U.PORTO
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Thick cylinders

Complements of Machine Elements

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Mestrado em Engenharia Mecânica

In all things, success depends on previous preparation. And without such preparation there is sure to be failure.

Confucius, Analects - taken from Fundamentals of Machine Elements [1]

Recommended bibliography

- Vullo, “Circular cylinders and pressure vessels”, Springer, 2014.

- Timoshenko, “Strength of Materials. Part II: Advanced Theory and Problems”, Third Edition, CBS, 1986.

Hiperlink

Lecture 1

Summary

Introduction

The circular cylinders are divided into two families: thin wall and thick wall cylinders [2]. The cylinder can have open or closed ends.

The main practical applications are: “pressure vessels such as gas storage cylinders, food and beverage cans and bottles, fuel tanks, hydraulic actuators and gun barrels; press and shrink fits; hydraulic and pneumatic tubing used for delivery of pressurized fluid” [1].

Figure 1: Typical applications of circular cylinders.

Thin wall cylinders

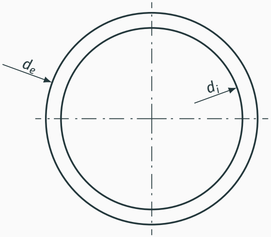


Figure 2: Thin wall cylinder.

$$t = \frac{d_e - d_i}{2}$$

If the wall thickness is small in comparison with the inner diameter, it is assumed that the stresses are uniformly distributed through it.

A thin wall cylinder should follow the following relation:

$$\frac{t}{d_i} \leq 0.051$$

$$K = \frac{d_e}{d_i} \leq 1.101$$

The reason behind such limits is presented ahead. Other sources state a value of $\frac{t}{d_i} < 0.1$ [3], $\frac{t}{d_i} < 0.083$ [2], $\frac{t}{d_i} < 0.05$ [4] and $\frac{t}{d_i} < 0.025$ [1].

Thin wall cylinders – hoop stress

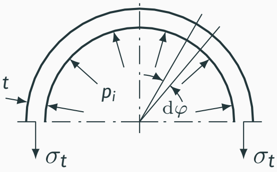


Figure 3: Thin wall cylinder with internal pressure[1] [3].

For internal pressure $p_i \neq 0$ and external pressure $p_o = 0$:

$$2\sigma_t t = 2\int_0^{\frac{\pi}{2}} p_i \sin\varphi r \,\mathrm{d}\varphi$$

$$2\sigma_t t = 2p_i r\left[-\cos\varphi\right]_0^{\frac{\pi}{2}} = 2p_i r$$

$$\sigma_t = \frac{p_i r}{t}$$

For internal pressure $p_i = 0$ and external pressure $p_o \neq 0$:

$$\sigma_t = -\frac{p_o r}{t}$$

These are the Mariotte's formulas for boilers.

Hoop stress is, in reality a function of the radius, but the differences between the values at the inner and outer radii are negligible.

[1]The inner radius is represented by r

Thin wall cylinders – axial stress

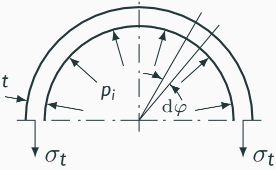


Figure 3: Thin wall cylinder.

If the free ends are closed by two heads, the axial stress is:

For internal pressure $p_i \neq 0$ and external pressure $p_o = 0$:

$$2\pi r t \sigma_l = p_i \pi r^2$$

$$\sigma_l = \frac{p_i \pi r^2}{2\pi r t} = \frac{p_i r}{2t}$$

For internal pressure $p_i = 0$ and external pressure $p_o \neq 0$:

$$\sigma_t = -\frac{p_o r}{2t}$$

We can conclude that the hoop stress σ_t is approximately twice the axial stress σ_l.

Thin wall cylinders – radial stress

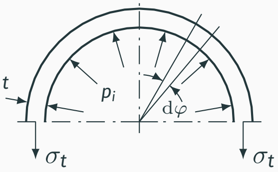


Figure 3: Thin wall cylinder.

For internal pressure p_i:

The radial stress in the inner wall:

$$\sigma_r = -p_i$$

The radial stress in the outer wall:

$$\sigma_r = 0$$

For external pressure p_o:

The radial stress in the inner wall:

$$\sigma_r = 0$$

The radial stress in the outer wall:

$$\sigma_r = -p_o$$

Thin wall cylinders – principal stresses and strains

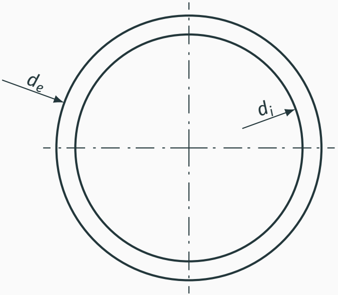


Figure 2: Thin wall cylinder.

The axisymmetry and the uniform distribution of stress through the wall thickness means that the radial, tangential and axial are the principal directions of both stresses and strains [2].

$$\sigma_1 = \sigma_t$$

$$\sigma_2 = \sigma_l$$

$$\sigma_3 = \sigma_r$$

For cylinders with open ends: $\sigma_2 = \sigma_l = 0$

Thin wall cylinders – design

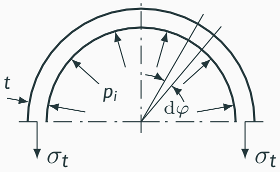


Figure 3: Thin wall cylinder.

