

Two Higgs Doublet Model(s)

PhD(c). Cristian Fernando Rodríguez Cruz¹
c.rodriguez45@uniandes.edu.co

Research advisors:

Prof. Andrés Florez¹ (ca.florez@uniandes.edu.co)

Prof. J. Jones-Pérez² (jones.j@pucp.edu.pe)

¹Universidad de los Andes

²Pontificia Universidad Católica del Perú

December 2, 2025

Outline

1 Introduction

2 On the Scalar Potential

3 Theoretical Constraints

- Unitarity
- Theoretical Constraints

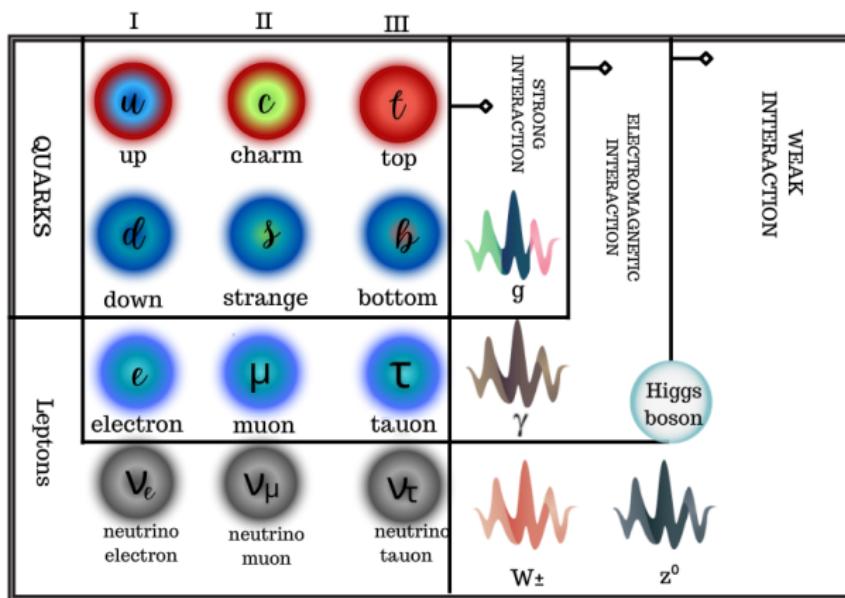
4 Experimental Constraints

5 STU Constraints

- FCNC-free Models
- Allowed Parameter Space for Type II THDM

6 Final Remarks and Next Steps

The Standard Model Higgs Mechanism



- One Higgs doublet provides EWSB
- Single physical Higgs boson discovered in 2012
- But is this the complete story?

Why Consider Extended Higgs Sectors?

BSM Motivations

■ **Hierarchy Problem**

Discrepancy between Higgs mass (~ 125 GeV) and Planck scale ($\sim 10^{19}$ GeV)

■ **Dark Matter Puzzle**

No viable DM candidate in SM (WIMP miracle suggests ~ 100 GeV scale)

■ **Baryogenesis Requirements**

Need strong 1st-order EW phase transition ($v_c/T_c > 1$)

■ **Flavor Anomalies**

Additional CP violation sources needed for B -physics observations

■ **Theoretical Naturalness**

SUSY, composite Higgs, etc. often require extended sectors

The Minimal Extension: Two Higgs Doublet Model (2HDM)

Simplest framework addressing some of these challenges

(Different types: Type I, II, Lepton-specific, Flipped with different phenomenology)

Purely Scalar Extension (arXiv:1106.0034)

- With Two Higgs doublets under $SU(2)_L \times U(1)_Y$, the most general potential is

$$V_{\text{tree}} = m_{11}^2 (\Phi_1^\dagger \Phi_1) + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + m_{22}^2 (\Phi_2^\dagger \Phi_2) + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 \quad (1)$$

$$+ \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} [\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}] \quad (2)$$

$$+ [-m_{12}^2 \Phi_1^\dagger \Phi_2 + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{h.c.}], \quad (3)$$

- 1 Typical mass-like terms and quartic self-interactions
- 2 Mixed terms between doublets (\mathbb{Z}_2 -Even)
- 3 Mixed terms between doublets (often \mathbb{Z}_2 -Odd and then forbidden, but m_{12}^2 still allowed as soft \mathbb{Z}_2 -symmetry-breaking term)

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- Written in Kibble parametrization, the two complex scalar doublets are:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}} (v_1 + \phi_1 + i a_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}} (v_2 + \phi_2 + i a_2) \end{pmatrix}.$$

Both doublets have a neutral part with a scalar component ϕ_i that acquire a VEV $v_i/\sqrt{2}$, a pseudoscalar component a_i , and have a charged part ϕ_i^+ .

- Electroweak constraint: $v_1^2 + v_2^2 = v^2 = (246 \text{ GeV})^2$

Minimization conditions

The minimization conditions around the vacuum Ω come from requiring

$$\frac{\partial V}{\partial \Phi_1} \Big|_{\Omega} = \frac{\partial V}{\partial \Phi_2} \Big|_{\Omega} = 0:$$

$$\frac{\partial V}{\partial \Phi_1} \Big|_{\Omega} = v_1 \left[m_{11}^2 + \frac{\lambda_1}{2} v_1^2 + \frac{\lambda_{345}}{2} v_2^2 \right] - m_{12}^2 v_2 + \frac{3\lambda_6}{2} v_1^2 v_2 + \frac{\lambda_7}{2} v_2^3 = 0 \quad (4)$$

$$\frac{\partial V}{\partial \Phi_2} \Big|_{\Omega} = v_2 \left[m_{22}^2 + \frac{\lambda_2}{2} v_2^2 + \frac{\lambda_{345}}{2} v_1^2 \right] - m_{12}^2 v_1 + \frac{\lambda_6}{2} v_1^3 + \frac{3\lambda_7}{2} v_1 v_2^2 = 0 \quad (5)$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$.

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where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$.

In the cases where $\lambda_6 = \lambda_7 = 0$ with the electroweak constraint,
 $v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$ ($\implies \tan \beta = v_2/v_1, \cos \beta = v_1/v, \sin \beta = v_2/v$),

$$m_{11}^2 = m_{12}^2 \tan \beta - \frac{1}{2} v^2 \left(\lambda_1 \cos^2 \beta + \lambda_{345} \sin^2 \beta \right) \quad (6)$$

$$m_{22}^2 = m_{12}^2 \cot \beta - \frac{1}{2} v^2 \left(\lambda_2 \sin^2 \beta + \lambda_{345} \cos^2 \beta \right) \quad (7)$$

Usually, you could see the short notation: $c_\beta = \cos \beta, s_\beta = \sin \beta, t_\beta = \tan \beta$.

