StanCon 2017

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StanCon 2017

- First annual conference for the Stan Probabilistic Programming Language
- Talks highlighted new developments, case-studies, and future directions
- Got to meet prominent applied bayesian statisticians

What is bayesian statistics

- A field of statistics devoted to assigning probabilities to parameters, and updating in response to new data
- Contrast with null hypothesis testing which assigns probabilities to data
- A logical consequence of two important concepts:
    1. The subjective interpretation of probability
    2. Bayes' Theorem

The subjective interpretation of probability

- In frequentist interpretation, probabilities represent the long term chances of an event occuring
- In the subjective interpration, probabilities represent degrees of belief about the chance of an event

Bayes Theorem

$$P(\theta|D) = \frac{\boxed{P(D|\theta)} * P(\theta)}{P(D)}$$

1. $P(D|\theta)$: The likelihood of the data given the parameters

Bayes Theorem

$$P(\theta|D) = \frac{P(D|\theta) * \boxed{P(\theta)}}{P(D)}$$

1. $P(D|\theta)$: The likelihood of the data given given the parameters
2. $P(\theta)$: The prior probability of the parameters

Bayes Theorem

$$P(\theta|D) = \frac{P(D|\theta) * P(\theta)}{\boxed{P(D)}}$$

1. $P(D|\theta)$: The likelihood of the data given given the parameters
2. $P(\theta)$: The prior probability of the parameters
3. $P(D)$: The probability of the data (normalizing constant)

Bayes Theorem

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1. $P(D|\theta)$: The likelihood of the data given given the parameters
2. $P(\theta)$: The prior probability of the parameters
3. $P(D)$: The probability of the data (normalizing constant)
4. $P(\theta|D)$: Posterior probability of the parameters

The Posterior

- The posterior is the goal of bayesian inference
- The posterior summarize our beliefs about the state of the system
- How do we get it?

Conjugate Bayesian Analysis

- If we carefully choose our models carefully the posterior can be computed directly from the prior and the data.

~~Conjugate Bayesian Analysis~~

- If we carefully choose our models carefully the posterior can be computed directly from the prior and the data.
- Only works for a small class of models
- We need something more general

General Solution

- Approximate
- Sample from the unnormalized posterior, estimate the normalizing constant
- Sampling is unbiased in ideal case, but can be troublesome in practice

What is bayesian statistics

bayesian statistics is the largest field devoted to ignoring a denominator
-unknown

History

- Metropolis-Hastings: During the Manhattan project early computers were used to sample from arbitrary distributions. The original algorithm is still in use today
- Gibbs-Sampling: A successor to Metropolis-Hastings became democratized in the form of **B**ayesian Inference **U**sing **G**ibs **S**ampling (BUGS), a software package that allowed users with personal computers to sample posteriors.
- Hamilitonian-Monte-Carlo: A tool from physics imported by U of T's own Radford Neal sped up sampling enough to be practical. Made available by the Stan programming language.

Stan

Stan

Stan Provides:

1. Model fitting
2. Posterior Sampling
3. Automatic differentiation
4. Variational Inference
5. Wrappers for R and Python

Conference Highlights

1. **Andrew Gelman's introduction and post-mortem of a flawed election model**

Key Points

- Good data is better than good methods
- Highlighted problem of using summarized and convenience data
- Discussed the need for higher level languages for writing models
- Emphasized three main objects of statistics
  1. Generalize from sample to population
  2. Generalize from controls to treatment
  3. Generalize from measurement to construct
- Stressed unbiased estimation is less important than typically taught in stats education. Publication biases and forking paths in modelling mean all estimates are biased.

Conference Highlights

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2. **Charles Margosian's introduction to ODEs with Stan**

Key Points

- Ordinary differential equation models are useful when you can only measure rates of change of your quantities of interest
- Frequently used in pharmacometric models, measuring movement of compounds between body compounds
- Stan now includes support for linear and non-linear bayesian estimation of ODEs

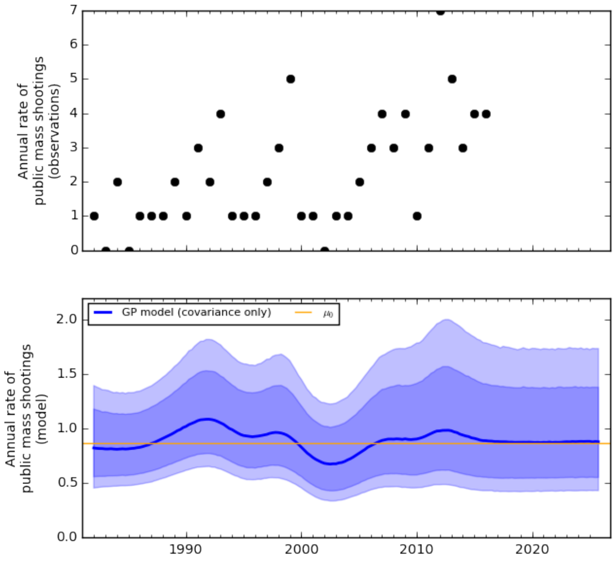
Conference Highlights

1. Andrew Gelman's introduction and post-mortem of a flawed election model
2. Charles Margosian's introduction to ODEs with Stan
3. **Rob Trangucci and Nathan Sanders used gaussian process regression for modelling time series data**

