MATH 330 – HW #22

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Proposition 10.21: Let $L = \lim_{k\to\infty} x_k$.
(i) If $(x_k)_{k=1}^{\infty}$ is increasing, then $x_k \leq L$ for all $k \in \mathbb{N}$.
Proof:

Assume $(x_n)_{k=1}^{\infty}$ is increasing and $\lim_{k\to\infty} x_k = L$. We will prove by contradction. Assume there exists $m \in \mathbb{N}$ such that $x_m > L$. Let $\varepsilon = x_m - L$. Then for any $M \in \mathbb{N}$, let k = $\max\{M, m\}$. So, in particular, $k \leq M$. Since the sequence is increasing, we have $x_k \leq x_n > L$. But then $|x_k - L| = x_k - L \geq x_n - L = \varepsilon$. Thus, the sequence $(x_k)_{k=1}^{\infty}$ does not converge to L, which is a contradction.

(i) If $(x_n)_{k=1}^{\infty}$ is decreasing, then $x_k \geq L$ for all $k \in \mathbb{N}$.
Proof:

Assume $(x_n)_{k=1}^{\infty}$ is decreasing and $\lim_{k\to\infty} x_k = L$. We will prove by contradiction. Assume there exists $m \in \mathbb{N}$ such that