Scalar Mass Matrices

Expanding at second order in the fields around the vacuum, setting $\lambda_6 = \lambda_7 = 0$, and using the minimization conditions, the mass matrices for scalar components become:

CP-even scalars (ϕ_1, ϕ_2):

$$(\phi_1 \quad \phi_2) \begin{pmatrix} m_{12}^2 t_\beta + \lambda_1 v^2 c_\beta^2 & -m_{12}^2 + \frac{\lambda_{345}}{2} v^2 s_{2\beta} \\ -m_{12}^2 + \frac{\lambda_{345}}{2} v^2 s_{2\beta} & m_{12}^2 / t_\beta + \lambda_2 v^2 s_\beta^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

CP-odd scalars (a_1, a_2):

$$(a_1 \quad a_2) \left[m_{12}^2 - \frac{1}{2} \lambda_5 v^2 s_{2\beta} \right] \begin{pmatrix} t_\beta & -1 \\ -1 & 1/t_\beta \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Charged scalars (ϕ_1^\pm, ϕ_2^\pm):

$$(\phi_1^+ \quad \phi_2^+) \left[m_{12}^2 - \frac{1}{4} (\lambda_4 + \lambda_5) v^2 s_{2\beta} \right] \begin{pmatrix} t_\beta & -1 \\ -1 & 1/t_\beta \end{pmatrix} \begin{pmatrix} \phi_1^- \\ \phi_2^- \end{pmatrix}$$

Diagonalization implies two rotation matrices with angles α and β :

$$\begin{pmatrix} h \\ H \end{pmatrix} = \mathcal{R}(\alpha) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \begin{pmatrix} A \\ G^0 \end{pmatrix} = \mathcal{R}(\beta) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \mathcal{R}(\beta) \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

Scalar Mass Spectrum

Diagonalization implies two rotation matrices with angles α and β :

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After diagonalization, we obtain physical states:

- **CP-even Higgs:** h (SM-like, 125 GeV), H (heavy scalar)
- **CP-odd Higgs:** A (pseudoscalar)
- **Charged Higgs:** H^\pm
- **Goldstone bosons:** G^\pm, G^0 (absorbed by W^\pm, Z^0)

with eigenvalues:

$$m_{H,h}^2 = \frac{1}{2} \left[M_{P,11}^2 + M_{P,22}^2 \pm \sqrt{(M_{P,11}^2 - M_{P,22}^2)^2 + 4(M_{P,12}^2)^2} \right]$$

$$m_A^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \lambda_5 v^2$$

$$m_{H^\pm}^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \frac{1}{2}(\lambda_4 + \lambda_5)v^2$$

where M_P^2 is the CP-even scalar mass matrix.

Gauge Interactions and Decoupling Limit

The gauge-kinetic Lagrangian is given as

$$\mathcal{L}_g = (D^\mu \Phi_1)^\dagger (D_\mu \Phi_1) + (D^\mu \Phi_2)^\dagger (D_\mu \Phi_2)$$

We obtain the neutral Higgs couplings to VV ($VV \equiv ZZ, WW$)

$$\begin{aligned} \mathcal{L}_g \supset & \frac{g^2 + g'^2}{8} v^2 ZZ \left(1 + 2 \frac{h}{v} y_h^V + 2 \frac{H}{v} y_H^V \right) \\ & + \frac{g^2}{4} v^2 W^+ W^- \left(1 + 2 \frac{h}{v} y_h^V + 2 \frac{H}{v} y_H^V \right) \end{aligned}$$

where $y_h^V = \sin(\beta - \alpha)$ and $y_H^V = \cos(\beta - \alpha)$.

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Decoupling Limit

The SM has been frustratingly successful in describing the Higgs boson properties. In order to get a SM-like Higgs boson, we require $\cos(\beta - \alpha) \approx 0$, which implies $\beta - \alpha \approx \pi/2$.

This is known as the **decoupling limit**.

Lambda Parameters in the Decoupling Limit (arXiv:1604.01406)

In the Decoupling limit the scalar mass spectrum simplifies and we can invert the relations to express the λ_i in terms of the physical masses:

$$\begin{aligned} v^2 \lambda_1 &= m_h^2 - \frac{t_\beta (m_{12}^2 - m_H^2 s_\beta c_\beta)}{c_\beta^2} \\ v^2 \lambda_2 &= m_h^2 - \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{t_\beta s_\beta^2} \\ v^2 \lambda_3 &= m_h^2 + 2m_{H^\pm}^2 - 2m_H^2 - \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta c_\beta} \\ v^2 \lambda_4 &= m_A^2 - 2m_{H^\pm}^2 + m_H^2 + \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta c_\beta} \\ v^2 \lambda_5 &= m_H^2 - m_A^2 + \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta c_\beta} \end{aligned} \tag{8}$$

where $m_h = 125$ GeV is the SM-like Higgs boson mass.

