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1 String Algorithms

1.1 String Alignment

```
int pd[ms][ms];

int edit_distance(string &a, string &b) {
    int n = a.size(), m = b.size();
    for(int i = 0; i <= n; i++) pd[i][0] = i;
    for(int j = 0; j <= m; j++) pd[0][j] = j;
    for(int i = 1; i <= n; i++) {
        for(int j = 1; j <= m; j++) {
            int del = pd[i][j-1] + 1;
```

```
            int ins = pd[i-1][j] + 1;
            int mod = pd[i-1][j-1] + (a[i-1] != b[j-1]);
            pd[i][j] = min(del, min(ins, mod));
        }
    }
    return pd[n][m];
}
```

1.2 KMP

```
string p, t;
int b[ms], n, m;

void kmpPreprocess() {
    int i = 0, j = -1;
    b[0] = -1;
    while(i < m) {
        while(j >= 0 && p[i] != p[j]) j = b[j];
        b[++i] = ++j;
    }
}

void kmpSearch() {
    int i = 0, j = 0, ans = 0;
    while(i < n) {
        while(j >= 0 && t[i] != p[j]) j = b[j];
        i++; j++;
        if(j == m) {
            //ocorrencia aqui comecando em i - j
            ans++;
            j = b[j];
        }
    }
}
```

1.3 Trie

```
int trie[ms][sigma], terminal[ms], z;

void init() {
    memset(trie[0], -1, sizeof trie[0]);
    z = 1;
}

int get_id(char c) {
    return c - 'a';
}

void insert(string &p) {
    int cur = 0;
    for(int i = 0; i < p.size(); i++) {
        int id = get_id(p[i]);
        if(trie[cur][id] == -1) {
            memset(trie[z], -1, sizeof trie[z]);
            trie[cur][id] = z++;
        }
        cur = trie[cur][id];
    }
    terminal[cur]++;
}

int count(string &p) {
    int cur = 0;
    for(int i = 0; i < p.size(); i++) {
        int id = get_id(p[i]);
        if(trie[cur][id] == -1) {
            return false;
        }
        cur = trie[cur][id];
    }
    return terminal[cur];
}
```

1.4 Aho-Corasick

```
// Construa a Trie do seu dicionario com o codigo acima

int fail[ms], q[ms], front, rear;

void buildFailure() {
    front = 0; rear = 0; q[rear++] = 0;
    while(front < rear) {
        int node = q[front++];
        for(int pos = 0; pos < sigma; pos++) {
            int &v = trie[node][pos];
            int f = node == 0 ? 0 : trie[fail[node]][pos];
            if(v == -1) {
                v = f;
            } else {
                fail[v] = f;
                q.push(v);
                // juntar as informacoes da borda para o V ja q um match em V implica um
                // match na borda
                terminal[v] += terminal[f];
            }
        }
    }
}

int search(string &txt) {
    int node = 0;
    int ans = 0;
    for(int i = 0; i < txt.length(); i++) {
        int pos = get_id(txt[i]);
        node = trie[node][pos];
        // processar informacoes no no atual
        ans += terminal[node];
    }
    return ans;
}
```

2 Data Structures

2.1 BIT - Binary Indexed Tree

```
int arr[ms], bit[ms], n;

void update(int v, int idx) {
    while(idx <= n) {
        bit[idx] += v;
        idx += idx & -idx;
    }
}

int query(int idx) {
    int r = 0;
    while(idx > 0) {
        r += bit[idx];
        idx -= idx & -idx;
    }
    return r;
}
```

2.2 Iterative Segment Tree

```
int n, t[2 * ms];

void build() {
    for(int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
}
```

```
void update(int p, int value) { // set value at position p
    for(t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
}

int query(int l, int r) {
    int res = 0;
    for(l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if(l&1) res += t[l++];
        if(r&1) res += t[--r];
    }
    return res;
}

// If is non-commutative
S query(int l, int r) {
    S resl, resr;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) resl = combine(resl, t[l++]);
        if (r&1) resr = combine(t[--r], resr);
    }
    return combine(resl, resr);
}
```

2.3 Iterative Segment Tree with Interval Updates

```
int n, t[2 * ms];

void build() {
    for(int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
}

void update(int l, int r, int value) {
    for(l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if(l&1) t[l++] += value;
        if(r&1) t[--r] += value;
    }
}

int query(int p) {
    int res = 0;
    for(p += n; p > 0; p >>= 1) res += t[p];
    return res;
}

void push() { // push modifications to leafs
    for(int i = 1; i < n; i++) {
        t[i<<1] += t[i];
        t[i<<1|1] += t[i];
        t[i] = 0;
    }
}
```

2.4 Recursive Segment Tree

```
int arr[4 * ms], seg[4 * ms], n;

void build(int idx = 0, int l = 0, int r = n - 1) {
    int mid = (l+r)/2, left = 2 * idx + 1, right = 2 * idx + 2;
    if(l == r) {
        seg[idx] = arr[l];
        return;
    }
    build(left, l, mid); build(right, mid + 1, r);
    seg[idx] = seg[left] + seg[right];
}

int query(int L, int R, int idx = 0, int l = 0, int r = n - 1) {
    int mid = (l+r)/2, left = 2 * idx + 1, right = 2 * idx + 2;
    if(R < l || L > r) return 0;
    if(L <= l && r <= R) return seg[idx];
    return query(L, R, left, l, mid) + query(L, R, right, mid + 1, r);
}
```

```

}

void update(int V, int I, int idx = 0, int l = 0, int r = n - 1) {
    int mid = (l+r)/2, left = 2 * idx + 1, right = 2 * idx + 2;
    if(l > I || r < I) return;
    if(l == r) {
        arr[l] = V;
        seg[idx] = V;
        return;
    }
    update(V, I, left, l, mid); update(V, I, right, mid + 1, r);
    seg[idx] = seg[left] + seg[right];
}

