hwk4-skeleton-code

March 7, 2019

```
In [3]: import numpy as np
        import matplotlib.pyplot as plt
        import sklearn
        import scipy.spatial
        import functools
        from scipy.spatial.distance import cdist
        import warnings
        warnings.filterwarnings('ignore')

        %matplotlib inline
```

6.2 Kernels and Kernel Machines

6.2.1

Write functions to compute the RBF and polynomial kernels.

```
In [2]: ### Kernel function generators
        def linear_kernel(X1, X2):
            """
            Computes the linear kernel between two sets of vectors.
            Args:
                X1 - an n1xd matrix with vectors x1_1,...,x1_n1 in the rows
                X2 - an n2xd matrix with vectors x2_1,...,x2_n2 in the rows
            Returns:
                matrix of size n1xn2, with x1_i^T x2_j in position i,j
            """
            return np.dot(X1,np.transpose(X2))

        def RBF_kernel(X1,X2,sigma):
            """
            Computes the RBF kernel between two sets of vectors
            Args:
                X1 - an n1xd matrix with vectors x1_1,...,x1_n1 in the rows
                X2 - an n2xd matrix with vectors x2_1,...,x2_n2 in the rows
                sigma - the bandwidth (i.e. standard deviation) for the RBF/Gaussian kernel
            Returns:
                matrix of size n1xn2, with exp(-||x1_i-x2_j||^2/(2 sigma^2)) in position i,j
```

```python
    """
    return np.exp(-cdist(X1,X2,'sqeuclidean')/(2*sigma**2))

def polynomial_kernel(X1, X2, offset, degree):
    """
    Computes the inhomogeneous polynomial kernel between two sets of vectors
    Args:
        X1 - an n1xd matrix with vectors x1_1,...,x1_n1 in the rows
        X2 - an n2xd matrix with vectors x2_1,...,x2_n2 in the rows
        offset, degree - two parameters for the kernel
    Returns:
        matrix of size n1xn2, with (offset + <x1_i,x2_j>)^degree in position i,j
    """
    return (offset + X1@X2.T)**degree
```

6.2.2

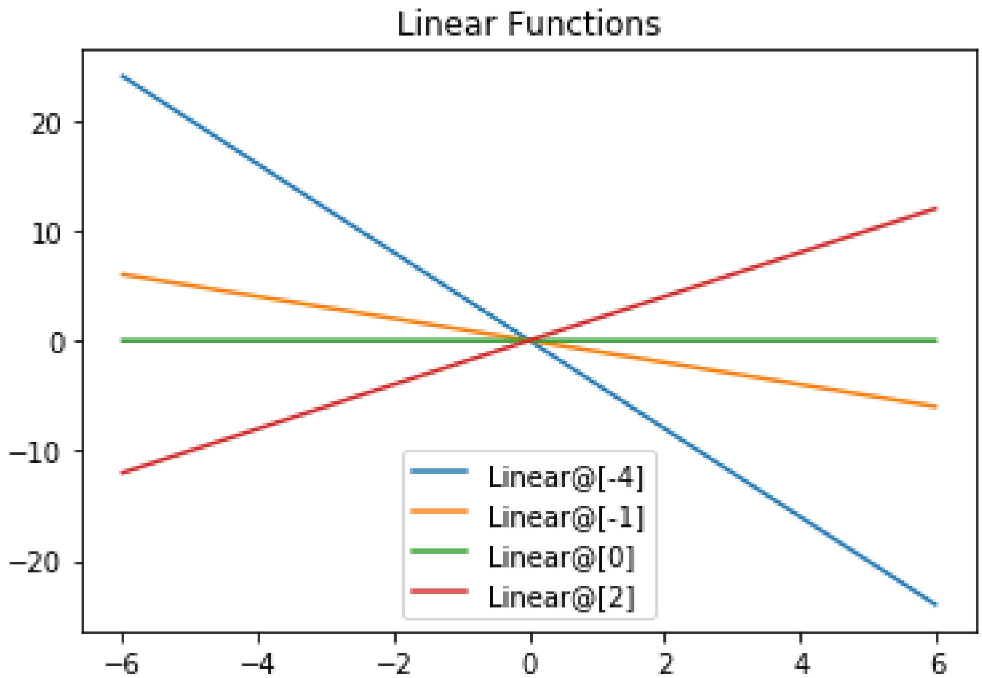
Use the linear kernel function defined in the code to compute the kernel matrix on the set of points $x_0 \in \mathcal{D}_X = \{-4, -1, 0, 2\}$. Include both the code and the output.

```
In [3]: x0 = np.array([[-4],[-1],[0],[2]])
        linear_kernel(x0,x0)
```

```
Out[3]: array([[16,  4,  0, -8],
               [ 4,  1,  0, -2],
               [ 0,  0,  0,  0],
               [-8, -2,  0,  4]])
```

