

MATH 330 – HW #28

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Proposition 13.10: A subset of a countable set is countable.

Proof: Assume $B \subseteq C$, where C is a countable set.

If $B = \emptyset$, then B is obviously countable.

If B is nonempty, then there exists a surjection $\mathbb{N} \rightarrow B$, by Prop. 13.9. The surjection $\mathbb{N} \rightarrow B$ can compose the surjection $B \rightarrow A$, $x \mapsto \begin{cases} x & \text{if } x \in A \\ a_0 & \text{otherwise,} \end{cases}$ for a fixed $a_0 \in A$. This composition give a surjection $\mathbb{N} \rightarrow A$, and by Prop. 13.9, A is countable. \square