COMP - 285

Advanced Analysis of Algorithms

Welcome to COMP 285

Lecture 1: CS Job Hunting (Resume), Pseudocode and Recursion

Chris Lucas (cflucas@ncat.edu)

HW0 is out!

Due Thursday @ 11:59PM !!!!

Homework Assignments

Homework 0: Logistics + Getting to Know You

Release: Aug 18, 3:30 PM - Due: Aug 25, 11:59 PM

- PDF Version: [Link]
- Repl.it: [Starter code]

Big Questions!

08/23/22 - Session

- How to get started with career prep?
- How to write a compelling technical resume?
- How to multiply integers?
- How to conquer? Do we divide?
- How fast is fast enough?

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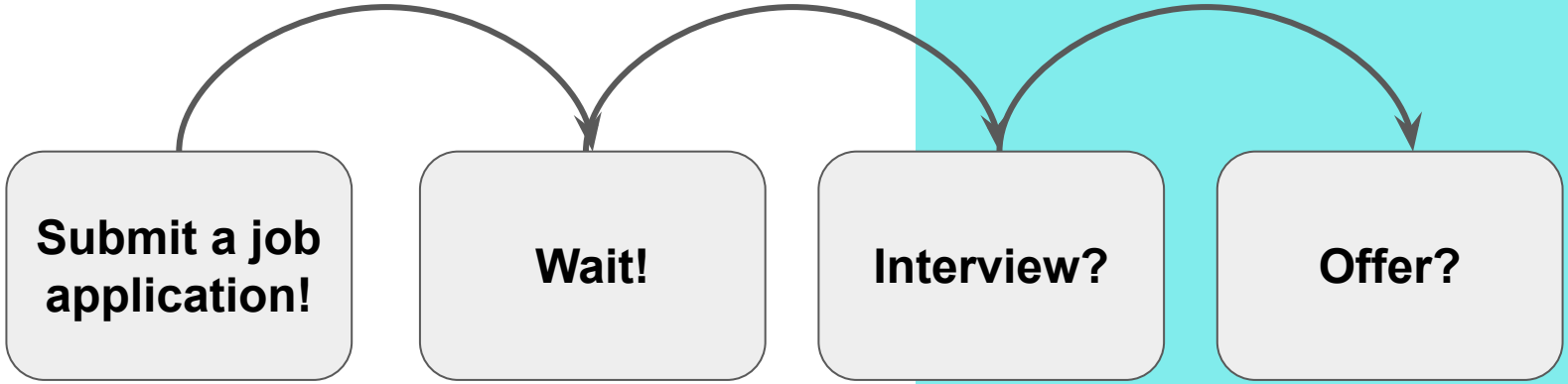
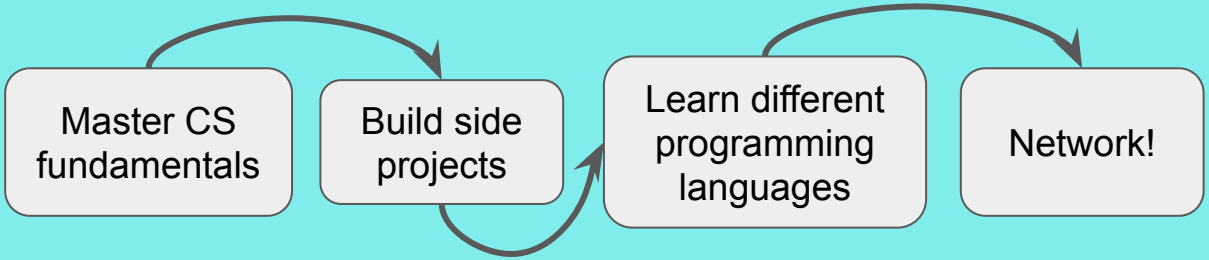
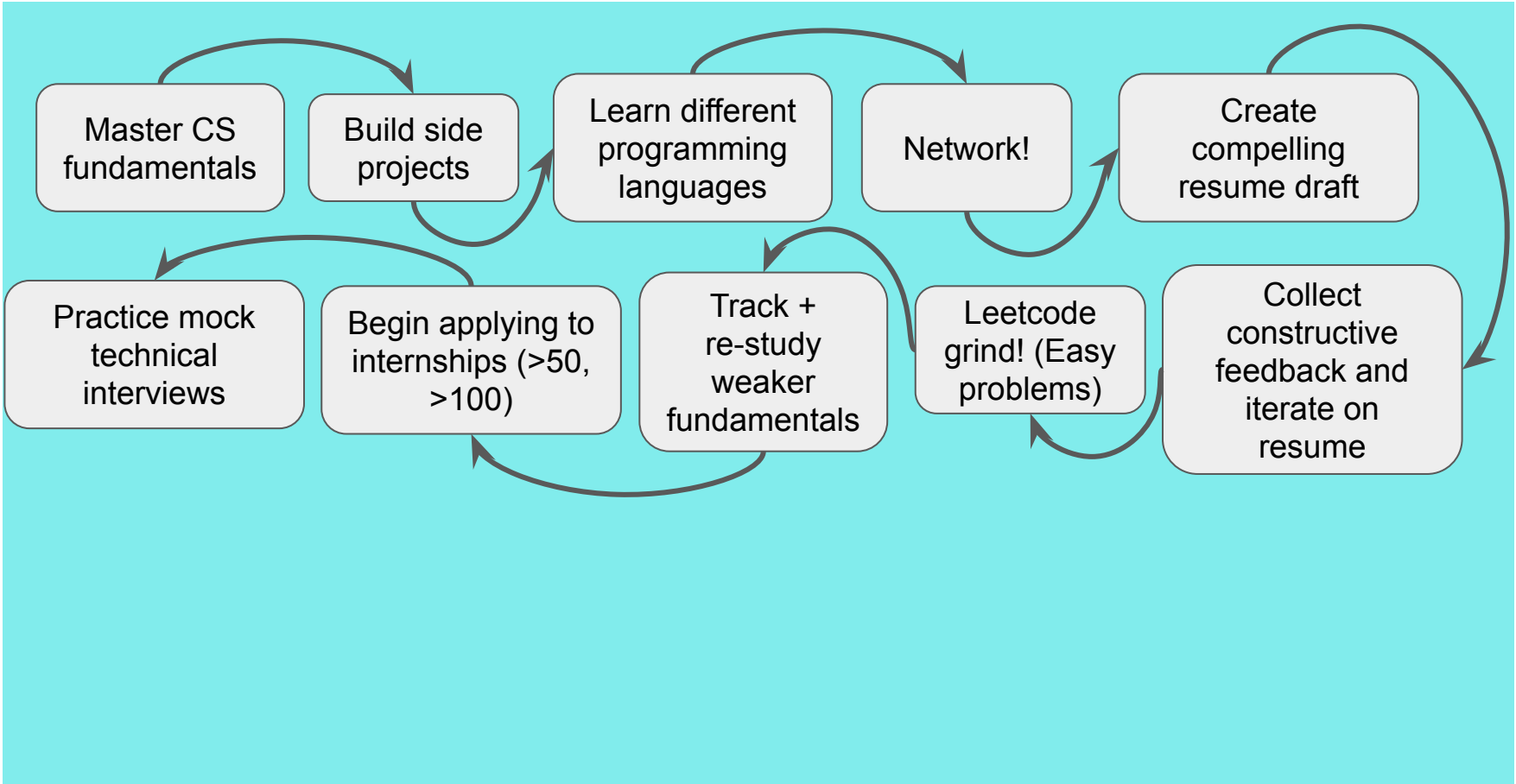
Recruitment Process?




Master CS fundamentals
Build side projects
Learn different programming languages
Network!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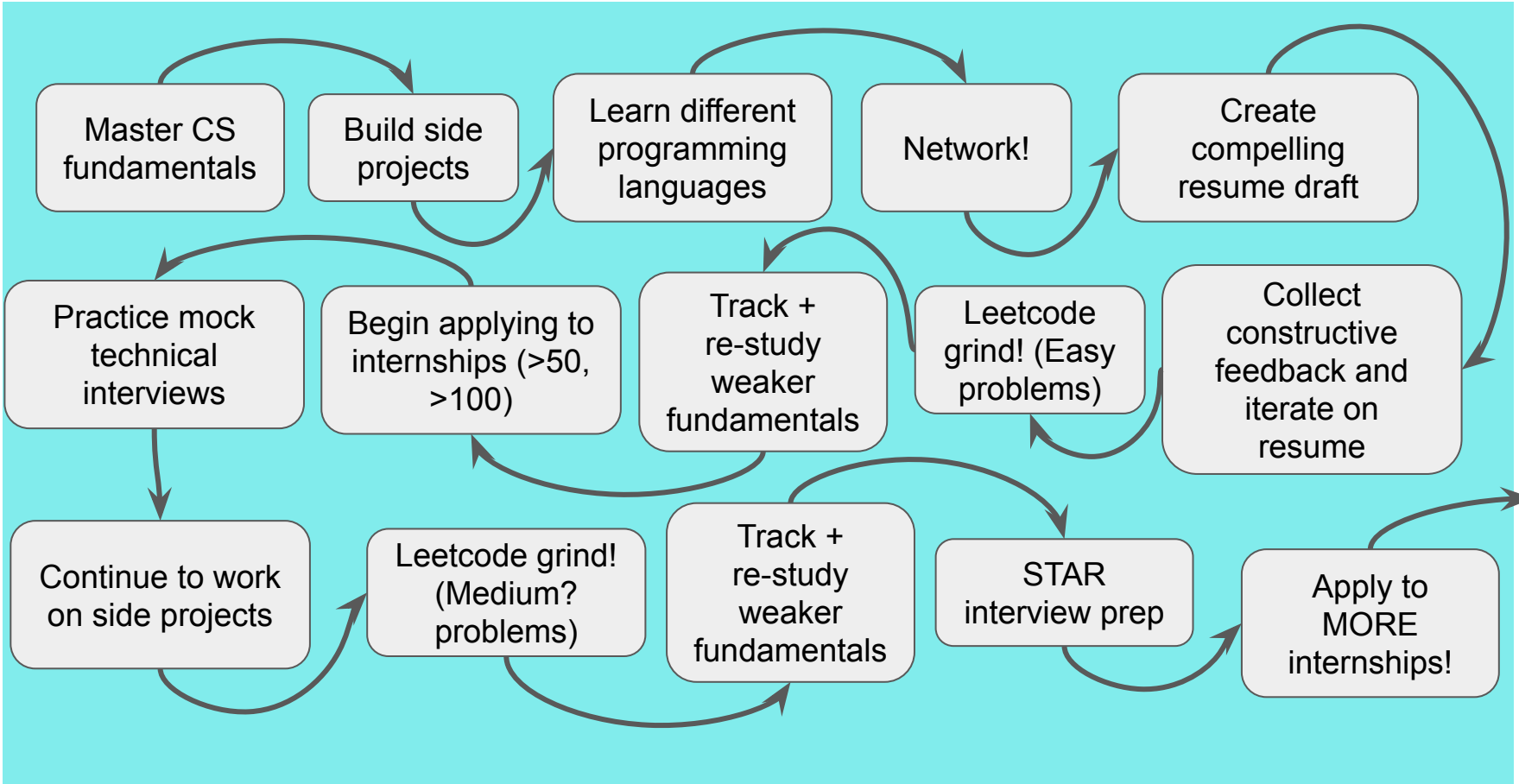
Create compelling resume draft
Practice mock technical interviews
Begin applying to internships (>50, >100)
Track + re-study weaker fundamentals
Leetcode grind! (Easy problems)
Collect constructive feedback and iterate on resume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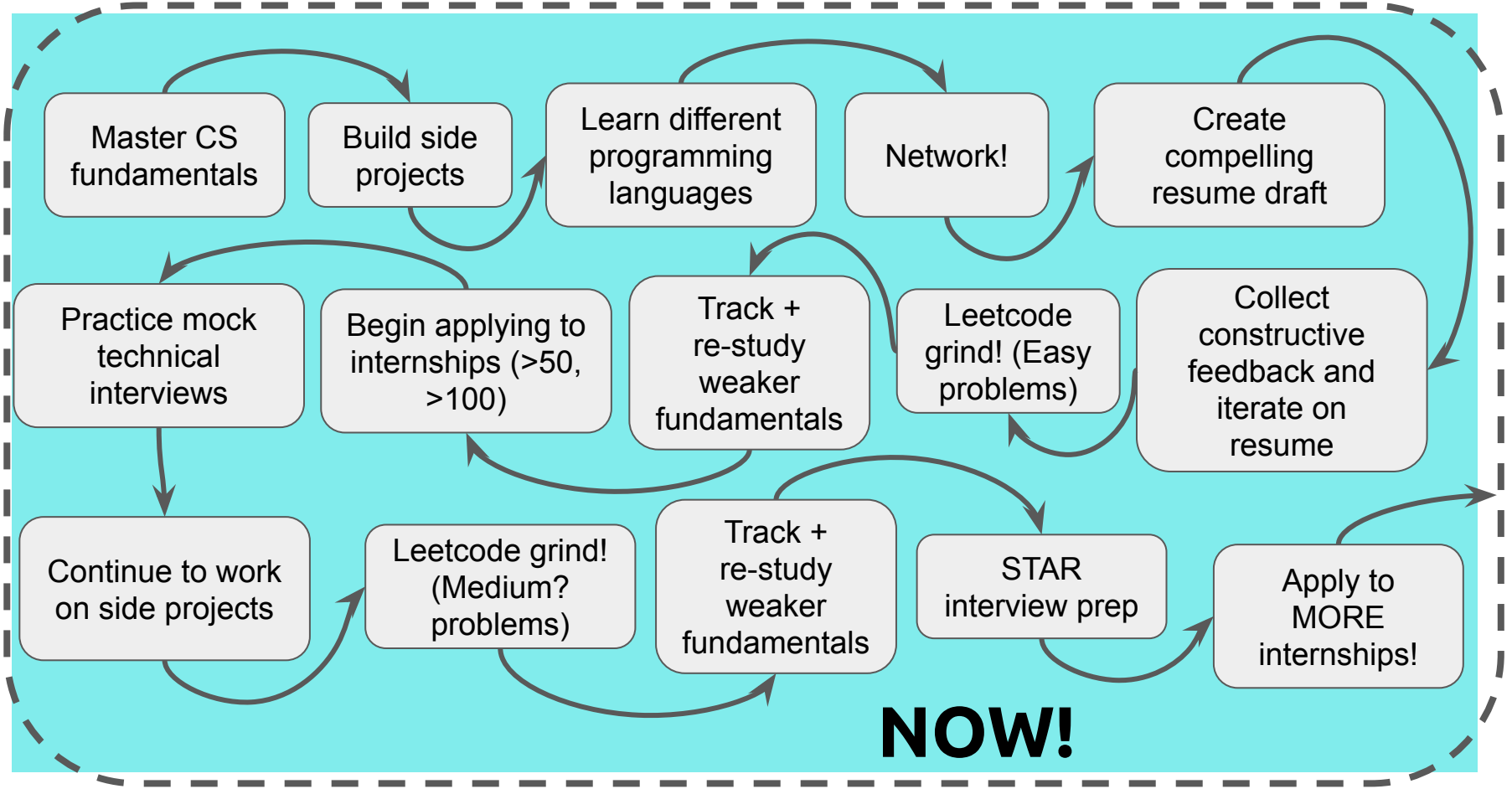
STAR interview prep
Apply to MORE internships!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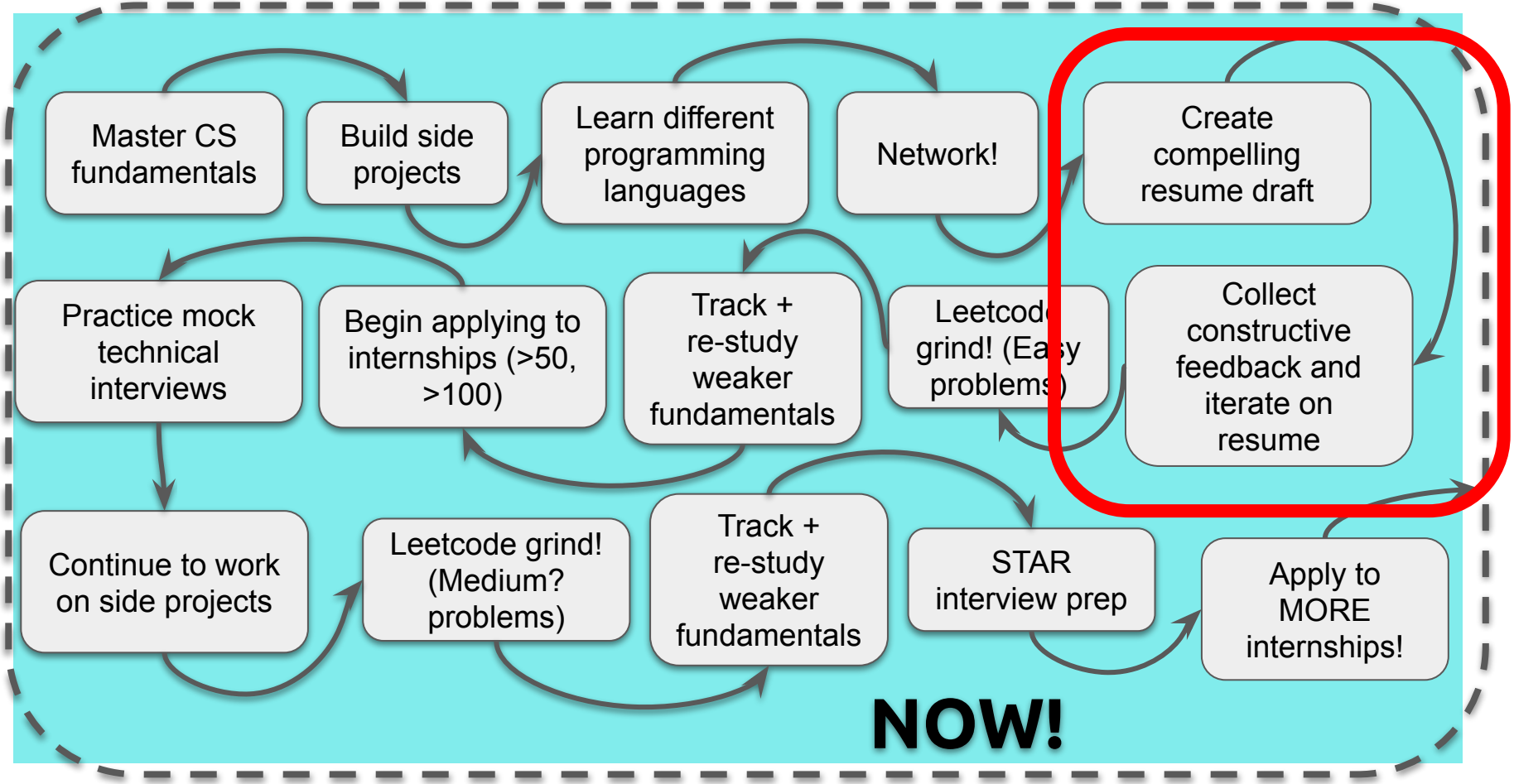
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Resumes!!!