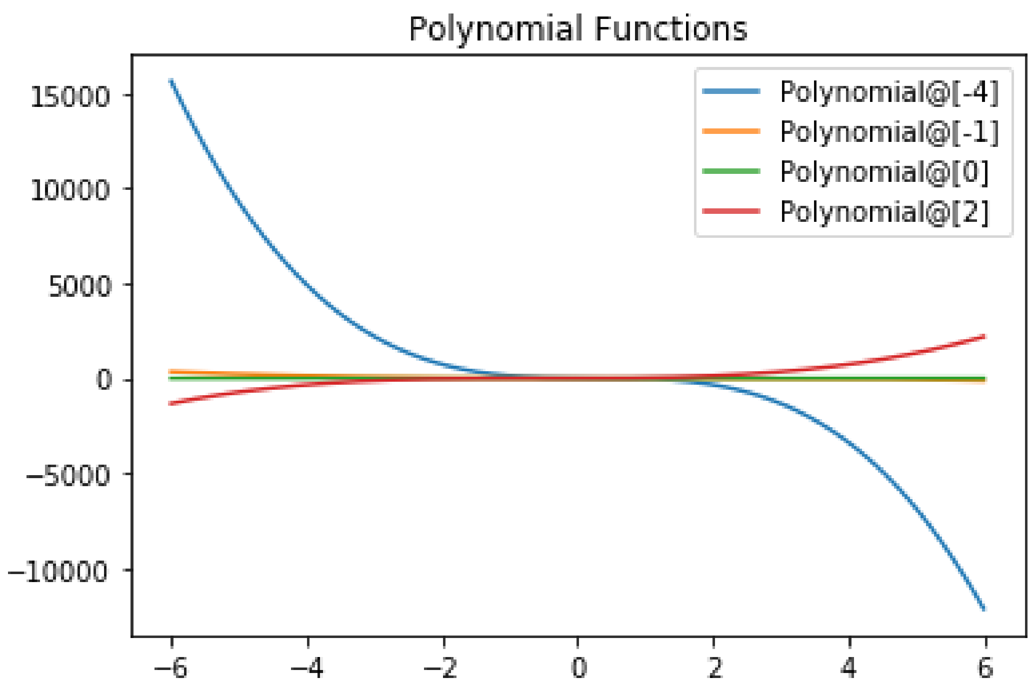
6.2.3.a

In [4]:
```python
# PLot kernel machine functions
plot_step = .01
xpts = np.arange(-6.0, 6, plot_step).reshape(-1,1)
prototypes = np.array([-4,-1,0,2]).reshape(-1,1)

# Linear kernel
y = linear_kernel(prototypes, xpts)
for i in range(len(prototypes)):
    label = "Linear@"+str(prototypes[i,:])
    plt.plot(xpts, y[i,:], label=label)
plt.legend(loc = 'best')
plt.title('Linear Functions')
plt.show()
```



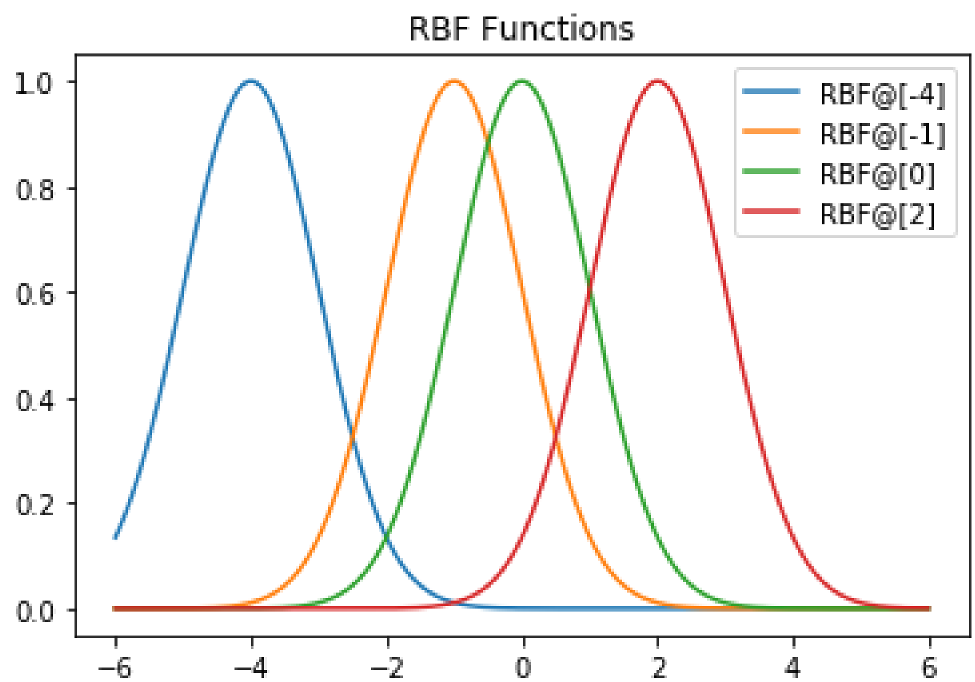
6.2.3.b

```
In [5]: # PLot kernel machine functions
        plot_step = .01
        xpts = np.arange(-6.0, 6, plot_step).reshape(-1,1)
        prototypes = np.array([-4,-1,0,2]).reshape(-1,1)

        # Linear kernel
        y = polynomial_kernel(prototypes, xpts, 1,3)
        for i in range(len(prototypes)):
            label = "Polynomial@"+str(prototypes[i,:])
            plt.plot(xpts, y[i,:], label=label)
        plt.legend(loc = 'best')
        plt.title('Polynomial Functions')
        plt.show()
```



6.2.3.c

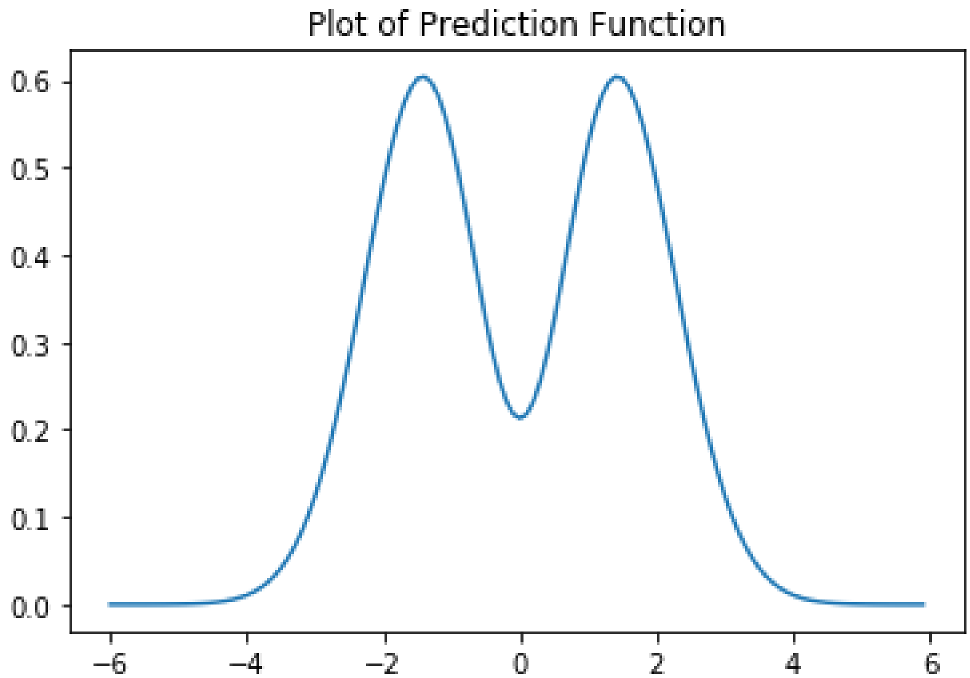
```
In [6]: # PLot kernel machine functions
        plot_step = .01
        xpts = np.arange(-6.0, 6, plot_step).reshape(-1,1)
        prototypes = np.array([-4,-1,0,2]).reshape(-1,1)

        # Linear kernel
        y = RBF_kernel(prototypes, xpts, 1)
        for i in range(len(prototypes)):
            label = "RBF@"+str(prototypes[i,:])
            plt.plot(xpts, y[i,:], label=label)
        plt.legend(loc = 'best')
        plt.title('RBF Functions')
        plt.show()
```