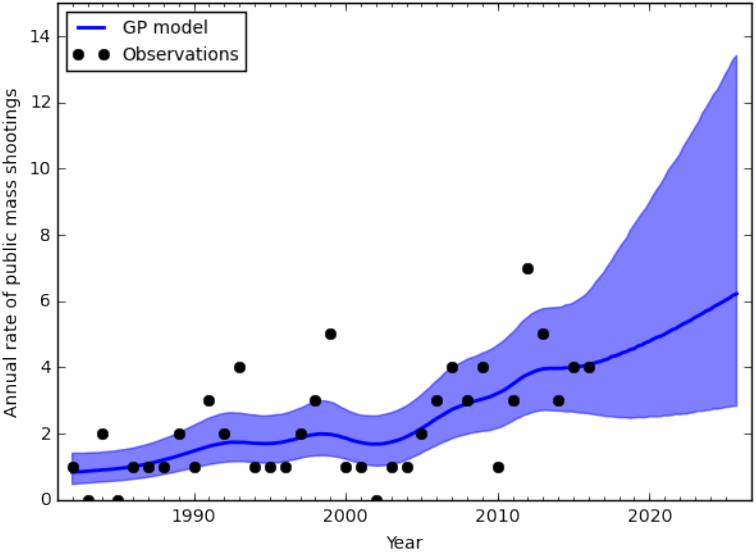
Key Points

- Gaussian processes describe a distribution of functions, covered by a mean and covariance function.
- Rob Trangucci introduced the new gaussian process functions in Stan
- Nathan Sanders demonstrated using them to fit mass-shooting data to assess the chance that the tend of mass shootings is increasing
- Modelled as a poisson regression, allowing the residuals to be modelled by a gaussian process
- Allowing the length scale of the gaussian process to vary meant the model could range from uncorrelated residuals (standard poisson) to fully modelled by the covariance

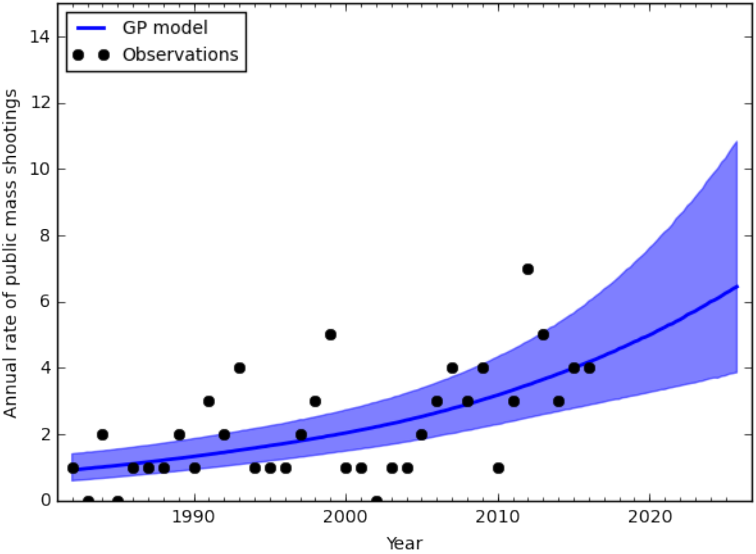
Raw Data and Covariance Only



Short Length Scale



Long Length Scale



Conference Highlights

1. Andrew Gelman's introduction and post-mortem of a flawed election model
2. Charles Margosian's introduction to ODEs with Stan
3. Rob Trangucci and Nathan Sanders used gaussian process regression for modelling time series data
4. **Michael Betancourt's introduction to Hamiltonian Monte Carlo**

Introduction to Hamiltonian Monte Carlo

- Statistics comes down to estimating expectations from probability distributions

$$\mathbb{E}|f(x)| = \int f(x)P(x)dx$$

- To compute expectations quickly you need to focus on regions of parameter space where $f(x)P(x)$ is high

Estimating Expectations

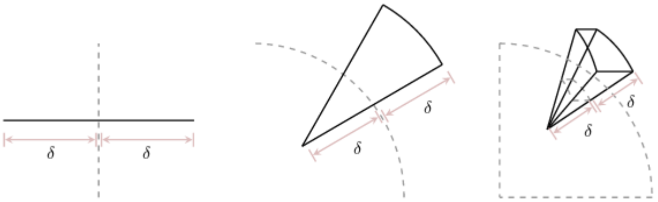
$$\mathbb{E}|f(x)| = \int f(x)P(x)dx$$

- In most cases the exact form of $f(x)$ is difficult to capitalize on
- So you decide to focus on where $P(x)$ is high... Right?

