

Desculpa, Beto - ICPC Library

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1 Data Structures

1.1 Color Update

```

template<class Info>
struct ColorUpdate {
    struct Range {
        int l, r;
        Info value;
        Range(int l = 0, int r = 0, Info value = 0) : l(l), r(r),
            value(value) {}
    };
    bool operator < (const Range &r) const { return l < r.l; }
};

set<Range> ranges;

void update(int l, int r, Info value) {
    if (l > r) return;

    auto it = ranges.lower_bound(l);
    if (it != ranges.begin()) {
        it--;
        if (it->r >= l) {
            auto cur = *it;
            ranges.erase(it);
            ranges.insert(Range(cur.l, l - 1, cur.value));
            ranges.insert(Range(l, cur.r, cur.value));
        }
    }
}

```

```

    }

    it = ranges.upper_bound(r);
    if (it != ranges.begin()) {
        it--;
        if (it->r > r) {
            auto cur = *it;
            ranges.erase(it);
            ranges.insert(Range(cur.l, r, cur.value));
            ranges.insert(Range(r + 1, cur.r, cur.value));
        }
    }

    ranges.erase(ranges.lower_bound(l), ranges.upper_bound(r));
    ranges.insert(Range(l, r, value));
}

Info get(int pos) {
    auto it = ranges.upper_bound(pos);
    if (it == ranges.begin()) return -1;
    it--;
    if (it->r < pos) return -1;
    return it->value;
}
};

```

1.2 Persistent seg tree

```

const int BUFFER_SIZE = 2e6;

//lembra do vazio
struct node_t {
    node_t *l, *r;

    node_t() {
        l = nullptr;
        r = nullptr;
    }

    node_t(node_t* L, node_t* R) {
        l = L;
        r = R;
    }

    node_t(node_t L, node_t R) {
    }
};

template <typename val_t, typename n_t = node_t>
struct persistent_tree_t {
private:
    vector<n_t*> versions;
    n_t buffer[BUFFER_SIZE];
    int n, buffer_ptr;

    n_t* NewNode() {
        buffer[buffer_ptr] = n_t();
        return &buffer[buffer_ptr++];
    }

    n_t* NewNode(val_t x) {
        buffer[buffer_ptr] = n_t(x);
        return &buffer[buffer_ptr++];
    }

    n_t* NewNode(n_t *L, n_t *R) {
        buffer[buffer_ptr] = n_t(L, R);
    }
};

```

```

    return &buffer[buffer_ptr++];
}

void clear() {
    versions.clear();
    buffer_ptr = 0;
}

n_t* build(int l, int r, vector<val_t> &base) {
    if (l == r) {
        return NewNode(base[l]);
    } else {
        int mid = (l + r) >> 1;
        return NewNode(build(l, mid, base), build(mid + 1, r, base));
    }
}

n_t* build(int l, int r) {
    if (l == r) {
        return NewNode();
    } else {
        int mid = (l + r) >> 1;
        return NewNode(build(l, mid), build(mid + 1, r));
    }
}

//lembra da folha
n_t* modify(n_t* cur_node, int l, int r, int id, val_t new_val) {
    if (l == r) {
        return new_val;
    } else {
        int mid = (l + r) >> 1;
        if (mid < id) {
            return NewNode(cur_node->l, modify(cur_node->r, mid + 1, r, id, new_val));
        } else {
            return NewNode(modify(cur_node->l, l, mid, id, new_val), cur_node->r);
        }
    }
}

n_t get(n_t* cur_node, int l, int r, int a, int b) {
    if (l > b || r < a) {
        return n_t();
    }
    if (l >= a && r <= b) {
        return *cur_node;
    }
    int mid = (l + r) >> 1;
    return n_t(get(cur_node->l, l, mid, a, b), get(cur_node->r, mid + 1, r, a, b));
}

public:
persistent_tree_t(vector<val_t> &base) {
    clear();
    n = (int) base.size() - 1;
    versions.push_back(build(l, n, base));
}

persistent_tree_t(int n = 1) {
    clear();
    versions.push_back(build(l, n));
}

void re_create(vector<val_t> &base) {
    clear();
    n = (int) base.size() - 1;
    versions.push_back(build(l, n, base));
}

void modify(int vers, int id, val_t x) {
    versions.push_back(modify(versions[vers], l, n, id, x));
}

```

```

    }

    n_t get(int vers, int l, int r) {
        return get(versions[vers], l, n, l, r);
    }

    int cur_version() {
        return (int) versions.size() - 1;
    }
};

```

1.3 Seg tree

```

template<typename Info>
class SegmentTree {
private:
    int n;
    vector<Info> tree;

    Info merge(Info &l, Info &r) {
        // ...
    }

public:
    SegmentTree() {}
    SegmentTree(int k) {
        n = k + 1;
        tree.resize(n << 1);
    }
    SegmentTree(vector<Info> v) {
        n = (int) v.size();
        tree.resize(n << 1);
        for(int i = 0; i < n; i++) {
            tree[i + n] = v[i];
        }
        for(int i = n - 1; i >= 1; i--) {
            tree[i] = merge(tree[i + i], tree[i + i + 1]);
        }
    }

    void init(int k) {
        n = k + 1;
        tree.resize(n << 1);
    }

    void up(int pos, Info val) {
        pos += n;
        tree[pos] = val;
        while(pos > 1) {
            pos >>= 1;
            tree[pos] = merge(tree[pos + pos], tree[pos + pos + 1]);
        }
    }

    Info get(int l, int r) {
        Info ans = ; //colocar a base
        for(l += n, r += n + 1; l < r; l >>= 1, r >>= 1) {
            if (l & 1) {
                ans = merge(ans, tree[l++]);
            }
            if (r & 1) {
                ans = merge(ans, tree[--r]);
            }
        }
        return ans;
    }
};

