Interference effects

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Two body scattering

CM-Frame

Consider the process

$$A(\vec{\boldsymbol{p}}_1) + B(\vec{\boldsymbol{p}}_2) \longrightarrow C(\vec{\boldsymbol{p}}_3) + D(\vec{\boldsymbol{p}}_4), \tag{1}$$

From the Golden Rule, the cross section is given by

$$\sigma = \frac{S(2\pi)^4}{4\sqrt{(\vec{\boldsymbol{p}}_1 \cdot \vec{\boldsymbol{p}}_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \frac{d^3\vec{\boldsymbol{p}}_3}{(2\pi)^3 2E_3} \frac{d^3\vec{\boldsymbol{p}}_4}{(2\pi)^3 2E_4}. \tag{2}$$

But, in the CM frame, $\vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{p}}_2 = 0$, where

$$\sqrt{(\vec{\boldsymbol{p}}_1 \cdot \vec{\boldsymbol{p}}_2)^2 - (m_1 m_2)^2} = E_1 E_2 \left|\vec{\boldsymbol{p}}_1\right|, \tag{3}$$

$$\delta^{(4)}(p_1 + p_2 - p_3 - p_4) = \delta\left(E_1 + E_2 - E_3 - E_4\right) \delta^{(3)}(\vec{\boldsymbol{p}}_3 + \vec{\boldsymbol{p}}_4). \tag{4}$$

Thus

$$\sigma = \left(\frac{1}{8\pi}\right)^2 \frac{S}{(E_1 E_2)\left|\vec{\boldsymbol{p}}_1\right|} \int |\mathcal{M}|^2 \frac{\delta\left(E_1 + E_2 - \sqrt{\vec{\mathbf{p}}_3^2 + m_3^2} - \sqrt{\vec{\mathbf{p}}_3^2 + m_4^2}\right)}{\sqrt{\vec{\mathbf{p}}_3^2 + m_3^2}\sqrt{\vec{\mathbf{p}}_3^2 + m_4^2}} \mathrm{d}\vec{\mathbf{p}}_3 \tag{5}$$

Integrating over the radial part $|\vec{\mathbf{p}}_3|$, we get

$$\sigma = \left(\frac{1}{8\pi}\right)^2 \frac{S|\vec{\boldsymbol{p}}_3|}{(E_1+E_2)^2|\vec{\boldsymbol{p}}_1|} \int |\mathcal{M}|^2 \mathrm{d}\Omega, \tag{6}$$

with

$$|\vec{\boldsymbol{p}}_3| = \frac{1}{2}\frac{\sqrt{((E_1+E_2)^2 - m_3^2 - m_4^2)^2 - 4m_3^2m_4^2}}{E_1+E_2}, \tag{7}$$

the outgoing momentum in the CM frame.

We prefer work with differential cross section as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{S}{(E_1+E_2)^2}\frac{|\vec{\boldsymbol{p}}_3|}{|\vec{\boldsymbol{p}}_1|}|\mathcal{M}|^2. \tag{8}$$

Note that at this point, we don't need to know the explicit form of the matrix element $\mathcal{M}$, so it is a generic result.

Defining $\sqrt{s} = E_1 + E_2$, we have

$$|\vec{\boldsymbol{p}}_3| = \frac{1}{2}\frac{\sqrt{(s - m_3^2 - m_4^2)^2 - 4m_3^2m_4^2}}{\sqrt{s}}, \quad |\vec{\boldsymbol{p}}_1| = \frac{1}{2}\frac{\sqrt{(s - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2}}{\sqrt{s}}. \tag{9}$$

so the differential cross section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{S}{s}\sqrt{\frac{(s-(m_3+m_4)^2)(s-(m_3-m_4)^2)}{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}|\mathcal{M}|^2. \tag{10}$$

In general, there are three Lorentz-invariant useful kinematical variables to describe the scattering process, known as Mandelstam variables:

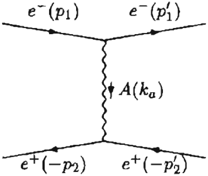
$$\hat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2 = m_1^2 + m_2^2 + 2p_1^\mu p_{2\mu} = m_3^2 + m_4^2 + 2p_3^\mu p_{4\mu}, \tag{11}$$

