Introduction to simulation code and related physics – version 1

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1 Physics

Pressure amplitude Notice that sound waves carry energy. We define the intensity I as the rate at which energy E flows through a unit area A perpendicular to the direction of travel of the wave. $I = P/A = E/(At)$ where P is the power of sound. For a point source, energy spreads out in all directions. Area of a sphere is $A = 4\pi r^2$, so $I \propto 1/r^2$. Because $I = \frac{p^2}{2\rho v}$ where ρ is the density of air and v is the speed of sound, $p \propto \sqrt{I}$. As a result, $p \propto 1/r$, which means at a given point that has distance r from the point source, the pressure at that point will be attenuated by r. For harmonic vibrations with time dependence $exp(-i\omega t)$, $p = i\omega\delta_0\vec{V}$, where p is excess pressure (ie, the difference between actual pressure and pressure of the gas at rest), δ_0 is the density of gas at rest, and $\vec{V}$ is the velocity potential, pressure and velocity potential have a linear relationship. Then, $\vec{V} \propto 1/r$, too.

In the problem that we are investigating, assuming the original pressure from the sonar source is p_0. Then the pressure of the signal has been attenuated to p_0/r after it travels r meters and arrives at the leaf. In the past, while we were calculating the pressure amplitude of the leaf beampattern, we assumed the pressure of incoming wave was 1. So now the leaf beampattern should times p_0/r to get the real beampattern. After the leaf reflects the sound, the leaf itself becomes a point source. As a result, what the bat receives at the place of the sonar will be leaf beampattern$/r$. In a word, the amplitude the bat receives at the sonar from ith leaf will be

$$\text{Amp\_i} = S(\theta_i^s, \phi_i^s, frq_i, pos_i)L_i(\theta_{0i}, a_i, frq_i)/(kr_i^2). \tag{1}$$

The choice of time-harmonic fields (with time-dependence factor $e^{-i\omega t}$ omitted throughout) is justified by the fact that this is an important case in practice.

Several concepts

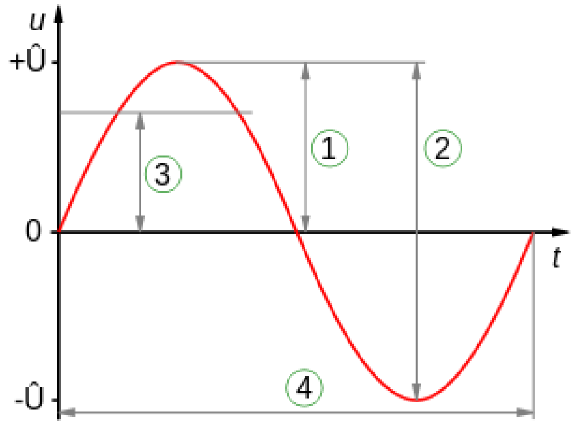


Figure 1: Amplitude of a wave

(1) Sound energy E. Sound energy is a form of energy associated with the vibration or disturbance of matter. The SI unit of sound energy is the joule (J).

(2) Power of sound P. Sound power or acoustic power is the rate at which sound energy is emitted, reflected, transmitted or received, per unit time. The SI unit of sound power is the watt (W). $P = E/t$. It is equivalent to an amount of energy consumed per unit time.

(3) Sound intensity I. Sound intensity or acoustic intensity is defined as the sound power per unit area. $I = P/A$. The SI unit of sound intensity is the watt per square metre (W/m2).

(4) Amplitude of sound. Amplitude is the objective measurement of the degree of change (positive or negative) in atmospheric pressure caused by sound waves. In figure 2, (1), (2), and (3) are both amplitude but with different names. They are peak amplitude, peak-to-peak amplitude, and root-mean-square amplitude, respectively. (4) is wave period.