```

2.5 Segment Tree with Lazy Propagation

```

int arr[4 * ms], seg[4 * ms], lazy[4 * ms], n;

void build(int idx = 0, int l = 0, int r = n - 1) {
    int mid = (l+r)/2, left = 2 * idx + 1, right = 2 * idx + 2;
    if(l == r) {
        seg[idx] = arr[l];
        return;
    }
    build(left, l, mid); build(right, mid + 1, r);
    seg[idx] = seg[left] + seg[right];
}

void propagate(int idx, int l, int r, int left, int right) {
    if(lazy[idx]) {
        seg[idx] += lazy[idx] * (r - l + 1);
        if(l < r) {
            lazy[left] += lazy[idx];
            lazy[right] += lazy[idx];
        }
        lazy[idx] = 0;
    }
}

int query(int L, int R, int idx = 0, int l = 0, int r = n - 1) {
    int mid = (l+r)/2, left = 2 * idx + 1, right = 2 * idx + 2;
    propagate(idx, l, r, left, right);
    if(R < l || L > r) return 0;
    if(L <= l && r <= R) return seg[idx];
    return query(L, R, left, l, mid) + query(L, R, right, mid + 1, r);
}

void update(int V, int L, int R, int idx = 0, int l = 0, int r = n - 1) {
    int mid = (l+r)/2, left = 2 * idx + 1, right = 2 * idx + 2;
    propagate(idx, l, r, left, right);
    if(l > R || r < L) return;
    if(L <= l && r <= R) {
        lazy[idx] += V;
        propagate(idx, l, r, left, right);
        return;
    }
    update(V, L, R, left, l, mid); update(V, L, R, right, mid + 1, r);
    seg[idx] = seg[left] + seg[right];
}

```

2.6 Color Updates Structure

```

struct range {
    int l, r;
    int v;

    range(int l = 0, int r = 0, int v = 0) : l(l), r(r), v(v) {}

    bool operator < (const range &a) const {
        return l < a.l;
    }
};

```

```

set<range> ranges;

vector<range> update(int l, int r, int v) { // [l, r)
    vector<range> ans;
    if(l >= r) return ans;
    auto it = ranges.lower_bound(l);
    if(it != ranges.begin()) {
        it--;
        if(it->r > l) {
            auto cur = *it;
            ranges.erase(it);
            ranges.insert(range(cur.l, l, cur.v));
            ranges.insert(range(l, cur.r, cur.v));
        }
    }
    it = ranges.lower_bound(r);
    if(it != ranges.begin()) {
        it--;
        if(it->r > r) {
            auto cur = *it;
            ranges.erase(it);
            ranges.insert(range(cur.l, r, cur.v));
            ranges.insert(range(r, cur.r, cur.v));
        }
    }
    for(it = ranges.lower_bound(l); it != ranges.end() && it->l < r; it++) {
        ans.push_back(*it);
    }
    ranges.erase(ranges.lower_bound(l), ranges.lower_bound(r));
    ranges.insert(range(l, r, v));
    return ans;
}

int query(int v) { // Substituir -1 por flag para quando nao houver resposta
    auto it = ranges.upper_bound(v);
    if(it == ranges.begin()) {
        return -1;
    }
    it--;
    return it->r >= v ? it->v : -1;
}

```

2.7 Policy Based Structures

```

#include <ext/pb_ds/assoc_container.hpp> // Common file
#include <ext/pb_ds/tree_policy.hpp> // Including tree_order_statistics_node_update

using namespace __gnu_pbds;

typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;

ordered_set X;
X.insert(1);
X.find_by_order(0);
X.order_of_key(-5);
end(X), begin(X);

```

2.8 Heavy Light Decomposition

```

#include <bits/stdc++.h>
#define all(foo) foo.begin(), foo.end()
using namespace std;
const int N = 112345, inf = 0x3f3f3f3f;

int k, adj[N], ant[2*N], to[2*N];

void add(int a, int b, bool f = 1) {
    ant[k] = adj[a], adj[a] = k, to[k] = b;
    k++;
    if(f) add(b, a, 0);
}