6.2.3.d

```
In [7]: class Kernel_Machine(object):
            def __init__(self, kernel, prototype_points, weights):
                """
                Args:
                    kernel(X1,X2) - a function return the cross-kernel matrix between rows of X1
                    prototype_points - an Rxd matrix with rows mu_1,...,mu_R
                    weights - a vector of length R with entries w_1,...,w_R
                """

                self.kernel = kernel
                self.prototype_points = prototype_points
                self.weights = weights

            def predict(self, X):
                """
                Evaluates the kernel machine on the points given by the rows of X
                Args:
                    X - an nxd matrix with inputs x_1,...,x_n in the rows
                Returns:
                    Vector of kernel machine evaluations on the n points in X.  Specifically, jt
                        Sum_{i=1}^R w_i k(x_j, mu_i)
                """
                return self.kernel(X, self.prototype_points)@self.weights
```

```
In [8]: from functools import partial
        kernel = partial(RBF_kernel, sigma=1)
        x = np.array([[-1],[0],[1]])
        w = np.array([1,-1,1])
        model = Kernel_Machine(kernel=kernel, prototype_points=x, weights=w)
        x_pred = np.arange(-6.0, 6, 0.1).reshape(-1,1)
        y_pred = model.predict(x_pred)
        plt.plot(x_pred, y_pred)
        plt.title("Plot of Prediction Function")
        plt.show()
```


Plot of Prediction Function
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−6
−4
−2
0
2
4
6

6.3 Kernel Ridge Regression

Load train & test data; Convert to column vectors so it generalizes well to data in higher dimensions.

```
In [5]: data_train,data_test = np.loadtxt("krr-train.txt"),np.loadtxt("krr-test.txt")
        x_train, y_train = data_train[:,0].reshape(-1,1),data_train[:,1].reshape(-1,1)
        x_test, y_test = data_test[:,0].reshape(-1,1),data_test[:,1].reshape(-1,1)
```

6.3.1

Plot the training data

```
In [10]: plt.scatter(x_train, y_train)
         plt.xlabel('X')
         plt.ylabel('Y')
         plt.title('Plot of Training Data')
         plt.show()
```

6.3.2

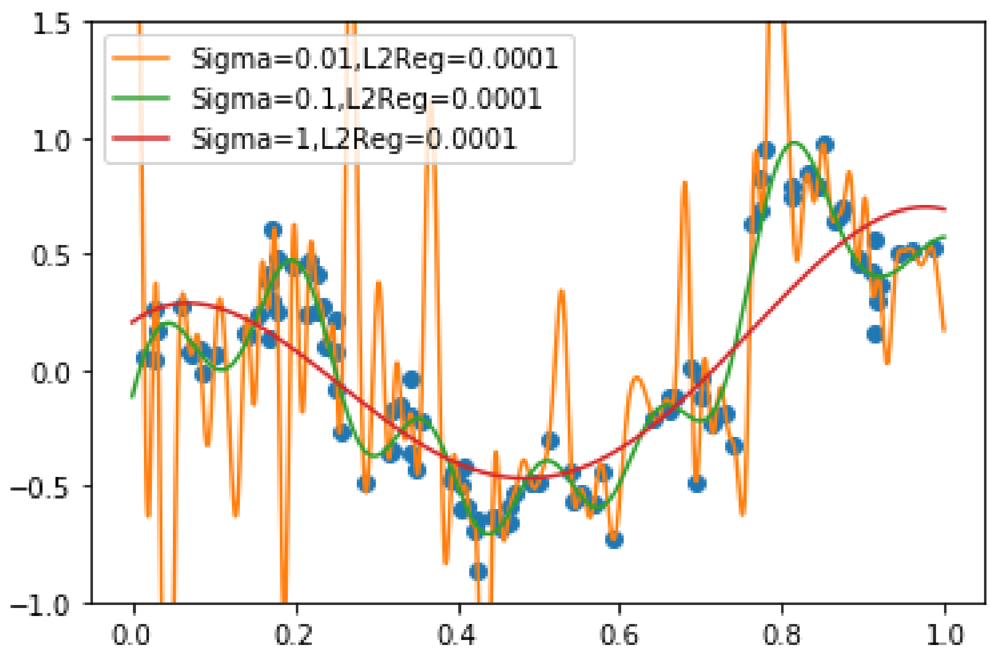
Complete the function train_kernel_ridge_regression

```
In [11]: def train_kernel_ridge_regression(X, y, kernel, l2reg):
             K = kernel(X,X)
             alpha = np.linalg.inv(l2reg*np.eye(len(X)) + K)@y
             return Kernel_Machine(kernel, X, alpha)
```

6.3.3

Use the code provided to plot your fits to the training data for the RBF kernel with a fixed regularization parameter of 0.0001 for 3 different values of sigma: 0.01, 0.1, and 1.0. What values of sigma do you think would be more likely to over fit, and which less?

```
In [12]: plot_step = .001
         xpts = np.arange(0 , 1, plot_step).reshape(-1,1)
         plt.plot(x_train,y_train,'o')
         l2reg = 0.0001
         for sigma in [.01,.1,1]:
             k = functools.partial(RBF_kernel, sigma=sigma)
             f = train_kernel_ridge_regression(x_train, y_train, k, l2reg=l2reg)
             label = "Sigma="+str(sigma)+",L2Reg="+str(l2reg)
             plt.plot(xpts, f.predict(xpts), label=label)
         plt.legend(loc = 'best')
         plt.ylim(-1,1.5)
         plt.show()
```

