# Two Higgs Doublet Model(s)

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# Outline

1 Introduction

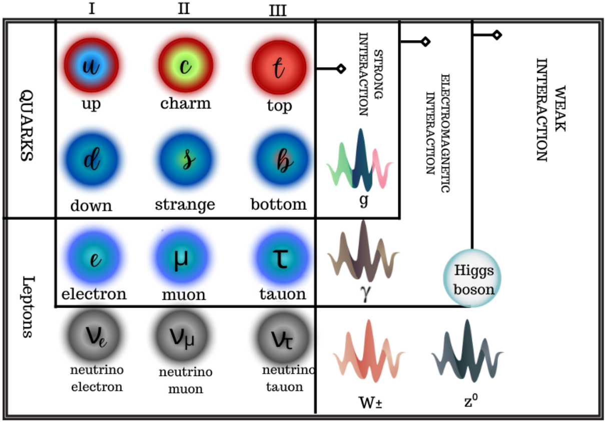
2 On the Scalar Potential

3 Theoretical Constrains
- Unitarity
- Theoretical Constraints

4 Experimental Constraints
- FCNC-free Models
- Allowed Parameter Space for Type II THDM

5 Final Remarks

# The Standard Model Higgs Mechanism




- One Higgs doublet provides EWSB
- Single physical Higgs boson discovered in 2012
- But is this the complete story?

# Why Consider Extended Higgs Sectors?

## BSM Motivations

- **Hierarchy Problem**
  Discrepancy between Higgs mass ($\sim$125 GeV) and Planck scale ($\sim 10^{19}$ GeV)
- **Dark Matter Puzzle**
  No viable DM candidate in SM (WIMP miracle suggests $\sim$100 GeV scale)
- **Baryogenesis Requirements**
  Need strong 1st-order EW phase transition ($v_c/T_c > 1$)
- **Flavor Anomalies**
  Additional CP violation sources needed for $B$-physics observations
- **Theoretical Naturalness**
  SUSY, composite Higgs, etc. often require extended sectors

## The Minimal Extension: Two Higgs Doublet Model (2HDM)

**Simplest framework** addressing some of these challenges

(Different types: Type I, II, Lepton-specific, Flipped with different phenomenology)

# Purely Scalar Extension (ar$\chi$iv:1106.0034)

- With Two Higgs doublets under $SU(2)_L \times U(1)_Y$, the most general potential is

$$V_{\text{tree}} = m_{11}^2 \left(\Phi_1^\dagger \Phi_1\right) + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1\right)^2 + m_{22}^2 \left(\Phi_2^\dagger \Phi_2\right) + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2\right)^2 \tag{1}$$

$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right) \left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right) \left(\Phi_2^\dagger \Phi_1\right) + \frac{1}{2} \left[\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right] \tag{2}$$

$$+ \left[-m_{12}^2 \Phi_1^\dagger \Phi_2 + \lambda_6 \left(\Phi_1^\dagger \Phi_1\right) \left(\Phi_1^\dagger \Phi_2\right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2\right) \left(\Phi_1^\dagger \Phi_2\right) + \text{ h.c. }\right], \tag{3}$$

1. Typical mass-like terms and quartic self-interactions
2. Mixed terms between doublets ($\mathbb{Z}_2$-Even)
3. Mixed terms between doublets (often $\mathbb{Z}_2$-Odd and then forbidden, but $m_{12}^2$ still allowed as soft $\mathbb{Z}_2$-symmetry-breaking term)

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- Written in Kibble parametrization, the two complex scalar doublets are:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}} \left(v_1 + \phi_1 + \mathrm{i}a_1\right) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}} \left(v_2 + \phi_2 + \mathrm{i}a_2\right) \end{pmatrix}.$$

Both doublets have a neutral part with a scalar component $\phi_i$ that acquire a VEV $v_i/\sqrt{2}$, a pseudoscalar component $a_i$, and have a charged part $\phi_i^+$.