```

```

}

int par[N], h[N], big[N], sz[N];
void dfs(int v, int p, int hght){
    sz[v] = 1, par[v] = p, h[v] = hght, big[v] = -1;
    for(int i = adj[v]; ~i; i = ant[i]){
        if(to[i] != p){
            dfs(to[i], v, hght+1);
            sz[v] += sz[to[i]];
            if(big[v] == -1 || sz[big[v]] < sz[to[i]]) big[v] = to[i];
        }
    }
}

int chainNo, chain[N], ind[N], chainSz[N], head[N];
vector<int> tree[N];
vector<int> st[N];

void upd(int p, int value, vector<int> &tree){
    int n = tree.size() >> 1;
    for(tree[p += n] = value; p > 1; p >>= 1) tree[p >> 1] = min(tree[p], tree[p ^ 1]);
}

int rmq(int l, int r, vector<int> &tree){
    int res = inf;
    int n = tree.size() >> 1;
    for(l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if(l & 1) res = min(res, tree[l++]);
        if(r & 1) res = min(res, tree[--r]);
    }
    return res;
}

void HLD(int v, int p, int cIn){
    if(cIn == 0) head[chainNo] = v;
    ind[v] = cIn;
    chain[v] = chainNo;
    st[chainNo].push_back(v);
    if(~big[v]){
        HLD(big[v], v, cIn+1);
    } else {
        int n = chainSz[chainNo] = st[chainNo].size();
        tree[chainNo].resize(2*n);
        fill(all(tree[chainNo]), inf);
        chainNo++;
    }
    for(int i = adj[v]; ~i; i = ant[i]){
        if(to[i] != p && to[i] != big[v]){
            HLD(to[i], v, 0);
        }
    }
}

int up(int v){
    return (head[chain[v]] != v) ? head[chain[v]] : (par[v] != -1 ? par[v] : v);
}

int LCA(int a, int b){
    while(chain[a] != chain[b]){
        if(par[a] == -1 || h[up(a)] < h[up(b)]) swap(a, b);
        a = up(a);
    }
    return h[a] < h[b] ? a : b;
}

int queryUp(int a, int b){
    int ans = -1, curr;
    while(chain[a] != chain[b]){
        curr = rmq(0, ind[a]+1, tree[chain[a]]);
        if(curr != inf) ans = st[chain[a]][curr];
        a = par[head[chain[a]]];
    }
    curr = rmq(ind[b], ind[a]+1, tree[chain[a]]);
    if(curr != inf) ans = st[chain[a]][curr];
    return ans;
}

int main(){
    int n, q;

```

```

scanf("%d %d", &n, &q);
memset(adj, -1, sizeof adj);
for(int i = 0; i < n-1; i++){
    int a, b;
    scanf("%d %d", &a, &b);
    add(a, b);
}
dfs(1, -1, 0);
HLD(1, -1, 0);
for(int i = 0; i < q; i++){
    int o, v;
    scanf("%d %d", &o, &v);
    if(o){
        printf("%d\n", queryUp(v, 1));
    } else {
        int ans = rmq(ind[v], ind[v]+1, tree[chain[v]]);
        upd(ind[v], (ans == inf) ? ind[v] : inf, tree[chain[v]]);
    }
}
}
}

```

3 Graph Algorithms

3.1 Dinic Max Flow

```

const int ms = 1e3; // Quantidade maxima de vertices
const int me = 1e5; // Quantidade maxima de arestas

int adj[ms], to[me], ant[me], wt[me], z, n;
int copy_adj[ms], fila[ms], level[ms];

void clear() {
    memset(adj, -1, sizeof adj);
    z = 0;
}

int add(int u, int v, int k) {
    to[z] = v;
    ant[z] = adj[u];
    wt[z] = k;
    adj[u] = z++;
    swap(u, v);
    to[z] = v;
    ant[z] = adj[u];
    wt[z] = 0;
    adj[u] = z++;
}

int bfs(int source, int sink) {
    memset(level, -1, sizeof level);
    level[source] = 0;
    int front = 0, size = 0, v;
    fila[size++] = source;
    while(front < size) {
        v = fila[front++];
        for(int i = adj[v]; i != -1; i = ant[i]) {
            if(wt[i] && level[to[i]] == -1) {
                level[to[i]] = level[v] + 1;
                fila[size++] = to[i];
            }
        }
    }
    return level[sink] != -1;
}

int dfs(int v, int sink, int flow) {
    if(v == sink) return flow;
    int f;
    for(int &i = copy_adj[v]; i != -1; i = ant[i]) {
        if(wt[i] && level[to[i]] == level[v] + 1 &&
            (f = dfs(to[i], sink, min(flow, wt[i])))) {
            wt[i] -= f;
            wt[i ^ 1] += f;
            return f;
        }
    }
}

```

```

    }
    return 0;
}

int maxflow(int source, int sink) {
    int ret = 0, flow;
    while(bfs(source, sink)) {
        memcpy(copy_adj, adj, sizeof adj);
        while((flow = dfs(source, sink, 1 << 30)) {
            ret += flow;
        }
    }
    return ret;
}

```

3.2 Articulation Points/Bridges/Biconnected Components

```

const int ms = 1e3; // Quantidade maxima de vertices
const int me = 1e5; // Quantidade maxima de arestas

int adj[ms], to[me], ant[me], z, n;
int idx[ms], bc[me], ind, nbc, child, st[me], top;

void generateBc(int edge) {
    while(st[--top] != edge) {
        bc[st[top]] = nbc;
    }
    bc[edge] = nbc++;
}

int dfs(int v, int par = -1) {
    int low = idx[v] = ind++;
    for(int i = adj[v]; i > -1; i = ant[i]) {
        if(idx[to[i]] == -1) {
            if(par == -1) child++;
            st[top++] = i;
            int temp = dfs(to[i], v);
            if(par == -1 && child > 1 || ~par && temp >= idx[v]) generateBc(i);
            if(temp >= idx[v]) art[v] = true;
            if(temp > idx[v]) bridge[i] = true;
            low = min(low, temp);
        } else if(to[i] != par && idx[to[i]] < low) {
            low = idx[to[i]];
            st[top++] = i;
        }
    }
    return low;
}

void biconnected() {
    ind = 0;
    nbc = 0;
    top = -1;
    memset(idx, -1, sizeof idx);
    memset(art, 0, sizeof art);
    memset(bridge, 0, sizeof bridge);
    for(int i = 0; i < n; i++) if(idx[i] == -1) {
        child = 0;
        dfs(i);
    }
}