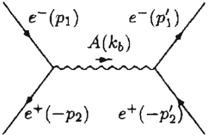
$$\hat{t} = (p_1 - p_3)^2 = (p_2 - p_4)^2 = m_1^2 + m_3^2 - 2p_1^\mu p_{3\mu} = m_2^2 + m_4^2 - 2p_2^\mu p_{4\mu}, \tag{12}$$

$$\hat{u} = (p_1 - p_4)^2 = (p_2 - p_3)^2 = m_1^2 + m_4^2 - 2p_1^\mu p_{4\mu} = m_2^2 + m_3^2 - 2p_2^\mu p_{3\mu}. \tag{13}$$

In the CM-frame, $\hat{s} = s = (E_1 + E_2)^2$.

At tree level, there are two Feynman diagrams that contribute to the process





$$\left[\bar{u}_1'(ie\gamma^\mu)u_1\right] iD_{\mu\nu}(k_a) \left[\bar{v}_2(ie\gamma^\nu)v_2'\right]$$

$$= -ie^2 \frac{\left[\bar{u}_1'\gamma^\mu u_1\right]\left[\bar{v}_2\gamma_\mu v_2'\right]}{k_a^2}$$

$$k_a = p_1 - p_1' = p_2' - p_2$$

$$\left[\bar{v}_2(ie\gamma^\mu)u_1\right] iD_{\mu\nu}(k_b) \left[\bar{u}_1'(ie\gamma^\nu)v_2'\right]$$

$$= -ie^2 \frac{\left[\bar{v}_2\gamma^\mu u_1\right]\left[\bar{u}_1'\gamma_\mu v_2'\right]}{k_b^2}$$

$$k_b = p_1 + p_2 = p_2' + p_1'$$

The fermionic exchange between the initial positron and the final electron is the same in both diagrams, so we have a relative minus sign between the two contributions.

$$i\mathcal{M} = ie^2 \left(\frac{\left[\bar{u}_1'\gamma^\mu u_1\right]\left[\bar{v}_2\gamma_\mu v_2'\right]}{(p_1 - p_1')^2} - \frac{\left[\bar{v}_2\gamma^\mu u_1\right]\left[\bar{u}_1'\gamma_\mu v_2'\right]}{(p_1 + p_2)^2} \right), \tag{14}$$

QED $e^+e^- \longrightarrow e^+e^-$ scattering

In terms of the Mandelstam variables, we have

$$i\mathcal{M} = ie^2 \left(\frac{\left[\bar{u}_1' \gamma^\mu u_1\right] \left[\bar{v}_2 \gamma_\mu v_2'\right]}{\hat{t}} - \frac{\left[\bar{v}_2 \gamma^\mu u_1\right] \left[\bar{u}_1' \gamma_\mu v_2'\right]}{\hat{s}} \right). \tag{15}$$

So, the mean square of the matrix element is

$$\overline{|\mathcal{M}|^2} = \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^2}{4} \left(\frac{T_{11}}{\hat{t}^2} + \frac{T_{22}}{\hat{s}^2} - \frac{T_{12} + T_{21}}{\hat{s}\hat{t}} \right) \tag{16}$$

with

$$T_{11} = 32 \left[(\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2)^2 + (\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2')^2 + 2m^2(m^2 - \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_1') \right], \tag{17}$$

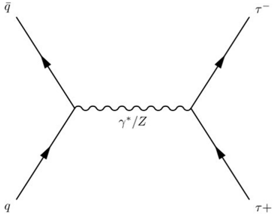
$$T_{22} = 32 \left[(\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_1')^2 + (\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2')^2 + 2m^2(m^2 + \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2) \right], \tag{18}$$

$$-T_{12} = -T_{21} = 32 \left[(\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2') + m^2 \left(\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2' + \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_1' \right) + m^4. \right] \tag{19}$$

Photon and Z-boson interference, $q\bar{q} \longrightarrow \tau^+\tau^-$

The squared matrix element can be written as

$$\begin{aligned}|\mathcal{M}|^2 &= \left|\mathcal{M}_{\gamma^*} + \mathcal{M}_Z\right| \\ &= \left|\mathcal{M}_{\gamma^*}\right|^2 + |\mathcal{M}_Z|^2 + 2\,\mathrm{Re}\left(\mathcal{M}^*_{\gamma^*}\mathcal{M}_Z\right).\end{aligned}$$



In madgraph, with the sm model, we have

- `q q~ → Z → ta+ ta-` for $|\mathcal{M}_Z|^2$.
- `q q~ → a → ta+ ta-` for $\left|\mathcal{M}_{\gamma^*}\right|^2$.
- `q q~ → ta+ ta- / h QED=2` for $|\mathcal{M}|^2$.

