counting

Monday, November 21, 2022 8:39 PM

1.

a. unique subset (don't share exact same letters w/ any other set)

unsal $\binom{4}{4}$ + uu_ _ _ $\binom{4}{3}$ + uuu_ _ → $\binom{4}{2}$ = 1 + 4 + 6 = **11**

1

nal, sal, lns, asn

al, sl, ln, an, sn, as

b. determine restraint effects & enumerate ~~Permutations~~ brute force

3 cases: 1 'u' — 2 'u's — 3 'u's

unsal → Find all perms → $5 \times 4 \times 3 \times 2 \times 1$ = 5!

2 'u's: uu_ _ _ ($4 \times 3 \times 2$) → $10 \times 4!$

3 'u's: uuu_ _ (4×3) → 10×12

$5! + 10 \times 4! + 10 \times 12$ = **480**

$\binom{5}{2}$:
uu_ _ _, u_u_ _, u_ _u_, u_ _ _u, _uu_ _, _u_u_, _u_ _u, _ _uu_, _ _u_u, _ _ _uu

$\binom{5}{3}$:
uuu_ _, uu_u_, uu_ _u, u_u_u, u_ _uu, u_uu_, _uuu_, _uu_u, _u_uu, _ _uuu

2. 1st 2 cards: $\binom{13}{2}$ (number of card values) × $\binom{4}{2}$ × $\binom{4}{2}$ (number of cards w/ same value) × $\binom{11}{1}$ (choose non-pair card) × $\binom{4}{1}$ (possible suits of final card) = 123,552

