Clutches and Brakes

Complements of Machine Elements

Carlos Fernandes

2023

Mestrado em Engenharia Mecânica

Nothing has such power to broaden the mind as the ability to investigate systematically and truly all that comes under thy observation in life.

Marcus Aurelius

Recommended bibliography

- Robert C. Juvinall, Kurt M. Marshek; Fundamentals of machine component design. ISBN: 0-0471-52989-3

- Sharpe, C. (1983). Clutches and brakes.

- Day, A. (2014). Braking of Road Vehicles. In Braking of Road Vehicles. https://doi.org/10.1016/C2011-0-07386-6

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Lecture 1

Summary

Introduction

Brakes and clutches are examples of machine elements that uses friction in a useful way [1].

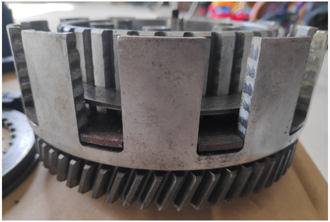
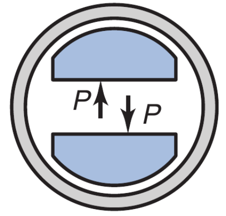


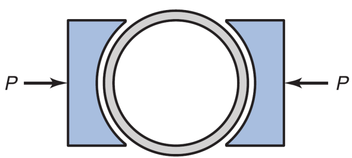
Figure 1: Clutch from a Suzuki GT 380 motorcycle.

Figure 2: Drum brake.

Types of clutch and brake



(a) Internal, expanding



(b) External, contracting

Figure 3: Rim type [1].

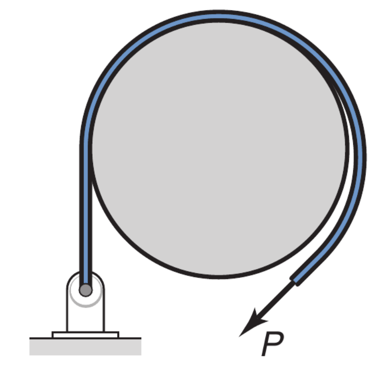
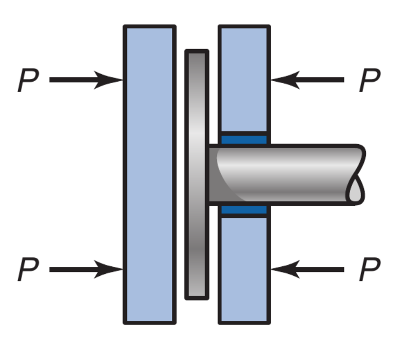
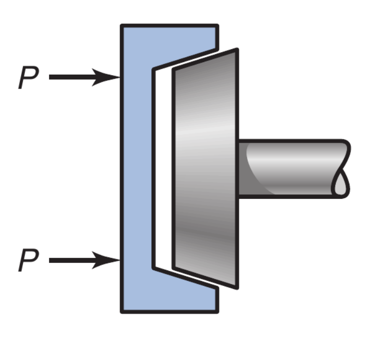


Figure 4: Band brake [1].

Types of clutch and brake



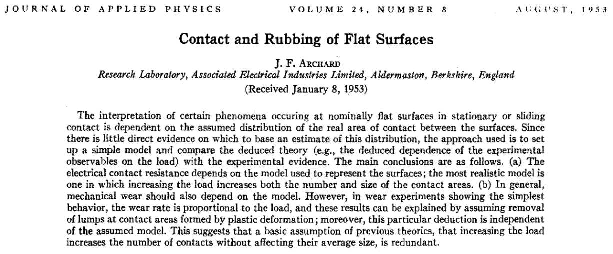
(a) Thrust



(b) Cone

Figure 5: Disc type [1].

