# Tragedy of Commons Explorable Explanation

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## Introduction

The goal of our explorable explanation was to explain the complex concept of the tragedy of commons. This is a theory originally proposed by William Lloyd which claims that when individuals act independently to serve their self-interest, it is always to the detriment of the common good because the shared resource is depleted or spoiled.

The tragedy of commons has many applications in environmental science, especially regarding sustainable development. These are increasingly relevant issues in today's socio political discourse; phenomena such as global warming and overfishing are threatening the future wellbeing of people all around the globe.

Let us expound upon one of the examples: global warming. The average temperature of the Earth is continuing to rise due to the increase in greenhouse gas emissions. This is because of the tragedy of commons; individuals are continuing to make decisions that serve their own selfish desires and needs (e.g., driving to work, using one-use plastic cutlery), which is to the detriment of the common good. Despite the overwhelming scientific consensus that human actions are exacerbating global warming, according to a Gallup poll held in 2017, around half of the American population are still not concerned about the consequences of global warming.[1] Instilling a proper understanding of tragedy of commons is crucial in shifting public perception about important environmental issues.

## Related Work

*The Evolution of Trust*

Many of the high-level design decisions that we made (particularly, the three main sections that we outlined in the first paragraph of the **Methods** section of this paper) were inspired by The Evolution of Trust, an interactive guide to game theory by Nicky Case. That related work did a great job at motivating the understanding of game theory through the use of an engaging interactive activity, giving a textual explanation to articulate the concept demonstrated in the earlier game, and then concluding by providing the player a more detailed analysis of the math going on behind each decision so that the player could gain a more nuanced and complex understanding of game theory.

*Tragedy of Commons Lesson Plan*

As shown in this lesson plan, the tragedy of commons is often explained in terms of an overfishing example. We have incorporated that into our first game.

[1] 2017 Gallup Poll

*Tragedy of Commons Formal Model by UC Berkeley*
We have gone through the formal model and mathematical equations for the tragedy of commons and used them to formalize the algorithm and logic for our interactive activities in the project.

## Methods

On the tech side, we used a combination of ReactJS, d3, and jQuery to develop this app.

The final explorable explanation consists of three main sections:

1. An interactive game with two rounds that allows the player to experience the phenomenon of the tragedy of commons.
2. A detailed explanation of tragedy of commons and the math behind it.
3. An opportunity to see how the utility score for each player changes as he/she harvests more fish.

This **Methods** section will give a detailed explanation of the algorithms and logic behind section 1 (the game) and section 3 (the calculation of the utility scores).

*Section 1 - The Logic for the Fishing Game*
The game starts off by outlining some basic rules about the consequences of harvesting a certain number of fish:

- If you fish one fish, you and your family die of starvation.
- If you fish two fish, you have enough food to feed your family.
- If you fish three fish, you have enough to eat and you can sell the extra fish for a profit.
- If you fish four fish, you can move your family into a larger house, and upgrade your fishing equipment.

After making the first round, if the player chose to only harvest 1 fish, he/she is informed that not only has their family starved, but despite not fishing much, other fisherman came along and overfished the lake anyway. This is meant to convey the futility of a single individual's actions in the face of the tragedy of commons - if everybody else acts for their own selfish good, it will still lead to collective detriment, even if there are a handful of people who make environmentally sustainable actions.

If the player decides to fish 2-4 fish, he/she moves onto the second round. The fish reproduce, so the number of fish doubles in the second round. The player is informed that more fishermen have come to town. No matter how many fish the player decides to harvest, it always ends in a bad ending - they either starve, or the other fisherman fish too much and the pond always ends up being overfished and unable to sustain its ecosystem.

*Section 2 - Calculation of the utility scores*
Our simple fishing game (section 1) served to illustrate Tragedy of the Commons. The increased fishermen in the town results in increased fishing. The lake loses the ability to replenish itself, and the fish supply disappears, and families lose their source of livelihood. This tragedy arises from the conflict among individuals with selfish interests sharing a scarce resource. Here, multiple parties are sharing a common good, so their own decisions affect the group as a whole.

