# hw3-solution-final

February 24, 2019

## 2. Calculating Subgradients

### 2.1

Suppose $f_1, \ldots, f_m : \mathbf{R}^d \to \mathbf{R}$ are convex functions, and

$$f(x) = \max_{i=1,\ldots,,m} f_i(x).$$

Let $k$ be any index for which $f_k(x) = f(x)$, and choose $g \in \partial f_k(x)$. Show that $g \in \partial f(x)$.

**Proof:** Since $g \in \partial f_k(x)$, we have,

$$f_k(x+v) \geq f_k(x) + g^T v \qquad \forall v \in \mathbf{R}^d$$

But

$$\begin{aligned} f(x) &= \max_{i=1,\ldots,,m} f_i(x) \\ &\geq f_i(x) && \forall i = 1, \ldots, , m \end{aligned}$$

Thus $\forall y \in \mathbf{R}^d$, we have $f(y) \geq f_k(y)$
Now observe that

$$\begin{aligned} f(x+v) &\geq f_k(x+v) \\ &\geq f_k(x) + g^T v \\ &= f(x) + g^T v && \text{since } f(x) = f_k(x) \end{aligned}$$

Thus $g \in \partial f(x)$.

### 2.2

Give a subgradient of $J(w) = \max\left\{0, 1 - yw^T x\right\}$.

From 2.1 we have

$$\partial J(w) = \begin{cases} -yx & yw^T x \leq 1 \\ 0 & otherwise \end{cases}$$

# 3. Perceptron

### 3.1

Show that if $\{x \mid w^T x = 0\}$ is a separating hyperplane for a training set $\mathcal{D} = ((x_1, y_1), \ldots, (x_n, y_n))$, then the average perceptron loss on $\mathcal{D}$ is 0. Thus any separating hyperplane of $\mathcal{D}$ is an empirical risk minimizer for perceptron loss.

**Proof** Assume $\{x \mid w^T x = 0\}$ is a separating hyperplane. Then $y_i w^T x_i > 0 \ \forall \ i \in \{1, \ldots, n\}$.
Now observe that

$$\begin{aligned}
\text{Average perceptron loss} &= \frac{1}{n} \sum_{i=1}^{n} max\{0, -\hat{y}_i y_i\} \\
&= \frac{1}{n} \sum_{i=1}^{n} max\{0, -y_i w^T x_i\} \\
&= \frac{1}{n} \sum_{i=1}^{n} 0 && \text{since } y_i w^T x_i > 0 \\
&= 0
\end{aligned}$$

### 3.2

Let $\mathcal{H}$ be the linear hypothesis space consisting of functions $x \mapsto w^T x$. Consider running stochastic subgradient descent (SSGD) to minimize the empirical risk with the perceptron loss. We'll use the version of SSGD in which we cycle through the data points in each epoch. Show that if we use a fixed step size 1, we terminate when our training data are separated, and we make the right choice of subgradient, then we are exactly doing the Perceptron algorithm.

**Proof** By 2.1 we have $\partial l(\hat{y}, y) = \begin{cases} -yx & yw^T x < 0 \\ 0 & otherwise \end{cases}$

With this choice of subgradient and a step size of 1 we can see that the update step is

$$w^{(k+1)} = \begin{cases} w^{(k)} + y_i x_i & y_i w^T x_i < 0 \\ w^{(k)} & otherwise \end{cases}.$$

Note that this is precisely the update step in the perceptron algorithm.

Now for the terminating conditions.

Note that when the training data is separated, we have $y_i w^T x_i > 0 \; \forall \; i \in \{1, \ldots, n\}$. In the perceptron algorithm this will result in all_correct = True and termination of that algorithm. Thus the terminating conditions are equivalent.

### 3.3

Suppose the perceptron algorithm returns $w$. Show that $w$ is a linear combination of the input points. That is, we can write $w = \sum_{i=1}^{n} \alpha_i x_i$ for some $\alpha_1, \ldots, \alpha_n \in \mathbf{R}$. The $x_i$ for which $\alpha_i \neq 0$ are called support vectors. Give a characterization of points that are support vectors and not support vectors.

**Proof** From 3.2 we have that the update step is

$$w^{(k+1)} = \begin{cases} w^{(k)} + y_i x_i & y_i w^T x_i < 0 \\ w^{(k)} & \textit{otherwise} \end{cases}.$$

Then since $w^{(0)} = 0$ and $y_i \in \{-1, 1\}$ it follows that can write $w = \sum_{i=1}^{n} \alpha_i x_i$ for some $\alpha_1, \ldots, \alpha_n \in \mathbf{R}$.

The support vectors are vectors that were misclassified at some point during the optimization. When this happens, $y_i w^T x_i < 0$ which results in $w^{(k+1)} = w^{(k)} + y_i x_i$ and so $\alpha_i \neq 0$.

