Math 330 – HW #23

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Project 10.20: Prove the analogous statement for decreasing bounded sequences. In summary, we then know that every monotonic bounded sequence converges.

Proof: We will prove that every decreasing bounded sequence converges. Suppose $(x_k)_{k=1}^{\infty}$ is a sequence that is decreasing and bounded and let A be a set. Then the set $A = \{x_k : k \in \mathbb{N}\} \subseteq \mathbb{R}$ is bounded. Thus, by Proposition 8.50, it has a greatest lower bound s. We will show that $\lim_{k\to\infty} x_k = s$.

Let $\varepsilon > 0$. Then $s + \varepsilon > s$, and so $s + \varepsilon$ is not a lower bound for A. Thus, there exists some $M \in \mathbb{N}$ such that $x_M < s + \varepsilon$. Since the sequence is decreasing, we have $x_m \leq x_M$ for all $m \geq M$. Therefore, for $m \geq M$, we have $s - \varepsilon < s \leq x_m \leq x_M < s + \varepsilon$, and so $|x_m - s| < \varepsilon$ by Proposition 10.8 (v).$\square$