```

1.4 Sparse table

```
template<class Info_t>
class SparseTable {
private:
    vector<int> log2;
    vector<vector<Info_t>> table;

    Info_t merge(Info_t &a, Info_t &b) {

    }

public:
    SparseTable(int n, vector<Info_t> v) {
        log2.resize(n + 1);
        log2[1] = 0;
        for (int i = 2; i <= n; i++) {
            log2[i] = log2[i >> 1] + 1;
        }
        table.resize(n + 1);
        for (int i = 0; i < n; i++) {
            table[i].resize(log2[n] + 1);
        }
        for (int i = 0; i < n; i++) {
            table[i][0] = v[i];
        }
        for (int i = 0; i < log2[n]; i++) {
            for (int j = 0; j < n; j++) {
                if (j + (1 << i) >= n) break;
                table[j][i + 1] = merge(table[j][i], table[j + (1 << i)][i]);
            }
        }

        int get(int l, int r) {
            int k = log2[r - l + 1];
            return merge(table[l][k], table[r - (1 << k) + 1][k]);
        }
    };
};
```

1.5 Sqrt

```
#include <bits/stdc++.h>
using namespace std;

const int block = 500;

struct Node_t {
    vector<char> value;
    int size;

    Node_t() {
        size = 0;
    }
};

int n;
string s;
vector<Node_t> buffer;
vector<int> pos;

void rebuild() {
    pos.clear();
    int i = 0;
    Node_t cur = Node_t();
    while (i < n) {
        if (i % block == 0 && i > 0) {
```

```
            pos.push_back(buffer.size());
            buffer.push_back(cur);
            cur = Node_t();
        }
        cur.size++;
        cur.value.push_back(s[i]);
        i++;
    }
    if (cur.size > 0) {
        pos.push_back(buffer.size());
        buffer.push_back(cur);
    }
}

void take() {
    s = "";
    for (int i = 0; i < (int) pos.size(); i++) {
        for (int j = 0; j < buffer[pos[i]].size; j++) {
            s.push_back(buffer[pos[i]].value[j]);
        }
    }
}

int split(int id) {
    int i = 0, on = 0;
    while (i < (int) pos.size() && on < id) {
        on += buffer[pos[i]].size;
        i++;
    }
    if (on == id) {
        return i;
    }
    i--;
    on -= buffer[pos[i]].size;
    Node_t new_l = Node_t(), new_r = Node_t();
    int x = 0;
    while (on < id) {
        new_l.value.push_back(buffer[pos[i]].value[x++]);
        new_l.size++;
        on++;
    }
    while (x < buffer[pos[i]].size) {
        new_r.value.push_back(buffer[pos[i]].value[x++]);
        new_r.size++;
    }
    pos[i] = buffer.size();
    buffer.push_back(new_l);
    pos.insert(pos.begin() + i + 1, buffer.size());
    buffer.push_back(new_r);
    return i + 1;
}

//[l, r], [0...]
void rotate(int l, int r, int k) {
    int id_l = split(l);
    int id_mid = split(l + k);
    int id_r = split(r + 1);
    vector<int> to_put;
    for (int i = id_l; i < id_mid; i++) {
        to_put.push_back(pos[i]);
    }
    for (int i = id_mid; i < id_r; i++) {
        pos[id_l + i - id_mid] = pos[i];
    }
    for (int i = 0; i < id_mid - id_l; i++) {
        pos[id_l + id_r - id_mid + i] = to_put[i];
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);
```

```

cin >> s;
n = (int) s.size();
rebuild();
int m;
cin >> m;
vector<int> l(m), r(m), k(m);
for (int i = 0; i < m; i++) {
    cin >> l[i] >> r[i] >> k[i];
    l[i]--;
    r[i]--;
}
for (int i = m - 1; i >= 0; i--) {
    if (i % block == 0) {
        take();
        rebuild();
    }
    rotate(l[i], r[i], k[i]);
}
take();
cout << s << endl;
return 0;
}

```

1.6 Treap

```

mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());

template <typename Info_t>
class Treap {
private:
    struct node_t {
        long long int prior;
        int size;
        Info_t value;
        node_t *l, *r;

        node_t(Info_t a) {
            value = a;
            prior = rng();
            l = nullptr;
            r = nullptr;
            size = 1;
        }
    };

    typedef node_t* pnode_t;

    int sz(pnode_t t) {
        if (t) return t->size;
        return 0;
    }

    void upd_sz(pnode_t t) {
        if (t) {
            t->size = 1 + sz(t->l) + sz(t->r);
        }
    }

    void split(pnode_t cur, Info_t key, pnode_t &l, pnode_t &r) {
        if (cur == nullptr) {
            l = r = nullptr;
        } else if (cur->value < key) {
            l = cur;
            split(cur->r, key, l->r, r);
            upd_sz(l);
        } else {
            r = cur;
            split(cur->l, key, l, r->l);
            upd_sz(r);
        }
    }
}

