COMP 285 (NC A&T, Spr '22) Weekly Quiz 9

Reporting Issues If you find any issues with the solutions, reach out to Chi Wang (author) or Luis Perez (reviewer).

1

We are computing the longest common subsequence between two strings of length four $S = X_1X_2X_3X_4$ and $T = Y_1Y_2Y_3Y_4$. We fill the array C where $C_{i,j}$ is the length of the longest common subsequence between the prefix of length i from S and the prefix of length j from T. The array C can be found below with some entries masked. What can be said about X_1 and Y_4 (the first character of S and the last character of Y)?

	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	1
2	0	0	0	1	1
3	0	0	1	1	♡
4	0	1	♣	♠	♢

Solution

The characters are equal. We can see this by looking at row 1. It is all 0's until we get to column 4, which means that X_1 matched none of the characters in Y except for Y_4.

2

What can be said about X_2 and Y_3?

Solution

By a similar argument, if we look at row labeled 2, it is all 0s until we reach the column labeled 3. As such, this is where it matched with Y_3. So the two characters are equal.

3

What can be said about X_2 and Y_4?

Solution

Suppose X_2 matches Y_4. By row 2, we know that X_2 matches Y_3. However, that would mean Y_3 and Y_4 are the same character, but if we look at row 1, we see that X_1 only matches Y_4, which means $Y_4 \neq Y_3$, as asuch $X_2 \neq Y_4$.

4

What is the value of the heart?

Solution

We're basically asking whether X_3 matches Y_4. We know that Y_4 does not match any other characters in Y. However, we see that X_3 matches Y_2. As such, X_3 cannot match Y_4.
This means the hearth has a value of 1 (max of the up and left cells).

5

What is the value of the spade?

Solution

We're asking if X_4 matches Y_3. We know already that Y_3 does not match any other characters in Y (this is clear from row labeled 2). However, X_4 matches Y_1. As such, X_4 does not match Y_3.
Therefore, the spade has a value of 1 (max of the up and left cells).

6

What is the value of the clubs?

Solution

This is asking if X_4 matches Y_2. We know Y_2 matches X_3 and X_4 matches Y_1. However, we know X_3 does not match Y_1. Therfore X_4 doe snot match Y_2.
Therefore, the clubs has a value of 1 (max of the up and left cells).

7

What is the value of the diamonds?

Solution

This is asking if Y_4 matches X_4. We know X_4 matches Y_1. However, from before, we know that Y_4 in unique in Y, as such, Y_4 doe snot match X_4.
Therefore, the diamons has a value of 1 (max of the up and left cells).

8

Consider the LCS problem from lecture 25 and our dynamic programming algorithm for it. Given input strings of lengths m and n, what is the memory complexity of this algorithm?

Solution

$O(mn)$ because we fill in an $m \times n$ array and each cell takes $O(1)$ time to fill.

9

When we are filling up the i-th row of our dynamic programming table C, what rows do we need tohave access to?

Solution

We need to access the value in the i-th row and $(i-1)$-th row only. This is clear from the recursive definition of the algorithm given by:

$$C[i][j] = \begin{cases} 1 + C[i-1][j-1] & X[i] == Y[j] \\ \max\{C[i-1][j], C[i][j-1]\} & \end{cases}$$

10

Given the observation above can we optimize our space complexity further?

Solution

Yes we can reduce the memory complexity to $O(\min\{m, n\})$