MATH 330 – HW #21

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Let A be a subset of $\mathbb{R}$. We define $-A = \{-a : a \in A\}$.
Question Let A be bounded subset of $\mathbb{R}$. Prove that
(1) $\inf(-A) = -\sup(A)$

Proof: We want to prove $\inf(-A) \leq -\sup(A)$ **(a)** and $\inf(-A) \geq -\sup(A)$ **(b)** then it implies that $\inf(-A) = -\sup(A)$.

(a) We know the $\inf(-A) \leq$ for all $-a \in -A$. Therefore, $-\inf(-A)$ $\geq a$. In other words, $-\inf(-A)$ is an upper bound for A. Hence, $\sup(A) \leq$ $-\inf(-A)$, which is equivalent to, $\inf(-A) \leq -\sup(A)$.

(b) Since A is bounded above, then $a \leq \sup(A)$. We can compute that, $-a \geq -\sup(A)$. Therefore, $-\sup(A)$ is a lower bound for $-A$ which implies that $\inf(-A) \geq -\sup(A)$.

Finally, since $\inf(-A) \leq -\sup(A)$ and $\inf(-A) \geq -\sup(A)$ implies that $\inf(-A) = -\sup(A)$. $\square$

(2) $\sup(-A) = -\inf(A)$

Proof: We want to prove $\sup(-A) \leq -\inf(A)$ **(a)** and $\sup(-A) \geq$ $-\inf(A)$ **(b)** then it implies that $\inf(-A) = -\sup(A)$.

(a) We know the $\sup(-A) \geq$ for all $-a \in -A$. Therefore, $-\sup(-A)$ $\leq a$. In other words, $-\sup(-A)$ is a lower bound for A. Hence, $-\sup(-A)$ $\leq \inf(A)$, which is equivalent to, $\sup(-A) \leq -\inf(A)$.

(b) Since A is bounded below, then $\inf(A) \leq$ for all $a \in A$. We can compute that, $-\inf(A) \geq -a$. Therefore, $-\inf(A)$ is an upperbound for $-A$ which implies that $\sup(-A) \geq -\inf(A)$.

Finally, since **(a)** and **(b)** hold, then both statements imply that $\sup(-A)$ $= -\inf(A)$. $\square$