EVERYONE CAN WRITE A POLISHED AND COMPELLING RESUME!

EVERYONE CAN WRITE A POLISHED AND COMPELLING RESUME!

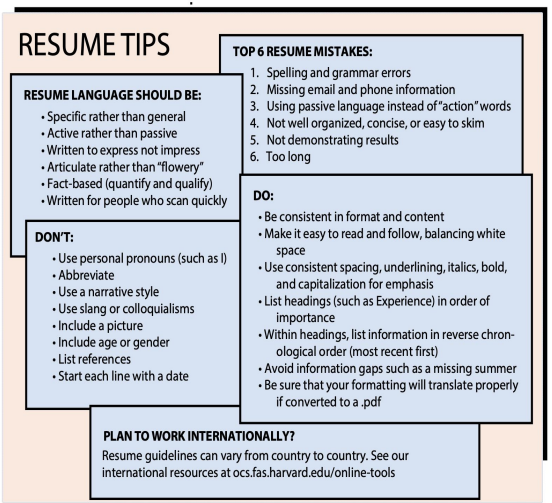
(it will just take some time and effort)

What makes a strong, technical resume?

What makes a ~~strong, technical~~ resume?

What makes a ~~strong, technical~~ resume?

- 1 page !
- Personal information (name, phone # and/or email)
- No pictures, selfies
- 10-12pt font, Times New Roman, Arial, etc.
- Standardized sections: Education, Experience, Other, Software Projects, Skills & Interests
- CONCISE! SKIMMABLE!



*https://hwpi.harvard.edu/files/ocs/files/undergrad_resumes_and_cover_letters.pdf

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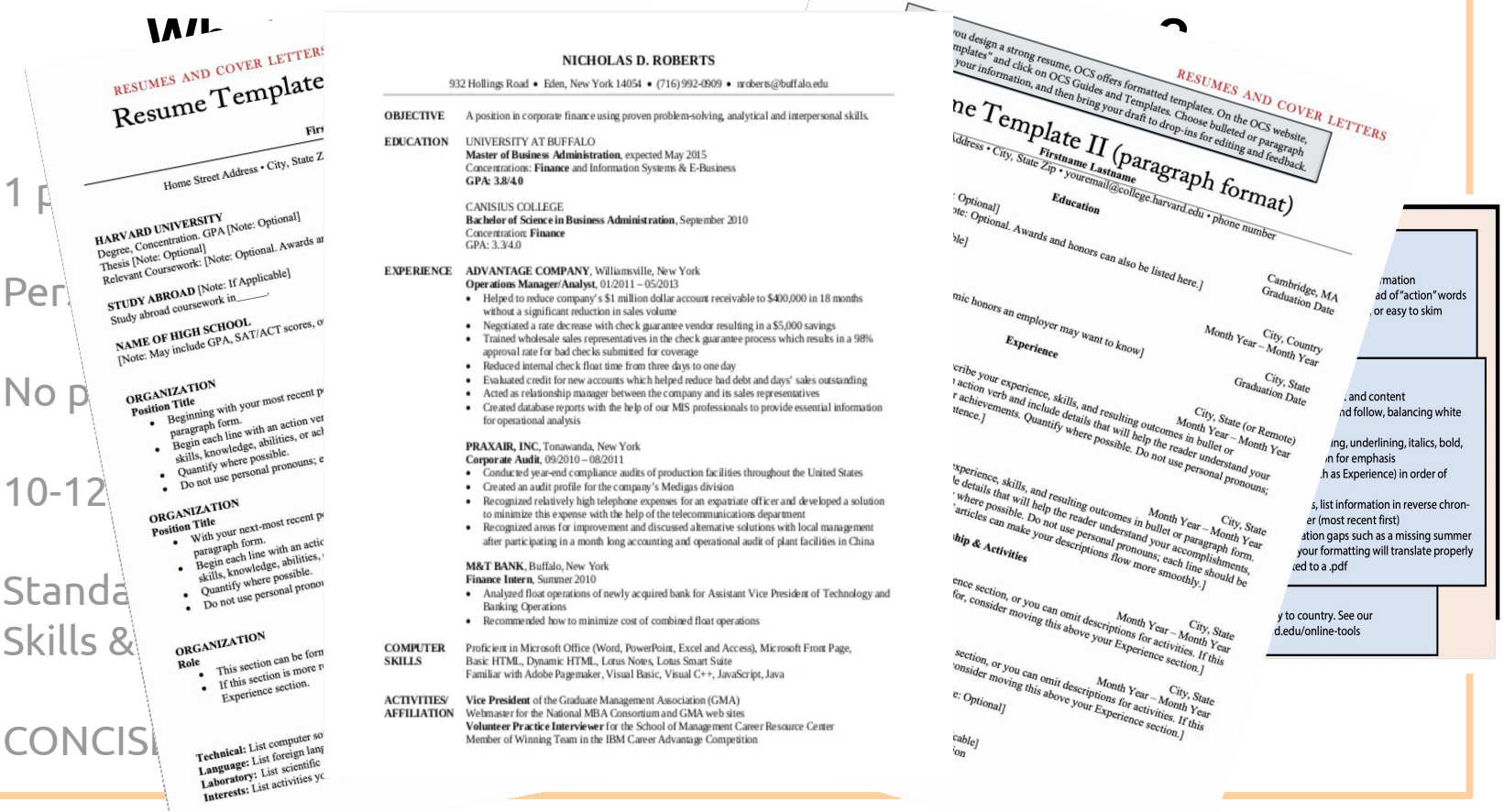

RESUMES AND COVER LETTERS

Resume Template

Home Street Address • City, State Z

HARVARD UNIVERSITY
Degree, Concentration. GPA [Note: Optional]
Thesis [Note: Optional]
Relevant Coursework: [Note: Optional. Awards ar

STUDY ABROAD [Note: If Applicable]
Study abroad coursework in_____.

NAME OF HIGH SCHOOL
[Note: May include GPA, SAT/ACT scores, o

ORGANIZATION
Position Title
- Beginning with your most recent p
  paragraph form.
- Begin each line with an action ver
  skills, knowledge, abilities, or acl
- Quantify where possible.
- Do not use personal pronouns; e

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ORGANIZATION
Role
- This section can be form
- If this section is more r
  Experience section.

Technical: List computer so
Language: List foreign lang
Laboratory: List scientific
Interests: List activities yc

NICHOLAS D. ROBERTS

932 Hollings Road • Eden, New York 14054 • (716) 992-0909 • nroberts@buffalo.edu

OBJECTIVE A position in corporate finance using proven problem-solving, analytical and interpersonal skills.

EDUCATION
UNIVERSITY AT BUFFALO
Master of Business Administration, expected May 2015
Concentrations: **Finance** and Information Systems & E-Business
GPA: 3.8/4.0

CANISIUS COLLEGE
Bachelor of Science in Business Administration, September 2010
Concentration: **Finance**
GPA: 3.3/4.0

EXPERIENCE
ADVANTAGE COMPANY, Williamsville, New York
Operations Manager/Analyst, 01/2011 – 05/2013
- Helped to reduce company's $1 million dollar account receivable to $400,000 in 18 months without a significant reduction in sales volume
- Negotiated a rate decrease with check guarantee vendor resulting in a $5,000 savings
- Trained wholesale sales representatives in the check guarantee process which results in a 98% approval rate for bad checks submitted for coverage
- Reduced internal check float time from three days to one day
- Evaluated credit for new accounts which helped reduce bad debt and days' sales outstanding
- Acted as relationship manager between the company and its sales representatives
- Created database reports with the help of our MIS professionals to provide essential information for operational analysis

PRAXAIR, INC, Tonawanda, New York
Corporate Audit, 09/2010 – 08/2011
- Conducted year-end compliance audits of production facilities throughout the United States
- Created an audit profile for the company's Medigas division
- Recognized relatively high telephone expenses for an expatriate officer and developed a solution to minimize this expense with the help of the telecommunications department
- Recognized areas for improvement and discussed alternative solutions with local management after participating in a month long accounting and operational audit of plant facilities in China

M&T BANK, Buffalo, New York
Finance Intern, Summer 2010
- Analyzed float operations of newly acquired bank for Assistant Vice President of Technology and Banking Operations
- Recommended how to minimize cost of combined float operations

COMPUTER SKILLS Proficient in Microsoft Office (Word, PowerPoint, Excel and Access), Microsoft Front Page, Basic HTML, Dynamic HTML, Lotus Notes, Lotus Smart Suite
Familiar with Adobe Pagemaker, Visual Basic, Visual C++, JavaScript, Java

ACTIVITIES/ AFFILIATION **Vice President** of the Graduate Management Association (GMA)
Webmaster for the National MBA Consortium and GMA web sites
Volunteer Practice Interviewer for the School of Management Career Resource Center
Member of Winning Team in the IBM Career Advantage Competition

RESUMES AND COVER LETTERS

ou design a strong resume, OCS offers formatted templates. On the OCS website, mplates" and click on OCS Guides and Templates. Choose bulleted or paragraph your information, and then bring your draft to drop-ins for editing and feedback.

ne Template II (paragraph format)

Firstname Lastname
ddress • City, State Zip • youremail@college.harvard.edu • phone number

Education

Optional]
te: Optional. Awards and honors can also be listed here.]
Cambridge, MA
Graduation Date
mic honors an employer may want to know]
City, Country
Month Year – Month Year

Experience

City, State
Graduation Date
cribe your experience, skills, and resulting outcomes in bullet or
action verb and include details that will help the reader understand your
r achievements. Quantify where possible. Do not use personal pronouns;
tence.]
City, State (or Remote)
Month Year – Month Year

xperience, skills, and resulting outcomes in bullet or paragraph form.
le details that will help the reader understand your accomplishments,
where possible. Do not use personal pronouns; each line should be
articles can make your descriptions flow more smoothly.]
City, State
Month Year – Month Year

hip & Activities

ence section, or you can omit descriptions for activities. If this
for, consider moving this above your Experience section.]
City, State
Month Year – Month Year

section, or you can omit descriptions for activities. If this
onsider moving this above your Experience section.]
City, State
Month Year – Month Year

e: Optional]

cable]
ion

mation
ad of "action" words
, or easy to skim

and content
nd follow, balancing white

ng, underlining, italics, bold,
n for emphasis
h as Experience) in order of

s, list information in reverse chron-
er (most recent first)
ation gaps such as a missing summer
your formatting will translate properly
ed to a .pdf

y to country. See our
d.edu/online-tools

*https://hwpi.harvard.edu/f[illegible]/files/undergrad_resumes_and_cover_letters.pdf