Archard's law



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Contact and Rubbing of Flat Surfaces

J. F. ARCHARD

Research Laboratory, Associated Electrical Industries Limited, Aldermaston, Berkshire, England

(Received January 8, 1953)

The interpretation of certain phenomena occuring at nominally flat surfaces in stationary or sliding contact is dependent on the assumed distribution of the real area of contact between the surfaces. Since there is little direct evidence on which to base an estimate of this distribution, the approach used is to set up a simple model and compare the deduced theory (e.g., the deduced dependence of the experimental observables on the load) with the experimental evidence. The main conclusions are as follows. (a) The electrical contact resistance depends on the model used to represent the surfaces; the most realistic model is one in which increasing the load increases both the number and size of the contact areas. (b) In general, mechanical wear should also depend on the model. However, in wear experiments showing the simplest behavior, the wear rate is proportional to the load, and these results can be explained by assuming removal of lumps at contact areas formed by plastic deformation; moreover, this particular deduction is independent of the assumed model. This suggests that a basic assumption of previous theories, that increasing the load increases the number of contacts without affecting their average size, is redundant.

Figure 6: Archard's paper from 1953 [2].

According to Archard's law, the wear rate ($\dot{h}$) is proportional to the product of contact pressure (p) and velocity (v):

$$\frac{h}{t} = C \cdot p \cdot v$$

Commonly represented as:

$$V = \frac{K}{H} \cdot F \cdot L$$

F	normal force	N
H	hardness of the softer material	$\mathrm{N/m^2}$
K	wear rate	-
L	length of sliding	m
V	volume of worn material	$\mathrm{m^3}$

Archard's law

Dividing by the contact surface area:

$$V = \frac{K}{H} \cdot F \cdot L \Leftrightarrow h = \frac{K}{H} \cdot p \cdot L$$

Deriving in order to time:

$$\dot{h} = \frac{K}{H} \cdot p \cdot v$$

For a circular disc $v = \omega \cdot r$

$$\dot{h} = \frac{K}{H} \cdot p \cdot \omega \cdot r$$

We can conclude that the wear rate is proportional to the normal pressure and the radius.

A typical way to characterize the wear rate is to use a pin-on-disc test according to ASTM G 99 – 04 [3].

Thrust pad clutches and brakes

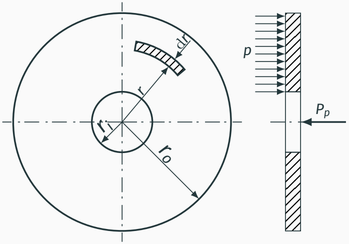


Figure 7: Thrust disc clutch with uniform pressure [1, 4].

The elemental area is:

$$\mathrm{d}A = r\,\mathrm{d}\theta\,\mathrm{d}r$$

The elemental normal force $\mathrm{d}P$:

$$\mathrm{d}P = p\,\mathrm{d}A = p \cdot r\,\mathrm{d}\theta\,\mathrm{d}r$$

The coefficient of friction is μ.

The friction force is $F = \mu \cdot P$ and the friction torque is given by:

$$\boxed{T = \int r\,\mathrm{d}F = \int \mu \cdot r\,\mathrm{d}P = \int\int \mu \cdot p \cdot r^2\,\mathrm{d}r\,\mathrm{d}\theta}$$

Uniform pressure model

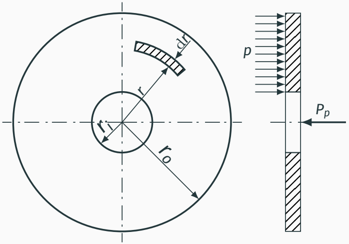


Figure 7: Thrust disc clutch with uniform pressure [1, 4].

The contact pressure for new thrust disc is assumed uniform over the surfaces:

$$p = \frac{P_p}{\pi \cdot \left(r_o^2 - r_i^2\right)}$$

The friction torque is given by:

$$T_p = \int_0^{2\pi} \int_{r_i}^{r_o} \mu \cdot p \cdot r^2 \, \mathrm{d}r \, \mathrm{d}\theta$$

After integration:

$$T_p = \frac{2 \cdot \pi \cdot \mu \cdot p}{3} \cdot \left(r_o^3 - r_i^3\right)$$

Substituting the value of contact pressure p into previous equation:

$$\boxed{T_p = \mu \cdot P_p \cdot \frac{2}{3} \cdot \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}}$$

Disc wear

Since the sliding distance is proportional to the radius, the outside of the disc has a larger sliding distance than the inner radius. If the pressure and the hardness are constant, then more wear will occur on the outside of the disc. This will cause a redistribution of the pressure. So, after some run-in (initial wear) takes place, it can be assumed that a constant wear will occur.

