COMP - 285

Analysis of Algorithms

Welcome to COMP 285

Lecture 15: Graph DFS + Topological Sort

Lecturer: Chris Lucas (cflucas@ncat.edu)

Midterm Grades Released!

Midpoint Grades Submitted!

HW4 grades by EoW!

HW5 due in 1 week!

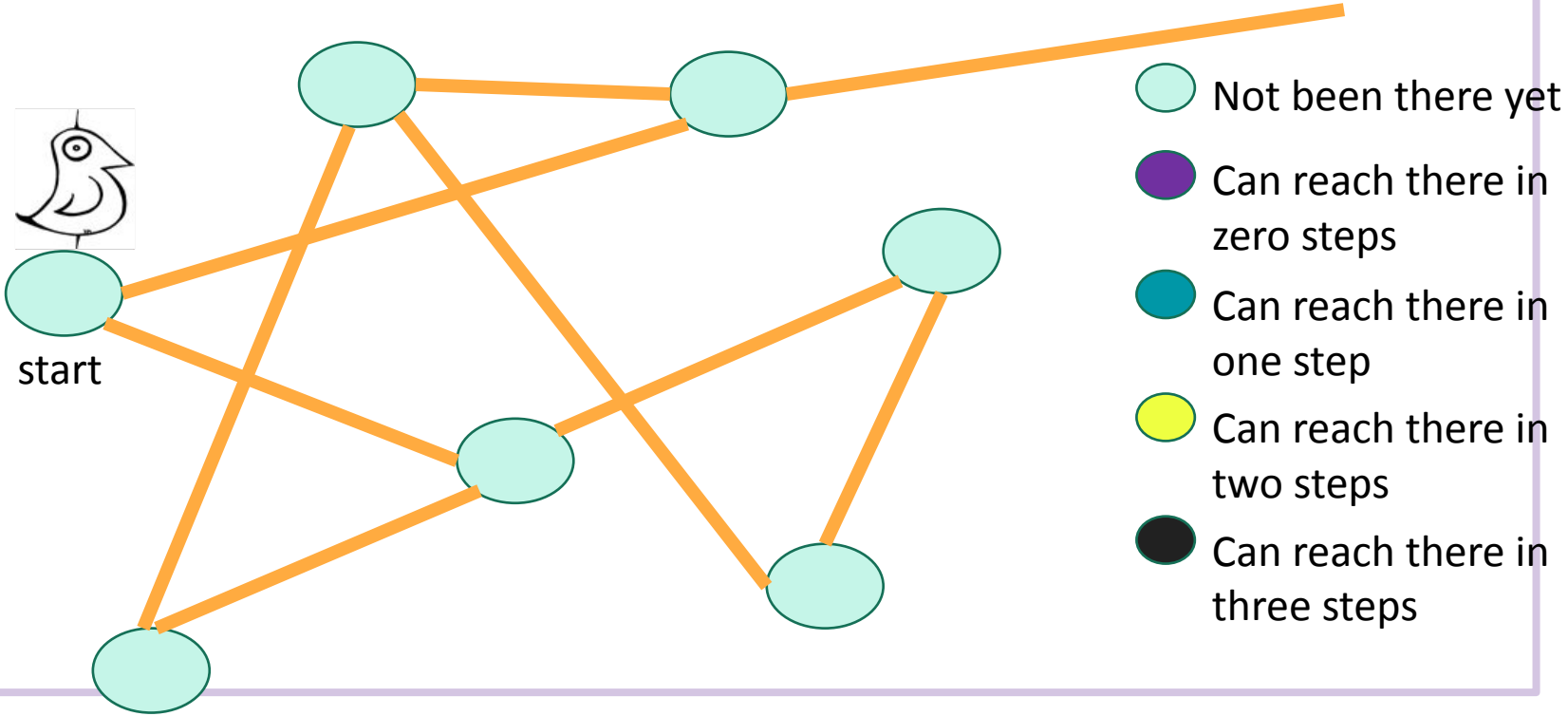
10/25 @ 11:59PM ET

Quiz!

www.comp285-fall22.ml

Recall where we ended last lecture…

Breadth-First Search: Exploring the world with a bird's-eye view



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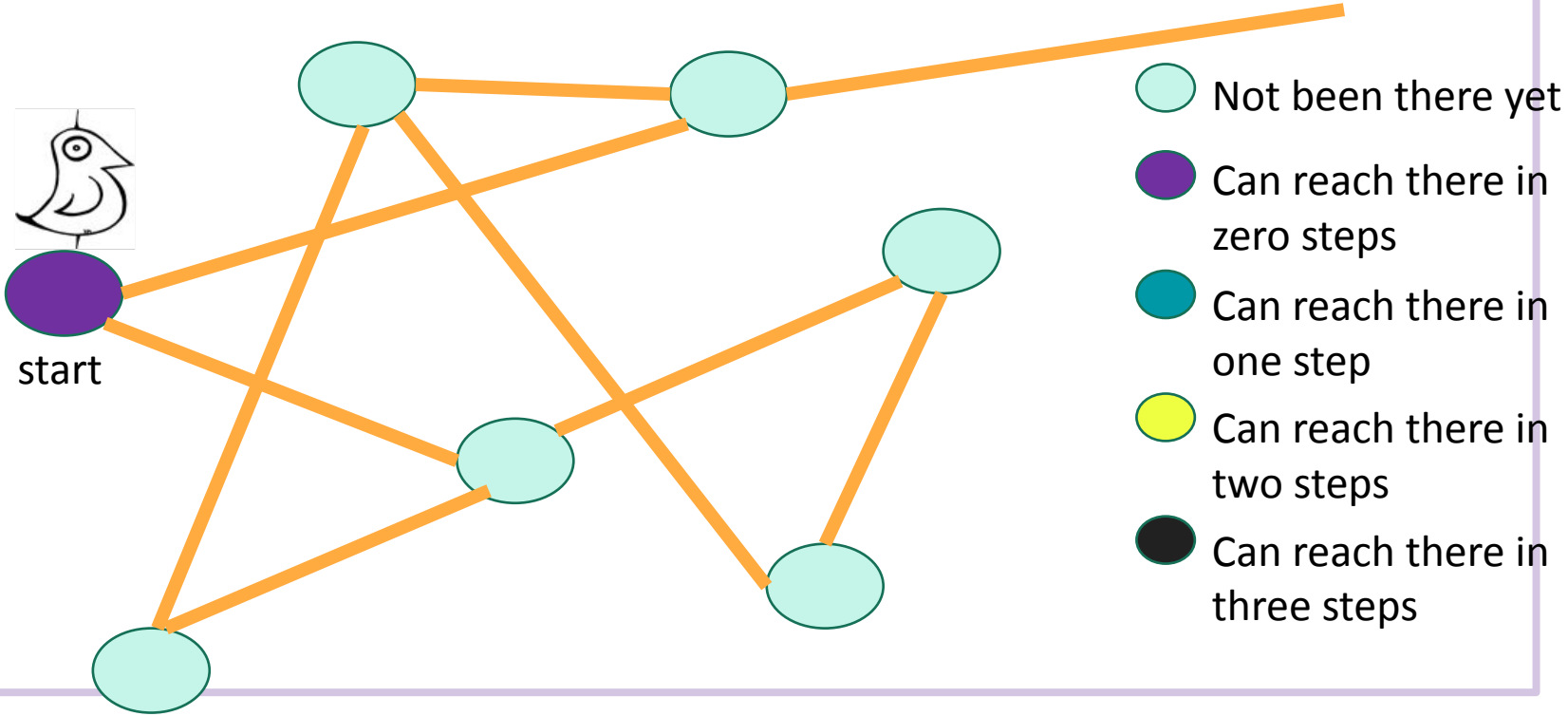

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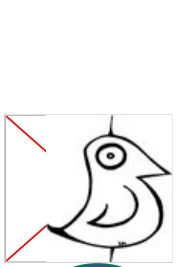


start

Not been there yet

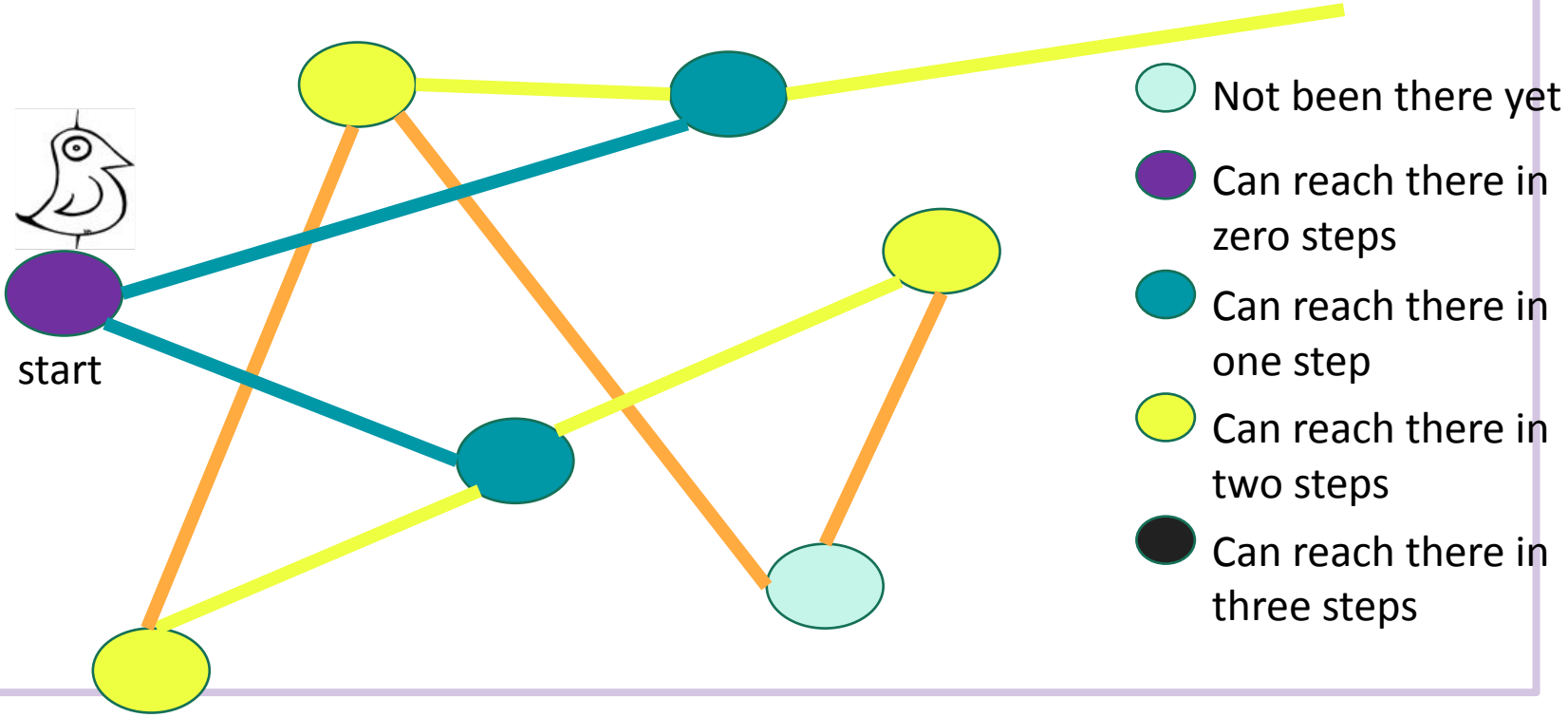
Can reach there in zero steps

Can reach there in one step

Can reach there in two steps

Can reach there in three steps

Breadth-First Search: Exploring the world with a bird's-eye view



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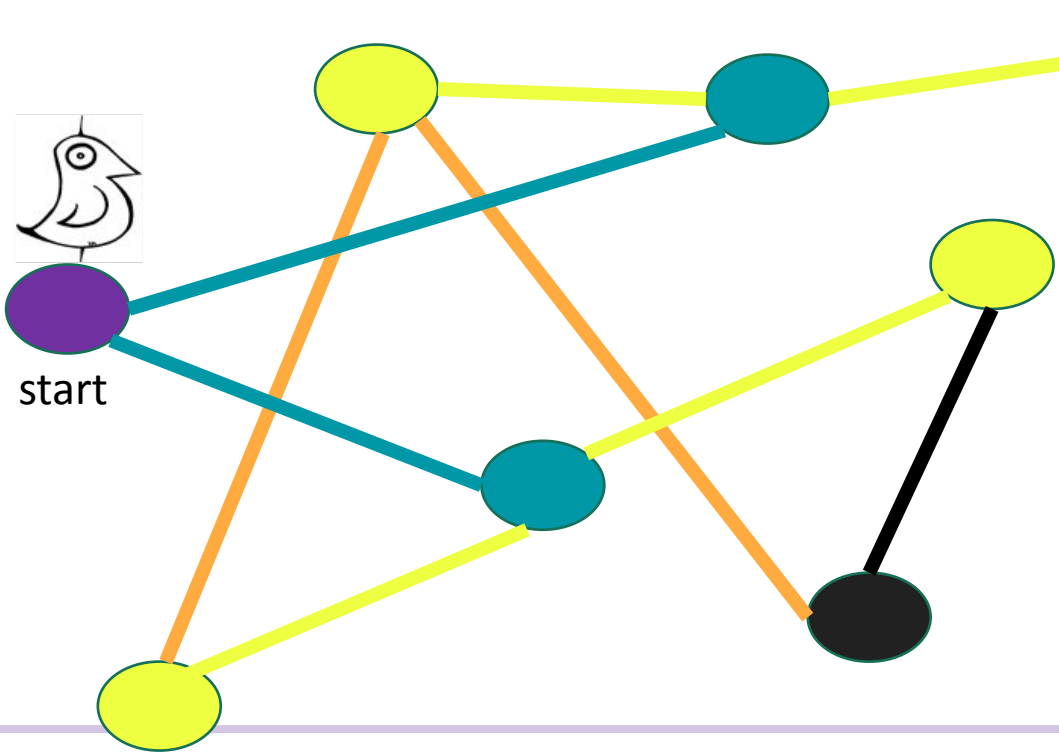

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World:

explored!

