U.PORTO
FEUP FACULDADE DE ENGENHARIA
UNIVERSIDADE DO PORTO

Curved Beams

Complements of Machine Elements

Carlos Fernandes

2023

Mestrado em Engenharia Mecânica

Ein Mann, der konstruieren will, Der schau erst mal und denke!

A man who wants to design, He first looks and thinks!

From Gustav Niemann, Hans Winter, Bernd-Robert Höhn, “Maschinenelemente: Band I Konstruktion und Berechnung von Verbindungen”

Recomended bibliograhy

- Timoshenko, “Strength of Materials. Part I: Elementary Theory and Problems”, Third Edition, CBS, vol.1, 1986.
- Timoshenko, “Resistência de materiais”, Ao Livro Técnico, vol.1, 1969.
- Robert C. Juvinall, Kurt M. Marshek; “Fundamentals of machine component design”, Wiley, 2017.

Contents

Lecture 1

Summary

Introduction

Machine frames, springs, clips and fasteners frequently are curved shapes [1].

A lifting crane hook as presented in Figure 1 is a typical curved beam.

Several lifting hook geometries are described in:

- DIN 15401-1, Lifting hooks for lifting appliances; Single hooks; Unmachined parts, November 1982.

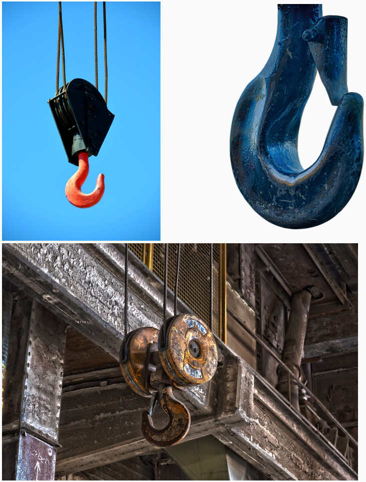


Figure 1: Lifting crane hook - typically treated as a curved beam.

Pure bending loading - straight beams

Figure 2 shows a straight beam loaded *only* in bending; hence the term, “pure bending” [2].

Figure 2 presents typical sections with two axes of symmetry.

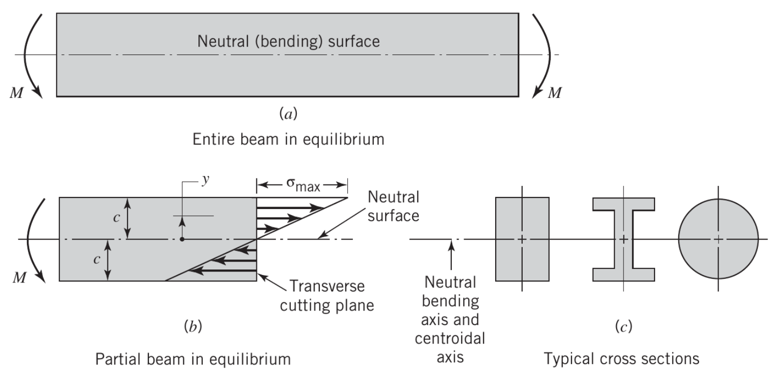


Figure 2: Pure bending of sections with two axes of symmetry [2].

Pure bending loading - straight beams

Figure 3 presents typical sections with one axis of symmetry.

Tensile stresses exist above the neutral axis and compressive stresses below.

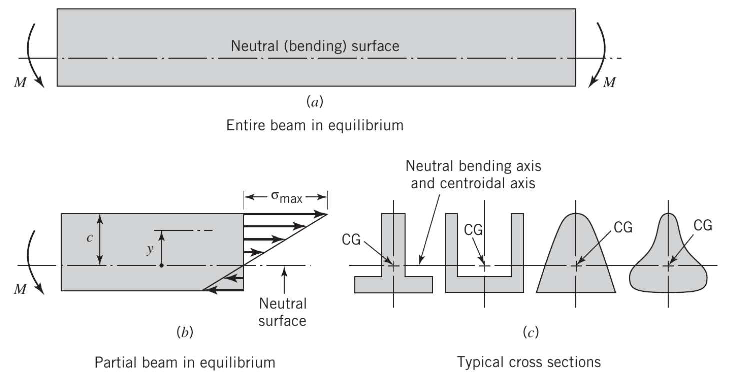


Figure 3: Pure bending of sections with one axis of symmetry [2].

Pure bending loading - straight beams

The bending stresses or normal stresses (axial stresses) are given by:

$$\sigma = \frac{My}{I}$$

I – second moment of area of the cross section;
y – distance from the neutral axis.

Equation applies to any cross section with the following limitations [2]:

1. The bar must be initially straight and loaded in a plane of symmetry.
2. The material must be homogeneous, and all stresses must be within the elastic range.
3. The section for which stresses are calculated must not be too close to significant stress raisers or to regions where external loads are applied (Saint-Venant's Principle).

Pure bending loading - curved beams

Hypothesis[1] [3]:

- *center line* of the beam is a plane curve;
- cross sections have an axis of symmetry in this plane;
- beam is submitted to the action of forces acting in the plane of symmetry;
- bending takes place in the plane of symmetry;
- cross sections originally plane and normal to the center line remain so after bending.