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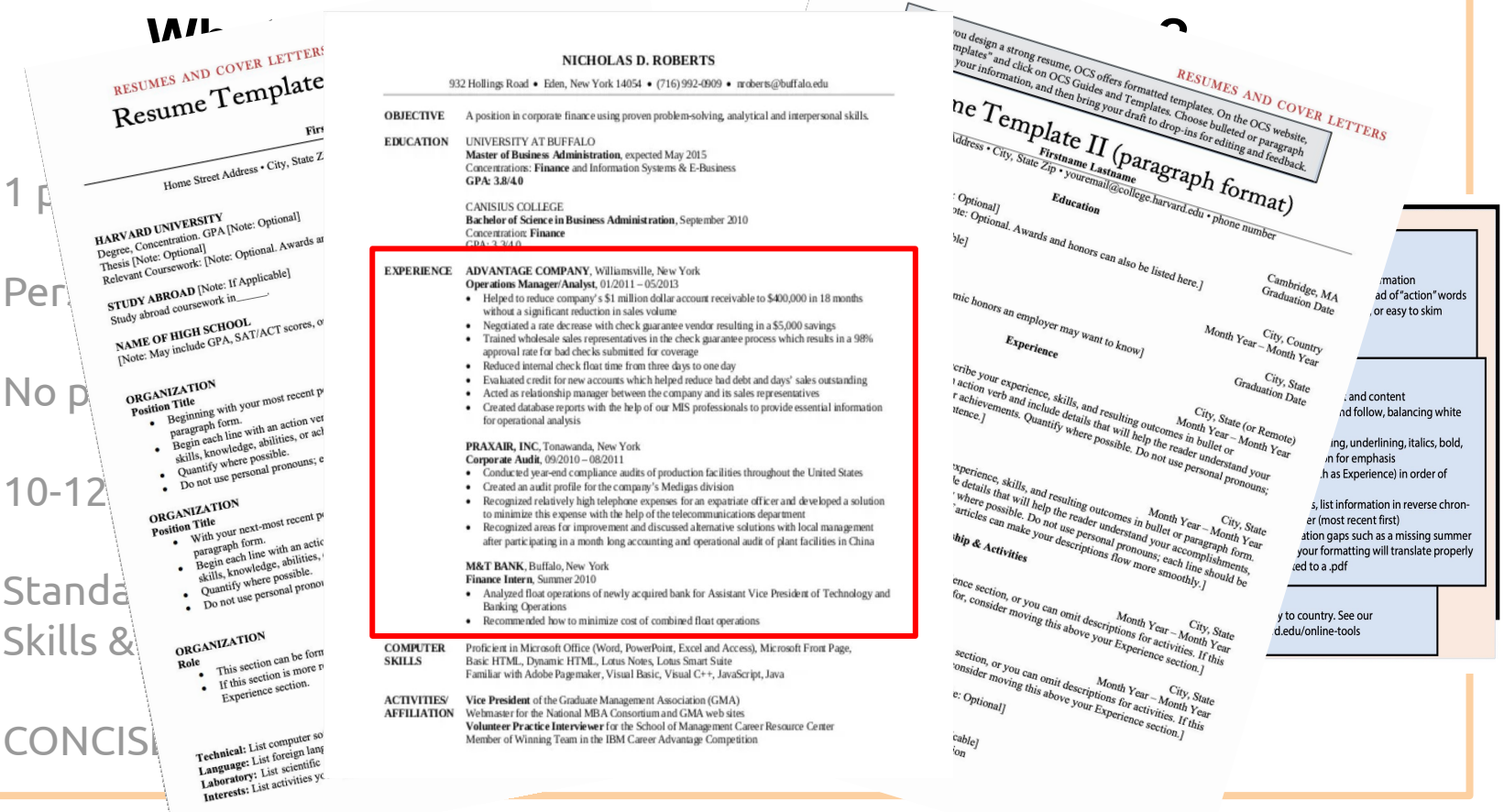

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What makes a strong, technical resume?

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CASHIER, CHICK-FIL-A, GREENSBORO, NC | 06/17 - 09/17

• I would help customers by taking their orders then would ensure they received their correct order.

CENTRE-STORE STOCKER, FOOD LION, GREENSBORO, NC | 06/18 - 09/18

• I would find and retrieve stock then place it on allocated shelves in an efficient way.

“RATE EM” IOS APP, NCAT HACKATHON, GREENSBORO, NC | 10/19

• We worked on an app to rate professors and cafeteria food on-campus using “Rate Me” tokens.

Using pronouns

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How to quantify? Why does it matter?

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What does this “token” mean?

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Using pronouns

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• I would help customers by tak rrect order.

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CENTRE

• I would way.

"RATE EM" ENSBORO, NC | 10/19

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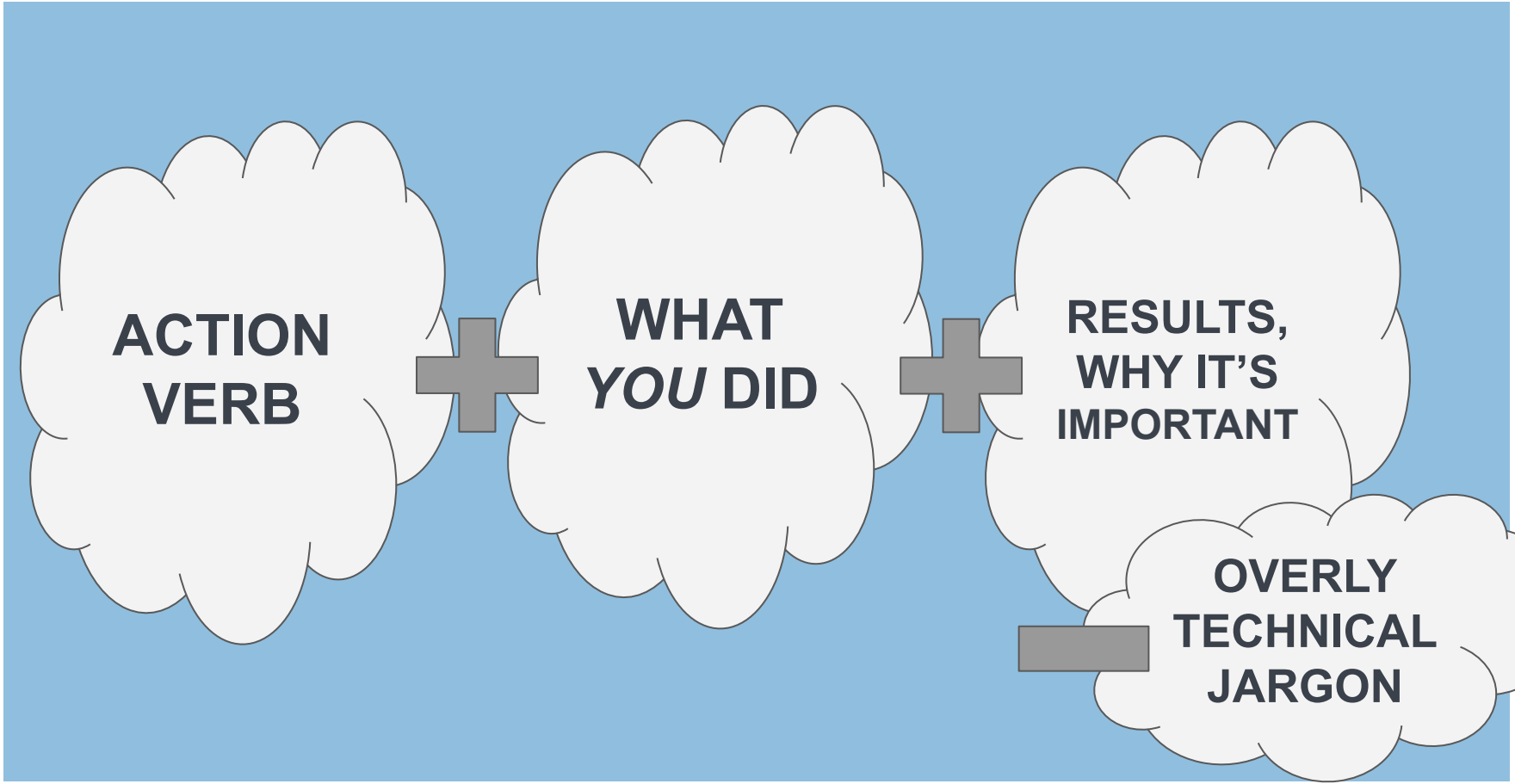
How to quantify? Why does it matter?

What did *I* do? Was I good at the job?

What does this "token" mean?

What was my impact?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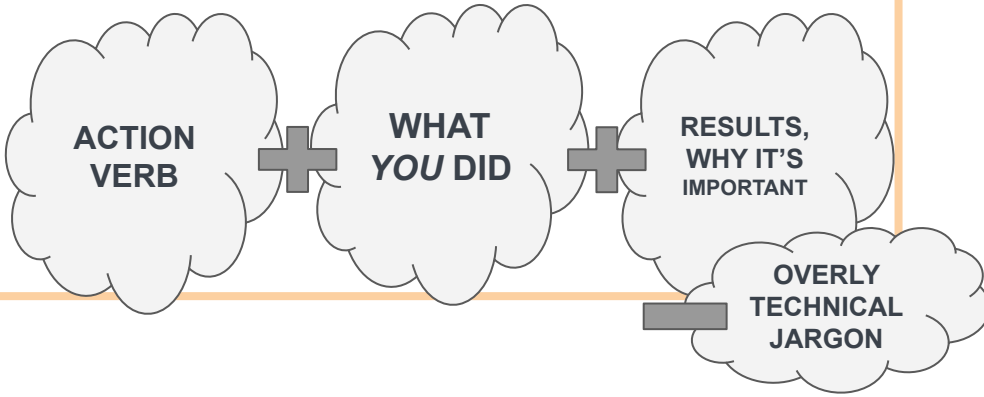

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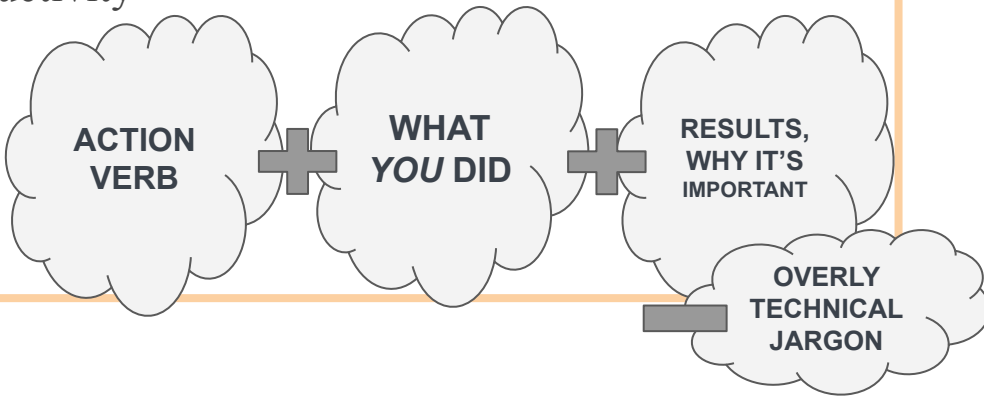
- I would help customers by taking their orders then would ensure they received their correct order.

- Utilized F2F order management system to successfully input over 50 orders/hr then mentored two new hires to achieve same level of productivity



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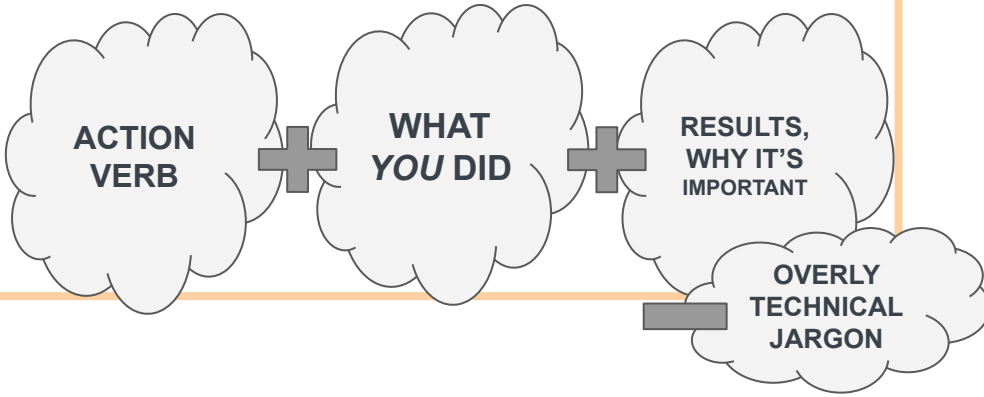

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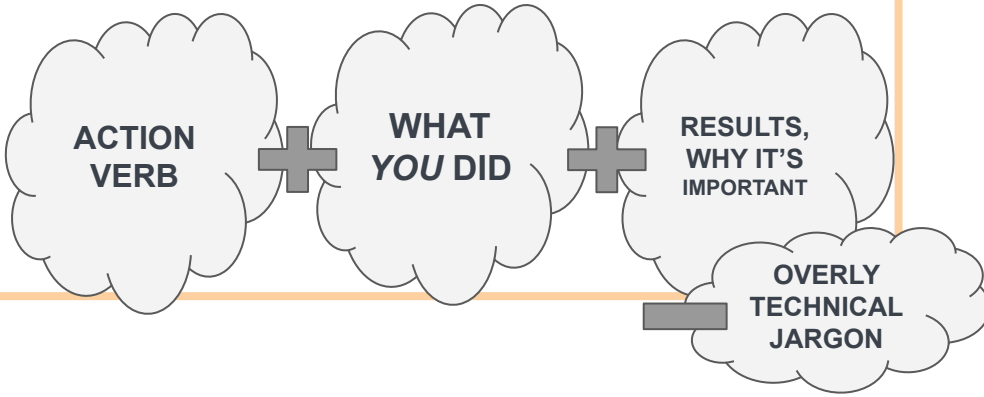
- I would find and retrieve stock then place it on allocated shelves in an efficient way.