```

3.3 SCC - Strongly Connected Components / 2SAT

```

vector<int> g[ms];
int idx[ms], low[ms], z, comp[ms], ncomp, st[ms], top;

int dfs(int u) {
    if(~idx[u]) return idx[u] ? idx[u] : z;
    low[u] = idx[u] = z++;
    st.push(u);
}

```

```

for(int v : g[u]) {
    low[u] = min(low[u], dfs(v));
}
if(low[u] == idx[u]) {
    idx[st.top()] = 0;
    st.pop();
    while(st.top() != u) {
        int v = st.top();
        st.pop();
        idx[v] = 0;
        low[v] = low[u];
        comp[v] = ncomp;
    }
    comp[u] = ncomp++;
}
return low[u];
}

bool solveSat() {
    memset(idx, -1, sizeof idx);
    ind = 1; top = -1;
    for(int i = 0; i < n; i++) dfs(i);
    for(int i = 0; i < n; i++) if(comp[i] == comp[i^1]) return false;
    return true;
}

// Operacoes comuns de 2-sat
// ~v = "nao v"
#define trad(v) (v<0?((~v)*2)^1:v*2)
void addImp(int a, int b) { g[trad(a)].push(trad(b)); }
void addOr(int a, int b) { addImp(~a, b); addImp(~b, a); }
void addEqual(int a, int b) { addOr(a, ~b); addOr(~a, b); }
void addDiff(int a, int b) { addEqual(a, ~b); }
// valoracao: value[v] = comp[trad(v)] < comp[trad(~v)]

```

3.4 LCA - Lowest Common Ancestor

```

int par[ms][mlg], lvl[ms];

void dfs(int v, int p, int l = 0) {
    lvl[v] = l;
    par[v][0] = p;
    for(int i = adj[v]; i > -1; i = ant[i]) {
        if(to[i] != p) dfs(to[i], v, l + 1);
    }
}

void processAncestors(int root = 0) {
    dfs(root, root);
    for(int k = 1; k <= mlg; k++) {
        for(int i = 0; i < n; i++) {
            par[i][k] = par[par[i][k-1]][k-1];
        }
    }
}

int lca(int a, int b) {
    if(lvl[b] > lvl[a]) swap(a, b);
    for(int i = mlg; i >= 0; i--) {
        if(lvl[a] - (1 << i) >= lvl[b]) a = par[a][i];
    }
    if(a == b) return a;
    for(int i = mlg; i >= 0; i--) {
        if(par[a][i] != par[b][i]) a = par[a][i], b = par[b][i];
    }
    return par[a][0];
}

```

4 Math

4.1 Discrete Logarithm

```
ll discreteLog(ll a, ll b, ll m) {
    // a^ans == b mod m
    // ou -1 se nao existir
    ll cur = a, on = 1;
    for(int i = 0; i < 100; i++) {
        cur = cur * a % m;
    }
    while(on * on <= m) {
        cur = cur * a % m;
        on++;
    }
    map<ll, ll> position;
    for(ll i = 0, x = 1; i * i <= m; i++) {
        position[x] = i * on;
        x = x * cur % m;
    }
    for(ll i = 0; i <= on + 20; i++) {
        if(position.count(b)) {
            return position[b] - i;
        }
        b = b * a % m;
    }
    return -1;
}
```

4.2 GCD - Greatest Common Divisor

```
ll gcd(ll a, ll b) {
    while(b) a %= b, swap(a, b);
    return a;
}
```

4.3 Extended Euclides

```
// euclides estendido: acha u e v da equacao:
// u * x + v * y = gcd(x, y);
// u eh inverso modular de x no modulo y
// v eh inverso modular de y no modulo x

pair<ll, ll> euclides(ll a, ll b) {
    ll u = 0, oldu = 1, v = 1, oldv = 0;
    while(b) {
        ll q = a / b;
        oldv = oldv - v * q;
        oldu = oldu - u * q;
        a = a - b * q;
        swap(a, b);
        swap(u, oldu);
        swap(v, oldv);
    }
    return make_pair(oldu, oldv);
}
```

4.4 Fast Exponentiation

```
const ll mod = 1e9+7;

ll fExp(ll a, ll b) {
    ll ans = 1;
    while(b) {
```

```
        if(b & 1) ans = ans * a % mod;
        a = a * a % mod;
        b >>= 1;
    }
    return ans;
}
```

4.5 Matrix Fast Exponentiation

```
const ll mod = 1e9+7;
const int m = 2; // size of matrix

struct Matrix {
    ll mat[m][m];
    Matrix operator * (const Matrix &p) {
        Matrix ans;
        for(int i = 0; i < m; i++)
            for(int j = 0; j < m; j++)
                for(int k = 0; k < m; k++)
                    ans.mat[i][j] = (ans.mat[i][j] + mat[i][k] * p.mat[k][j]) % mod;
        return ans;
    }
};

Matrix fExp(Matrix a, ll b) {
    Matrix ans;
    for(int i = 0; i < m; i++) for(int j = 0; j < m; j++)
        ans.mat[i][j] = i == j;
    while(b) {
        if(b & 1) ans = ans * a;
        a = a * a;
        b >>= 1;
    }
    return ans;
}
```