```

```

void merge(pnode_t l, pnode_t r, pnode_t &ans) {
    if (l == nullptr) {
        ans = r;
    } else if (r == nullptr) {
        ans = l;
    } else {
        if (l->prior >= r->prior) {
            ans = l;
            merge(l->r, r, ans->r);
        } else {
            ans = r;
            merge(l, r->l, ans->l);
        }
    }
    upd_sz(ans);
}

public:
    pnode_t root;

    Treap() {
        root = nullptr;
    }

    bool search(Info_t x, pnode_t cur) {
        if (cur == nullptr) {
            return false;
        }
        if (cur->value > x) {
            return search(x, cur->l);
        } else if (cur->value < x) {
            return search(x, cur->r);
        } else {
            return true;
        }
    }

    void insert(Info_t x) {
        //if it cannot contain equal elements
        if (search(x, root)) {
            return;
        }
        pnode_t l, r;
        split(root, x, l, r);
        merge(l, new node_t(x), root);
        merge(root, r, root);
    }

    void erase(Info_t x) {
        pnode_t l, mid, r;
        split(root, x, l, mid);
        split(mid, x + 1, mid, r);
        merge(l, r, root);
        if (mid != nullptr) {
            assert(mid->size == 1 && mid->value == x);
            delete mid;
        }
    }

    int get(Info_t x, pnode_t cur) {
        if (!cur) {
            return 0;
        }
        if (cur->value < x) {
            return 1 + sz(cur->l) + get(x, cur->r);
        } else {
            return get(x, cur->l);
        }
    }
};

//implicit with reversion
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());

```

```

template <typename Info_t>
class Treap {
private:
    struct node_t {
        long long int prior;
        int size;
        Info_t value;
        node_t *l, *r;
        bool rev;

        node_t(Info_t a) {
            prior = rng();
            l = nullptr;
            r = nullptr;
            size = 1;
            value = a;
            rev = false;
        }
    };

    typedef node_t* pnode_t;

    int sz(pnode_t t) {
        if (t) return t->size;
        return 0;
    }

    void upd_sz(pnode_t t) {
        if (t != nullptr) {
            t->size = 1 + sz(t->l) + sz(t->r);
        }
    }

    void push(pnode_t t) {
        if (t != nullptr && t->rev) {
            t->rev = false;
            swap(t->l, t->r);
            if (t->l) {
                t->l->rev ^= true;
            }
            if (t->r) {
                t->r->rev ^= true;
            }
        }
    }

    void split(pnode_t cur, int key, int add, pnode_t &l, pnode_t &r) {
        if (cur == nullptr) {
            l = r = nullptr;
            return;
        }
        push(cur);
        int cur_key = add + sz(cur->l);
        if (cur_key < key) {
            l = cur;
            split(cur->r, key, 1 + sz(cur->l) + add, l->r, r);
            upd_sz(r);
            upd_sz(l);
        } else {
            r = cur;
            split(cur->l, key, add, l, r->l);
            upd_sz(r);
            upd_sz(l);
        }
    }

    void merge(pnode_t l, pnode_t r, pnode_t &ans) {
        push(l);
        push(r);
        if (l == nullptr) {
            ans = r;
        } else if (r == nullptr) {
            ans = l;
        } else {
            if (l->prior >= r->prior) {
                ans = l;
                merge(l->r, r, ans->r);
            } else {
                ans = r;
                merge(l, r->l, ans->l);
            }
        }
        upd_sz(ans);
    }

public:
    pnode_t root;

    Treap() {
        root = nullptr;
    }

    bool search(Info_t x, pnode_t cur) {
        if (cur == nullptr) {
            return false;
        }
        if (cur->value > x) {
            return search(x, cur->l);
        } else if (cur->value < x) {
            return search(x, cur->r);
        } else {
            return true;
        }
    }

    void insert(Info_t x, int id = -1) {
        if (id == -1) {
            id = get_less_than(x, root);
        }
        pnode_t l, r;
        split(root, id, 0, l, r);
        merge(l, new node_t(x), root);
        merge(root, r, root);
    }

    void erase(int id) {
        pnode_t l, mid, r;
        split(root, id, 0, l, mid);
        split(mid, l, 0, mid, r);
        merge(l, r, root);
        if (mid != nullptr) {
            delete mid;
        }
    }

    int get_less_than(Info_t x, pnode_t cur) {
        if (!cur) {
            return 0;
        }
        if (cur->value < x) {
            return 1 + sz(cur->l) + get_less_than(x, cur->r);
        } else {
            return get_less_than(x, cur->l);
        }
    }

    void print(pnode_t cur) {
        if (cur == nullptr) {
            return;
        }
        print(cur->l);
        cout << cur->value << ' ';
        print(cur->r);
    }

    void reverse(int L, int R) {
        pnode_t l, mid, r;
        split(root, L, 0, l, mid);
        split(mid, R - L + 1, 0, mid, r);
        assert(mid->rev == false);
    }