- Electroweak constraint: $v_1^2 + v_2^2 = v^2 = (246\ \text{GeV})^2$

# Minimization conditions

The minimization conditions around the vacuum $\Omega$ come from requiring $\frac{\partial V}{\partial \Phi_1}\Big|_\Omega = \frac{\partial V}{\partial \Phi_2}\Big|_\Omega = 0$:

$$\frac{\partial V}{\partial \Phi_1}\Big|_\Omega = v_1 \left[ m_{11}^2 + \frac{\lambda_1}{2} v_1^2 + \frac{\lambda_{345}}{2} v_2^2 \right] - m_{12}^2 v_2 + \frac{3\lambda_6}{2} v_1^2 v_2 + \frac{\lambda_7}{2} v_2^3 = 0 \tag{4}$$

$$\frac{\partial V}{\partial \Phi_2}\Big|_\Omega = v_2 \left[ m_{22}^2 + \frac{\lambda_2}{2} v_2^2 + \frac{\lambda_{345}}{2} v_1^2 \right] - m_{12}^2 v_1 + \frac{\lambda_6}{2} v_1^3 + \frac{3\lambda_7}{2} v_1 v_2^2 = 0 \tag{5}$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$.

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where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$.

In the cases where $\lambda_6 = \lambda_7 = 0$ with the electroweak constraint, $v^2 = v_1^2 + v_2^2 = (246\text{ GeV})^2$ ( $\implies \tan\beta = v_2/v_1, \cos\beta = v_1/v, \sin\beta = v_2/v$),

$$m_{11}^2 = m_{12}^2 \tan\beta - \frac{1}{2} v^2 \left(\lambda_1 \cos^2\beta + \lambda_{345} \sin^2\beta\right) \tag{6}$$

$$m_{22}^2 = m_{12}^2 \cot\beta - \frac{1}{2} v^2 \left(\lambda_2 \sin^2\beta + \lambda_{345} \cos^2\beta\right) \tag{7}$$

Usually, you could see the short notation: $c_\beta = \cos\beta$, $s_\beta = \sin\beta$, $t_\beta = \tan\beta$.

# Scalar Mass Matrices

Expanding at second order in the fields around the vacuum, setting $\lambda_6 = \lambda_7 = 0$, and using the minimization conditions, the mass matrices for scalar components become:

**CP-even scalars ($\phi_1, \phi_2$):**

$$\begin{pmatrix}\phi_1 & \phi_2\end{pmatrix} \begin{pmatrix} m_{12}^2 t_\beta + \lambda_1 v^2 c_\beta^2 & -m_{12}^2 + \frac{\lambda_{345}}{2} v^2 s_{2\beta} \\ -m_{12}^2 + \frac{\lambda_{345}}{2} v^2 s_{2\beta} & m_{12}^2/t_\beta + \lambda_2 v^2 s_\beta^2 \end{pmatrix} \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}$$

**CP-odd scalars ($a_1, a_2$):**

$$\begin{pmatrix}a_1 & a_2\end{pmatrix} \left[m_{12}^2 - \tfrac{1}{2}\lambda_5 v^2 s_{2\beta}\right] \begin{pmatrix} t_\beta & -1 \\ -1 & 1/t_\beta \end{pmatrix} \begin{pmatrix}a_1 \\ a_2\end{pmatrix}$$

**Charged scalars ($\phi_1^\pm, \phi_2^\pm$):**

$$\begin{pmatrix}\phi_1^+ & \phi_2^+\end{pmatrix} \left[m_{12}^2 - \tfrac{1}{4}(\lambda_4 + \lambda_5) v^2 s_{2\beta}\right] \begin{pmatrix} t_\beta & -1 \\ -1 & 1/t_\beta \end{pmatrix} \begin{pmatrix}\phi_1^- \\ \phi_2^-\end{pmatrix}$$

Diagonalization implies two rotation matrices with angles $\alpha$ and $\beta$:

$$\begin{pmatrix} h \\ H \end{pmatrix} = \mathcal{R}(\alpha) \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}, \quad \begin{pmatrix} A \\ G^0 \end{pmatrix} = \mathcal{R}(\beta) \begin{pmatrix}a_1 \\ a_2\end{pmatrix}, \quad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \mathcal{R}(\beta) \begin{pmatrix}\phi_1^\pm \\ \phi_2^\pm\end{pmatrix}$$