4.6 FFT - Fast Fourier Transform

```
typedef complex<double> Complex;
typedef long double ld;
typedef long long ll;

const int ms = 2*(maxn);
const ld pi = acos(-1.0);

int rbit[1 << 23];
Complex a[ms], b[ms];

void calcReversedBits(int n) {
    int lg = 0;
    while(1 << (lg + 1) < n) {
        lg++;
    }
    for(int i = 1; i < n; i++) {
        rbit[i] = (rbit[i >> 1] >> 1) | ((i & 1) << lg);
    }
}

void fft(Complex a[], int n, bool inv = false) {
    for(int i = 0; i < n; i++) {
        if(rbit[i] > i) swap(a[i], a[rbit[i]]);
    }
    double ang = inv ? -pi : pi;
    for(int m = 1; m < n; m += m) {
        Complex d(cos(ang/m), sin(ang/m));
        for(int i = 0; i < n; i += m+m) {
            Complex cur = 1;
            for(int j = 0; j < m; j++) {
                Complex u = a[i + j], v = a[i + j + m] * cur;
                a[i + j] = u + v;
```

```

        a[i + j + m] = u - v;
        cur += d;
    }
}

if(inv) {
    for(int i = 0; i < n; i++) a[i] /= n;
}

void multiply(ll x[], ll y[], ll ans[], int nx, int ny, int &n) {
    n = nx + ny;
    calcReversedBits(n);
    for(int i = 0; i < nx; i++) {
        a[i] = Complex(x[i]);
    }
    for(int i = 0; i < ny; i++) {
        b[i] = Complex(y[i]);
    }
    fft(a, n); fft(b, n);
    for(int i = 0; i < n; i++) {
        a[i] = a[i] * b[i];
    }
    fft(a, n, true);
    for(int i = 0; i < n; i++) {
        ans[i] = ll(a[i].real() + 0.5);
    }
}

```

5 Geometry

5.1 Geometry

```

const double inf = 1e100, eps = 1e-9;

struct PT {
    double x, y;
    PT(double x = 0, double y = 0) : x(x), y(y) {}
    PT operator + (const PT &p) { return PT(x + p.x, y + p.y); }
    PT operator - (const PT &p) { return PT(x - p.x, y - p.y); }
    PT operator * (double c) { return PT(x * c, y * c); }
    PT operator / (double c) { return PT(x / c, y / c); }
    bool operator <(const PT &p) const {
        if(fabs(x - p.x) >= eps) return x < p.x;
        return fabs(y - p.y) >= eps && y < p.y;
    }
    bool operator ==(const PT &p) const {
        return fabs(x - p.x) < eps && fabs(y - p.y) < eps;
    }
};

double dot(PT p, PT q) { return p.x * q.x + p.y * q.y; }
double dist2(PT p, PT q) { return dot(p - q, p - q); }
double dist(PT p, PT q) { return hypot(p.x - q.x, p.y - q.y); }
double cross(PT p, PT q) { return p.x * q.y - p.y * q.x; }

// Rotaciona o ponto CCW ou CW ao redor da origem
PT rotateCCW90(PT p) { return PT(-p.y, p.x); }
PT rotateCW90(PT p) { return PT(p.y, -p.x); }
PT rotateCCW(PT p, double d) {
    return PT(p.x * cos(t) - p.y * sin(t), p.x * sin(t) + p.y * cos(t));
}

// Projeta ponto c na linha a - b assumindo a != b
PT projectPointLine(PT a, PT b, PT c) {
    return a + (b - a) * dot(c - a, b - a) / dot(b - a, b - a);
}

// Projeta ponto c no segmento a - b
PT projectPointSegment(PT a, PT b, PT c) {
    double r = dot(b - a, b - a);
    if(fabs(r) < eps) return a;
    r = dot(c - a, b - a) / r;