# 4. The Data

### 4.1

Load all the data and randomly split it into 1500 training examples and 500 validation examples.

```
In [27]: from load import shuffle_data
         from sklearn.model_selection import train_test_split

         # I added a return statement to shuffle_data
         data = shuffle_data()
         train, val = train_test_split(data, train_size=1500, test_size=500)
         x_train = [l[:-1] for l in train ]
         y_train = [l[-1] for l in train]
         x_val = [l[:-1] for l in val ]
         y_val = [l[-1] for l in val]
```

# 5. Sparse Representations

## 5.1

Write a function that converts an example (e.g. a list of words) into a sparse bag-of-words representation.

```
In [13]: from collections import Counter

         def to_sparse(word_list):
             return Counter(word_list)
```

# 6. Support Vector Machine via Pegasos

## 6.1

Consider the "stochastic" SVM objective function, which is the SVM objective function with a single training point[1]: $J_i(w) = \frac{\lambda}{2}\|w\|^2 + \max\left\{0, 1 - y_i w^T x_i\right\}$. The function $J_i(\theta)$ is not differentiable everywhere. Give an expression for the gradient of $J_i(w)$ where it's defined, and specify where it is not defined.

**Solution** $\nabla J_i(w) = \begin{cases} \lambda w - y_i x_i & y_i w^T x_i < 1 \\ \lambda w & y_i w^T x_i > 1 \\ undefined & y_i w^T x_i = 1 \end{cases}$

[1]Recall that if $i$ is selected uniformly from the set $\{1, \ldots, m\}$, then this stochastic objective function has the same expected value as the full SVM objective function.

### 6.2

Show that a subgradient of $J_i(w)$ is given by

$$g = \begin{cases} \lambda w - y_i x_i & \text{for } y_i w^T x_i < 1 \\ \lambda w & \text{for } y_i w^T x_i \geq 1. \end{cases}$$

You may use the following facts without proof: 1) If $f_1, \ldots, f_m : \mathbf{R}^d \to \mathbf{R}$ are convex functions and $f = f_1 + \ldots + f_m$, then $\partial f(x) = \partial f_1(x) + \ldots + \partial f_m(x)$. 2) For $\alpha \geq 0$, $\partial(\alpha f)(x) = \alpha \partial f(x)$.

**Solution** Let $f_1(w) = \frac{\lambda}{2}\|w\|^2$ and $f_2(w) = max\{0, 1 - y_i w^T x_i\}$. Note that $J_i(w) = f_1(w) + f_2(w)$.
Now $\partial f_1(w) = \lambda w$.

From the results of question 2.1 we also have $\partial f_2(w) = \begin{cases} -y_i x_i & \text{for } y_i w^T x_i < 1 \\ 0 & \text{for } y_i w^T x_i \geq 1. \end{cases}$

Then from fact 1 we can conclude that $g = \begin{cases} \lambda w - y_i x_i & \text{for } y_i w^T x_i < 1 \\ \lambda w & \text{for } y_i w^T x_i \geq 1. \end{cases}$

**6.3**

Show that if your step size rule is $\eta_t = 1/(\lambda t)$, then doing SGD with the subgradient direction from the previous problem is the same as given in the pseudocode.

**Solution** Observe that

$$\begin{aligned}
w_{t+1} &= w_t - \eta_t g(w_t) \\
&= \begin{cases} w_t - \eta_t \lambda w_t + \eta_t y_i x_i & \text{for } y_i w_t^T x_i < 1 \\ w_t - \eta_t \lambda w_t & \text{for } y_i w_t^T x_i \geq 1. \end{cases} \\
&= \begin{cases} w_t(1 - \eta_t) + \eta_t y_i x_i & \text{for } y_i w_t^T x_i < 1 \\ w_t(1 - \eta_t) & \text{for } y_i w_t^T x_i \geq 1. \end{cases}
\end{aligned}$$

This is equivalent to the update step in the Pegasos algorithm.

### 6.4

Implement the Pegasos algorithm to run on a sparse data representation.

```
In [44]: def sparse_multiply(c, x):
             # multiply values in dictionary x by scaler c
             # we'll use counters to be consistent
             out = Counter()
             for k, v in x.items():
                 out[k] = c*v
             return out
```

```
In [65]: from util import dotProduct, increment

         def pegasos_svm(x, y, lambda_reg, n_epochs=10):
             t=0
             w = Counter()
             for epoch in range(n_epochs):
                 for x_i, y_i in zip(x, y):
                     t += 1
                     eta = 1/(t*lambda_reg)
                     scale = (1 - eta * lambda_reg)
                     # Rescale w
                     for k, v in w.items():
                         w[k] = scale * v
                     # Apply gradient if needed
                     if y_i * dotProduct(w, x_i) < 1:
                         support_grad = sparse_multiply(eta * y_i, x_i)
                         increment(w, 1, support_grad)
             return w
```

### 6.5

#### 6.5.1

Verify that the Pegasos update step is equivalent to:

$$\begin{aligned} s_{t+1} &= (1 - \eta_t \lambda)\, s_t \\ W_{t+1} &= W_t + \frac{1}{s_{t+1}} \eta_t y_j x_j. \end{aligned}$$

Where $s_{t+1} = (1 - \eta_t \lambda)\, s_t$

**Proof** Observe that

$$\begin{aligned} w_{t+1} &= s_{t+1} W_{t+1} \\ &= s_{t+1} \left( W_t + \frac{1}{s_{t+1}} \eta_t y_j x_j \right) \\ &= s_{t+1} W_t + \eta_t y_j x_j \\ &= (1 - \eta_t \lambda) s_t W_t + \eta_t y_j x_j \\ &= (1 - \eta_t \lambda) w_t + \eta_t y_j x_j \end{aligned}$$

This is the same as the update step in the Pegasos algorithm.

