The Longest Tennis Match in History

Probability and Statistics for Data Science

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Tennis

Match divided into sets, divided into games, divided into points

- First player to win at least 4 points and lead by 2, wins game
- First player to win at least 6 games and lead by 2, wins set
- Exception: Tie-break (more on this later)
- In Grand Slams (e.g. Wimbledon), first player to win 3 sets wins the match
- Wimbledon 2023 final: 1-6, 7-6, 6-1, 3-6, 6-4

Longest matches before 2010

Smith/van Dillen vs Cornejo/Fillol: 7-9, 37-39, 8-6, 6-1, 6-3

Gonzales vs Pasarell: 22-24, 1-6, 16-14, 6-3, 11-9

Lara/Loyo-Mayo vs Santana/Garcia: 10-12, 24-22, 11-9, 3-6, 6-2

Drysdale/Moore vs Emerson/Barnes: 29-31, 8-6, 3-6, 8-6, 6-2

Wimbledon 2010 First Round

$70 + 68 = 138$ games! (8 hours 11 minutes!)

Why did this happen?

Why had it never happened before? (previous longest: 76 games)

Player who starts the point (serves) has an advantage over player who returns

Image credit: y.caradec (Wikipedia)

Our match

Out of the 183 games, serving player won 180

Why did this happen?

Because both players:

- Served very well
- Returned very badly

As a result, they *took turns* winning their serves for many games

How can we make this precise?

Estimate

$$\mathrm{P}\,(\text{138 or more games in a set})$$

under reasonable assumptions

Probability quantifies how often such a long set should happen

Challenge:

How to estimate this probability

Strategy: First estimate probability of winning a point

Point win %

			Our match	
	Serve	Return	Serve	Return
Djokovic	68	42		
de Minaur	64	40		
Rune	65	37		
Munar	61	39		
Isner	**72**	**30**	76.2	21.3
Mahut	**64**	**34**	78.7	23.8

Source: atptour.com

Challenge

We have estimated

$$\mathrm{P}\,(\text{Isner wins serve point}) = 0.762$$

$$\mathrm{P}\,(\text{Mahut wins serve point}) = 0.787$$

How do we compute

$$\mathrm{P}\,(\text{138 or more games in a set})\ ?$$

Let's try

Assumptions: Points are independent and identically distributed

$$\mathrm{P}\left(\text{138 or more games}\right) = \mathrm{P}\left(\text{138 games}\right) + \mathrm{P}\left(\text{139 games}\right) + \cdots$$

$$\begin{aligned}
&\mathrm{P}\left(\text{138 games}\right) \\
&= \mathrm{P}\left(\text{Isner wins 1st game}\right)\mathrm{P}\left(\text{Mahut wins 2nd game}\right)\cdots\mathrm{P}\left(\text{Mahut wins 138th game}\right) \\
&+ \mathrm{P}\left(\text{Isner wins 1st game}\right)\mathrm{P}\left(\text{Isner wins 2nd game}\right)\cdots\mathrm{P}\left(\text{Mahut wins 138th game}\right) \\
&+ \cdots \\
&+ \mathrm{P}\left(\text{Mahut wins 1st game}\right)\mathrm{P}\left(\text{Isner wins 2nd game}\right)\cdots\mathrm{P}\left(\text{Isner wins 138th game}\right)
\end{aligned}$$

$$\begin{aligned}
&\mathrm{P}\left(\text{Isner wins 1st game}\right) \\
&= \mathrm{P}\left(\text{Isner wins 1st point}\right)\mathrm{P}\left(\text{Isner wins 2nd point}\right)\cdots\mathrm{P}\left(\text{Isner wins 4th point}\right) \\
&+ \mathrm{P}\left(\text{Isner wins 1st point}\right)\mathrm{P}\left(\text{Mahut wins 2nd point}\right)\cdots\mathrm{P}\left(\text{Isner wins 5th point}\right) \\
&+ \cdots
\end{aligned}$$

Not impossible, but painful...

However, simulating is easy!

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```
def simulate_game(win_prob_serve):
    points_1 = 0
    points_2 = 0
    while (points_1 < 4 and points_2 < 4)
              or np.abs(points_1 - points_2) < 2:
        if rng.random() < win_prob_serve:
            points_1 += 1
        else:
            points_2 += 1
   if points_1 > points_2:
        # Player who serves wins
        outcome = 1
    else:
        # Player who receives wins
        outcome = 0
    return outcome
```

Simulating is easy!

