bayesian_regression_support

April 5, 2019

0.1 Recreating figure 3.7 from Bishop's "Pattern Recognition and Machine Learning."

This notebook provides scaffolding for your exploration Bayesian Linear Gaussian Regression, as described in Lecture. In particular, through this notebook you will reproduce several variants of figure 3.7 from Bishop's book.

0.2 Instructions:

0.2.1 5.1-3:

Implement the functions in `problem` -- completed implementations of these functions are needed to generate the plots.

```
In [3]: from support_code import *
        from problem import *
```

0.3 Instructions (continued):

0.3.1 5.4:

If your implementations are correct, then the next few code blocks in this notebook will generate the required variants of Bishop's figure. These are the same figures that you would obtain if you ran `python problem.py` from the command line -- this notebook is just provided as additional support.

```
In [4]: # Generate our simulated dataset
        # Note we are using sigma == 0.2

        np.random.seed(46134)
        actual_weights = np.matrix([[0.3], [0.5]])
        data_size = 40
        noise = {"mean":0, "var":0.2 ** 2}
        likelihood_var = noise["var"]
        xtrain, ytrain = generate_data(data_size,
                                       noise,
                                       actual_weights)
```

Next, we generate the plots using 3 different prior covariance matrix. In the main call to `problem.py`, this is done in a loop -- here we wrap the loop body in a short helper function.

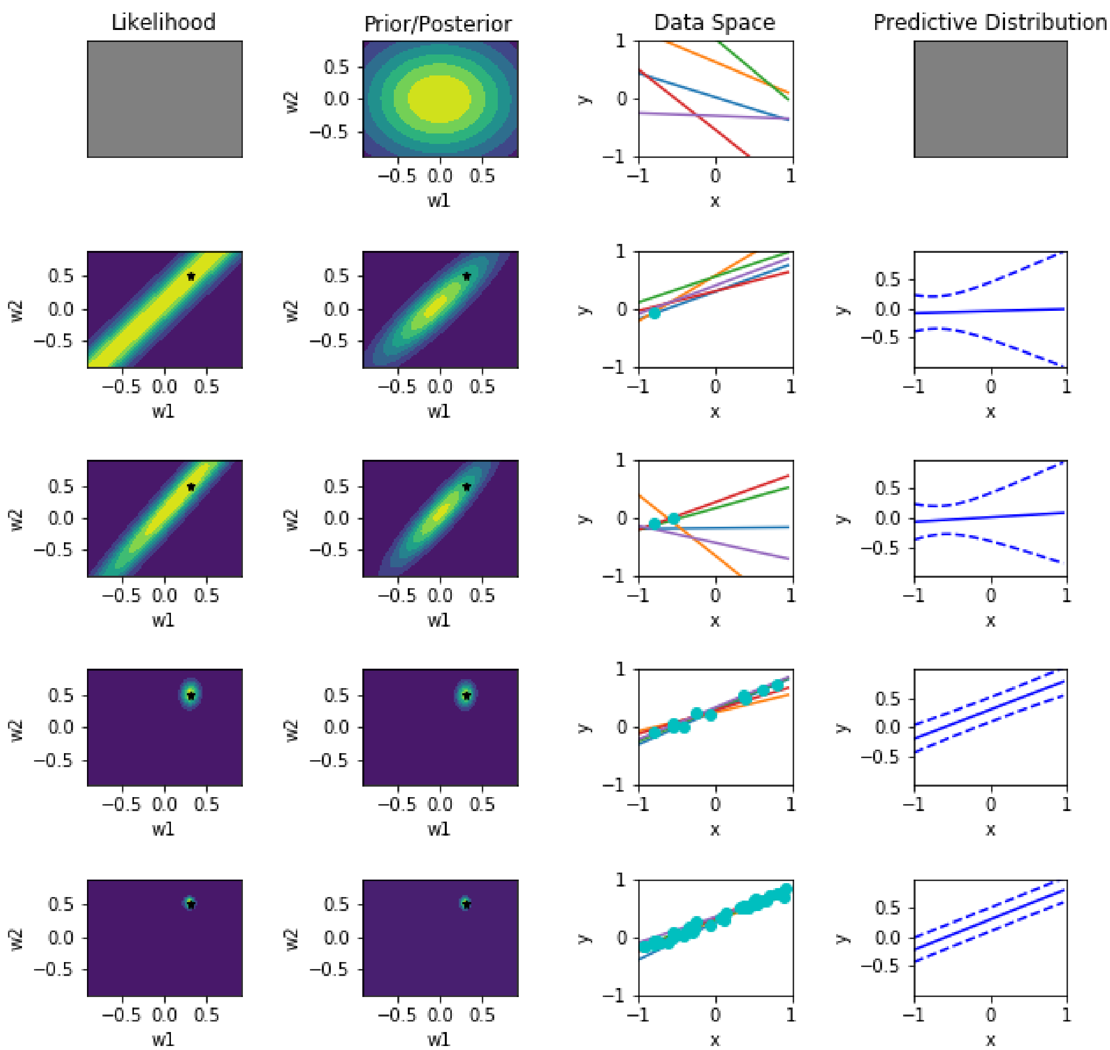
```
In [9]: def make_plot_given_sigma(sigma_squared):
            prior = {"mean":np.matrix([[0], [0]]),
                     "var":matlib.eye(2) * sigma_squared}

            make_plots(actual_weights,
                       xtrain,
                       ytrain,
                       likelihood_var,
                       prior,
                       likelihood_func,
                       get_posterior_params,
                       get_predictive_params,
                       sigma_squared)
```

```
In [8]: sigmas = [1/2, 1/(2**5), 1/(2**10)]
```