#### 6.5.2

Implement the Pegasos algorithm with the $(s, W)$ representation described above.

```
In [69]: def pegasos_svm_modified(x, y, lambda_reg, n_epochs=10):
             t = 1
             s = 1
             W = Counter()
             for epoch in range(n_epochs):
                 for x_i, y_i in zip(x, y):
                     t += 1
                     eta = 1/(t*lambda_reg)
                     s = (1 - eta * lambda_reg) * s
                     if y_i * s * dotProduct(W, x_i) < 1:
                         support_grad = sparse_multiply(eta * y_i, x_i)
                         increment(W, 1/s, support_grad)
             return sparse_multiply(s,W)
```

### 6.6

Run both implementations of Pegasos on the training data for a couple epochs (using the bag-of-words feature representation described above). Make sure your implementations are correct by verifying that the two approaches give essentially the same result. Report on the time taken to run each approach.

In [31]:
```
x_train_sparse = [to_sparse(x) for x in x_train]
x_val_sparse = [to_sparse(x) for x in x_val]
```

In [96]:
```
from timeit import default_timer as timer

n_epochs = 2
lambda_reg = 1

start = timer()
w1 = pegasos_svm(x_train_sparse, y_train, lambda_reg, n_epochs)
end = timer()
delta_t = end-start
print("Standard algorithm took {} seconds to complete {} epochs".format(delta_t, n_epoc

start = timer()
w2 = pegasos_svm_modified(x_train_sparse, y_train, lambda_reg, n_epochs)
end = timer()
delta_t = end-start
print("Modified algorithm took {} seconds to complete {} epochs".format(delta_t, n_epoc
```

```
Standard algorithm took 20.150097092962824 seconds to complete 2 epochs
Modified algorithm took 0.5985467850696295 seconds to complete 2 epochs
```

In [91]:
```
# Check for similar results
w1_values = list(w1.values())
w2_values = list(w2.values())
print(w1_values[:3])
print(w2_values[:3])
```

```
[0.0013333333333333372, -0.012999999999999979, -0.00133333333333333]
[0.001332889036987675, -0.012995668110629811, -0.0013328890369876678]
```

### 6.7

Write a function that takes a sparse weight vector $w$ and a collection of $(x, y)$ pairs, and returns the percent error when predicting $y$ using $sign(w^T x)$. In other words, the function reports the 0-1 loss of the linear predictor $x \mapsto w^T x$.

```
In [100]: def percent_error(x, y, w):
              n_wrong = 0
              for x_, y_ in zip(x,y):
                  y_pred = dotProduct(x_,w)
                  if y_pred * y_ < 0:
                      n_wrong += 1
              return n_wrong / len(y)
```

### 6.8

Using the bag-of-words feature representation described above, search for the regularization parameter that gives the minimal percent error on your test set. (You should now use your faster Pegasos implementation, and run it to convergence.) A good search strategy is to start with a set of regularization parameters spanning a broad range of orders of magnitude. Then, continue to zoom in until you're convinced that additional search will not significantly improve your test performance. Once you have a sense of the general range of regularization parameters that give good results, you do not have to search over orders of magnitude every time you change something (such as adding a new feature)