```

```

mid->rev = true;
merge(l, mid, root);
merge(root, r, root);
}

Info_t get(int pos) {
    pnode_t l, mid, r;
    split(root, pos, 0, l, mid);
    split(mid, 1, 0, mid, r);
    Info_t ans = mid->value;
    merge(l, mid, root);
    merge(root, r, root);
    return ans;
}
};

```

2 Graph Algorithms

2.1 arborescence

```

using ll = long long;
using i3 = pair<ll, pair<int, int>>;

struct UnionFind {
    vector<int> par;
    UnionFind(int n) : par(n) {
        for (int i = 0; i < n; ++i) par[i] = i;
    }
    int find(int i) { return (par[i] == i ? i : (par[i] = find(par[i]))); }
    void merge(int i, int j) {
        i = find(i); j = find(j);
        par[j] = i;
    }
};

template <class T>
struct SkewHeap {
    priority_queue<T, vector<T>, greater<T>> heap;
    ll lazy;

    SkewHeap() { lazy = 0; }
    void insert(T v) { heap.push(v); }
    void adjust(ll x) { lazy += x; }
    bool empty() { return heap.empty(); }
    T apply(T ans, ll lazy) {
        ans.first += lazy;
        return ans;
    }
    T pop_min() {
        T ans = heap.top();
        ans = apply(ans, lazy);
        heap.pop();
        return ans;
    }
    void absorb(SkewHeap<T>& o) {
        if (o.heap.size() > heap.size()) {
            swap(o.heap, heap);
            swap(o.lazy, lazy);
        }
        ll diff = o.lazy - lazy;
        while (!o.heap.empty()) {
            T it = o.heap.top();
            o.heap.pop();
            insert(apply(it, diff));
        }
    }
};

struct edge { int to; int v; ll w; int id;};

```

```

ll get_arborescence(int n, vector<vector<edge>> &E, vector<edge>& all_edges,
    vector<int>& ant) {
    ant.assign(n, -1);
    UnionFind uf(n);
    vector<SkewHeap<i3>> sk(n);
    for (int i = 1; i < n; ++i)
        for (const edge &e : E[i])
            sk[i].insert({e.w, {e.v, e.id}});
    ll ans = 0LL;
    vector<ll> best(n, -1);
    for (int i = 1, root = 0; i < n; ++i) {
        if (uf.find(i) == root) continue;
        vector<int> st; st.push_back(i);
        while (true) {
            int u = st.back(), v, id; ll w;
            if (sk[u].empty()) {
                return -1; //impossible
            }
            i3 ret = sk[u].pop_min();
            w = ret.first; v = ret.second.first; id = ret.second.second;
            v = uf.find(v);
            if (v == u) continue;
            ant[all_edges[id].to] = id;
            ans += (best[u] = w);
            if (v == root) break;
            if (best[v] == -1) {
                st.push_back(v);
            } else {
                while (true) {
                    sk[st.back()].adjust(-best[st.back()]);
                    if (st.back() != u) sk[u].absorb(sk[st.back()]);
                    if (uf.find(st.back()) == v) break;
                    else uf.merge(st.back(), v), v = uf.find(v), st.pop_back();
                }
                swap(sk[u], sk[v]);
                st.pop_back(), st.push_back(v);
            }
        }
        while (!st.empty()) uf.merge(root, st.back()), st.pop_back();
        root = uf.find(root);
    }
    return ans;
}

```

2.2 Centroid Decomposition

```

//Centroid decomposition1

void dfsSize(int v, int pa) {
    sz[v] = 1;
    for(int u : adj[v]) {
        if (u == pa || rem[u]) continue;
        dfsSize(u, v);
        sz[v] += sz[u];
    }
}

int getCentroid(int v, int pa, int tam) {
    for(int u : adj[v]) {
        if (u == pa || rem[u]) continue;
        if (2 * sz[u] > tam) return getCentroid(u, v, tam);
    }
    return v;
}

void decompose(int v, int pa = -1) {
    //cout << v << ' ' << pa << '\n';
    dfsSize(v, pa);
}

```

```

    int c = getCentroid(v, pa, sz[v]);
    //cout << c << '\n';
    par[c] = pa;
    rem[c] = 1;
    for(int u : adj[c]) {
        if (!rem[u] && u != pa) decompose(u, c);
    }
    adj[c].clear();
}

//Centroid decomposition2

void dfsSize(int v, int par) {
    sz[v] = 1;
    for(int u : adj[v]) {
        if (u == par || removed[u]) continue;
        dfsSize(u, v);
        sz[v] += sz[u];
    }
}

int getCentroid(int v, int par, int tam) {
    for(int u : adj[v]) {
        if (u == par || removed[u]) continue;
        if (2 * sz[u] > tam) return getCentroid(u, v, tam);
    }
    return v;
}

void setDis(int v, int par, int nv, int d) {
    dis[v][nv] = d;
    for(int u : adj[v]) {
        if (u == par || removed[u]) continue;
        setDis(u, v, nv, d + 1);
    }
}

void decompose(int v, int par, int nv) {
    dfsSize(v, par);
    int c = getCentroid(v, par, sz[v]);
    ct[c] = par;
    removed[c] = 1;
    setDis(c, par, nv, 0);
    for(int u : adj[c]) {
        if (!removed[u]) {
            decompose(u, c, nv + 1);
        }
    }
}

