$$

$$F_t = \frac{2M_t}{d_w} = \frac{2P}{\omega \cdot d_w} = 7795\,\mathrm{N}$$

Normal:

$$F_n = \frac{F_t}{\cos\alpha} = 8296\,\mathrm{N}$$

Radial:

$$F_r = F_t \cdot \tan\alpha = 2837\,\mathrm{N}$$

Bearing forces (*BL* – left bearing, *BR* – right bearing):

$$\sum \vec{F} = \vec{0} \Leftrightarrow \vec{R}_{BL} + \vec{R}_{BR} = \vec{F}_G$$

$$\sum \vec{M}_{BL} = \vec{0} \Leftrightarrow \vec{r}_{BR} \times \vec{R}_{BR} + \vec{r}_G \times \vec{F}_G = 0$$

$$\vec{R}_{BL} = \begin{cases} 1153 \\ 0 \\ 3167 \end{cases} \mathrm{N}$$

$$\vec{R}_{BR} = \begin{cases} 1685 \\ 0 \\ 4628 \end{cases} \mathrm{N}$$

Bending and torque diagram

At this moment the calculation performed both a basic calculation and a strength calculation with the default settings. We will avoid for now the strength calculation and check if the calculation is according to the operating conditions that we established.

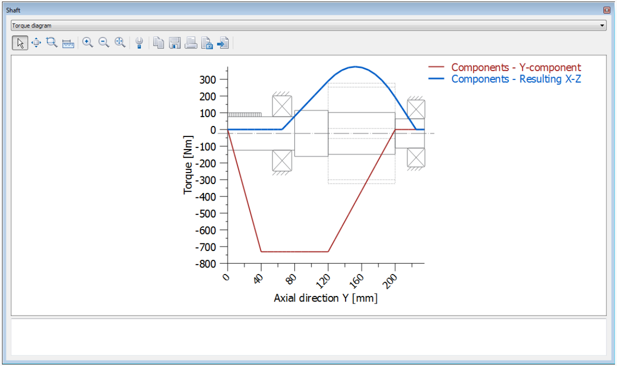


Figure 27: Torque (Y) and resultant bending moment ($X - Z$) diagram.

Displacement diagram

If you check the results along Y, the axis direction, no force or displacement should occur, because the load is zero in the Y direction. You can also check the critical speed of the shaft which is $n = 41\,530\,\mathrm{rpm}$.

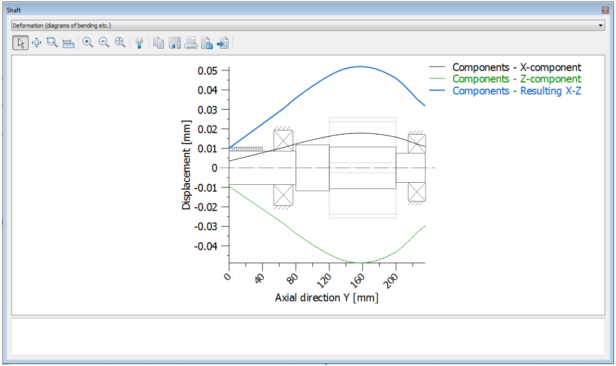


Figure 28: Resultant displacement of the shaft ($X - Z$).

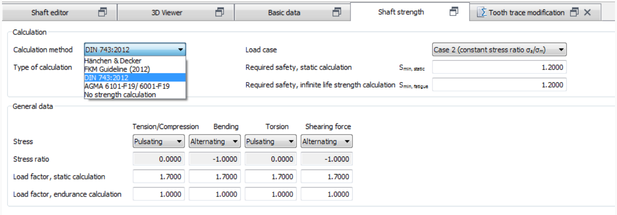


Figure 29: Select a strength calculation method: DIN 743:2012.

The other strength calculation methods are out of the scope of the course.

Now we should also define (an infinite life verification will be done):

- Load case:
  - Case 1 - the stress amplitude changes while the mean equivalent stress remains constant during a variation of the operational working load.
  - Case 2 - the stress ratio between mean equivalent stress and effective stress amplitude remains constant during a variation of the operational working load.
- The stress behavior due to tension, bending (alternating), torsion (constant) and shearing force;
- Load factor K_A

Equivalent stress

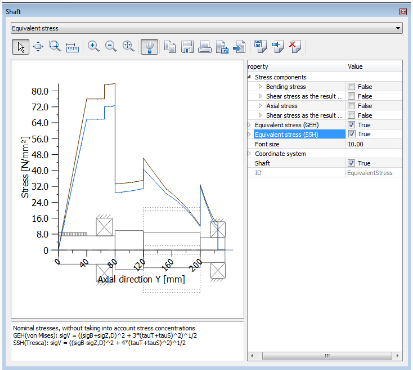


Figure 30: Equivalent stress along Y according to von Mises (blue) and Tresca (brown) without stress concentration factors - equations on the figure.

Report: safety factors, critical sections

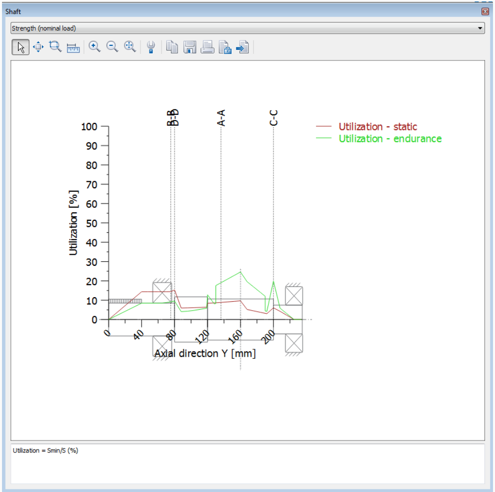


Figure 31: Percentage of utilization of the safety factors.

Results:

Cross section	βσ	KF σ	K2d	SD	SS
A-A	2.98	1.00	0.87	6.70	16.12
B-B	2.69	1.00	0.89	13.06	8.32
C-C	2.33	0.90	0.90	5.91	19.47
D-D	1.77	0.87	0.89	12.53	7.84
Required safeties:				1.20	1.20

Abbreviations:
βσ: Notch factor, bending
KF σ: Surface factor
K2d: size factor bending
SD: Safety endurance limit
SS: Safety against yield point

Figure 32: Stress concentration factors on the 4 cross sections defined

Assignments

Assignment A4

Available on the Course Contents:

“Assignments” proposed for Complements of Machine Elements

References

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