```
def simulate_set(win_prob_serve_1,win_prob_serve_2):
  games_1 = 0
  games_2 = 0
  who_serves = 1
  if rng.random() < 0.5:
    who_serves = 2
  while (games_1 < 6 and games_2 < 6)
            or np.abs(games_1 - games_2) < 2:
    if who_serves == 1:
      if simulate_game(win_prob_serve_1)==1:
        games_1 += 1
      else:
        games_2 += 1
      who_serves = 2
```

Simulating is easy!

```
    else:
      if simulate_game(win_prob_serve_2)==1:
        games_2 += 1
      else:
        games_1 += 1
      who_serves = 1
  return [games_1,games_2]
```

Intuitive definition of probability

$$\mathrm{P}(\text{event}) = \frac{\text{number of times event occurs}}{\text{total repetitions}}$$

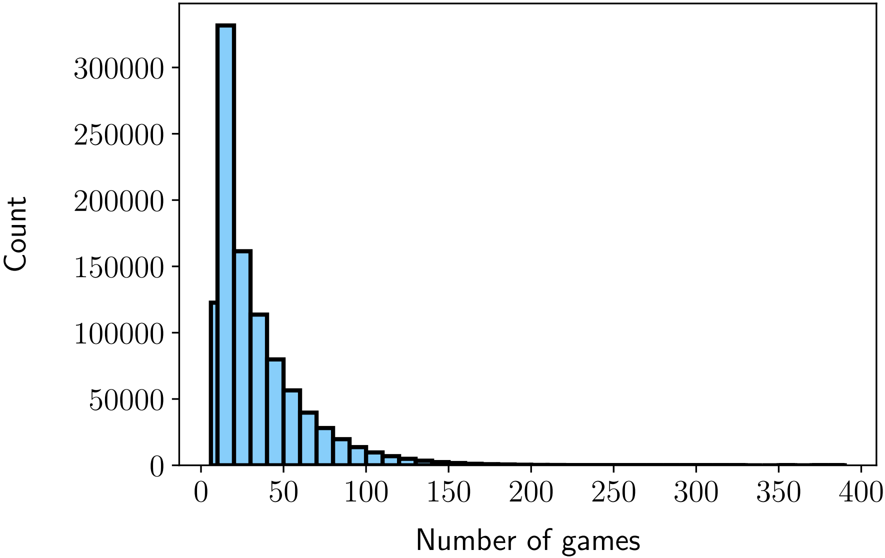
Monte Carlo method

1. Generate m simulations
2. Compute the fraction of simulation where event happens,

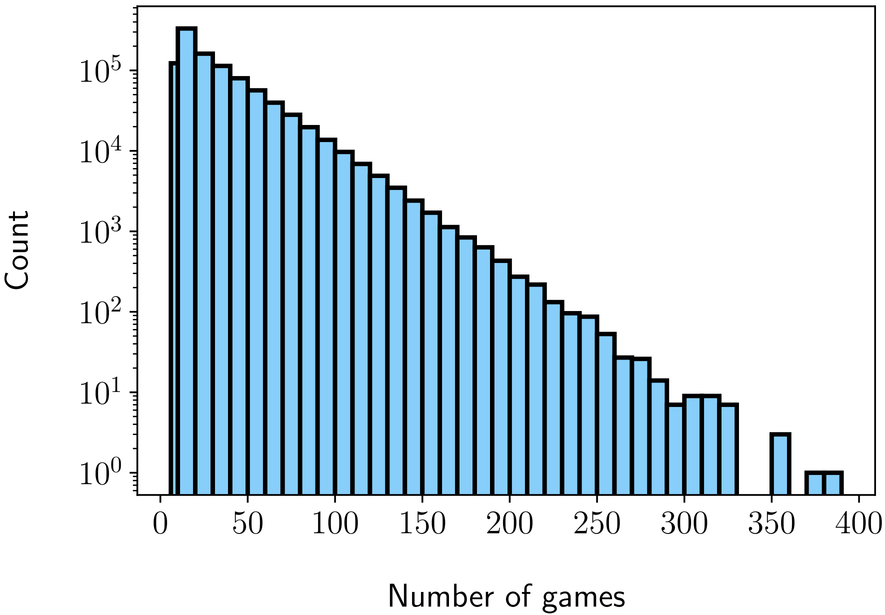
$$\mathrm{P}_{\mathsf{MC}}\,(\text{event}) := \frac{\text{number of times event occurs}}{m}$$

Converges to true probability as $m \to \infty$ by law of large numbers

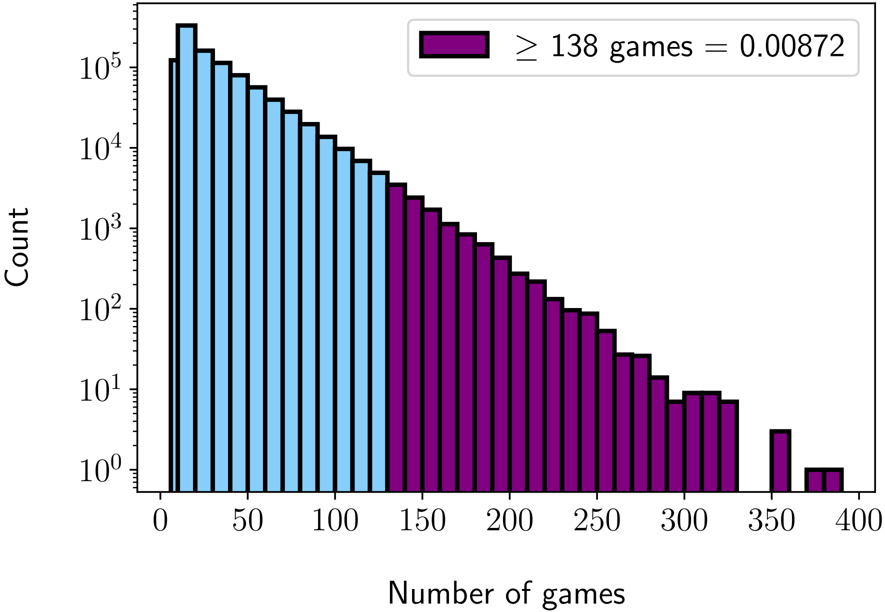