```

3 Dynamic Programming

3.1 CHT

```

typedef long double double_t;
typedef long long int ll;

class HullDynamic {
public:
    const double_t inf = 1e9;

    struct Line {
        ll m, b;
        double_t start;
        bool is_query;

        Line() {}
    }

```

```

    Line(ll _m, ll _b, double_t _start, bool _is_query) : m(_m), b(_b),
        start(_start), is_query(_is_query) {}

    ll eval(ll x) {
        return m * x + b;
    }

    double_t intersect(const Line& l) const {
        return (double_t) (l.b - b) / (m - l.m);
    }

    bool operator< (const Line& l) const {
        if (is_query == 0) return m > l.m;
        return (start < l.start);
    }
};

typedef set<Line>::iterator iterator_t;

bool has_prev(iterator_t it) {
    return (it != hull.begin());
}

bool has_next(iterator_t it) {
    return (++it != hull.end());
}

bool irrelevant(iterator_t it) {
    if (!has_prev(it) || !has_next(it)) return 0;
    iterator_t prev = it, next = it;
    prev--;
    next++;
    return next->intersect(*prev) <= it->intersect(*prev);
}

void update_left(iterator_t it) {
    if (it == hull.begin()) return;
    iterator_t pos = it;
    --it;
    vector<Line> rem;
    while(has_prev(it)) {
        iterator_t prev = it;
        --prev;
        if (prev->intersect(*pos) <= prev->intersect(*it)) {
            rem.push_back(*it);
        } else {
            break;
        }
        --it;
    }
    double_t start = pos->intersect(*it);
    Line f = *pos;
    for (Line r : rem) hull.erase(r);
    hull.erase(f);
    f.start = start;
    hull.insert(f);
}

void update_right(iterator_t it) {
    if (!has_next(it)) return;
    iterator_t pos = it;
    ++it;
    vector<Line> rem;
    while(has_next(it)) {
        iterator_t next = it;
        ++next;
        if (next->intersect(*pos) <= pos->intersect(*it)) {
            rem.push_back(*it);
        } else {
            break;
        }
        ++it;
    }
}

```

```

double_t start = pos->intersect(*it);
Line f = *it;
for (Line r : rem) hull.erase(r);
hull.erase(f);
f.start = start;
hull.insert(f);
}

void insert_line(ll m, ll b) {
    Line f(m, b, -inf, 0);
    iterator_t it = hull.lower_bound(f);
    if (it != hull.end() && it->m == f.m) {
        if (it->b <= f.b) {
            return;
        } else if (it->b > f.b) {
            hull.erase(it);
        }
    }
    hull.insert(f);
    it = hull.lower_bound(f);
    if (irrelevant(it)) {
        hull.erase(it);
        return;
    }
    update_left(it);
    it = hull.lower_bound(f);
    update_right(it);
}

ll get(ll x) {
    Line f(0, 0, x, 1);
    iterator_t it = hull.upper_bound(f);
    assert(it != hull.begin());
    --it;
    return it->m * x + it->b;
}

```

```

private:
    set<Line> hull;
};

```

```

//////////
//Linear

```

```

class HullDynamic {
public:
    const double_t inf = 1e18;

    struct Line {
        ll m, b;
        double_t start;
        bool is_query;

        Line() {}

        Line(ll _m, ll _b, double_t _start, bool _is_query) : m(_m), b(_b),
            start(_start), is_query(_is_query) {}

        ll eval(ll x) {
            return m * x + b;
        }

        double_t intersect(const Line& l) const {
            return (double_t) (l.b - b) / (m - l.m);
        }

        bool operator< (const Line& l) const {
            if (is_query == 0) return m > l.m;
            return (start < l.start);
        }
    };
};

```

```

typedef vector<Line>::iterator iterator_t;

void insert(ll m, ll b) {
    Line f(m, b, -inf, 0);
    while((int) hull.size() > 1 && f.intersect(hull[(int) hull.size() -
        2]) <= hull.back().start) {
        hull.pop_back();
    }
    double_t st = -inf;
    if (!hull.empty()) {
        st = f.intersect(hull.back());
    }
    f.start = st;
    hull.push_back(f);
    /*for (auto it : hull) {
        cout << it.m << ' ' << it.b << ' ' << it.start << '\n';
    }
    cout << '\n';*/
}

ll get(ll x) {
    pt = min(pt, (int) hull.size() - 1);
    while(pt < (int) hull.size() - 1) {
        if (hull[pt + 1].start <= x) {
            pt++;
        } else {
            break;
        }
    }
    //cout << hull[pt].m << ' ' << hull[pt].b << '\n';
    return hull[pt].m * x + hull[pt].b;
}

private:
    vector<Line> hull;
};