[1] Timoshenko, S., Strength of Materials. Part I: Elementary Theory and Problems, Third Edition, CBS, page 362.

Pure bending loading - curved beams

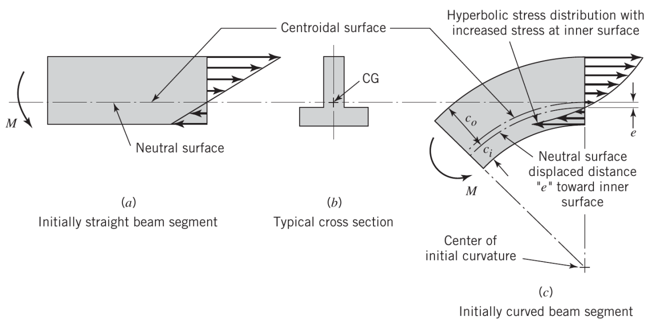


Figure 4: Effect of initial curvature, pure bending of sections with one axis of symmetry [2].

The shortest path along the length of a curved beam is at the inside surface. A consideration of the relative stiffness suggests that the stresses at the inside surface are ***greater*** than indicated by the straight-beam equations.

Pure bending loading - curved beams

For a curved beam with *pure bending* loading in the plane of curvature, the bending stresses are just approximately in accordance with $\sigma = \frac{M \cdot y}{I}$.

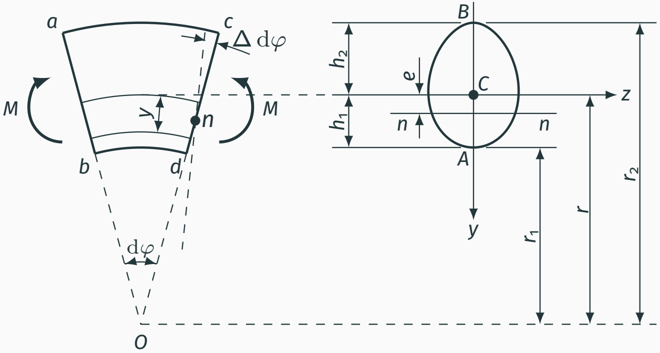


Figure 5: Curved beam under “pure bending” [3].

Pure bending loading - curved beams

The strain on the fiber shown at distance y from the centroidal axis is:

$$\epsilon = \frac{\delta}{l} = \frac{(y - e)\,\Delta\,\mathrm{d}\varphi}{(r - y)\,\mathrm{d}\varphi}$$

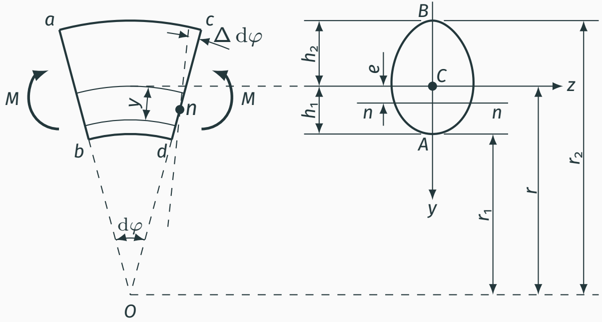


Figure 5: Curved beam under “pure bending” [3].

Pure bending loading - curved beams

Applying Hooke's law[2]:

$$\sigma = E\frac{(y-e)\,\Delta\,\mathrm{d}\varphi}{(r-y)\,\mathrm{d}\varphi}$$

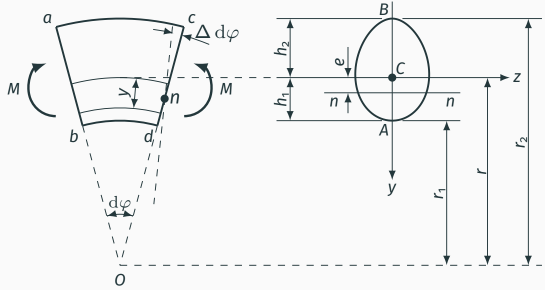


Figure 5: Curved beam under "pure bending" [3].

[2]Assuming no radial pressure between the longitudinal fibers.

Pure bending loading - curved beams

From statics, the sum of forces should be zero. Taking note that Young modulus $E \neq 0$:

$$\sum F = 0 : \int \sigma \, \mathrm{d}A = \underbrace{\frac{E\Delta \, \mathrm{d}\varphi}{\mathrm{d}\varphi}}_{\neq 0} \underbrace{\int \frac{y-e}{r-y} \, \mathrm{d}A}_{=0} = 0$$

$$\int \frac{y-e}{r-y} \, \mathrm{d}A = \underbrace{\int \frac{y}{r-y} \, \mathrm{d}A}_{\text{dimensions of area}} - e \int \frac{1}{r-y} \, \mathrm{d}A = 0$$

Expressing the first integral as a *modified area* of the cross section **mA**, where **m** is a number to be determined for each cross section:

$$\int \frac{y}{r-y} \, \mathrm{d}A = mA$$

Pure bending loading - curved beams

$$\int \frac{y-e}{r-y}\,\mathrm{d}A = \underbrace{\int \frac{y}{r-y}\,\mathrm{d}A}_{mA} - e\underbrace{\int \frac{1}{r-y}\,\mathrm{d}A}_{(m+1)\frac{A}{r}} = 0$$