First covariance matrix:

$$\Sigma_0 = \frac{1}{2}I, \qquad I \in \mathbb{R}^{2\times 2}$$

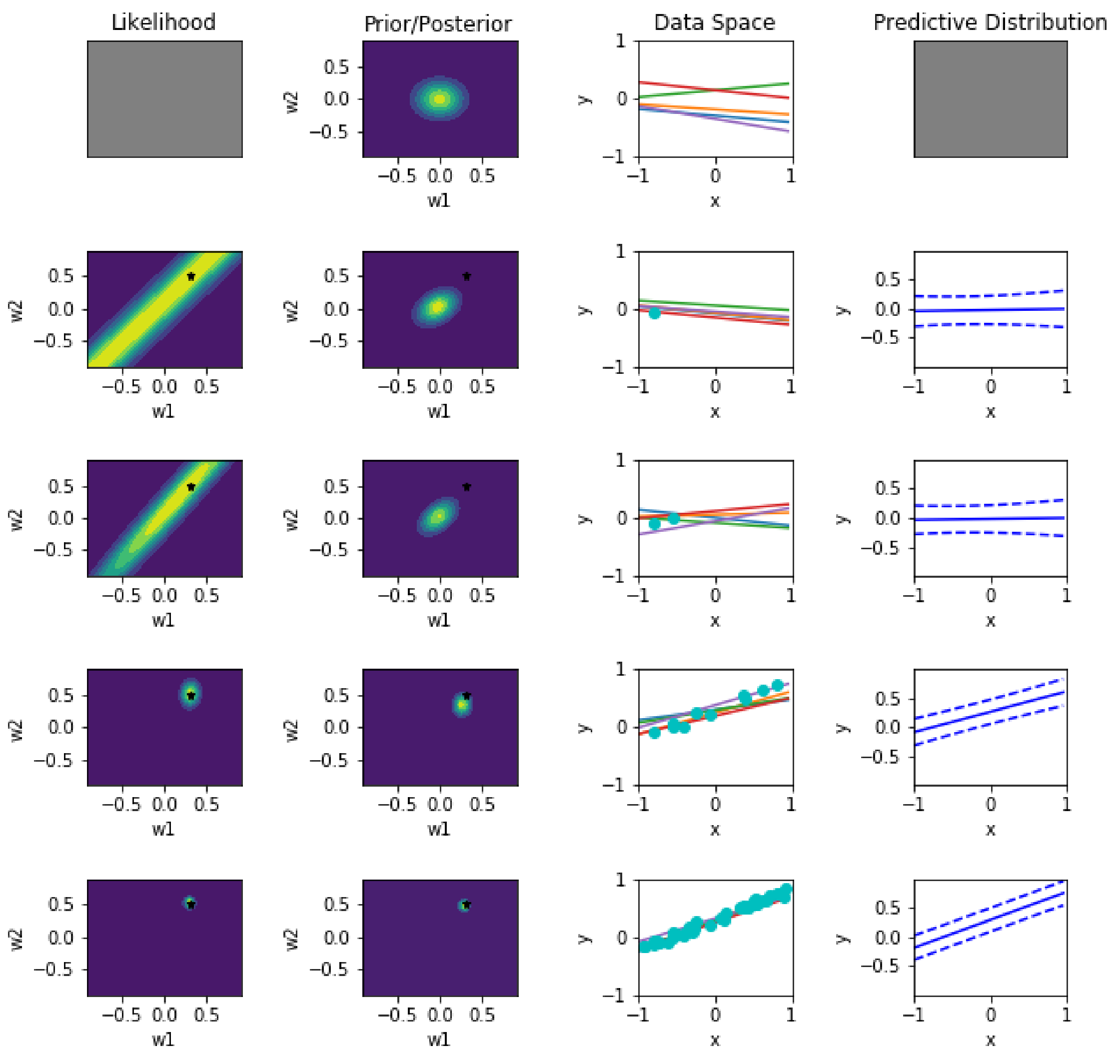
```
In [7]: try:
            make_plot_given_sigma(sigmas[0])
        except NameError:
            print('If not yet implemented, implement functions in problem.py.')
            print('If you have implemented, remove this try/except.')
```