One million simulations



One million simulations



$\mathrm{P}\,(\text{138 or more games}) = 0.872\%$



Side note: Allows to predict match outcome!

Based on the one million simulations

$$\mathrm{P}\left(\text{Isner wins match}\right) = 0.387$$

Challenge:

Requires estimating point win % before the match

Estimated serve-point win %

Do we trust these estimates?

$$\mathrm{P}\left(\text{Isner wins serve point}\right) = 0.762$$

$$\mathrm{P}\left(\text{Mahut wins serve point}\right) = 0.787$$

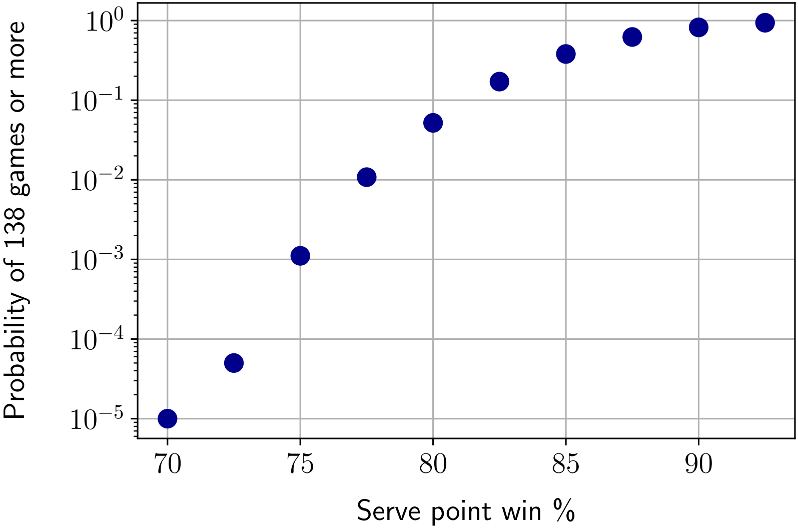
Depends on sample size

We can use 0.95 confidence intervals to quantify uncertainty

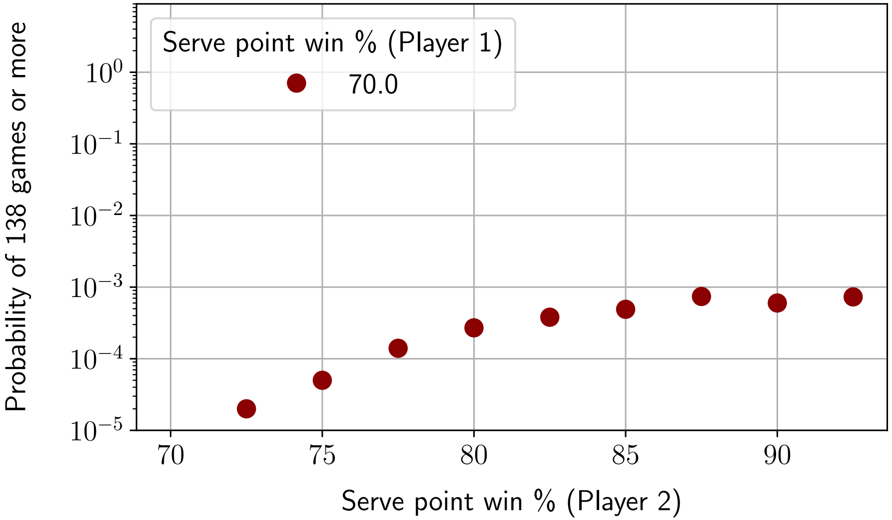
Assuming points are independent

- Isner: [71.7, 80.6] ($n = 491$)
- Mahut: [74.3, 83.2] ($n = 489$)

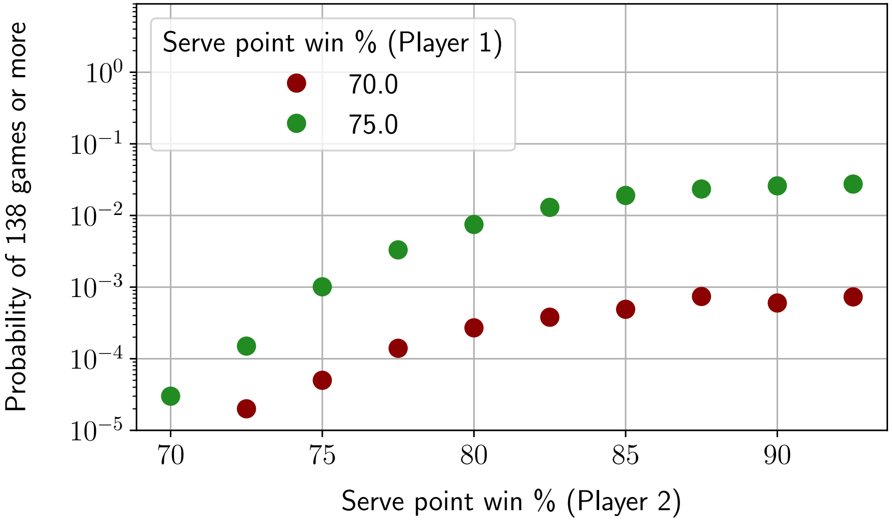
If both players have same serve-point win %



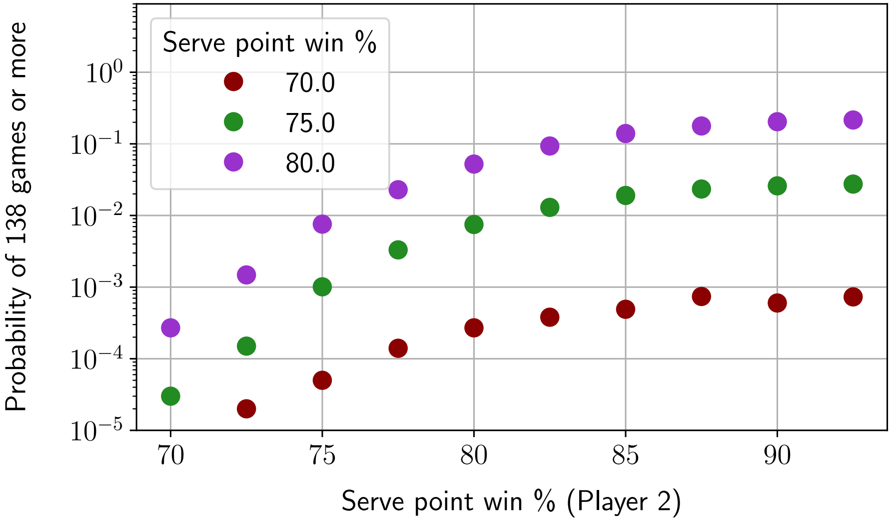
Different serve-point win %



Different serve-point win %



Different serve-point win %



Why did this happen only once?

Strategy:

Determine how many similar match-ups there were before 2010