According to the SNCTTI code [3, 5], the radial stress is assumed to have the following value:

$$\sigma_r = \frac{-p_i + 0}{2} = -\frac{p_i}{2}$$

Using a failure criteria (Tresca):

$$\sigma_e = \sigma_1 - \sigma_3 = \sigma_y$$

With $\sigma_1 = \sigma_t = \frac{p_i r}{t}$ and $\sigma_3 = -\frac{p_i}{2}$:

$$\frac{p_i r}{t} + \frac{p_i}{2} = \frac{\sigma_y}{S_F}$$

S_F is a safety factor. The cylinder thickness t is given by:

$$t = \frac{p_i r}{\frac{\sigma_y}{S_F} - 0.5 p_i}$$

ASME Boiler & Pressure Vessel Code [6] suggests the following:

$$t = \frac{p_i r}{\frac{\sigma_y}{S_F} - 0.6 p_i}$$

Thick wall cylinders

Thick-walled circular cylinders, by convention, satisfy any one of the following equivalent conditions (Bickell and Ruiz 1967, Iurzolla 1981, Burr 1982, Ventsel and Krauthammer 2001, Ugural and Fenster 2003):

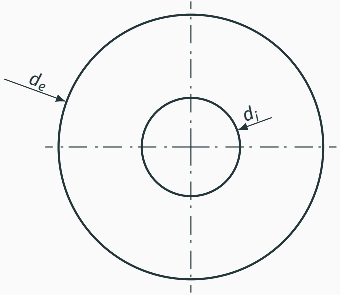


Figure 4: Thick wall cylinder.

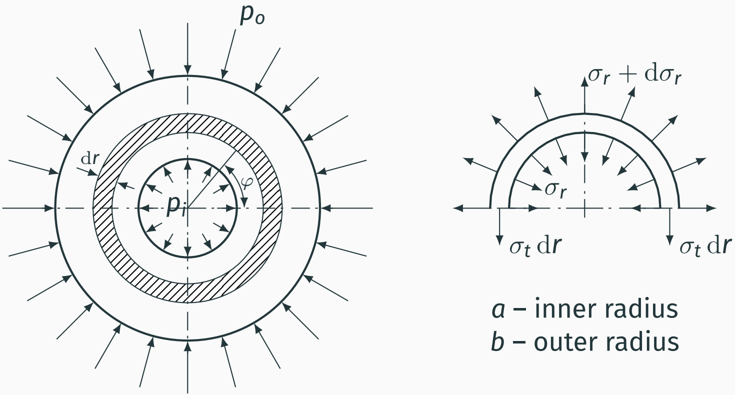
$$t = \frac{d_e - d_i}{2}$$

$$\frac{t}{d_i} > 0.051$$

$$K = \frac{d_e}{d_i} > 1.101$$

Other sources state a value of $\frac{t}{d_i} > 0.1$ [3], $\frac{t}{d_i} > 0.083$ [2], $\frac{t}{d_i} > 0.05$ [4] and $\frac{t}{d_i} > 0.025$ [1].

Thick wall cylinders



a – inner radius
b – outer radius

Figure 5: Thick wall cylinder with internal and external pressure [3].

Presentation of the topic taken from the Course Notes by Paulo M.S.T. Castro [3]. An equivalent treatment can be found in Féodossiev (pages 284–298) [7] or Timoshenko (pages 205–214) [8].

Thick wall cylinders – hoop and radial stress

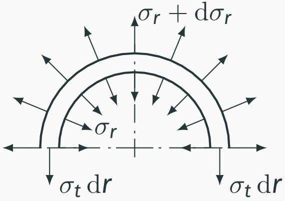


Figure 6: Infinitesimal element of the thick wall cylinder [3].

Static equilibrium written along the tangential direction:

$$2\sigma_t\,\mathrm{d}r + 2r\sigma_r - 2\left(\sigma_r + \mathrm{d}\sigma_r\right)\left(r + \mathrm{d}r\right) = 0$$

Disregarding higher order terms:

$$\sigma_t - \sigma_r - r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} = 0$$

Thick wall cylinders – hoop and radial stress

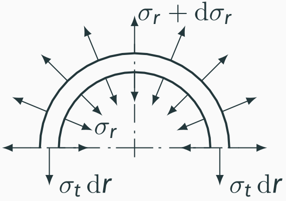


Figure 7: Infinitesimal element of the thick wall cylinder [3].

The equilibrium equation is:

$$\sigma_t - \sigma_r - r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} = 0$$

Using Hooke's law and the fact that the axial strain is constant:

$$\varepsilon_l = \frac{1}{E}\left[\sigma_l - \nu\left(\sigma_t + \sigma_r\right)\right]$$

In previous equation, if the axial stress $\sigma_l = 0$:

$$\sigma_t + \sigma_r = 2C_1 \Leftrightarrow \sigma_t = 2C_1 - \sigma_r$$

Replacing into equilibrium equation:

$$2C_1 - \sigma_r - \sigma_r - r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} = 0$$

$$2\sigma_r + r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} = 2C_1$$

Thick wall cylinders – hoop and radial stress

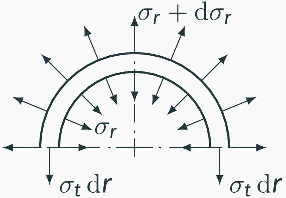


Figure 7: Infinitesimal element of the thick wall cylinder [3].

$$2\sigma_r + r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} = 2C_1$$

$$\underbrace{2r\sigma_r + r^2\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}}_{\frac{\mathrm{d}}{\mathrm{d}r}(r^2\sigma_r)} = 2rC_1$$

$$2rC_1 = \frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\sigma_r\right)$$

Integrating the equation:

$$r^2\sigma_r = r^2C_1 + C_2$$

The radial and the hoop stress:

$$\begin{cases} \sigma_r = C_1 + \frac{C_2}{r^2} \\ \sigma_t = C_1 - \frac{C_2}{r^2} \end{cases}$$