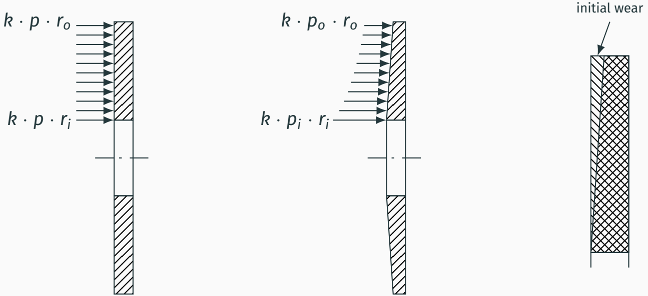


Figure 8: Thrust disc clutch [1, 4].

Uniform wear model

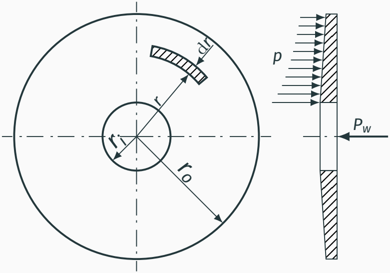


Figure 9: Thrust disc clutch [1, 4].

Assuming a constant wear along the disc surfaces, the outer radius should have a lower pressure.

$$\dot{h} = \frac{K}{H} \cdot p \cdot \omega \cdot r$$

So, p is inversely proportional to the sliding speed (or radius):

$$p = \frac{\dot{h} \cdot H}{K \cdot \omega \cdot r} = \frac{c}{r}$$

For $r = r_i$ the pressure is maximum, so:

$$c = p_{max} \cdot r_i$$

Uniform wear model

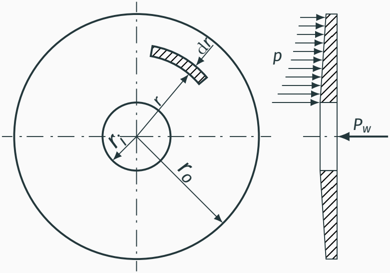


Figure 9: Thrust disc clutch [1, 4].

Recall that the pressure is a function of the radius:

$$p = \frac{c}{r}$$

$$P_w = 2 \cdot \pi \int_{r_i}^{r_o} p \cdot r \, \mathrm{d}r = 2 \cdot \pi \int_{r_i}^{r_o} c \, \mathrm{d}r$$

The axial force is then:

$$P_w = 2 \cdot \pi \cdot c \cdot (r_o - r_i)$$

For $c = p_{max} \cdot r_i$, the axial force is:

$$P_w = 2 \cdot \pi \cdot p_{max} \cdot r_i \cdot (r_o - r_i)$$

Uniform wear model

For a constant wear assumption, the friction torque is given by:

$$T_w = \int_0^{2\pi} \int_{r_i}^{r_o} \mu \cdot p \cdot r^2 \, \mathrm{d}r \, \mathrm{d}\theta = 2 \cdot \pi \int_{r_i}^{r_o} \mu \cdot \frac{c}{r} \cdot r^2 \, \mathrm{d}r = \frac{2 \cdot \pi \cdot \mu \cdot c}{2} \cdot \left(r_o^2 - r_i^2\right)$$

Knowing that $c = p_{max} \cdot r_i$:

$$T_w = \pi \cdot \mu \cdot r_i \cdot p_{max} \cdot \left(r_o^2 - r_i^2\right)$$

Recall that the axial force is:

$$P_w = 2 \cdot \pi \cdot p_{max} \cdot r_i \cdot (r_o - r_i)$$

Combining both definitions, the friction torque is evaluated as:

$$\boxed{T_w = F_w \cdot r_m = \mu \cdot P_w \cdot \frac{r_o + r_i}{2}}$$

Dimensionless torque and friction radius

Uniform pressure model

$$T_p = \mu \cdot P_p \cdot \frac{2}{3} \cdot \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}$$

The torque can be written as function of a friction radius r_f:

$$T_p = \mu \cdot P_p \cdot r_f \Rightarrow \boxed{r_f = \frac{2}{3} \cdot \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}}$$