```

```

    if(r < 0) return a;
    if(r > 1) return b;
    return a + (b - a) * r;
}

// Calcula distancia entre o ponto c e o segmento a - b
double distancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, projectPointSegment(a, b, c)));
}

// Calcula distancia entre o ponto (x, y, z) e o plano ax + by + cz = d
double distancePointPlane(double x, double y, double z, double a, double b, double c, double d) {
    return abs(a * x + b * y + c * z - d) / sqrt(a * a + b * b + c * c);
}

// Determina se as linhas a - b e c - d sao paralelas ou colineares
bool linesParallel(PT a, PT b, PT c, PT d) {
    return abs(cross(b - a, c - d)) < eps;
}

bool linesCollinear(PT a, PT b, PT c, PT d) {
    return linesParallel(a, b, c, d) && abs(cross(a - b, a - c)) < eps && abs(cross(c - d, c - a)) < eps;
}

// Determina se o segmento a - b intersecta com o segmento c - d
bool segmentsIntersect(PT a, PT b, PT c, PT d) {
    if(linesCollinear(a, b, c, d)) {
        if(dist2(a, c) < eps || dist2(a, d) < eps || dist2(b, c) < eps || dist2(b, d) < eps)
            return true;
        if(dot(c - a, c - b) > 0 && dot(d - a, d - b) > 0 && dot(c - b, d - b) > 0) return
            false;
        return true;
    }
    if(cross(d - a, b - a) * cross(c - a, b - a) > 0) return false;
    if(cross(a - c, d - c) * cross(b - c, d - c) > 0) return false;
    return true;
}

// Calcula a intersecao entre as retas a - b e c - d assumindo que uma unica intersecao
// existe
// Para intersecao de segmentos, cheque primeiro se os segmentos se intersectam e que nao
// paralelos
PT computeLineIntersection(PT a, PT b, PT c, PT d) {
    b = b - a; d = d - c; c = c - a;
    assert(cross(b, d) != 0); // garante que as retas nao sao paralelas, remover pra evitar
    tle
    return a + b * cross(c, d) / cross(b, d);
}

// Calcula centro do circulo dado tres pontos
PT computeCircleCenter(PT a, PT b, PT c) {
    b = (a + b) / 2;
    c = (a + c) / 2;
    return computeLineIntersection(b, b + rotateCW90(a - b), c, c + rotateCW90(a - c));
}

// Determina se o ponto esta num poligono possivelmente nao-convexo
// Retorna 1 para pontos estritamente dentro, 0 para pontos estritamente fora do poligono
// e 0 ou 1 para os pontos restantes
// Eh possivel converter num teste exato usando inteiros e tomando cuidado com a divisao
// e entao usar testes exatos para checar se esta na borda do poligono
bool pointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i + 1) % p.size();
        if((p[i].y <= q.y && q.y < p[j].y || p[j].y <= q.y && q.y < p[i].y) &&
            q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
            c = !c;
    }
    return c;
}

// Determina se o ponto esta na borda do poligono
bool pointOnPolygon(const vector<PT> &p, PT q) {
    for(int i = 0; i < p.size(); i++)
        if(dist2(projectPointSegment(p[i], p[(i + 1) % p.size()], q), q) < eps)
            return true;
}

```

```

    return false;
}

// Calcula intersecao da linha a - b com o circulo centrado em c com raio r > 0
vector<PT> circleLineIntersection(PT a, PT b, PT c, double r) {
    vector<PT> ans;
    b = b - a;
    a = a - c;
    double x = dot(b, b);
    double y = dot(a, b);
    double z = dot(a, a) - r * r;
    double w = y * y - x * z;
    if (w < -eps) return ans;
    ans.push_back(c + a + b * (-y + sqrt(w + eps)) / x);
    if (w > eps)
        ans.push_back(c + a + b * (-y - sqrt(w)) / x);
    return ans;
}

// Calcula intersecao do circulo centrado em a com raio r e o centrado em b com raio R
vector<PT> circleCircleIntersection(PT a, PT b, double r, double R) {
    vector<PT> ans;
    double d = sqrt(dist2(a, b));
    if (d > r + R || d + min(r, R) < max(r, R)) return ans;
    double x = (d * d - R * R + r * r) / (2 * d);
    double y = sqrt(r * r - x * x);
    PT v = (b - a) / d;
    ans.push_back(a + v * x + rotateCCW90(v) * y);
    if (y > 0)
        ans.push_back(a + v * x - rotateCCW90(v) * y);
    return ans;
}

// Calcula a area ou o centroide de um poligono (possivelmente nao-convexo)
// assumindo que as coordenadas estao listada em ordem horaria ou anti-horaria
// O centroide eh equivalente a o centro de massa ou centro de gravidade
double computeSignedArea(const vector<PT> &p) {
    double area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i + 1) % p.size();
        area += p[i].x * p[j].y - p[j].x * p[i].y;
    }
    return area / 2.0;
}

double computeArea(const vector<PT> &p) {
    return abs(computeSignedArea(p));
}

PT computeCentroid(const vector<PT> &p) {
    PT c(0,0);
    double scale = 6.0 * ComputeSignedArea(p);
    for(int i = 0; i < p.size(); i++) {
        int j = (i + 1) % p.size();
        c = c + (p[i] + p[j]) * (p[i].x * p[j].y - p[j].x * p[i].y);
    }
    return c / scale;
}

// Testa se o poligono listada em ordem CW ou CCW eh simples (nenhuma linha se intersecta)
bool isSimple(const vector<PT> &p) {
    for(int i = 0; i < p.size(); i++) {
        for(int k = i + 1; k < p.size(); k++) {
            int j = (i + 1) % p.size();
            int l = (k + 1) % p.size();
            if (i == l || j == k) continue;
            if (segmentsIntersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

```

5.2 Convex Hull

```
vector<PT> convexHull(vector<PT> p) {
```

```

    int n = p.size(), k = 0;
    vector<PT> h(2 * n);
    sort(p.begin(), p.end());
    for(int i = 0; i < n; i++) {
        while(k >= 2 && cross(h[k - 1] - h[k - 2], p[i] - h[k - 2]) <= 0) k--;
        h[k++] = p[i];
    }
    for(int i = n - 2, t = k + 1; i >= 0; i--) {
        while(k >= t && cross(h[k - 1] - h[k - 2], p[i] - h[k - 2]) <= 0) k--;
        h[k++] = p[i];
    }
    h.resize(k);
    return h;
}