# Scalar Mass Spectrum

Diagonalization implies two rotation matrices with angles $\alpha$ and $\beta$:

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After diagonalization, we obtain physical states:

- **CP-even Higgs**: $h$ (SM-like, 125 GeV), $H$ (heavy scalar)
- **CP-odd Higgs**: $A$ (pseudoscalar)
- **Charged Higgs**: $H^\pm$
- **Goldstone bosons**: $G^\pm$, $G^0$ (absorbed by $W^\pm$, $Z^0$)

with eigenvalues:

$$m_{H,h}^2 = \frac{1}{2}\left[M_{P,11}^2 + M_{P,22}^2 \pm \sqrt{(M_{P,11}^2 - M_{P,22}^2)^2 + 4(M_{P,12}^2)^2}\right]$$

$$m_A^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \lambda_5 v^2$$

$$m_{H^\pm}^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \frac{1}{2}(\lambda_4 + \lambda_5) v^2$$

where $M_P^2$ is the CP-even scalar mass matrix.

# Gauge Interactions and Decoupling Limit

The gauge-kinetic Lagrangian is given as

$$\mathcal{L}_g = (D^\mu \Phi_1)^\dagger (D_\mu \Phi_1) + (D^\mu \Phi_2)^\dagger (D_\mu \Phi_2)$$

We obtain the neutral Higgs couplings to $VV(VV \equiv ZZ, WW)$

$$\begin{aligned}\mathcal{L}_g \supset & \frac{g^2 + g'^2}{8} v^2 ZZ \left(1 + 2\frac{h}{v} y_h^V + 2\frac{H}{v} y_H^V\right) \\ & + \frac{g^2}{4} v^2 W^+ W^- \left(1 + 2\frac{h}{v} y_h^V + 2\frac{H}{v} y_H^V\right)\end{aligned}$$

where $y_h^V = \sin(\beta - \alpha)$ and $y_H^V = \cos(\beta - \alpha)$.

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where $y_h^V = \sin(\beta - \alpha)$ and $y_H^V = \cos(\beta - \alpha)$.

## Decoupling Limit

The SM has been frustratingly successful in describing the Higgs boson properties. In order to get a SM-like Higgs boson, we require $\cos(\beta - \alpha) \approx 0$, which implies $\beta - \alpha \approx \pi/2$.
This is known as the **decoupling limit**.

# Lambda Parameters in the Decoupling Limit (ar$\chi$iv:1604.01406)

In the Decoupling limit the scalar mass spectrum simplifies and we can invert the relations to express the $\lambda_i$ in terms of the physical masses:

$$\begin{aligned}
v^2\lambda_1 &= m_h^2 - \frac{t_\beta\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{c_\beta^2} \\
v^2\lambda_2 &= m_h^2 - \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{t_\beta s_\beta^2} \\
v^2\lambda_3 &= m_h^2 + 2m_{H\pm}^2 - 2m_H^2 - \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta} \\
v^2\lambda_4 &= m_A^2 - 2m_{H\pm}^2 + m_H^2 + \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta} \\
v^2\lambda_5 &= m_H^2 - m_A^2 + \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta}
\end{aligned} \tag{8}$$

where $m_h = 125$ GeV is the SM-like Higgs boson mass.

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v^2\lambda_3 &= m_h^2 + 2m_{H\pm}^2 - 2m_H^2 - \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta} \\
v^2\lambda_4 &= m_A^2 - 2m_{H\pm}^2 + m_H^2 + \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta} \\
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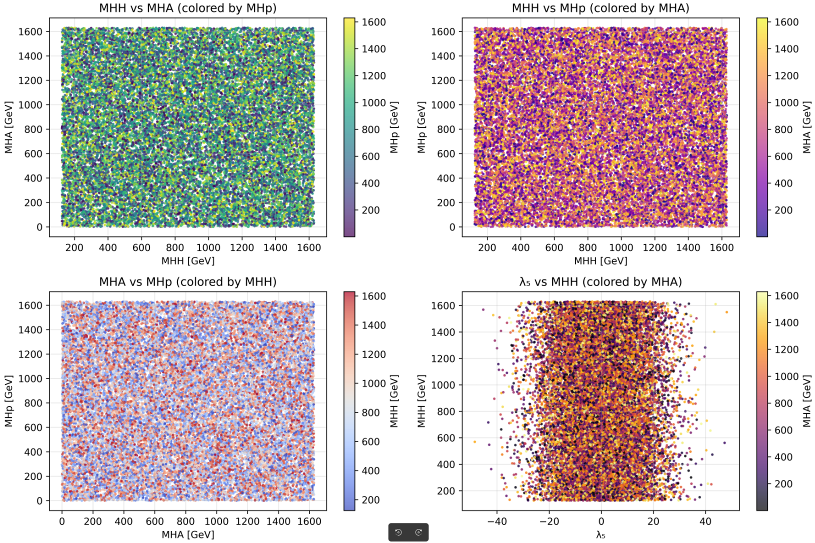
\end{aligned} \tag{8}$$