Compute probability of ≥ 138 games happening exactly once

Similar match-ups

We identify strong servers / weak returners (*servebots*)

According to ChatGPT:

John Newcombe (1967-1979), Stan Smith (1968-1978), Clark Graebner (1965-1975), Roscoe Tanner (1972-1985), Kevin Curren (1980-1993), Slobodan Zivojinovic (1981-1997), Goran Ivanisevic (1990-2004), Richard Krajicek (1989-2003), Greg Rusedski (1991-2007), Marc Rosset (1988-2005), Mark Philippoussis (1994-2008), Wayne Arthurs (1990-2007), Joachim Johansson (2000-2009), Ivo Karlovic (2000-2022), John Isner (2007-2023), Mario Ancic (2001-2011), Sam Querrey (2006-2022), Taylor Dent (1998-2010), Nicolas Mahut (2000-2023)

Tie breaks

- Played when set score is 6 - 6
- Decisive game that ends the set
- Widespread since 1970s, except for final deciding set in some cases (e.g. Wimbledon)

Important reason why there haven't been more very long sets

How many sets without tie-breaks between

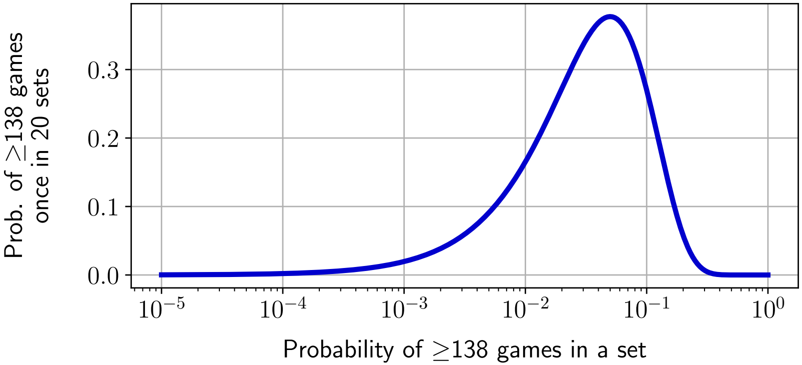