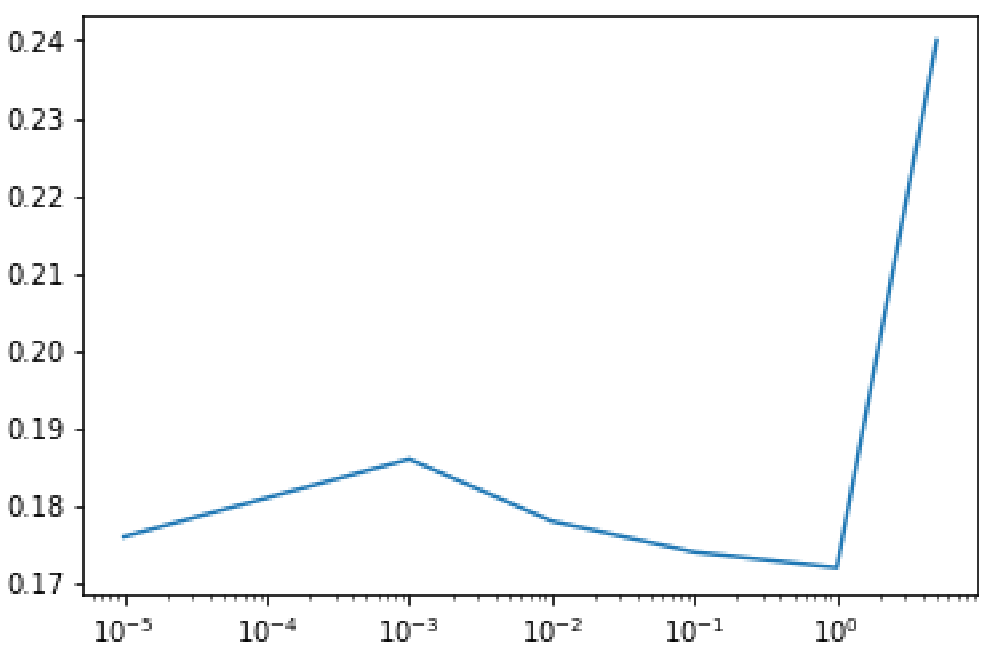
```
In [165]: from tqdm import tqdm
          import matplotlib.pyplot as plt
          import matplotlib
          matplotlib.rcParams['figure.figsize'] = [15, 10]
```

```
In [117]: lambdas = [1e-5, 1e-3, 1e-2, 1e-1, 1, 5]
          errors = []

          for l in tqdm(lambdas):
              w = pegasos_svm_modified(x_train_sparse, y_train, l, n_epochs=1000)
              errors.append(percent_error(x_val_sparse, y_val, w))
```

```
100%|| 6/6 [22:46<00:00, 269.07s/it]
```

```
In [118]: plt.plot(lambdas, errors)
          plt.xscale('log')
```

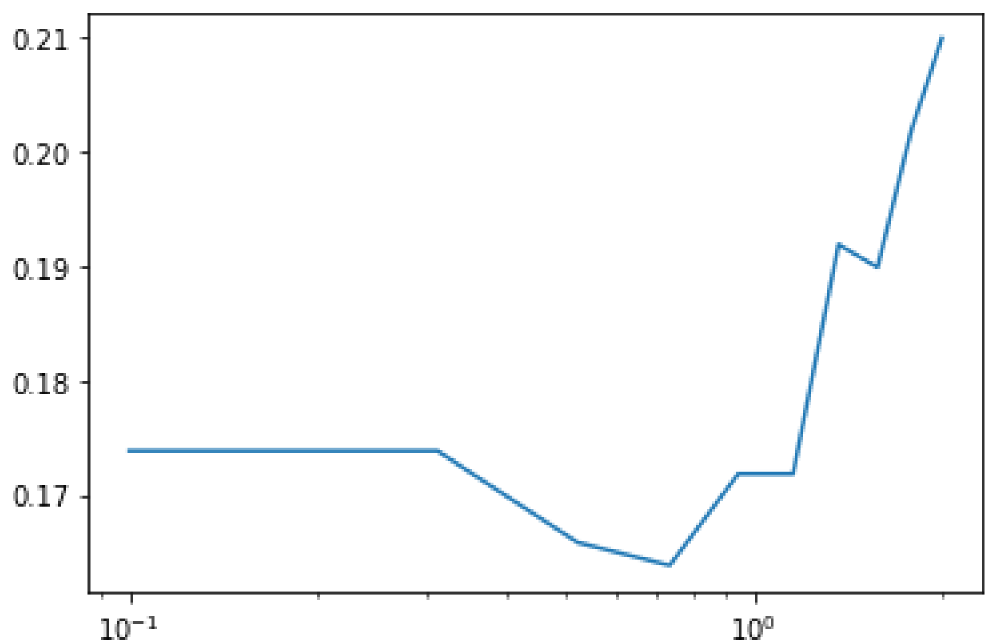


In [119]:
```
lambdas = np.linspace(0.1, 2, 10)
errors = []

for l in tqdm(lambdas):
    w = pegasos_svm_modified(x_train_sparse, y_train, l, n_epochs=1000)
    errors.append(percent_error(x_val_sparse, y_val, w))
```

```
100%|| 10/10 [1:40:36<00:00, 647.60s/it]
```

