Fuzzy Path Tracking Control of a Vehicle

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Abstract: This paper deals with the Path Following (PF) problem of a car. A unified kinematics model is derived for this problem. The mobile target configuration is represented by the motion of a reference car which has the same kinematics constrains as the real one. The generated kinematics model is added to the car dynamics one in order to obtain a general state representation for the PF problem. Firstly, this problem is solved by means of a state feedback control law, then by a fuzzy control one. The stability analysis of the fuzzy control system of the vehicle dynamics is discussed using Lyapunov's approach and convex optimization techniques based on Linear Matrix Inequalities (LMI). Finally, simulation results are given to demonstrate the controller's effectiveness.

1. INTRODUCTION

Recently, fuzzy control has become a popular research in the control engineering. The fuzzy logic controller has made itself available not only in the laboratory work but also in industrial applications [1-11], mostly based on the knowledge and experience of a human operator. In recent years, theoretical developments of fuzzy control have been proposed and the construction and the use of fuzzy controllers have explored [12-16]. These works are essentially based on a fuzzy model of the process and on Lyapunov stability to design the fuzzy control law.
One important application of fuzzy control is in vehicles : maritime, space and ground vehicles. In [1], Waneck proposed a fuzzy controller for an autonomous boat without initially having to develop nonlinear dynamics model of the vehicle. Sugeno [2] [3] has designed a fuzzy controller based on fuzzy modeling of human operator's control actions, to navigate and to park a car. Larkin [4] has proposed a fuzzy controller for aircraft flight control where the fuzzy rules are generated by interrogating an experienced pilot and asking him a number of highly structured questions. In [5], the authors have designed an autopilot for ships by translating the steering behaviour of a human controller into a fuzzy mathematical model. In [6], A fuzzy control that uses rules based on a skilled human operator's experience is applied to automatic train operations. Nguyen and Widrows [7] have developed a neural network controller for the truck backer upper to a loading dock problem from an arbitrary initial position by manipulating the steering. Kong and Kosko [8] have proposed a fuzzy control strategy for the same problem. In [9], Wang has solved the same problem by generating fuzzy rules using learning algorithms. However, all the above studies do not treat the PF problem and have not analyzed stability of the control systems.
Normally, vehicles are used to transport goods or passengers. Most of them are manually controlled. But there are situations where manual control is not desirable. For example in a polluted environment such as chemical factories and nuclear power stations. In such situations, the necessity of auto-guided vehicles arises.
The local asymptotic stability using a state feedback control law is shown in this study. This approach, however, generally only renders a local result. This paper focuses on the design of fuzzy controller of vehicle using nonlinear dynamics model to treat nonlinearities of control systems, after have presented a unified control scheme for the PF problem. A representation of the configuration error in the basis of the frame linked to the car target configuration is used.
The paper is organized as follows: in section 2, the car dynamics model is presented, thereafter, a kinematics model based on mobile target configuration tracking is derived for the PF problem. Section 3, is devoted to the state feedback control law synthesis. The fuzzy control for the PF problem is presented in section 4, where the stability analysis of fuzzy control system using Lyapunov's approach and convex optimization techniques based on Linear Matrix Inequalities (LMI) are also presented. In section 5, simulation results are given to highlight the effectiveness of the proposed control laws. Section 6, concludes the paper.

2- PROBLEM STATEMENT

2.1- car dynamics model

Generally, the dynamics models of the vehicles is a Multi-Input Multi-Output (MIMO) system. Such dynamics models have been developed [23][24], and are used by car constructors [25] in order to simulate the vehicle behavior. Consider the dynamics equations describing the vehicle, as introduced in [18][24][26]:

$$
\begin{aligned}
\dot{u} &= v.r - f.g + \left(\frac{f.k_1 - k_2}{M}\right)u^2 + \frac{Cf}{M}\left(\frac{v}{u} + \frac{a.r}{u}\right)\delta + \frac{1}{M}T \\
\dot{v} &= -u.r - \left(\frac{Cf + Cr}{M}\right).\frac{v}{u} + \left(\frac{b.Cr - a.Cf}{M}\right).\frac{r}{u} + \frac{Cf}{M}\delta + \frac{1}{M}T\delta \qquad (1) \\
\dot{r} &= -\frac{f.M.h}{I_z}.u.r + \left(\frac{b.Cr - a.Cf}{I_z}\right).\frac{v}{u} - \left(\frac{b^2.Cr + a^2.Cf}{I_z}\right).\frac{r}{u} + \frac{a.Cf}{I_z}\delta + \frac{a}{I_z}T\delta \\
\dot{\varphi} &= r
\end{aligned}
$$