Photon and Z-boson interference, $q\bar{q} \longrightarrow \tau^+\tau^-$

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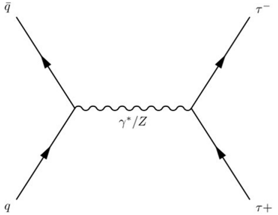
$$



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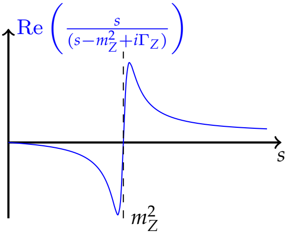
- `q q~ → ta+ ta- / h QED=2` for $|\mathcal{M}|^2$.

For the case $q_R\bar{q}_L \longrightarrow \tau_L^+\tau_R^-$, the amplitudes are

$$\left|\mathcal{M}_{\gamma^*}\right|^2 = e^4\left[Q^{(f)}Q^{(q)}\right]^2[1+\cos\theta]^2$$

$$|\mathcal{M}_Z|^2 = \frac{s^2 g_Z^4\left[g_R^{(f)}g_R^{(q)}\right]^2}{(s-m_Z^2)^2 + (m_Z\Gamma_Z)^2}[1+\cos\theta]^2$$

$$\mathcal{M}^*_{\gamma^*}\mathcal{M}_Z = \frac{g_Z^2 e^2 Q^{(f)}Q^{(q)}g_R^{(f)}g_R^{(q)}}{(s-m_Z^2+i\Gamma_Z)}s\,(1+\cos\theta)^2$$



The amplitude for the $gg(\to \Phi) \longrightarrow t\bar{t}$ process is

$$\mathcal{M}^{\Phi}_{gg\to t\bar{t}} = -\sum_{\Phi} \frac{\mathcal{M}_{gg\Phi}\hat{s}\mathcal{M}_{\Phi t\bar{t}}}{\hat{s} - m_{\Phi}^2 + im_{\Phi}\Gamma_{\Phi}} + \mathcal{M}^{\text{QCD}}_{gg\to t\bar{t}} \,, \tag{20}$$

And the BSM contributions $\frac{\mathrm{d}\hat{\sigma}_S}{\mathrm{d}z}$, $\frac{\mathrm{d}\hat{\sigma}_I}{\mathrm{d}z}$ to the QCD XS $\frac{\mathrm{d}\hat{\sigma}_B}{\mathrm{d}z}$ are

$$\frac{\mathrm{d}\hat{\sigma}_S}{\mathrm{d}z} = \frac{3\alpha_s^2 G_F^2 m_t^2}{8192\pi^3}\hat{s}^2 \sum_{\Phi} \frac{\hat{\beta}_t^{p_\Phi} \left|\hat{g}^2_{\Phi tt} A^{\Phi}_{1/2}(\tau_t)\right|^2}{\left(s - M_{\Phi}^2\right)^2 + \Gamma_{\Phi}^2 M_{\Phi}^2}$$

$$\frac{\mathrm{d}\hat{\sigma}_I}{\mathrm{d}z} = -\frac{\alpha_s^2 G_F m_t^2}{64\sqrt{2}\pi} \frac{1}{1 - \hat{\beta}_t^2 z^2} \mathrm{Re}\left[\sum_{\Phi} \frac{\hat{\beta}_t^{p_\Phi} \hat{g}^2_{\Phi tt} A^{\Phi}_{1/2}(\tau_t)}{s - M_{\Phi}^2 + i\Gamma_{\Phi} M_{\Phi}}\right]$$

where $\hat{\beta}_t = \sqrt{1 - 4m_t^2/\hat{s}}$, $\tau_t = 4m_t^2/\hat{s}$, $z = \cos\theta$, and $p_\Phi \in \{3, 1\}$.