Dividing by the outer radius $2 \cdot \mu \cdot P_p \cdot r_o$ and making $\beta = \frac{r_i}{r_o}$:

$$\boxed{\overline{T_p} = \frac{T_p}{2 \cdot \mu \cdot P_p \cdot r_o} = \frac{(1 - \beta^3)}{3 \cdot (1 - \beta^2)}}$$

Dimensionless torque and friction radius

Uniform wear model

$$T_w = \mu \cdot P_w \cdot \frac{r_o + r_i}{2}$$

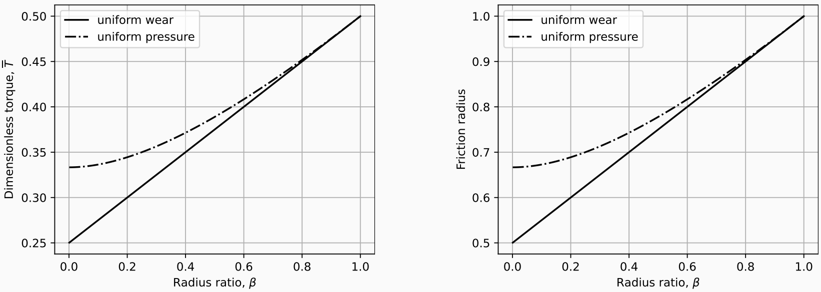
The torque can be written as function of a friction radius r_f:

$$T_w = \mu \cdot P_w \cdot r_f \Rightarrow \boxed{r_f = \frac{r_o + r_i}{2}}$$

Dividing by the outer radius $2 \cdot \mu \cdot P_w \cdot r_o$ and making $\beta = \frac{r_i}{r_o}$:

$$\boxed{\overline{T_w} = \frac{T_w}{2 \cdot \mu \cdot P_w \cdot r_o} = \frac{1 + \beta}{4}}$$

Dimensionless torque and friction radius



(a) Dimensionless torque vs. radius ratio

(b) Friction radius vs. radius ratio

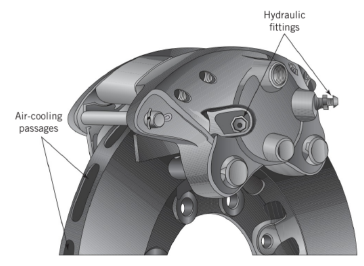
Figure 10: Comparison between uniform wear and uniform pressure models.

For the same dimensionless torque: **uniform wear model** requires a larger β and implies a more conservative design.

Lecture 2

Summary

Caliper type disc brakes



(a) Caliper disc brake, hidraulically operated [5]

(b) Axle with MacPherson suspension and disc brake

Figure 11: Caliper type disc brake [6].

Caliper type disc brakes

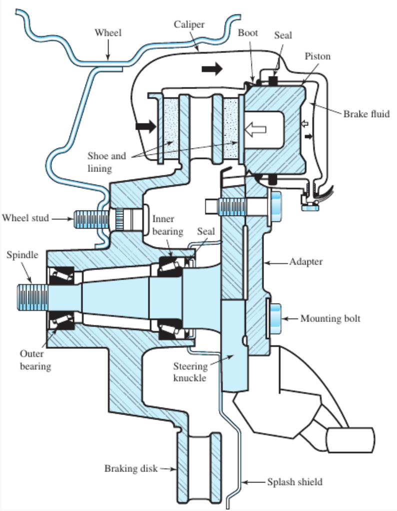


Figure 12: An automotive disc brake [7].

Caliper type disc brakes

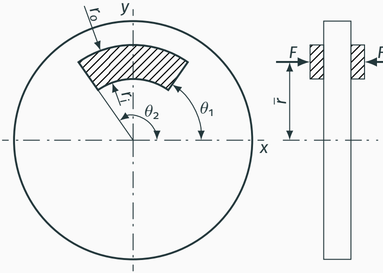


Figure 13: Geometry of contact area of a caliper brake. [7].

The caliper type disc brake is a disc type brake with partial contact.