(5) Gain. In electromagnetics, an antenna's power gain or simply gain is a key performance figure which combines the antenna's directivity and electrical efficiency, where directivity is a figure of merit for an antenna. It measures the power density the antenna radiates in the direction of its strongest emission, versus the power density radiated by an ideal isotropic radiator (which emits uniformly in all directions) radiating the same total power. The diagram showing directivity is shown in figure 2.

(6) Beam. A narrow, propagating stream of particles or energy.

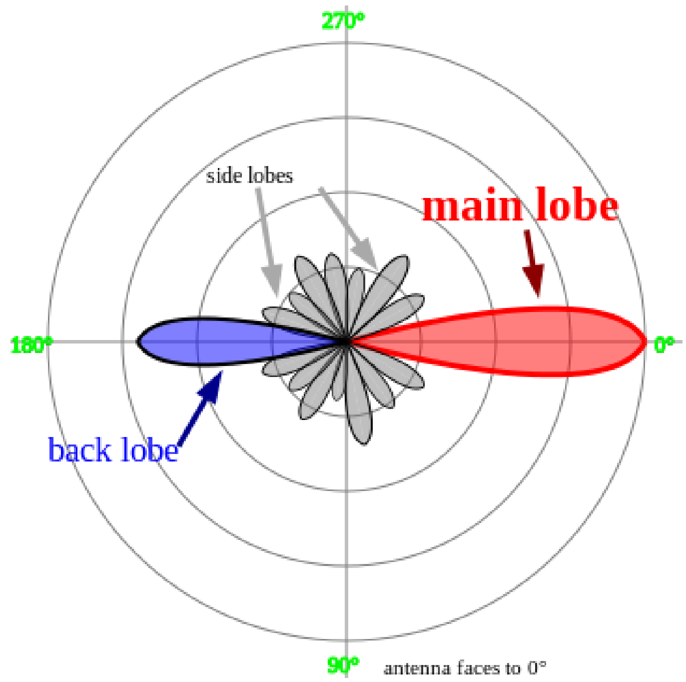


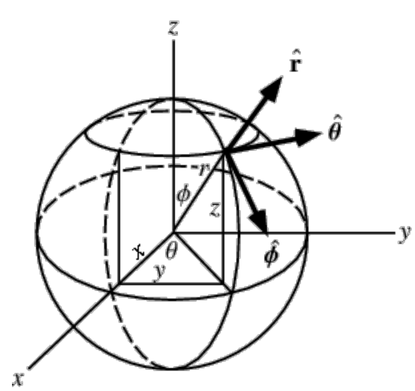
Figure 2: Diagram showing directivity

2 Parameterization

Several assumptions:

(a) There is only one coordinates. The sonar is fixed to the origin of this coordinates, and it emits sound toward the -x axis.

(b) When I refer to elevation and azimuth, they are defined as the figure shows. The elevation starts from z axis. In MATLAB, the elevation starts from x-y plane.



The following general parameterization can be used to describe the parameters in leaf beampattern $L_i(\theta_{0i}, a_i, frq_i)$ in equation (1):

(1) frq_i: the frequency of the sonar beampattern between 60kHz and 80kHz.

(2) θ_{0i}: known parameter: incident angle, which will be decided by the distribution. Now we are using uniform distribution (function rand() in MATLAB) to assign incident angle for each leaf.

(3) a_i: the radius of the leaf

We use subscript i to discriminate each leaf.

The following describes the sonar beampattern $S(\theta_i^s, \phi_i^s, frq_i, pos_i)$

(1) θ^s: the azimuth of the line between the origin(sonar) and the leaf center, and can be derived from the coordinate of leaf center, (x, y, z).

(2) ϕ^s: the elevation of the line between the origin(sonar) and the leaf center, and can be derived from the coordinate of leaf center, (x, y, z). ϕ^s and θ^s both determine the amplitude of the wave which the leaf receives.

(3) frq_i: the frequency of sonar beampattern.