- Managed stock of 300+ items and was recognized by senior management for ability to debug inconsistencies in the inventory counts logged in tracking software versus actual supply



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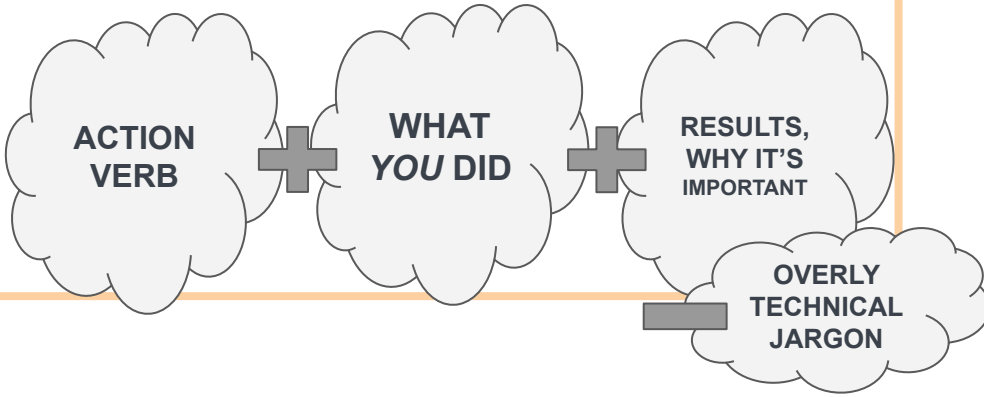

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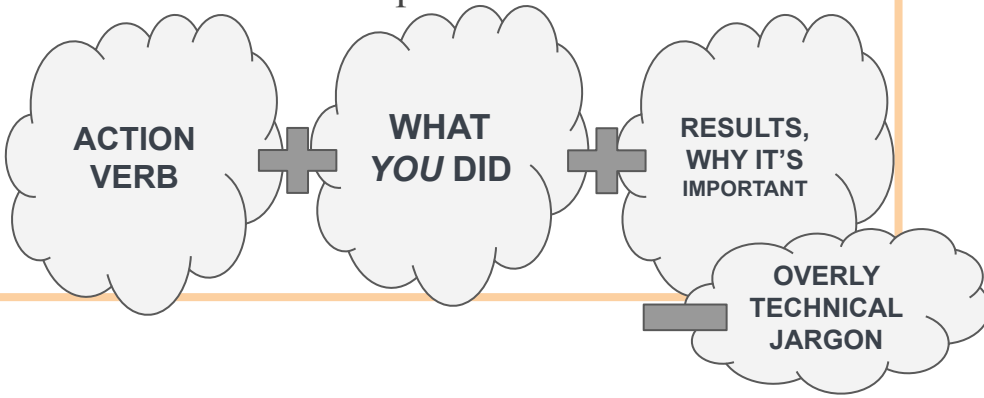
- Developed iOS app to allow students to rate +250 professors and +7 on-campus cafeterias, requiring data from multiple university APIs; managed team of 4 and won third place award from Citadel sponsor



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- Consider your part time work experience!

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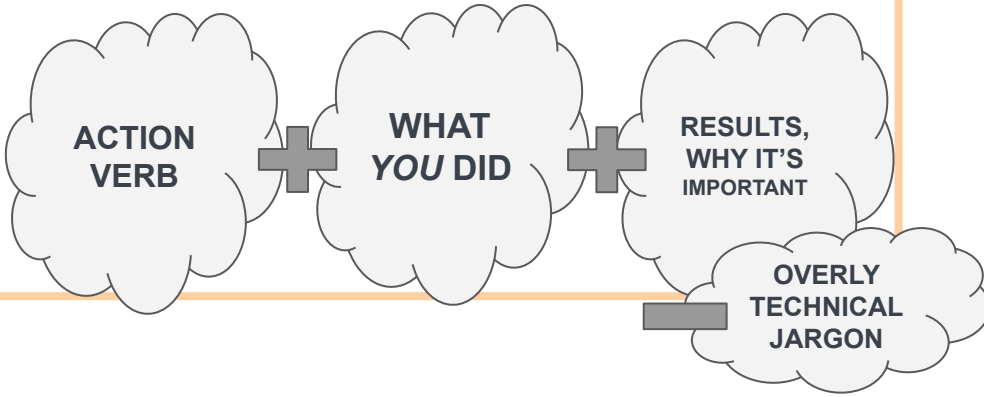
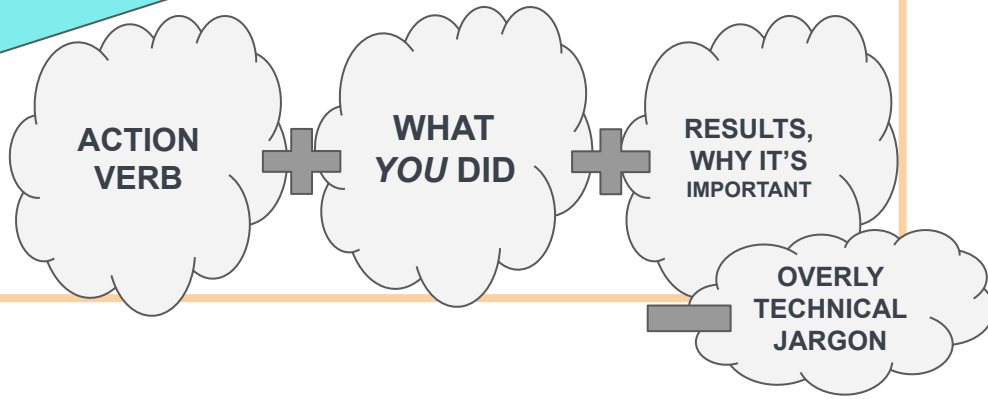
- Consider side projects! (Github, Kaggle, Leetcode)
- Consider your extracurriculars here at NCAT! (leadership positions, impact that you’ve had, etc.)
- Consider your course projects and assignments! (Turn into side project!)
- Consider your part time work experience!
- Take time to breathe and reflect! Everyone starts somewhere and you have more experiences than you think!

EVERYONE CAN WRITE A POLISHED AND COMPELLING RESUME!



HW 1!!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Big Questions!

08/23/22 - Session

- How to get started with career prep?
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- How to multiply integers?
- How to conquer? Do we divide?
- How fast is fast enough?

Recall where we ended last lecture…

Constant

5s

x

5s

2x

5s

3x

Linear

5s

x

10s

2x

15s

3x

Quadratic

5s

x

20s

2x

45s

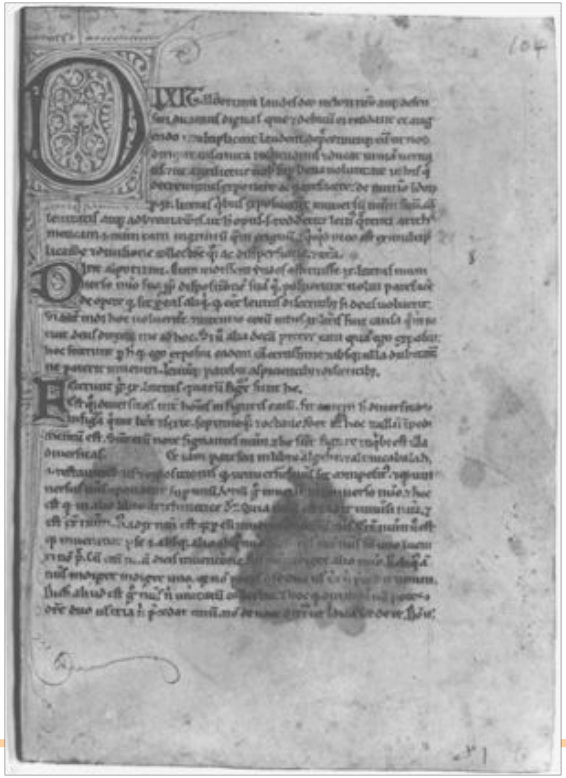
3x

What’s an algorithm anyhow?

- Al-Khwarizmi was a 9th-century scholar, born in present-day Uzbekistan, who studied and worked in Baghdad during the Abbassid Caliphate.
- Among many other contributions in mathematics, astronomy, and geography, **he wrote a book about how to multiply with Arabic numerals.**
- His ideas came to Europe in the 12th century.
-
-

What's an algorithm anyhow?

- Originally, "Algorisme" [old French] referred to just the Arabic number system, but eventually it came to mean "Algorithm" as we know today.



This was kind of a big deal!

$$\mathrm{XLIV} \times \mathrm{XCVII} = ?$$

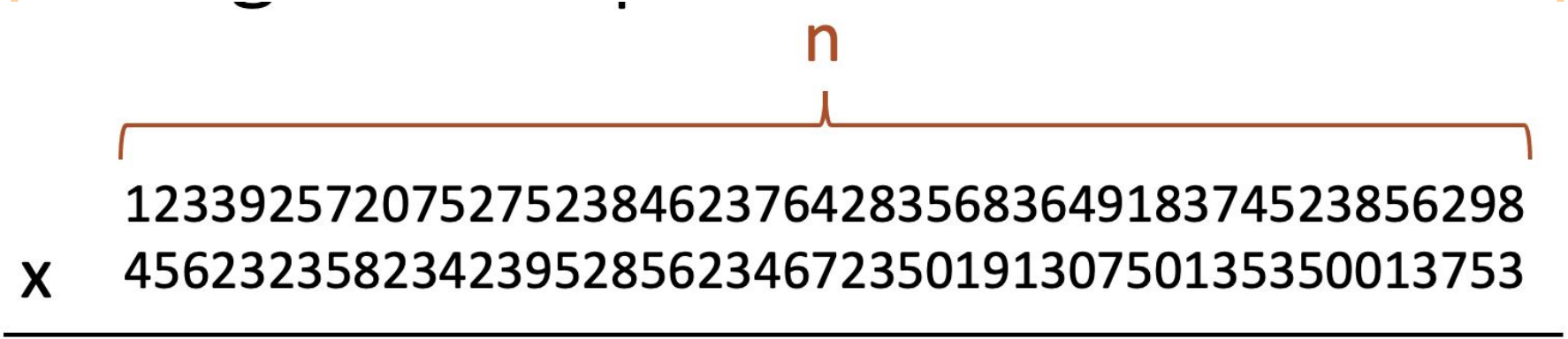
Integer multiplication!

$$\begin{array}{r} 44 \\ \times\ 97 \\ \hline \end{array}$$

Integer multiplication!

$$
\begin{array}{rr}
 & 12345678959314 13 \\
\times & 4563823520395533 \\
\hline
\end{array}
$$

How fast is this anyway?



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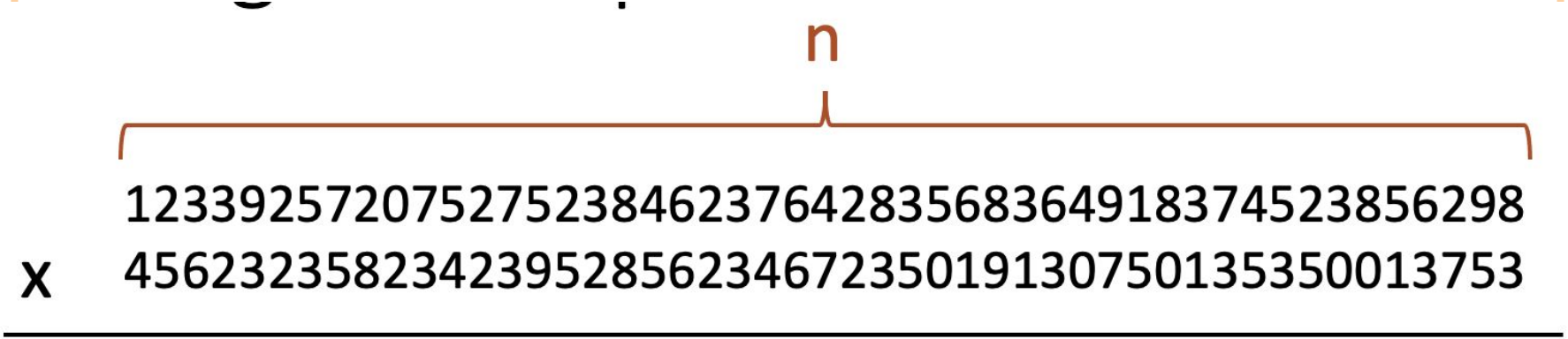


(How many one-digit operations?)

How fast is this anyway?

About n^2 one-digit operations

(How many one-digit operations?)

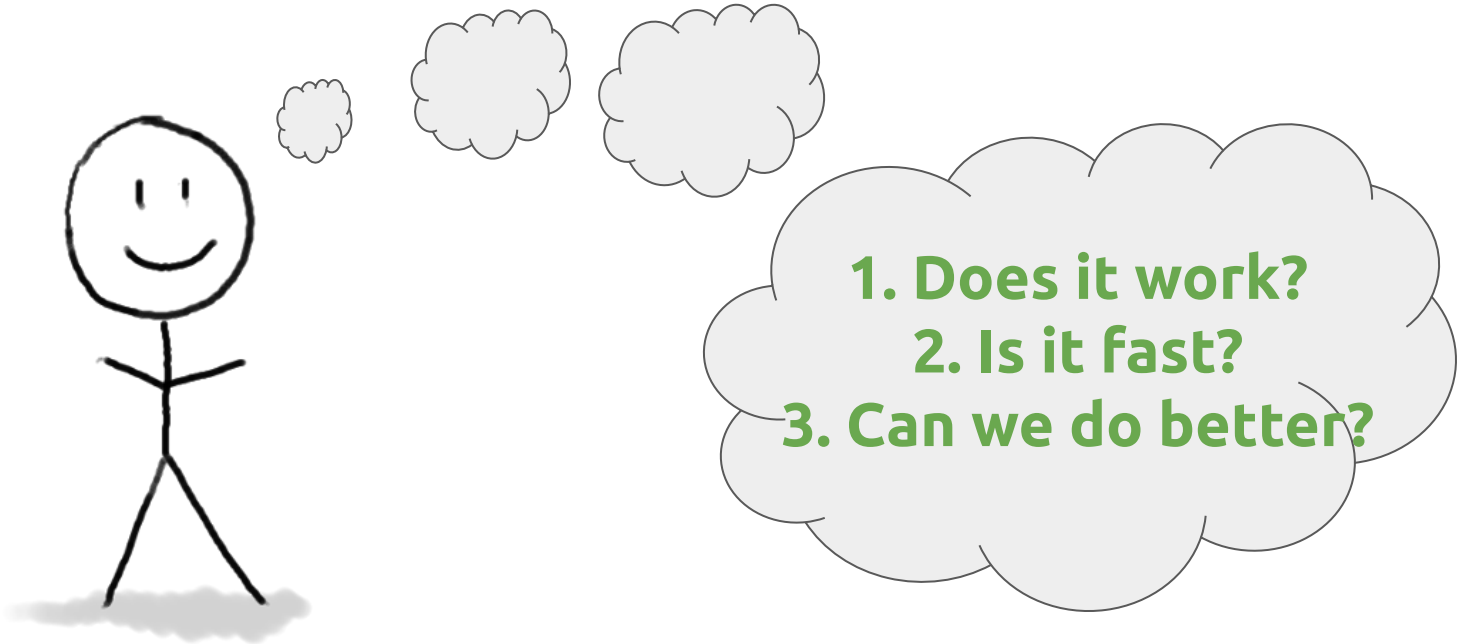
How fast is this anyway?