```

5.3 ClosestPair

```

double closestPair(vector<PT> p) {
    int n = p.size(), k = 0;
    sort(p.begin(), p.end());
    double d = inf;
    set<PT> ptsInv;
    for(int i = 0; i < n; i++) {
        while(k < i && p[k].x < p[i].x - d) {
            ptsInv.erase(swapCoord(p[k++]));
        }
        for(auto it = ptsInv.lower_bound(PT(p[i].y - d, p[i].x - d));
            it != ptsInv.end() && it->x <= p[i].y + d; it++) {
            d = min(d, !p[i] - swapCoord(*it));
        }
        ptsInv.insert(swapCoord(p[i]));
    }
    return d;
}

```

5.4 Intersection Points

```

// Utiliza uma seg tree

int intersectionPoints(vector<pair<PT, PT>> v) {
    int n = v.size();
    vector<pair<int, int>> events, vertInt;
    for(int i = 0; i < n; i++) {
        if(v.first.x == v.second.x) { // Segmento Vertical
            int y0 = min(v.first.y, v.second.y), y1 = max(v.first.y, v.second.y);
            events.push_back({v.first.x, vertInt.size()}); // Tipo = Indice no array
            vertInt.push_back({y0, y1});
        } else { // Segmento Horizontal
            int x0 = min(v.first.x, v.second.x), x1 = max(v.first.x, v.second.x);
            events.push_back({x0, -1}); // Inicio de Segmento
            events.push_back({x1, inf}); // Final de Segmento
        }
    }
    sort(events.begin(), events.end());
    int ans = 0;
    for(int i = 0; i < events.size(); i++) {
        int t = events[i].second;
        if(t == -1) {
            segUpdate(events[i].first, 1);
        } else if(t == inf) {
            segUpdate(events[i].first, 0);
        } else {
            ans += segQuery(vertInt[t].first, vertInt[t].second);
        }
    }
    return ans;
}

```

5.5 Maximum Segments Overlap (Sweep Line)


```
int maxSegOverlap(vector<pair<int, int>> v) {
    int n = v.size();
    vector<pair<int, int>> events(n * 2);
    // tipo 1 = Inicio de segmento
    // tipo 0 = Fim de segmento
    for(int i = 0; i < n; i++) {
        events[i*2] = {v[i].first, 1};
        events[i*2+1] = {v[i].second, 0};
    }
    sort(events.begin(), events.end());
    int qnt = 0, ans = 0;
    for(int i = 0; i < 2 * n; i++) {
        if(events[i].second) {
            ans = max(++qnt, ans);
        } else {
            --qnt;
        }
    }
    return ans;
}
```

6 Miscellaneous

6.1 LIS - Longest Increasing Subsequence

```
int arr[ms], lisArr[ms], n;
// int bef[ms], pos[ms];

int lis() {
    int len = 1;
    lisArr[0] = arr[0];
    // bef[0] = -1;
    for(int i = 1; i < n; i++) {
        // upper_bound se non-decreasing
        int x = lower_bound(lisArr, lisArr + len, arr[i]) - lisArr;
        len = max(len, x + 1);
        lisArr[x] = arr[i];
        // pos[x] = i;
        // bef[i] = x ? pos[x-1] : -1;
    }
    return len;
}

vi getLis() {
    int len = lis();
    vi ans;
    for(int i = pos[lisArr[len - 1]]; i >= 0; i = bef[i]) {
        ans.push_back(arr[i]);
    }
    reverse(ans.begin(), ans.end());
    return ans;
}
```

6.2 Binary Search

```
int smallestSolution() {
    int x = -1;
    for(int b = z; b >= 1; b /= 2) {
        while(!ok(x+b)) x += b;
    }
    return x + 1;
}

int maximumValue() {
    int x = -1;
    for(int b = z; b >= 1; b /= 2) {
        while(f(x+b) < f(x+b+1)) x += b;
    }
    return x + 1;
}
```

6.3 Ternary Search

```
// R
for(int i = 0; i < LOG; i++){
    long double m1 = (A * 2 + B) / 3.0;
    long double m2 = (A + 2 * B) / 3.0;

    if(f(m1) > f(m2))
        A = m1;
    else
        B = m2;
}
ans = f(A);

// Z
while(B - A > 4){
    int m1 = (A + B) / 2;
    int m2 = (A + B) / 2 + 1;
    if(f(m1) > f(m2))
        A = m1;
    else
        B = m2;
}
ans = inf;
for(int i = A; i <= B; i++) ans = min(ans, f(i));
```

7 Teoremas e formulas uteis

7.1 Grafos

Formula de Euler: $V - E + F = 2$ (para grafo planar)
 Handshaking: Numero par de vertices tem grau impar
 Kirchhoff's Theorem: Monta matriz onde $M_{i,i} = \text{Grau}[i]$ e $M_{i,j} = -1$ se houver aresta $i-j$ ou 0
 caso contrario, remove uma linha e uma coluna qualquer e o numero de spanning trees
 nesse grafo eh o det da matriz

Grafo contem caminho hamiltoniano se:
 Dirac's theorem: Se o grau de cada vertice for pelo menos $n/2$
 Ore's theorem: Se a soma dos graus que cada par nao-adjacente de vertices for pelo menos n

Trees:
 Tem Catalan(N) Binary trees de N vertices
 Tem Catalan(N-1) Arvores enraizadas com N vertices
 Caley Formula: n^{n-2} arvores em N vertices com label
 Prufer code: Cada etapa voce remove a folha com menor label e o label do vizinho eh
 adicionado ao codigo ate ter 2 vertices