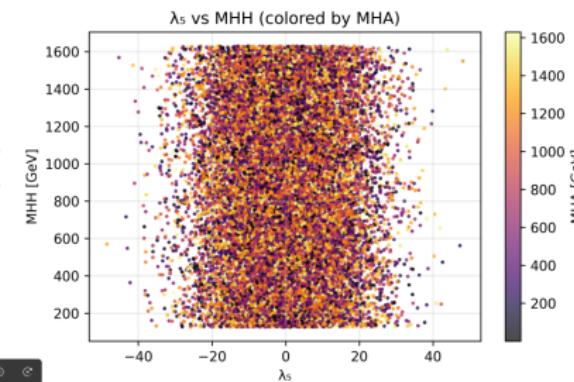
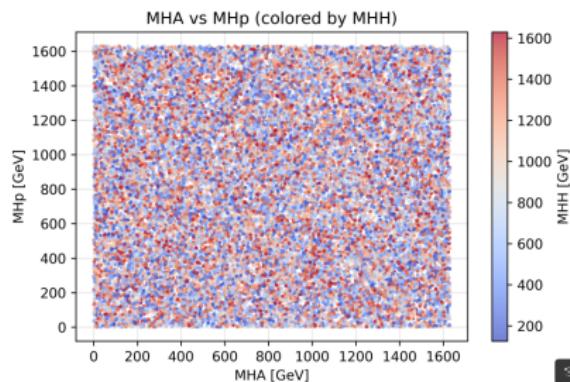
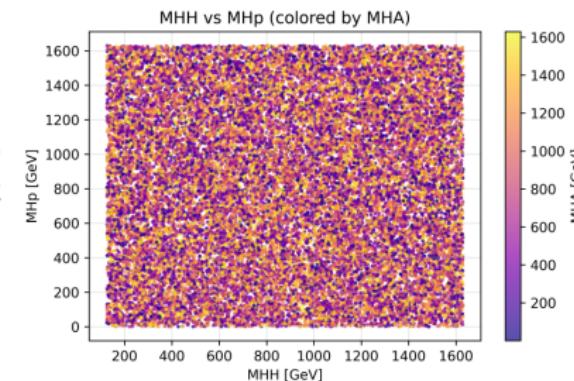
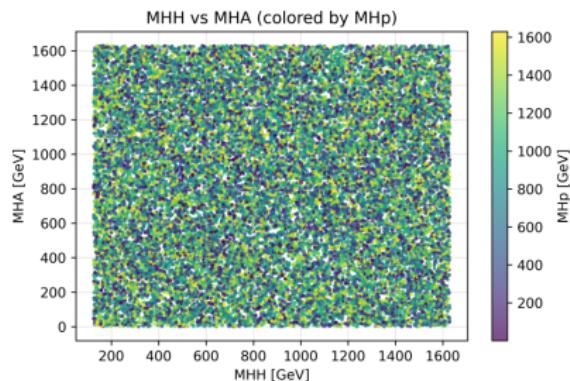
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 v^2 \lambda_2 &= m_h^2 - \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{t_\beta s_\beta^2} \\
 v^2 \lambda_3 &= m_h^2 + 2m_{H^\pm}^2 - 2m_H^2 - \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta c_\beta} \\
 v^2 \lambda_4 &= m_A^2 - 2m_{H^\pm}^2 + m_H^2 + \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta c_\beta} \\
 v^2 \lambda_5 &= m_H^2 - m_A^2 + \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta c_\beta}
 \end{aligned} \tag{8}$$

where $m_h = 125$ GeV is the SM-like Higgs boson mass.
So we have the following free parameters:

- Physical masses: m_H, m_A, m_{H^\pm}
- Mixing angle: β (or $\tan \beta$).
- Soft \mathbb{Z}_2 -breaking term: m_{12}^2 (I prefer λ_5)

Initial State of the Scan ($\tan \beta = 10$ fixed)

Unitarity Constraints (arXiv:1805.07306)

From quantum mechanics, the S-matrix must be unitary:

$$S = 1 + iT \quad \Rightarrow \quad T^\dagger T = -i(T - T^\dagger)$$

For scattering processes $s_1 s_2 \rightarrow s_3 s_4$, this implies bounds on amplitudes.

The matrix element for the process is defined as:

$$\langle \{s_1, s_2\} | iT | \{s_3, s_4\} \rangle \equiv i\mathcal{M}\delta^4(k_3 + k_4 - p_1 - p_2)(2\pi)^4$$

From the scattering theory, the amplitude \mathcal{M} can be expanded in partial waves:

$$\mathcal{M} = \sum_{J=0}^{\infty} (2J+1) a_J P_J(\cos \theta) \implies a_J \equiv \frac{1}{32\pi} \sqrt{\frac{4|\mathbf{p}^{\text{in}}| |\mathbf{p}^{\text{out}}|}{2^{\delta_{12}} 2^{\delta_{34}} S}} \int_{-1}^1 d(x) \mathcal{M} P_J(x)$$

where P_J are Legendre polynomials and a_J are the partial wave amplitudes.
So, we have

$$-\frac{i}{2} (a_J - a_J^\dagger) \geq a_J a_J^\dagger \implies \text{Re}(a_J) \leq \frac{1}{2} \quad (9)$$

Unitarity Constraints (arXiv:1805.07306)

At large energies, the dominant contributions are the spherically symmetric, $J = 0$, partial wave amplitudes and $s \approx |\mathbf{p}|^2$. Thus,

$$\text{Re}\{a_0\} \sim \frac{1}{16\pi} \sqrt{2^{-\delta_{12}-\delta_{34}}} Q_{1234} \leq \frac{1}{2} \quad (10)$$

where Q_{1234} is the quartic coupling of the process $s_1 s_2 \rightarrow s_3 s_4$. Considering all the possible processes,

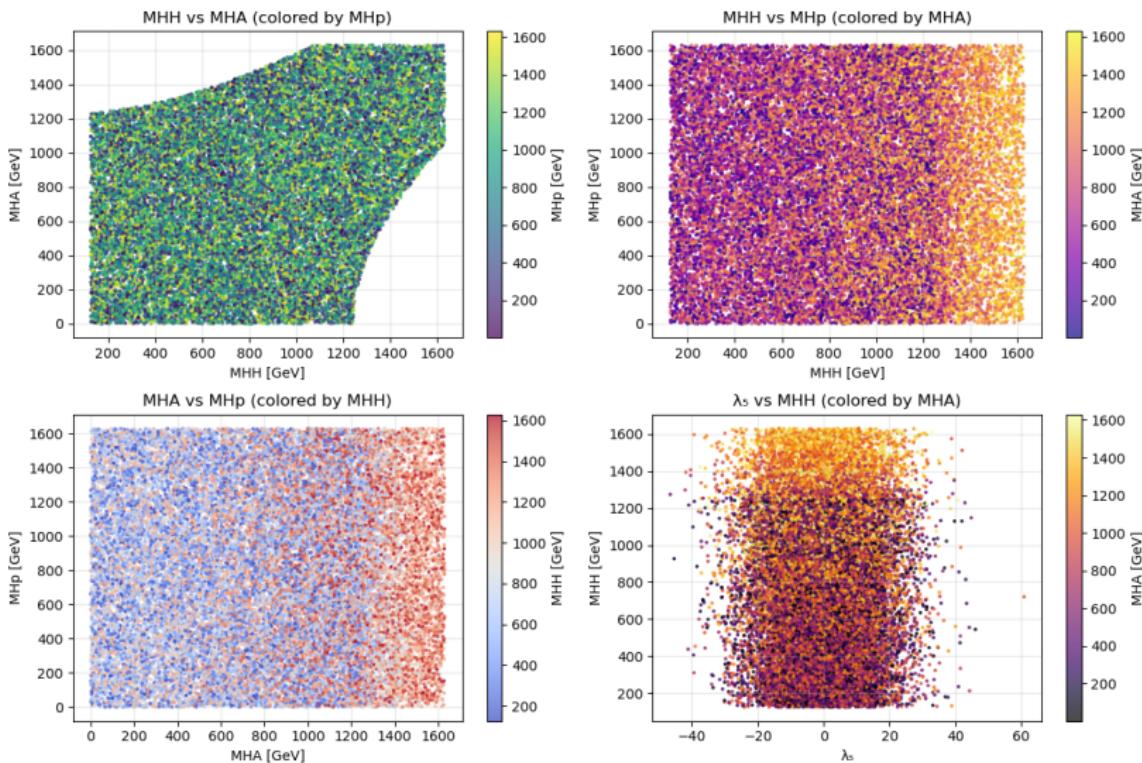
$$|a_{\pm}|, |b_{\pm}|, |c_{\pm}|, |e_{\pm}|, |f_{\pm}|, |g_{\pm}| \leq 8\pi$$