Thick wall cylinders – hoop and radial stress

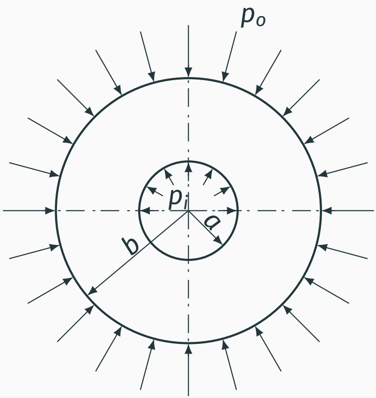


Figure 8: Thick wall cylinder with internal and external pressure.

$$\begin{cases} \sigma_r = C_1 + \frac{C_2}{r^2} \\ \sigma_t = C_1 - \frac{C_2}{r^2} \end{cases}$$

The pressure boundary conditions is then taken into account to determine the integration constants C_1 and C_2:

$$\begin{cases} \sigma_r = -p_i & r = a \\ \sigma_r = -p_o & r = b \end{cases}$$

$$\begin{cases} -p_i = C_1 + \frac{C_2}{a^2} \\ -p_o = C_1 + \frac{C_2}{b^2} \end{cases}$$

Thick wall cylinders – Lamé equations

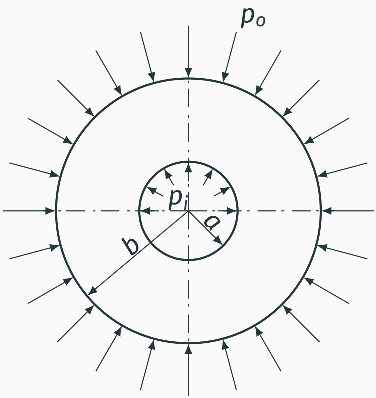


Figure 9: Thick wall cylinder with internal and external pressure.

The following solution was first presented by Lamé and Clapeyron[2]:

$$\begin{cases} \sigma_r = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} - \frac{(p_i - p_o) a^2 b^2}{r^2 (b^2 - a^2)} \\ \sigma_t = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{(p_i - p_o) a^2 b^2}{r^2 (b^2 - a^2)} \end{cases}$$

[2] Lamé and Clapeyron, “Mémoire sur l’equilibre intérieur des corpes solides homogènes”, Mém. divers savans, 1833.

Thick wall cylinders – $p_o = 0$

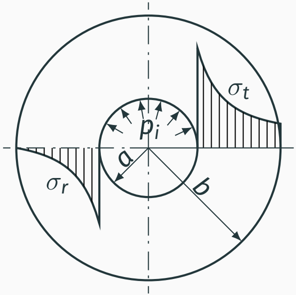


Figure 10: Hoop and radial stresses for a thick wall cylinder with internal pressure [3].

For internal pressure $p_i \neq 0$ and $p_o = 0$:

$$\begin{cases} \sigma_t = p_i \frac{a^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right) \\ \sigma_r = p_i \frac{a^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right) \end{cases}$$

The radial stress at the inner radius is $-p_i$ and at the outer radius is $p_o = 0$.

The hoop stress at inner and outer radius:

$$\begin{cases} (\sigma_t)_{max} = p_i \frac{b^2 + a^2}{b^2 - a^2} & \text{inner radius} \\ (\sigma_t)_{min} = p_i \frac{2a^2}{b^2 - a^2} & \text{outer radius} \end{cases}$$

Thick wall cylinders – $p_i = 0$

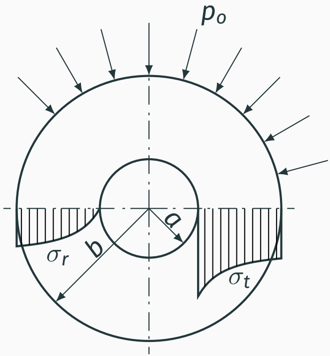


Figure 11: Hoop and radial stresses for a thick wall cylinder with external pressure [3].

For external pressure $p_o \neq 0$ and $p_i = 0$:

$$\begin{cases} \sigma_t = -p_o \frac{b^2}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right) \\ \\ \sigma_r = -p_o \frac{b^2}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right) \end{cases}$$

The radial stress at inner radius is $p_i = 0$ and at outer radius is $-p_o$.

The hoop stress at inner and outer radius:

$$\begin{cases} (\sigma_t)_{max} = -p_o \frac{2b^2}{b^2 - a^2} & \text{inner radius} \\ (\sigma_t)_{min} = -p_o \frac{b^2 + a^2}{b^2 - a^2} & \text{outer radius} \end{cases}$$

Yield criteria

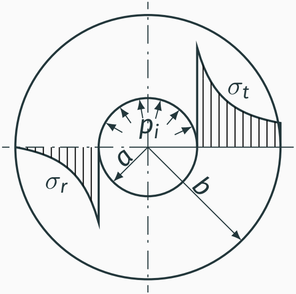


Figure 10: Hoop and radial stresses for a thick wall cylinder with internal pressure [3].

For a thick wall cylinder with internal pressure, the Yield starts at the inner radius.