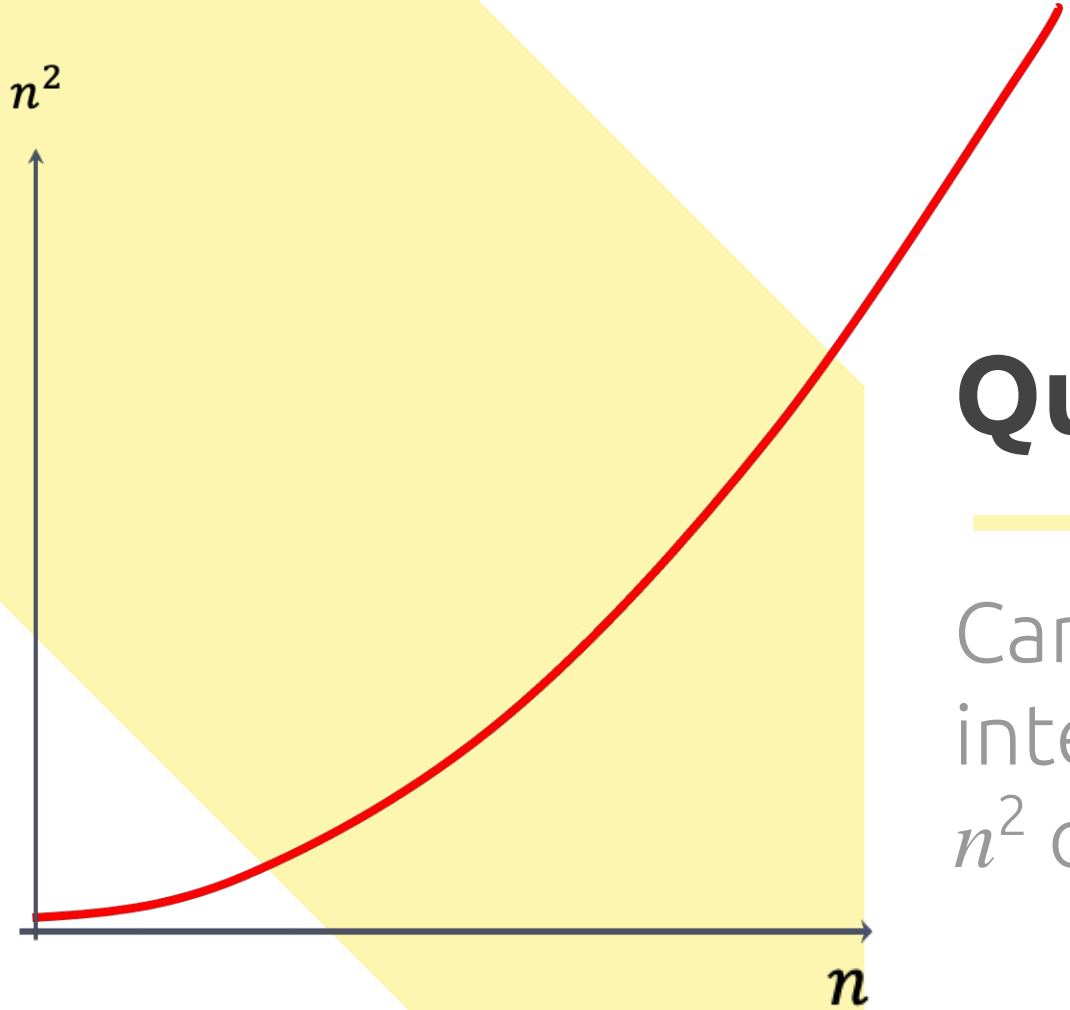
About n^2 one-digit operations

*Multiply each one of the n-digits in the first number with each one of the n-digits in the second number (n * n)*

(How many one-digit operations?)

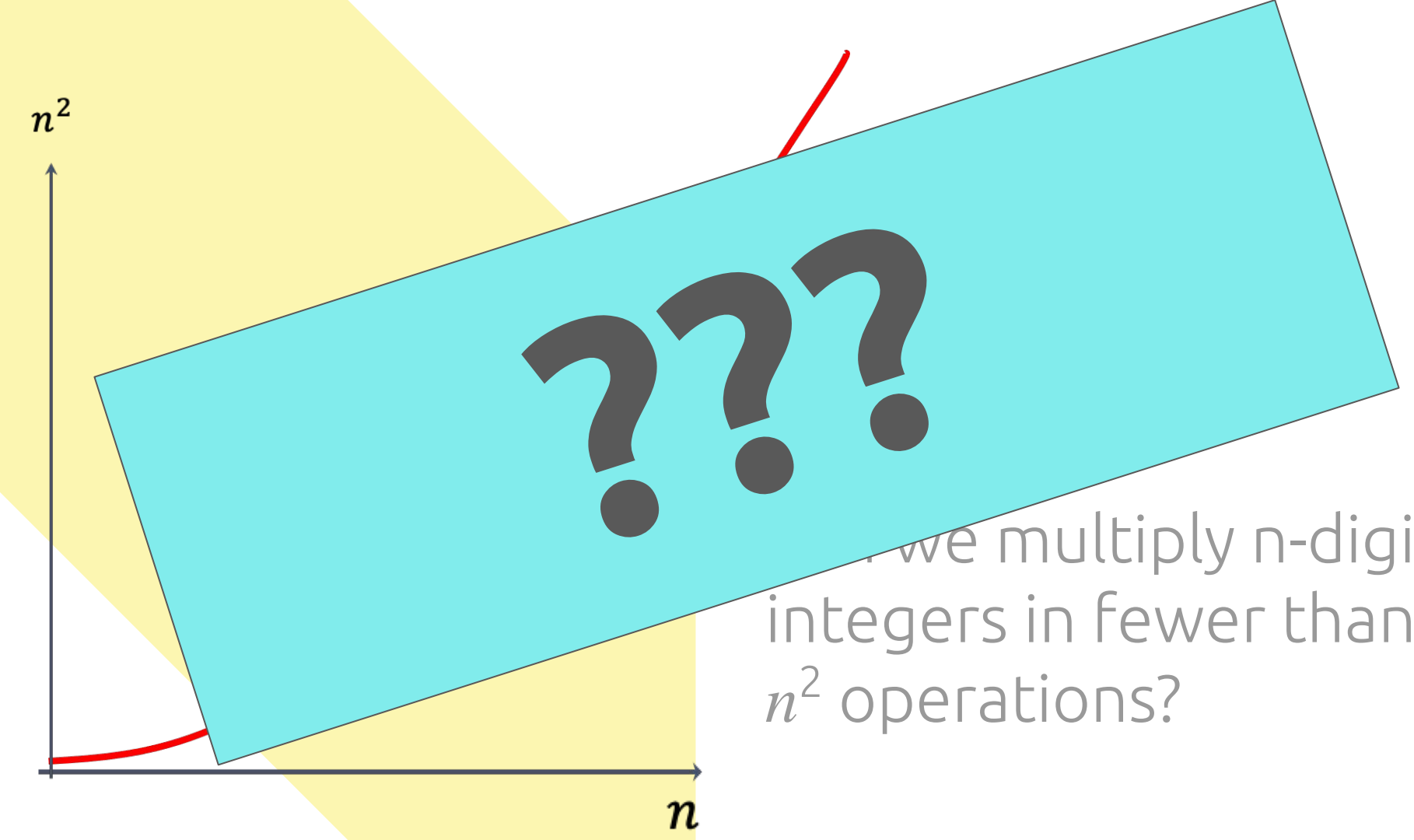
Our Guiding Questions…





Question?

Can we multiply n-digit integers in fewer than n^2 operations?



we multiply n-digit integers in fewer than n^2 operations?

Big Questions!

08/23/22 - Session

- How to get started with career prep?
- How to write a compelling technical resume?
- How to multiply integers?
- How to conquer? Do we divide?
- How fast is fast enough?

Can we do better?
Let's dig into our algorithmic toolkit!

A technique to know! - Divide & Conquer

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Break problem up into smaller (easier) sub-problems

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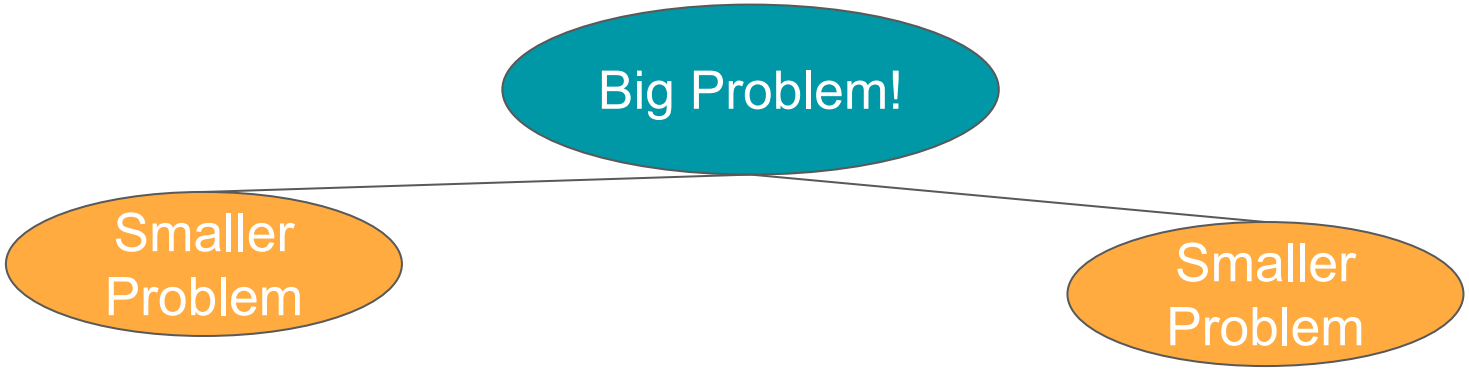

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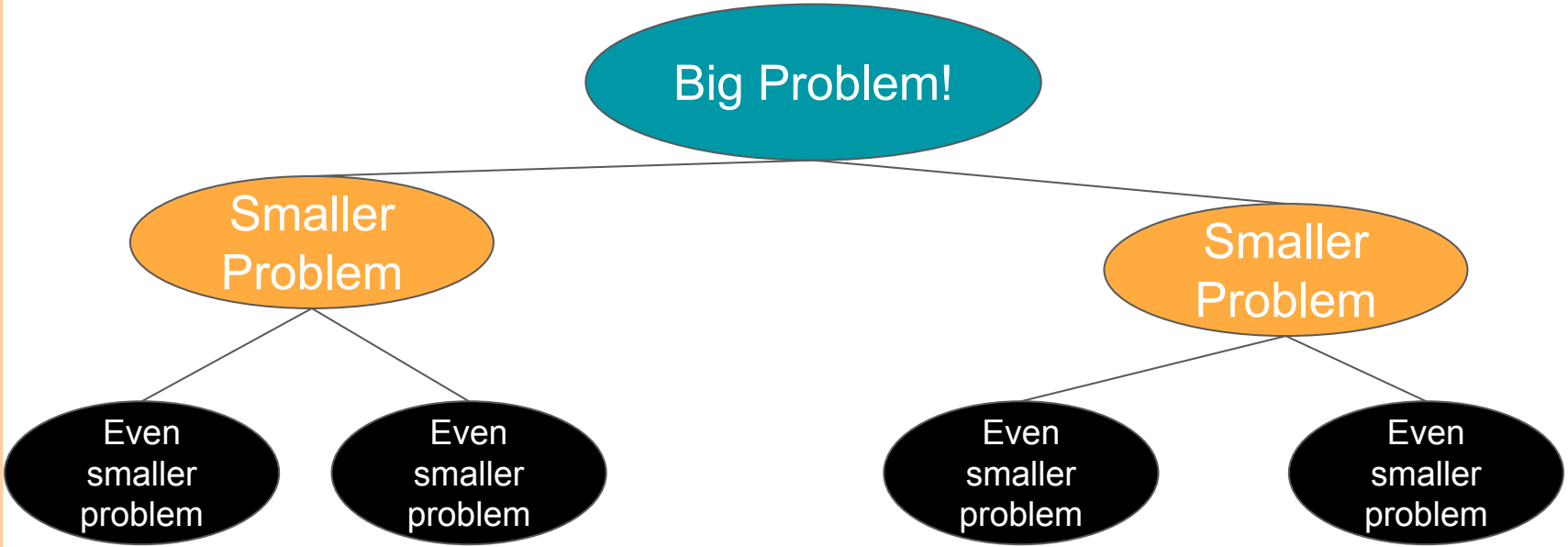
Break problem up into smaller (easier) sub-problems



Can we do this for multiplication of integers?

12 * 34

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12 = 1 * 10 + 2
34 = 3 * 10 + 4

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Can we do this for multiplication of integers?