In [121]:
```
plt.plot(lambdas, errors)
plt.xscale('log')
```



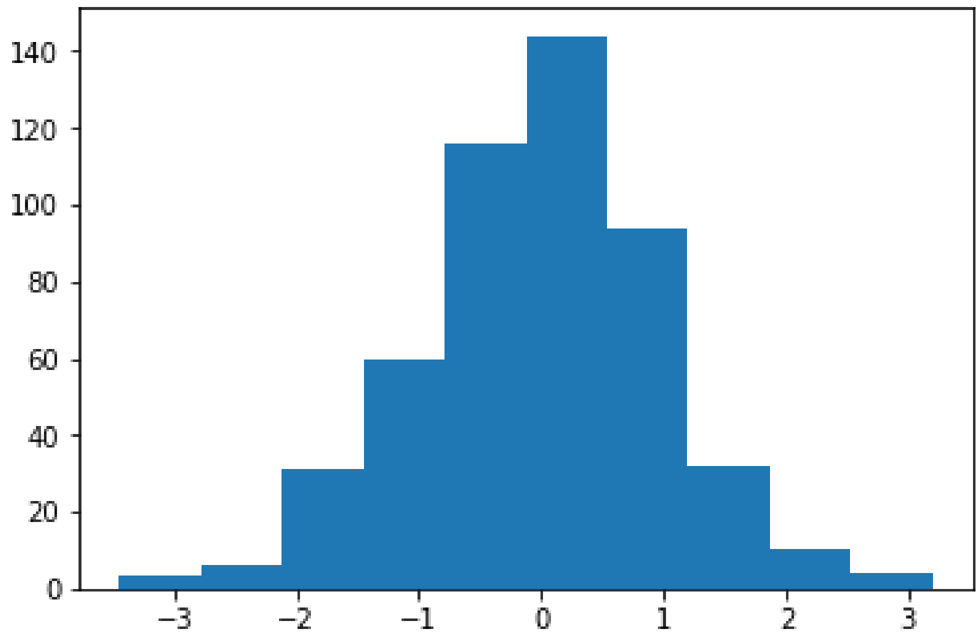

In [130]:
```
best_lambda = lambdas[np.argmin(errors)]
w = pegasos_svm_modified(x_train_sparse, y_train, best_lambda, n_epochs=1000)
print("The best value of lambda is {} with a validation error of {}".format(best_lambd
```

```
The best value of lambda is 0.7333333333333333 with a validation error of 0.164
```

### 6.9

Recall that the "score" is the value of the prediction $f(x) = w^T x$. We like to think that the magnitude of the score represents the confidence of the prediction. This is something we can directly verify or refute. Break the predictions into groups based on the score (you can play with the size of the groups to get a result you think is informative). For each group, examine the percentage error. You can make a table or graph. Summarize the results. Is there a correlation between higher magnitude scores and accuracy?

In [134]:
```
# First lets look at the range of predicted values
ax = plt.hist(y_pred)
```




In [170]:
```
from sklearn.metrics import accuracy_score

def standard_error(y, y_pred):
    p = accuracy_score(y, y_pred)
    n = len(y)
    return np.sqrt(p*(1-p)/n)

# Try with splitting into 3 groups
# |y| < 1
# 1 < |y| < 2
# |y| > 2

score = np.array([dotProduct(w, x) for x in x_val_sparse])
score_mag = np.abs(score)
```

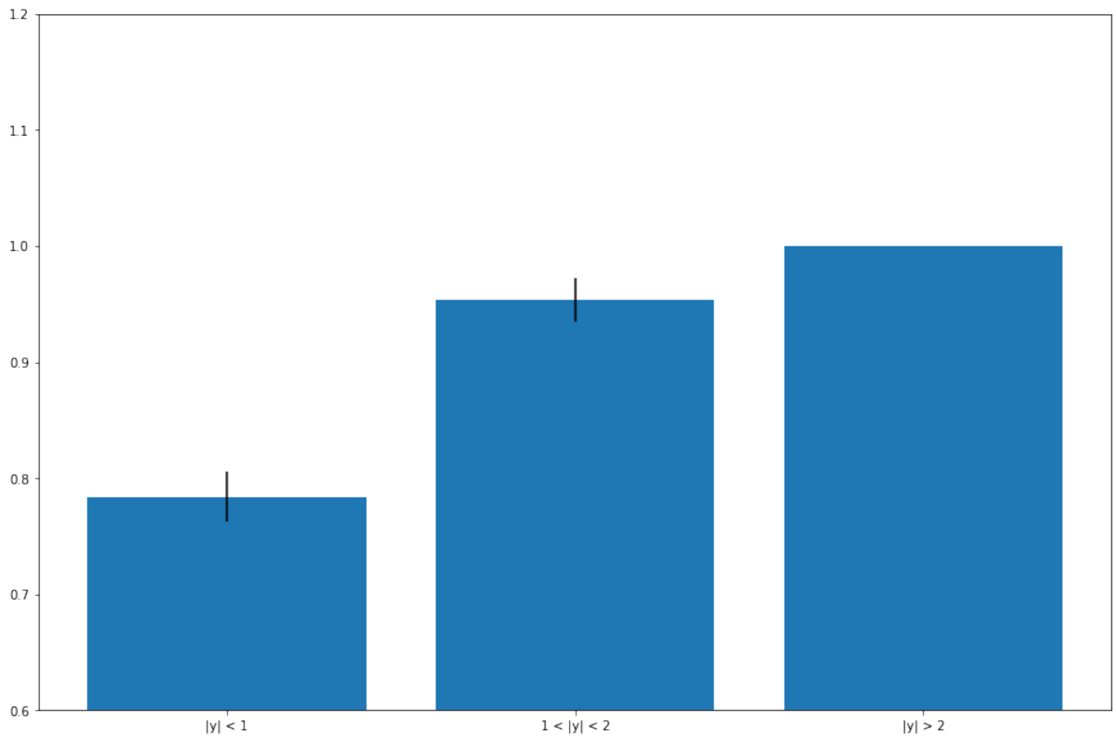
```
idx_1 = np.argwhere(score_mag < 1)
idx_2 = np.argwhere((score_mag > 1) & (score_mag < 2))
idx_3 = np.argwhere(score_mag > 2)

y_pred_1 = np.sign(score[idx_1])
y_pred_2 = np.sign(score[idx_2])
y_pred_3 = np.sign(score[idx_3])
y_preds = [y_pred_1, y_pred_2, y_pred_3]

y_1 = np.array(y_val)[idx_1]
y_2 = np.array(y_val)[idx_2]
y_3 = np.array(y_val)[idx_3]
y_trues = [y_1, y_2, y_3]

accuracies = [accuracy_score(y_pred, y_true) for y_pred, y_true in zip(y_preds, y_true
errors = [standard_error(y_pred, y_true) for y_pred, y_true in zip(y_preds, y_trues)]
```

```
In [172]: ax = plt.bar(["|y| < 1", "1 < |y| < 2", "|y| > 2"], accuracies, yerr=errors)
          _ = plt.ylim(0.6,1.2)
```