John Newcombe (1967-1979), Stan Smith (1968-1978), Clark Graebner (1965-1975), Roscoe Tanner (1972-1985), Kevin Curren (1980-1993), Slobodan Zivojinovic (1981-1997), Goran Ivanisevic (1990-2004), Richard Krajicek (1989-2003), Greg Rusedski (1991-2007), Marc Rosset (1988-2005), Mark Philippoussis (1994-2008), Wayne Arthurs (1990-2007), Joachim Johansson (2000-2009), Ivo Karlovic (2000-2022), John Isner (2007-2023), Mario Ancic (2001-2011), Sam Querrey (2006-2022), Taylor Dent (1998-2010), Nicolas Mahut (2000-2023)

According to ChatGPT: Only 20

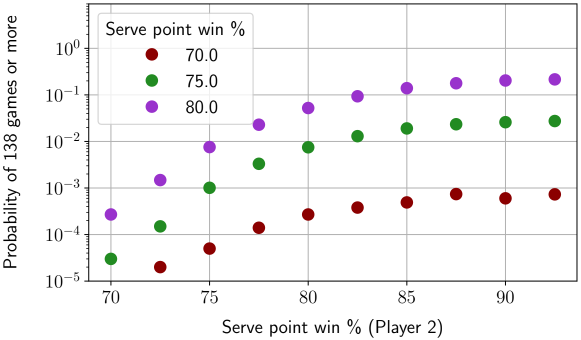
$\mathrm{P}\,(\geq 138 \text{ games exactly once})$

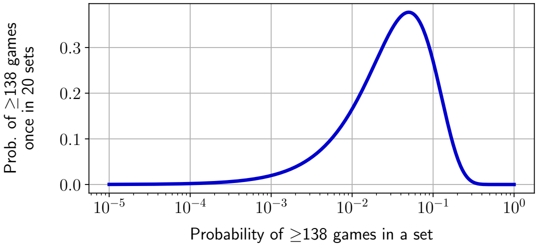
If $\mathrm{P}\,(138 \text{ or more games}) := \theta$

$$\mathrm{P}\,(\geq 138 \text{ games once in 20 sets}) = 20\,\theta\,(1-\theta)^{19}$$



Isner: [71.7, 80.6] Mahut: [74.3, 83.2]





Conclusion

A match with 138 (or more) games occurs with *non-negligible probability* if both players are strong servers / weak returners

Consequently, given number of sets without tie-breaks between such players observing *just one* such game is plausible

Under the assumption that points are independent and identically distributed (e.g. ignoring fatigue)