where the eigenvalues are:

$$\begin{aligned} a_{\pm} &= \frac{3}{2} (\lambda_1 + \lambda_2) \pm \sqrt{\frac{9}{4} (\lambda_1 - \lambda_2)^2 + (2\lambda_3 + \lambda_4)^2} & e_{\pm} &= \lambda_3 + 2\lambda_4 \pm 3\lambda_5 \\ b_{\pm} &= \frac{1}{2} (\lambda_1 + \lambda_2) \pm \sqrt{\frac{1}{4} (\lambda_1 - \lambda_2)^2 + \lambda_4^2} & f_{\pm} &= \lambda_3 \pm \lambda_4 \\ c_{\pm} &= \frac{1}{2} (\lambda_1 + \lambda_2) \pm \sqrt{\frac{1}{4} (\lambda_1 - \lambda_2)^2 + \lambda_5^2} & g_{\pm} &= \lambda_3 \pm \lambda_5 \end{aligned} \quad (11)$$

It is just a safety check, not the strict theory limitation.

Example: $|f_+| = |\lambda_3 + \lambda_4| \approx |m_h^2 + m_A^2 - m_H^2| / v^2 < 8\pi$



Vacuum Stability Constraints (arXiv:2203.07244)

The potential must be bounded from below at large field values.

The dominant quartic terms in the potential are:

$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + \lambda_3 X_1^2 X_2^2 + \lambda_4 X_1^2 X_2^2 \rho^2 + \lambda_5 X_1^2 X_2^2 \rho^2 \cos 2\theta.$$

where the fields have been parametrized as

$$\Phi_1^\dagger \Phi_1 = X_1^2, \quad \Phi_2^\dagger \Phi_2 = X_2^2, \quad \Phi_1^\dagger \Phi_2 = X_1 X_2 \rho e^{i\theta} \quad \text{with} \quad 0 \leq \rho \leq 1.$$

Here, ρ is the normalized magnitude of the inner product between Φ_1 and Φ_2 , obtained from the Cauchy-Schwarz inequality:

$$|\Phi_1^\dagger \Phi_2| \leq \sqrt{(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)} \Rightarrow \rho = \frac{|\Phi_1^\dagger \Phi_2|}{X_1 X_2} \in [0, 1].$$

Geometrically, ρ plays the role of the cosine of the angle between the two Higgs doublets in field space: $\rho = 1$ means they are aligned, while $\rho = 0$ means they are orthogonal.

Theoretical Constraints

V_4 must have a minimum for all directions in field space, i.e., for all $X_1, X_2 \geq 0$ and $\rho \in [0, 1]$.

The term with θ is $\lambda_5 X_1^2 X_2^2 \rho^2 \cos 2\theta$.

- If $\lambda_5 > 0$, the minimum occurs at $\cos \theta = -1$,
- else if $\lambda_5 < 0$, the minimum occurs at $\cos \theta = 1$.
- In any case, the term becomes $-|\lambda_5| X_1^2 X_2^2 \rho^2$ at the minimum.

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At the minimum, the term with ρ is $(\lambda_4 - |\lambda_5|) X_1^2 X_2^2 \rho^2$.

- If $\lambda_4 - |\lambda_5| > 0$, the minimum occurs at $\rho = 0$, then

$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + \lambda_3 X_1^2 X_2^2 \quad (12)$$

and we require $\lambda_1 > 0$, $\lambda_2 > 0$, and $\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0$.

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V_4 must have a minimum for all directions in field space, i.e., for all $X_1, X_2 \geq 0$ and $\rho \in [0, 1]$.

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and we require $\lambda_1 > 0, \lambda_2 > 0$, and $\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0$.

- else if $\lambda_4 - |\lambda_5| < 0$, the minimum occurs at $\rho = 1$, then

$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + (\lambda_3 + \lambda_4 - |\lambda_5|) X_1^2 X_2^2 \quad (13)$$

and we require $\lambda_1 > 0, \lambda_2 > 0, \lambda'_3 + \sqrt{\lambda_1 \lambda_2} > 0$, with $\lambda'_3 = \lambda_3 + \lambda_4 - |\lambda_5|$.