Breadth-First Search Pseudocode

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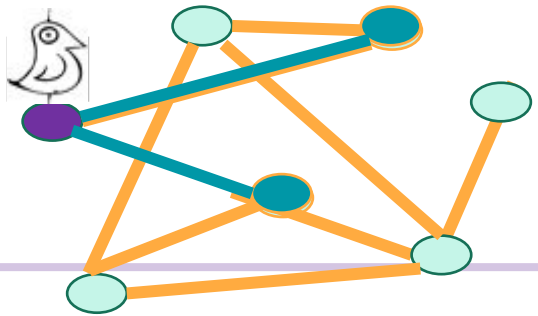
algorithm bfs
    Input: undirected graph G = (V,E), s and d
    Output: true/false if path from s to d

    frontier = Queue of integers
    visited = {} // empty hash set
    frontier.add(s)
    visited.insert(s)
    while not frontier.empty()
      currNode = frontier.remove()
      if currNode == d
        return true
      for each neighbor of currNode
        if neighbor not in visited
          visited.insert(neighbor)
          frontier.add(neighbor)
    return false
```



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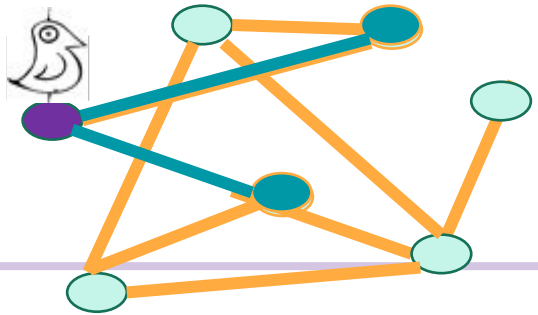
```

Runtime: O(V + E)

Why? What does this signify?



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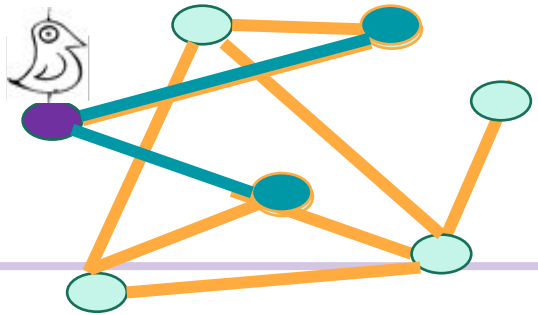
Runtime: O(V + E)

Traversing every edge and vertex!



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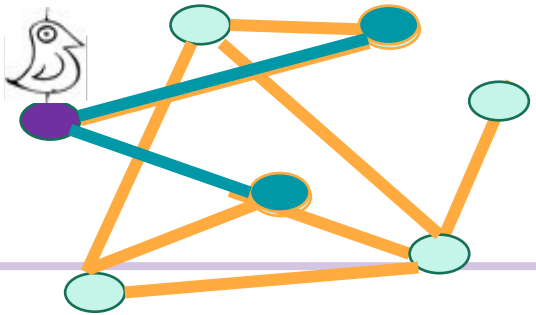
```

Runtime: O(V + E)

Space Complexity: ???



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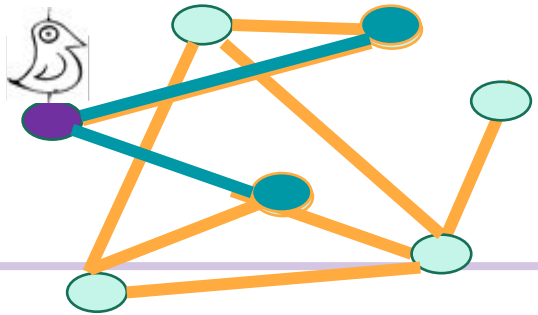
```

Runtime: O(V + E)

Space Complexity: O(V)



Summary of Breadth-First Search

- BFS can be written such that starting from a node v:
  - You find a path to every other vertex in G (if one exists)
  - You find a path to a specific destination node w
  - (Depends on your use case)
- BFS is guaranteed to find the shortest path between nodes
- BFS running time is O(V + E): traversing every node and every edge
- Note that BFS also works on directed graphs.

Big Questions!

- What about Depth-first search (DFS)?
- What's a DAG?
- What about topological sorting (TopoSort)?

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- What about Depth-first search (DFS)?
- What's a DAG?
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Analogy

Analogy

BFS

Analogy

BFS

DFS

Breadth/Depth-First Search Pseudocode

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Breadth/Depth-First Search Pseudocode

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          frontier.add(neighbor)
    return false
```

```
algorithm DFS
   Input: undirected graph G = (V,E), s and d
   Output: true/false if path from s to d

    frontier = Stack of integers
    visited = {} // empty hash set
    frontier.add(s)
    visited.insert(s)
    while not frontier.empty()
      currNode = frontier.remove()
      if currNode == d
        return true
      for each neighbor of currNode
        if neighbor not in visited
          visited.insert(neighbor)
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    return false
```

https://visualgo.net/en/dfsbfs

One More DFS Implementation…

algorithm DFS

```
Input: undirected graph G = (V, E), int s and int t
Output: whether or not there's a path from s to t

visited = new boolean array of size |V|
return dfsHelper(G, s, t, visited)
```

algorithm DFSHelper

```
Input: undirected graph G = (V, E), s, t, and visited
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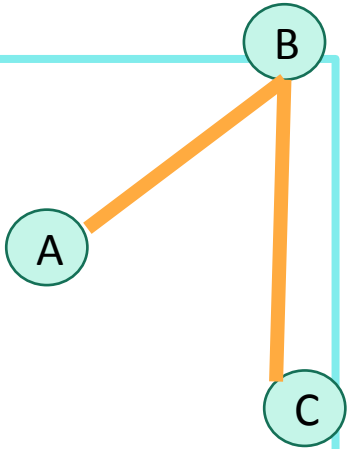

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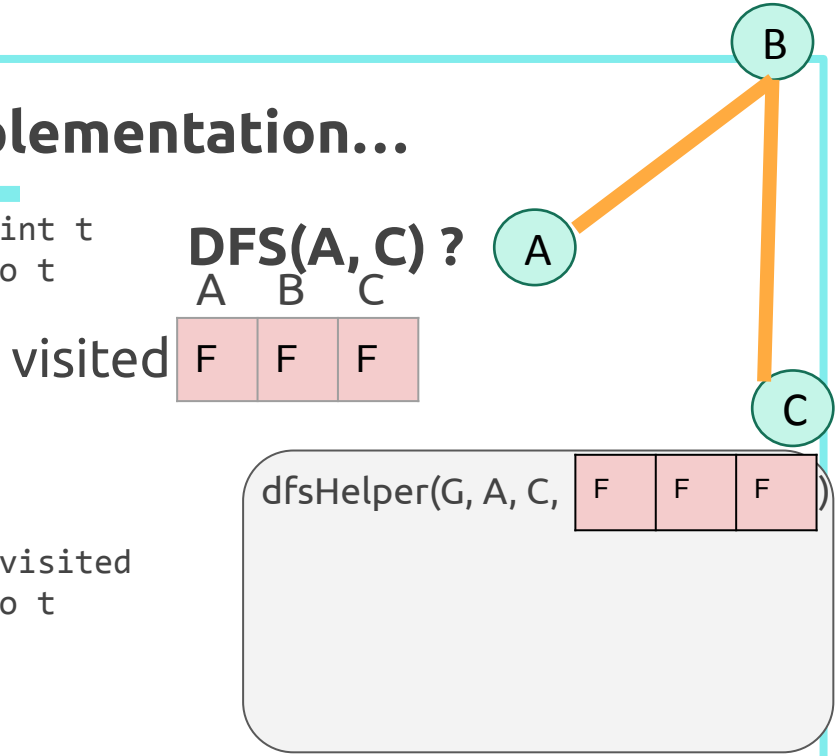

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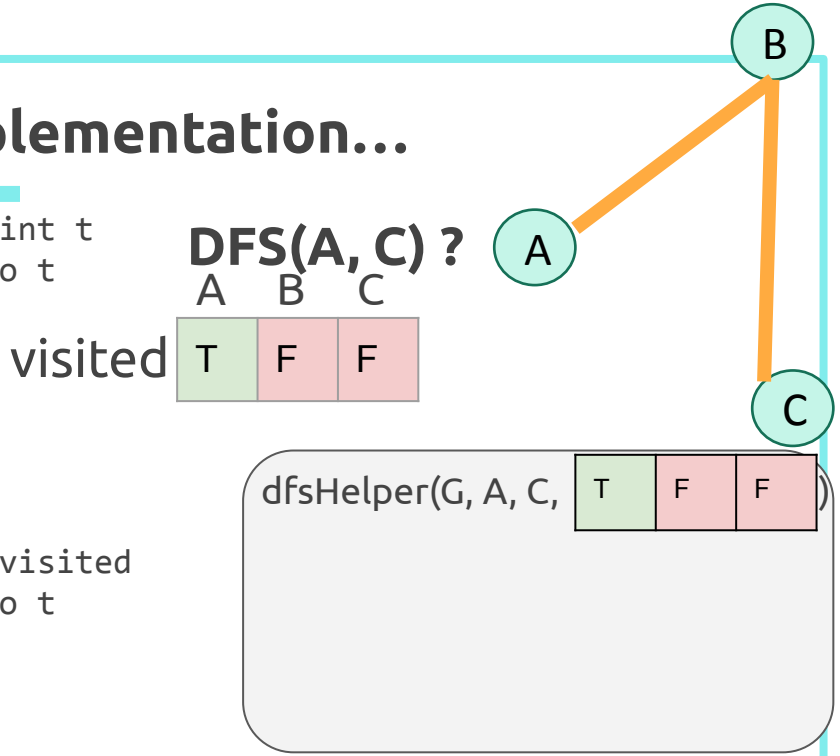

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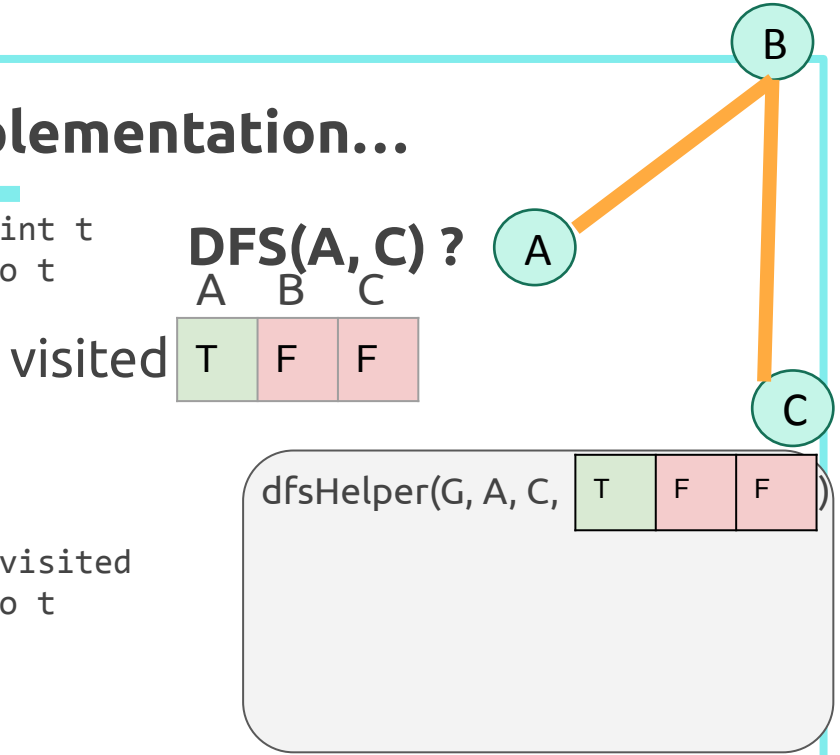