where $m_h = 125$ GeV is the SM-like Higgs boson mass.
So we have the following free parameters:

- Physical masses: $m_H, m_A, m_{H\pm}$
- Mixing angle: $\beta$ (or $\tan\beta$).
- Soft $\mathbb{Z}_2$-breaking term: $m_{12}^2$ ( I prefeer $\lambda_5$ )

# Initial State of the Scan ($\tan\beta = 10$ fixed)



# Unitarity Constraints (ar$\chi$iv:1805.07306)

From quantum mechanics, the S-matrix must be unitary:

$$S = 1 + iT \quad \Rightarrow \quad T^\dagger T = -i(T - T^\dagger)$$

For scattering processes $s_1 s_2 \to s_3 s_4$, this implies bounds on amplitudes.

The matrix element for the process is defined as:

$$\langle \{s_1, s_2\} | iT | \{s_3, s_4\} \rangle \equiv i\mathcal{M} \delta^4(k_3 + k_4 - p_1 - p_2)(2\pi)^4$$

From the scattering theory, the amplitude $\mathcal{M}$ can be expaded in partial waves:

$$\mathcal{M} = \sum_{J=0}^{\infty} (2J+1) a_J P_J(\cos\theta) \implies a_J \equiv \frac{1}{32\pi} \sqrt{\frac{4\,|\mathbf{p}^{\text{in}}|\,|\mathbf{p}^{\text{out}}|}{2^{\delta_{12}} 2^{\delta_{34}} s}} \int_{-1}^{1} d(x) \mathcal{M} P_J(x)$$

where $P_J$ are Legendre polynomials and $a_J$ are the partial wave amplitudes.
So, we have

$$-\frac{i}{2}\left(a_J - a_J^\dagger\right) \geq a_J a_J^\dagger \implies \text{Re}(a_J) \leq \frac{1}{2} \tag{9}$$

# Unitarity Constraints (ar$\chi$iv:1805.07306)

At large energies, the dominant contributions are the spherically symmetric, $J=0$, partial wave amplitudes and $s \approx |\mathbf{p}|^2$. Thus,

$$\mathrm{Re}\{a_0\} \sim \frac{1}{16\pi}\sqrt{2^{-\delta_{12}-\delta_{34}}}Q_{1234} \leq \frac{1}{2} \tag{10}$$

where $Q_{1234}$ is the quartic coupling of the process $s_1 s_2 \to s_3 s_4$.
Considering all the possible processes,

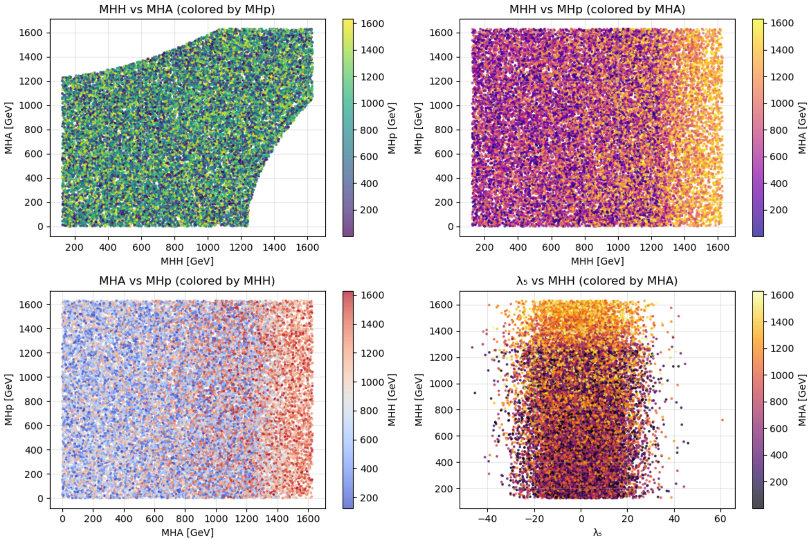
$$|a_\pm|, |b_\pm|, |c_\pm|, |e_\pm|, |f_\pm|, |g_\pm| \leq 8\pi$$