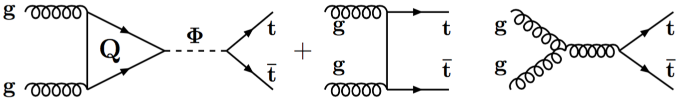
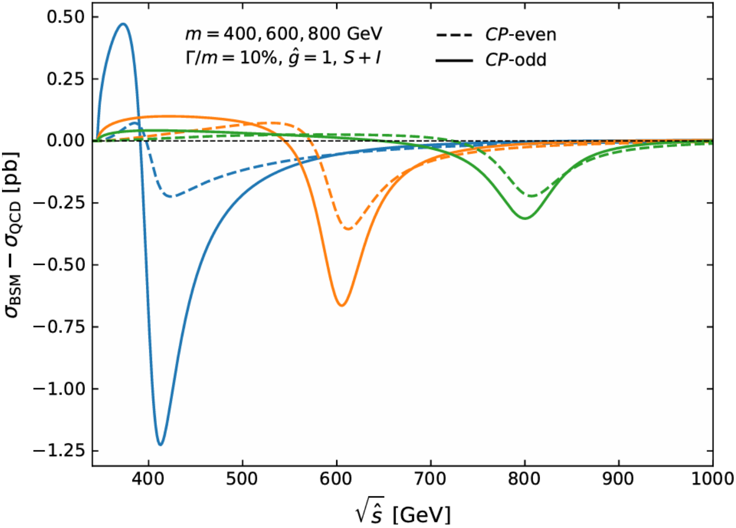
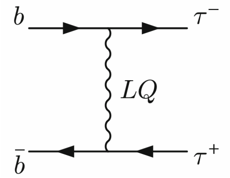
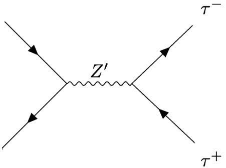


Figure 1: *Feynman diagrams for the signal process $gg \to \Phi \to t\bar{t}$ and the QCD process $gg \to t\bar{t}$ that is the dominant background at the LHC. The state Φ may represent either a CP–even state H or a CP–odd A state.*


$m = 400, 600, 800$ GeV
$\Gamma/m = 10\%$, $\hat{g} = 1$, $S + I$
CP-even
CP-odd
$\sigma_{\mathrm{BSM}} - \sigma_{\mathrm{QCD}}$ [pb]
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
400
500
600
700
800
900
1000
$\sqrt{\hat{s}}$ [GeV]

Interferences between the Z' and the Vector Leptoquark





The amplitudes has the form

$$\mathcal{M}_{LQ} \sim \frac{1}{t - m_{LQ}^2 + i m_{LQ}\Gamma_{LQ}},$$

$$\mathcal{M}_{Z'} \sim \frac{1}{s - m_{Z'}^2 + i m_{Z'}\Gamma_{Z'}},$$

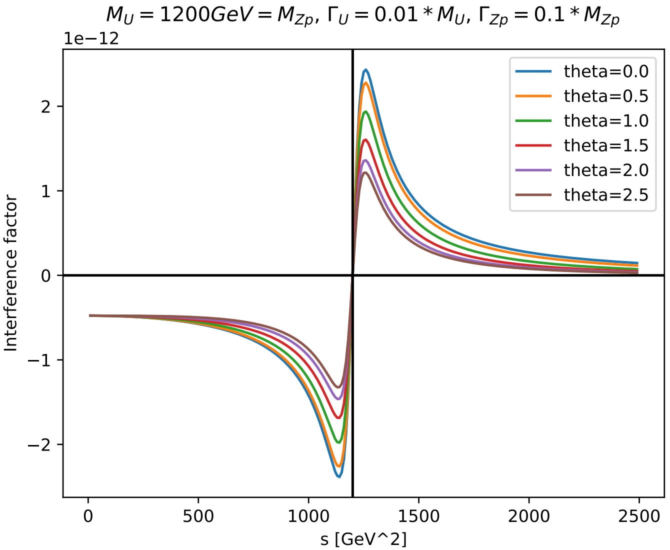
so the interference has the form

$$\sim \frac{m_{LQ}m_{Z'}\Gamma_{LQ}\Gamma_{Z'} - (t - m_{LQ}^2)(s - m_{Z'}^2)}{\left[(t - m_{LQ}^2)^2 + m_{LQ}^2\Gamma_{LQ}^2\right]\left[(s - m_{Z'}^2)^2 + m_{Z'}^2\Gamma_{Z'}^2\right]}.$$

