Sound Analysis and Synthesis

DAAP - Homework 1

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The purpose of this script is to estimate the Room Impulse Response (RIR) of a small reverberant environment by applying the Wiener filter to the source signal. The signal must be reconstructed by correctly applying Overlap-and-Add (OLA) algorithm to each filtered frame.

1 Wiener Filters and Steepest Descent

In order to normalize the auto-correlation function and the cross-correlation vector, we tried several techniques and we choose to divide them by the length of the signal because it showed a good *cost function ratio*.

For the Steepest Descent algorithm (computationally more efficient than solving WH equations), we inizialized the tap-weight vector with a null vector, because we didn't have any prior knowledge about the location of the minimum point of the error-performance.

After implementing both the Wiener-Hopf equations and the Steepest Descent algorithm we computed both the theoretical and the actual minimum Mean-squared error. The results are the following:

Theoretical minimum MSE

$$J_{min} = \sigma_d^2 - \mathbf{p}^H \mathbf{R}^{-1} \mathbf{p} = 0.0013$$

Actual minimum MSE

$$J(\mathbf{w}) = J_{min} + (\mathbf{w} - \mathbf{w}_o)^H \mathbf{R} (\mathbf{w} - \mathbf{w}_o) = 0.0013$$

Cost function ratio

$$ratio = \frac{J(\mathbf{w})}{J_{min}} = 1.0068$$

We decided to put μ, the *step-size parameter* in the equation of the SD iterative algorithm equal to $0.95\mu_{max}$, which gives the ratio close to 1, as hoped.

We chose an high value for μ because it's inversely proportional to τ_j, the *global time constant* and so this means taking less time in convergence. The convergence rate depends also on the *smaller eigenvalue* λ_{min}, so in this particular case τ_j is already high because λ_{min} tends to zero.

Otherwise we decided not to have a value equal to μ_{max}, even if it would have taken a bit less in time, because in that case the transient behaviour is underdamped, so the trajectory shows oscillations.

2 Overlap-and-Add

All windows satisfying the COLA condition perfectly reconstruct the original signal, so there is no constraint on the window type, only that the window overlap-adds to a constant for the hop-size used. Since M is even, we can use Hanning window at 75% overlap ($R = M/4$).

3 Results and Conclusions

The algorithms above have been implemented using MATLAB by considering a a real-valued source signal $x(n)$, the signal $y(n)$ captured by the microphone. The RIR $g(n)$ is provided for visual reference only and this is the reason why it is shown in the plot below.

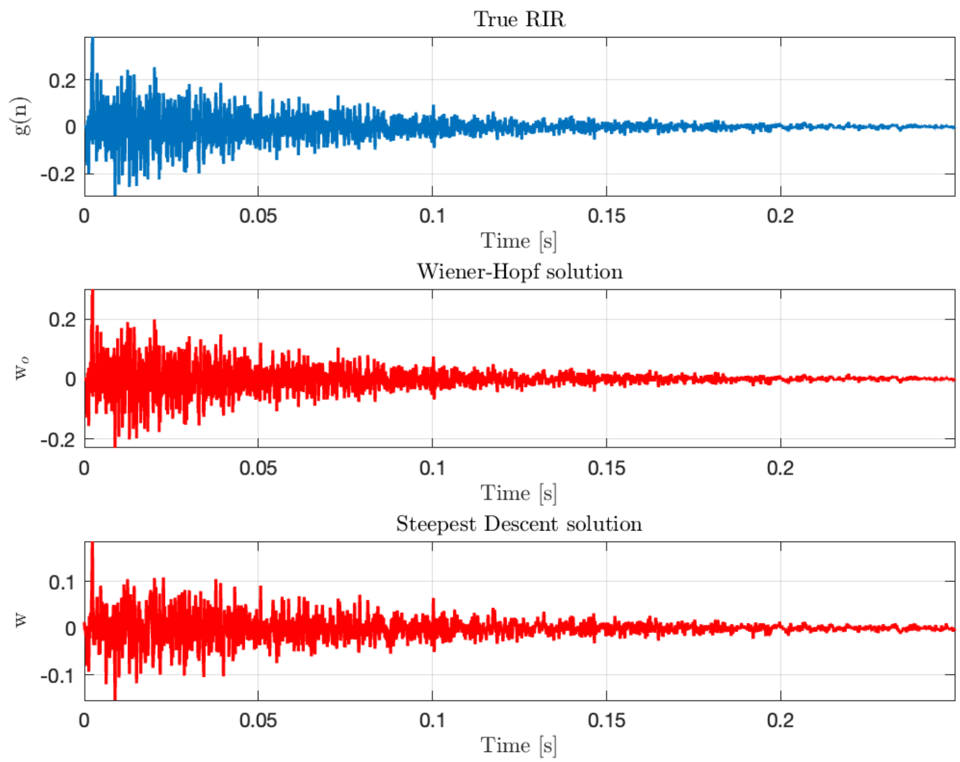

Figure 1: plots for the true RIR and the filter taps obtained via WH solution and SD algorithm

The result analysis from the outputs of wiener filter and SD algorithm shows that Wiener algorithm provides better performance, but it has high degree of complexity. That is because SD algorithm, even if it still requires knowledge of $\mathbf{R}$ and $\mathbf{p}$, is a practical method of obtaining estimates of the filter taps w in real time without matrix inversion in the closed-form WH solution. These estimates improve gradually with time as taps are adjusted and the filter learns the characteristics of the signals.

In conclusion, the simplicity of the SD algorithm makes it better for real-time systems. When the input signal and the desired response are stationary, the method of steepest descent converges to the optimum Wiener solution, but in general it never reaches the theoretical optimum (the Wiener solution) as we can see also in this case.