Flow:
 Max Edge-disjoint paths: Max flow com arestas com peso 1
 Max Node-disjoint paths: Faz a mesma coisa mas separa cada vertice em um com as arestas de
 chegadas e um com as arestas de saida e uma aresta de peso 1 conectando o vertice com
 aresta de chegada com ele mesmo com arestas de saida
 Konig's Theorem: minimum node cover = maximum matching se o grafo for bipartido
 Min Node disjoint path cover: formar grafo bipartido de vertices duplicados, onde aresta sai
 do vertice tipo A e chega em tipo B, entao o path cover eh $N - \text{matching}$
 Min General path cover: Mesma coisa mas colocando arestas de A pra B sempre que houver
 caminho de A pra B
 Dilworth's Theorem: Min General Path cover = Max Antichain (set de vertices tal que nao
 existe caminho no grafo entre vertices desse set)
 Hall's marriage: um grafo tem um matching completo do lado X se para cada subconjunto W de X,
 $|W| \leq |\text{vizinhosW}|$ onde $|\text{W}|$ eh quantos vertices tem em W

7.2 Math

Goldbach's: todo numero par $n > 2$ pode ser representado com $n = a + b$ onde a e b sao primos
 Twin prime: existem infinitos pares $p, p + 2$ onde ambos sao primos
 Legendre's: sempre tem um primo entre n^2 e $(n+1)^2$
 Lagrange's: todo numero inteiro pode ser inscrito como a soma de 4 quadrados

Zeckendorf's: todo numero pode ser representado pela soma de dois numeros de fibonnacis diferentes e nao consecutivos
 Euclid's: toda tripla de pitagoras primitiva pode ser gerada com
 $(n^2 - m^2, 2nm, n^2 + m^2)$ onde n, m sao coprimos e um deles eh par
 Wilson's: n eh primo quando $(n-1)! \bmod n = n - 1$
 Mcnugget: Para dois coprimos x, y o maior inteiro que nao pode ser escrito como $ax + by$ eh $(x-1)(y-1)/2$

Fermat: Se p eh primo entao $a^{p-1} \equiv 1 \pmod{p}$
 Se x e m tambem forem coprimos entao $x^k \equiv 1 \pmod{m}$ se $k \equiv 0 \pmod{\phi(m)}$
 Euler's theorem: $x^{\phi(m)} \equiv 1 \pmod{m}$ onde $\phi(m)$ eh o totiente de euler

Chinese remainder theorem:
 Para equacoes no formato $x \equiv a_1 \pmod{m_1}, \dots, x \equiv a_n \pmod{m_n}$ onde todos os pares m_1, \dots, m_n sao coprimos
 Deixe $X_k = m_1 m_2 \dots m_n / m_k$ e $X_k^{-1} \pmod{m_k}$ = inverso de $X_k \pmod{m_k}$, entao
 $x = \text{somatorio com } k \text{ de } 1 \text{ ate } n \text{ de } a_k X_k X_k^{-1} \pmod{m_k}$
 Para achar outra solucao so somar $m_1 m_2 \dots m_n$ a solucao existente

Catalan number: exemplo expressoes de parenteses bem formadas
 $C_0 = 1, C_n = \text{somatorio de } i=0 \rightarrow n-1 \text{ de } C_i C_{n-1-i}$
 outra forma: $C_n = \frac{(2n)!}{(n+1)! n!}$
 Bertrand's ballot theorem: p votos tipo A e q votos tipo B com $p > q$, prob de em todo ponto ter mais As do que Bs antes dele = $(p-q)/(p+q)$
 Se puder empates entao prob = $(p+1-q)/(p+1)$, para achar quantidade de possibilidades nos dois casos basta multiplicar por $(p+q)$

Hockey-stick: Somatorio de $i = r \rightarrow n$ de $\binom{i}{r} = \binom{n+1}{r+1}$

Vandermonde: $\sum_{k=0}^m \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}$

Burnside lemma: colares diferentes nao contando rotacoes quando m = cores e n = comprimento
 $(m^n + \text{somatorio } i=1 \rightarrow n-1 \text{ de } m^{\gcd(i, n)})/n$

Distribuicao uniforme $a, a+1, \dots, b$ Expected[X] = $(a+b)/2$
 Distribuicao binomial com n tentativas de probabilidade p , X = sucessos:
 $P(X = x) = p^x (1-p)^{n-x} \binom{n}{x}$ e $E[X] = np$
 Distribuicao geometrica onde continuamos ate ter sucesso, X = tentativas:
 $P(X = x) = (1-p)^{x-1} p$ e $E[X] = 1/p$

7.3 Geometry

Formula de Euler: $V - E + F = 2$
 Pick Theorem: Para achar pontos em coords inteiras num poligono Area = $i + b/2 - 1$ onde i eh o o numero de pontos dentro do poligono e b de pontos no perimetro do poligono
 Two ears theorem: Todo poligono simples com mais de 3 vertices tem pelo menos 2 orelhas, vertices que podem ser removidos sem criar um crossing, remover orelhas repetidamente triangula o poligono
 Incentro triangulo: $(a(X_a, Y_a) + b(X_b, Y_b) + c(X_c, Y_c))/(a+b+c)$ onde a = lado oposto ao vertice a , incentro eh onde cruzam as bissetrizes, eh o centro da circunferencia inscrita e eh equidistante aos lados