The actuating force is:

$$F = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} p \cdot r \, \mathrm{d}r \, \mathrm{d}\theta$$

The friction torque:

$$T = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} \mu \cdot p \cdot r^2 \, \mathrm{d}r \, \mathrm{d}\theta$$

Caliper type disc brakes

Uniform pressure model

The friction torque is:

$$T = \mu \cdot F \cdot r_e$$

The equivalent radius r_e:

$$r_e = \frac{T}{\mu \cdot F} = \frac{\int_{r_i}^{r_o} p \cdot r^2 \,\mathrm{d}r}{\int_{r_i}^{r_o} p \cdot r \,\mathrm{d}r} = \frac{2}{3} \cdot \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}$$

The locating coordinate $\bar{r}$ of the activating force is found by taking moments about the x axis:

$$\bar{r} = \frac{M_x}{F} = \frac{2}{3} \cdot \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \cdot \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1}$$

Caliper type disc brakes

Uniform wear model

The friction torque is:

$$T = \mu \cdot F \cdot r_e$$

The equivalent radius r_e:

$$r_e = \frac{T}{\mu \cdot F} = \frac{\int_{r_i}^{r_o} p \cdot r^2 \, \mathrm{d}r}{\int_{r_i}^{r_o} p \cdot r \, \mathrm{d}r} = \frac{r_o + r_i}{2}$$

The locating coordinate $\bar{r}$ of the activating force is found by taking moments about the x axis:

$$\bar{r} = \frac{M_x}{F} = \frac{r_o + r_i}{2} \cdot \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1}$$

Cone clutches and brakes

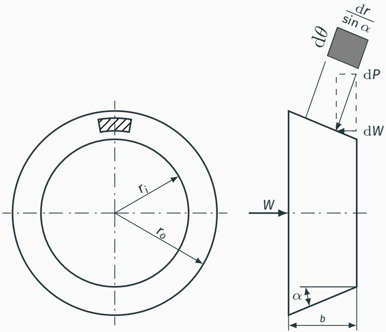


Figure 14: Forces acting on elements of a cone clutch [1, 4].

The half cone angle is α.

The elemental area is:

$$dA = r\,d\theta\,\frac{dr}{\sin\alpha}$$

The normal force P:

$$dP = p\,dA = p \cdot r\,d\theta\,\frac{dr}{\sin\alpha}$$

The thrust force:

$$dW = dP\sin\alpha = p \cdot r\,dr\,d\theta$$

Uniform pressure model

$$T = \int \mu \cdot r \,\mathrm{d}P = \frac{2 \cdot \pi}{\sin \alpha} \cdot \int_{r_i}^{r_o} \mu \cdot p \cdot r^2 \,\mathrm{d}r$$

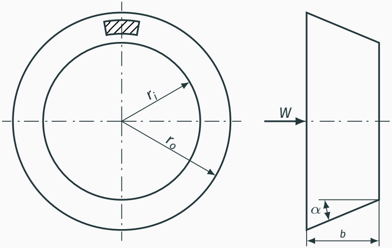


Figure 15: Cone of a clutch [1, 4].

For new, flat and aligned discs:

$$p = \frac{W}{\pi \cdot \left(r_o^2 - r_i^2\right)}$$

$$T_p = \frac{2 \cdot \pi \cdot p \cdot \mu}{3 \cdot \sin \alpha} \cdot \left(r_o^3 - r_i^3\right)$$

$$\boxed{T_p = \frac{2 \cdot \mu \cdot W}{3 \cdot \sin \alpha} \cdot \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}}$$

Uniform wear model

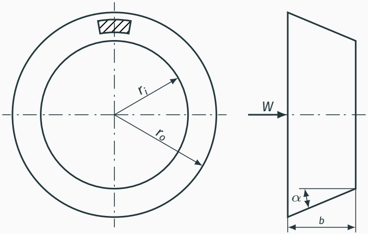


Figure 15: Cone of a clutch [1, 4].