The second integral may be transformed in the following way:

$$\int \frac{1}{r-y}\,\mathrm{d}A = \int \frac{y+r-y}{r\,(r-y)}\,\mathrm{d}A = \underbrace{\int \frac{y}{r\,(r-y)}\,\mathrm{d}A}_{\frac{mA}{r}} + \underbrace{\int \frac{1}{r}\,\mathrm{d}A}_{\frac{A}{r}} = (m+1)\,\frac{A}{r}$$

Finally, we can write $\boxed{mA - e\,(m+1)\,\frac{A}{r} = 0}$, so:

$$e = r\frac{m}{m+1} \quad \texttt{or} \quad m = \frac{e}{r-e}$$

Pure bending loading - curved beams

The sum of moments should also be zero:

$$\sum M_C = 0 : \int \sigma y \,\mathrm{d}A = \frac{E\Delta \,\mathrm{d}\varphi}{\mathrm{d}\varphi} \int \frac{(y-e)\,y}{r-y} \,\mathrm{d}A - M = 0$$

$$M = \frac{E\Delta \,\mathrm{d}\varphi}{\mathrm{d}\varphi} \left(\int \underbrace{\frac{y^2}{r-y}}_{-y+r\frac{y}{(r-y)}} \,\mathrm{d}A - \underbrace{e \int \frac{y}{r-y} \,\mathrm{d}A}_{meA} \right) = \frac{E\Delta \,\mathrm{d}\varphi}{\mathrm{d}\varphi} (mrA - meA)$$

$$\int \frac{y^2}{r-y} \,\mathrm{d}A = \underbrace{-\int y \,\mathrm{d}A}_{=0^3} + r \int \frac{y}{r-y} \,\mathrm{d}A = 0 + rmA$$

[3] The first moment of area about the centroidal axis is zero

Pure bending loading - curved beams

$$M = \frac{E\Delta\,\mathrm{d}\varphi}{\mathrm{d}\varphi}\left(mrA - meA\right) \Leftrightarrow \frac{\Delta\,\mathrm{d}\varphi}{\mathrm{d}\varphi} = \frac{M}{E\left(mrA - meA\right)}$$

Recall that:

$$\sigma = E\frac{(y-e)\,\Delta\,\mathrm{d}\varphi}{(r-y)\,\mathrm{d}\varphi} = E\frac{(y-e)}{(r-y)}\frac{M}{E\left(mrA - meA\right)} \quad \text{and} \quad m = \frac{e}{r-e}$$

$$\sigma = \frac{(y-e)}{(r-y)}\frac{M}{\frac{e}{r-e}rA - \frac{e}{r-e}eA} = \frac{M\,(y-e)\,(r-e)}{Ae\,(r-y)\,(r-e)}$$

$$\boxed{\sigma = \frac{M\,(y-e)}{Ae\,(r-y)}}$$

Pure bending loading - curved beams

$$\sigma_A = \frac{M\,(h_1 - e)}{eAr_1} \qquad \sigma_B = \frac{-M\,(h_2 + e)}{eAr_2}$$

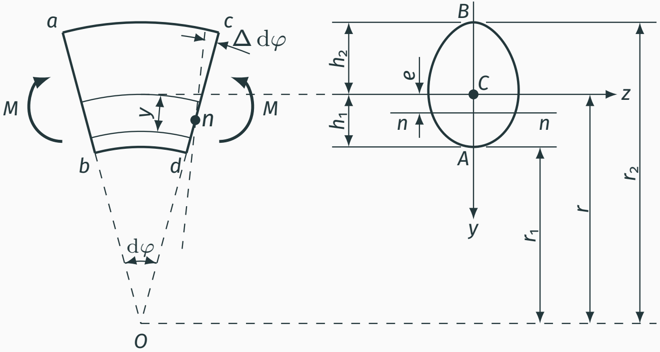


Figure 5: Curved beam under “pure bending” [3].

Particular cases of curved beams

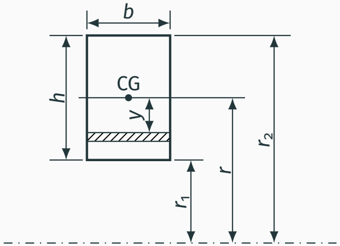


Figure 6: Curved beam with rectangular cross section [3].

$$mA = \int \frac{y}{r-y}\,\mathrm{d}A = b\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{y-r+r}{r-y}\,\mathrm{d}y = br\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{1}{r-y}\,\mathrm{d}y - bh$$

$$mA = br\ln\frac{r_2}{r_1} - bh \Leftrightarrow m = \frac{r}{h}\ln\frac{r_2}{r_1} - 1$$

Particular cases of curved beams

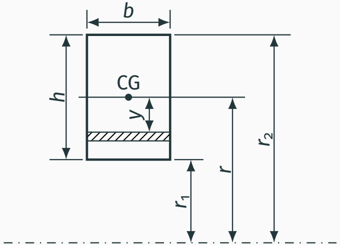


Figure 6: Curved beam with rectangular cross section [3].