$12 = 1 * 10 + 2$

$34 = 3 * 10 + 4$

$$\begin{aligned} 12 * 34 &= (1 * 10 + 2)(3 * 10 + 4) \\ &= (1 * 10)(3 * 10) + 2*(3 * 10) + 4 * (1 * 10) + 2 * 4 \\ &= (1 * 3) * 100 + (2 * 3 + 4 * 1) * 10 + 2 * 4 \end{aligned}$$

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One 2-digit problem -> Four 1-digit problems

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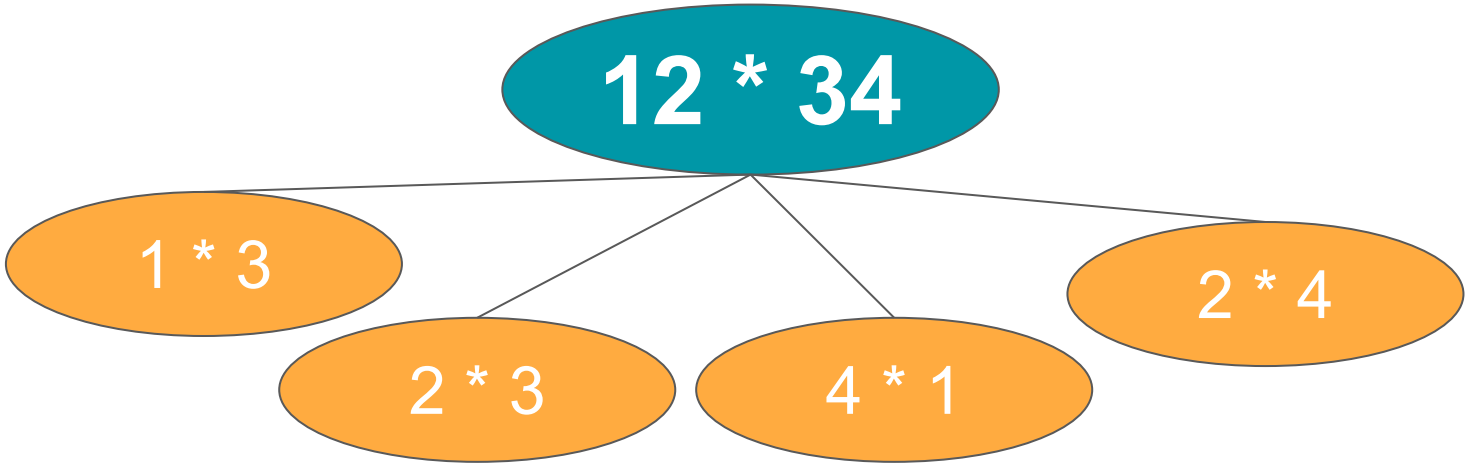

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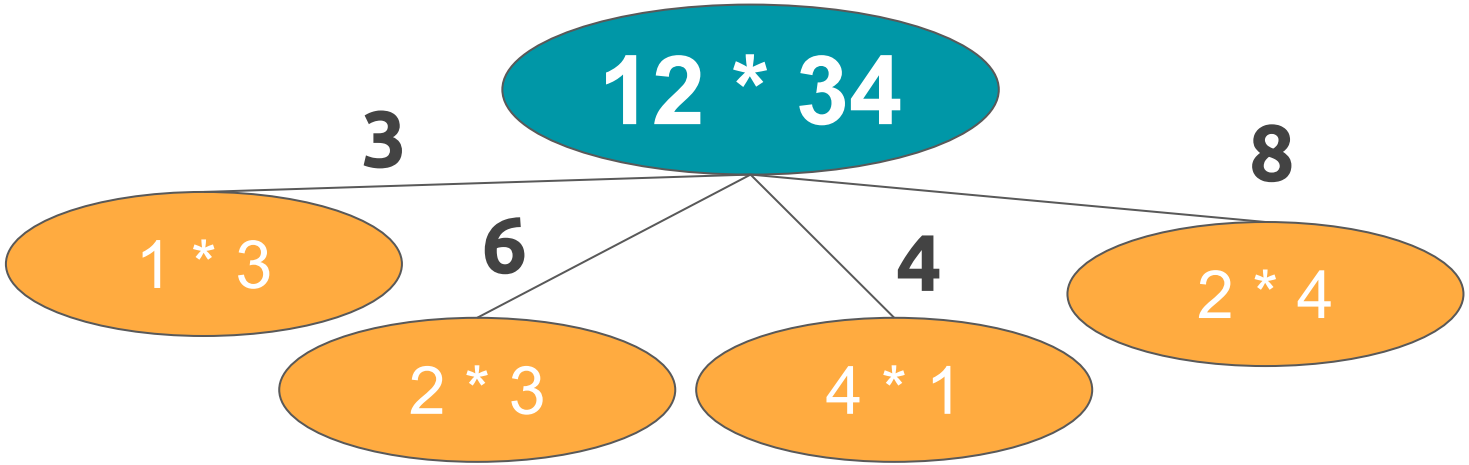

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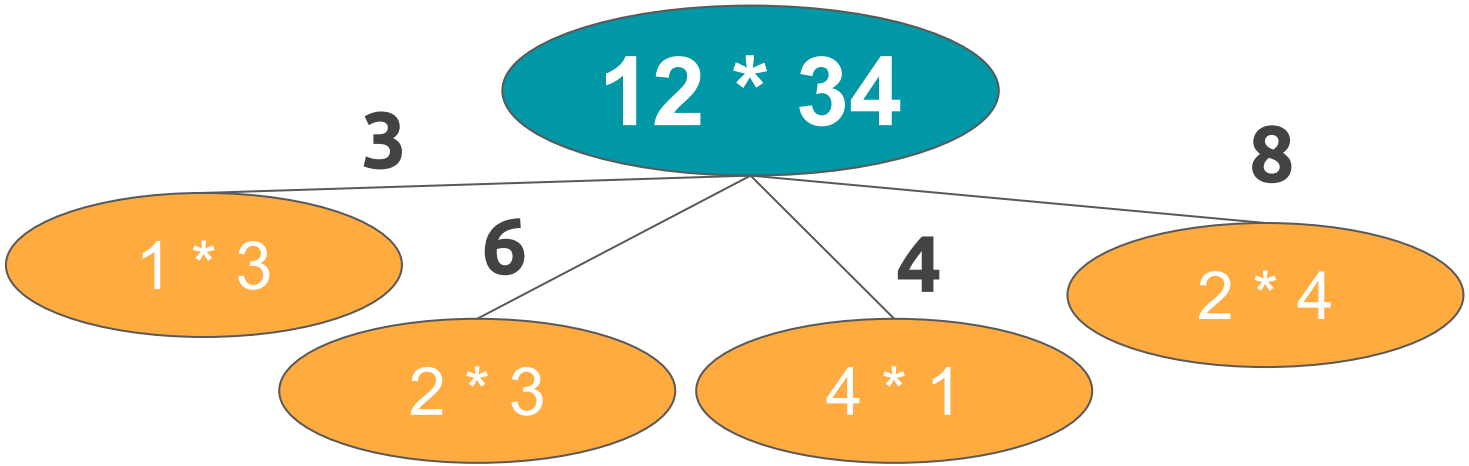
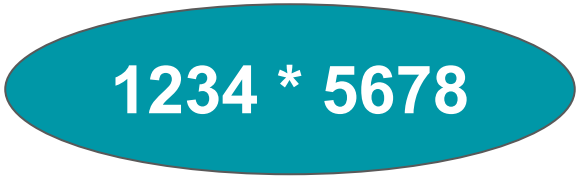


$$= (1 * 3) * 100 + (2 * 3 + 4 * 1) * 10 + 2 * 4$$
$$= 3 * 100 + (6 + 4) * 10 + 8$$
$$= 408\ !$$

Can we do this for multiplication of integers? (pt. 2)



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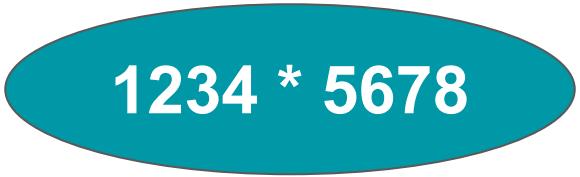
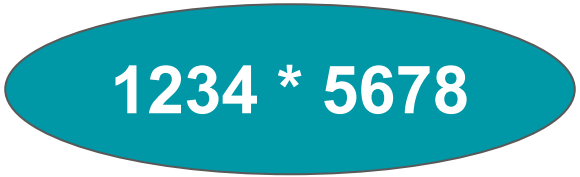


1234 = 12 * 100 + 34
5678 = 56 * 100 + 78

1234 * 5678 => ???

Can we do this for multiplication of integers? (pt. 2)



1234 = 12 * 100 + 34
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1234 * 5678 =>
= (12*100 + 34)(56*100+78)
= (12*100)(56*100)+(12*100*78)+(56*100*34)+34*78
= (12*56)*10000 + (12*78 + 56*34)*100 + 34*78

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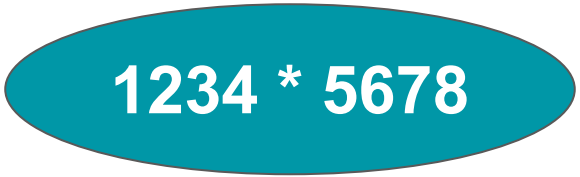

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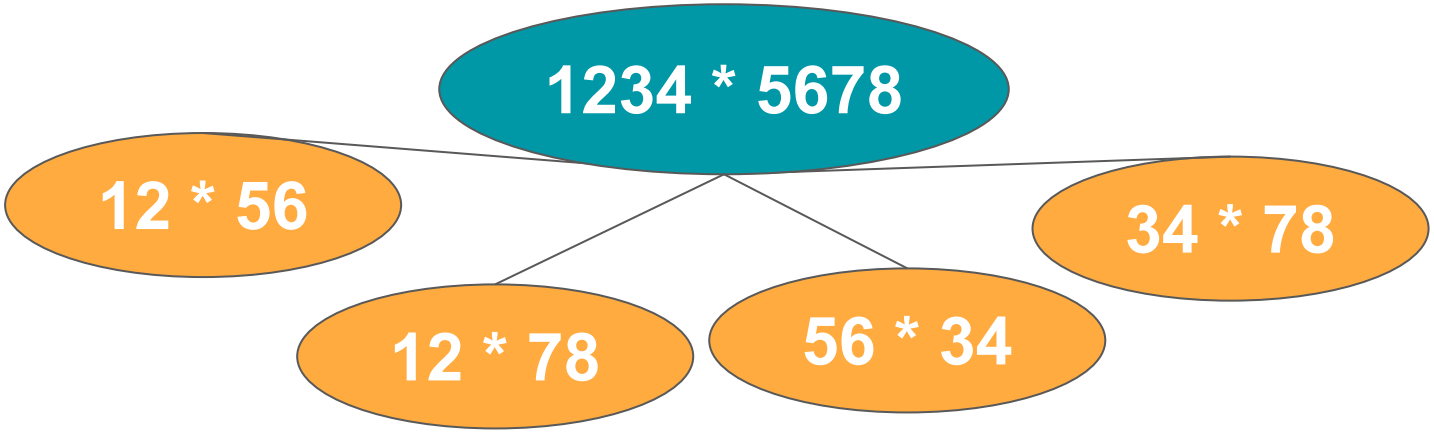

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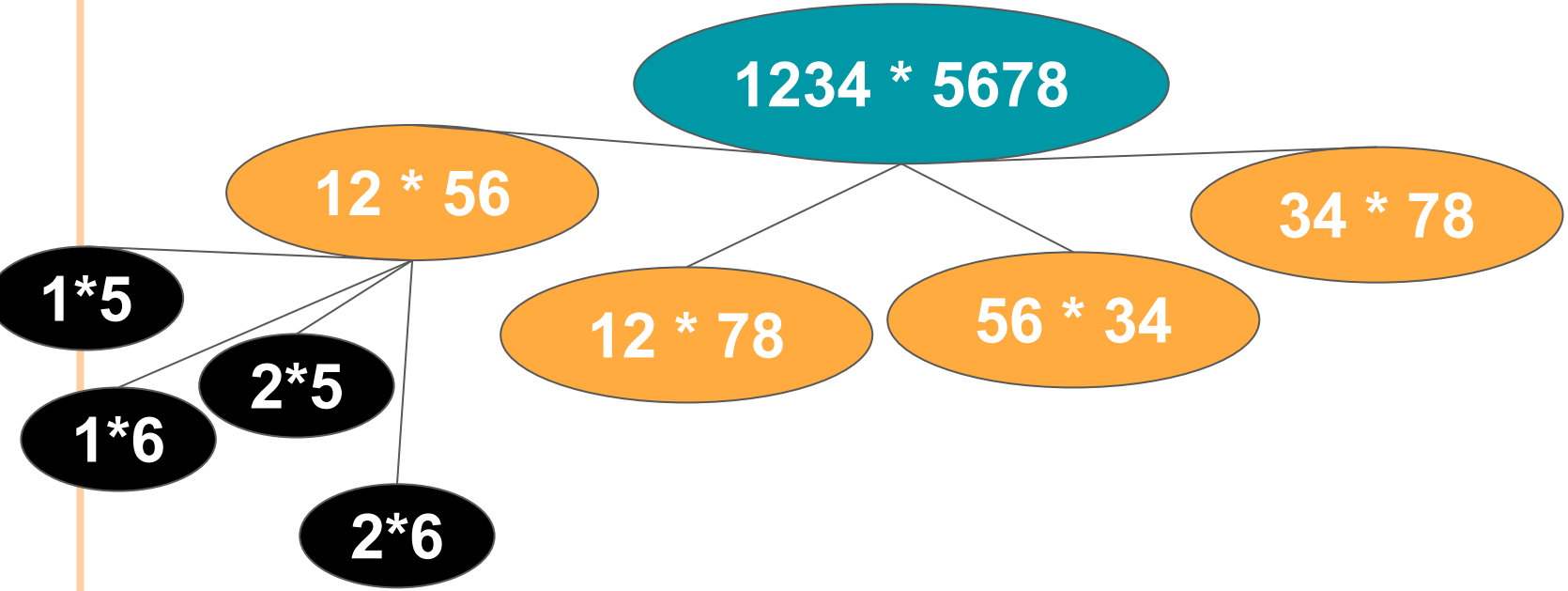