The actuating force is given by:

$$W = 2{\cdot}\pi{\cdot}c{\cdot}\int_{r_i}^{r_o} \mathrm{d}r\,\mathrm{d}\theta = 2{\cdot}\pi{\cdot}c{\cdot}(r_o - r_i)$$

$$c = \frac{W}{2 \cdot \pi \cdot (r_o - r_i)}$$

The friction torque is given by:

$$T_w = \frac{2 \cdot \pi \cdot \mu \cdot c}{\sin\alpha} \cdot \int_{r_i}^{r_o} r\,\mathrm{d}r$$

$$T_w = \frac{\pi \cdot \mu \cdot c}{\sin\alpha} \cdot \left(r_o^2 - r_i^2\right)$$

Combining the previous equations:

$$\boxed{T_w = \frac{\mu \cdot W}{2 \cdot \sin\alpha} \cdot (r_o + r_i)}$$

Cone clutch

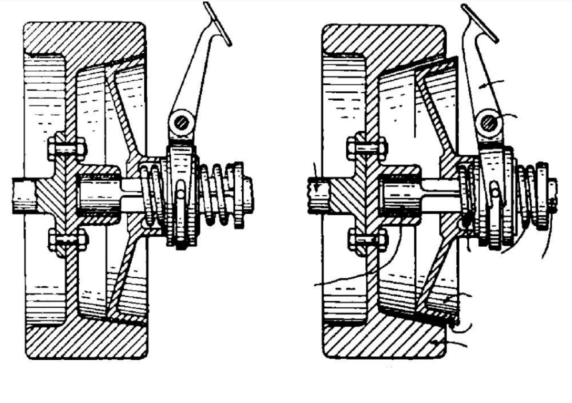


Figure 16: Cone clutch [4].

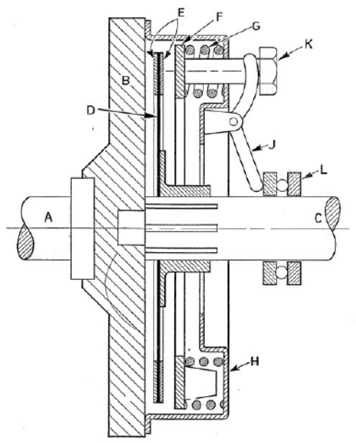


Figure 17: Basic friction clutch with 1 plate [4].

Drive shaft A is connected to disc B. Disc B have transnational motion over the driven shaft. Disc B rotates with the driven shaft.

Basic friction clutch

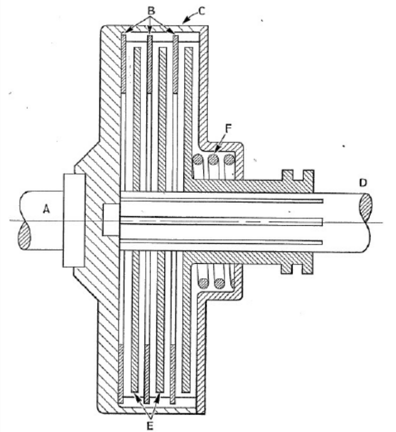


Figure 18: Multi disc friction clutch [4].

The transmitted torque is calculated multiplying the torque transmitted by one disc by the number of discs in contact.

Drum brake

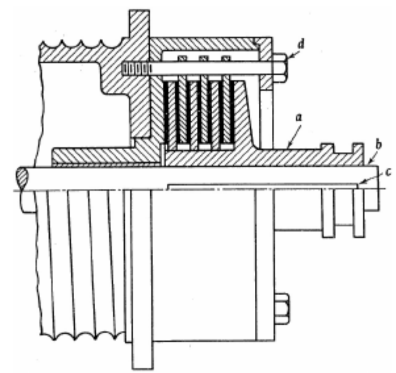


Figure 19: Drum brake for lifting equipment [4].

The shaft *a* is mounted with a key *c* to shaft *b*. It allows axial movement that removes the contact between the friction discs.

Braking torque

The braking torque (T) due to inertia (I):

$$T = I \cdot \alpha = I \cdot \frac{2 \cdot \pi \cdot n}{60 \cdot t} \Leftrightarrow t = \frac{I}{T} \cdot \frac{2 \cdot \pi \cdot n}{60}$$

Recall the angular acceleration:

$$\alpha = \dot{\omega} = \ddot{\theta} = \frac{2 \cdot \pi \cdot n}{60 \cdot t}$$

The angular position is calculated from the angular acceleration:

$$\theta = \frac{1}{2} \cdot \alpha \cdot t^2 = \frac{1}{2} \cdot \frac{2 \cdot \pi \cdot n}{60 \cdot t} \cdot t^2 = \frac{1}{2} \cdot \frac{2 \cdot \pi \cdot n}{60} \cdot t$$

