MATH 330 – HW #22 Revision

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November 28, 2017

Proposition 10.21: Let $L = \lim_{k\to\infty} x_k$.
(i) If $(x_k)_{k=1}^{\infty}$ is increasing, then $x_k \leq L$ for all $k \geq 0$.
Proof:

Assume $(x_k)_{k=1}^{\infty}$ is increasing and $L = \lim_{k\to\infty} x_k$. We will prove by contradiction. Assume there exists $m \in \mathbb{N}$ such that $x_m > L$. Let $\varepsilon = x_m - L$. Then for any $M \in \mathbb{N}$ such that $m \geq M$, let k = $\max\{M, m\}$. So, in particular, $k \geq M$. Since the sequence is increasing, we have $x_k \leq x_m > L$. But then $|x_k - L| = x_k - L \geq x_m - L = \varepsilon$. Thus, the sequence $(x_k)_{k=1}^{\infty}$ does not converge to L, then $L \neq \lim_{k\to\infty} x_k$ (implied by Proposition 10.16), which is a contradiction. $\square$

(ii) If $(x_k)_{k=1}^{\infty}$ is decreasing, then $x_k \geq L$ for all $k \geq 0$.
Proof:

Assume $(x_k)_{k=1}^{\infty}$ is decreasing and $L = \lim_{k\to\infty} x_k$. We will prove by contradiction. Assume there exists $m \in \mathbb{N}$ such that $x_m < L$. Let $\varepsilon = L + x_m$. Then for any $M \in \mathbb{N}$ such that $m \geq M$, let k = $\max\{M, m\}$. So, in particular, $k \leq M$. Since the sequence is decreasing, we have $x_k \geq x_m < L$. But then $|L + x_k| = L + x_k \leq L + x_m = \varepsilon$. Thus, the sequence $(x_k)_{k=1}^{\infty}$ does not converge to L, then $L \neq \lim_{k\to\infty} x_k$ (implied by Proposition 10.16), which is a contradiction. $\square$