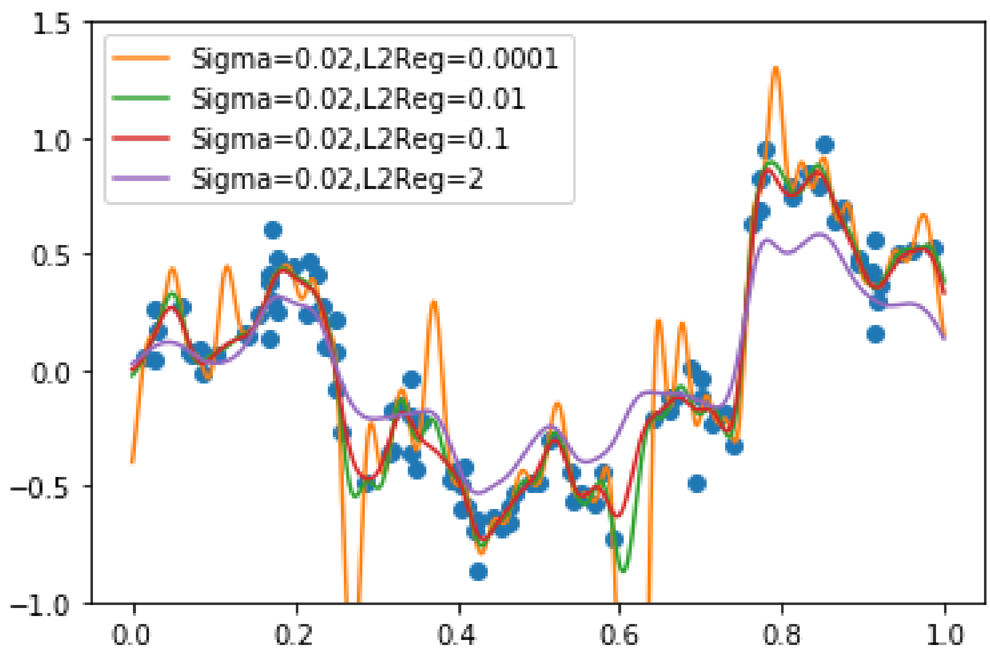


Lower values of σ are more prone to overfitting. With lower values of σ, the final prediction function is highly dependednt on local values of the training data.

6.3.4

Use the code provided to plot your fits to the training data for the RBF kernel with a fixed sigma of 0.02 and 4 different values of the regularization parameter λ: 0.0001, 0.01, 0.1, and 2.0. What happens to the prediction function as $\lambda \to \infty$?

```
In [13]: plot_step = .001
         xpts = np.arange(0 , 1, plot_step).reshape(-1,1)
         plt.plot(x_train,y_train,'o')
         sigma= .02
         for l2reg in [.0001,.01,.1,2]:
             k = functools.partial(RBF_kernel, sigma=sigma)
             f = train_kernel_ridge_regression(x_train, y_train, k, l2reg=l2reg)
             label = "Sigma="+str(sigma)+",L2Reg="+str(l2reg)
             plt.plot(xpts, f.predict(xpts), label=label)
         plt.legend(loc = 'best')
         plt.ylim(-1,1.5)
         plt.show()
```



As $\lambda \to \infty$, the weights become smaller, eventually reaching 0. At this point the prediction function is just 0 for all x.

In [ ]:
```
def sigmoid_kernel(X1, X2, eta, nu):
    return np.tanh(eta*X1@X2.T+nu)
```

In [30]:
```
from sklearn.base import BaseEstimator, RegressorMixin, ClassifierMixin

class KernelRidgeRegression(BaseEstimator, RegressorMixin):
    """sklearn wrapper for our kernel ridge regression"""

    def __init__(self, kernel="RBF", sigma=1, degree=2, offset=1, l2reg=1, eta=1, nu=1)
        self.kernel = kernel
        self.sigma = sigma
        self.degree = degree
        self.offset = offset
        self.l2reg = l2reg
        self.eta = eta
        self.nu = nu

    def fit(self, X, y=None):
        """
        This should fit classifier. All the "work" should be done here.
        """
        if (self.kernel == "linear"):
            self.k = linear_kernel
        elif (self.kernel == "RBF"):
            self.k = functools.partial(RBF_kernel, sigma=self.sigma)
        elif (self.kernel == "polynomial"):
            self.k = functools.partial(polynomial_kernel, offset=self.offset, degree=se
        elif self.kernel == 'sigmoid':
            self.k = functools.partial(sigmoid_kernel, eta=self.eta, nu=self.nu)
        else:
            raise ValueError('Unrecognized kernel type requested.')

        self.kernel_machine_ = train_kernel_ridge_regression(X, y, self.k, self.l2reg)

        return self

    def predict(self, X, y=None):
        try:
            getattr(self, "kernel_machine_")
        except AttributeError:
            raise RuntimeError("You must train classifer before predicting data!")

        return(self.kernel_machine_.predict(X))

    def score(self, X, y=None):
        # get the average square error
        return(((self.predict(X)-y)**2).mean())
```

In [6]:
```
from sklearn.model_selection import GridSearchCV,PredefinedSplit
```

```
from sklearn.model_selection import ParameterGrid
from sklearn.metrics import mean_squared_error,make_scorer
import pandas as pd

test_fold = [-1]*len(x_train) + [0]*len(x_test)   #0 corresponds to test, -1 to train
predefined_split = PredefinedSplit(test_fold=test_fold)
```

In [55]:
```
param_grid = [{'kernel': ['RBF'],'sigma':np.linspace(0.01,1,25), 'l2reg': np.logspace(1
              {'kernel':['polynomial'],'offset':np.linspace(-5,5,25), 'degree':[2,3,4,5
              {'kernel':['linear'],'l2reg': np.linspace(0.1,10,50)}]
kernel_ridge_regression_estimator = KernelRidgeRegression()
grid = GridSearchCV(kernel_ridge_regression_estimator,
                    param_grid,
                    cv = predefined_split,
                    scoring = make_scorer(mean_squared_error,greater_is_better = False)
                    ,n_jobs = -1  #should allow parallelism, but crashes Python on my m
                    ,verbose=0
                   )
_ = grid.fit(np.vstack((x_train,x_test)),np.vstack((y_train,y_test)))
```