3. bars = 6

stars = 16

∴ | ∴ | ∴ | ∴ | · | ∴ ~~|~~

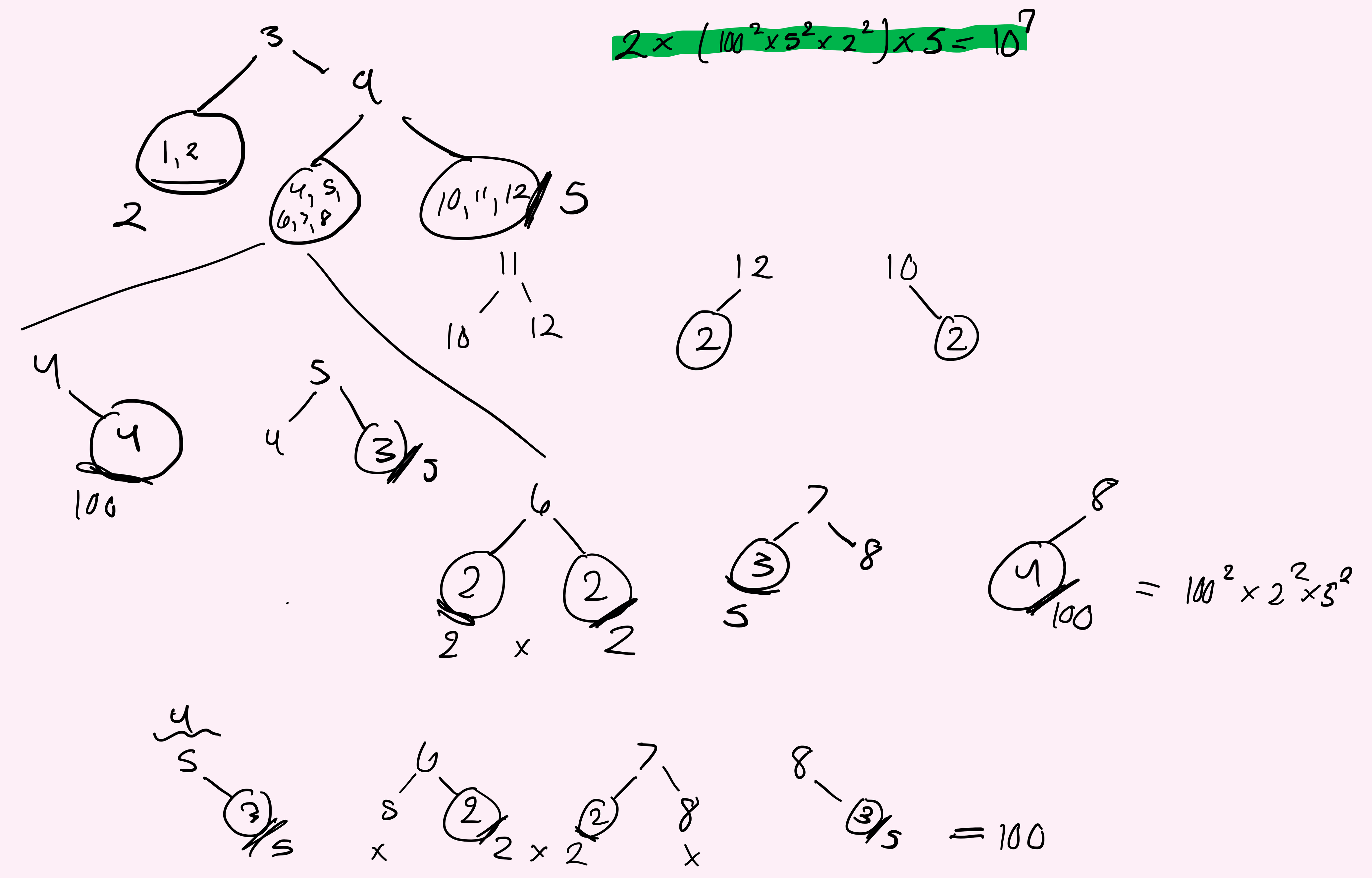
• Break it up into

$\binom{16\ \text{*'s}}{6\ \text{|'s}} + \binom{15\ \text{*'s}}{6\ \text{|'s}}$

$\binom{16-1+6}{6} + \binom{15-1+6}{6}$ = **93,024**

4.

$2 \times (100^2 \times 5^2 \times 2^2) \times 5 = 10^7$



56

5. 4 nurses → 9 + 5 + 4 + 3 + 2 + 1 + 1 = 25

10 people

(7, 1, 1, 1)
(6, 2, 1, 1)
(5, 2, 2, 1) (5, 3, 1, 1)
(4, 2, 2, 2) (4, 3, 2, 1) (4, 4, 1, 1)
(3, 3, 2, 2) (3, 3, 3, 1)
= 9

9 people

(6, 1, 1, 1)
(5, 2, 1, 1)
(4, 2, 2, 1)
(3, 2, 2, 2) (3, 3, 2, 1) = 5

8 people

(5, 1, 1, 1)
(4, 2, 1, 1)
(3, 2, 2, 1)
(2, 2, 2, 2) = 4

7 people

(4, 1, 1, 1)
(3, 2, 1, 1) = 3
(2, 2, 2, 1)

6 people

(3, 1, 1, 1)
(2, 2, 1, 1) = 2

5 people

(2, 1, 1, 1) = 1

4 people

(1, 1, 1, 1) = 1

3 nurses → 8 + 7 + 5 + 4 + 3 + 2 + 1 + 1 = 31

10 people

(8, 1, 1)
(7, 2, 1)
(6, 3, 1) (6, 2, 2)
(5, 4, 1) (5, 3, 2)
(4, 4, 2) (4, 3, 3) = 8

9 people

(7, 1, 1)
(6, 2, 1)
(5, 2, 2) (5, 1, 3)
(4, 3, 2) (4, 4, 1) = 7
(3, 3, 3)

8 people

(6, 1, 1)
(5, 2, 1)
(4, 2, 2) (4, 1, 3) = 5
(3, 3, 2)

7 people

(5, 1, 1)
(4, 2, 1)
(3, 3, 1) (3, 2, 2) = 4

6 people

(4, 1, 1)
(3, 2, 1)
(2, 2, 2) = 3

5 people

(3, 1, 1)
(2, 2, 1) = 2

4 people

(1, 2, 1) = 1

3 people

(1, 1, 1) = 1