$$m = \frac{r}{h} \ln \frac{r_2}{r_1} - 1$$

$$e = r\frac{m}{m+1} = r\frac{\frac{r}{h}\ln\frac{r_2}{r_1} - 1}{\frac{r}{h}\ln\frac{r_2}{r_1}} = r - \frac{h}{\ln\frac{r_2}{r_1}}$$

Particular cases of curved beams

For small values of $\frac{h}{r}$ the distance **e** is small in comparison with h. To have sufficient accuracy for e it is necessary to take $\ln\frac{r_2}{r_1}$ with a high degree of accuracy.

$$\ln\frac{r_2}{r_1} = \ln\frac{r+\frac{h}{2}}{r-\frac{h}{2}} = \ln\frac{\frac{2r+h}{2}}{\frac{2r-h}{2}} = \ln\frac{2r+h}{2r-h} = \ln\frac{1+\frac{h}{2r}}{1-\frac{h}{2r}}$$

A Taylor series expansion around zero (Maclaurin) can be employed:

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{(iv)}(0) + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + O(x^6)$$

Particular cases of curved beams

Performing the same for $\ln(1-x)$:

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5} + O(x^6)$$

Recalling that $x = \frac{h}{2r}$:

$$\ln\left(1+\frac{h}{2r}\right) = \frac{h}{2r} - \frac{1}{2}\left(\frac{h}{2r}\right)^2 + \frac{1}{3}\left(\frac{h}{2r}\right)^3 - \frac{1}{4}\left(\frac{h}{2r}\right)^4 + \frac{1}{5}\left(\frac{h}{2r}\right)^5 + \dots$$

$$\ln\left(1-\frac{h}{2r}\right) = -\frac{h}{2r} - \frac{1}{2}\left(\frac{h}{2r}\right)^2 - \frac{1}{3}\left(\frac{h}{2r}\right)^3 - \frac{1}{4}\left(\frac{h}{2r}\right)^4 - \frac{1}{5}\left(\frac{h}{2r}\right)^5 + \dots$$

Particular cases of curved beams

Now remember that $\ln \frac{r_2}{r_1} = \ln r_2 - \ln r_1$:

$$\ln \frac{1+\frac{h}{2r}}{1-\frac{h}{2r}} = \frac{h}{2r} - \cancel{\frac{1}{2}\left(\frac{h}{2r}\right)^2} + \frac{1}{3}\left(\frac{h}{2r}\right)^3 - \cancel{\frac{1}{4}\left(\frac{h}{2r}\right)^4} + \frac{1}{5}\left(\frac{h}{2r}\right)^5 + \frac{h}{2r} + \cancel{\frac{1}{2}\left(\frac{h}{2r}\right)^2} + \frac{1}{3}\left(\frac{h}{2r}\right)^3 + \cancel{\frac{1}{4}\left(\frac{h}{2r}\right)^4} + \frac{1}{5}\left(\frac{h}{2r}\right)^5 + \dots$$

$$\ln \frac{1+\frac{h}{2r}}{1-\frac{h}{2r}} = \frac{h}{r}\left[1 + \frac{1}{3}\left(\frac{h}{2r}\right)^2 + \frac{1}{5}\left(\frac{h}{2r}\right)^4 + \dots\right]$$

And substituting in m:

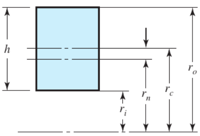
$$m = \frac{r}{h}\ln\frac{r_2}{r_1} - 1 = \frac{1}{3}\left(\frac{h}{2r}\right)^2 + \frac{1}{5}\left(\frac{h}{2r}\right)^4 + \dots$$

Taking only the first term:

$$m \approx \frac{h^2}{12r^2} \quad \text{and} \quad e \approx \frac{h^2}{12r}$$

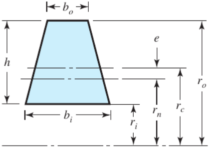
Formulas for sections of Curved Beams

Formulas taken from [1]. See also [4, 5]



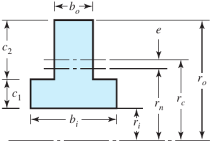
$$r_c = r_i + \frac{h}{2}$$

$$r_n = \frac{h}{\ln(r_o/r_i)}$$



$$r_c = r_i + \frac{h}{3}\,\frac{b_i + 2b_o}{b_i + b_o}$$

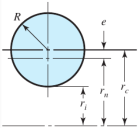
$$r_n = \frac{A}{b_o - b_i + [(b_i r_o - b_o r_i)/h]\ln(r_o/r_i)}$$



$$r_c = r_i + \frac{b_i c_1^2 + 2b_o c_1 c_2 + b_o c_2^2}{2(b_o c_2 + b_i c_1)}$$

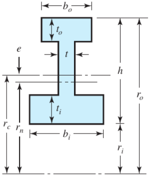
$$r_n = \frac{b_i c_1 + b_o c_2}{b_i \ln[(r_i + c_1)/r_i] + b_o \ln[r_o/(r_i + c_1)]}$$