High Dimensional Geometry

- Unfortunately no!
- In higher dimensions the volume occupied by the mode is small
- Shrinks exponentially as number of dimensions increases

High Dimensional Geometry



(a) (b) (c)

High Dimensional Geometry

$$\mathbb{E}|f(x)| = \int f(x)P(x)dx$$

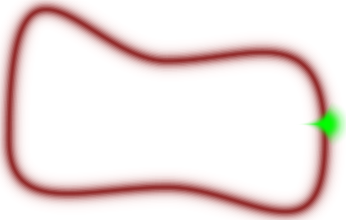
- In higher dimensions the volume occupied by the mode is small
- Shrinks exponentially as number of dimensions increases
- Integration volume dx comes to dominate the integral

Typical Set

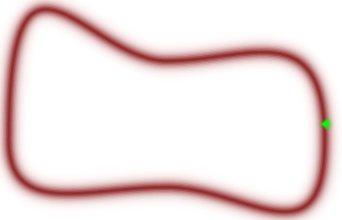
$$\mathbb{E}|f(x)| = \int f(x) \boxed{P(x)dx}$$

- As the dimensions increase, the region where dx is large and $P(x)$ isn't negligible contracts to thin shell.
- This shell is called the typical set
- Markov Chain Monte Carlo aims to explore this set

Typical Set



(a)

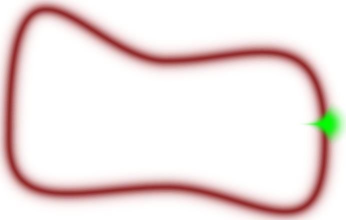


(b)

Metropolis-Hastings

- Metropolis-Hastings aims to explore the typical set by proposing new positions x adding gaussian noise to the current position
- If the proposed new location is has a higher $P(x)$ the markov chain moves toward it, if it's low it moves toward it with probability proportional to $P(x)$.
- Since the typical set is narrow, proposed locations are almost always going to fall into zones with very low $P(x)$
- Solution is to reduce step size, but this means the typical set gets explored very slowly.

Metropolis-Hastings



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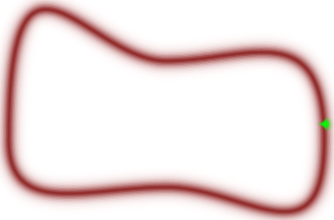
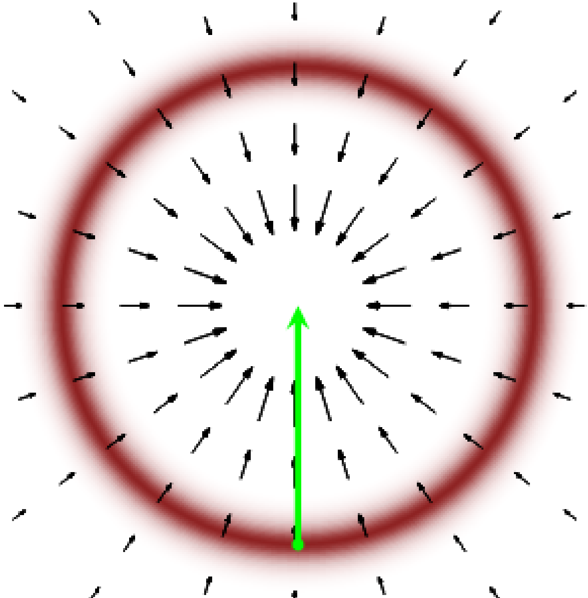


(b)

Hamiltonian Monte Carlo

- Use information about the shape of the distribution to better explore the typical set
- Imagine the the markov chain as a particle orbiting areas of high probability mass
- The gradient of the probability function points toward the mode (gravity)

Hamiltonian Monte Carlo



Hamiltonian Monte Carlo

- Given a certain amount of energy the markov chain can orbit stably
- The amount of energy determines where in the probability mass the chain orbits
- Drawing energies from the correct distribution allows you to explore the typical set.
- Big improvements to come with better choices of energy distribution

Take-Aways

- Bayesian statistics is getting more practical every day
- Hybrid parametric non-parametric techniques could be highly beneficial for modelling a variety of phenomena
- Many tools to take the pain out of model fitting exist (rstanarm, rethinking), so don't be afraid to try them out.
- Never underestimate a good intuitive explanation

More Information

Talks: `http://mc-stan.org/events/stancon`

Notebooks with code:

`https://github.com/stan-dev/stancon_talks`

Betancourt's Conceptual Introduction to HMC:

`https://arxiv.org/pdf/1701.02434.pdf`

Questions?