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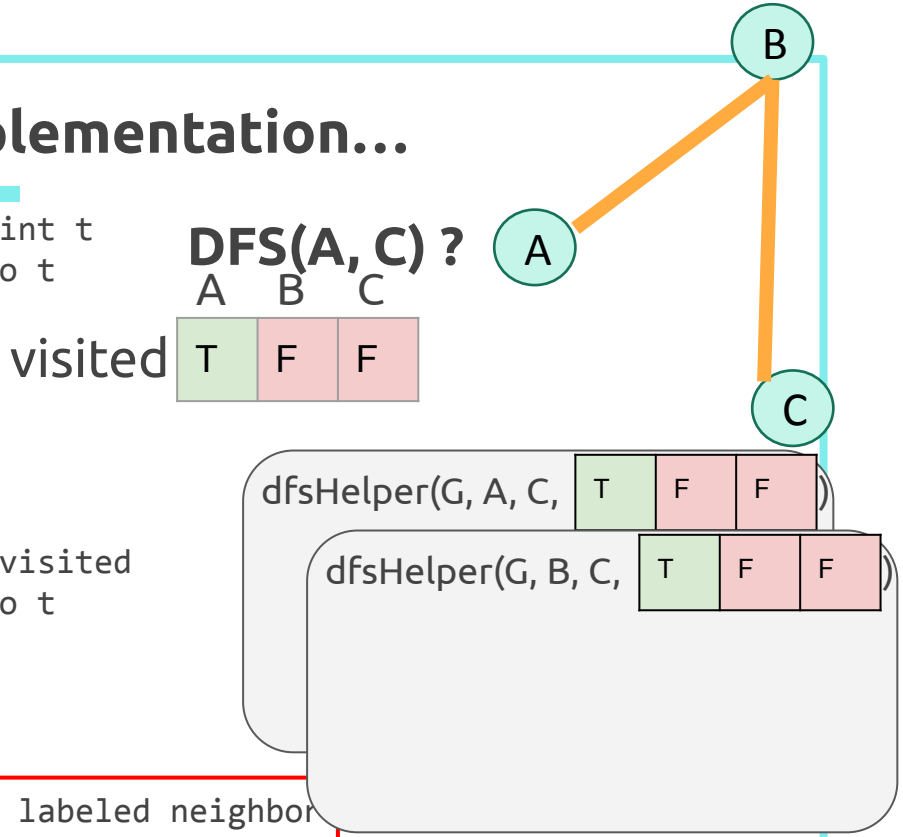

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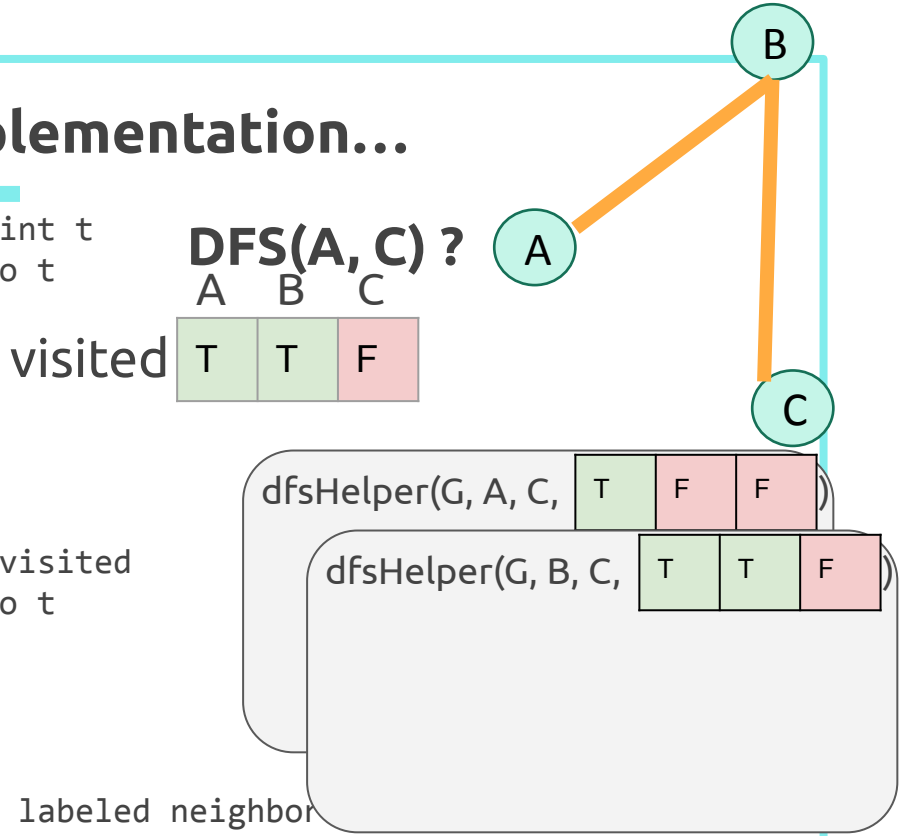

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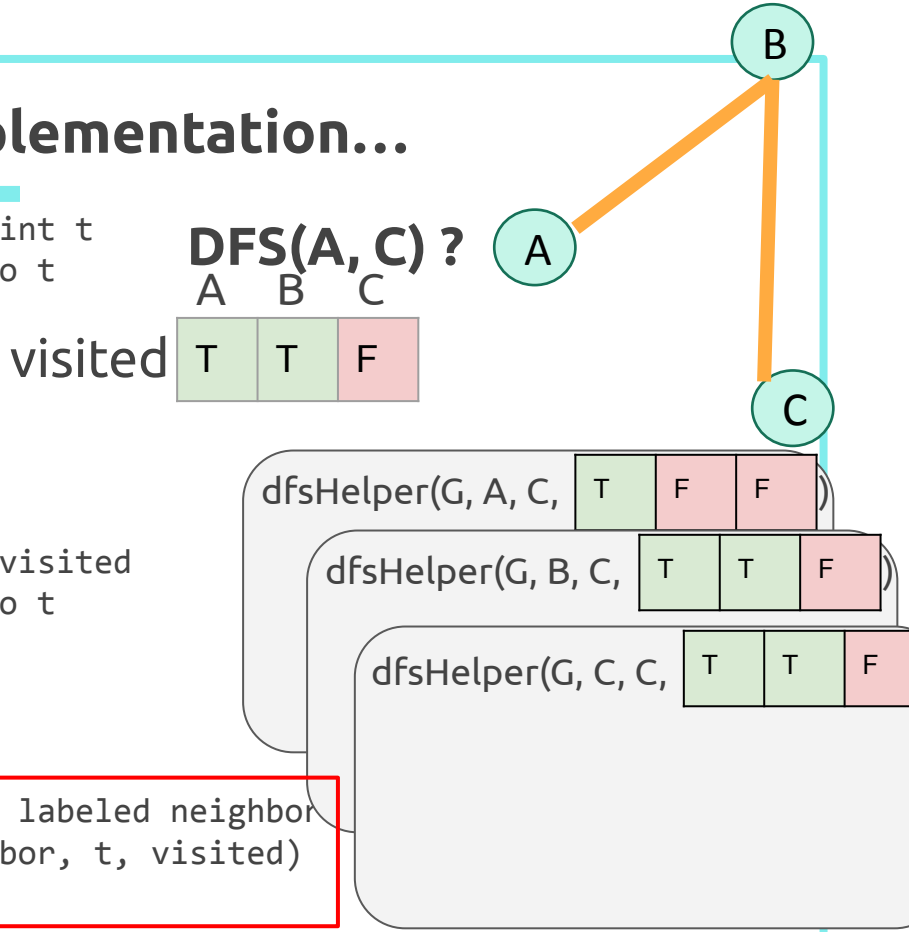

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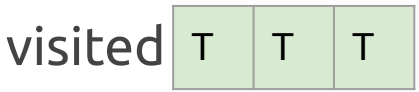

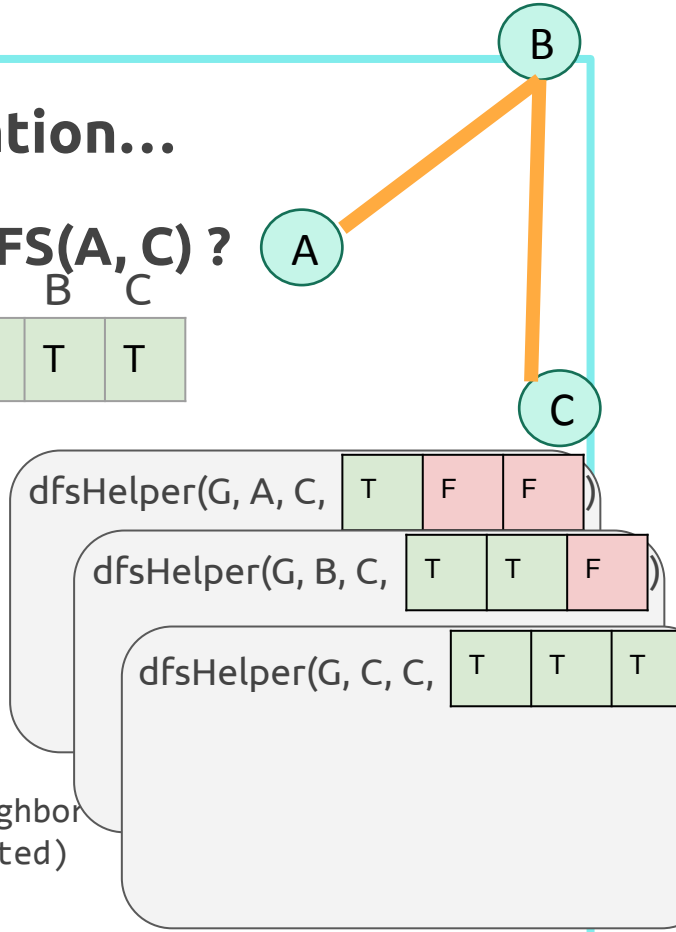

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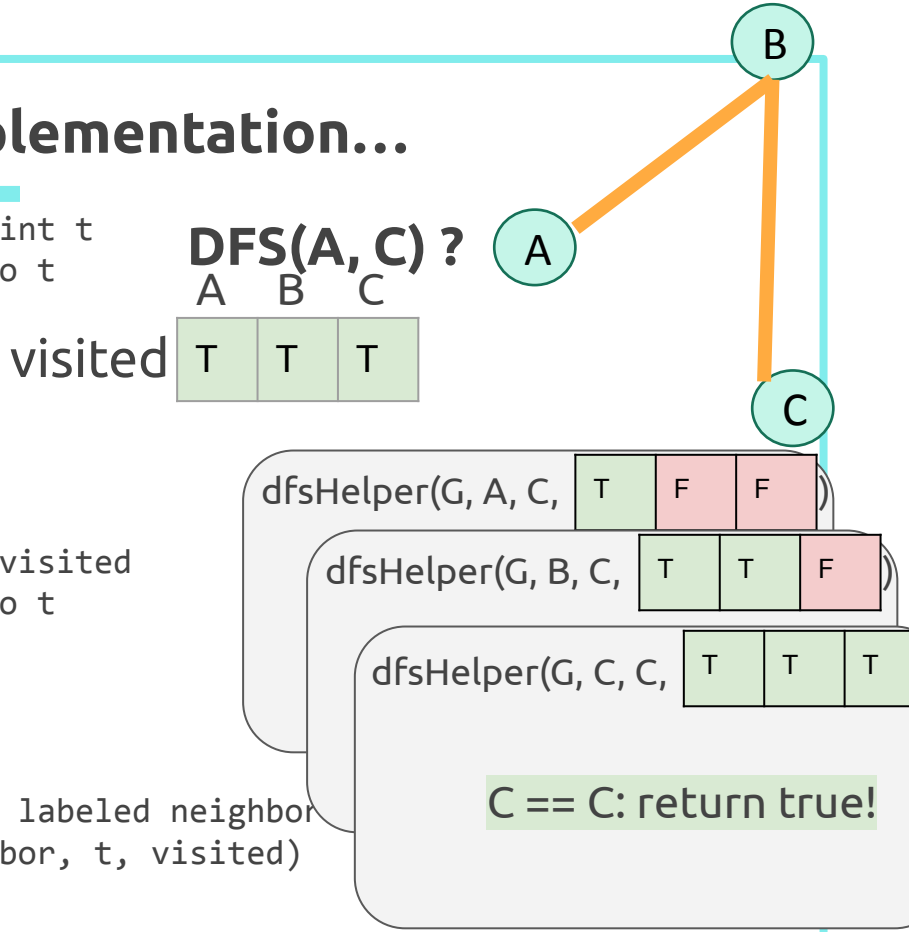