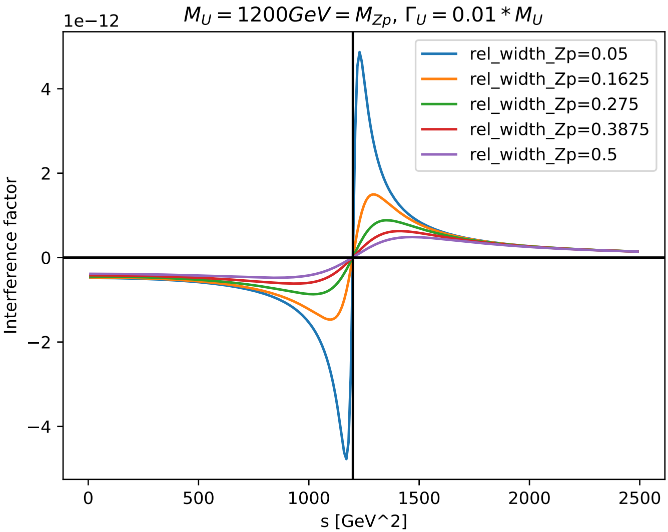
with $t = (p_1 - p_3)^2 = (E_1 - E_3)^2 - (\vec{\mathbf{p}}_1 - \vec{\mathbf{p}}_3)^2$ and in CM frame, as $m_3 = m_4$ and $m_1 = m_2$ with $E_1 = E_2 = E_3 = E_4 = E$, we have

$$t = -(\vec{\mathbf{p}}_1 - \vec{\mathbf{p}}_3)^2 = -\vec{\mathbf{p}}_1^2 - \vec{\mathbf{p}}_3^2 + 2\vec{\mathbf{p}}_1\vec{\mathbf{p}}_3 \tag{21}$$

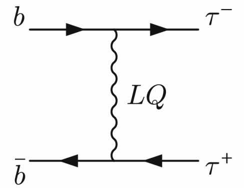
where $\vec{\mathbf{p}}_1^2 = E^2 - m_b^2$ and $\vec{\mathbf{p}}_3^2 = E^2 - m_\tau^2$.

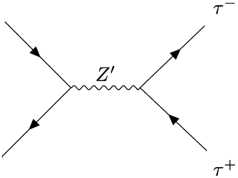


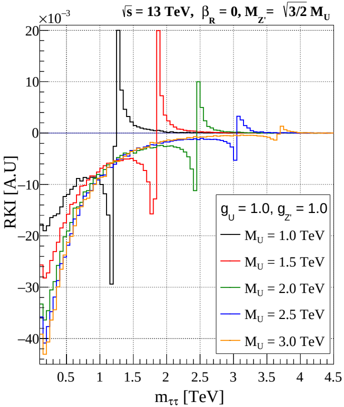
$M_U = 1200GeV = M_{Zp}$, $\Gamma_U = 0.01 * M_U$, $\Gamma_{Zp} = 0.1 * M_{Zp}$
1e−12
2
1
0
−1
−2
Interference factor
0
500
1000
1500
2000
2500
s [GeV^2]
theta=0.0
theta=0.5
theta=1.0
theta=1.5
theta=2.0
theta=2.5


$M_U = 1200GeV = M_{Zp}$, $\Gamma_U = 0.01 * M_U$
1e−12
rel_width_Zp=0.05
rel_width_Zp=0.1625
rel_width_Zp=0.275
rel_width_Zp=0.3875
rel_width_Zp=0.5
4
2
0
−2
−4
Interference factor
0
500
1000
1500
2000
2500
s [GeV^2]

Interferences between the Z' and the Vector Leptoquark







$$\mathrm{RKI} = \frac{1}{\sigma_{LQ+Z'}} \frac{\mathrm{d}}{\mathrm{d}m} \left[\sigma_{LQ+Z'} - \left(\sigma_{LQ} + \sigma_{Z'} \right) \right]$$