where the eigenvalues are:

$$\begin{aligned} a_\pm &= \frac{3}{2}(\lambda_1+\lambda_2) \pm \sqrt{\frac{9}{4}(\lambda_1-\lambda_2)^2+(2\lambda_3+\lambda_4)^2} & \mathrm{e}_\pm &= \lambda_3+2\lambda_4 \pm 3\lambda_5 \\ b_\pm &= \frac{1}{2}(\lambda_1+\lambda_2) \pm \sqrt{\frac{1}{4}(\lambda_1-\lambda_2)^2+\lambda_4^2} & \mathrm{f}_\pm &= \lambda_3 \pm \lambda_4 \\ c_\pm &= \frac{1}{2}(\lambda_1+\lambda_2) \pm \sqrt{\frac{1}{4}(\lambda_1-\lambda_2)^2+\lambda_5^2} & \mathrm{g}_\pm &= \lambda_3 \pm \lambda_5 \end{aligned} \tag{11}$$

**It is just a safety check, not the strict theory limitation.**

# Example: $|f_+| = |\lambda_3 + \lambda_4| \approx \left|m_h^2 + m_A^2 - m_H^2\right| / v^2 < 8\pi$



# Vacuum Stability Constraints (ar$\chi$iv:2203.07244)

The potential must be bounded from below at large field values.

The dominant quartic terms in the potential are:

$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + \lambda_3 X_1^2 X_2^2 + \lambda_4 X_1^2 X_2^2 \rho^2 + \lambda_5 X_1^2 X_2^2 \rho^2 \cos 2\theta.$$

where the fields have been parametrized as

$$\Phi_1^\dagger \Phi_1 = X_1^2, \quad \Phi_2^\dagger \Phi_2 = X_2^2, \quad \Phi_1^\dagger \Phi_2 = X_1 X_2 \rho\, e^{i\theta} \quad \text{with} \quad 0 \le \rho \le 1.$$

Here, $\rho$ is the normalized magnitude of the inner product between $\Phi_1$ and $\Phi_2$, obtained from the Cauchy–Schwarz inequality:

$$|\Phi_1^\dagger \Phi_2| \le \sqrt{(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)} \;\Rightarrow\; \rho = \frac{|\Phi_1^\dagger \Phi_2|}{X_1 X_2} \in [0, 1].$$

Geometrically, $\rho$ plays the role of the cosine of the angle between the two Higgs doublets in field space: $\rho = 1$ means they are aligned, while $\rho = 0$ means they are orthogonal.

$V_4$ must have a minimum for all directions in field space, i.e., for all $X_1, X_2 \geq 0$ and $\rho \in [0, 1]$.

The term with $\theta$ is $\lambda_5 X_1^2 X_2^2 \rho^2 \cos 2\theta$.

- If $\lambda_5 > 0$, the minimum occurs at $\cos\theta = -1$,
- else if $\lambda_5 < 0$, the minimum occurs at $\cos\theta = 1$.
- In any case, the term becomes $-|\lambda_5| X_1^2 X_2^2 \rho^2$ at the minimum.

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At the minimum, the term with $\rho$ is $(\lambda_4 - |\lambda_5|)\, X_1^2 X_2^2 \rho^2$.