Second covariance matrix:

$$\Sigma_0 = \frac{1}{2^5} I, \qquad I \in \mathbb{R}^{2\times 2}$$

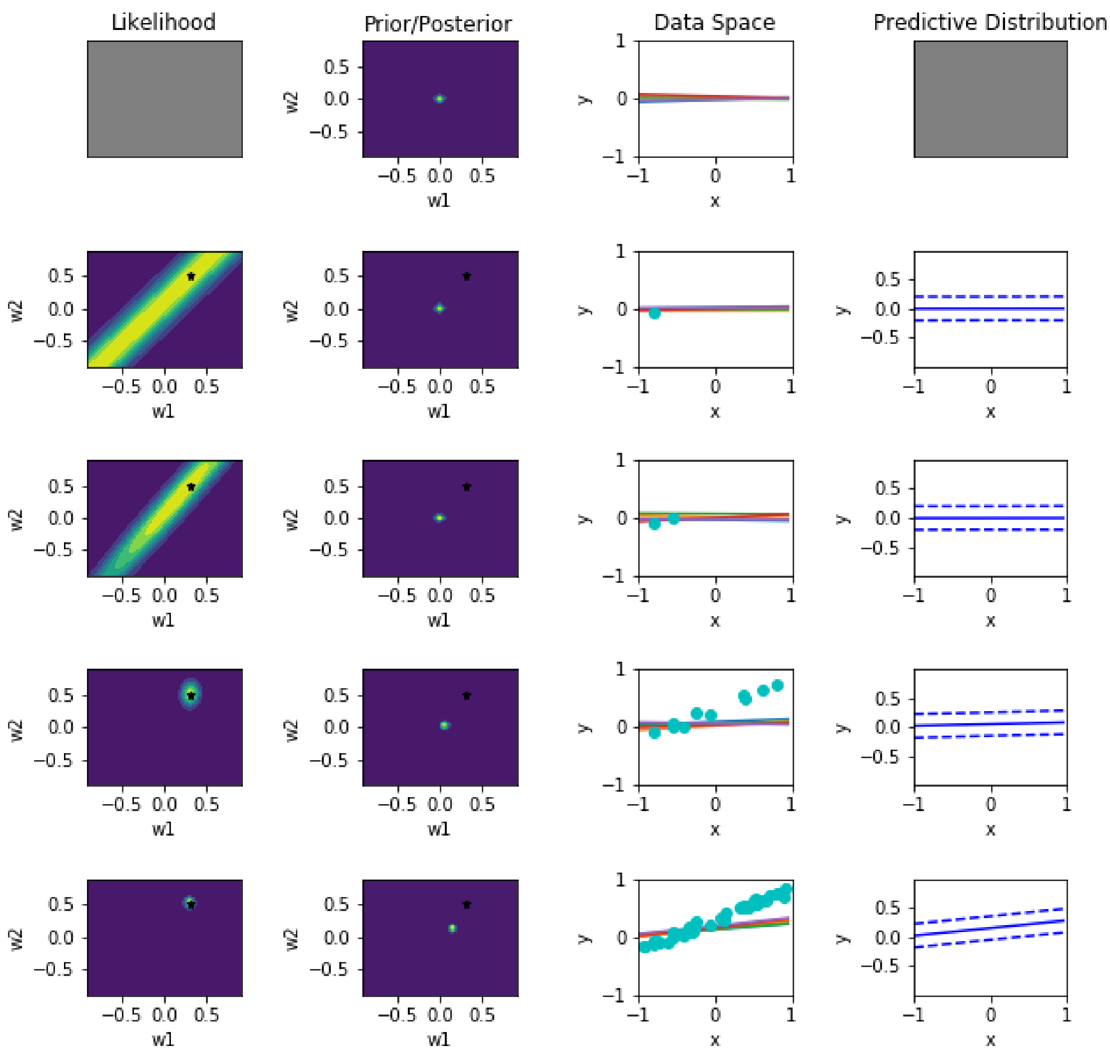
In [8]:
```
try:
    make_plot_given_sigma(sigmas[1])
except NameError:
    print('If not yet implemented, implement functions in problem.py.')
    print('If you have implemented, remove this try/except.')
```