This dynamics model is MIMO system with 4 state variables and two control actions. These variables correspond to longitudinal and lateral displacement (x, y) of the car's center M, longitudinal and lateral velocity (u, v), the angular velocity r about the vertical axe, and the orientation angle φ of the car with respect to the abscises axis $\vec{i}_0$. The control actions of the vehicle are the force of propulsion or the force of braking T and the steering angle δ (see Figure 1). Constants of the system are indicated in appendix.

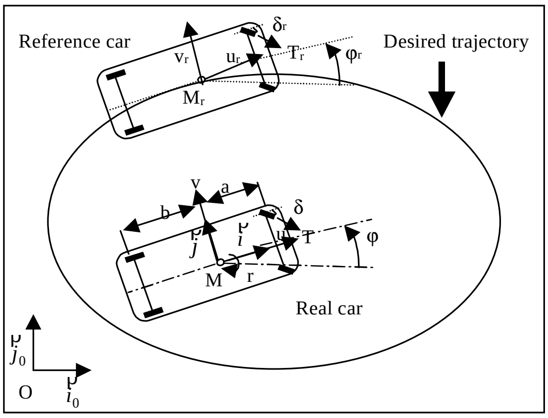

Figure. 1. Path following representation

2.2 Path following problem

As shown in Figure 1, the target configuration is represented by a reference car with the same kinematics constraints as the real one.

Let $(u,v,\dot{\varphi})$ and $(u_r,v_r,\dot{\varphi}_r)$ be respectively the longitudinal, lateral, angular velocity of the real and the reference car.

Our objective is to determine the necessary controls T and δ to superpose the real car and the reference one by vanishing the error configuration $(x_e,y_e,\varphi_e)^T$, where (x_e,y_e) represent the coordinates of the position error vector $\overrightarrow{MM}_r$ in the frame $R_1(M,\vec{i},\vec{j})$ linked to the real car, and $\varphi_e=\varphi-\varphi_r$ denotes the orientation error between both cars.

The position error vector can be written in the mobile frame R_1 as follows:

$$\overrightarrow{MM}_r = x_e\vec{i} + y_e\vec{j} \tag{2}$$

Differentiating (2) with respect to time yields:

$$\frac{d\overrightarrow{MM}_r}{dt} = \dot{x}_e\vec{i} + \dot{y}_e\vec{j} + x_e\dot{\varphi}\vec{j} - y_e\dot{\varphi}\vec{i} \tag{3}$$

Furthermore, we have:

$$\frac{d\overrightarrow{MM}_r}{dt} = \frac{d\overrightarrow{OM}_r}{dt} - \frac{d\overrightarrow{OM}}{dt} \tag{4}$$

Where:

$$\frac{d\overrightarrow{OM_r}}{dt} = u_r\cos\varphi_e\,\vec{i} - u_r\sin\varphi_e\,\vec{j} + v_r\sin\varphi_e\,\vec{i} + v_r\cos\varphi_e\,\vec{j} \tag{5}$$

$$\frac{d\overrightarrow{OM}}{dt} = u\vec{i} + v\vec{j} \tag{6}$$

Substituting (3), (5) and (6) in (4), one obtain:

$$\begin{aligned} \dot{x}_e &= u_r\cos\varphi_e + v_r\sin\varphi_e - u + y_e\dot{\varphi} \\ \dot{y}_e &= -u_r\sin\varphi_e + v_r\cos\varphi_e - v - x_e\dot{\varphi} \end{aligned} \tag{7}$$