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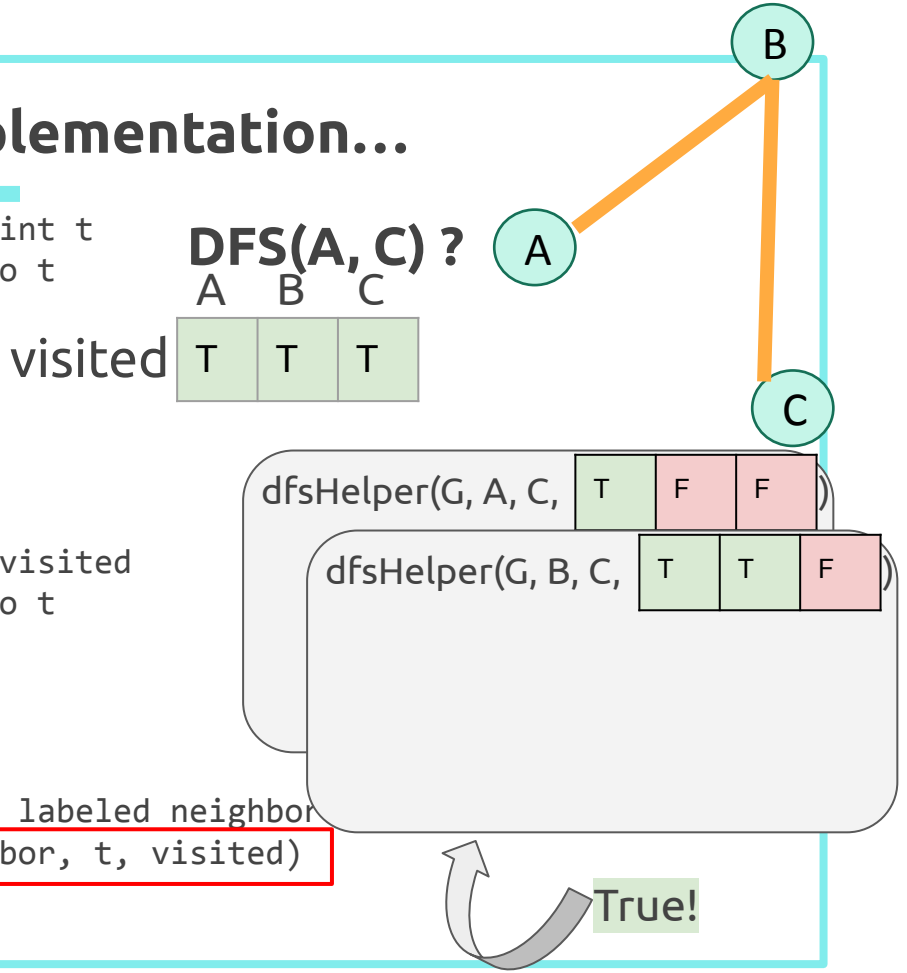

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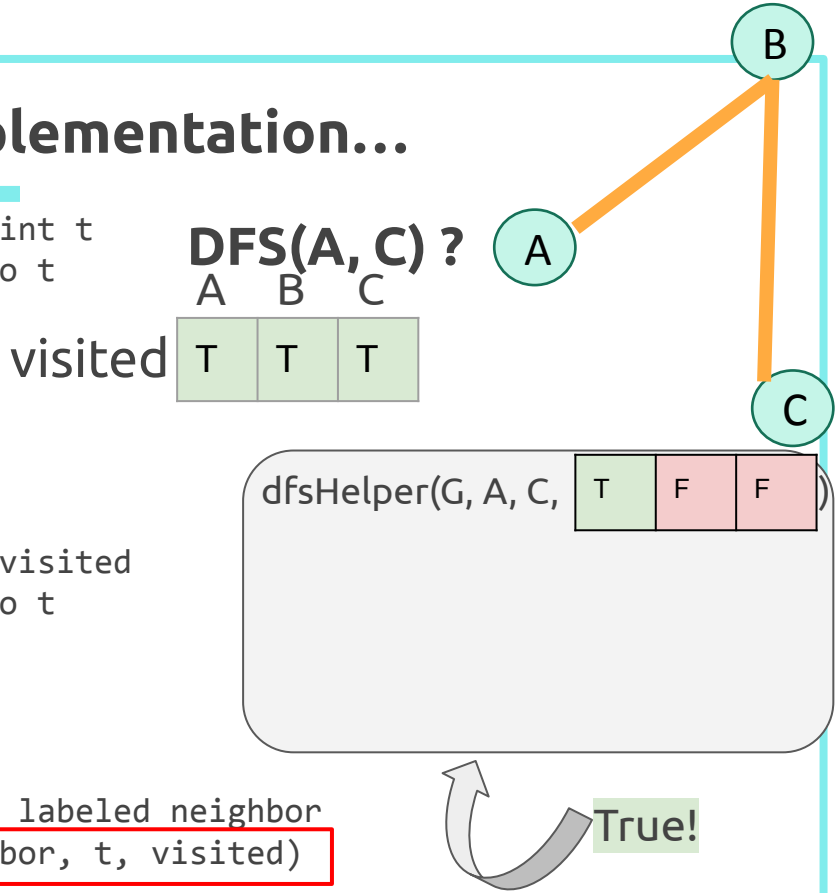

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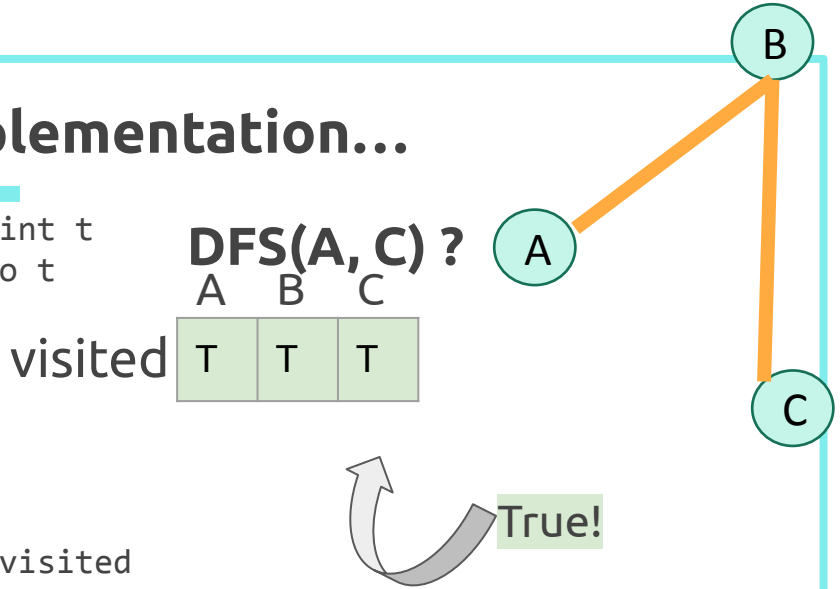

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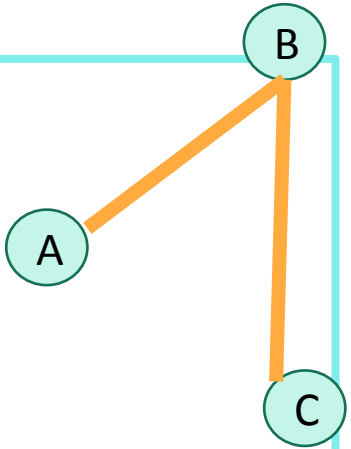

DFS(A, C) is True!

Kahoot!

www.kahoot.it, Code: XXX YYY

Enter your @aggies.ncat email

One More DFS Implementation…

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 Input: undirected graph G = (V, E), int s and int t
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```

Time Complexity? O(V + E)
Space Complexity? O(V)

```
algorithm DFSHelper
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Depth-First Search Pseudocode

```
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 Input: undirected graph G, int s and int t
 Output: whether or not there's a path from s to t

 visited = new boolean array of size |V|
 return dfsHelper(G, s, t, visited)


algorithm DFSHelper
 Input: undirected graph G = (V, E), s, t, and
visited
 Output: whether or not there's a path from s to t

 visited[s] = true
 if s == t
   return true
 for each neighbor of s, starting from smallest
labeled neighbor
   if !visited[neighbor] and dfsHelper(G,
neighbor, t, visited)
     return true
 return false
```

```
algorithm DFS
   Input: undirected graph G = (V,E), s and d
   Output: true/false if path from s to d

    frontier = Stack of integers
    visited = {} // empty hash set
    frontier.add(s)
    visited.insert(s)
    while not frontier.empty()
      currNode = frontier.remove()
      if currNode == d
        return true
      for each neighbor of currNode
        if neighbor not in visited
          visited.insert(neighbor)
          frontier.add(neighbor)
    return false
```

Depth-First Search Pseudocode

```
algorithm DFS
 Input: undirected graph G, int s and int t
 Output: whether or not there's a path from s to t

 visited = new boolean array of size |V|
 return dfsHelper(G, s, t, visited)


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Time Complexity? O(V + E)
Space Complexity? O(V)