Third covariance matrix:

$$\Sigma_0 = \frac{1}{2^{10}} I, \qquad I \in \mathbb{R}^{2\times 2}$$

```
In [9]: try:
            make_plot_given_sigma(sigmas[2])
        except NameError:
            print('If not yet implemented, implement functions in problem.py.')
            print('If you have implemented, remove this try/except.')
```



0.4 Instructions (continued):

0.4.1 5.5:

The likelihood function is unaffected by the strength of the prior. As more samples are drawn, the likelihood function becomes more centered around the true parameters and its variance decreases.

The posterior distribution is strongly affected by the strength of the prior. In the final plot, the prior is so strong, that even after many observations, our parameter distribution has not come close to the true values.

The predictive distribution is also strongly affected by the strength of the prior. With a weak prior, there is high variance around unseen values. With a strong prior, although the variance is reduced, the predictive distribution is not able to converge to the true values.

0.5 Instructions (continued):

0.5.1 5.6:

For question (6), find the MAP solution for the first prior covariance $\left(\frac{1}{2}I\right)$ by completing the implementation below. In addition, be sure to justify the value for the regularization coefficient (in `sklearn` named `alpha`) in your written work.

```
In [1]: from sklearn.linear_model import Ridge
```

```
In [15]: n = len(ytrain)
         alpha = 1/(2*n*sigmas[0])
         #alpha = 9999 # Change to the correct value
         ridge = Ridge(alpha=alpha,
                       fit_intercept=False,
                       solver='cholesky')

         ridge.fit(xtrain, ytrain)
```

```
Out[15]: Ridge(alpha=0.025, copy_X=True, fit_intercept=False, max_iter=None,
            normalize=False, random_state=None, solver='cholesky', tol=0.001)
```

If alpha is set correctly, ridge.coef_ will equal the prior mean/MAP estimate returned by the next two cells.

```
In [16]: ridge.coef_
```

```
Out[16]: array([[0.30085783, 0.52614072]])
```

```
In [14]: prior = {"mean":np.matrix([[0], [0]]),
                  "var":matlib.eye(2) * sigmas[0]}


         post = get_posterior_params(xtrain, ytrain, prior,
                                     likelihood_var = 0.2**2)
         post[0].ravel()
```

```
Out[14]: matrix([[0.30052135, 0.52406189]])
```