In our fishing game example, each fisherman is a rational player so wants to maximize his own utility, so will fish more whenever possible. (Even if you weren't completely selfish in your fishing decisions, the other fishermen were). However, each additional fish fished degrades the quality of Common Lake, and the reduces the number of fish available in the next fishing season. In a selfish situation, all fishermen would seek to maximize his own harvest of fish.

Let us formalize this model now to demonstrate the positive and negative effects of increasing an extra fish. Let $n$ be the number of fisherman, each able to choose the amount of fish they can fish. Let $K$ be the total amount of fish available. Each player $i = 1, \ldots, n$ chooses the amount of fish to to fish. Let $k_i$ denote the amount fished by the $i$-th player. Then the amount of fish left in the pool, after all the fishermen have fished is:

$$K - \sum_{i=1}^{n} k_i.$$

For simplicity, let the utility functions be logarithms. Player $i$ incurs a benefit equal to $ln(k_i)$ for fishing $k_i$ fish. Each player also enjoys an added benefit, from the remaining fish, sustaining the ecosystem. This gives us an additional benefit of:

$$\ln\left(K - \sum_{i=1}^{n} k_i\right).$$

Then, the value for each player for player $i$ from fishing $k_i$ fish contingent on the actions of the other players is given by the utility function:

$$v_i(k_i, k_{-i}) = \ln(k_i) + \ln\left(K - \sum_{j=1}^{n} k_j\right).$$

where $k_{-i}$ are the actions of all the other players.

This demonstrates both the positive and negative effect of fishing one more fish. The fisherman gains a positive utility of gaining one more fish, which incurs a negative effect of that commons from not having that extra benefit based the remaining fish left. Since this effect is based on the actions of all the fishermen, the negative effects of overfishing are also shared among the fishermen. Here, the social and private incentives differ. To avoid the tragic destruction of the fish ecosystem, all the fishermen must cooperate and reduce their collect fishing. However, a single fishermen has limited incentive to reduce their own fishing, because each fishermen's harvest is only part of the problem. Each fishermen utilizes the common resource, but by over utilizing such a resource, he diminishes the utility that the other fishermen can get.

The question now arises: is there is a solution to avoid this tragedy?

To find this solution, we must first define the idea of the Nash equilibrium. This is the solution where an individual maximizes their personal good (value function) for all actions of the other players. The Nash equilibrium is the solution that is optimal for all players, where each player is doing the best he possibly can given the choices of the other players. Then, no player has an incentive to deviate from his choose strategy given the strategy of the other players.

1. Player 1 finds the choices that maximizes his value function given every possible action combination of the other n-1 players. This is the maximum point of the value function of player 1 and will be a function of the actions of the other parties.
2. Repeat step 1 for all N players.
3. There are n equations here with n unknowns; the intersection point is the Nash equilibrium.

Following this iterative procedure, we can come up with a process to find a Nash equilibrium to tack the Tragedy of the Commons.

To find the maximum point, we can take the first order derivative of the maximum function. This point is

$$k_i = \frac{K - \sum_{j \neq i} k_j}{2}$$

For n players, we can construct the linear system

$$\begin{bmatrix} 2 & 1 & \ldots & 1 \\ 1 & 2 & \ldots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \ldots & 2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix} = \begin{bmatrix} K \\ K \\ \vdots \\ K \end{bmatrix}$$

By solving the linear system, we get that the solution is

$$\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix} = \begin{bmatrix} \frac{K}{n+1} \\ \frac{K}{n+1} \\ \vdots \\ \frac{K}{n+1} \end{bmatrix}$$

This means that the optimal amount for the player to fish is the total number of fish available divided by the number of players plus one. The plus one comes from the indirect impact of a player's consumption.