In [ ]:
```
pd.set_option('display.max_rows', 20)
df = pd.DataFrame(grid.cv_results_)
# Flip sign of score back, because GridSearchCV likes to maximize,
# so it flips the sign of the score if "greater_is_better=FALSE"
df['mean_test_score'] = -df['mean_test_score']
df['mean_train_score'] = -df['mean_train_score']
cols_to_keep = ["param_degree", "param_kernel","param_l2reg" ,"param_offset","param_sigm
        "mean_test_score","mean_train_score"]
df_toshow = df[cols_to_keep].fillna('-')
df_toshow = df_toshow.sort_values(by=["mean_test_score"])
```

6.3.5

Perform hyperparameter search and provide a summary of the results for each kernel

Best params for RBF

```
In [57]: df_toshow[df_toshow.param_kernel=='RBF'].head()
```

```
Out[57]:    param_degree param_kernel  param_l2reg param_offset param_sigma  \
         1             -          RBF     1.000023            -     0.05125
         26            -          RBF     3.359892            -     0.05125
         2             -          RBF     1.000023            -      0.0925
         4             -          RBF     1.000023            -       0.175
         3             -          RBF     1.000023            -     0.13375

             mean_test_score  mean_train_score
         1          0.015609          0.018215
         26         0.025470          0.031895
         2          0.026406          0.029071
         4          0.028863          0.040367
         3          0.028890          0.034944
```

Best params for Polynomial Kernel

```
In [58]: df_toshow[df_toshow.param_kernel=='polynomial'].head()
```

```
Out[58]:      param_degree param_kernel  param_l2reg param_offset param_sigma  \
         3042            6   polynomial      0.05125      2.08333           -
         3069            6   polynomial      0.09250      2.91667           -
         3095            6   polynomial      0.13375      3.33333           -
         3121            6   polynomial      0.17500         3.75           -
         3096            6   polynomial      0.13375         3.75           -

               mean_test_score  mean_train_score
         3042         0.032453          0.049113
         3069         0.032517          0.048018
         3095         0.032600          0.048699
         3121         0.032672          0.048877
         3096         0.032692          0.046910
```

Best params for Linear

```
In [59]: df_toshow[df_toshow.param_kernel=='linear'].head()
```

```
Out[59]:      param_degree param_kernel  param_l2reg param_offset param_sigma  \
         3644            -       linear     3.938776            -           -
         3643            -       linear     3.736735            -           -
         3645            -       linear     4.140816            -           -
         3642            -       linear     3.534694            -           -
```

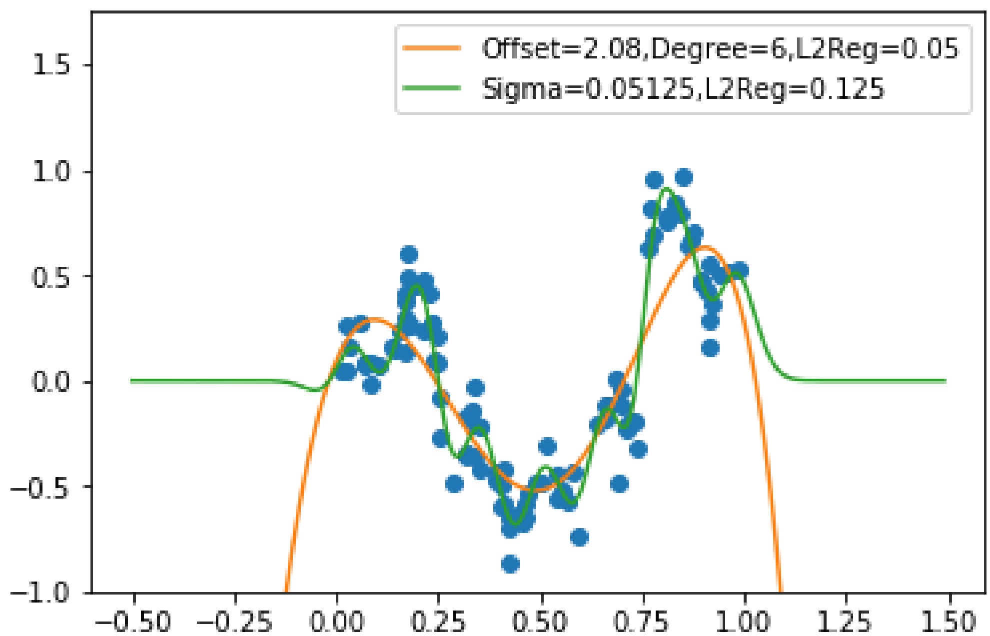
3646	-	linear	4.342857	-	-

	mean_test_score	mean_train_score
3644	0.16451	0.206561
3643	0.16451	0.206556
3645	0.16451	0.206567
3642	0.16451	0.206551
3646	0.16451	0.206572

6.3.6

Plot the best fitting prediction functions using the polynomial and RBF kernels.

```
In [60]: ## Plot the best polynomial and RBF fits you found
         plot_step = .01
         xpts = np.arange(-.5 , 1.5, plot_step).reshape(-1,1)
         plt.plot(x_train,y_train,'o')
         #Plot best polynomial fit
         offset= 2.08
         degree = 6
         l2reg = 0.05
         k = functools.partial(polynomial_kernel, offset=offset, degree=degree)
         f = train_kernel_ridge_regression(x_train, y_train, k, l2reg=l2reg)
         label = "Offset="+str(offset)+",Degree="+str(degree)+",L2Reg="+str(l2reg)
         plt.plot(xpts, f.predict(xpts), label=label)
         #Plot best RBF fit
         sigma = 0.05125
         l2reg= 0.125000
         k = functools.partial(RBF_kernel, sigma=sigma)
         f = train_kernel_ridge_regression(x_train, y_train, k, l2reg=l2reg)
         label = "Sigma="+str(sigma)+",L2Reg="+str(l2reg)
         plt.plot(xpts, f.predict(xpts), label=label)
         plt.legend(loc = 'best')
         plt.ylim(-1,1.75)
         plt.show()
```



The RBF kernel fits the data much better. Another thing to note is that outside the domain of x values that the model saw, it predicts approximately 0 while the polynomial kernel quickly goes to $-\infty$ on both ends. This means that most likely, the RBF would perform better with predicting new data that fell outside the range of observed values.

