MATH 330 – HW #25 Revision Part 2

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Proposition 11.7: The rational number $\frac{m}{n} \in \mathbb{Q}$ is positive if and only if either $m > 0$ and $n > 0$, or $m < 0$ and $n < 0$.

Proof: Assume that $m > 0$ and $n > 0$, where $m, n \in \mathbb{Z}$. Proposition 8.40(i) implies that n, $\frac{1}{n}$, m, and $\frac{1}{m}$ all have the same sign. Since both m and n are positive, then $\frac{m}{n} = m \cdot \frac{1}{n} > 0$, by Axiom 8.26(ii).

If $m < 0$ and $n < 0$, then $-m > 0$ and $-n > 0$. Which implies $-\frac{1}{n} > 0$. Since $-m > 0$ and $-\frac{1}{n} > 0$, the multiplication of two negative real numbers is positive, so $\frac{m}{n}$ is positive.

Conversely, suppose that m and n have opposite signs. Again, Proposition 8.40(i) implies that n and $\frac{1}{n}$ have the same sign, so m and $\frac{1}{n}$ have opposite signs. If $m > 0$ and $\frac{1}{n} < 0$, then $\frac{m}{n} < 0$ and if $m < 0$ and $\frac{1}{n} > 0$, then $\frac{m}{n} < 0$, both by Prop. 8.32(iii). In both cases, $\frac{m}{n}$ is negative. $\square$