Using a failure criteria (Tresca):

$$\sigma_e = \sigma_1 - \sigma_3 = \sigma_y$$

With $\sigma_1 = \sigma_{t_a} = p_i \frac{b^2+a^2}{b^2-a^2}$ and $\sigma_3 = -p_i$:

$$p_i \frac{b^2 + a^2}{b^2 - a^2} + p_i = \sigma_y$$

Using $K = \frac{b}{a}$:

$$\frac{p_i}{\sigma_y} = \frac{b^2 - a^2}{2b^2} = \frac{K^2 - 1}{2K^2}$$

Yield criteria

Other criteria for yielding of the internal wall due to internal pressure [2]:

Thick wall

Maximum normal stress

$\sigma_y = \sigma_{max} = \sigma_t = p_i \frac{b^2+a^2}{b^2-a^2}$ $\qquad \frac{p_i}{\sigma_y} = \frac{K^2-1}{K^2+1}$

Maximum principal strain:

$\sigma_y = E\varepsilon_{max} = \sigma_t - \nu\left(\sigma_r + \sigma_l\right)$ $\qquad \frac{p_i}{\sigma_y} = \frac{K^2-1}{(1-2\nu)+K^2(1+\nu)}$

Maximum distortion energy (von Mises):

$2\sigma_y^2 = \left(\sigma_t - \sigma_l\right)^2 + \left(\sigma_l - \sigma_r\right)^2 + \left(\sigma_t - \sigma_r\right)^2$ $\qquad \frac{p_i}{\sigma_y} = \frac{K^2-1}{\sqrt{3}K^2}$

Thin wall

Maximum normal stress (Hamburg formula):

$\sigma_y = \sigma_{max} = \sigma_t = \frac{p_i r}{t}$ $\qquad \frac{p_i}{\sigma_y} = K - 1$

Thin wall vs. thick wall

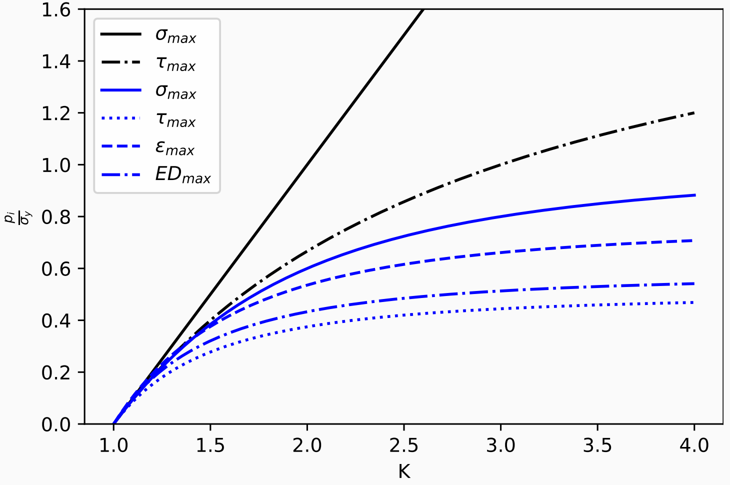


Figure 12: Several failure criteria for thin-wall (black) and thick-wall (blue) cylinders to define the beginning of yield due to internal pressure.

Thin wall vs. thick wall

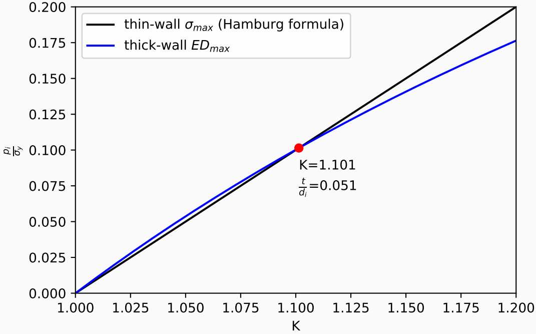


Figure 13: Maximum energy distortion for thick-wall vs. maximum normal stress (Hamburg formula) for thin-wall cylinder.

For $K \leq 1.101$ the maximum distortion energy criteria results in smaller wall thickness. This is the justification behind the international convention that sets the thickness limit for a thin-walled circular cylinder [2].

Thin wall vs. thick wall

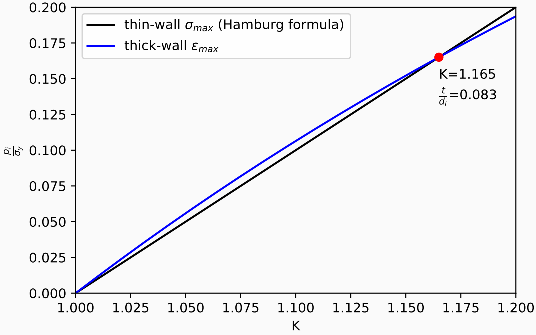


Figure 14: Maximum principal strain for thick-wall vs. maximum normal stress (Hamburg formula) for thin-wall cylinder.

Another thickness limit for a thin-walled circular cylinder can be set using the maximum strain theory: $K \leq 1.165$.

Autofrettage

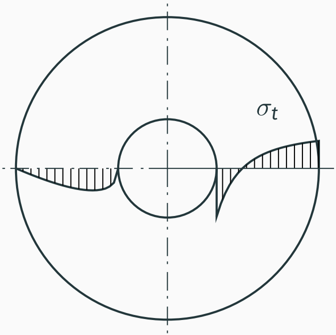


Figure 15: Permanent stress distribution after *autofrettage* process of a thick wall cylinder [7, 3].

In high pressure applications, apart from using compound cylinders, *autofrettage* is used, consisting of preloading the cylinder with an internal pressure greater than the yield, so as to obtain plastic deformations in the internal layers of the cylinder. When the pressure is gone, elastic extension stresses are retained in the outer layers and compressive stresses appear in the inner layers [7].

Fully plastic cylinder

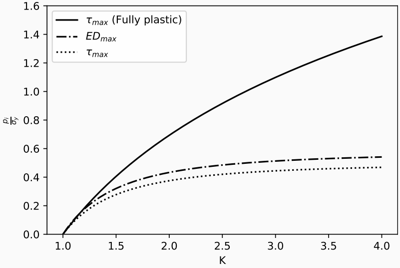


Figure 16: Internal pressure needed to reach a fully plastic cylinder using maximum shear stress theory [3].

