Complements of Machine Elements
Rotating Cylinders

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1 Flywheel stresses

A flywheel made of low-carbon steel has an outside radius of 150 mm and an inside radius of 25 mm. The flywheel is to be assembled (press fit) onto a shaft. The radial interference between the flywheel and shaft is 50 µm, and the shaft will operate at a speed of 5000 rpm [1]. Calculate:

1.1 The circumferential and radial stresses on the flywheel inner radius

The diameters of the composite cylinder are $a = 0\,\text{mm}$, $b = 25\,\text{mm}$ and $c = 150\,\text{mm}$. For low-carbon steel, $E = 207\,\text{GPa}$, $\nu = 0.3$, and $\rho = 7860\,\text{kg}\,\text{m}^{-3}$. The interference pressure yields:

$$p_c = \frac{E \cdot \delta}{b} \cdot \left[\frac{\left(c^2 - b^2\right)\left(b^2 - a^2\right)}{2b^2\left(c^2 - a^2\right)} \right] = \frac{E \cdot \delta}{2b} \cdot \left(\frac{c^2 - b^2}{c^2} \right) \tag{1}$$

$$p_c = \frac{207 \times 10^3 \times 0.050}{25} \times \left(\frac{150^2 - 25^2}{150^2} \right) = 201.25\,\text{MPa} \tag{2}$$

The angular speed of the system is:

$$\omega = \frac{2 \cdot \pi \cdot n}{60} = \frac{2 \times \pi \times 5000}{60} = 523.6\,\text{rad}\,\text{s}^{-1} \tag{3}$$

For such angular speed ω and considering that the flywheel and shaft are both the same material, the hoop stress can be obtained by:

$$\sigma_t = \frac{3 + \nu}{8} \cdot \rho \cdot \omega^2 \cdot \left(a^2 + c^2 + \frac{a^2 \cdot c^2}{r^2} - \frac{1 + 3 \cdot \nu}{3 + \nu} \cdot r^2 \right) + p_c \cdot \frac{c^2 + b^2}{c^2 - b^2} \tag{4}$$

$$
\begin{aligned}
\sigma_t &= \frac{3+0.3}{8} \times 7.86 \times 10^{-9} \times 523.6^2 \\
&\quad \times \left(150^2 - \frac{1+3\times 0.3}{3+0.3} \times 25^2\right) + 201.25 \times \frac{150^2+25^2}{150^2-25^2} \\
&= 232.43\,\mathrm{MPa}
\end{aligned} \tag{5}
$$

$$
\sigma_r = \frac{3+\nu}{8} \cdot \rho \cdot \omega^2 \cdot \left(a^2 + c^2 - \frac{a^2 \cdot c^2}{r^2} - r^2\right) - p_c \tag{6}
$$

$$
\sigma_r = \frac{3+0.3}{8} \times 7.86 \times 10^{-9} \times 523.6^2 \times \left(150^2 - 25^2\right) - 201.25 = -180.81\,\mathrm{MPa} \tag{7}
$$

1.2 The speed at which the flywheel will break loose from the shaft

The flywheel breaks free when the radial stress at the inner radius b is reduced to zero at a certain angular speed ω:

$$
\sigma_r = 0 = \frac{3+0.3}{8} \times 7.86 \times 10^{-9} \times \omega^2 \times \left(150^2 - 25^2\right) - 201.25 \tag{8}
$$

After solving the equation, the angular speed $\omega = 1684\,\mathrm{rad\,s^{-1}}$, corresponding to 16 080 rpm.

2 Flywheel design

The output, or load torque, of a flywheel used in a punch press for each revolution of the shaft is 12 Nm from zero to π and from $3\pi/2$ to 2π and 144 Nm from π to $3\pi/2$. The coefficient of fluctuation is $C_f = 0.05$ about an average speed of 600 rpm. Assume that the flywheel's solid disk is made of low-carbon steel of constant 25 mm thickness [1]. Determine the following:

2.1 The average load or output torque

Using the load or output torque variation for one cycle, the average output torque is:

$$
2 \times \pi \times \overline{T} = 12 \times \pi + 144 \times \frac{\pi}{2} + 12 \times \frac{\pi}{2} \Leftrightarrow \overline{T} = 6 + 36 + 3 = 45\,\mathrm{Nm} \tag{9}
$$

2.2 The locations $\theta_{\omega_{min}}$ and $\theta_{\omega_{max}}$

$$\theta_{\omega_{max}} = \pi \tag{10}$$

$$\theta_{\omega_{min}} = \frac{3\pi}{2} \tag{11}$$

2.3 The energy fluctuation required

The kinetic energy for one cycle is given by:

$$E_{max} - E_{min} = \int_{\theta_{\omega_{min}}}^{\theta_{\omega_{max}}} T_{avg} - T_l \, \mathrm{d}\theta = \int_{\frac{3\pi}{2}}^{\pi} (45 - 144) \, \mathrm{d}\theta = 155.5 \, \mathrm{N\,m} \tag{12}$$

2.4 The outside diameter of the flywheel

The average angular speed can be expressed as:

$$\omega_{avg} = \frac{2 \cdot \pi \cdot n_{avg}}{60} = 62.83 \, \mathrm{rad\,s^{-1}} \tag{13}$$

The required mass moment of inertia is estimated using:

$$E_{max} - E_{min} = I \cdot C_f \cdot \omega_{avg}^2 \Leftrightarrow I = \frac{E_{max} - E_{min}}{C_f \cdot \omega_{avg}^2} = \frac{155.5}{0.05 \times 62.83^2} = 0.7879 \, \mathrm{kg\,m^2} \tag{14}$$

According to the definition of the mass moment of inertia for a solid round disc:

$$I = \frac{m \cdot d^2}{8} = \frac{\pi \cdot d^2 \cdot t \cdot \rho}{4} \cdot \frac{d^2}{8} = \frac{\pi \cdot \rho \cdot t \cdot d^4}{32} \tag{15}$$

Considering a low carbon steel with $\rho = 7860 \, \mathrm{kg\,m^{-3}}$:

$$I = \frac{\pi \times 7860 \times 0.025 \times d^4}{32} = 19.29 \times d^4 \tag{16}$$

Making equations (14) and (16) equal:

$$19.29 \times d^4 = 0.7879 \tag{17}$$

The diameter of the flywheel should be $d = 449.5 \, \mathrm{mm}$

References

[1] Osgood, Carl and Fatigue Design: *Fundamentals of Machine Elements*. 2014, ISBN 9781482247503.