

Interference effects

Cristian Fernando Rodríguez Cruz

Authors:

A. Flórez¹, **C. Rodríguez**¹, J. Reyes-Vega¹,
J. Jones-Pérez².

¹Universidad de los Andes

²Pontificia Universidad Católica del Perú

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Two body scattering

CM-Frame

Consider the process

$$A(\vec{p}_1) + B(\vec{p}_2) \longrightarrow C(\vec{p}_3) + D(\vec{p}_4), \quad (1)$$



From the Golden Rule, the cross section is given by

$$\sigma = \frac{S(2\pi)^4}{4\sqrt{(\vec{p}_1 \cdot \vec{p}_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \frac{d^3 \vec{p}_3}{(2\pi)^3 2E_3} \frac{d^3 \vec{p}_4}{(2\pi)^3 2E_4}. \quad (2)$$

But, in the CM frame, $\vec{p}_1 + \vec{p}_2 = 0$, where

$$\sqrt{(\vec{p}_1 \cdot \vec{p}_2)^2 - (m_1 m_2)^2} = E_1 E_2 |\vec{p}_1|, \quad (3)$$

$$\delta^{(4)}(p_1 + p_2 - p_3 - p_4) = \delta(E_1 + E_2 - E_3 - E_4) \delta^{(3)}(\vec{p}_3 + \vec{p}_4). \quad (4)$$

Thus

$$\sigma = \left(\frac{1}{8\pi}\right)^2 \frac{S}{(E_1 E_2) |\vec{p}_1|} \int |\mathcal{M}|^2 \frac{\delta(E_1 + E_2 - \sqrt{\vec{p}_3^2 + m_3^2} - \sqrt{\vec{p}_3^2 + m_4^2})}{\sqrt{\vec{p}_3^2 + m_3^2} \sqrt{\vec{p}_3^2 + m_4^2}} d\vec{p}_3 \quad (5)$$

Two body scattering

CM-Frame

Integrating over the radial part $|\vec{p}_3|$, we get

$$\sigma = \left(\frac{1}{8\pi}\right)^2 \frac{S|\vec{p}_3|}{(E_1 + E_2)^2 |\vec{p}_1|} \int |\mathcal{M}|^2 d\Omega, \quad (6)$$

with

$$|\vec{p}_3| = \frac{1}{2} \frac{\sqrt{((E_1 + E_2)^2 - m_3^2 - m_4^2)^2 - 4m_3^2 m_4^2}}{E_1 + E_2}, \quad (7)$$

the outgoing momentum in the CM frame.

We prefer work with differential cross section as

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{S}{(E_1 + E_2)^2} \frac{|\vec{p}_3|}{|\vec{p}_1|} |\mathcal{M}|^2. \quad (8)$$

Note that at this point, we don't need to know the explicit form of the matrix element \mathcal{M} , so it is a generic result.

Two body scattering

CM-Frame

Defining $\sqrt{s} = E_1 + E_2$, we have

$$|\vec{p}_3| = \frac{1}{2} \frac{\sqrt{(s - m_3^2 - m_4^2)^2 - 4m_3^2 m_4^2}}{\sqrt{s}}, \quad |\vec{p}_1| = \frac{1}{2} \frac{\sqrt{(s - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2}}{\sqrt{s}}. \quad (9)$$

so the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{S}{s} \sqrt{\frac{(s - (m_3 + m_4)^2)(s - (m_3 - m_4)^2)}{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)}} |\mathcal{M}|^2. \quad (10)$$

In general, there are three Lorentz-invariant useful kinematical variables to describe the scattering process, known as Mandelstam variables:

$$\hat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2 = m_1^2 + m_2^2 + 2p_1^\mu p_{2\mu} = m_3^2 + m_4^2 + 2p_3^\mu p_{4\mu}, \quad (11)$$

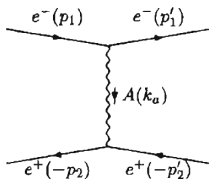
$$\hat{t} = (p_1 - p_3)^2 = (p_2 - p_4)^2 = m_1^2 + m_3^2 - 2p_1^\mu p_{3\mu} = m_2^2 + m_4^2 - 2p_2^\mu p_{4\mu}, \quad (12)$$

$$\hat{u} = (p_1 - p_4)^2 = (p_2 - p_3)^2 = m_1^2 + m_4^2 - 2p_1^\mu p_{4\mu} = m_2^2 + m_3^2 - 2p_2^\mu p_{3\mu}. \quad (13)$$

In the CM-frame, $\hat{s} = s = (E_1 + E_2)^2$.