```

3.2 Slope Trick

```

#include <limits>
#include <queue>
#include <utility>

// Thanks WeakestTopology

template <typename T>
struct SlopeTrick {
    static constexpr T inf = std::numeric_limits<T>::max() / 2;

    T cost = 0, left_offset = 0, right_offset = 0;
    using Pair = std::pair<T, T>;
    std::priority_queue<Pair> left;
    std::priority_queue<Pair, std::vector<Pair>, std::greater<Pair>> right;

    SlopeTrick() {
        left.emplace(-inf, 0);
        right.emplace(+inf, 0);
    }

    void rebalance() {
        while (true) {
            auto [xl, sl] = left.top();
            xl -= left_offset;
            auto [xr, sr] = right.top();
            xr += right_offset;
            if (xl <= xr) break;
            left.pop();
            right.pop();
            T take = std::min(sl, sr);
            cost += take * (xl - xr);
            sl -= take;

```



```

    sr -= take;
    left.emplace(xr + left_offset, take);
    right.emplace(xl - right_offset, take);
    if (sl) {
        left.emplace(xl + left_offset, sl);
    }
    if (sr) {
        right.emplace(xr - right_offset, sr);
    }
}

// Adds  $f(x) = s * \max(a - x, 0)$ .
void add_left(T a, T s) {
    left.emplace(a + left_offset, s);
    rebalance();
}

// Adds  $f(x) = s * \max(x - a, 0)$ .
void add_right(T a, T s) {
    right.emplace(a - right_offset, s);
    rebalance();
}

// Adds  $f(x) = s * \text{abs}(x - a)$ .
void add_abs(T a, T s) {
    add_left(a, s);
    add_right(a, s);
}

void relax_left(T offset) {
    assert(offset >= 0);
    left_offset += offset;
}

void relax_right(T offset) {
    assert(offset >= 0);
    right_offset += offset;
}

void relax(T offset) {
    assert(offset >= 0);
    left_offset += offset;
    right_offset += offset;
}
};

```

4 Math

4.1 Dirichlet Convolution

```

#include <bits/stdc++.h>
using namespace std;

/*
    fib(m + n) = fib(m) * fib(n + 1) + fib(m - 1) * fib(n)
    gcd(fib(n), fib(m)) = fib(gcd(n, m))
*/

/*
    Dirichlet Convolution

    (f * g)(n) = sum{d | n}{f(d) * g(n / d)}
    mi * 1 = e => (g = f * 1 <=> f = g * mi)
    phi * 1 = Id
    sf(n) = (s(f * g)(n) - sum{2 <= d <= n}{sf(floor(n / d)) * g(d)}) / g(1)
*/
struct ModularArithmeticFunctionPrefixSum {
    unordered_map<long long, long long> dp;
    vector<long long> precalculatedPrefixSumF;

```

```

    long long mod;

    ModularArithmeticFunctionPrefixSum(vector<long long>
        precalculatedPrefixSumF, long long mod) : precalculatedPrefixSumF(
        precalculatedPrefixSumF), mod(mod) {}

    long long fexp(long long b, long long e = -1) {
        if (e < 0) e += mod - 1;
        long long ans = 1;
        while (e > 0) {
            if (e & 1) ans = ans * b % mod;
            b = b * b % mod;
            e >>= 1;
        }
        return ans;
    }

    long long prefixG(long long x) {
        // implement here s_g(x)
        // example:
        // g = 1
        if (x <= 0) {
            return 0;
        }
        return x % mod;
    }

    const long long inv2 = fexp(2);

    long long prefixFG(long long x) {
        // implement here s_(f * g)(x)
        // example:
        // f = phi
        // g = 1
        // f * g = Id
        if (x <= 0) {
            return 0;
        }
        x %= mod;
        return x * (x + 1) % mod * inv2 % mod;
    }

    long long prefixF(long long x) {
        if (x < precalculatedPrefixSumF.size()) {
            return precalculatedPrefixSumF[x];
        }
        if (dp.count(x)) {
            return dp[x];
        }
        long long ans = prefixFG(x);
        for (long long lowerBound = 2, upperBound; lowerBound <= x; lowerBound
            = upperBound + 1) {
            upperBound = x / (x / lowerBound);
            long long intervalG = (prefixG(upperBound) - prefixG(lowerBound - 1)
                + mod) % mod;
            ans = (ans - prefixF(x / lowerBound) % mod * intervalG % mod + mod) %
                mod;
        }
        long long g1 = (prefixG(1) - prefixG(0) + mod) % mod;
        ans = ans * fexp(g1) % mod;
        assert(g1 * fexp(g1) % mod == 1);
        dp[x] = ans;
        return ans;
    }
};

const long long mod = 998244353L;
long long acc[2][2], base[2][2], temp[2][2];

long long fib(long long x) {
    if (x == 0) return 0;

    base[0][0] = 1;
    base[0][1] = 1;
    base[1][0] = 1;