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    return false
```

Time Complexity? ???
Space Complexity? ???

Kahoot!

www.kahoot.it, Code: XXX YYY

Enter your @aggies.ncat email

Depth-First Search Pseudocode

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          frontier.add(neighbor)
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```

Time Complexity? O(V+E)
Space Complexity? O(V)

Application: Flood Fill

- Microsoft Paint Bucket Tool
- Note that images can be represented as a grid of pixels
- To fill every connected pixel that is the same color as the “seed”, we can use a BFS or DFS
- We look up, down, left, and right, and explore the next node only if it is the same color as the seed

Big Questions!

- What about Depth-first search (DFS)?
- What's a DAG?
- What about topological sorting (TopoSort)?

Directed Acyclic Graphs (DAGs)

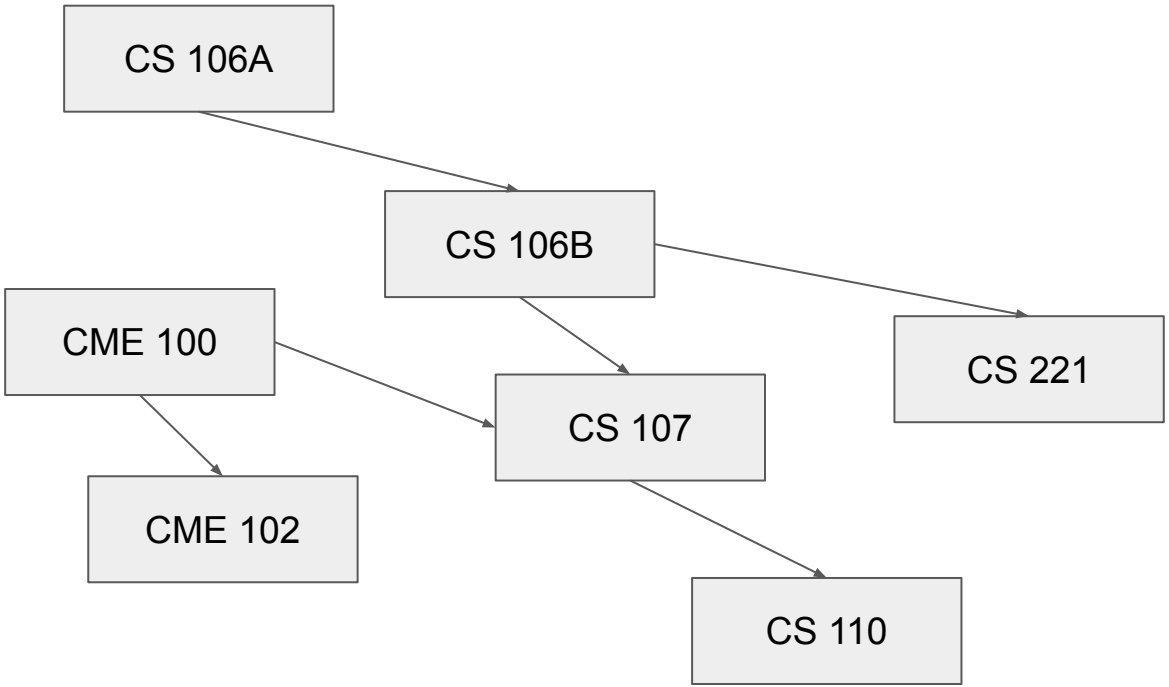
When we have a graph that is both directed and has no loops (acyclic), we call it a Directed Acyclic Graph (a.k.a DAG).

They’re useful for representing real-world dependencies.

Directed graphs (not just DAGs) have nodes that are:

- Sources: no inbound edges (i.e. no arrows pointing to it)
- Sinks: no outbound edges (i.e. no arrows pointing away from it)

Course Prerequisites



Kahoot!

www.kahoot.it, Code: XXX YYYY

Enter your @aggies.ncat email

Polls

Is this a DAG?

Yes

Which Nodes are source nodes?

3, 5

Which Nodes are sink nodes?

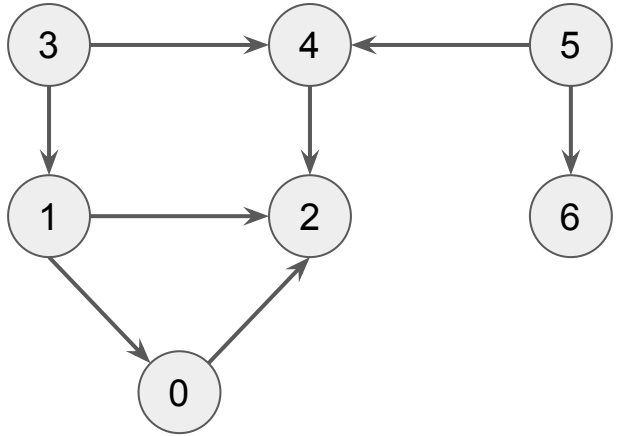
