MATH 330 – HW #18

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Proposition 9.11: If a function is bijective then its inverse is unique.
Proof:
Let f: be a bijective function from $A \to B$. Suppose g_1 and g_2 are both inverses to f. Then,

$$g_1 = g_1 \circ i_B = g_2 \circ (f \circ g_2) = (g_1 \circ f) \circ g_2 = i_A \circ g_2 = g_2,$$

proving that there is only one unique inverse for a function. $\square$

Proposition 9.12: Let A and B be sets. There exists an injection from $A \to B$ if and only if there exists a surjection from $B \to A$.
Proof:
We want to prove if there exists an injection from $A \to B$, then there exits a surjection from $B \to A$ and if there exits a surjection from $B \to A$, then there exits an injection from $A \to B$. Suppose $f : A \to B$ is an injection. Then by Proposition 9.10 (i), f has a left inverse $g : B \to A$. So,

$$g \circ f = id_A$$

This implies that g has a right inverse, and thus g is surjective by Proposition 9.10 (ii). Similary, if $g : B \to A$ is surjective, then g has a right inverse $f : A \to B$. Thus, f has a left inverse, f is injective. $\square$