```

```

base[1][1] = 0;
acc[0][0] = 1;
acc[0][1] = 0;
acc[1][0] = 0;
acc[1][1] = 1;

x--;
while (x > 0) {
    if (x & 1) {
        for (int i = 0; i < 2; i++) {
            for (int j = 0; j < 2; j++) {
                temp[i][j] = 0;
                for (int k = 0; k < 2; k++) {
                    temp[i][j] = (temp[i][j] + base[i][k] * acc[k][j] % mod) % mod;
                }
            }
        }
        for (int i = 0; i < 2; i++) {
            for (int j = 0; j < 2; j++) {
                acc[i][j] = temp[i][j];
            }
        }
    }
    x >>= 1;

    for (int i = 0; i < 2; i++) {
        for (int j = 0; j < 2; j++) {
            temp[i][j] = 0;
            for (int k = 0; k < 2; k++) {
                temp[i][j] = (temp[i][j] + base[i][k] * base[k][j] % mod) % mod;
            }
        }
    }
    for (int i = 0; i < 2; i++) {
        for (int j = 0; j < 2; j++) {
            base[i][j] = temp[i][j];
        }
    }
}

return acc[0][0];
}

int main() {
    ios::sync_with_stdio(false); cin.tie(nullptr);

    const int mx = 5000000;
    vector<int> primes, phi(mx);
    vector<bool> isComposite(mx, false);

    phi[1] = 1;

    for (int i = 2; i < mx; i++) {
        if (!isComposite[i]) {
            primes.push_back(i);
            phi[i] = i - 1;
        }
        //if (i < 30) cout << i << ' ' << phi[i] << endl;
        for (int p: primes) {
            if (i * p >= mx) break;
            isComposite[i * p] = true;
            if (i % p == 0) {
                phi[i * p] = p * phi[i];
                break;
            } else {
                phi[i * p] = phi[i] * phi[p];
            }
        }
    }

    vector<long long> sumPhi(mx);
    sumPhi[0] = phi[0];

```

```

for (int i = 1; i < mx; i++) {
    sumPhi[i] = (1ll * phi[i] + sumPhi[i - 1]) % mod;
}

ModularArithmeticFunctionPrefixSum phiCalculator(sumPhi, mod);

//cout << phiCalculator.prefixF(1ll * 10000000000) << endl;

long long n;
cin >> n;

long long ans = 0;
for (long long lowerBound = 1, upperBound; lowerBound <= n; lowerBound =
    upperBound + 1) {
    upperBound = n / (n / lowerBound);
    long long fibSum = (fib(upperBound + 2) - 1 - fib(lowerBound + 1) + 1 +
        mod) % mod;
    long long lim = n / lowerBound;
    assert(n / lowerBound == n / upperBound);

    ans = (ans + (2 * phiCalculator.prefixF(lim) - 1 + mod) % mod * fibSum
        % mod) % mod;
}

cout << ans << endl;

return 0;
}

```

4.2 Miller and Rho

```

typedef long long int ll;

bool overflow(ll a, ll b) {
    return b && (a >= (1ll << 62) / b);
}

ll add(ll a, ll b, ll md) {
    return (a + b) % md;
}

ll mul(ll a, ll b, ll md) {
    if (!overflow(a, b)) return (a * b) % md;
    ll ans = 0;
    while(b) {
        if (b & 1) ans = add(ans, a, md);
        a = add(a, a, md);
        b >>= 1;
    }
    return ans;
}

ll fexp(ll a, ll e, ll md) {
    ll ans = 1;
    while(e) {
        if (e & 1) ans = mul(ans, a, md);
        a = mul(a, a, md);
        e >>= 1;
    }
    return ans;
}

ll my_rand() {
    ll ans = rand();
    ans = (ans << 31) | rand();
    return ans;
}

ll gcd(ll a, ll b) {
    while(b) {
        ll t = a % b;
        a = b;
        b = t;
    }
}

```

```

    }
    return a;
}

bool miller(ll p, int iteracao) {
    if(p < 2) return 0;
    if(p % 2 == 0) return (p == 2);
    ll s = p - 1;
    while(s % 2 == 0) s >>= 1;
    for(int i = 0; i < iteracao; i++) {
        ll a = rand() % (p - 1) + 1, temp = s;
        ll mod = fexp(a, temp, p);
        while(temp != p - 1 && mod != 1 && mod != p - 1) {
            mod = mul(mod, mod, p);
            temp <<= 1;
        }
        if(mod != p - 1 && temp % 2 == 0) return 0;
    }
    return 1;
}

ll rho(ll n) {
    if (n == 1 || miller(n, 10)) return n;
    if (n % 2 == 0) return 2;
    while(1) {
        ll x = my_rand() % (n - 2) + 2, y = x;
        ll c = 0, cur = 1;
        while(c == 0) {
            c = my_rand() % (n - 2) + 1;
        }
        while(cur == 1) {
            x = add(mul(x, x, n), c, n);
            y = add(mul(y, y, n), c, n);
            y = add(mul(y, y, n), c, n);
            cur = gcd((x >= y ? x - y : y - x), n);
        }
        if (cur != n) return cur;
    }
}

```

5 String Algorithms

5.1 Hash

```

using ull = uint64_t;

const ull MOD = (1LL << 61) - 1;
const int base = 31;
const int MAX_POT = 1000100;

ull p[MAX_POT];

ull modMul(uint64_t a, uint64_t b) {
    ull l1 = (uint32_t)a, h1 = a >> 32, l2 = (uint32_t)b, h2 = b >> 32;
    ull l = l1 * l2, m = l1 * h2 + l2 * h1, h = h1 * h2;
    ull ret = (l & MOD) + (l >> 61) + (h << 3) + (m >> 29) + ((m << 35) >> 3)
        + 1;
    ret = (ret & MOD) + (ret >> 61);
    ret = (ret & MOD) + (ret >> 61);
    return ret - 1;
}

void buildHashPot() {
    p[0] = 1;
    for (int i = 1; i < MAX_POT; i++) p[i] = modMul(base, p[i - 1]);
}

struct Hash {
    ull val;
    int size;