There appears to be some correlation between the score magnitude and the accurace of the predictions.

## 7. Error Analysis

### 7.1

Investigate the feature importances of two misclasified points.

```
In [201]: import pandas as pd

          def get_feature_importances(x, w):
              # Get the feature weights of features in x soreted by magnitude
              features = [(word, count*w[word]) for word, count in x.items()]
              sorted_features = sorted(features, key=lambda x: np.abs(x[1]), reverse=True)
              return pd.DataFrame(sorted_features)
```

```
In [190]: y_pred = np.sign(score)
          incorrect_examples = np.array(x_val_sparse)[y_val != y_pred]
          incorrect_examples_raw = np.array(x_val)[y_val != y_pred]
          y_true_incorrect = np.array(y_val)[y_val != y_pred]
```

#### First example

```
In [202]: feature_importances_1 = get_feature_importances(incorrect_examples[0], w)
          print("True label was {}".format(y_true_incorrect[0]))
          feature_importances_1[:10]
```

```
True label was -1
```

Out[202]:

| | 0 | 1 |
|---|---|---|
| 0 | and | 0.329479 |
| 1 | to | -0.246941 |
| 2 | is | 0.198787 |
| 3 | be | -0.167227 |
| 4 | very | 0.160676 |
| 5 | with | 0.142685 |
| 6 | many | 0.121658 |
| 7 | even | -0.106560 |
| 8 | the | 0.101324 |
| 9 | on | -0.093114 |

```
In [197]: print(' '.join(incorrect_examples_raw[0]))
```

```
in 1970s many european intellectuals especially those on the left political hemisphere became ob
```

**Analysis** The true label here is -1 which means the model predicted this as a positive review. Looking at the feature importance we see that many words that we would consider neutral contributed greatly to the positive clasification. These words include "and", "is", and "the". A possible explanation is that, by chance, these words have a small positive weight in the weight vector.

However, since they are so common, their effects accumulate and eventually dominate the classification especially in longer reviews such as this one. In fact, let's look at the weight of some of these words in the weight vector.

```
In [207]: print(w['as'])
          print(w['is'])
          print(w['the'])
          print(w['bad'])
```

```
0.015632716850917223
0.02839816288607632
0.003166361525456027
-0.1164499223667763
```

As we can see, these words have much smaller weights especially when compared to a more strongly polarized word such as "bad". Removal of these common words ("stopwords") could helpin improving the model. Using tf-idf could also help here.

Another thing to note is the strong positive contribution by the word "very". It is probably true that, in general, the word "very" is used in a positive manner. However in this review at one point it is preceded by a "not". Introducing bigrams may give the model some more context into how the words were used and increase performance.

**Second example**

```
In [222]: feature_importances_2 = get_feature_importances(incorrect_examples[15], w)
          print("True label was {}".format(y_true_incorrect[15]))
          feature_importances_2[:15]
```

```
True label was -1
```

```
Out[222]:          0         1
          0       and  0.278790
          1        is  0.198787
          2        be -0.133782
          3      with  0.118904
          4        on -0.093114
          5      this -0.092986
          6      from  0.086779
          7        to -0.082314
          8        or -0.071262
          9         a  0.065479
          10      the  0.063327
          11   enough -0.061673
          12        ? -0.059773
          13     last -0.057795
          14       an -0.051053
```

```
In [341]: print(' '.join(incorrect_examples_raw[15]))
```

```
silly performances and some huge gaps in logic mar an otherwise interesting tale of an eclectic
```

**Analysis** Again we see strong contributions by common, neutral words such as "and", "is", and "with". One thing I noticed is that many of these incorrectly classified examples are very long. It appears that the model has a hard time classifying long reviews due to the large number of these common words. The prediction is dominated by whatever direction these words just happen to point. Again, removal of stopwords or using tf-idf could help here.