2, 6

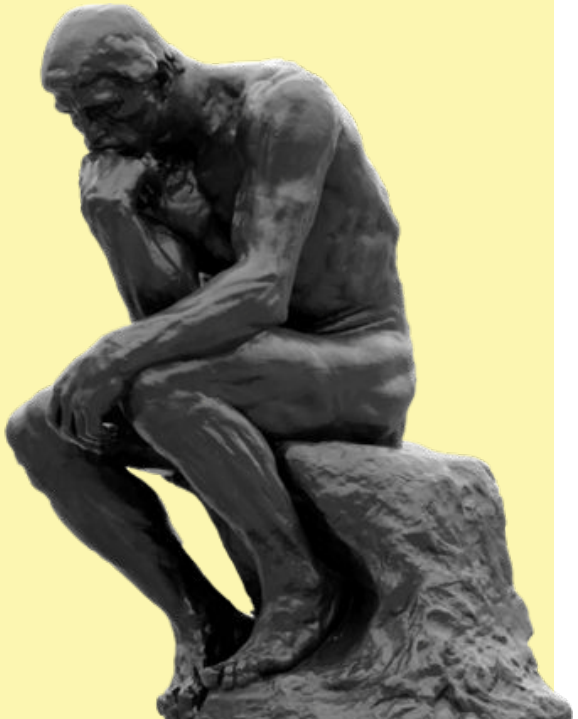
If we ran DFS starting at 4 and followed only the outbound edges, what would we print out?

4, 2

If we ran DFS starting at 3 and only followed the outbound edges, what would we print out? (Ties broken numerically, smallest node value first)

3, 1, 0, 2, 4





Big Questions!

- What about Depth-first search (DFS)?
- What's a DAG?
- What about topological sorting (TopoSort)?

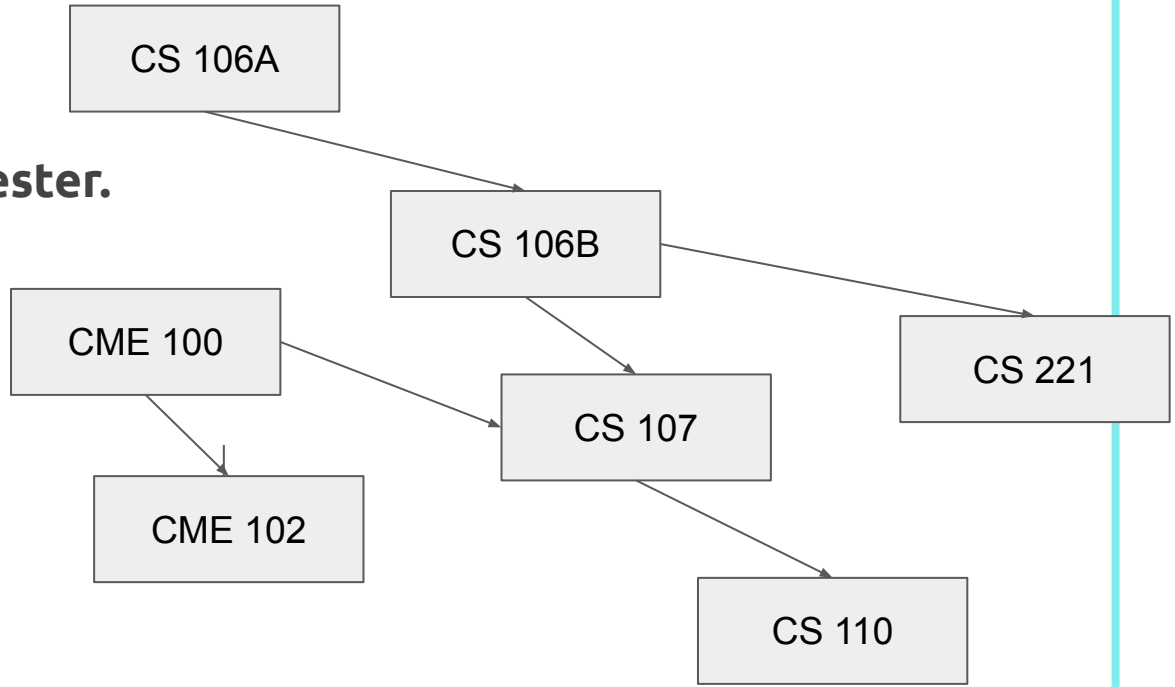
Topological Sort

- "A topological sort of a DAG G = (V, E) is a linear ordering of all its vertices such that if G contains an edge (u, v) then u appears before v in the ordering. (If the graph contains a cycle,then no linear ordering is possible.)"
  - Fancy way of saying “give me one valid ordering of a DAG’s nodes.”
  - If U -> V in graph, means node U must occur before node V in order
  - Only possible to calculate this for DAGs
- A valid topological sort will have all "pre-req" nodes appear before the nodes that depend on those pre-reqs.
- "Sort" is misleading. Don't confuse this with the sorts we saw before. Topological “ordering” is another name that may be less confusing.

Topological Sort Example #1

What's one valid course ordering?
Assume 1 course per semester.

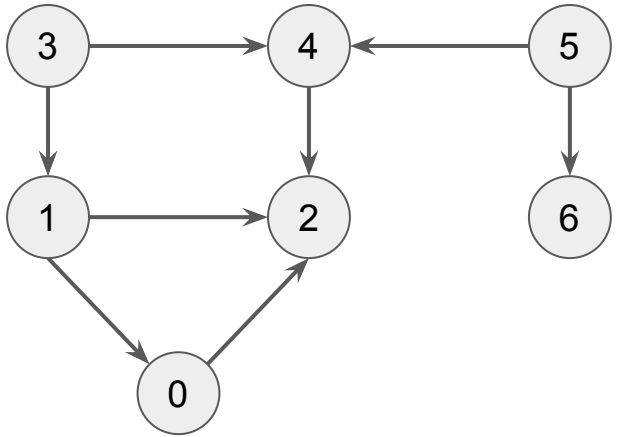
CME 100
CME 102
CS 106A
CS 106B
CS 107
CS 221
CS 110



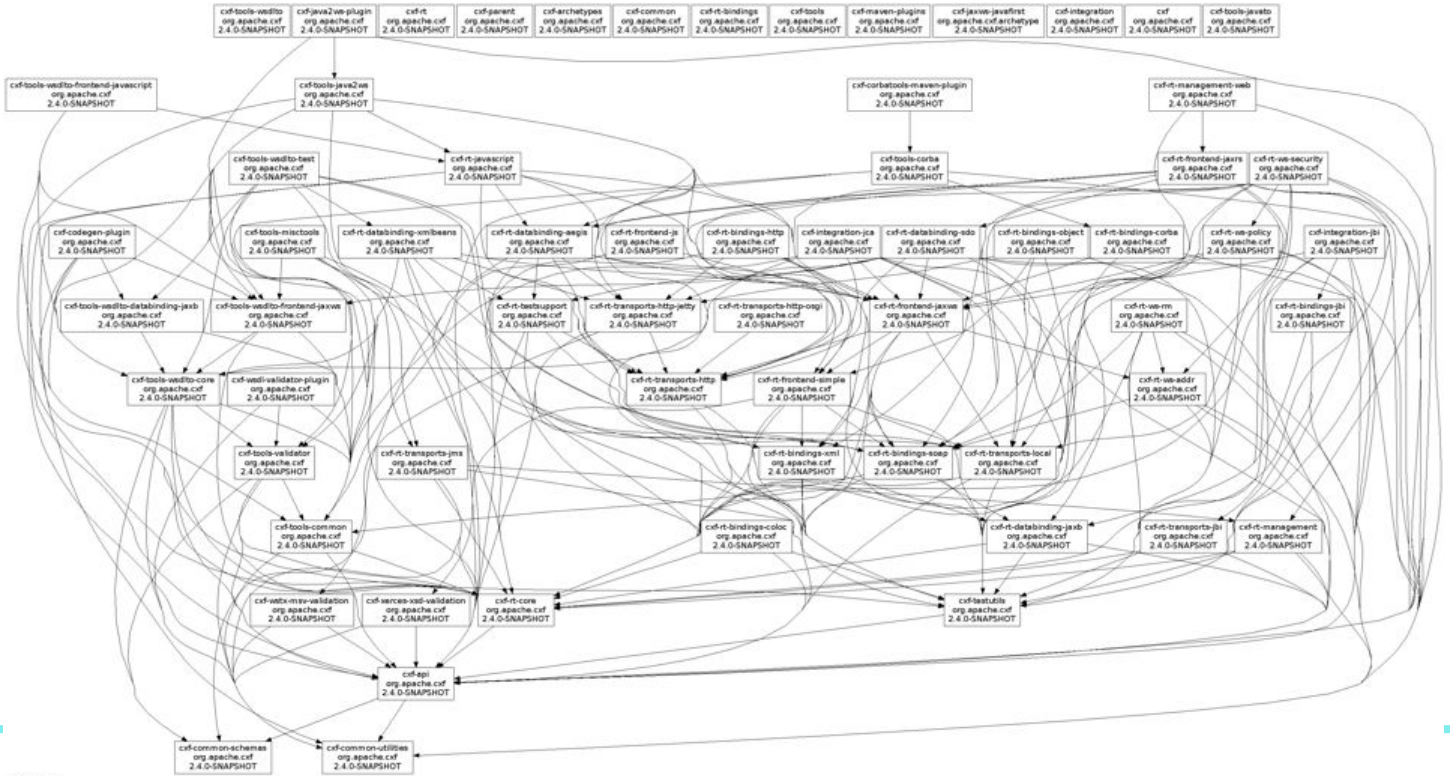
Topological Sort Example #2

What's one valid ordering?

5, 4, 6, 3, 1, 0, 2