In Summary, the bounded from below conditions are:

$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0, \quad \lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} > 0.$

Electroweak Vacuum Stability (arXiv:1303.5098)

- The scalar potential can have multiple minima - we must ensure our EW-breaking vacuum is the **global minimum**
- Metastability could lead to dangerous vacuum decay via quantum tunneling.
- To guarantee the selected vacuum is truly the lowest-energy state, we require:

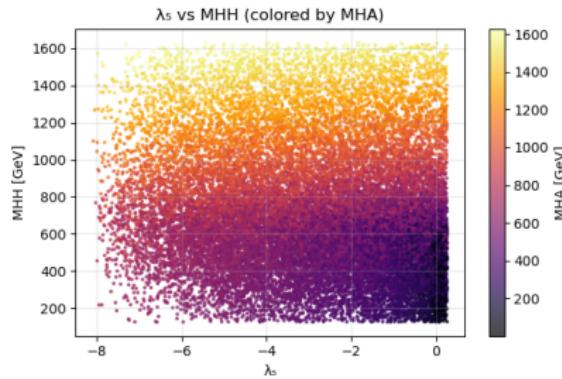
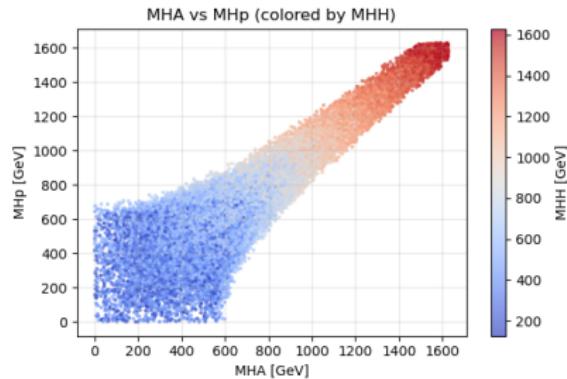
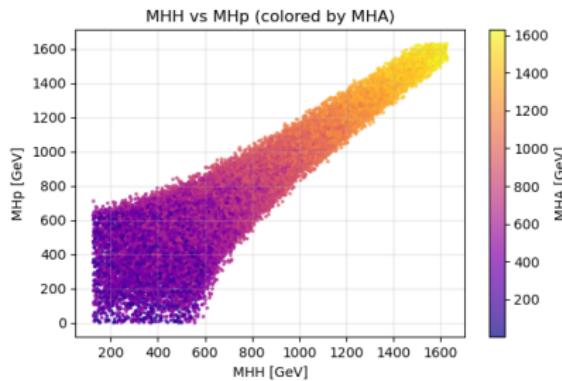
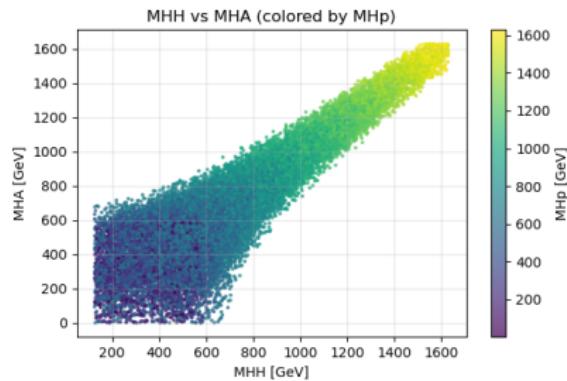
$$m_{12}^2 \left(m_{11}^2 - k^2 m_{22}^2 \right) (\tan \beta - k) > 0$$

where $k \equiv (\lambda_1/\lambda_2)^{1/4}$ compares the quartic couplings.

- The details of this condition, requires a mapping of the potential as a Minkowskian manifold and stability of the Landau-Ginzburg effective potential, are given in arXiv:1303.5098.

Theoretical Constraints

All the theoretical constraints



Oblique Parameters (PDG 2024, eq 10.98)

W -boson mass at one-loop level could have small corrections to the mass:

$$m_W^2 = m_W^2(\text{SM}) + \frac{\alpha c_W^2}{c_W^2 - s_W^2} m_Z^2 \left(-\frac{1}{2}S + c_W^2 T + \frac{c_W^2 - s_W^2}{4s_W^2} U \right)$$

This corrections are parametrized by the oblique parameters S , T , and U which are complicated loop-function that we calculated in Spheno.

The accuracy of the W boson mass requires :

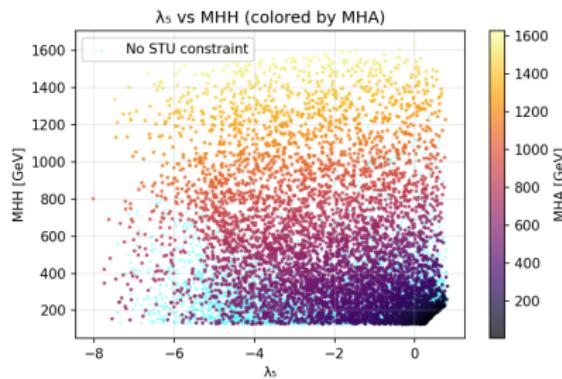
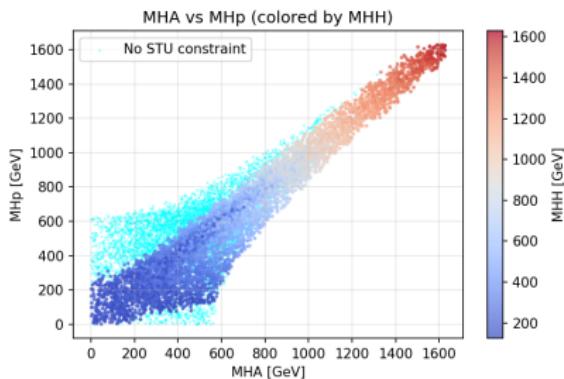
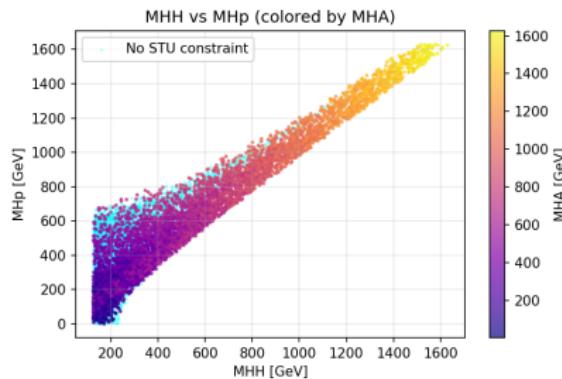
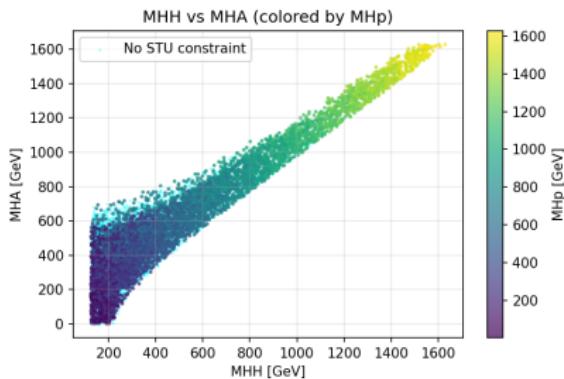
$$S = -0.04 \pm 0.10, \tag{14}$$

$$T = +0.01 \pm 0.12, \tag{15}$$

$$U = -0.01 \pm 0.09 \tag{16}$$

These parameters are crucial for precision tests of the Standard Model and for constraining new physics scenarios.