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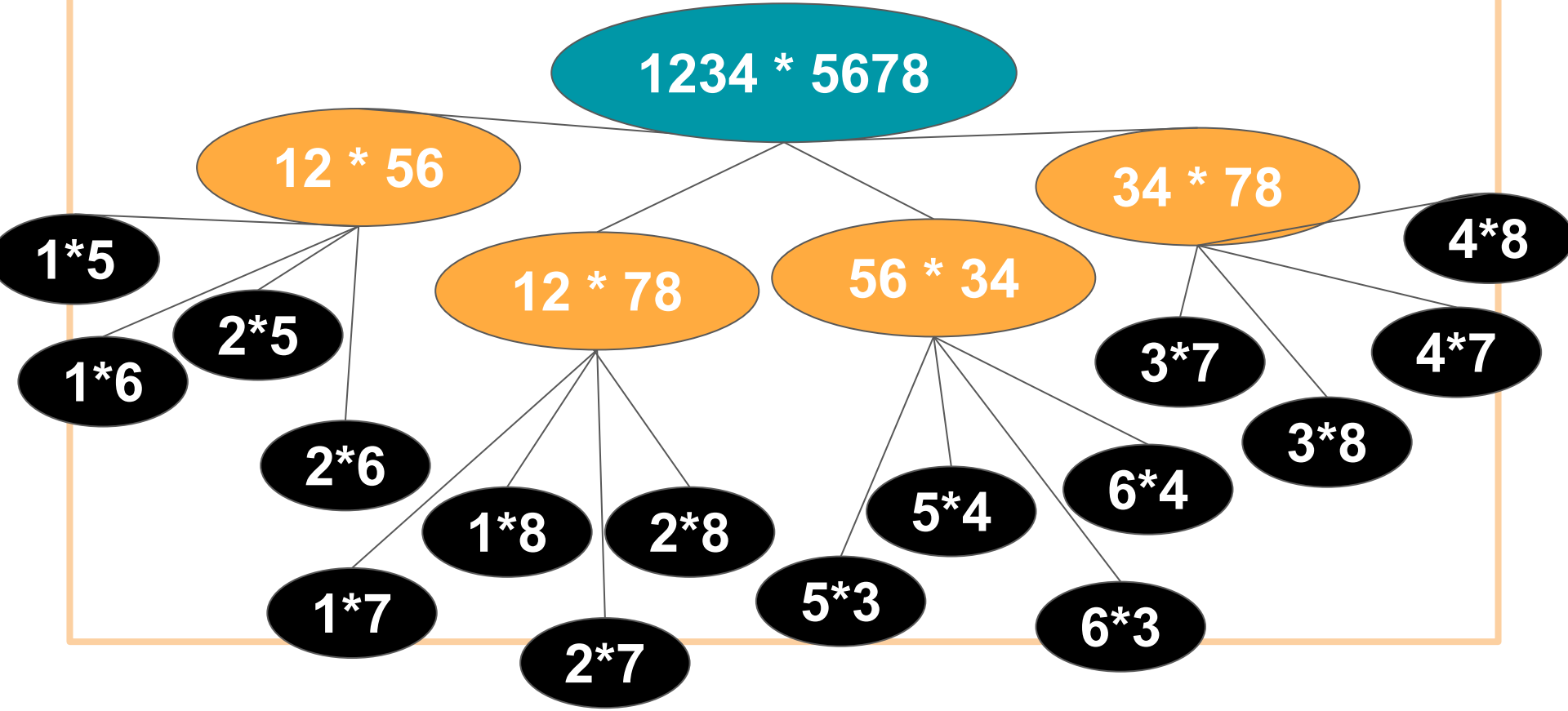

And in general!

Break up an n-digit integer x:

$[x_1, x_2, \ldots, x_n] = [x_1, x_2, \ldots x_{n/2}]*10^{n/2} + [x_{n/2+1}, x_{n/2+2}, \ldots, x_n]$

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One n-digit problem -> Four (n/2)-digit problems

So what is our algorithm in pseudocode?

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x,y are n-digit numbers

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return answer

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Multiply(*x, y***):**

    If n =1**:**

        return x*y

Base case: we have 1-digit multiplication, cannot break into subproblems

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Compute a, b, c, d from x, y

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Calculate *xy* using results

answer = $ac\ 10^n + (ad + bc)\ 10^{n/2} + bd$

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Let’s code it!!!

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return answer

we have 1-digit multiplication,
ak into subproblems

1. Does it work?
2. Is it fast?
3. Can we do better?

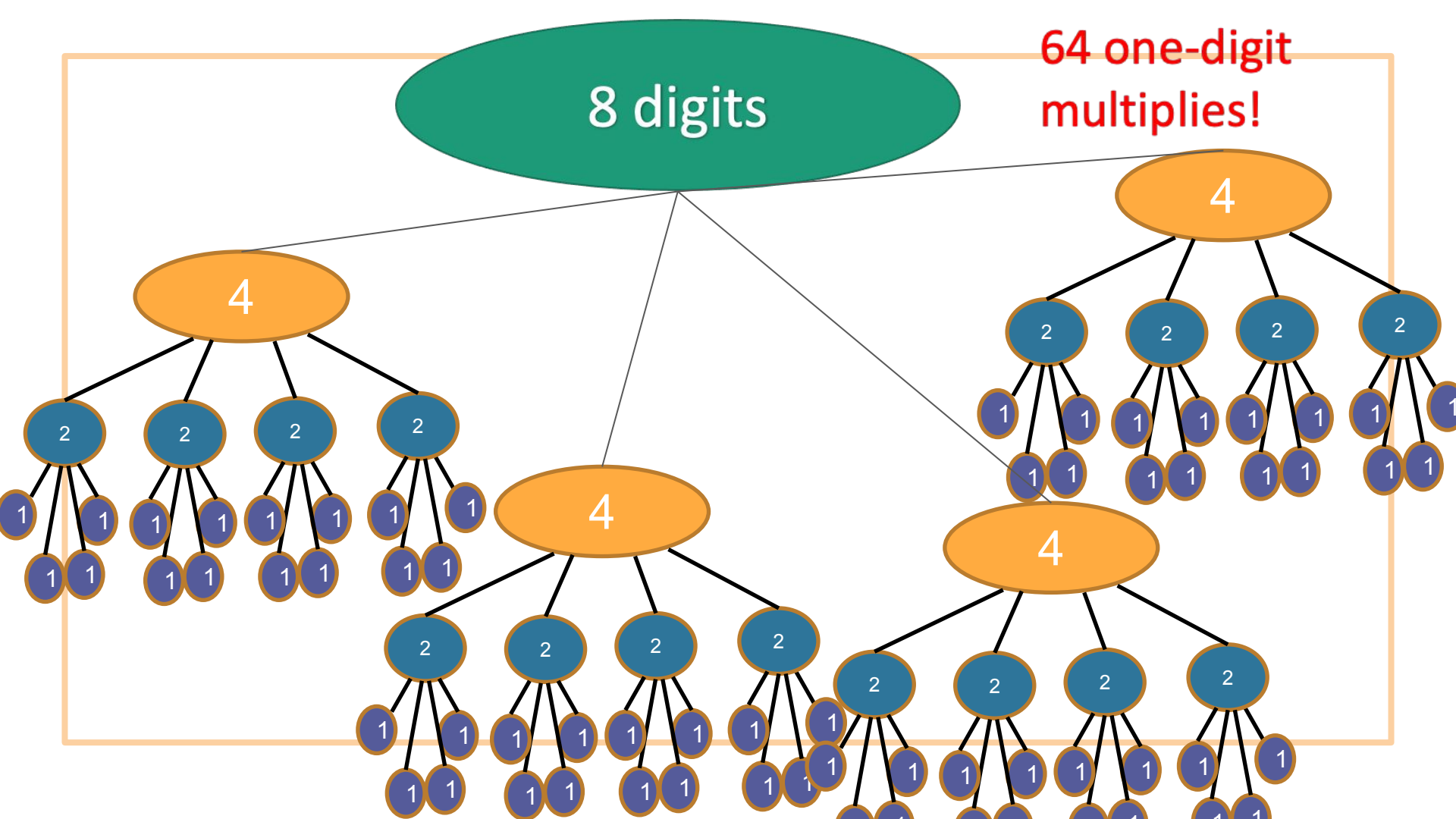
Is it fast?

- We saw that multiplying 2 digit numbers took 4 multiplications.

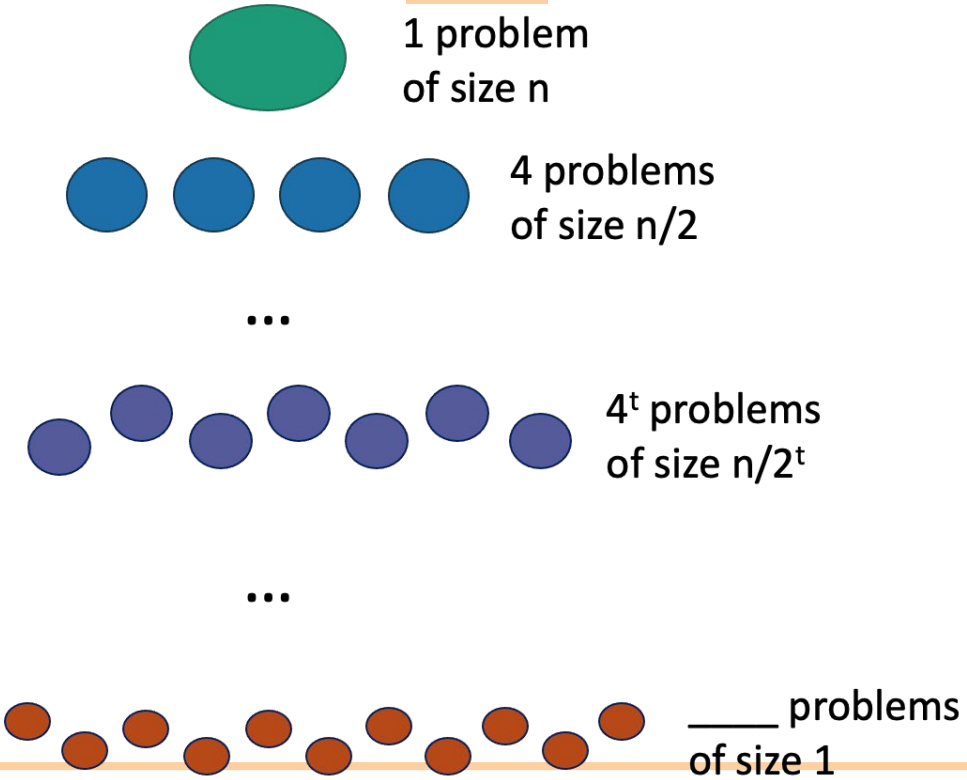
- We saw that multiplying 4 digit numbers takes 16 multiplications.

- What about 8 digit numbers? What about 16 digit numbers?

- **How many operations as a function of n?**

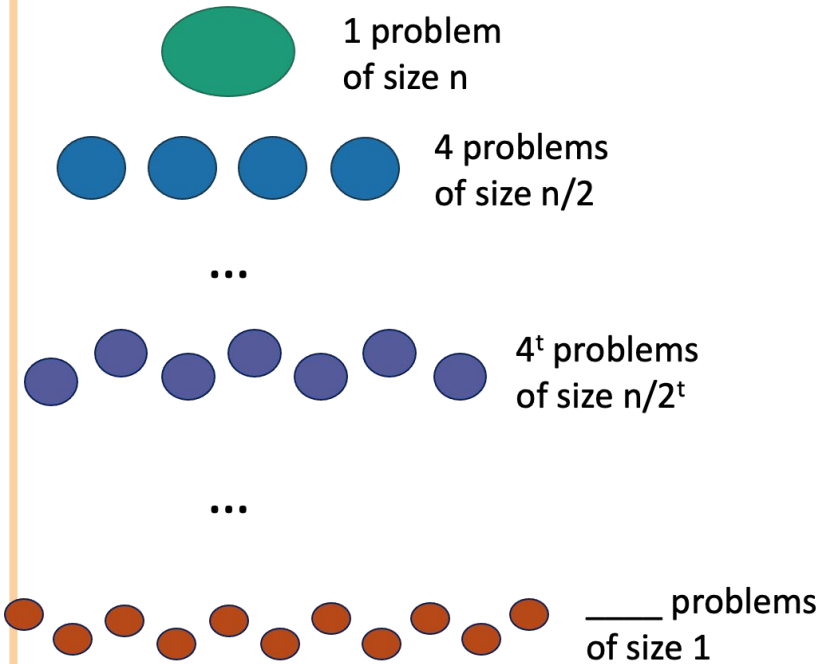

8 digits
64 one-digit multiplies!
4
4
4
4
2
1

There are ??? 1-digit multiplications



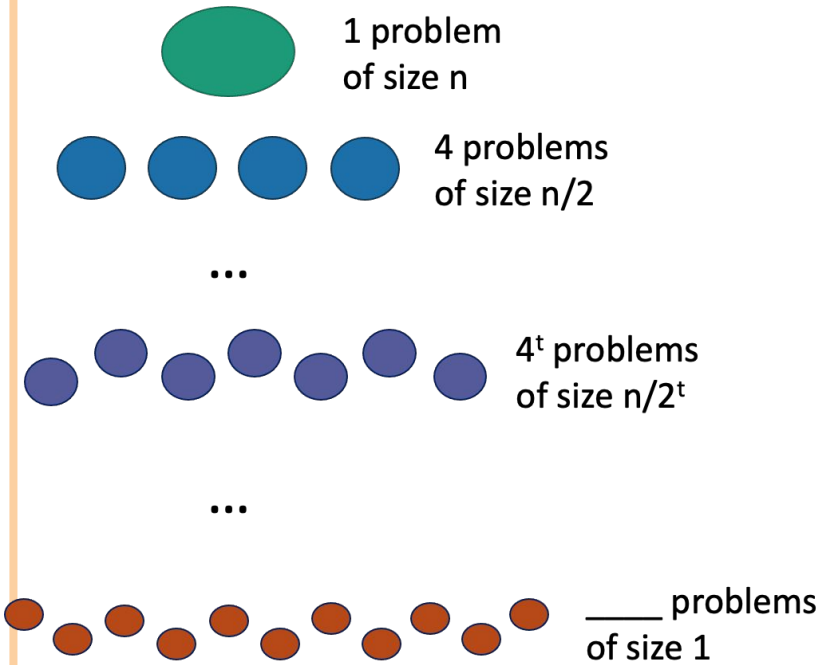
Note: this is just a cartoon – I'm not going to draw all 4^t circles!

How many problems on the last level?



