MATH 330 – HW #29

Cristobal Forno

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Question: Let $z = x + yi$ be a complex number, where $x, y \in \mathbb{R}$.
(a) $|\bar{z}| = |z|$.

Proof: We have the equation $|\bar{z}| = |z|$. If we square both sides, we have $|\bar{z}|^2 = |z|^2$. This can be computed as $|x - yi|^2 = |x + yi|^2$, where $x, y \in \mathbb{R}$. By Proposition C.9 (iii),

$$\begin{aligned}
|x + yi|^2 &= (x + yi)(x - yi) && (1)\\
&= x^2 - (yi)^2 && (2)\\
&= |x|^2 - |yi|^2 && (3)\\
&= |x - yi|^2 && (4)
\end{aligned}$$

(3) is computed by Proposition 10.6. Since $|x - yi| = |\bar{z}|$, we have prove that $|z| = |\bar{z}|$. $\square$
(b) If $z \neq 0$, then $z^{-1} = \frac{\bar{z}}{|z|^2}$

Proof: Assume $z \neq 0$, where $z \in \mathbb{C}$. Let $z = x + yi$, where $x, y \in \mathbb{R}$.Then there exists an inverse of z, z^{-1}.

$$z^{-1} = \frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{\bar{z}}{x^2+y^2} = \frac{\bar{z}}{x^2+(-1)y^2} = \frac{\bar{z}}{x^2-y^2i^2} = \frac{\bar{z}}{z^2} = \frac{\bar{z}}{|z|^2}$$

The last equality is computed from Proposition 10.6. $\square$