6.3.7

The Bayes prediction function is

$$f^* = E(Y|X) = f(x)$$

The risk of this function is

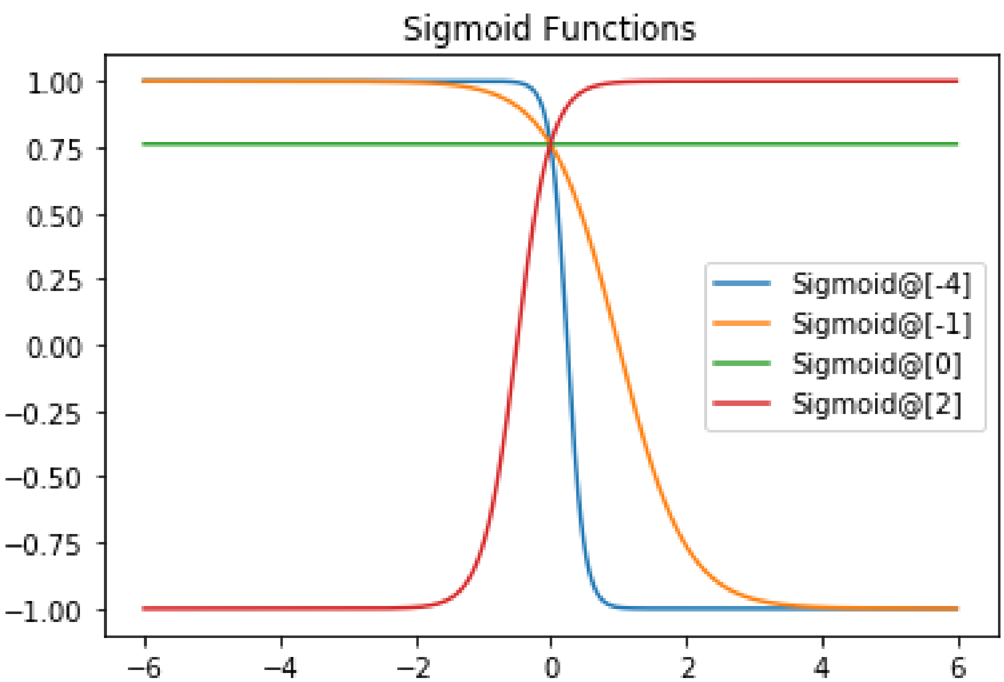
$$\begin{aligned}
R(f^*) &= E\big[l(f^*(x), y)\big] \\
&= E\big[(f^*(x) - f(x) - \epsilon)^2\big] \\
&= E\big[(f(x) - f(x) - \epsilon)^2\big] \\
&= E\big[\epsilon^2\big] \\
&= Var(\epsilon) \\
&= 0.1^2
\end{aligned}$$

6.3.8

Attempt to improve performance by using different kernel functions.

```
In [61]: def sigmoid_kernel(X1, X2, eta, nu):
             return np.tanh(eta*X1@X2.T+nu)
```

```
In [62]: # PLot kernel machine functions
         plot_step = .01
         xpts = np.arange(-6.0, 6, plot_step).reshape(-1,1)
         prototypes = np.array([-4,-1,0,2]).reshape(-1,1)

         # Sigmoid kernel
         y = sigmoid_kernel(prototypes, xpts, 1,1)
         for i in range(len(prototypes)):
             label = "Sigmoid@"+str(prototypes[i,:])
             plt.plot(xpts, y[i,:], label=label)
         plt.legend(loc = 'best')
         plt.title('Sigmoid Functions')
         plt.show()
```




```
In [63]: param_grid = {'eta': np.linspace(0, 10, 10),
                       'nu': np.logspace(-2, 0, 10),
                      'l2reg': np.logspace(-7,-5, 15),
                       'kernel': ['sigmoid']}
```

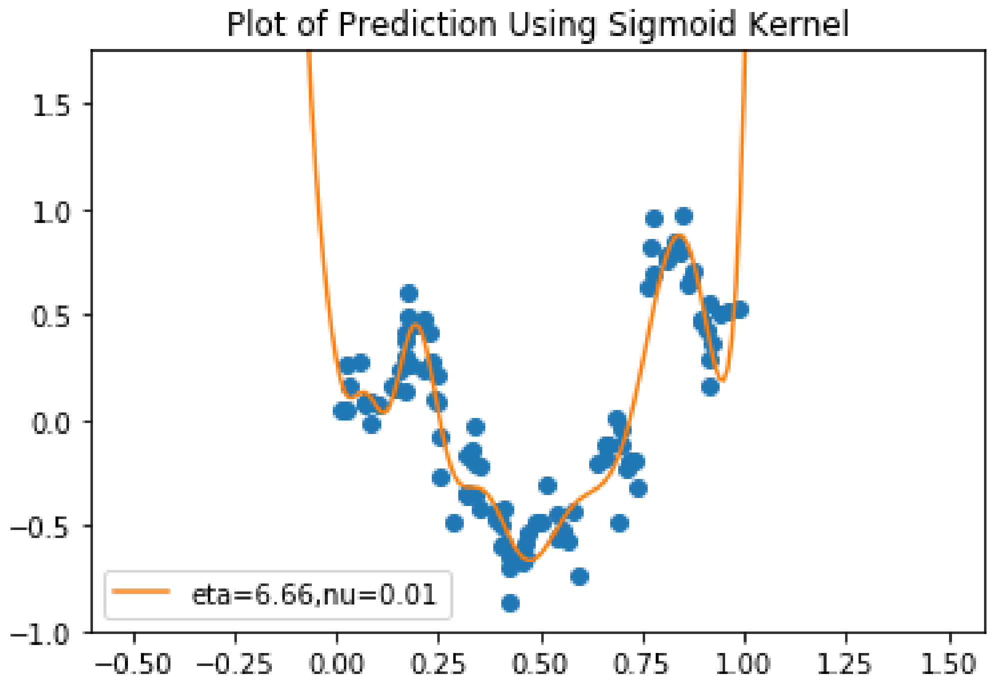
```
kernel_ridge_regression_estimator = KernelRidgeRegression()
grid = GridSearchCV(kernel_ridge_regression_estimator,
                    param_grid,
                    cv = predefined_split,
                    scoring = make_scorer(mean_squared_error,
                                          greater_is_better = False)
                    ,n_jobs = -1
                    ,verbose=0
                   )
_ = grid.fit(np.vstack((x_train,x_test)),np.vstack((y_train,y_test)))
```