Formulas for sections of Curved Beams



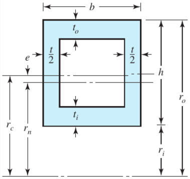
$$r_c = r_i + R$$

$$r_n = \frac{R^2}{2\left(r_c - \sqrt{r_c^2 - R^2}\right)}$$



$$r_c = r_i + \frac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t) + t_o(b_o - t)(h - t_o/2)}{t_i(b_i - t) + t_o(b_o - t) + ht}$$

$$r_n = \frac{t_i(b_i - t) + t_o(b_o - t) + ht_o}{b_i \ln\frac{r_i + t}{r_i} + t \ln\frac{r_o - t_o}{r_i + t_i} + b_o \ln\frac{r_o}{r_o - t_o}}$$



$$r_c = r_i + \frac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b - t) + t_o(b - t)(h - t_o/2)}{ht + (b - t)(t_i + t_o)}$$

$$r_n = \frac{(b - t)(t_i + t_o) + ht}{b\left(\ln\frac{r_i + t_i}{r_i} + \ln\frac{r_o}{r_o - t_o}\right) + t \ln\frac{r_o - t_o}{r_i + t_i}}$$

Stress concentration factors for curved beams

The bending stress can be determined based on stress concentration factors and the equation for straight beams:

$$\sigma = K_t \frac{Mc}{I}$$

As stated by [5], the concept was proposed in "A simple method of determining stress in curved flexural member" [6].

$$K_t = 1 + 0.5 \frac{I}{bc^2} \left[\frac{1}{r-c} + \frac{1}{r} \right]$$

For circular and elliptical sections use 1.05 instead of 0.5 in the formula.

I is the second moment of area as used in straight-beam formula
b is the maximum breadth of the cross section
$c = r - r_1$ is the distance from centroidal axis to inside fiber
r is the radius of curvature of centroidal axis of the beam

Stress concentration factors for curved beams

Graphical method:

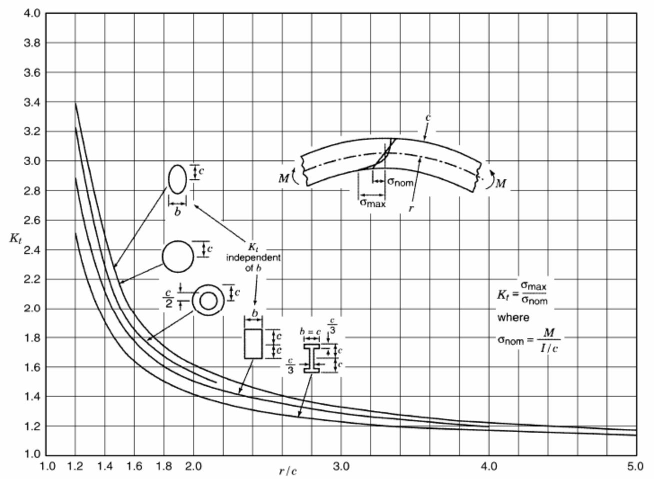


Figure 7: Stress concentration factors K_t for a curved beam in bending [7].

Stress concentration factors for curved beams

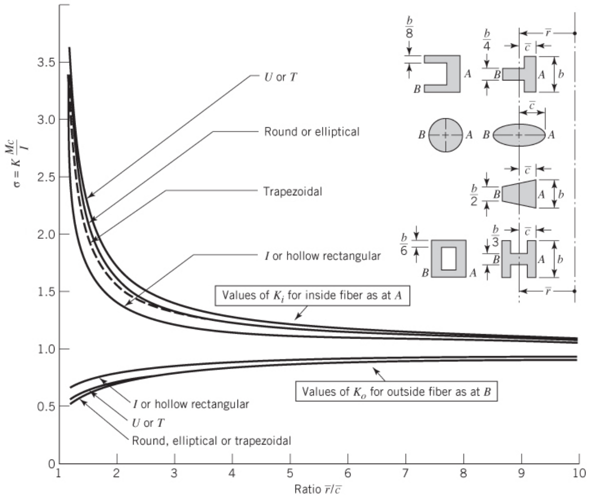


Figure 8: Effect of curvature on bending stresses: stress concentration factor for *inner* and *outer* fibers [2].

Lecture 2

Summary

Bending of curved beams due to forces

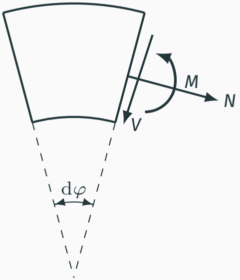


Figure 9: Bending due to forces [3].

Typically, the shear stress is not considered when the height is small compared with beam length.

