Interference effects

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August 15, 2024

Consider the process

$$A(\vec{\boldsymbol{p}}_1) + B(\vec{\boldsymbol{p}}_2) \longrightarrow C(\vec{\boldsymbol{p}}_3) + D(\vec{\boldsymbol{p}}_4), \tag{1}$$

From the Golden Rule, the cross section is given by

$$\sigma = \frac{S(2\pi)^4}{4\sqrt{(\vec{\boldsymbol{p}}_1 \cdot \vec{\boldsymbol{p}}_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \frac{d^3\vec{\boldsymbol{p}}_3}{(2\pi)^3 2E_3} \frac{d^3\vec{\boldsymbol{p}}_4}{(2\pi)^3 2E_4}. \tag{2}$$

But, in the CM frame, $\vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{p}}_2 = 0$, where

$$\sqrt{(\vec{\boldsymbol{p}}_1 \cdot \vec{\boldsymbol{p}}_2)^2 - (m_1 m_2)^2} = E_1 E_2 \left|\vec{\boldsymbol{p}}_1\right|, \tag{3}$$

$$\delta^{(4)}(p_1 + p_2 - p_3 - p_4) = \delta\left(E_1 + E_2 - E_3 - E_4\right) \delta^{(3)}(\vec{\boldsymbol{p}}_3 + \vec{\boldsymbol{p}}_4). \tag{4}$$

Thus

$$\sigma = \left(\frac{1}{8\pi}\right)^2 \frac{S}{(E_1 E_2)\left|\vec{\boldsymbol{p}}_1\right|} \int |\mathcal{M}|^2 \frac{\delta\left(E_1 + E_2 - \sqrt{\vec{\mathbf{p}}_3^2 + m_3^2} - \sqrt{\vec{\mathbf{p}}_3^2 + m_4^2}\right)}{\sqrt{\vec{\mathbf{p}}_3^2 + m_3^2}\sqrt{\vec{\mathbf{p}}_3^2 + m_4^2}} \mathrm{d}\vec{\mathbf{p}}_3 \tag{5}$$

Two body scattering
CM-Frame

Integrating over the radial part $|\vec{\mathbf{p}}_3|$, we get

$$\sigma = \left(\frac{1}{8\pi}\right)^2 \frac{S|\vec{p}_3|}{(E_1+E_2)^2|\vec{p}_1|} \int |\mathcal{M}|^2 \mathrm{d}\Omega, \tag{6}$$

with

$$|\vec{p}_3| = \frac{1}{2}\frac{\sqrt{((E_1+E_2)^2 - m_3^2 - m_4^2)^2 - 4m_3^2 m_4^2}}{E_1+E_2}, \tag{7}$$

the outgoing momentum in the CM frame.

We prefer work with differential cross section as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{S}{(E_1+E_2)^2}\frac{|\vec{p}_3|}{|\vec{p}_1|}|\mathcal{M}|^2. \tag{8}$$

Note that at this point, we don't need to know the explicit form of the matrix element $\mathcal{M}$, so it is a generic result.

Defining $\sqrt{s} = E_1 + E_2$, we have

$$|\vec{\boldsymbol{p}}_3| = \frac{1}{2}\sqrt{\frac{(s - m_3^2 - m_4^2)^2 - 4m_3^2m_4^2}{s}}, \quad |\vec{\boldsymbol{p}}_1|\frac{1}{2}\sqrt{\frac{(s - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2}{s}}. \tag{9}$$

so the differential cross section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{S}{s}\sqrt{\frac{(s - (m_3 + m_4)^2)(s - (m_3 - m_4)^2)}{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)}}|\mathcal{M}|^2. \tag{10}$$

In general, there are three Lorentz-invariant useful kinematical variables to describe the scattering process, known as Mandelstam variables:

$$\hat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2, \tag{11}$$

$$\hat{t} = (p_1 - p_3)^2 = (p_2 - p_4)^2, \tag{12}$$

$$\hat{u} = (p_1 - p_4)^2 = (p_2 - p_3)^2. \tag{13}$$

In the CM-frame, $\hat{s} = s$, $\hat{t} = -2|\vec{\boldsymbol{p}}_1||\vec{\boldsymbol{p}}_3|\cos\theta$, and $\hat{u} = -2|\vec{\boldsymbol{p}}_2||\vec{\boldsymbol{p}}_3|\cos\theta$.