Can’t always eyeball it…



Topological Sort Algorithm

- Note that some valid orderings laid out each “child” until we reached a sink.

- We can actually perform a DFS, starting at source nodes and **reverse the DFS finishing times** to get a valid topological sort.

Topological Sort Algorithm

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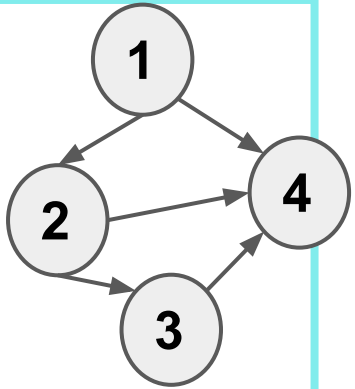
How do we know which v in G(V, E) are source nodes?

Graph Representation Examples



destination

start

0	**1**	**0**	**1**
0	**1**	**1**	**1**
0	**0**	**0**	**1**
0	**0**	**0**	**0**

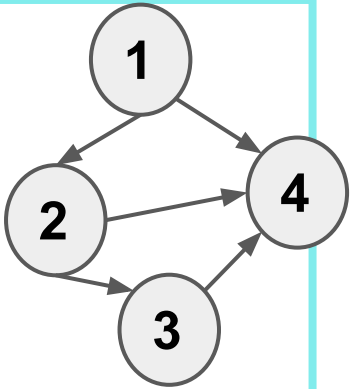
Adjacency Matrix

1: [2, 4]
2: [4, 2, 3]
3: [4]
4: []

Adjacency List

How do we know which v in G(V, E) are source nodes?

Graph Representation Examples



destination

start

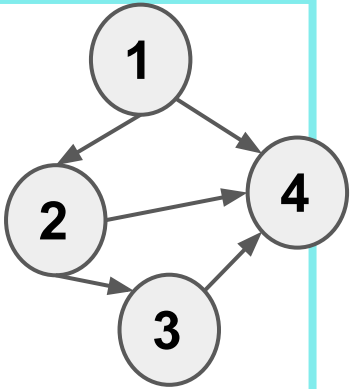
0	1	0	1
0	1	1	1
0	0	0	1
0	0	0	0

Adjacency Matrix

- **Sources**: ???
- **Sinks**: ???

How do we know which v in G(V, E) are source/sink nodes?

Graph Representation Examples



destination

start

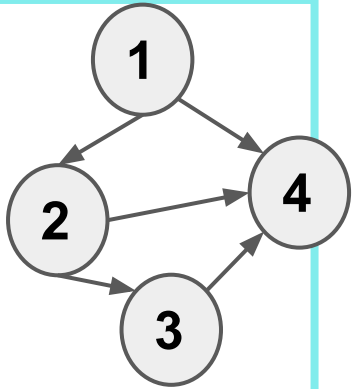
0	1	0	1
0	1	1	1
0	0	0	1
0	0	0	0

Adjacency Matrix

- **Sources**: Columns of all 0s
- **Sinks**: Rows of all 0s

How do we know which v in G(V, E) are source/sink nodes?

Graph Representation Examples



- **Sources**: ???
- **Sinks**: ???

1: [2, 4]
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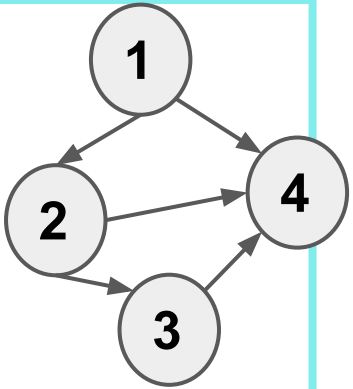
Adjacency List

How do we know which v in G(V, E) are source nodes?