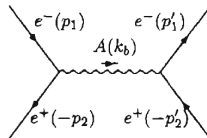
QED $e^+e^- \rightarrow e^+e^-$ scattering

At tree level, there are two Feynman diagrams that contribute to the process



$$\begin{aligned} & [\bar{u}'_1 (ie\gamma^\mu) u_1] iD_{\mu\nu}(k_a) [\bar{v}_2 (ie\gamma^\nu) v'_2] \\ &= -ie^2 \frac{[\bar{u}'_1 \gamma^\mu u_1] [\bar{v}_2 \gamma_\mu v'_2]}{k_a^2} \end{aligned}$$

$$k_a = p_1 - p'_1 = p'_2 - p_2$$



$$\begin{aligned} & [\bar{v}_2 (ie\gamma^\mu) u_1] iD_{\mu\nu}(k_b) [\bar{u}'_1 (ie\gamma^\nu) v'_2] \\ &= -ie^2 \frac{[\bar{v}_2 \gamma^\mu u_1] [\bar{u}'_1 \gamma_\mu v'_2]}{k_b^2} \end{aligned}$$

$$k_b = p_1 + p_2 = p'_2 + p'_1$$

The fermionic exchange between the initial positron and the final electron is the same in both diagrams, so we have a relative minus sign between the two contributions.

$$i\mathcal{M} = ie^2 \left(\frac{[\bar{u}'_1 \gamma^\mu u_1] [\bar{v}_2 \gamma_\mu v'_2]}{(p_1 - p'_1)^2} - \frac{[\bar{v}_2 \gamma^\mu u_1] [\bar{u}'_1 \gamma_\mu v'_2]}{(p_1 + p_2)^2} \right), \quad (14)$$

QED $e^+e^- \rightarrow e^+e^-$ scattering

In terms of the Mandelstam variables, we have

$$i\mathcal{M} = ie^2 \left(\frac{[\bar{u}'_1 \gamma^\mu u_1] [\bar{v}_2 \gamma_\mu v'_2]}{\hat{t}} - \frac{[\bar{v}_2 \gamma^\mu u_1] [\bar{u}'_1 \gamma_\mu v'_2]}{\hat{s}} \right). \quad (15)$$

So, the mean square of the matrix element is

$$\overline{|\mathcal{M}|^2} = \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^2}{4} \left(\frac{T_{11}}{\hat{t}^2} + \frac{T_{22}}{\hat{s}^2} - \frac{T_{12} + T_{21}}{\hat{s}\hat{t}} \right) \quad (16)$$

with

$$T_{11} = 32 \left[(\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2)^2 + (\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}'_2)^2 + 2m^2(m^2 - \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}'_1) \right], \quad (17)$$

$$T_{22} = 32 \left[(\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}'_1)^2 + (\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}'_2)^2 + 2m^2(m^2 + \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2) \right], \quad (18)$$

$$-T_{12} = -T_{21} = 32 \left[(\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}'_2) + m^2 (\vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}'_2 + \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}'_1) + m^4 \right]. \quad (19)$$

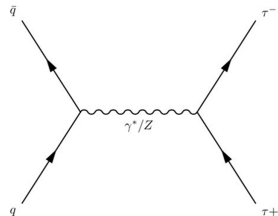
Photon and Z-boson interference, $q\bar{q} \rightarrow \tau^+\tau^-$

The squared matrix element can be written as

$$\begin{aligned} |\mathcal{M}|^2 &= |\mathcal{M}_{\gamma^*} + \mathcal{M}_Z| \\ &= |\mathcal{M}_{\gamma^*}|^2 + |\mathcal{M}_Z|^2 + 2 \operatorname{Re}(\mathcal{M}_{\gamma^*}^* \mathcal{M}_Z). \end{aligned}$$

In madgraph, with the sm model, we have

- $q \bar{q} \rightarrow Z \rightarrow \tau^+ \tau^-$ for $|\mathcal{M}_Z|^2$.
- $q \bar{q} \rightarrow \gamma^* \rightarrow \tau^+ \tau^-$ for $|\mathcal{M}_{\gamma^*}|^2$.
- $q \bar{q} \rightarrow \tau^+ \tau^-$ / h QED=2 for $|\mathcal{M}|^2$.



Photon and Z-boson interference, $q\bar{q} \rightarrow \tau^+\tau^-$

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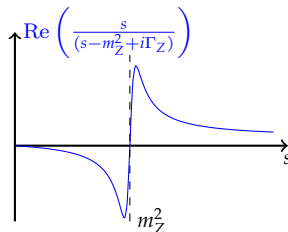
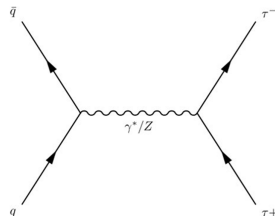
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- $q\bar{q} \rightarrow \tau^+\tau^-$ / h QED=2 for $|\mathcal{M}|^2$.