- If $\lambda_4 - |\lambda_5| > 0$, the minimum occurs at $\rho = 0$, then

$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + \lambda_3 X_1^2 X_2^2 \tag{12}$$

  and we require $\lambda_1 > 0$, $\lambda_2 > 0$, and $\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0$.

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At the minimum, the term with $\rho$ is $(\lambda_4 - |\lambda_5|)\, X_1^2 X_2^2 \rho^2$.

- If $\lambda_4 - |\lambda_5| > 0$, the minimum occurs at $\rho = 0$, then

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  and we require $\lambda_1 > 0$, $\lambda_2 > 0$, and $\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0$.
- else if $\lambda_4 - |\lambda_5| < 0$, the minimum occurs at $\rho = 1$, then

$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + (\lambda_3 + \lambda_4 - |\lambda_5|)\, X_1^2 X_2^2 \tag{13}$$

  and we require $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3' + \sqrt{\lambda_1 \lambda_2} > 0$, with $\lambda_3' = \lambda_3 + \lambda_4 - |\lambda_5|$.

In Summary, the bounded from below conditions are:

$$\boxed{\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0, \quad \lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} > 0.}$$

# Electroweak Vacuum Stability (ar$\chi$iv:1303.5098)

- The scalar potential can have multiple minima - we must ensure our EW-breaking vacuum is the **global minimum**

- Metastability could lead to dangerous vacuum decay via quantum tunneling.

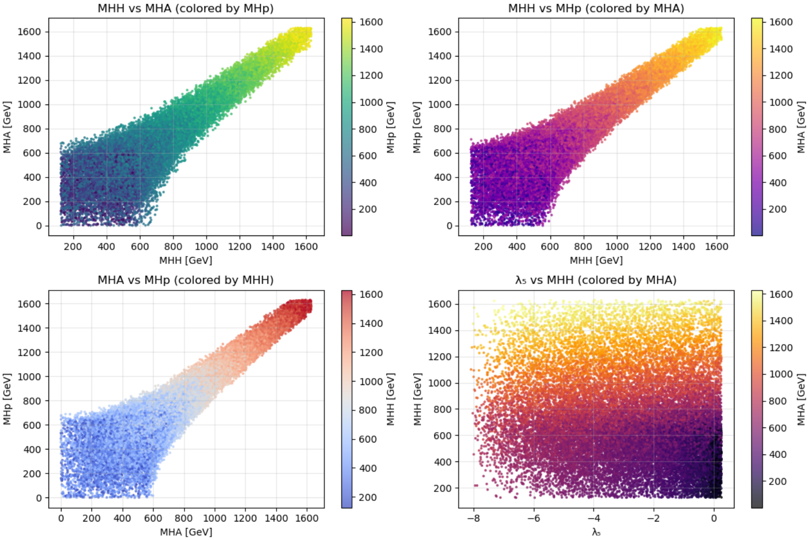
- To guarantee the selected vacuum is truly the lowest-energy state, we require:

$$m_{12}^2 \left(m_{11}^2 - k^2 m_{22}^2\right) (\tan\beta - k) > 0$$

where $k \equiv (\lambda_1/\lambda_2)^{1/4}$ compares the quartic couplings.

- The details of this condition, requires a mapping of the potential as a Minkowskian manifold and stability of the Landau-Ginzburg effective potential, are given in ar$\chi$iv:1303.5098.

# All the theoretical constraints



# Oblique Parameters (PDG 2024, eq 10.98)

$W$-boson mass at one-loop level could have small corrections to the mass:

$$m_W^2 = m_W^2(\mathrm{SM}) + \frac{\alpha c_W^2}{c_W^2 - s_W^2} m_Z^2 \left( -\frac{1}{2} S + c_W^2 T + \frac{c_W^2 - s_W^2}{4 s_W^2} U \right)$$