The stresses due to the normal load on the cross section is given by:

$$\sigma = \frac{N}{A}$$

Recall that the bending stress for a curved beam is given by:

$$\sigma = \frac{M\left(y-e\right)}{Ae\left(r-y\right)}$$

Combining bending stresses with normal load stresses:

$$\sigma = \frac{M\left(y-e\right)}{Ae\left(r-y\right)} + \frac{N}{A}$$

A crane hook

The theory of curved beams described above is applied in designing *crane hooks*.

$$\sigma = \frac{M\,(y-e)}{Ae\,(r-y)} + \frac{F}{A}$$

For circular cross sections:

$$\sigma_{max} = \frac{F}{A}\frac{h}{2mr_1}$$

$$\sigma_{min} = -\frac{F}{A}\frac{h}{2mr_2}$$

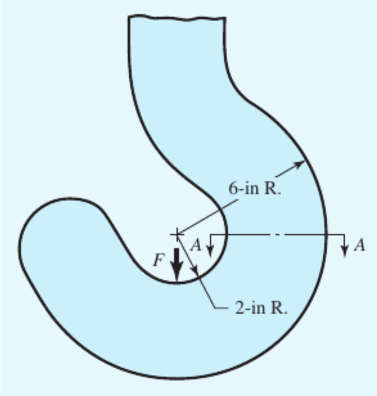
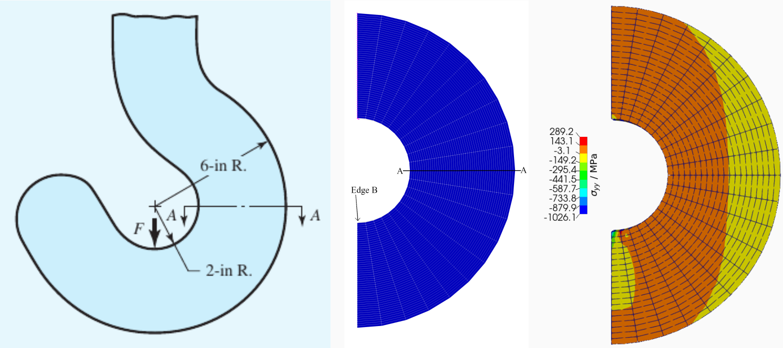


Figure 10: A crane hook [1].

Finite Element Method solution



(a) Crane hook geometry [1]. **(b)** FEM mesh. **(c)** Normal stress allong *yy* axis

Figure 11: FEM 3D calculation with CalculiX [8] for a crane hook (typically treated as a curved beam) - distributed load over the edge *B*.

Finite Element Method solution

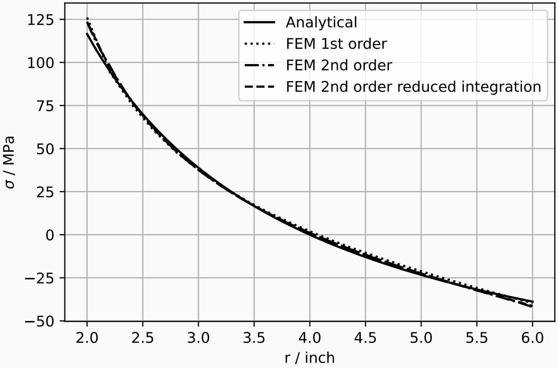


Figure 12: Normal stress along section $A-A$: comparison between analytical equation and FEM results.

Deflection of thin curved beams

The deflection of curved beams can be calculated with Castigliano's theorem.

Considering the simplest case in which the cross-sectional dimensions of the beam are small in comparison with the radius of its center line.

The strain energy of bending is given by the equation:

$$U = \int_0^s \frac{M^2}{2EI}\,\mathrm{d}s$$

The deflection of the point of application of any load P in the direction of the load is:

$$\delta = \frac{\partial U}{\partial P}$$

Example: deflection of thin curved beam

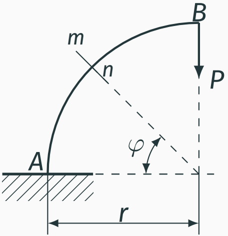


Figure 13: Curved beam example [3].

The bending moment at any cross section *mn* is:

$$M = -Pr\cos\varphi$$

$$U = \int_0^s \frac{M^2}{2EI}\,\mathrm{d}s = \int_0^{\frac{\pi}{2}} \frac{M^2}{2EI} \underbrace{r\,\mathrm{d}\varphi}_{\mathrm{d}s} = \int_0^{\frac{\pi}{2}} \frac{(-Pr\cos\varphi)^2}{2EI} r\,\mathrm{d}\varphi$$

Example: deflection of thin curved beam

$$U = \int_0^{\frac{\pi}{2}} \frac{(-Pr\cos\varphi)^2}{2EI} r \, \mathrm{d}\varphi$$

$$\delta = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \int_0^{\frac{\pi}{2}} \frac{(-Pr\cos\varphi)^2}{2EI} r \, \mathrm{d}\varphi$$

$$\delta = \frac{1}{EI} \int_0^{\frac{\pi}{2}} Pr^3 \cos^2\varphi \, \mathrm{d}\varphi = \frac{Pr^3}{EI} \int_0^{\frac{\pi}{2}} \frac{1+\cos 2\varphi}{2} \, \mathrm{d}\varphi$$

The vertical displacement of the end B is:

$$\delta = \frac{\pi}{4} \frac{Pr^3}{EI}$$

Example: deflection of thin curved beam

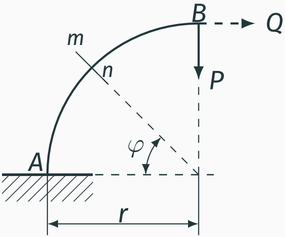


Figure 14: Curved beam example – fictitious horizontal load [3].