Equilibrium equation:

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} + \frac{\sigma_r - \sigma_t}{r} = 0$$

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} - \frac{\sigma_y}{r} = 0$$

After integration:

$$\sigma_r = \sigma_y \ln r + C_1$$

Boundary conditions: $\sigma_r = -p_i$ for $r = a$; $\sigma_r = 0$ for $r = b$.

$$p_i = \sigma_y \ln \frac{b}{a} \Leftrightarrow \frac{p_i}{\sigma_y} = \ln K$$

Tresca criteria:

$$\sigma_y = \sigma_1 - \sigma_3 = \sigma_t - \sigma_r$$

Finite Element Method solution

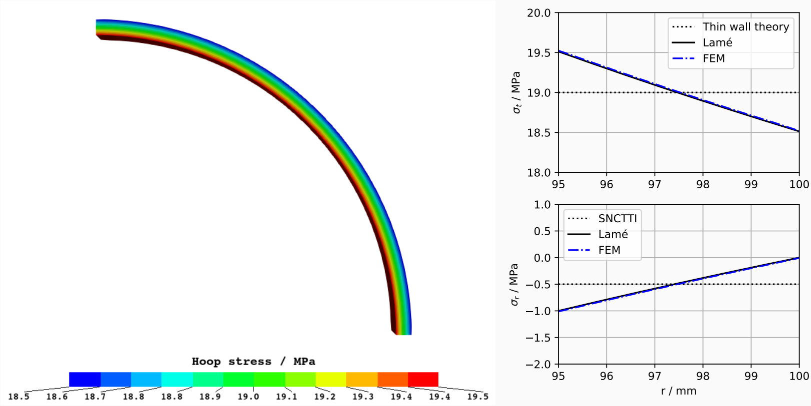
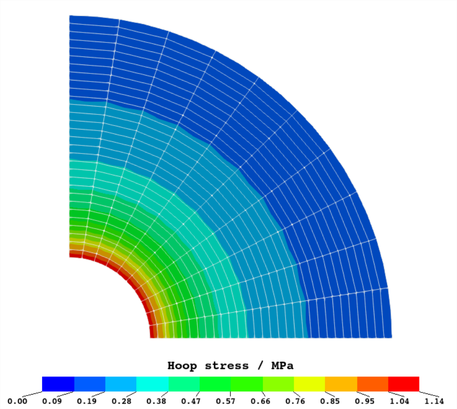


Figure 17: FEM 3D calculation with CalculiX [9] for a thin wall cylinder with internal pressure $p_i = 1\,\mathrm{MPa}$ (FEM calculation done with C3D20 elements for representation purposes)

Finite Element Method solution



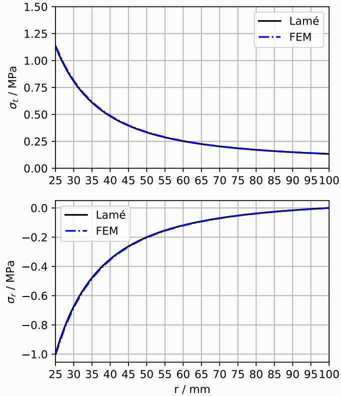


Figure 18: FEM 3D calculation with CalculiX [9] for a thick wall cylinder with internal pressure $p_i = 1\,\mathrm{MPa}$ (FEM calculation done with C3D20 elements for representation purposes).

Lecture 2

Sumário

Compound cylinders

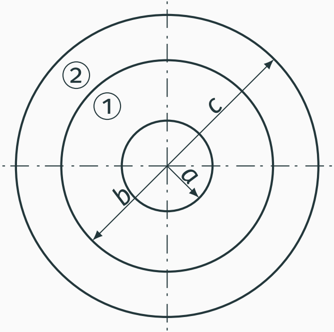


Figure 19: Compound cylinder.

The compound cylinders are structures consisting of concentric thick-walled cylinders assembled with a radial interference fit.

The cylinders can be:

- *press fit* assembled forcing them together with a press;
- *shrink fit*:
  - the outer cylinder **2** is preheated and expands to fit over the inner cylinder
  - the inner cylinder **1** is precooled and contracts.

Compound cylinders

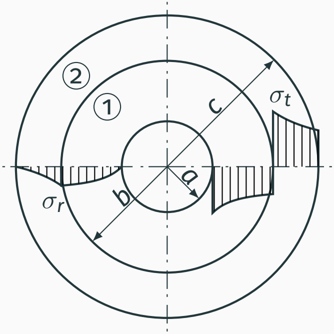


Figure 20: Compound cylinder.

Tangential stress:

- cylinder 1 outer radius:

$$\sigma_{t1} = -p_c \frac{b^2 + a^2}{b^2 - a^2}$$

- cylinder 2 inner radius:

$$\sigma_{t2} = p_c \frac{c^2 + b^2}{c^2 - b^2}$$

How to determine the contact pressure p_c?

Compound cylinders

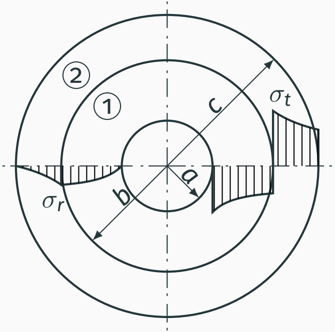


Figure 20: Compound cylinder [3].