Braking torque

The energy dissipated is calculated by the product of the braking torque and the angle:

$$E = T \cdot \theta = T \cdot \left(\frac{1}{2} \cdot \frac{2 \cdot \pi \cdot n}{60} \cdot t \right) = T \cdot \frac{2 \cdot \pi \cdot n}{120} \cdot t$$

Replacing the value of time t into previous equation:

$$E = T \cdot \frac{2 \cdot \pi \cdot n}{120} \cdot t = T \cdot \frac{2 \cdot \pi \cdot n}{120} \cdot \frac{I}{T} \cdot \frac{2 \cdot \pi \cdot n}{60}$$

The energy dissipated is then a function of inertia I and rotational speed n:

$$E = I \cdot \frac{\pi^2 \cdot n^2}{1800}$$

The temperature is a critical point of clutch and brake design. The high friction required is converted into heat resulting in very high temperature in the lining material. The thermal effects accelerate wear and can compromise performance and life.

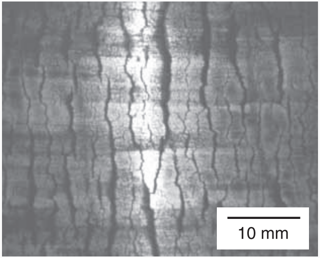


Figure 20: Brake drum surface showing a high level of heat checking [1].

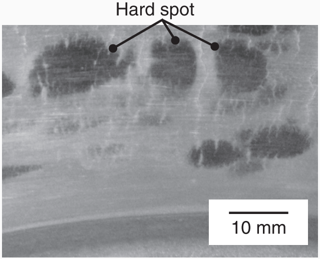


Figure 21: Hard spot on a brake drum [1].

Thermal considerations

Neglecting radiation, the first law of thermodynamics requires that:

$$E_f = E_c + E_h + E_s$$

If conduction and convection are negligible, the temperature rise is given by:

$$\Delta_T = \frac{E_f}{c_p \cdot m_a}$$

This gives the instantaneous temperature rise because the friction energy E_f is directly dissipated on the contacting surface and does not have time to be conducted or conveyed away. In reality the contacting area moves out of contact and can cool.

A more complex model is needed to cover the heating and cooling of the system.

Δ_T	temperature rise	°C
c_p	specific heat of the material	J/(kg °C)
m_a	mass of the clutch or brake	kg
E_f	friction energy	J
E_c	energy transferred by conduction	J
E_h	energy transferred by convection	J
E_s	energy stored in the solid	J

Calculation examples

Optimum size of a thrust disc clutch

A single set of thrust disc clutches is to be designed for use in an engine with a maximum torque of 150 N m. A woven fiber reinforced polymer will contact steel in a dry environment (values in Table below). A safety factor of 1.5 is assumed in order to account for slippage at full engine torque. The outside diameter should be as small as possible.

- Determine the appropriate values for r_o, r_i and P?

Friction material	μ	p_{max} / kPa	Maximum bulk temperature / °C
Molded	0.25-0.45	1030-2070	204-260
Woven	0.25-0.45	345-690	204-260
Sintered metal	0.15-0.45	1030-2070	204-677
Cork	0.30-0.50	55-95	82
Wood	0.20-0.30	345-620	93
Cast iron; hard steel	0.15-0.25	690-1720	260

Optimum size of a thrust disc clutch

We should make the calculation assuming the average coefficient of friction to estimate the average performance. But the smallest contact pressure is selected for a longer life.

Considering **uniform wear model** and the safety factor 1.5, the torque is given by:

$$S_F \cdot T_w = \pi \cdot \mu \cdot r_i \cdot p_{max} \cdot \left(r_o^2 - r_i^2\right) \Leftrightarrow r_i \cdot \left(r_o^2 - r_i^2\right) = \frac{S_F \cdot T_w}{\pi \cdot \mu \cdot p_{max}}$$

$$r_i \cdot \left(r_o^2 - r_i^2\right) = \frac{1.5 \times 150}{\pi \times 0.35 \times 345 \times 10^3} = 5.931 \times 10^{-4} \mathrm{m}^3$$