The total amount of fish consumed is $\frac{n}{n+1}K$ .
The payout or utility gained for each individual player for fishing K/(n+1) is:

$$\log\left(\frac{K}{n+1}\right)+\log\left(K-\frac{n}{n+1}K\right)$$

However, the Nash equilibrium does not taken into account player cooperation. Each player still acts selfishly. What happens if players cooperate? Can we find a socially optimal equilibrium? This is known as the Pareto optimal. This is outcome where any deviation (to acquire extra benefit) from the strategy will result in a loss for someone else.

The social welfare function w is the sum of all the value functions of the players.

$$\begin{aligned} w(v_1,\ldots,v_n) &= \sum_{i=1}^{n} v_i \\ &= \sum_{i=1}^{n} \ln(k_i) + n\ln\left(K-\sum_{i=1}^{n} k_i\right). \end{aligned}$$

Taking the first-order conditions for the problem to maximize this function we get

$$\frac{\partial w(k_1,\ldots,k_n)}{\partial k_i} = \frac{1}{k_i} - \frac{n}{K-\sum_{j=1}^{n} k_j} = 0$$

for $i = 1, \ldots, n$.

$$\frac{1}{k^{PO}} - \frac{n}{K-nk^{PO}} = 0$$

or

$$k^{PO} = \frac{K}{2n}$$

Then the Pareto optimal consumption of fish overall is equal to K/2 for *n* fishermen with each individual player fishing K/(2n) fish.

The payout or utility gained for each individual player from fishing K/(2n) fish is:

$$\log\left(\frac{K}{2}n\right)+\log\left(\frac{K}{2}\right)$$

## Results

The algorithms and logic described in the **Methods** section resulted in the following:

1. A game that demonstrates the consequences of the tragedy of commons through an overfishing scenario.
2. An interactive activity where users can see the calculations of the utility scores for every fishing decision that is made.

*The Game*

Using the logic that was described in detail in the **Methods** section, we have produced a game where any combination of decisions leads to a bad ending. No matter what the player does, all of the other fisherman acting selfishly will result in the pond being overfished and consequently the ecosystem being ravaged. This was to ensure that the player enters the explanation for the tragedy of commons after experiencing the detrimental consequences of that phenomenon in action; it doesn't do much to motivate an understanding of the tragedy of commons if the player ends up getting a good ending in the initial game.

*The Interactive Activity*

Using the complex equations described in the **Methods** section, the user can now simply press "Harvest fish for fisherman 1" or "Harvest fish for fisherman 2" and see the resultant utility score as one harvests more and more fish. This quantifies the consequences of the tragedy of commons without necessitating the player to have a thorough understanding of all the steps taken to calculate that score. The quantification of the tragedy of commons also adds more credibility to the analysis, and helps ground the abstract concept into specific examples.

## Discussion

Our most challenging problem was **determining the ending logic for the game**. At first, we thought of implementing a mix of good, bad, and neutral endings that the player could reach after the two rounds of fishing. However, we realized that this would make it difficult to construct our narrative; we want to player to always reach a bad ending, so that they recognize the futility of their decisions in the face of the mass of other individuals who are acting for their own selfish good. That would then demonstrate how the tragedy of commons plays out in an actual example. If the player decides to harvest just enough fish for their family for both rounds (2 fish) and then gets a good ending, then the viewer is no longer motivated to understand the tragedy of commons.

## Future Work

There are many possible directions that this explorable explanation could take from here:

- Implement an interaction with a more complex example of the tragedy of commons. The fishing example is a great way to introduce somebody to the concept, but there is ample opportunity to gain a more nuanced, complex understanding of the tragedy of commons.
- Instead of providing a theoretical example, give a couple of real-life scenarios in which the tragedy of commons is in action. This will help motivate the relevance of tragedy of commons.
- Provide more interactive and/or visual components to the textual explanation of the tragedy of commons