In [64]:
```
df2 = pd.DataFrame(grid.cv_results_)
# Flip sign of score back, because GridSearchCV likes to maximize,
# so it flips the sign of the score if "greater_is_better=FALSE"
df2['mean_test_score'] = -df2['mean_test_score']
df2['mean_train_score'] = -df2['mean_train_score']
cols_to_keep = ["param_eta", "param_nu","mean_test_score",
                "mean_train_score", "param_l2reg"]
df2 = df2[cols_to_keep]
df2 = df2.sort_values(by=["mean_test_score"])
df2.head()
```

Out[64]:

	param_eta	param_nu	mean_test_score	mean_train_score	param_l2reg
900	6.66667	0.01	0.020141	0.025288	1e-07
901	6.66667	0.016681	0.020214	0.025650	1e-07
902	6.66667	0.0278256	0.020612	0.026430	1e-07
1354	10	0.0774264	0.021767	0.025143	1e-07
1362	10	0.0278256	0.021950	0.024905	1.3895e-07

In [65]:
```
plot_step = .01
xpts = np.arange(-.5 , 1.5, plot_step).reshape(-1,1)
plt.plot(x_train,y_train,'o')
#Plot best sigmoid fit
eta = 6.66
nu= 0.01
l2reg = 1e-7
k = functools.partial(sigmoid_kernel, eta=eta, nu=nu)
f = train_kernel_ridge_regression(x_train, y_train, k, l2reg=l2reg)
label = "eta="+str(eta)+",nu="+str(nu)
plt.plot(xpts, f.predict(xpts), label=label)
plt.legend(loc = 'best')
plt.ylim(-1,1.75)
plt.title("Plot of Prediction Using Sigmoid Kernel")
plt.show()
```



This didn't improve the test score over using RBF (RBF=0.015, Sigmoid=0.02). However it still fits the data very well.

6.3.9

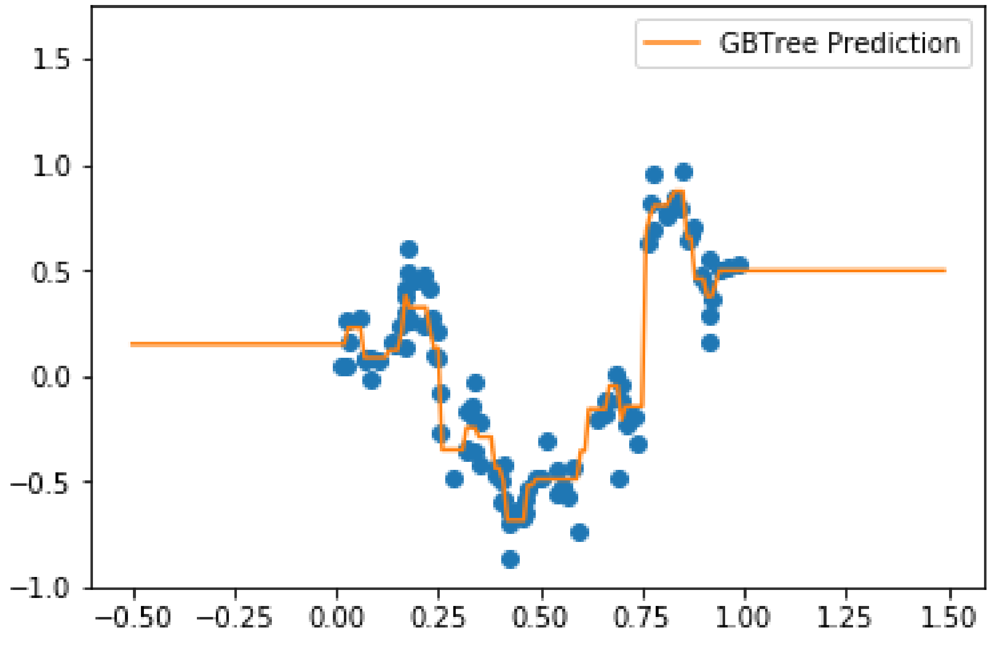
Use any machine learning model to get the best performance you can.

```
In [7]: from sklearn.ensemble import GradientBoostingRegressor
        from sklearn.model_selection import RandomizedSearchCV
        from scipy.stats import randint
        from scipy.stats.distributions import uniform
        param_dist = {'learning_rate': uniform(0.01,1),
                      'n_estimators': randint(1,50),
                      'min_samples_leaf': randint(1,5),
                      'max_depth': randint(2,5)}
        grid = RandomizedSearchCV(GradientBoostingRegressor(),
                                  param_dist,
                                  cv = predefined_split,
                                  scoring = make_scorer(mean_squared_error,
                                                        greater_is_better = False)
                                  ,n_jobs = -1
                                  ,verbose=0
                                  , n_iter=500
                                 )
        _ = grid.fit(np.vstack((x_train,x_test)),np.vstack((y_train,y_test)))
```

```
In [73]: pd.set_option('display.max_rows', 20)
         df2 = pd.DataFrame(grid.cv_results_)
         # Flip sign of score back, because GridSearchCV likes to maximize,
         # so it flips the sign of the score if "greater_is_better=FALSE"
         df2['mean_test_score'] = -df2['mean_test_score']
         df2['mean_train_score'] = -df2['mean_train_score']
         cols_to_keep = ["param_learning_rate", "param_n_estimators",
                         "param_min_samples_leaf", 'param_max_depth',
                         "mean_test_score","mean_train_score"]
         df_toshow2 = df2[cols_to_keep].fillna('-')
         df_toshow2 = df_toshow2.sort_values(by=["mean_test_score"])
         df_toshow2.head()
```

```
Out[73]:     param_learning_rate  param_n_estimators  param_min_samples_leaf  \
         99             0.270287                  25                       3
         308            0.086171                  43                       2
         355            0.211756                  43                       1
         64             0.218163                  24                       2
         9              0.082174                  30                       2

              param_max_depth  mean_test_score  mean_train_score
         99                 4         0.394587          0.032204
         308                4         0.395288          0.054924
         355                4         0.395995          0.008076
         64                 4         0.407881          0.039017
         9                  4         0.408592          0.088934
```

```
In [8]: plot_step = .01
        xpts = np.arange(-.5 , 1.5, plot_step).reshape(-1,1)
        plt.plot(x_train,y_train,'o')
        f = grid.best_estimator_
        plt.plot(xpts, f.predict(xpts), label='GBTree Prediction')
        plt.legend()
        plt.ylim(-1,1.75)
        plt.show()
```