The strain of the cylinder 2 inner radius is:

$$\varepsilon_{t2} = \frac{2\pi\left(b + \delta_2\right) - 2\pi b}{2\pi b} = \frac{\delta_2}{b}$$

The Hooke's law:

$$\varepsilon_{t2} = \frac{1}{E_2}\left(\sigma_{t2} - \nu\sigma_{r2}\right)$$

$$\delta_2 = \frac{bp_c}{E_2}\left(\frac{c^2 + b^2}{c^2 - b^2} + \nu\right)$$

$$\delta_1 = -\frac{bp_c}{E_1}\left(\frac{b^2 + a^2}{b^2 - a^2} - \nu\right)$$

Compound cylinder

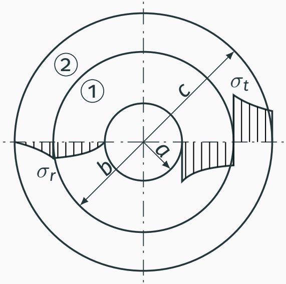


Figure 20: Compound cylinder [3, 7].

$$\delta_2 = \frac{bp_c}{E_2}\left(\frac{c^2 + b^2}{c^2 - b^2} + \nu\right)$$

$$\delta_1 = -\frac{bp_c}{E_1}\left(\frac{b^2 + a^2}{b^2 - a^2} - \nu\right)$$

Since $\delta = |\delta_2| + |\delta_1| = \delta_2 - \delta_1$ and for equal material $E_1 = E_2 = E$:

$$\boxed{p_c = \frac{E\delta}{b}\left[\frac{(c^2 - b^2)(b^2 - a^2)}{2b^2(c^2 - a^2)}\right]}$$

Compound cylinder

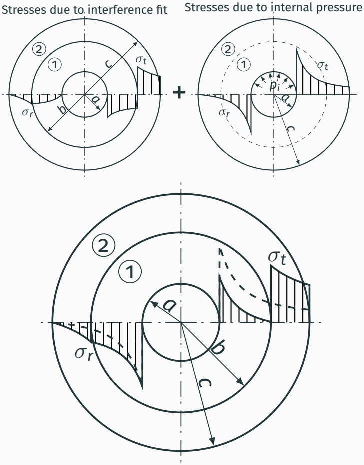


Figure 21: Compound cylinder with internal pressure [3].

The hoop stresses on the inner cylinder are now lower. So, the compound cylinder must withstand higher internal pressure than a single cylinder with the same dimensions.

The design is now based on the inner radius of the outer cylinder **2** where the stress state is more demanding, but still better than in the single cylinder with the same dimensions.

Optimizing two circular cylinders with interference fit

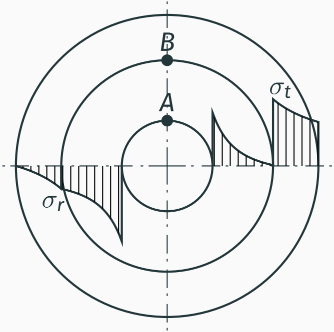


Figure 22: Compound cylinder with internal pressure [3, 7].

The stress state is now worse in the outer cylinder. Assuming the same material and the same yield strength for both cylinders.

Using Tresca criteria:

$$\sigma_e = \sigma_1 - \sigma_3 = \sigma_t - \sigma_r$$

In point A:

$$\sigma_{e_A} = p_i \frac{c^2 + a^2}{c^2 - a^2} - p_c \frac{2b^2}{b^2 - a^2} - (-p_i)$$

$$\sigma_{e_A} = p_i \frac{2c^2}{c^2 - a^2} - p_c \frac{2b^2}{b^2 - a^2}$$

Optimizing two circular cylinders with interference fit

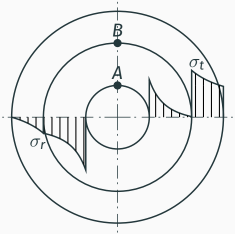


Figure 22: Compound cylinder with internal pressure [3, 7].

Using Tresca criteria in point B:

$$\sigma_{e_B} = p_c \frac{c^2 + b^2}{c^2 - b^2} + p_i \frac{a^2}{c^2 - a^2}\left(1 + \frac{c^2}{b^2}\right) - p_i \frac{a^2}{c^2 - a^2}\left(1 - \frac{c^2}{b^2}\right) - (-p_c)$$

$$\sigma_{e_B} = \frac{2c^2}{b^2}\left(p_i \frac{a^2}{c^2 - a^2} + p_c \frac{b^2}{c^2 - b^2}\right)$$

Optimizing two circular cylinders with interference fit

In order to have the same equivalent stress in both cylinders, we should do:

$$\sigma_{e_A} = \sigma_{e_B}$$

Replacing p_c into the equality $\sigma_{e_A} = \sigma_{e_B}$ we get the interference value:

$$\delta = \frac{2p_i}{E} \frac{bc^2\left(b^2 - a^2\right)}{b^2\left(c^2 - b^2\right) + c^2\left(b^2 - a^2\right)}$$

Recall that the contact pressure is:

$$p_c = \frac{E\delta}{b}\left[\frac{\left(c^2 - b^2\right)\left(b^2 - a^2\right)}{2b^2\left(c^2 - a^2\right)}\right]$$

Replacing p_c into σ_{e_A}:

$$\sigma_e = p_i \frac{2c^2}{c^2 - a^2}\left(1 - \frac{1}{\frac{b^2}{b^2 - a^2} + \frac{c^2}{c^2 - b^2}}\right)$$

The equation has a minimum for $b = \sqrt{ac}$ known as *Gadoline conditions*[3] [7, 3, 10]:

$$\sigma_e^{min} = p_i \frac{c}{c - a}$$

[3] Féodossiev, V., Résistance des Matériaux, 2 édition, Éditions MIR, Moscou, 1971, chpater IX, page 296.

