

# MATH 330 – HW #25

Cristobal Forno

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**Proposition 11.7:** The rational number  $\frac{m}{n} \in \mathbb{Q}$  is positive if and only if either  $m > 0$  and  $n > 0$ , or  $m < 0$  and  $n < 0$ .

**Proof:** Assume that  $m > 0$  and  $n > 0$ , where  $m, n \in \mathbb{Z}$ . Proposition 8.40(i) implies that  $n$ ,  $\frac{1}{n}$ ,  $m$ , and  $\frac{1}{m}$  all have the same sign. Since both  $m$  and  $n$  are positive, then  $\frac{m}{n} = m \cdot \frac{1}{n} > 0$ , by Axiom 8.26(ii).

If  $m < 0$  and  $n < 0$ , then  $-m > 0$  and  $-n > 0$ . Which implies  $-\frac{1}{n} > 0$ . Since  $-m > 0$  and  $-\frac{1}{n} > 0$ , the multiplication of two negative real numbers is positive, so  $\frac{m}{n}$  is positive.

Conversely, suppose that  $m$  and  $n$  have opposite signs. Again, Proposition 8.40(i) implies that  $n$  and  $\frac{1}{n}$  have the same sign, so  $m$  and  $\frac{1}{n}$  have opposite signs. If  $m > 0$  and  $\frac{1}{n} < 0$ , then  $\frac{m}{n} < 0$  and if  $m < 0$  and  $\frac{1}{n} > 0$ , then  $\frac{m}{n} < 0$ , both by Prop. 8.32(iii). In both cases,  $\frac{m}{n}$  is negative.  $\square$