For the case $q_R\bar{q}_L \rightarrow \tau_L^+\tau_R^-$, the amplitudes are

$$\begin{aligned} |\mathcal{M}_{\gamma^*}|^2 &= e^4 [Q^{(f)} Q^{(q)}]^2 [1 + \cos \theta]^2 \\ |\mathcal{M}_Z|^2 &= \frac{s^2 g_Z^4 [\mathcal{g}_R^{(f)} \mathcal{g}_R^{(q)}]^2}{(s - m_Z^2)^2 + (m_Z \Gamma_Z)^2} [1 + \cos \theta]^2 \\ \mathcal{M}_{\gamma^*}^* \mathcal{M}_Z &= \frac{g_Z^2 e^2 Q^{(f)} Q^{(q)} \mathcal{g}_R^{(f)} \mathcal{g}_R^{(q)}}{(s - m_Z^2 + i\Gamma_Z)} s (1 + \cos \theta)^2 \end{aligned}$$



$gg(\rightarrow \Phi) \longrightarrow t\bar{t}$ BSM scalar interference

The amplitude for the $gg(\rightarrow \Phi) \longrightarrow t\bar{t}$ process is

$$\mathcal{M}_{gg \rightarrow t\bar{t}}^{\Phi} = - \sum_{\Phi} \frac{\mathcal{M}_{gg\Phi} \hat{s} \mathcal{M}_{\Phi t\bar{t}}}{\hat{s} - m_{\Phi}^2 + im_{\Phi} \Gamma_{\Phi}} + \mathcal{M}_{gg \rightarrow t\bar{t}}^{\text{QCD}}, \quad (20)$$

And the BSM contributions $\frac{d\hat{\sigma}_S}{dz}$, $\frac{d\hat{\sigma}_I}{dz}$ to the QCD XS $\frac{d\hat{\sigma}_B}{dz}$ are

$$\frac{d\hat{\sigma}_S}{dz} = \frac{3\alpha_s^2 G_F^2 m_t^2}{8192\pi^3} \hat{s}^2 \sum_{\Phi} \frac{\hat{\beta}_t^{p_{\Phi}} \left| \hat{g}_{\Phi t\bar{t}}^2 A_{1/2}^{\Phi}(\tau_t) \right|^2}{(s - M_{\Phi}^2)^2 + \Gamma_{\Phi}^2 M_{\Phi}^2}$$

$$\frac{d\hat{\sigma}_I}{dz} = - \frac{\alpha_s^2 G_F^2 m_t^2}{64\sqrt{2}\pi} \frac{1}{1 - \hat{\beta}_t^2 z^2} \text{Re} \left[\sum_{\Phi} \frac{\hat{\beta}_t^{p_{\Phi}} \hat{g}_{\Phi t\bar{t}}^2 A_{1/2}^{\Phi}(\tau_t)}{s - M_{\Phi}^2 + i\Gamma_{\Phi} M_{\Phi}} \right]$$

where $\hat{\beta}_t = \sqrt{1 - 4m_t^2/\hat{s}}$, $\tau_t = 4m_t^2/\hat{s}$, $z = \cos \theta$, and $p_{\Phi} \in \{3, 1\}$.

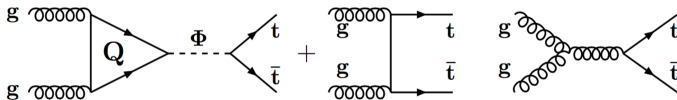
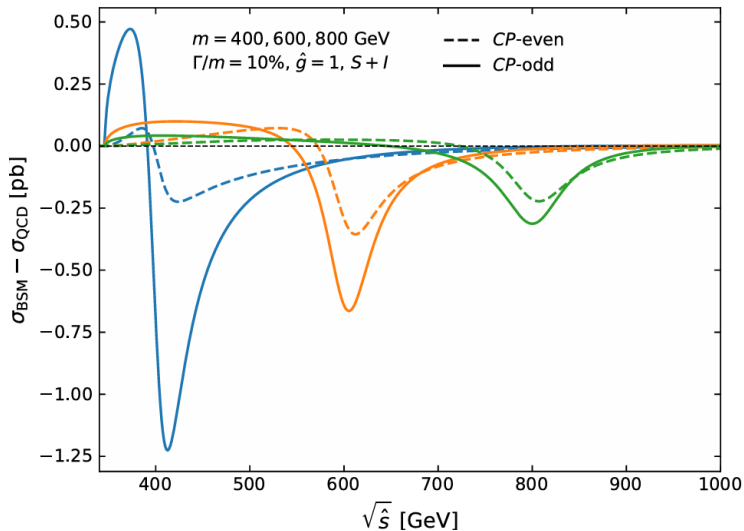
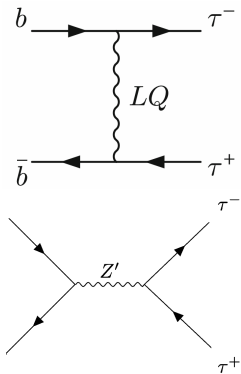


Figure 1: *Feynman diagrams for the signal process $gg \rightarrow \Phi \rightarrow t\bar{t}$ and the QCD process $gg \rightarrow t\bar{t}$ that is the dominant background at the LHC. The state Φ may represent either a CP-even state H or a CP-odd A state.*

$gg(\rightarrow \Phi) \longrightarrow t\bar{t}$
BSM scalar interference



Interferences between the Z' and the Vector Leptoquark



$$RKI = \frac{1}{\sigma_{LQ+Z'}} \frac{d}{dm} \left[\sigma_{LQ+Z'} - \left(\sigma_{LQ} + \sigma_{Z'} \right) \right]$$