If required, the horizontal displacement of the end *B* can be determined using a fictitious load *Q*.

$$M = -\left[Pr\cos\varphi + Qr\left(1 - \sin\varphi\right)\right]$$

Example: deflection of thin curved beam

$$U = \int_0^{\frac{\pi}{2}} \frac{M^2 r}{2EI}\,\mathrm{d}\varphi$$

$$\delta = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\int_0^{\frac{\pi}{2}} \frac{M^2}{2EI} r\,\mathrm{d}\varphi = \frac{1}{EI}\int_0^{\frac{\pi}{2}} M\frac{\partial M}{\partial Q} r\,\mathrm{d}\varphi$$

Taking $Q = 0$ in M:

$$\delta = \frac{1}{EI}\int_0^{\frac{\pi}{2}} Pr^3 \cos\varphi\,(1 - \sin\varphi)\,\mathrm{d}\varphi$$

The horizontal displacement of end B is:

$$\delta = \frac{Pr^3}{2EI}$$

Example: Thin ring

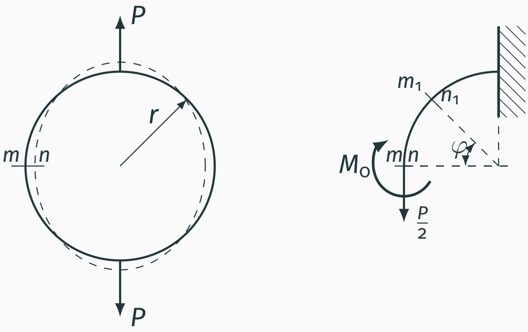


Figure 15: Thin ring [3].

Due to symmetry only one quadrant of the ring need be considered. There are no shear stresses over the cross section *mn* and the tensile force on this cross section is $\frac{P}{2}$.

Example: thin ring

The magnitude of the bending moment acting on this cross section is statically indeterminate and may be found by the Castigliano theorem. From the condition of symmetry it is known that section *mn* does not rotate during the bending of the ring.

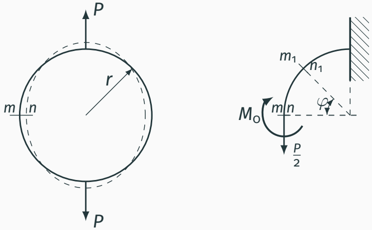


Figure 16: Thin ring [3].

$$\frac{\partial U}{\partial M_0} = 0$$

$$M = M_0 - \frac{P}{2} r \left(1 - \cos\varphi\right)$$

$$\frac{\mathrm{d}M}{\mathrm{d}M_0} = 1$$

$$0 = \frac{\mathrm{d}}{\mathrm{d}M_0} \int_0^{\frac{\pi}{2}} \frac{M^2 r}{2EI} \,\mathrm{d}\varphi = \frac{1}{EI} \int_0^{\frac{\pi}{2}} M \frac{\mathrm{d}M}{\mathrm{d}M_0} r \,\mathrm{d}\varphi$$

Example: Thin ring

$$0 = \frac{1}{EI} \int_0^{\frac{\pi}{2}} \left[M_0 - \frac{P}{2} r \left(1 - \cos\varphi\right) \right] r \, \mathrm{d}\varphi \Leftrightarrow M_0 = \frac{Pr}{2} \left(1 - \frac{2}{\pi}\right)$$

The equation for the bending moment for any cross section is:

$$M = M_0 - \frac{P}{2} r \left(1 - \cos\varphi\right) \quad \text{with} \quad M_0 = \frac{Pr}{2} \left(1 - \frac{2}{\pi}\right)$$

$$M = \frac{P}{2} r \left(\cos\varphi - \frac{2}{\pi}\right)$$

For $\frac{\pi}{2}$, point of application of force P:

$$M = \frac{Pr}{\pi} = -0.318Pr$$

Example: Thin ring – change in vertical diameter

The minus sign indicate that the bending moments at the points of application of the forces P tend to increase the curvature. The total strain energy stored in the ring:

$$U = 4\int_0^{\frac{\pi}{2}} \frac{M^2 r}{2EI}\,\mathrm{d}\varphi$$

The increase in the vertical diameter is:

$$\delta = \frac{\partial U}{\partial P} = \frac{4}{EI}\int_0^{\frac{\pi}{2}} M\frac{\mathrm{d}M}{\mathrm{d}P} r\,\mathrm{d}\varphi = \frac{Pr^3}{EI}\int_0^{\frac{\pi}{2}}\left(\cos\varphi - \frac{2}{\pi}\right)^2 \mathrm{d}\varphi$$

The increase in vertical diameter:

$$\delta = \left(\frac{\pi}{4} - \frac{2}{\pi}\right)\frac{Pr^3}{EI} = 0.149\frac{Pr^3}{EI}$$

Example: Thin ring – change in horizontal diameter

Applying two oppositely fictitious forces Q are applied at the ends of the horizontal diameter:

$$M = \frac{P}{2} r \left(\cos\varphi - \frac{2}{\pi} \right) - \frac{Q}{2} r \sin\varphi \quad \text{and} \quad \frac{\partial M}{\partial Q} = -\frac{r}{2} \sin\varphi$$