# 8. Features

### 8.2 Try to get the best performance possible by generating lots of new features

We will try the following modifications all at once:

- Removal of stopwords
- Stemming
- Adding number of positive and negative words as a feature
- Bigrams
- Tf-idf

In [291]:
```
class DictTfidf:
    def __init__(self):
        self.logdf = {}
        self.n_docs = Counter()

    def fit(self, x):
        self.n_docs = Counter()
        for document in x:
            for word in document.keys():
                self.n_docs[word] += 1
        N = len(x)
        self.logdf = {key: np.log10(N/v) for key, v in self.n_docs.items()}

    def transform(self,x):
        x_t = []
        for document in x:
            doc_transformed = {}
            for word, count in document.items():
                if word in self.logdf:
                    doc_transformed[word] = count * self.logdf[word]
            x_t.append(doc_transformed)
        return x_t
```

In [226]:
```
# Load sets of positive and negative words
with open('positive-words.txt') as f:
    pos_words = {word.strip() for word in f.readlines()}
with open('negative-words.txt') as f:
    neg_words = {word.strip() for word in f.readlines()}
```

In [267]:
```
from nltk.stem import PorterStemmer
from nltk.corpus import stopwords
stopwords = set(stopwords.words('english'))
```

In [309]:
```
# Get number of positive and negative words
n_pos_train = [len([w for w in doc if w in pos_words]) for doc in x_train]
n_neg_train = [len([w for w in doc if w in neg_words]) for doc in x_train]
n_pos_val = [len([w for w in doc if w in pos_words]) for doc in x_val]
n_neg_val = [len([w for w in doc if w in neg_words]) for doc in x_val]
```

In [284]:
```
# Remove stopwords
x_train_featurized = [[w for w in doc if w not in stopwords] for doc in x_train]
x_val_featurized = [[w for w in doc if w not in stopwords] for doc in x_val]
```

In [285]:
```
# Stemming
ps = PorterStemmer()
x_train_featurized = [[ps.stem(w) for w in doc] for doc in x_train_featurized ]
x_val_featurized = [[ps.stem(w) for w in doc] for doc in x_val_featurized ]
```

In [286]:
```
# Bigrams
for doc in x_train_featurized:
    bigrams = [w1 + ' ' + w2 for w1, w2 in zip(doc[:-1], doc[1:])]
    doc += bigrams

for doc in x_val_featurized:
    bigrams = [w1 + ' ' + w2 for w1, w2 in zip(doc[:-1], doc[1:])]
    doc += bigrams
```

In [287]:
```
# Convert to dictionaries
x_train_featurized_sparse = [to_sparse(doc) for doc in x_train_featurized]
x_val_featurized_sparse = [to_sparse(doc) for doc in x_val_featurized]
```

In [293]:
```
# Apply tfidf
tfidf = DictTfidf()
tfidf.fit(x_train_featurized_sparse)
x_train_featurized_sparse = tfidf.transform(x_train_featurized_sparse)
x_val_featurized_sparse = tfidf.transform(x_val_featurized_sparse)
```

In [312]:
```
# Finally, add the numeber of positive and negative words
pos_key = 'N_POS'
neg_key = 'N_NEG'
for doc, n in zip(x_train_featurized_sparse, n_pos_train):
    doc[pos_key] = n
for doc, n in zip(x_train_featurized_sparse, n_neg_train):
    doc[neg_key] = n
for doc, n in zip(x_val_featurized_sparse, n_pos_val):
    doc[pos_key] = n
for doc, n in zip(x_val_featurized_sparse, n_neg_val):
    doc[neg_key] = n
```