(4) pos_i: the position of each leaf. It represents the coordinates of the leaf center (x, y, z). Thus, the distance r_i between the leaf and the sonar (at the origin) will be $\sqrt{x^2 + y^2 + z^2}$

Since a few parameters can be derived from other parameters, the independent parameters are described here:

(1) frq: the frequency of the sonar

(2) a: radius of leaf

(3) pos: position of leaf center, coordinate (x, y, z)

(4) θ_0: incident angle

Leaf beampattern The beampattern of a leaf can be calculated with complex equations. To save time, we can approximate it with easy ones. Cosine function is a good fit. For different ratios of frequency and leaf size, the beampattern of leaf will be different. The ratio is defined as $c = 2\pi a \cdot frq/v$, where v is 340m/s, the speed of sound, frq is the frequency of the sonar beampattern, and a is the radius of the disc (leaf). In a word, for one c, there is one leaf beampattern.

The fit function is $L_i(\theta_0, a, frq) = A(c(frq, a)) \cdot cos[B(c(frq, a)) \cdot \theta_0]$, where $c(frq, a) = 2\pi a \cdot frq/v$, $A(c(frq, a)) = 0.5003 \cdot c^2 + 0.6867$, $B(c(frq, a)) = 0.3999 \cdot c^{-0.9065} + 0.9979$, and θ_0 is the assigned incident angle. For each c value, the values of functions $A(c)$ and $B(c)$ are all determined, and thus with different incident angle you will get different scattered field from a leaf.

Sonar beampattern A particular example of a two-dimensional Gaussian function is

$$f(x, y) = A * exp(-(\frac{(x - x_0)^2}{2\sigma_x^2} + \frac{(y - y_0)^2}{2\sigma_y^2})) \tag{2}$$

The general form is

$$f(x, y) = A * exp(-(a(x - x_0)^2 + 2b(x - x_0)(y - y_0) + c(y - y_0)^2)) \tag{3}$$

where

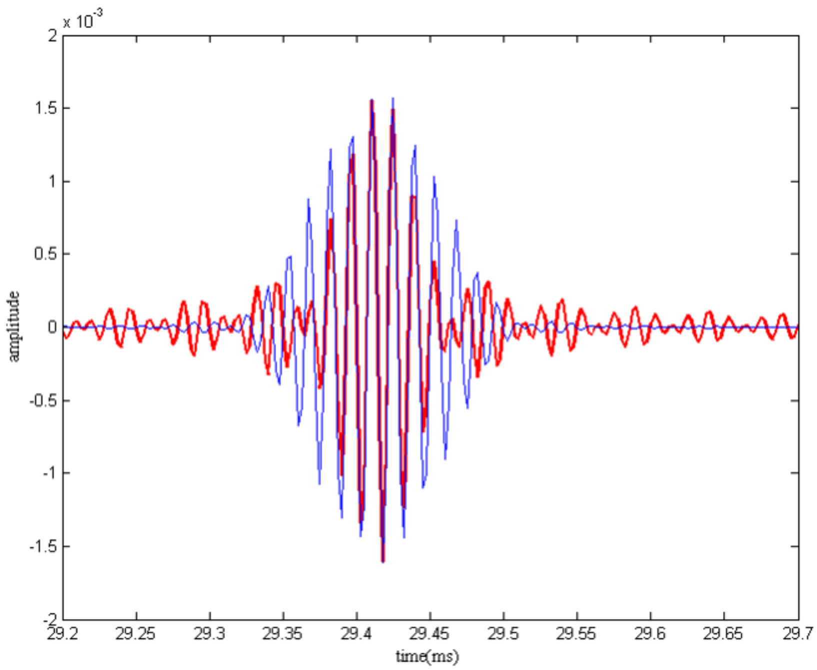
$$a = \frac{\cos^2\theta}{2\sigma_x^2} + \frac{\sin^2\theta}{2\sigma_y^2}, b = -\frac{\sin 2\theta}{4\sigma_x^2} + \frac{\sin 2\theta}{4\sigma_y^2}, c = \frac{\sin^2\theta}{2\sigma_x^2} + \frac{\cos^2\theta}{2\sigma_y^2} \tag{4}$$