Furthermore, from Figure 1, one has:

$$\dot{\varphi}_e = \dot{\varphi} - \dot{\varphi}_r \tag{8}$$

Finally, the state representation for the path following car problem can be written as follows:

$$\begin{aligned} \dot{u} &= v.r - f.g + \left(\frac{f.k_1 - k_2}{M}\right)u^2 + \frac{Cf}{M}\left(\frac{v}{u} + \frac{a.r}{u}\right)\delta + \frac{1}{M}T \\ \dot{v} &= -u.r - \left(\frac{Cf+Cr}{M}\right).\frac{v}{u} + \left(\frac{b.Cr-a.Cf}{M}\right).\frac{r}{u} + \frac{Cf}{M}\delta + \frac{1}{M}T\delta \\ \dot{r} &= -\frac{f.M.h}{I_z}u.r + \left(\frac{b.Cr-a.Cf}{I_z}\right).\frac{v}{u} - \left(\frac{b^2.Cr+a^2.Cf}{I_z}\right).\frac{r}{u} + \frac{a.Cf}{I_z}\delta + \frac{a}{I_z}T\delta \\ \dot{x}_e &= u_r\cos\varphi_e + v_r\sin\varphi_e - u + y_e\dot{\varphi} \\ \dot{y}_e &= -u_r\sin\varphi_e + v_r\cos\varphi_e - v - x_e\dot{\varphi} \\ \dot{\varphi}_e &= \dot{\varphi} - \dot{\varphi}_r \end{aligned} \tag{9}$$

Our control objective is to make the vehicle follow a desired trajectory, that is :

$$u \to u_r,\ v \to v_r,\ r \to r_r,\ x_e \to 0,\ y_e \to 0,\ \varphi_e \to 0$$

3 STATE FEEDBACK CONTROL LAW SYNTHESIS

The linearized model of system (9) about the equilibrium configuration $X_e=[u_{ei},v_{ei},r_{ei},0,0,0]^T$, $U=[T_{ei},\delta_{ei}]^T$ is given by the following linear system:

$$\dot{X}_e = AX_e + BU \tag{10}$$

where:

$$A = \begin{bmatrix} \frac{2u_{ei}.f.(k_1-k_2)}{M} - \frac{Cf(v_{ei}+ar_{ei})\delta_{ei}}{Mu_{ei}} & r_{ei} + \frac{Cf\delta_{ei}}{Mu_{ei}} & v_{ei} + \frac{aCf\delta_{ei}}{Mu_{ei}} & 0 & 0 & 0 \\ \left(\frac{Cf+Cr}{Mu_{ei}^2}\right)v_{ei} - \left(\frac{bCr-aCf}{Mu_{ei}^2}\right)r_{ei} - r_{ei} & -\left(\frac{Cf+Cr}{Mu_{ei}}\right) & \left(\frac{bCr-aCf}{Mu_{ei}}\right) - u_{ei} & 0 & 0 & 0 \\ -\frac{fMh}{I_z}r_{ei} - \left(\frac{b^2.Cr+a^2.Cf}{I_zu_{ei}}\right) + \left(\frac{bCr-aCf}{I_zu_{ei}}\right) & \left(\frac{bCr-aCf}{I_zu_{ei}}\right) & -\frac{fMh}{I_z}u_{ei} - \left(\frac{b^2.Cr+a^2.Cf}{I_zu_{ei}}\right) & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & r_{ei} & v_{ei} \\ 0 & -1 & 0 & -r_{ei} & 0 & -u_{ei} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{M} & \frac{Cf(v_{ei}+ar_{ei})}{Mu_{ei}} \\ \frac{\delta_{ei}}{M} & \frac{T_{ei}+Cf}{M} \\ \frac{a\delta_{ei}}{I_Z} & a.\frac{T_{ei}+Cf}{I_Z} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

we can easily check that the system (10) is controllable if u_{ei}, v_{ei}, r_{ei} are not null en same time. Therefore, we can not use this model for point stabilization problem.

The LQR method [27] is used to design a full state variable feedback controller of the form (11) that will guarantee the equilibrium point for this system is asymptotically stable.

$$U = -KX_e \tag{11}$$

A quadratic performance index J is minimized:

$$J = \int_{\infty}^{0}(X^TQX + U^TRU)dt$$

where Q and R are symmetric positive matrix to be chosen as of the control design process.