Taking $Q = 0$ in M:

$$\delta = \frac{4}{EI} \int_0^{\frac{\pi}{2}} M \frac{\partial M}{\partial Q} r \, \mathrm{d}\varphi = \frac{Pr^3}{EI} \int_0^{\frac{\pi}{2}} \left(-\cos\varphi \sin\varphi + \frac{2 \sin\varphi}{\pi} \right) \mathrm{d}\varphi$$

The decrease in horizontal diameter:

$$\delta = \left(\frac{2}{\pi} - \frac{1}{2} \right) \frac{Pr^3}{EI} = 0.137 \frac{Pr^3}{EI}$$

Bending of curved beams due to forces

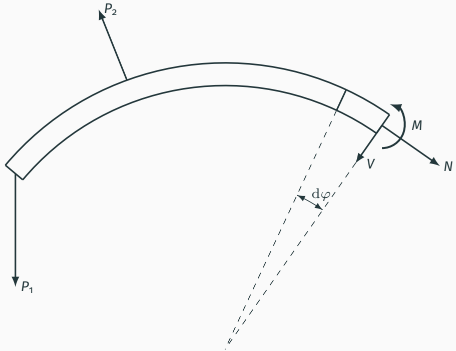


Figure 17: Bending due to forces acting in the plane of symmetry [3].

When the cross-sectional dimensions of a curved beam are not small in comparison with the radius of the center line, the shearing (V) and the longitudinal (N) forces must be taken into account.

Bending of curved beams due to forces

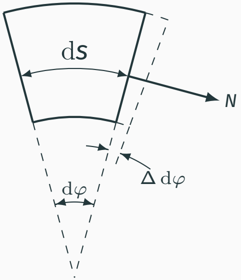


Figure 18: Bending due to forces [3].

The initial angle $\mathrm{d}\varphi$ will increase by the amount:

$$\Delta\,\mathrm{d}\varphi = \frac{N}{AE}\,\mathrm{d}\varphi = \frac{N\,\mathrm{d}s}{AEr}$$

The center line length $\mathrm{d}s$ will increase by:

$$\Delta\,\mathrm{d}s = \frac{N\,\mathrm{d}s}{AE}$$

The shear force V produces shear stresses and some warping of the cross section. It is assumed that the distribution of shear stresses along the cross section is the same as for straight beams. The relative radial displacement of two adjacent cross sections is $\frac{\alpha V}{AG}\,\mathrm{d}s$

Problems

Summary

Assignments

Assignments A1

Available on the Course Contents:

“Assignments” proposed for Complements of Machine Elements

Thin curved beam

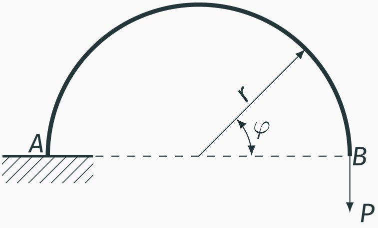


Figure 19: Thin curved beam [3].

Determine the vertical deflection of the end B of the thin curved beam of uniform cross section and semicircular center line.

Increase in distance

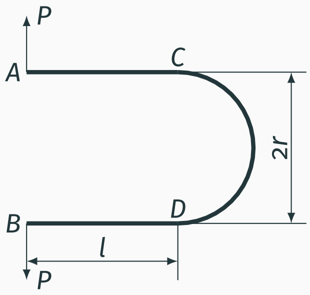


Figure 20: Increase in distance [3].

Determine the increase in distance between the ends A and B of a thin beam of uniform cross section consisting of a semicircular portion *CD* and two straight portions *AC* and *BD*.

Piston ring

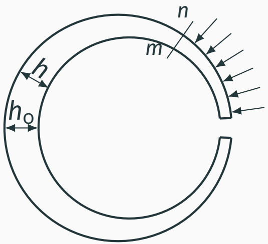


Figure 21: Piston ring [3].

A piston ring of a circular outer boundary has a rectangular cross section of constant width *b* and of a variable depth *h*. Determine the law of variation of *h* in order to obtain a ring which, when assembled with the piston in the cylinder, produces a uniformly distributed pressure on the cylinder wall.

Open S link

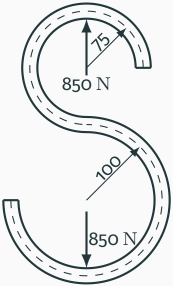


Figure 22: Open S link [4].

An open S link is made from a 25 mm diameter rod. Determine the maximum tensile stress and maximum shear stress.

Offset bar

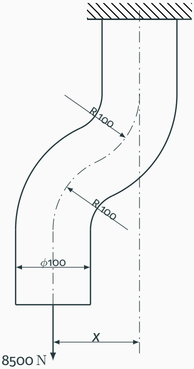


Figure 23: Curved beam geometry [4].

Consider the offset bar with circular cross section. The weight of the bar can be neglected. What is the maximum offset *x* if the allowable stress in tension is limited to 70 MPa.

References

References i

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