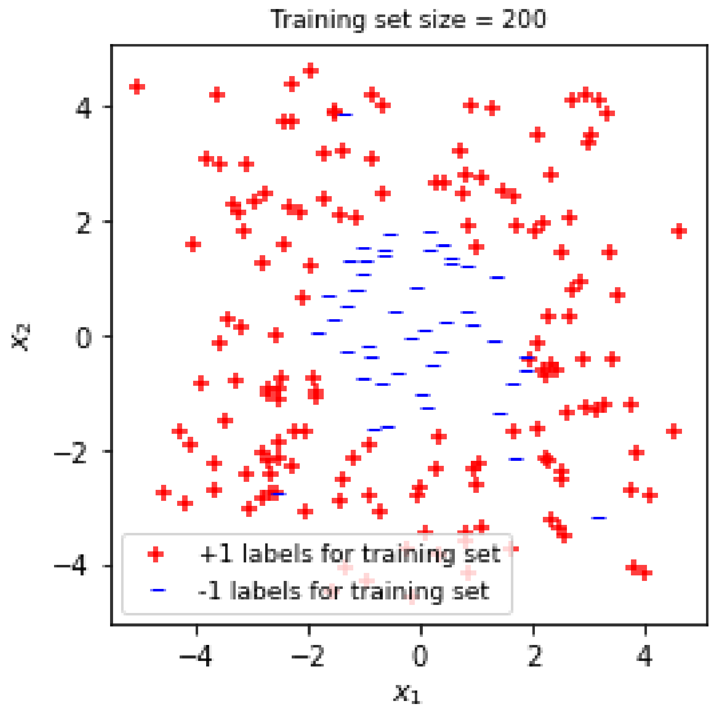
Usign gradient boosted trees resulted in an improved test score over RBF SVF. (RBF=0.15, GBT=0.013)

6.4 Kernalized Support Vector Machines with Kernalized Pegasos

6.4.1

Load the SVM training and test data from the zip file. Plot the training data using the code supplied. Are the data linearly separable? Quadratically separable? What if we used some RBF kernel?

In [69]:
```
# Load and plot the SVM data
#load the training and test sets
data_train,data_test = np.loadtxt("svm-train.txt"),np.loadtxt("svm-test.txt")
x_train, y_train = data_train[:,0:2], data_train[:,2].reshape(-1,1)
x_test, y_test = data_test[:,0:2], data_test[:,2].reshape(-1,1)

#determine predictions for the training set
yplus = np.ma.masked_where(y_train[:,0]<=0, y_train[:,0])
xplus = x_train[~np.array(yplus.mask)]
yminus = np.ma.masked_where(y_train[:,0]>0, y_train[:,0])
xminus = x_train[~np.array(yminus.mask)]

#plot the predictions for the training set
figsize = plt.figaspect(1)
f, (ax) = plt.subplots(1, 1, figsize=figsize)

pluses = ax.scatter (xplus[:,0], xplus[:,1], marker='+', c='r',
                     label = '+1 labels for training set')
minuses = ax.scatter (xminus[:,0], xminus[:,1], marker=r'$-$', c='b',
                      label = '-1 labels for training set')

ax.set_ylabel(r"$x_2$", fontsize=11)
ax.set_xlabel(r"$x_1$", fontsize=11)
ax.set_title('Training set size = %s'% len(data_train), fontsize=9)
ax.axis('tight')
ax.legend(handles=[pluses, minuses], fontsize=9)
plt.show()
```



The data is not linearly or quadratically seperable. However, an RBF kernel could separate it,

In [70]:
```
# Code to help plot the decision regions
# (Note: This ode isn't necessarily entirely appropriate for the questions asked. So th

sigma=1
k = functools.partial(RBF_kernel, sigma=sigma)
f = train_soft_svm(x_train, y_train, k, ...)

#determine the decision regions for the predictions
x1_min = min(x_test[:,0])
x1_max= max(x_test[:,0])
x2_min = min(x_test[:,1])
x2_max= max(x_test[:,1])
h=0.1
xx, yy = np.meshgrid(np.arange(x1_min, x1_max, h),
                     np.arange(x2_min, x2_max, h))

Z = f.predict(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)

#determine the predictions for the test set
y_bar = f.predict (x_test)
yplus = np.ma.masked_where(y_bar<=0, y_bar)
xplus = x_test[~np.array(yplus.mask)]
yminus = np.ma.masked_where(y_bar>0, y_bar)
xminus = x_test[~np.array(yminus.mask)]

#plot the learned boundary and the predictions for the test set
figsize = plt.figaspect(1)
f, (ax) = plt.subplots(1, 1, figsize=figsize)
decision =ax.contourf(xx, yy, Z, cmap=plt.cm.coolwarm, alpha=0.8)
pluses = ax.scatter (xplus[:,0], xplus[:,1], marker='+', c='b',
                     label = '+1 prediction for test set')
minuses = ax.scatter (xminus[:,0], xminus[:,1], marker=r'$-$', c='b',
                      label = '-1 prediction for test set')
ax.set_ylabel(r"$x_2$", fontsize=11)
ax.set_xlabel(r"$x_1$", fontsize=11)
ax.set_title('SVM with RBF Kernel: training set size = %s'% len(data_train), fontsize=9
ax.axis('tight')
ax.legend(handles=[pluses, minuses], fontsize=9)
plt.show()
```

```
---------------------------------------------------------------------------

NameError                                 Traceback (most recent call last)

<ipython-input-70-2d02ee1ae610> in <module>
  4 sigma=1
```

```
      5 k = functools.partial(RBF_kernel, sigma=sigma)
----> 6 f = train_soft_svm(x_train, y_train, k, ...)
      7
      8 #determine the decision regions for the predictions


    NameError: name 'train_soft_svm' is not defined
```