Cylindrical interference fits

The maximum torque that can be transmitted using a cylindrical interference fit is [3]:

$$M_t = F_c \cdot b = 2 \cdot p_c \cdot \pi \cdot b \cdot L \cdot b \cdot \mu$$

The axial force needed to disassemble the parts is:

$$F_a \geq 2 \cdot p_c \cdot \pi \cdot b \cdot L \cdot \mu$$

p_c is the contact pressure of the interference fit
b is the radius of the interference fit
L is the length of the shortest part on the interference fit
μ is the coefficient of friction

Cylindrical interference fits

The maximum shear stress of the interference fit is [1]:

$$\tau_{max} = p_c \cdot \mu$$

The circunferential force causes the following shear stress:

$$\tau_c = \frac{F_c}{A} = \frac{F_c}{2 \cdot \pi \cdot a \cdot L}$$

The axial force causes the following shear stress:

$$\tau_a = \frac{F_a}{A} = \frac{F_a}{2 \cdot \pi \cdot a \cdot L}$$

The axial and circunferential shear stresses are related to the maximum:

$$\tau_{max} = \sqrt{\tau_a^2 + \tau_c^2}$$

Cylindrical interference fits

Just a brief review of ISO 286. Typical interference fits: H7/p6, H7/r6, H7/s6

Basic hole	Tolerance classes for shafts																	
	Clearance fits							Transition fits				Interference fits						
H 6						g5	h5	js5	k5	m5		n5	p5					
H 7					f6	**g6**	**h6**	**js6**	**k6**	m6	**n6**		**p6**	**r6**	**s6**	t6	u6	x6
H 8				e7	**f7**		**h7**	js7	k7	m7					s7		u7	
			d8	**e8**	f8		h8											
H 9			d8	**e8**	f8		h8											
H10	b9	c9	d9	e9			h9											
H 11	**b11**	**c11**	d10				h10											

(a) Basic hole (H)

Basic shaft	Tolerance classes for holes																	
	Clearance fits							Transition fits				Interference fits						
h 5						G6	H6	JS6	K6	M6		N6	P6					
h 6					F7	**G7**	**H7**	**JS7**	**K7**	M7	**N7**		**P7**	**R7**	**S7**	T7	U7	X7
h 7				E8	**F8**		**H8**											
h 8			D9	E9	F9		H9											
h 9				E8	**F8**		**H8**											
			D9	**E9**	F9		**H9**											
	B11	C10	**D10**				H10											

(b) Basic shaft (h)

Figure 23: Preferable fits according to ISO 286 [11].

Practical example of an interference fit

Consider a cylindrical interference fit between a hub and a hollow shaft with a nominal diameter of 60 mm and H7/u6 interference fit according to ISO 286 standard. The shaft and the hub are made of C45 steel. The shaft inner diameter is 10 mm, the hub outer diameter is 90 mm.

Determine the minimum and maximum interference pressure p_c that can occur. What is the maximum, minimum and average transmittable torque if $\mu = 0.15$ and $L = 50\,\mathrm{mm}$?

$$\delta_{max} = D_{f_{shaft,max}} - D_{f_{hub,min}} \qquad \delta_{min} = D_{f_{shaft,min}} - D_{f_{hub,max}}$$

$$\delta_{max} = e_s - E_i = 0.106\,\mathrm{mm} \qquad \delta_{min} = e_i - E_s = 0.057\,\mathrm{mm}$$

$$p_{c,max} = 99.5\,\mathrm{MPa} \qquad p_{c,min} = 53.2\,\mathrm{MPa}$$

Standards about interference fits

The following standards apply:

- DIN 7190-1, August 2013, “Interference fits - Part 1: Calculation and design rules”
- DIN 7190-2, August 2013, “Interference fits - Part 2: Calculation and design rules for conical self-locking pressfits”
- NF E22-620, Mars 1980, “Assemblages frettés - Dimensions, tolérances et états de surface pour assemblages usuels”;
- NF E22-621, Mai 1980, “Assemblages frettés - Sur portée cylindrique - Fonction, réalisation, calcul”;

DIN 7190 - Nomenclature

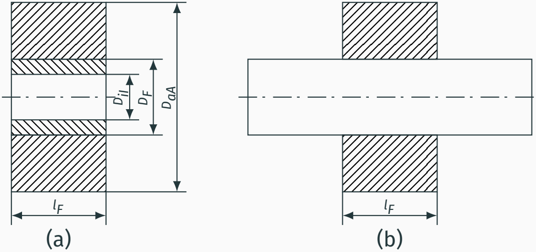


Figure 24: (a) computation model (b) real model.

The transmitted torque is:

$$T = \frac{\pi D_F^2}{2} l_F \nu_{ru} \frac{p_c}{S_R}$$

The axial force is $F_a = \pi D_F l_F \nu_{rl} \frac{p_c}{S_R}$

$S_R = 1.2$ – minimum safety factor against sliding to avoid micro gliding that may cause fretting
ν_{ru} – coefficient of adhesion along circumferential direction
ν_{rl} – coefficient of adhesion along longitudinal direction

DIN 7190 – coefficient of adhesion

The coefficient of adhesion for longitudinal fits are presented for loosening ν_{ll} or ν_{rl} slipping conditions [12]:

