Estimations

EDP 613

Week 9

A Note About The Slides

Currently the equations do not show up properly in Firefox. Other browsers such as Chrome and Safari do work.

A Note About Probability

We're going to introduce some concepts from Chapter 8 here.

Time to Transition!

Descriptive Statistics - Mathematical techniques for organizing and summarizing a set of numerical data

to

Inferential Statistics - Generalizing from a sample to a population

> - **Statistic** - Mathematical expression that describes some aspects of a set of scores for a sample
> - **Parameter** - Describes some aspect of a set of scores for a population

First a Brief Intro to Hypothesis Testing

- Formally - Testing an assumption about a population parameter
- In Better Terms - An assumption about a particular situation of the world that is testable

The Null Hypothesis

> - Represented as H_0
> - is basically what you expect to happen before you run an experiment
> - *You have to know this!*

The Alternative Hypothesis

- Represented as H_1 (or H_A)
- is basically what else could happen if what you expect doesn't occur
- *You don't have to know this!*

Tests of Statistical Significance

- Formally - Done to determine whether H_0 (or H_A) can be rejected
- In Better Terms - Test to figure out whether you can reasonably say if your initial assumption won't happen
- Basically if the results of a study don't reject the null hypothesis, then you aren't finding anything new or surprising

Statistical Error Types

A **(statistical) estimation** is a sample statistic is used to estimate the value of an unknown population parameter.

Decision	**Null is True**	**Null is False**
Reject Null	Type I Error (aka False Positive)	Correct Outcome (aka True Positive)
Fail to Reject Null	Correct Outcome (aka True Negative)	Type II Error (aka False Negative)

Alpha

> - Formally
>   - rejecting H_0 when it is true
>   - the probability of making a **Type I error**
> - In Better Terms - the chance of making the wrong decision when what was initially expected to happen actually occurs.
> - Given by α
> - Ranges from 0-1 like all other probabilities

- Default $\alpha = 0.05$
- But its context dependent!

Example

- If you're **analyzing airplane engine failures for**, you may want to lower the probability of making a wrong decision and use a **smaller** α
- If you're **analyzing paper airplanes for failures**, you might be willing accept the higher risk of making the wrong decision and use a **larger** α

Beta

- Formally
  - not rejecting the H_0 when H_1 is true
  - the probability of making a **Type II error**
- In Better Terms - the chance of making the wrong decision when an alternative actually occurs.
- Given by β
- Ranges from 0-1 like all other probabilities

Power

- $1 - \beta$ is called **statistical power**
  - extremely important!
  - Formally - the probability of NOT making a Type II error
  - In Better Terms - the chance that you can separate if an outcome is a result of something occurring vs. pure luck!

Decision Making

Reality	Did Not Reject Null	Rejected Null
Null is True	Correct decision 1-α Level of Confidence	Type I Error α Level of Significance
Null is False	Type II Error β Underpower	Correct Decision 1-β Statistical Power!

Decision Making

Null = "Forecast says its NOT going to rain" Alternative = "Something else will happen"

Reality	**Did not reject the forecast**	**Rejected forecast**
Forecast was right	Did not take an umbrella and you're dry	Took an umbrella AND you're dry but may look silly (or fancy)
Forecast was wrong	Did not take an umbrella AND you're wet	Took an umbrella AND you're dry

- You could have gotten wet from snow, a flood, etc.
- Important: **The alternative hypothesis does not mean the opposite!**

Estimation

- **(Statistical) Estimation** - a sample statistic is used to estimate the value of an unknown population parameter
    - **Point estimation** - use of sample data to calculate a single value
    - **Interval estimation** - use of sample data to calculate a possible range of values

Process: We are selecting a sample mean (Process)

Classification	Hypothesis Testing	Rejected forecast
Process	Determine the probability of getting that mean if the Null is true	Estimate the value of a population mean
Outcomes	Gain information about the population mean	Gain information about the population mean

Updating Estimation for Sample Means

- **Point estimation** - use of sample data to calculate a single **mean** value
    - Benefit - the sample mean will equal the population mean on average
    - Drawback - unable to figure out if a sample mean actually equals the population mean
- **Interval estimation** - use of sample data to calculate a possible range of **mean** values

The Characteristic of Hypothesis Testing and Estimation

Question	Hypothesis Testing	Point/Interval Estimation
Do we know the population mean?	Yes its the Null hypothesis	No we're trying to estimate it
What is the process use dto determine?	The chance of obtaining a sample mean	The value of a population mean
What is learned?	Whether the population mean is likely correct	The range of values within which the population mean is probably contained
What is our decision?	To retain or reject the null hypothesis	No actual decison

Confidence

- **Confidence Interval** - an interval that contains an unknown parameter (e.g. μ) with certain degree of confidence
- **Level of Confidence** - probability or likelihood that an interval estimate will contain an unknown population parameter

Determining the Confidence Interval

1. Calculate the standard error of the mean

$$\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{N}}$$

2. Decide on a level of confidence

Probability	z-score
0.90	1.645
0.95	1.96
0.99	2.576

Typical to have a 95% level of confidence ($\alpha = 0.05$).

Determining the Confidence Interval (continued)

3. `$CI = \overline{Y} \pm z(\sigma_{\overline{Y}})$`

4. Interpret the results.

Example

IQ scores in the general healthy population are approximately normally distributed with $100 \pm 15 (\mu \pm \sigma)$. In a sample of 100 students a sample mean IQ of 103 was recorded ($\overline{Y} = 103$).

1. $$\sigma_{\overline{Y}} = \frac{15}{\sqrt{100}} = 1.50$$

2. Want to find 90% confidence interval, so choose a 90% level of confidence.

$$z(\sigma\overline{Y}) = 1.645 \cdot 1.50 = 2.47$$

1. $$90\% \, CI = 103 \pm 2.47 = (105.47, 100.53)$$

2. We are 90% confident that the overall mean IQ is between 100.53 and 105.47.

That's it. Take a break before our R session!