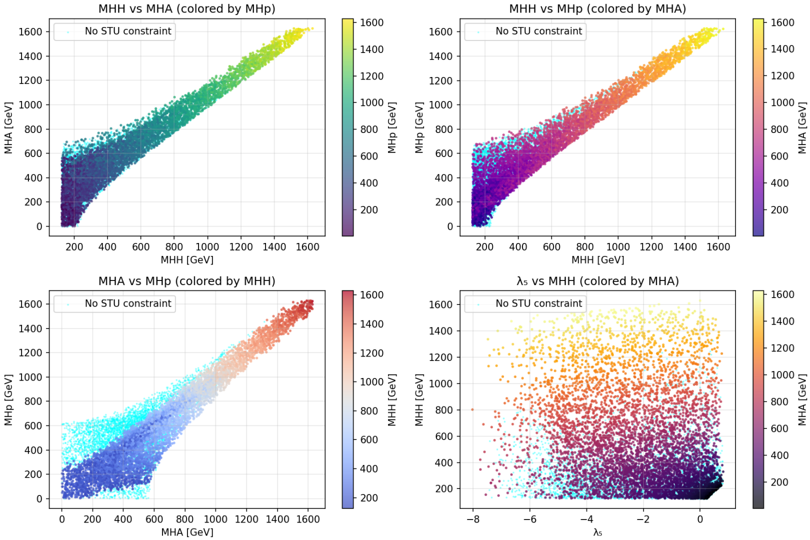
This corrections are parametrized by the oblique parameters $S$, $T$, and $U$ which are complicated loop-function that we calculated in Spheno.

The accuracy of the $W$ boson mass requires :

$$S = -0.04 \pm 0.10, \tag{14}$$
$$T = +0.01 \pm 0.12, \tag{15}$$
$$U = -0.01 \pm 0.09 \tag{16}$$

These parameters are crucial for precision tests of the Standard Model and for constraining new physics scenarios.

# Oblique Parameters Constraints



# Yukawa Sector (ar$\chi$iv:1106.0034)

In order to suppress flavor-changing neutral currents (FCNCs), we impose a $\mathbb{Z}_2$ symmetry on the Yukawa sector resulting in four types of Yukawa interactions:

| Model | $\Phi_2$ | $\Phi_1$ | $u_R^i$ | $d_R^i$ | $e_R^i$ |
|---|---|---|---|---|---|
| Type I | $+$ | $-$ | $+$ | $+$ | $+$ |
| Type II | $+$ | $-$ | $+$ | $-$ | $-$ |
| Lepton-specific | $+$ | $-$ | $+$ | $+$ | $-$ |
| Flipped | $+$ | $-$ | $+$ | $-$ | $+$ |

According to different charge assignments, there are four different models with Yukawa interactions:

$$-\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2 u_R + Y_{d2}\bar{Q}_L\Phi_2 d_R + Y_{\ell 2}\bar{L}_L\Phi_2 e_R + \text{ h.c. ( type I )},$$
$$-\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2 u_R + Y_{d1}\bar{Q}_L\Phi_1 d_R + Y_{\ell 1}\bar{L}_L\Phi_1 e_R + \text{ h.c. ( type II )},$$
$$-\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2 u_R + Y_{d1}\bar{Q}_L\Phi_2 d_R + Y_{\ell 1}\bar{L}_L\Phi_1 e_R + \text{ h.c. ( lepton specific )},$$
$$-\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2 u_R + Y_{d1}\bar{Q}_L\Phi_1 d_R + Y_{\ell 1}\bar{L}_L\Phi_2 e_R + \text{ h.c. ( flipped model) },$$

where $Q_L^T = (u_L, d_L)$, $L_L^T = (\nu_L, l_L)$, $\widetilde{\Phi}_{1,2} = i\tau_2\Phi^*_{1,2}$, and $Y_{u2}, Y_{d1,2}$ and $Y_{\ell 1,2}$ are $3\times 3$ matrices in family space.