We present a simulation results for path following problem of the vehicle using the proposed control law. The desired trajectory is a circle described by the motion of the vehicle (u_r=18.43 ,v_r=-0.55 ,r_r=0.23). Using the LQR design with:

$$Q=\begin{bmatrix}10&0&0&0&0&0\\0&10&0&0&0&0\\0&0&10&0&0&0\\0&0&0&10&0&0\\0&0&0&0&10&0\\0&0&0&0&0&10\end{bmatrix},\quad R=\begin{bmatrix}10&0\\0&5000\end{bmatrix}$$

we have:

K=[0.0003 0.000001 0.0004 -0.0002 0.0001 -0.0037;0.0048 0.0004 0.0241 -0.0026 0.0058 -0.2398].

Figure 2 and Figure 3 show, respectively, the asymptotic convergence of the state variables and the vehicle motion starting from (u_0=20,v_0=0,r_0=0,x_0=-10,y_0=-10,φ_0=0).

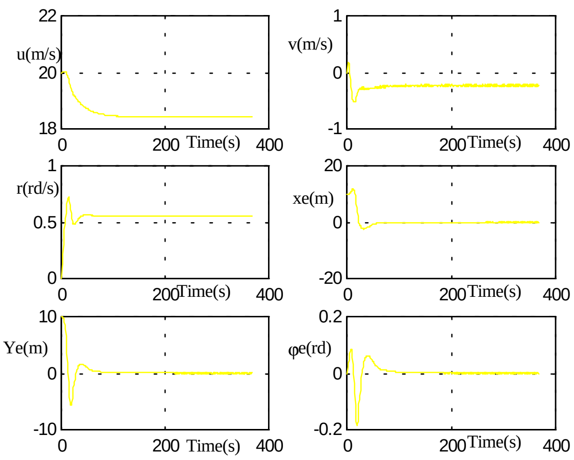

Figure 2. Time evolution of the state variables starting from the configuration (20,0,0,10,10,0)

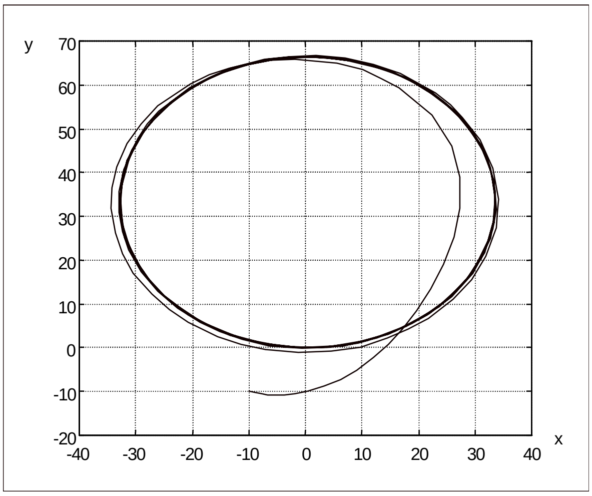

Figure 3 : vehicle trajectory starting from the configuration (20,0,0,10,10,0)

Thus, we have shown the local asymptotic stability using a state feedback control law. This approach, however, generally only renders a local result. The proof global stability requires the formulation of a suitable Lyapunov function. Therefore, we propose a method of nonlinear feedback control introducing fuzzy inference to treat nonlinearities of control systems.

4- ANALYSIS and DESIGN of FUZZY CONTROL SYSTEM

4.1 Fuzzy model of vehicle

The car dynamics model is nonlinear, the main idea is the linearize of the system about some operating point (x_{ei},u_{ei}). The nonlinear dynamics vehicle can be written as follows:

$$\dot{X}=F(X,U,t) \qquad (12)$$

Where $F=[F_1 \ldots F_6]$ is a 6 dimensional vector function of the state vector $X=[u,v,r,xe,ye,\varphi e]$, and control vector $U=[T,\delta]$. The dot ($\dot{}$) denotes differentiation with respect to time t. The functions F_i are continuous and continuously differentiable in their arguments. Assume that for a specified constant control $U=U_e$, there is a corresponding fixed point which satisfies:

$$F(X_e,U_e,t)=0$$

The proposed fuzzy system is described by fuzzy IF-THEN rules which locally represent linear model in the region of some operating point. The fuzzy system can be written as follows:

L^i : If X is $\sim(X_{ei},U_{ei})$ Then $\dot{\bar{X}}=A_i\bar{X}+B_i\bar{U}$

The fuzzy rule L^i represents the linearized system about the operating point (X_{ei},U_{ei}).

