MATH 330 – HW #28 Revision

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December 6, 2017

Proposition 13.10: A subset of a countable set is countable.
Proof: Assume $B \subseteq C$, where C is a countable set.

If $B = \emptyset$, then B is obviously countable.

If B is nonempty, then there exists a surjection $\mathbb{N} \to C$, by Prop. 13.9. The surjection $\mathbb{N} \to C$ can compose the surjection $C \to B$, $x \mapsto \begin{cases} x & \text{if } x \in B \\ b_0 & \text{otherwise,} \end{cases}$
for a fixed $b_0 \in B$. This composition give a surjection $\mathbb{N} \to B$, and by Prop. 13.9, B is countable. $\square$