Oblique Parameters Constraints



Yukawa Sector (arXiv:1106.0034)

In order to suppress flavor-changing neutral currents (FCNCs), we impose a \mathbb{Z}_2 symmetry on the Yukawa sector resulting in four types of Yukawa interactions:

Model	Φ_2	Φ_1	u_R^i	d_R^i	e_R^i
Type I	+	-	+	+	+
Type II	+	-	+	-	-
Lepton-specific	+	-	+	+	-
Flipped	+	-	+	-	+

According to different charge assignments, there are four different models with Yukawa interactions:

- $\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2u_R + Y_{d2}\bar{Q}_L\Phi_2d_R + Y_{\ell2}\bar{L}_L\Phi_2e_R + \text{h.c.}$ (type I),
- $\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2u_R + Y_{d1}\bar{Q}_L\Phi_1d_R + Y_{\ell1}\bar{L}_L\Phi_1e_R + \text{h.c.}$ (type II),
- $\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2u_R + Y_{d1}\bar{Q}_L\Phi_2d_R + Y_{\ell1}\bar{L}_L\Phi_1e_R + \text{h.c.}$ (lepton specific),
- $\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2u_R + Y_{d1}\bar{Q}_L\Phi_1d_R + Y_{\ell1}\bar{L}_L\Phi_2e_R + \text{h.c.}$ (flipped model) ,

where $Q_L^T = (u_L, d_L)$, $L_L^T = (\nu_L, l_L)$, $\tilde{\Phi}_{1,2} = i\tau_2\Phi_{1,2}^*$, and $Y_{u2}, Y_{d1,2}$ and $Y_{\ell1,2}$ are 3×3 matrices in family space.

Yukawa sector After EWSB

We can obtain the Yukawa couplings

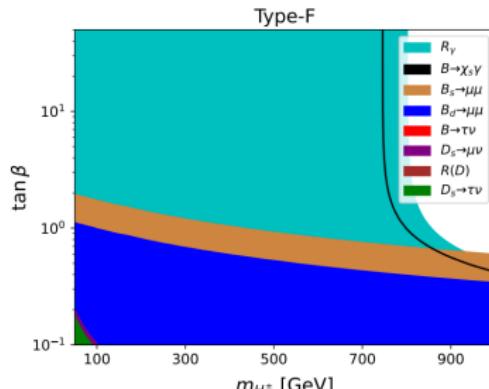
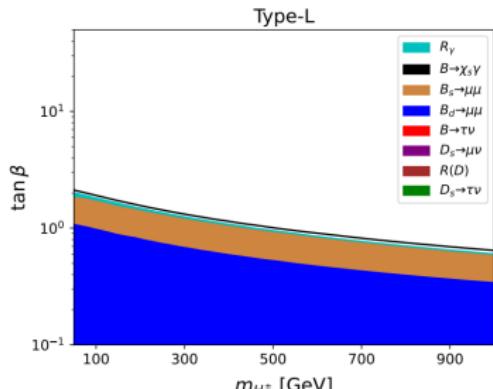
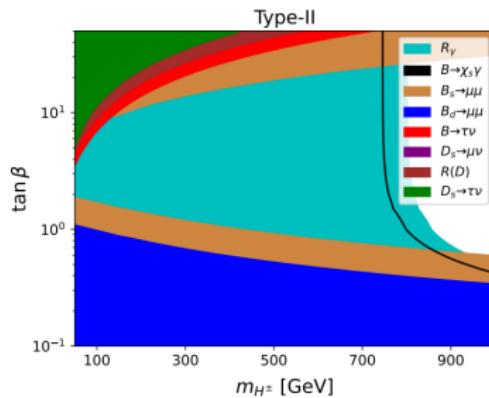
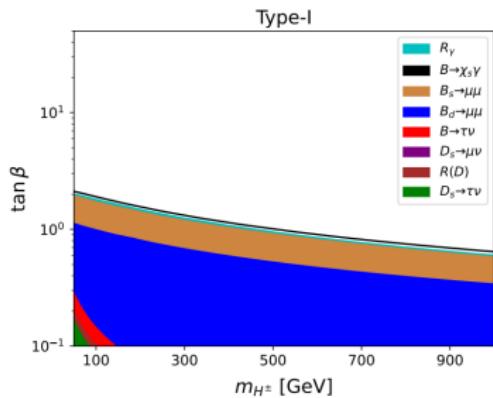
$$\begin{aligned}
 -\mathcal{L}_Y = & \frac{m_f}{v} y_h^f h \bar{f} f + \frac{m_f}{v} y_H^f H \bar{f} f \\
 & - i \frac{m_u}{v} \kappa_u A \bar{u} \gamma_5 u + i \frac{m_d}{v} \kappa_d A \bar{d} \gamma_5 d + i \frac{m_\ell}{v} \kappa_\ell A \bar{\ell} \gamma_5 \ell \\
 & + H^+ \bar{u} V_{CKM} \left(\frac{\sqrt{2} m_d}{v} \kappa_d P_R - \frac{\sqrt{2} m_u}{v} \kappa_u P_L \right) d + h.c. \\
 & + \frac{\sqrt{2} m_\ell}{v} \kappa_\ell H^+ \bar{\nu} P_R e + h.c.
 \end{aligned}$$

where $y_h^f = \sin(\beta - \alpha) + \cos(\beta - \alpha) \kappa_f$ and $y_H^f = \cos(\beta - \alpha) - \sin(\beta - \alpha) \kappa_f$. The values of κ_u , κ_d and κ_ℓ for the four models are