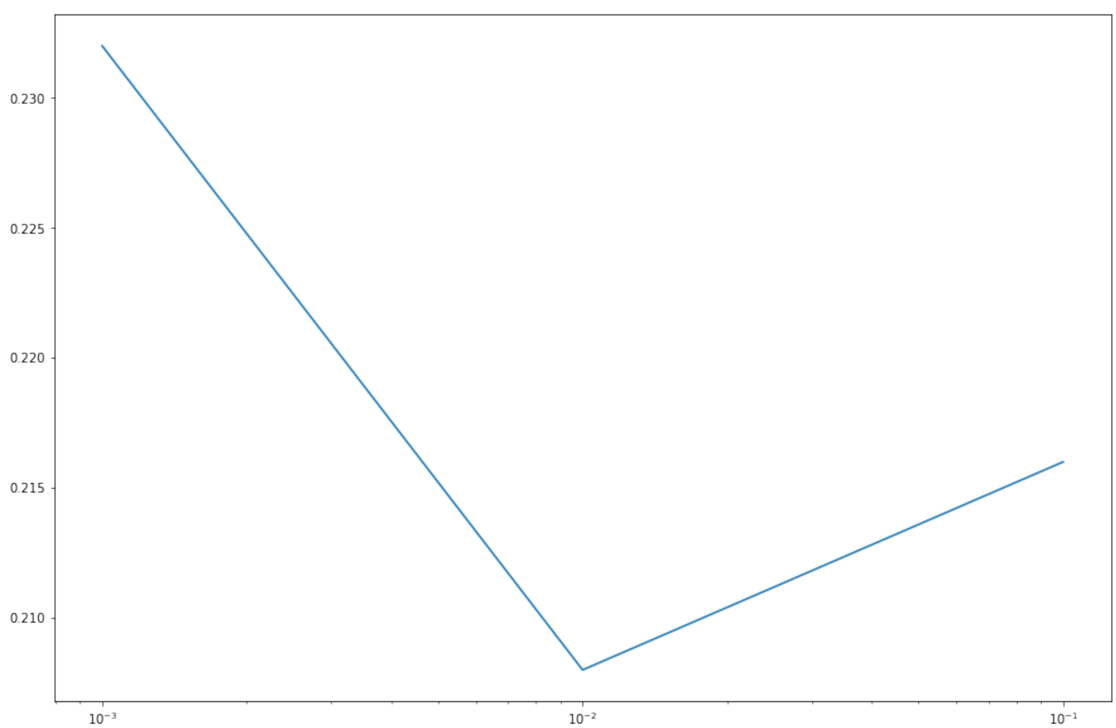
**First Param Search**

```
In [330]: #lambdas = np.linspace(0.01, 0.1, 10)
          lambdas = [0.001, 0.01, 0.1]
          errors = []
          ws = []

          for l in tqdm(lambdas):
              w = pegasos_svm_modified(x_train_featurized_sparse, y_train, l, n_epochs=1000)
              ws.append(w)
              errors.append(percent_error(x_val_featurized_sparse, y_val, w))
```

```
100%|| 3/3 [28:40<00:00, 568.25s/it]
```

```
In [331]: plt.plot(lambdas, errors)
          plt.xscale('log')
```



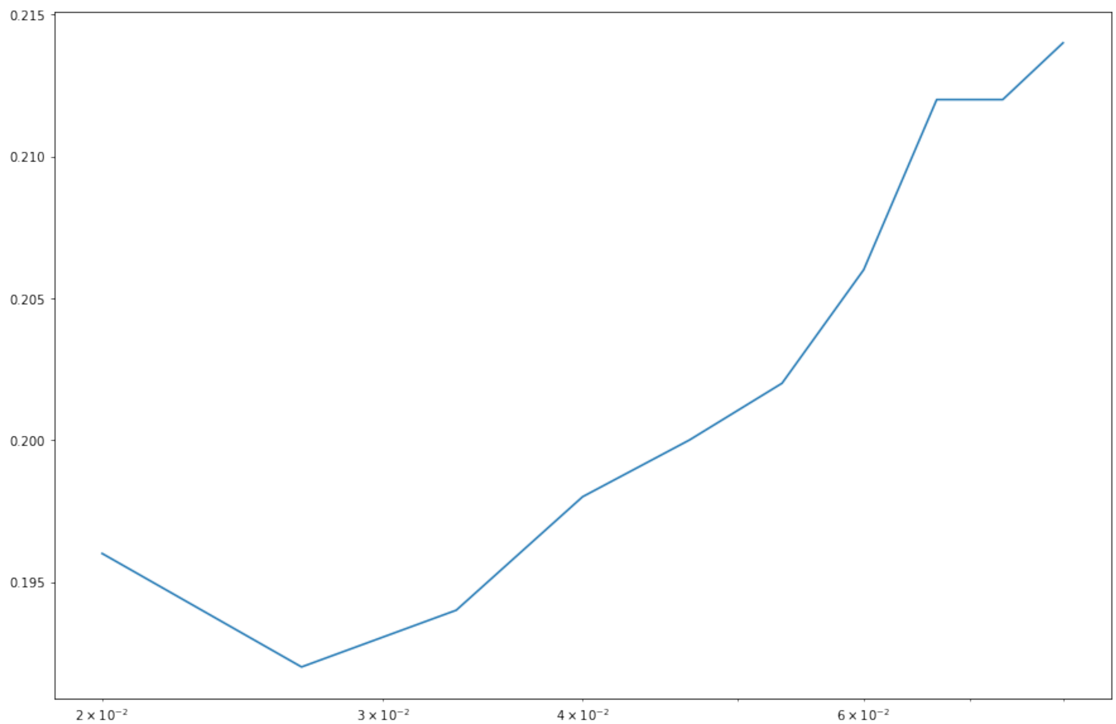
**Second Param Search**

```
In [333]: lambdas = np.linspace(0.02, 0.08, 10)
          errors = []
          ws = []

          for l in tqdm(lambdas):
              w = pegasos_svm_modified(x_train_featurized_sparse, y_train, l, n_epochs=500)
              ws.append(w)
              errors.append(percent_error(x_val_featurized_sparse, y_val, w))
```

```
100%|| 10/10 [48:54<00:00, 288.04s/it]
```

```
In [334]: plt.plot(lambdas, errors)
          plt.xscale('log')
```




Ok I'm not getting better results. Maybe I tried too many things at once.........