Material		Coefficient of adhesion			
Used	New	Dry		Lubricated	
		ν_{ll}	ν_{rl}	ν_{ll}	ν_{rl}
St 60-2	E 335	0.11	0.08	0.08	0.07
GS-60	GE 300	0.11	0.08	0.08	0.07
RSt37-2	S 235JRG2	0.10	0.09	0.07	0.06
GG-25	EN-GJL-250	0.12	0.11	0.06	0.05
GGG-60	EN-GJS-600-3	0.10	0.09	0.06	0.05
G-AlSi12(Cu)	EN AB-44000	0.07	0.06	0.05	0.04
G-CuPb10Sn	CB495K	0.07	0.06	-	-
TiAl6V4	TiAl6V4	-	-	0.05	-

DIN 7190 – coefficient of adhesion

The coefficient of adhesion for radial interference fits in the longitudinal (ν_{rl}) and tangential (ν_{ru}) directions subjected to slipping [12]:

Material pairing, lubrication and joining method	
Steel-steel pairing	
Oil pressure connection, normally joined with mineral oil	0.12
Oil pressure connection, degreased contact surfaces, joined with glycerin	0.18
Shrink fit, normally heating up to 300 $^\circ\mathrm{C}$ the external component in electrical oven	0.14
Shrink fit, degreased contact surfaces, heating up to 300 $^\circ\mathrm{C}$ in electrical oven	0.20
Steel-cast iron pairing	
Oil pressure connection, normally joined with mineral oil	0.10
Oil pressure connection, with degreased contact surfaces	0.16
Steel-MgAl pairing, dry	0.10-0.15
Steel-CuZn pairing, dry	0.17-0.25

Problems

Summary

Assignments

Assignment A2

Available on the Course Contents:

“Assignments” proposed for Complements of Machine Elements

Thin wall cylinder

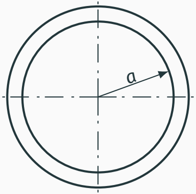


Figure 25: Thin wall cylinder [13].

Consider the thin wall cylinder in Figure 25. The inner radius is $a = 50\,\mathrm{mm}$. The yield strength of the material is $\sigma_y = 400\,\mathrm{MPa}$ and the internal pressure is $p_i = 2\,\mathrm{MPa}$. Use a safety factor of 3, consider that thin wall analysis is adequate and determine the wall thickness using the maximum shear stress theory. Repeat the calculation using the thick wall theory and quantify the difference on the wall thickness.

Hoop stresses in a thick wall cylinder

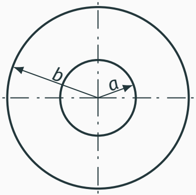


Figure 26: Thick wall cylinder [8].

Determine the tangential stresses at the inner and outer surfaces and at the middle thickness of the wall of a cylinder with inner radius $a = 100\,\mathrm{mm}$ and outer radius $b = 200\,\mathrm{mm}$, subjected to an internal pressure of $p_i = 200\,\mathrm{MPa}$. Repeat the calculation assuming $p_i = 0\,\mathrm{MPa}$ and $p_o = 200\,\mathrm{MPa}$ [8].

Compound cylinder

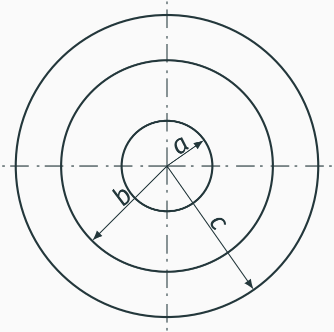


Figure 27: Compound cylinder [8].

Determine the stresses in the compound cylinder of Figure 27 subjected to an internal pressure $p_i = 200\,\mathrm{MPa}$ if $a = 100\,\mathrm{mm}$, $b = 150\,\mathrm{mm}$, $c = 200\,\mathrm{mm}$ and the interference is $\delta = 0.125\,\mathrm{mm}$. Consider that both materials are steel.

Hub and shaft

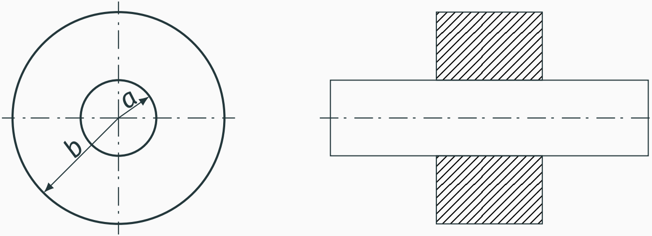


Figure 28: Hub and shaft [14].

For the hub and shaft in Figure 28, both in steel with $E = 206\,\mathrm{GPa}$, find the uniform pressure p_c if the radius of the shaft is $150\,\mathrm{mm}$ and the outer radius of the hub is $300\,\mathrm{mm}$. The initial difference in diameters between hub and shaft is $\delta = 0.3\,\mathrm{mm}$.

Two stiff shafts

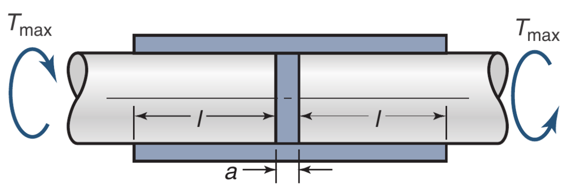


Figure 29: Two stiff shafts connected by a thin-wall tube [1].

Two stiff shafts are connected by a thin-walled elastic tube to a press-fit connection as shown in Figure 29. The contact pressure between the shafts and the tube is p_c.

The coefficient of friction is μ. Calculate the maximum torque T_{max} that can be transmitted through the press fit. Describe and calculate what happens if the torque decreases from T_{max} to θT_{max} where $0 < \theta < 1$.

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[11] *ISO 286-1:2010 Geometrical product specifications (GPS) - ISO code system for tolerances on linear sizes - Part 1: Basis of tolerances, deviations and fits.* **Standard, International Organization for Standardization, Geneva, Switzerland, 2007.**

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