Where
$$\bar{X}=X-X_{ei}$$
$$\bar{U}=U-U_{ei}$$

$$A_i=\frac{\partial F}{\partial X}\Big|_{Xei,Uei}=A \;;\; B_i=\frac{\partial F}{\partial U}\Big|_{Xei,Uei}=B$$

Where L^i(i=1,2,..n) denotes the i-th implication. (A_i,B_i) is the i-th local model of the fuzzy system. Let w_i be the membership function of the inferred fuzzy set corresponding to the operating regime (X_{ei},U_{ei}). The final state of the system is inferred by taking the weighted average of all local models.

$$\dot{\bar{X}}=\frac{\sum_{i=1}^{n}w_i(A_i\bar{X}+B_i\bar{U})}{\sum_{i=1}^{n}w_i} \qquad (13)$$

4.2 Design of fuzzy controller

We consider a finite number of operating regime (X_{ei},U_{ei}). In each one the system is characterized by the local linear model. For each model (A_i,B_i), we choose a corresponding linear controller having the following structure:

$$\bar{U}=-K_i^f(x-x^r) \qquad (14)$$

Where x^r is the reference model state and local feedback gain K_i^f is obtained by LQR method as shown in section 3. For each operating regime (X_{ei},U_{ei}), the system is characterized by the local model to a degree given by the membership function. It is natural to exploit the given partition of the operating range and for each operating regime to ascribe to the local controller a validity given by the same membership function. The overall controller is defined by liking the weighted average of all local controller. The membership function are used as smooth interpolations.

Finally the design method of fuzzy controller consists of rules of the type:

R^i : If T_r is $\sim(X_{ei},U_{ei})$ Then $\bar{U}=-K_i^f\bar{X}$

By using the previous rules, the final output of fuzzy control is calculated by:

$$\overline{U} = -\frac{\sum_{i=1}^{n} w_i K_i^f \overline{X}}{\sum_{i=1}^{n} w_i} \qquad (15)$$

4.3 Stability Analysis

In This section, we show an analysis technique of stability. Consider the dynamics system described by the following fuzzy system.

$$\dot{x} = \sum_{i=1}^{n} w_i (A_i \bar{x} + B_i \bar{u}) \Big/ \sum_{i=1}^{n} w_i \qquad (16)$$

Select the reference dynamics model described by:

$$\bar{x}^r = \sum_{i=1}^{n} w_i (A_i \bar{x}^r + B_i \bar{u}^r) \Big/ \sum_{i=1}^{n} w_i \qquad (17)$$

Define the difference between the system state vector and the reference model state vector as the error vector:

$$e_x = x - x^r \qquad (18)$$

Differentiating (18) with respect to time leads to:

$$\dot{e}_x = \sum_{i=1}^{n} w_i (A_i e_x + B_i e_u) / \sum_{i=1}^{n} w_i \qquad (19)$$

where:

$$e_u = u - u^r$$

Choosing the following fuzzy control law:

$$u = \sum_{i=1}^{n} w_i (u_{ei} - K_i^f (x - x_r)) \Big/ \sum_{i=1}^{n} w_i \qquad (20)$$

we obtain:

$$e_u = -\sum_{i=1}^{n} w_i K_i^f e_x \Big/ \sum_{i=1}^{n} w_i \qquad (21)$$

substituting (21) in (18), we obtain:

$$\dot{e}_x = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \left[A_i - B_i K_j^f\right] e_x}{\sum_{i=1}^{n} \sum_{j=1}^{n} w_j w_i} \qquad (22)$$

A sufficient stability condition derived by Tanaka and Sugeno [8], for ensuring stability of (22) is given as follows:

Theorem 1: The equilibrium of a fuzzy system (22) is asymptotically stable if there exists a common positive definite matrix P such that:

$(A_i - B_i K_i^f)^T P + P(A_i - B_i K_i^f) < 0$ for $i=1,2,...n$. (23)

$G_{ij}^T P + P G_{ij} < 0$ for $i<j<n$ (24)

where:

$$G_{ij} = \frac{[A_i - B_i K_j^f] + [A_j - B_j K_{if}]}{2}$$

This theorem reduces to the Lyapunov stability theorem for continuous time linear systems when n=1. The control design problem is to select K^f_i (i=1,2,...n) such that conditions (23) and (24) in Theorem 1 are satisfied.