And θ is the clockwise angle how much the blob is rotated. Since now the $\sigma_x = \sigma_y$, the general form becomes the particular one. When referring to measurements of power quantities, a ratio can be expressed as a level in decibels by evaluating ten times the base-10 logarithm of the ratio of the measured quantity to the reference level. Thus, the ratio of P (measured power) to P_0 (reference power) is represented by L_p, that ratio expressed in decibels, which is calculated using the formula:

$$L_p = 10\log_{10}(\frac{P}{P_0})\text{dB} \tag{5}$$

So the Half Power Beamwidth (HPBW) is $L_p = 10\log_{10}(\frac{P}{P_0})\text{dB} = 10\log_{10}(1/2)\text{dB} = -3\text{dB}$. The angle across the main lobe of an antenna pattern between the two directions at which the antenna's sensitivity is half its maximum value at the center of the lobe. Abbreviated HPBW. Here, in predict_gauss_sigma.m, the beamwidth is in degrees.

Window function in frequency domain By applying window function on frequency domain, the leakage in time domain is avoided.



Phase Phase changes due to propagation. If waves travel r meters, then the phase delay will be $\frac{r}{\lambda} \cdot 2\pi$, where λ is the wavelength ($\lambda f = v$, wavelength has a relationship with frequency and speed of sound). Because it is a round trip for the sound traveling from the sonar to the leaf and to the sonar again, the phase delay due to propagation is $\frac{2r}{\lambda} \cdot 2\pi$. Another part contributing to phase delay is the phase shift after the waves strike the leaf. The phase shift changes according to the incident angle. To make the computation efficient, the phase shift was also fit to a relatively easier function, error function $erf(PA(c) * (1.57 - \theta_0)) - PB(c)$, which is easier to be applied in MATLAB. Here, $PA(c) = 0.9824 \cdot c^{0.3523} - 0.9459$, $PB(c) = 2.6343$, and θ_0 is the incident angle. Then, the phase delay will be

$$\text{Phase\_delay} = -\frac{2r}{\lambda} \cdot 2\pi - \text{Phase\_shift}(\theta_0) \tag{6}$$

3 Simulation steps

The sonar has been put at the origin emitting sound towards the negative x direction. In the simplest situation, one leaf is put on the negative x axis facing the sonar. The position(x, y, z) and incident angle θ_0 of the leaf are given, and then from these four parameters we can get the distance between the leaf and sonar r, and the orientation of the line between sonar and leaf(θ^s, ϕ^s). Frequency goes from 60kHz to 80kHz. For each frequency, there will be one amplitude and one phase delay. Then the real part of frequency domain will be amplitude $*$ $\cos(\text{phase\_delay})$, and the imaginary part will be amplitude $* \sin(\text{phase\_delay})$. When all the data in the frequency domain are done, inverse Fourier transform is conducted to get the time response in time domain. In the process of inverse Fourier transform, a Hanning window is applied to the frequency domain especially between 60kHz and 80kHz to avoid the leakage in the time domain signal.

Multiple leaves There are two ways to calculate the time domain impulse from multiple leaves. First, do inverse Fourier transform for each leaf and then add their time domain impulse up. Second, to reduce calculation time, we can add all leaves' frequency domain up and do inverse Fourier transform once in the end and then get the total time domain impulse, because inverse Fourier transform is time consuming. The corresponding property of Fourier transform is: if $x_1[n]+x_2[n] = x_3[n]$, then $ReX_1[f]+ReX_2[f] = ReX_3[f]$ and $ImX_1[f]+ImX_2[f] = ImX_3[f]$. Here, $x[n]$ is the response in time domain. $ReX[f]$ and $ImX[f]$ are the real and imaginary parts in frequency domain.