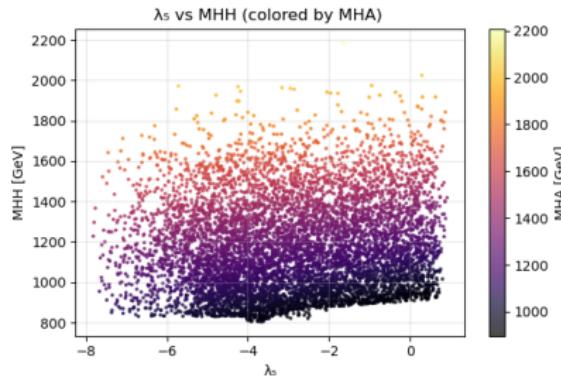
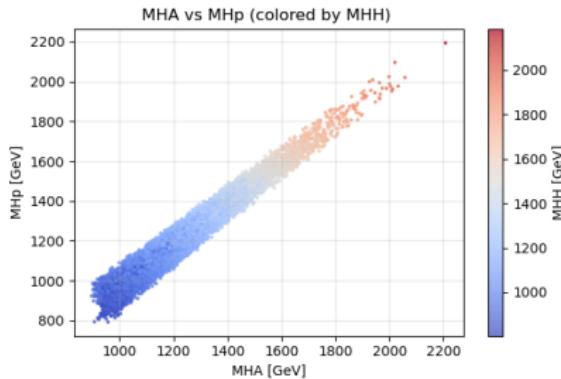
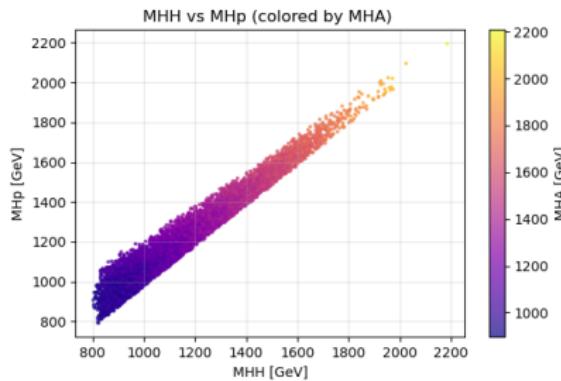
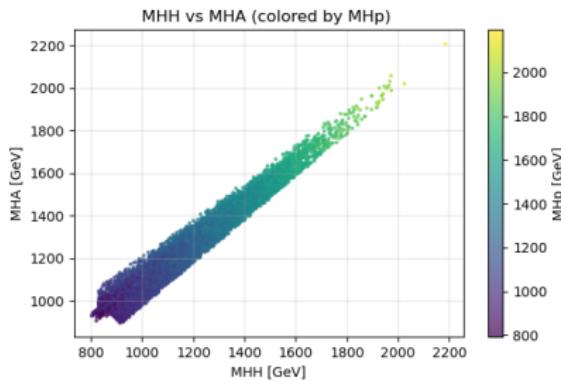
	type-I	type-II	lepton-specific	flipped
κ_u	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$
κ_d	$1/t_\beta$	$-t_\beta$	$1/t_\beta$	$-t_\beta$
κ_ℓ	$1/t_\beta$	$-t_\beta$	$-t_\beta$	$1/t_\beta$

These models naturally have preferential couplings to the third generation fermions, due to the Higgs Mechanism.

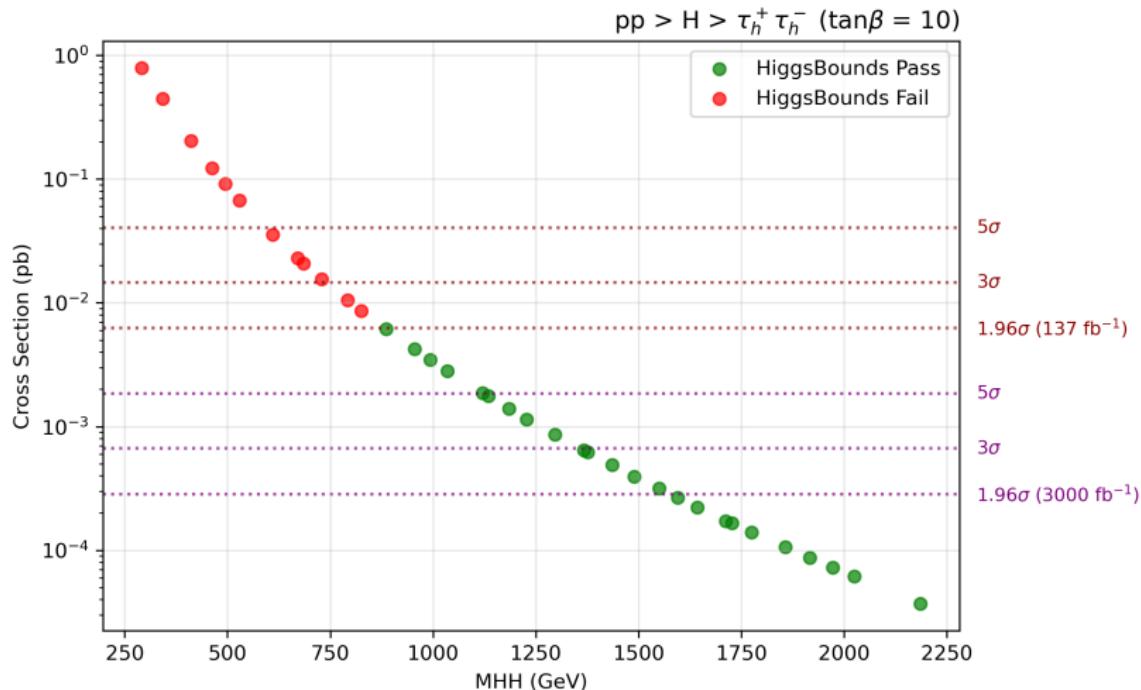
H^\pm Constraints (arXiv:2412.04572)