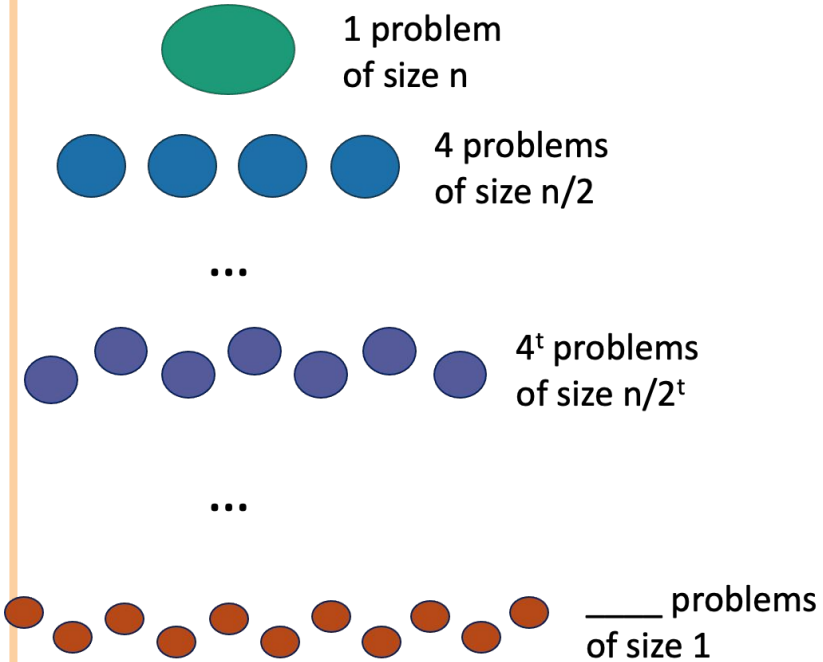
- For each "**level**", we multiply the number of problems by **4**
- So for the **t^{th}** level (where the first level is **t=0**), we will have **4^t** problems
- The problems on the **last** level are of size (1)

So what's the index of the last level?



- We start with **size n**
- Then go down to **size n/2**
- Then go down to **(n/2)/2 = n/4**
- Then **(n/4)/2 = n/8**
- And so on…
- On the last level, we must have **$n/2^t = 1$**

So what's the index of the last level?

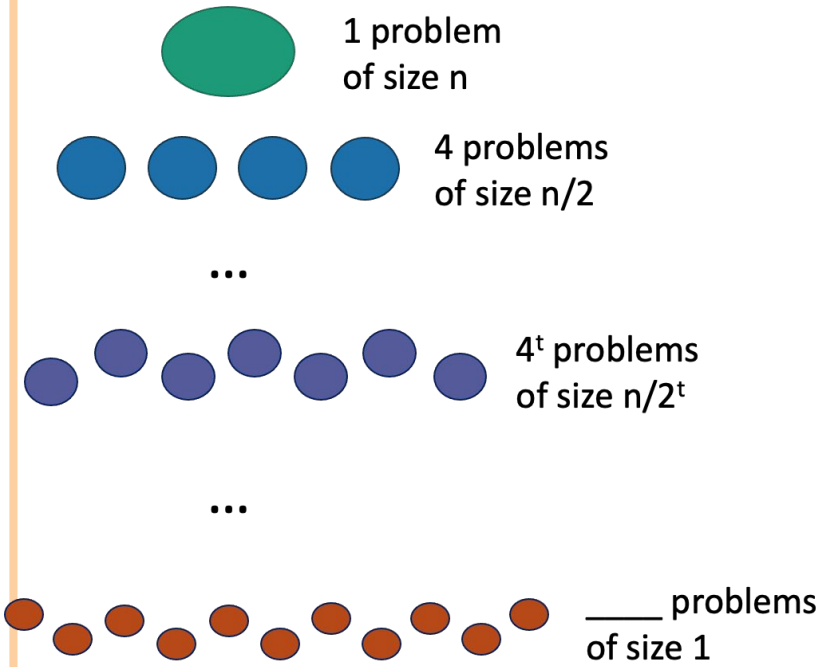


$$\frac{n}{2^t} = 1$$

$$\implies n = 2^t$$

$$\implies t = \log_2 n$$

So how many problems on the last level?



- $4^t = 4^{\log_2 n}$

$= (2^2)^{\log_2 n}$

$= 2^{2 \log_2 n}$

$= (2^{\log_2 n})^2$

$= n^2$

Darn, that's sad :(

Big Questions!

08/23/22 - Session

- How to get started with career prep?
- How to write a compelling technical resume?
- How to multiply integers?
- How to conquer? Do we divide?
- How fast is fast enough?

But wait!!!

Let's recall our algorithm!

Break up an n-digit integer:

$$[x_1 x_2 \cdots x_n] = [x_1 x_2 \cdots x_{n/2}] \times 10^{n/2} + [x_{n/2+1} x_{n/2+2} \cdots x_n]$$

$$x \times y = (a \times 10^{n/2} + b)(c \times 10^{n/2} + d)$$

$$= \underbrace{(a \times c)10^n}_{1} + \underbrace{(a \times d + c \times b)10^{n/2}}_{2,3} + \underbrace{(b \times d)}_{4}$$

What if we did less work?

Break up an n-digit integer:

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What if we recursed 3 times instead of 4?

Break up an n-digit integer:

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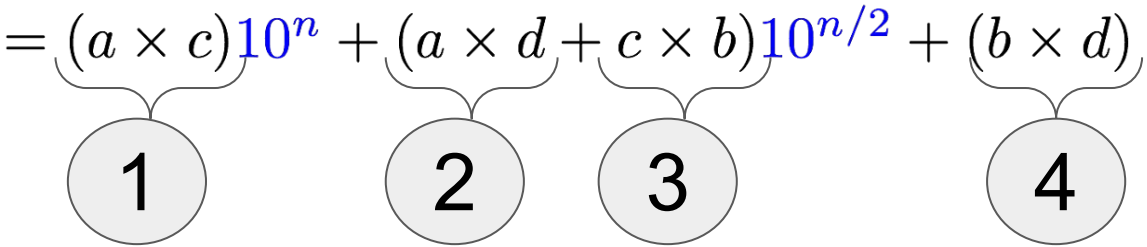
This is what Karatsuba figured out!

- It feels like we need four multiplications
  - ac
  - bd
  - bc
  - ad



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- It feels like we need four multiplications
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$$= \underbrace{(a \times c)}_{1}10^{n} + \underbrace{(a \times d}_{2} + \underbrace{c \times b)}_{3}10^{n/2} + \underbrace{(b \times d)}_{4}$$

- Karatsuba figured out that's not true!! You only need **three**!
  - ac
  - bd
  - (a+b)(c+d)

How does this work?

- **These are the “values” we need**
  - ac
  - bd
  - bc
  - ad

- **Karatsuba says**
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- **These are the “values” we need**
  - ac
  - bd
  - **bc**
  - **ad**

- **Karatsuba says**
  - ac (just do it!)
  - bd (just do it!)
  - we want **(ad + bc)**, could we compute it another way?

$$(a+b)(c+d) = ac + bd + bc + ad$$

How does this work?

- **These are the “values” we need**
  - ac
  - bd
  - **bc**
  - **ad**

- **Karatsuba says**
  - ac (just do it!)
  - bd (just do it!)
  - Do (a+b)(c+d) instead! Then subtract ac and bd!

$$(a+b)(c+d) = ac + bd + bc + ad$$

So what is our algorithm in pseudocode?

x,y are n-digit numbers

Multiply(x, y**):**

If n =1**:**

Base case: we have 1-digit multiplication, cannot break into subproblems

return x*y

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Compute *ac, ad, bc, bd* from recursion

ac = **Multiply(**a,c**)**

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Compute *ac, ad, bc, bd* from recursion

ac = **Karatsuba(**a,c**)**

bd = **Karatsuba(**b,d**)**

z = **Karatsuba(**a+b,c+d**)**

Recursive cases

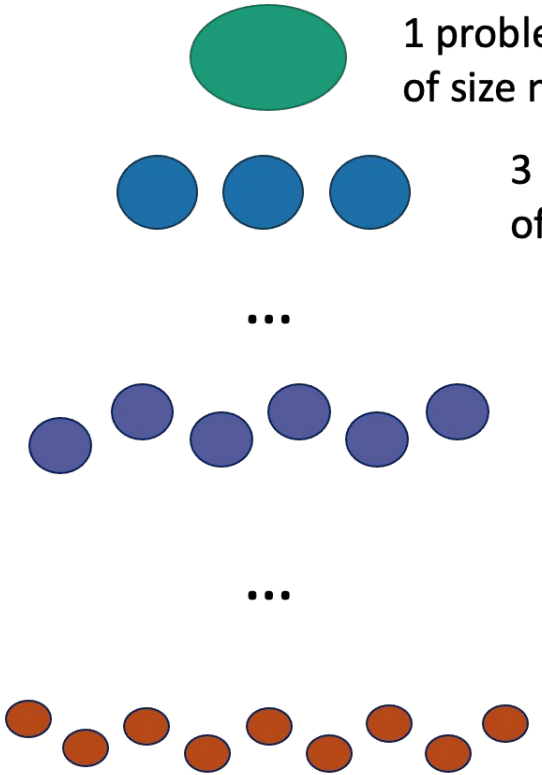
Calculate *xy* using results

answer = $ac\,10^n + (z - ac - bd)10^{n/2} + bd$

return answer

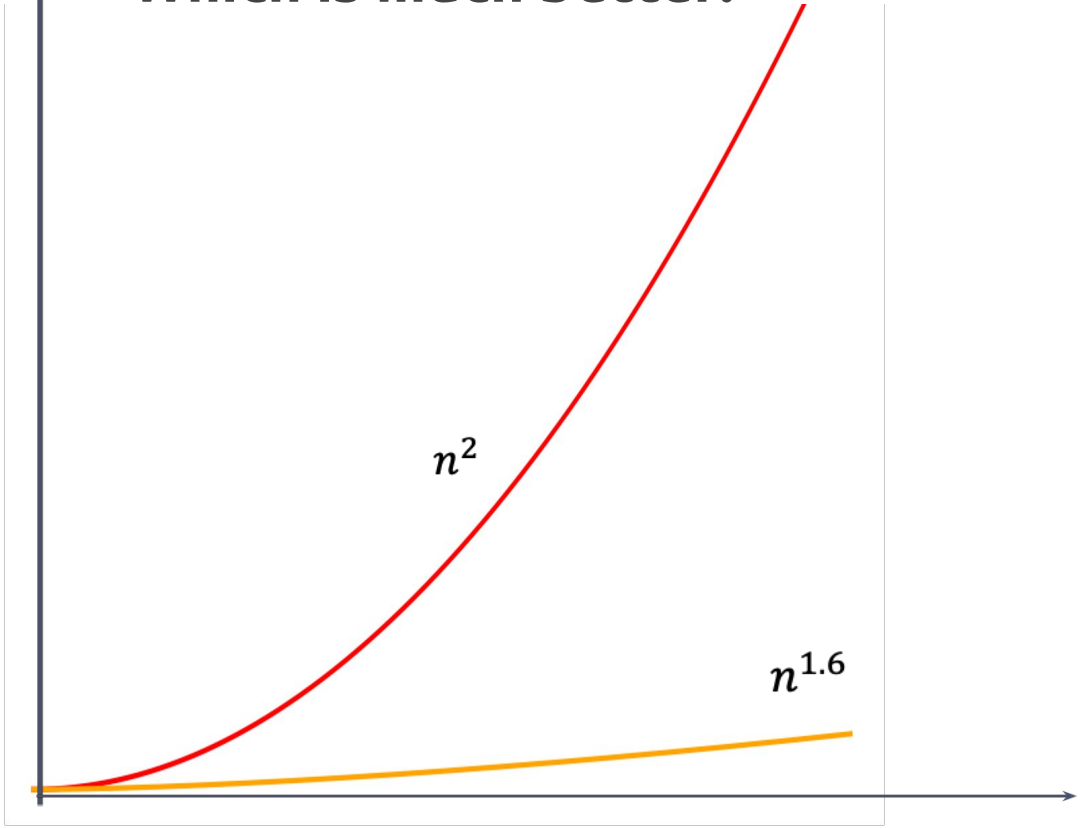
Let’s code it!!!

What's the running time?



$$
\begin{aligned}
3^t &= 3^{\log_2 n} \\
&= \left(2^{\log_2 3}\right)^{\log_2 n} \\
&= \left(2^{\log_2 n}\right)^{\log_2 3} \\
&= n^{\log_2 3} \\
&\approx n^{1.6}
\end{aligned}
$$

Which is much better!



Can we do even better?

- **Toom-Cook** (1963): instead of breaking into three n/2 sized problems, break into five n/3-sized problems.
  - Runs in $O(n^{1.465})$
- **Schönhage–Strassen (1971)**
  - Runs in I(n log (n))log log (n)
- **Furer** (2007)
  - Runs in n log(n) * $2^{O(\log^*(n))}$
- **Harvey and van der Hoeven** (2019)
  - Runs in time O(n log (n))

Can you?

- **Describe**
  - The components of a compelling, technical resume
  - The Karatsuba Integer Multiplication optimization
- **Practice**
  - Applying action verb + what *you* did + results/significance formula to your resume
  - Designing an efficient algorithm for integer multiplication

How was the pace today?

Wrap-Up

- **Everyone** can write a polished and compelling resume.
  - Action verb + what *you* did + results

- **Divide and conquer** is a tool we use to decompose large problems into smaller, easier-to-solve subproblems.

- **Karatsuba optimization**: $n^{1.6}$ is better than n^2 operations; we reduced the number of nodes we needed to execute.

Announcements

- **HW 0 is out!**
  - Due Thursday 08/25 @ 11:59PM
- **Google Tech Exchange**
  - Deadline Monday 09/12

Next time!

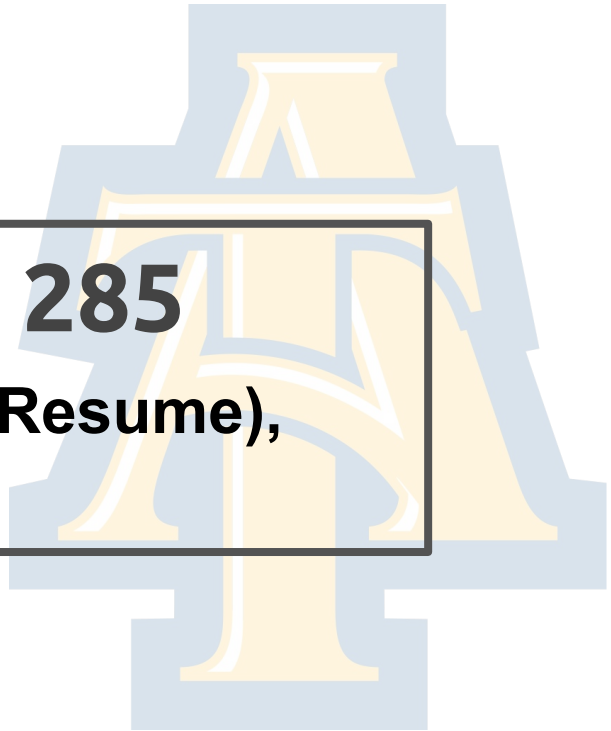
- (More) formal big-O introduction!
- Space/time complexity practice
- Space/time complexity with recursion.

COMP - 285

Advanced Analysis of Algorithms

Welcome to COMP 285

Lecture 1: CS Job Hunting (Resume), Pseudocode and Recursion



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