Graph Representation Examples



- **Sources**: Node that doesn't appear in any list
- **Sinks**: Node that has empty list

1: [2, 4]
2: [4, 2, 3]
3: [4]
4: []

Adjacency List

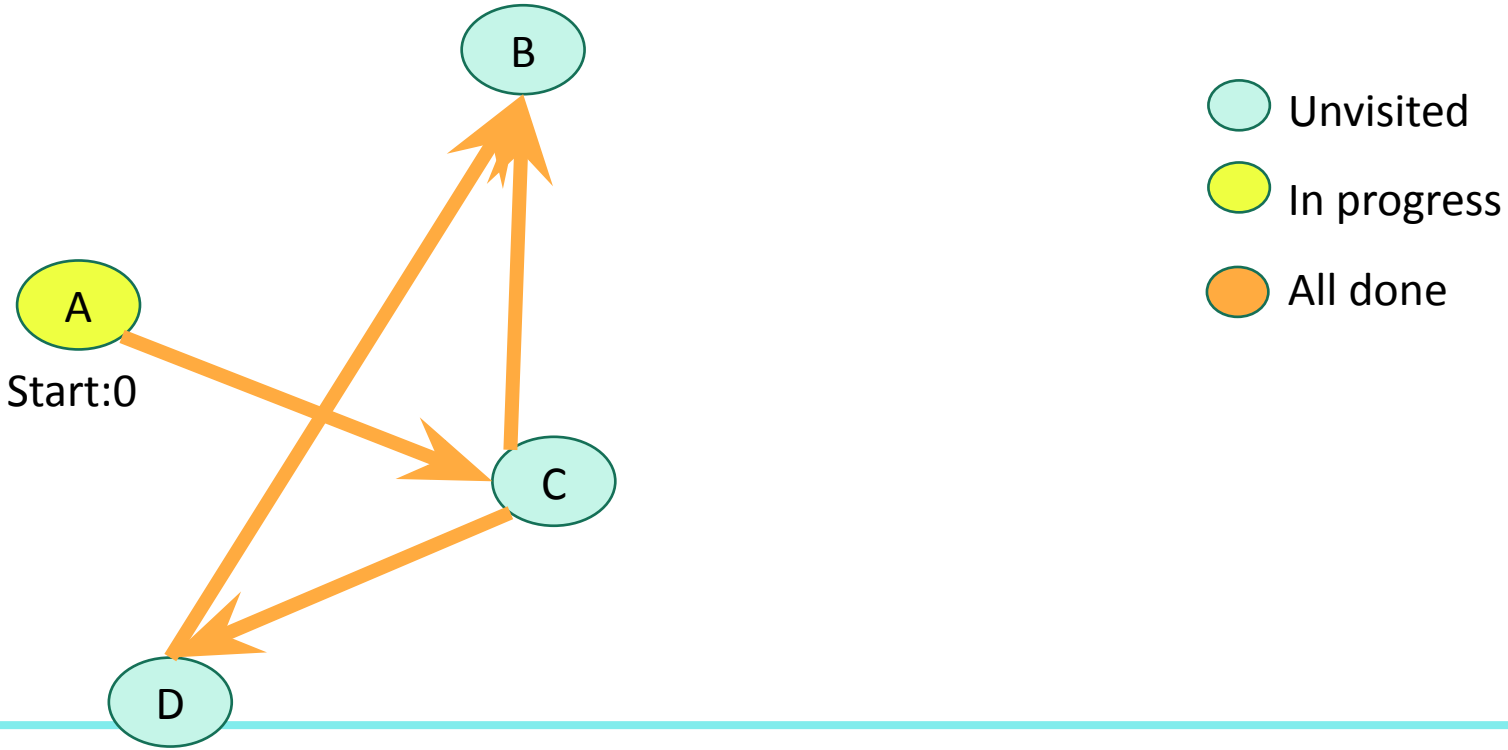
How do we know which v in G(V, E) are source nodes?

Topological Sort Algorithm

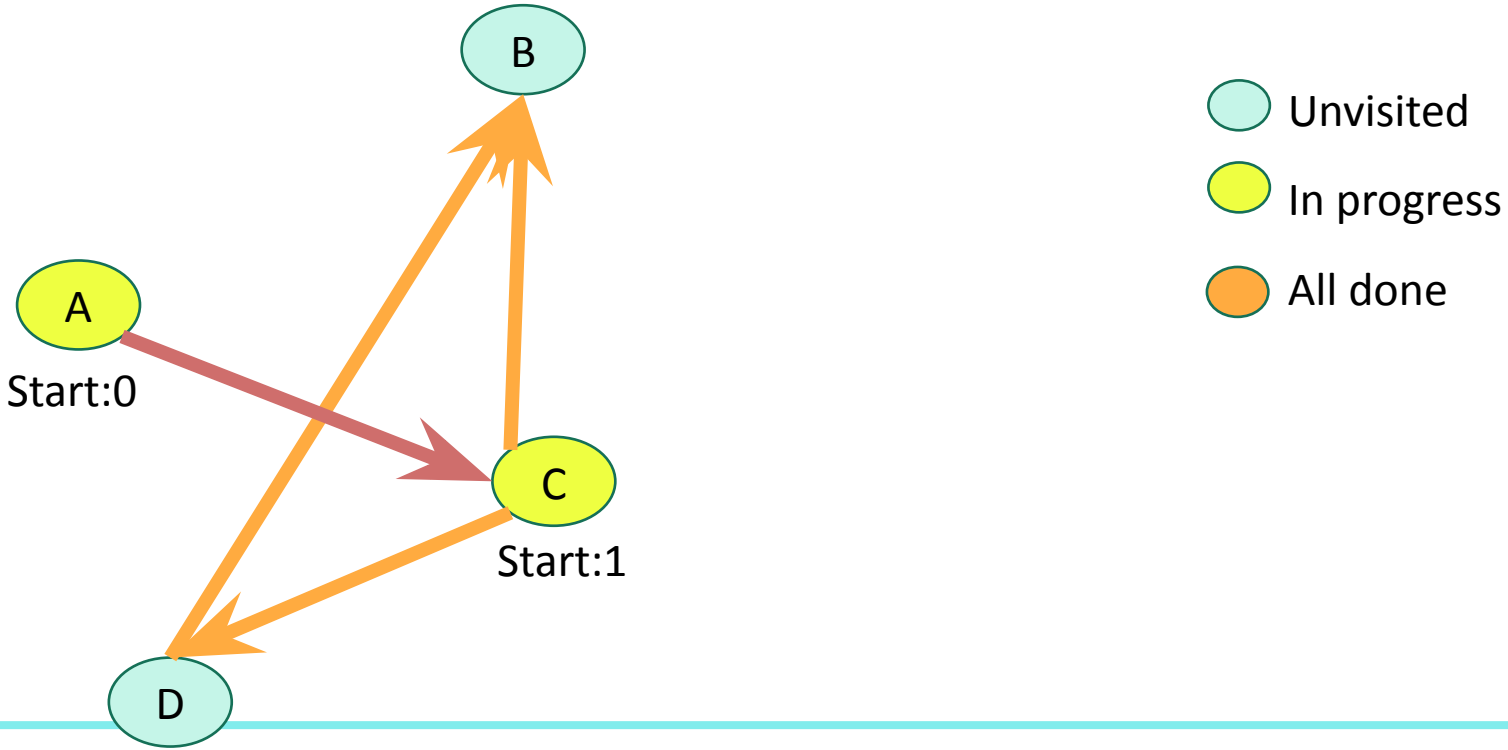
- Note that some valid orderings laid out each “child” until we reached a sink.

- We can actually perform a DFS, starting at source nodes and **reverse the DFS finishing times** to get a valid topological sort.

TopoSort Example



TopoSort Example



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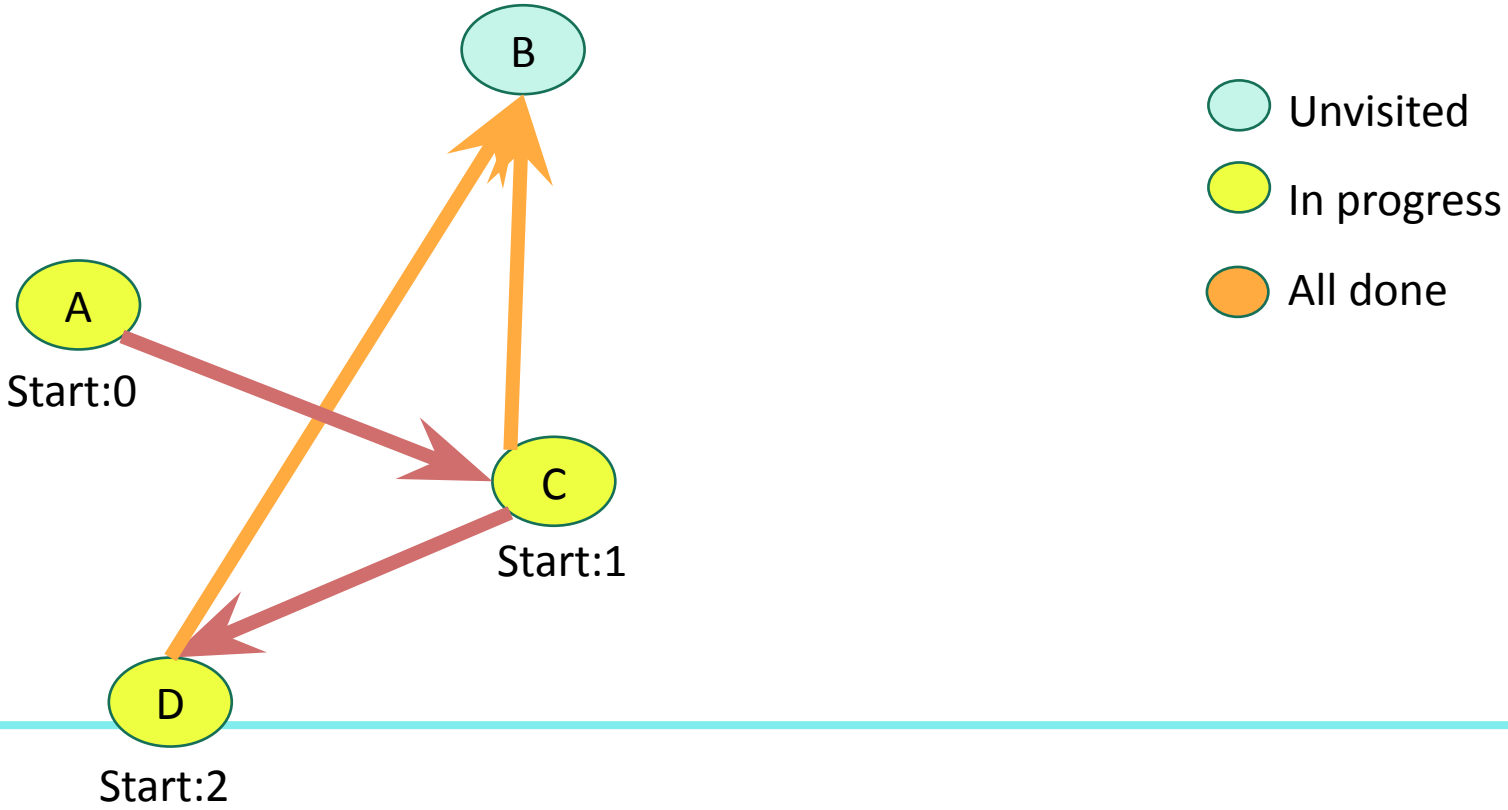
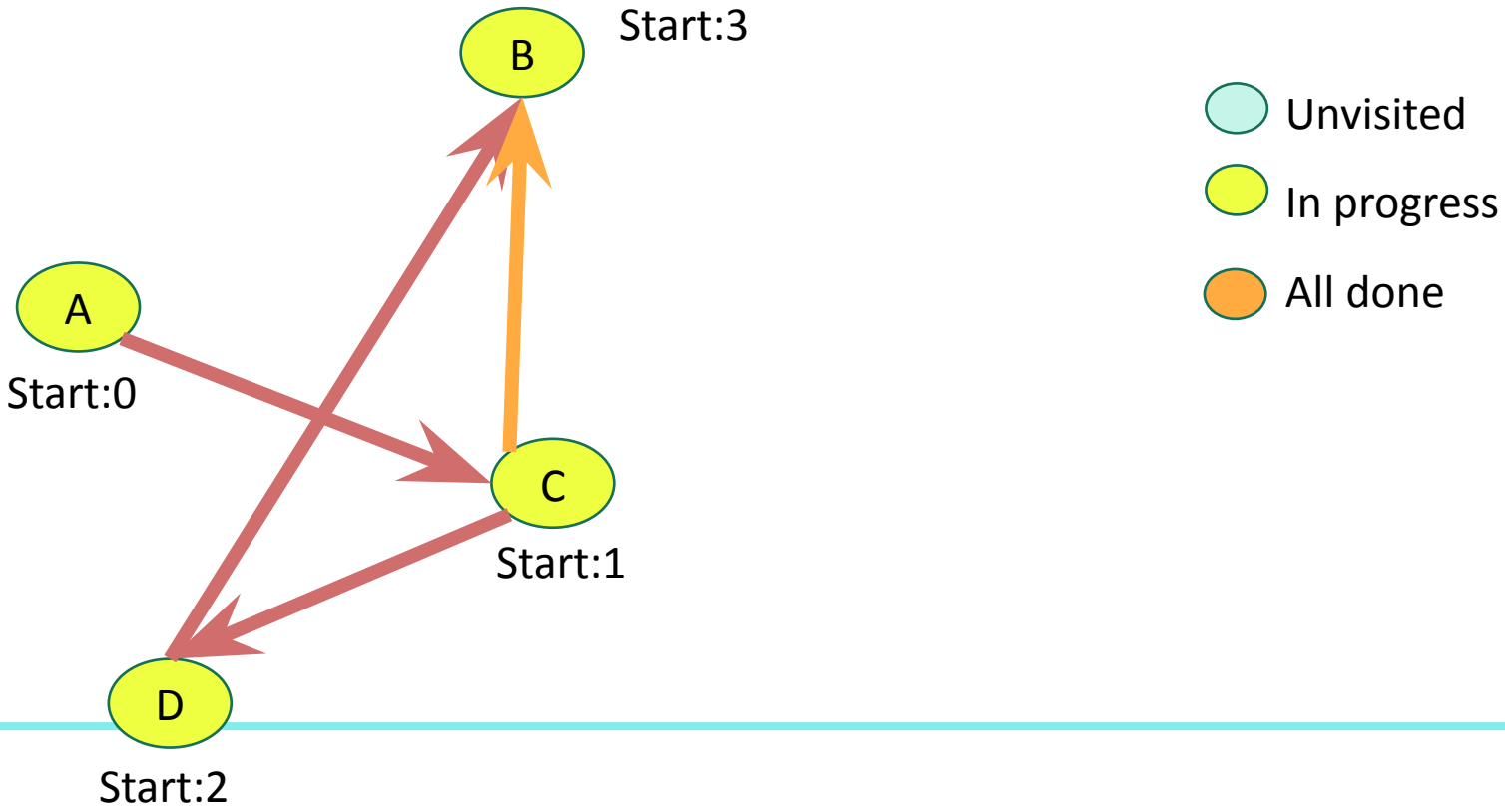
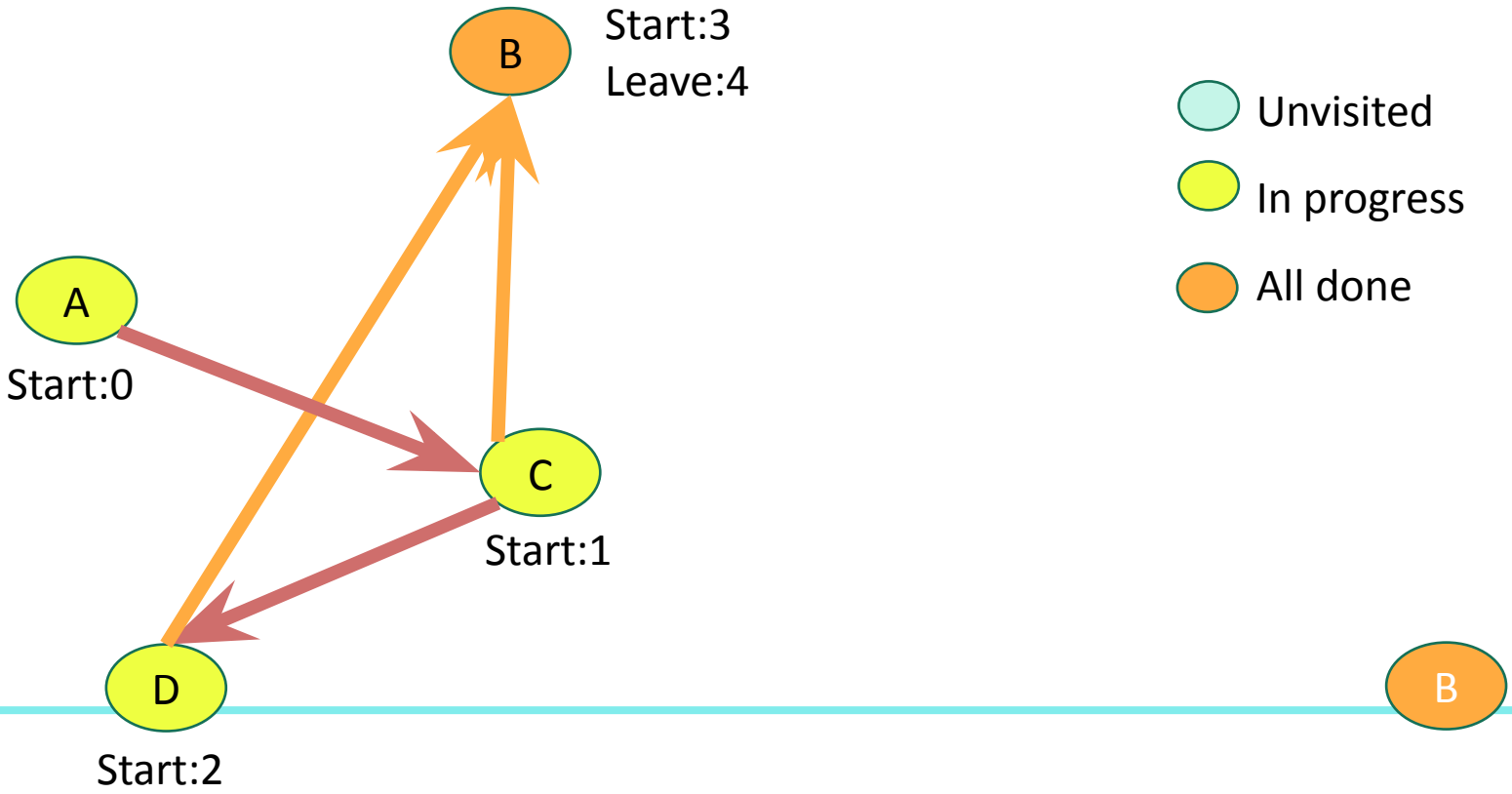


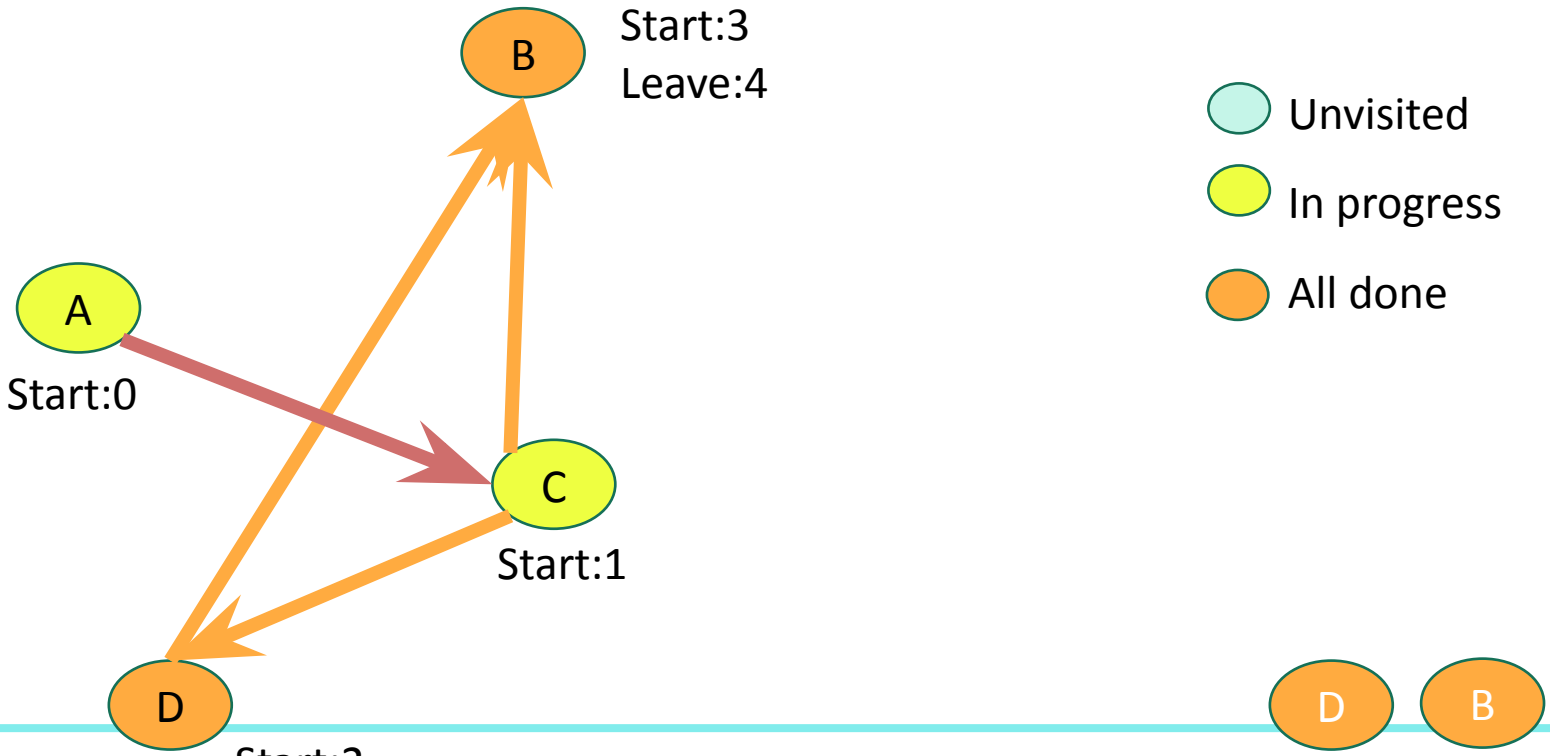
TopoSort Example



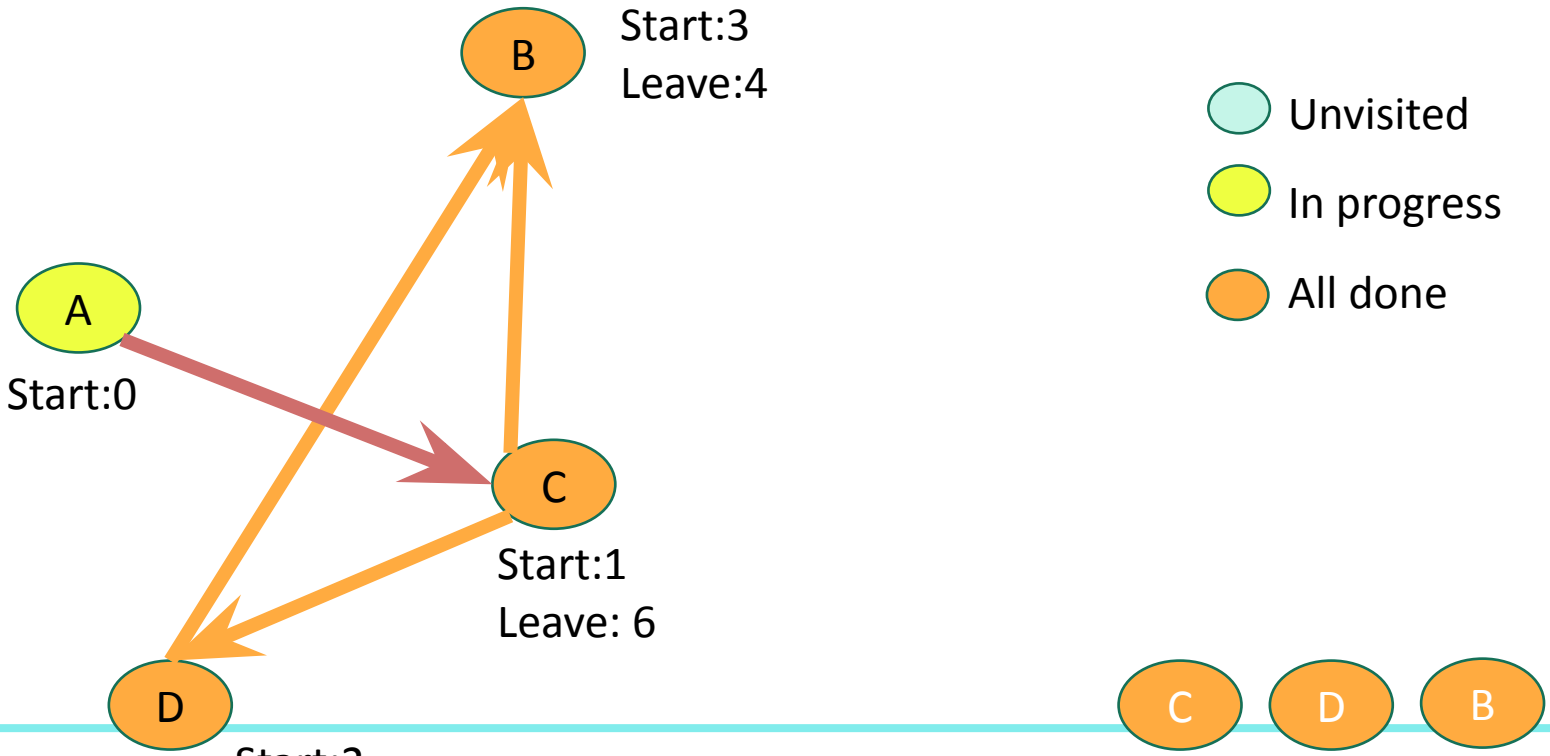
TopoSort Example



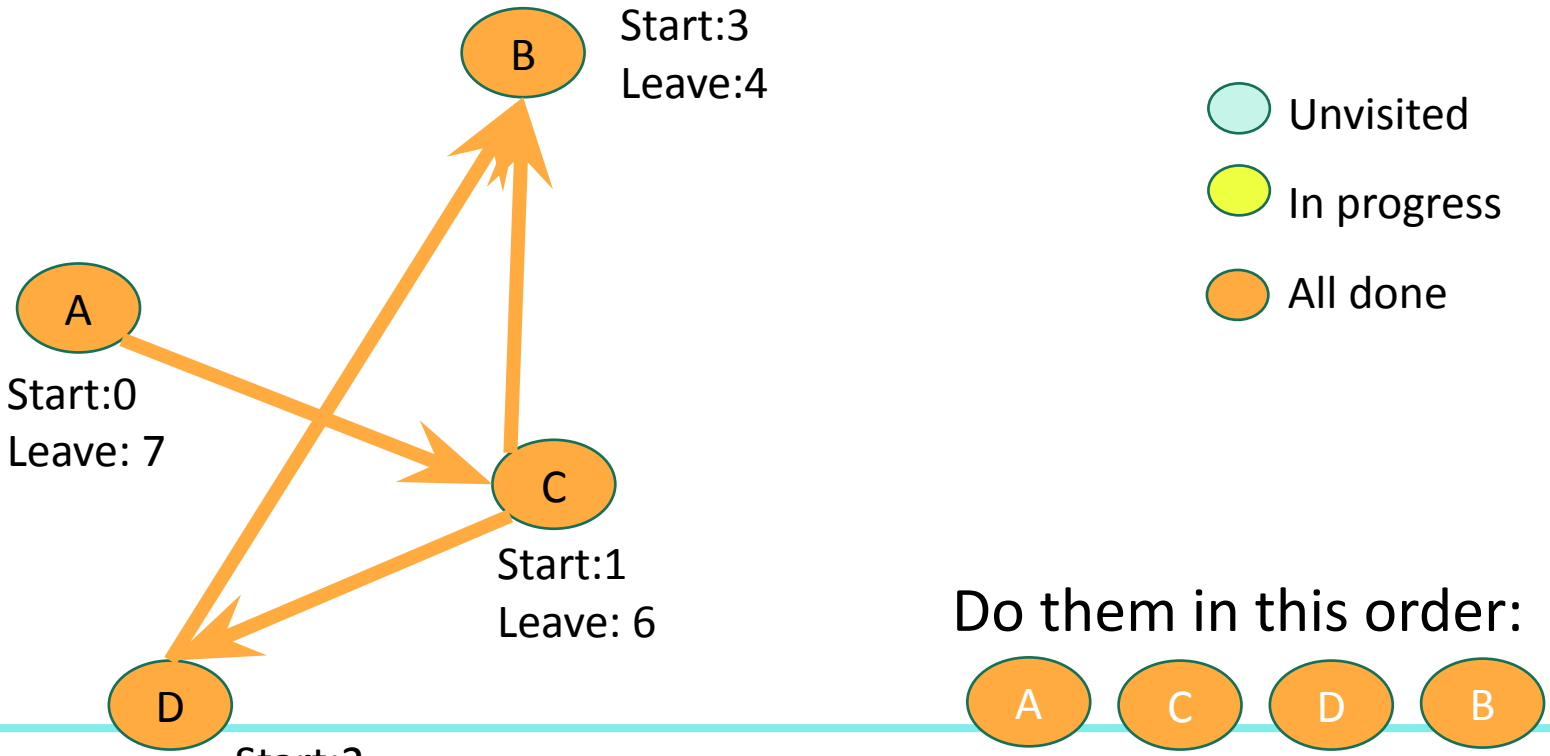
TopoSort Example



TopoSort Example



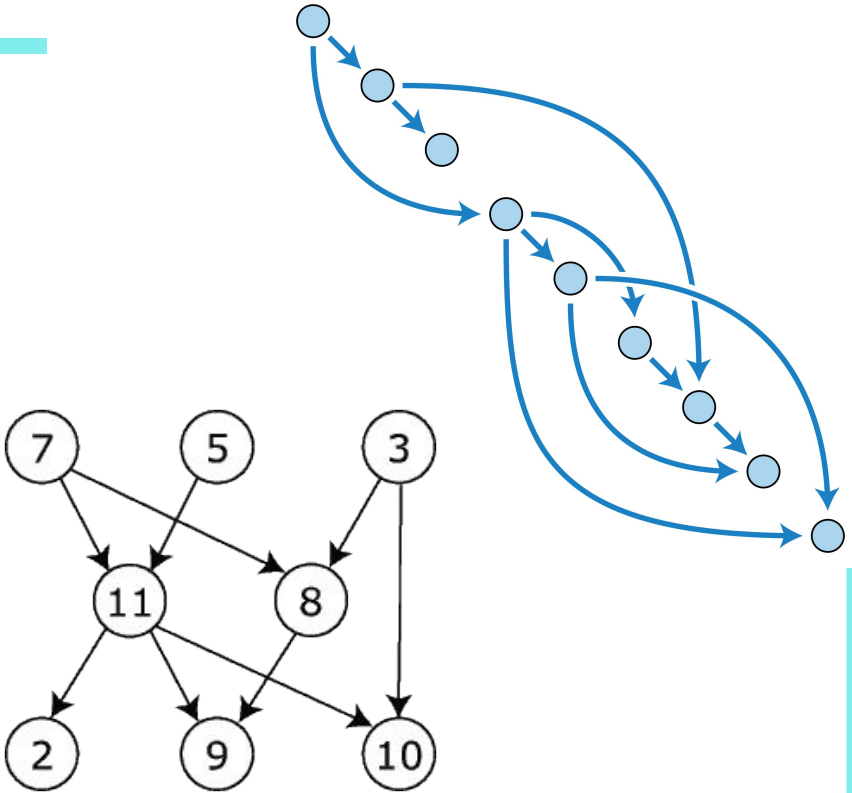
TopoSort Example



TopoSort Code

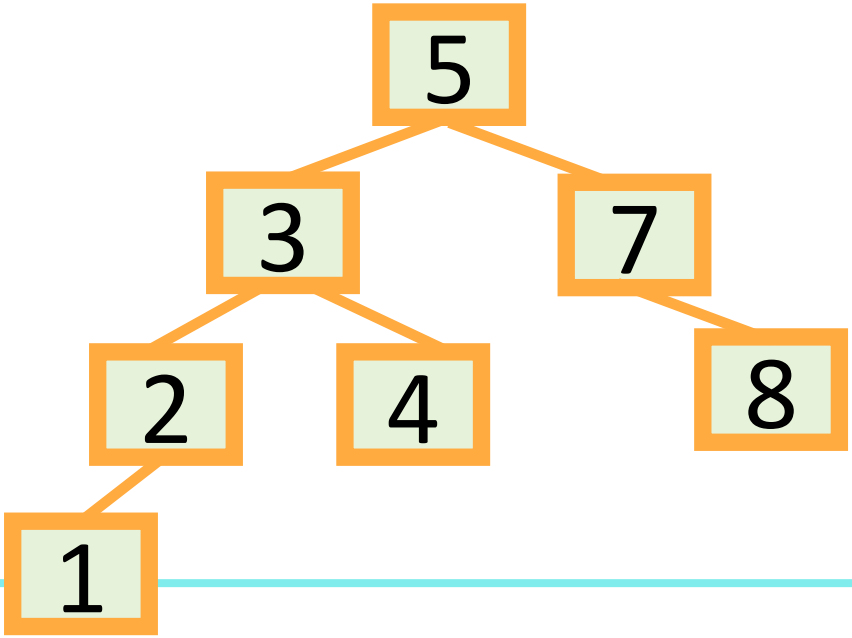
For implementation,

see repl.it



Another use of DFS that we've already seen…

- In-order enumeration of binary search trees



Perform DFS and print a node's label when you are done with the left child and before you begin the right child.

COMP - 285

Analysis of Algorithms

Welcome to COMP 285

Lecture 15: Graph DFS + Topological Sort

Lecturer: Chris Lucas (cflucas@ncat.edu)