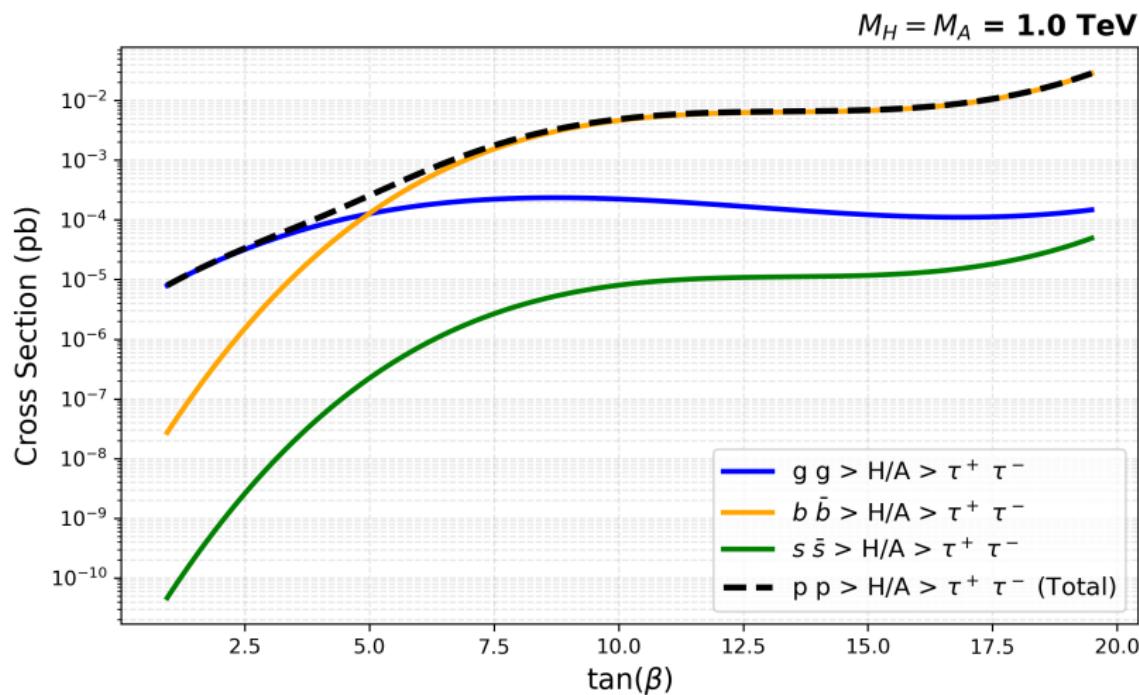
$m_{H^\pm} \gtrsim 750 \text{ GeV} + \text{HB constraints}$ (Work in progress)



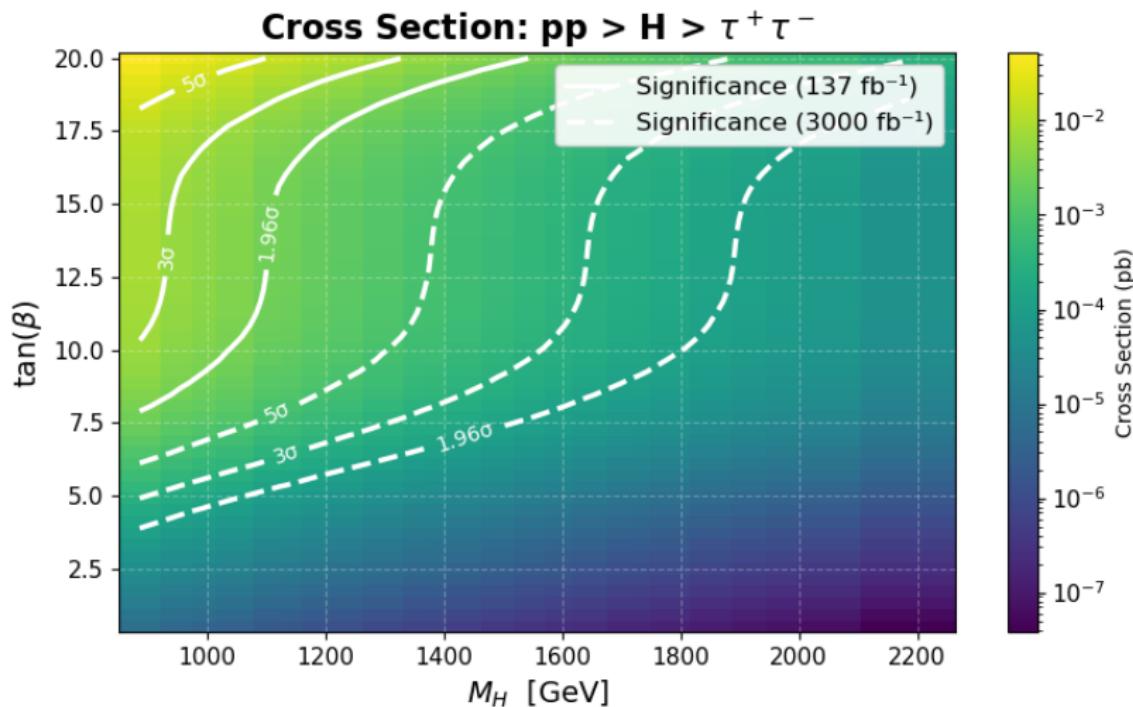
LHC production cross-section (hadronic channel) (work in progress)



$\tan \beta$ effects on the cross-section (work in progress)



High-Lumi Projections (work in progress)



Summary

Key Parts from THDM Analysis

- **Theoretical Framework:** Extended SM with two Higgs doublets Φ_1, Φ_2
 - 5 physical scalars: h (125 GeV), $H, A, H^\pm + 3$ Goldstones
 - Decoupling limit: $\cos(\beta - \alpha) \approx 0$ for SM-like behavior
- **Constraints to the Scalar Potential:**
 - Vacuum stability: $\lambda_1, \lambda_2 > 0; \sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 > \max(\lambda_4, |\lambda_5|)$ bounded-from-below conditions
 - Unitarity: $|a_\pm|, |b_\pm|, |c_\pm|, |e_\pm|, |f_\pm|, |g_\pm| \leq 8\pi$
 - EW precision: Oblique parameters S, T, U within 1σ
- **Type II THDM Phenomenology:**
 - Enhanced τ, b couplings: $\kappa_d = \kappa_\ell = \tan \beta$
 - Suppressed top couplings: $\kappa_u = 1/\tan \beta$
 - Strong H^\pm constraints: $m_{H^\pm} \gtrsim 750 \text{ GeV}, \tan \beta \in [1, 15]$
- **Experimental Constraints On the Yukawa Sector:**
 - LHC searches exclude regions via $H/A \rightarrow \tau\tau, H \rightarrow WW/ZZ$
 - Allowed parameter space identified for future studies

Main Achievement

Systematic parameter scan establishing **viable THDM Type II regions**
consistent with all theoretical and experimental constraints

Thank you for your attention!

Questions?

Details to be added based on specific analysis requirements