```

```

    Hash(ull val = 0, int size = 0): val(val), size(size) {}
};

// maior pot vai ficar no l
Hash combine(Hash l, Hash r) {
    ull val = modMul(p[r.size], l.val);
    return Hash(val + r.val, l.size + r.size);
}

struct StringHash {
    // 0 based
    // pref[i] = hash da string 0..i, maior pot ta no 0
    // suff[i] = hash da string i..n-1, maior pot ta no n-1
    int n;
    vector<Hash> pref, suff;

    ull getInt(char c) {
        return c - 'a' + 1;
    }

    StringHash(string &s) {
        // remember to call it in the main
        // buildHashPot();
        n = s.size();
        pref.resize(n);
        suff.resize(n);
        pref[0] = Hash(getInt(s[0]), 1);
        for (int i = 1; i < n; i++) {
            Hash cur = Hash(getInt(s[i]), 1);
            pref[i] = combine(pref[i - 1], cur);
        }
        suff[n - 1] = Hash(getInt(s[n - 1]), 1);
        for (int i = n - 2; i >= 0; i--) {
            Hash cur = Hash(getInt(s[i]), 1);
            suff[i] = combine(suff[i + 1], cur);
        }
    }

    // [l, r]
    // maior pot vai estar no l
    Hash getHash(int l, int r) {
        ull res = pref[r].val;
        if(l > 0) res = (res + MOD - modMul(p[r - l + 1], pref[l - 1].val)) %
            MOD;
        return Hash(res, r - l + 1);
    }

    // [l, r]
    // maior pot vai estar no r
    Hash getRevHash(int l, int r) {
        ull res = suff[l].val;
        if (r < n - 1) res = (res + MOD - modMul(p[r - l + 1], suff[r + 1].val)
            ) % MOD;
        return Hash(res, r - l + 1);
    }
};

```

5.2 Kmp Automaton

```

const int limit =

vector<vector<int>> build_automaton(string s) {
    s += '#'; //tem que ser diferente de todos os caracteres
    int n = (int) s.size();
    vector<vector<int>> ans(n, vector<int>(limit));
    vector<int> fail(n);
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < limit; j++) {
            if (i == 0) {
                if (s[i] == j + 'a') {

```

```

        ans[i][j] = i + 1;
    } else {
        ans[i][j] = 0;
    }
} else {
    if (s[i] == j + 'a') {
        ans[i][j] = i + 1;
    } else {
        ans[i][j] = ans[fail[i - 1]][j];
    }
}
}
if (i == 0) {
    continue;
}
int j = fail[i - 1];
while (j > 0 && s[i] != s[j]) {
    j = fail[j - 1];
}
fail[i] = j + (s[i] == s[j]);
return ans;
}
}

```

6 Miscellaneous

6.1 Operations

```

inline int add(int a, int b) {
    a += b;
    if (a >= mod) {
        a -= mod;
    }
    return a;
}

inline int sub(int a, int b) {
    a -= b;
    if (a < 0) {
        a += mod;
    }
    return a;
}

inline int mul(int a, int b) {
    int ans = 0;
    asm(
        "mull %%ebx;"
        "divl %3;"
        : "=d" (ans)
        : "a" (a), "b" (b), "c" (mod)
    );
}

```

```

    );
    return ans;
}

```

6.2 Count Inversions

```

typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;

/*
Without collisions. If there are collisions, it might be possible to use
less_equal as a comparator (the delete does not work as expected)
*/
int solve(vector<pair<int, int>> all) {
    sort(all.begin(), all.end());
    reverse(all.begin(), all.end());
    int ans = 0;
    ordered_set S;
    for (int i = 0; i < n; i++) {
        ans += S.order_of_key(all[i].second);
        S.insert(all[i].second);
    }
    return ans;
}

```

7 Teoremas e formulas uteis

7.1 Teoremas

Quadrangular inequality

- SE $\text{cost}(l, r) + \text{cost}(l + 1, r + 1) \leq \text{cost}(l, r + 1) + \text{cost}(l + 1, r)$
- E $f(l, r) = \text{dp}[l - 1] + \text{cost}(l, r)$
- Entao $f(l, r)$ tbm obedece a quadrangular inequality
- Isso implica que se existe um k tal que $f(l + 1, k) \leq f(l, k)$, entao $f(l + 1, k + 1) \leq f(l, k)$
- Ou seja, a partir do momento que o $(l + 1)$ fica melhor do que o l , ele sempre eh melhor.
- Com isso da pra otimizar $\text{dp}[x]$ para $O(n * \log)$, fazendo uma busca binaria pra descobrir quando $(l + 1)$ fica melhor do que l

Se $e \geq \log_2(m)$
 $n^e \bmod m = n^{(\phi(m) + e \bmod \phi(m)) \bmod m} \bmod m$
 Se $e < \log_2(m)$, so fazer o bruto

$\phi(\phi(m)) \leq m/2$