To check the stability of the fuzzy control system, it has long been considered difficult to find a common positive definite matrix P satisfying the conditions of (23) and (24). Most of the time, a trial and error type of procedure is used [9,10]. In [13], a procedure to construct a common P is given for second order fuzzy systems. We pointed out in [14] that the common P problem for fuzzy control systems can be solved numerically. To do this, a very important observation is that the stability condition of Theorem 1 is expressed in LMI [15]. To check stability, we need to find P satisfying the LMI.

P>0 $\quad (A_i - B_i K_i^f)^T P + P(A_i - B_i K_i^f) < 0$ for $i=1,2,...n$.

$G_{ij}^T P + P G_{ij} < 0$ for $i<j<n$

Or determination that no such P exists. This is a convex feasibility problem. Numerically, This feasibility problem can be solved very efficiently by means of the most powerful tools available to date in the mathematical programming literature.

5- SIMULATION RESULTS

For the path following control, the operating point must check the following equations:

$$\begin{aligned} & T_e + M.v_e.r_e - M.f.g + Cf.\delta_e(v_e + a.r_e)/u_e + u_e^2(f.k_1 - k_2) = 0 \\ & T_e\delta_e - M.u_e.r_e + Cf.\delta_e - (Cf+Cr)v_e/u_e + (b.Cr - a.Cf)r_e/u_e = 0 \\ & a.T_e.\delta_e - M.f.h.u_e.r_e + a.Cf.\delta_e - (a.Cf - b.Cr)v_e/u_e - (b^2.Cr + a^2.Cf)r_e/u_e = 0 \end{aligned} \qquad (25)$$

Two operating points are chosen:

$(x_{e1};u_{e1})$=(18.43,-0.23,0.55,0,0,0;400,5°)

$(x_{e2};u_{e2})$=(18.57,-1.26,2.79,0,0,0;400,25°)

Figure 4 shows the fuzzy sets of the operating regimes $(x_{e1};u_{e1})$ $(x_{e2};u_{e2})$.

Using the LQR design with:

$Q_1=Q_2=Q'$, $R_1=R_2=R'$

we obtain:

K^f_1=[-2.14e-6 5.23e-6 0.0001 -3.99e-5 0.0001 -0.0014; 0.3096 -0.0892 0.5244 -0.3133 0.3191 -4.3642].

K^f_2=[0.0001 5.731e-7 -0.0008 -0.0001 -0.0002 0.0075; 0.1684 -0.0224 1.5489 -0.1146 0.4323 -14.6242]

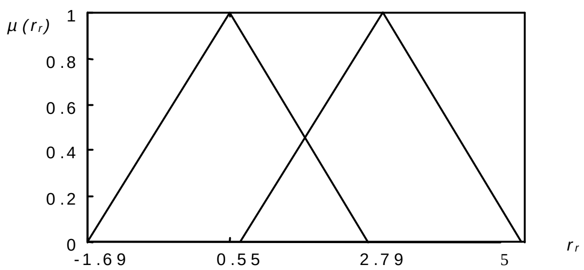

Figure 4: Memberships functions

The fuzzy model of the car constructed from 2 rules is stable if:

$$\begin{aligned} S_{11} &= G_{11}^T P + P G_{11} < 0 \\ S_{22} &= G_{22}^T P + P G_{22} < 0 \\ S_{12} &= G_{12}^T P + P G_{12} < 0 \end{aligned} \qquad (26)$$

For a common positive matrix P.

Using an LMI optimization algorithm, we obtain the following:

$$P = 1.0e\text{-}009 \cdot \begin{bmatrix} 0.00 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \\ 0.00 & 0.0033 & -0.0049 & -0.0015 & -0.0015 & -0.0013 \\ 0.00 & -0.0049 & 0.0096 & 0.0020 & 0.0032 & -0.0128 \\ 0.00 & -0.0015 & 0.0020 & 0.0051 & 0.0023 & 0.0246 \\ 0.00 & -0.0015 & 0.0032 & 0.0023 & 0.0139 & -0.0431 \\ 0.01 & -0.0013 & -0.0128 & 0.0246 & -0.0431 & 0.6662 \end{bmatrix}$$

It can be easily shown that the stability conditions (26) are satisfied.