Solving in order to r_o:

$$r_o = \sqrt{\frac{5.931 \times 10^{-4}}{r_i} + r_i^2}$$

Optimum size of a thrust disc clutch

The minimum outside radius is obtained by taking the derivative of the outside radius with respect to the inside radius and setting it equal to zero:

$$\frac{\mathrm{d}r_o}{\mathrm{d}r_i} = \frac{0.5}{\sqrt{\frac{5.931\times10^{-4}}{r_i} + r_i^2}} \left(-\frac{5.931\times10^{-4}}{r_i^2} + 2r_i \right) = 0$$

Finally $r_i = 66.69\,\mathrm{mm}$ and $r_o = \sqrt{\frac{5.931\times10^{-4}}{0.06669} + 0.06669^2} = 115.5\,\mathrm{mm}$

The maximum force is:

$$P = \frac{2 \cdot S_F \cdot T_w}{\mu \cdot (r_o + r_i)} = \frac{2 \times 1.5 \times 150}{0.35\,(0.1155 + 0.06669)} = 7057\,\mathrm{N}$$

Cone clutch

A cone clutch with the following dimensions: $D = 330\,\text{mm}$, $d = 306\,\text{mm}$ and $b = 60\,\text{mm}$. The clutch uses sintered metal on steel, with a coefficient of friction of $\mu = 0.26$, and the torque transmitted is $200\,\text{N m}$.

- Determine the minimum required actuating force and the associated contact pressure by using the uniform pressure and uniform wear models.

Friction material	μ	p_{max} / kPa	Maximum bulk temperature / °C
Molded	0.25-0.45	1030-2070	204-260
Woven	0.25-0.45	345-690	204-260
Sintered metal	0.15-0.45	1030-2070	204-677
Cork	0.30-0.50	55-95	82
Wood	0.20-0.30	345-620	93
Cast iron; hard steel	0.15-0.25	690-1720	260

Uniform wear model

The half-cone angle of the cone clutch is:

$$\tan\alpha = \frac{r_o - r_i}{b} = \frac{165 - 153}{60} = 0.2$$

The pressure is a maximum for $r = r_i$:

$$T_w = \frac{\pi \cdot \mu \cdot r_i \cdot p_{max}}{\sin\alpha} \cdot \left(r_o^2 - r_i^2\right) \Leftrightarrow p_{max} = \frac{T_w \cdot \sin\alpha}{\pi \cdot \mu \cdot r_i \cdot \left(r_o^2 - r_i^2\right)} = 82.25\,\text{kPa}$$

The actuating force is:

$$W = \frac{2 \cdot T_w \cdot \sin\alpha}{\mu \cdot (r_o + r_i)} = \frac{2 \times 200 \times \sin 11.31}{0.26 \times (0.165 + 0.153)} = 948.8\,\text{N}$$

Uniform pressure model

$$W = \frac{3 \cdot T_p \cdot \sin\alpha\left(r_o^2 - r_i^2\right)}{2 \cdot \mu \cdot \left(r_o^3 - r_i^3\right)}$$

Substituting the corresponding values:

$$W = \frac{3 \times 200 \times \sin 11.31 \times \left(0.165^2 - 0.153^2\right)}{2 \times 0.26 \times \left(0.165^3 - 0.153^3\right)} = 948.4\,\mathrm{N}$$

References

[1] Osgood, Carl and Fatigue Design: *Fundamentals of Machine Elements*.
2014, ISBN 9781482247503.

[2] Archard, J. F.: *Contact and Rubbing of Flat Surfaces*.
Journal of Applied Physics, 24(8):981–988, aug 1953, ISSN 0021-8979.

[3] G99-04, ASTM: *Standard test method for wear testing with a pin-on-disk apparatus*.
pages 1–5, 2004.

[4] Castro, Paulo M.S.T. de: *O Contacto num Colar de Apoio. Observações Sobre um Tipo de Embraiagem e de Freio.*, 2017.

References ii

[5] Juvinall, Robert C. and Kurt M. Marshek: *FUNDAMENTALS OF MACHINE COMPONENT DESIGN*.
Wiley, 2017.

[6] Day, Andrew: *Braking of Road Vehicles*.
2014, ISBN 9780123973382.

[7] Budynas, Richard G.: *Shigley's mechanical engineering design*.
McGraw-Hill, 2014, ISBN 9789339221638.