To highlight the effectiveness of the proposed control law, we present some simulation results for path following problem of the vehicle. One arbitrary chosen initial state $[u_0, v_0, r_0, x_0, y_0, \varphi_0] = (20,0,0,5,10,0)$ and two desired trajectories described respectively by the motion of the vehicle ($u_r=18.5$,$v_r=-0.34$, $r_r=0.78$) and ($u_r=18.57$,$v_r=-1.01$, $r_r=2.23$) are used to test this fuzzy control method. Figure 5 shows the asymptotic convergence of the state variables. The vehicle trajectories are shown in Figure 6. We see that this fuzzy control method successfully controls the vehicle to the desired trajectories starting from a arbitrary initial state.

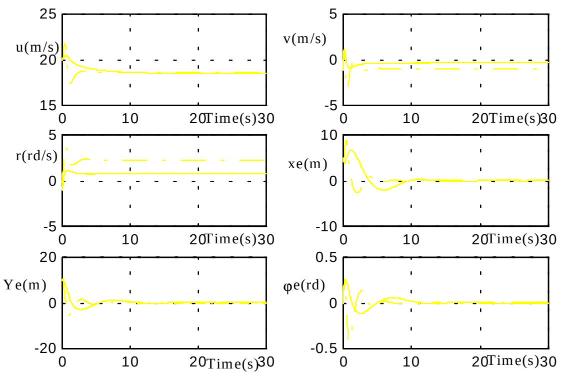

Figure 5. Time evolution of the states variables for the desired trajectories ($u_r=18.5$,$v_r=-0.34$, $r_r=0.78$) ___ and ($u_r=18.57$,$v_r=-1.01$, $r_r=2.23$). ----.

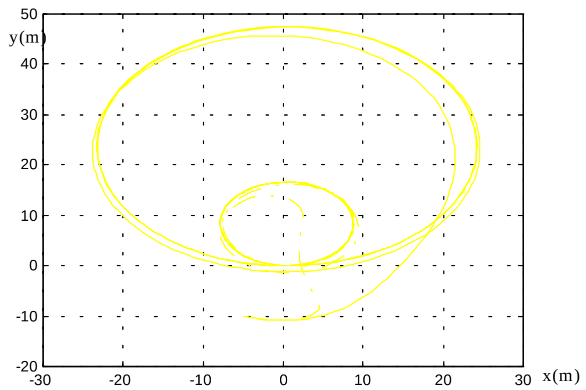

Figure 6. Motions vehicle to the desired trajectories ($u_r=18.5$,$v_r=-0.34$, $r_r=0.78$) ___ and ($u_r=18.57$,$v_r=-1.01$, $r_r=2.23$). ----.

6- CONCLUSION

In This paper, a general state representation is derived for the PF problem of a vehicle. This representation includes the car dynamics model and the kinematics position error one which is based on a vehicle target configuration tracking. Thereafter, a local state feedback controller has been proposed to solve the path following problem. This approach, however, generally only renders a local result. Therefore, we proposed a method of nonlinear feedback control introducing fuzzy inference to treat nonlinearities of control systems. Stability of the fuzzy control system has been analyzed using Lyapunov's method and convex optimization techniques based on (LMI). The designed fuzzy controller effectively achieves the PF movement control of the vehicle. We have shown that we can not use the generated global model for stabilizing the system to a fixed configuration. A future work is to derive a new vehicle model for the point stabilization problem.

Appendix

Symbol	Name	Value
a	Distance, c.g. to front axle	1050 mm
b	Distance, c.g. to rear axle	1630 mm
h	c.g. height	530 mm
M	Total mass	1480 kg
f	Nominal friction coefficient	0.02
Iz	Moment of inertia	2350 kg.m^2
g	Acceleration of gravity	8.81 m/sec^2
Cf	Front roll stiffness	135000 N/rd
Cr	Rear roll stiffness	95000 N/rd
k1	Portance parameter	0.005 N.sec/m
k2	Drag parameter	0.41 N.sec/m
fb	Distribution coefficient front/rear	0.6

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