# Yukawa sector After EWSB

We can obtain the Yukawa couplings

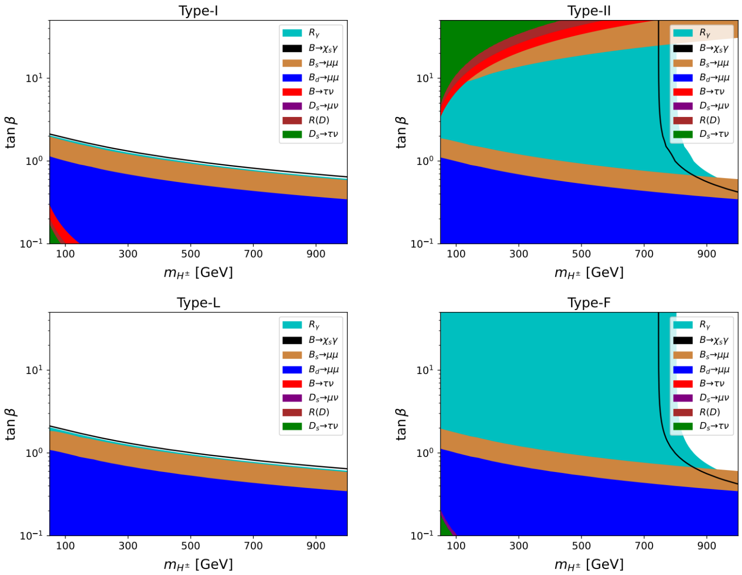
$$
\begin{aligned}
-\mathcal{L}_Y = & \frac{m_f}{v} y_h^f h\bar{f}f + \frac{m_f}{v} y_H^f H\bar{f}f \\
& - i\frac{m_u}{v}\kappa_u A\bar{u}\gamma_5 u + i\frac{m_d}{v}\kappa_d A\bar{d}\gamma_5 d + i\frac{m_\ell}{v}\kappa_\ell A\bar{\ell}\gamma_5\ell \\
& + H^+\bar{u}V_{\text{CKM}}\left(\frac{\sqrt{2}m_d}{v}\kappa_d P_R - \frac{\sqrt{2}m_u}{v}\kappa_u P_L\right) d + h.c. \\
& + \frac{\sqrt{2}m_\ell}{v}\kappa_\ell H^+\bar{\nu}P_R e + h.c.
\end{aligned}
$$

where $y_h^f = \sin(\beta-\alpha) + \cos(\beta-\alpha)\kappa_f$ and $y_H^f = \cos(\beta-\alpha) - \sin(\beta-\alpha)\kappa_f$. The values of $\kappa_u$, $\kappa_d$ and $\kappa_\ell$ for the four models are

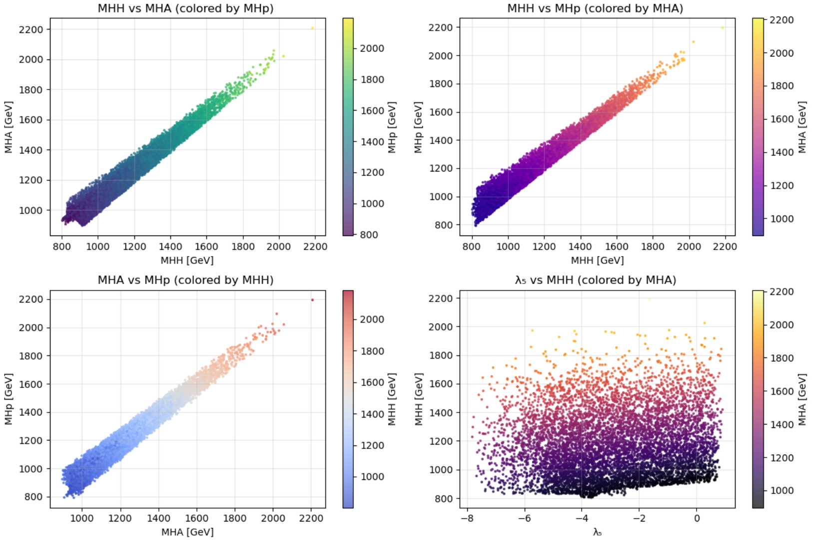
| | type-I | type-II | lepton-specific | flipped |
|---|---|---|---|---|
| $\kappa_u$ | $1/t_\beta$ | $1/t_\beta$ | $1/t_\beta$ | $1/t_\beta$ |
| $\kappa_d$ | $1/t_\beta$ | $-t_\beta$ | $1/t_\beta$ | $-t_\beta$ |
| $\kappa_\ell$ | $1/t_\beta$ | $-t_\beta$ | $-t_\beta$ | $1/t_\beta$ |

These models naturally have preferential couplings to the third generation fermions, due to the Higgs Mechanism.

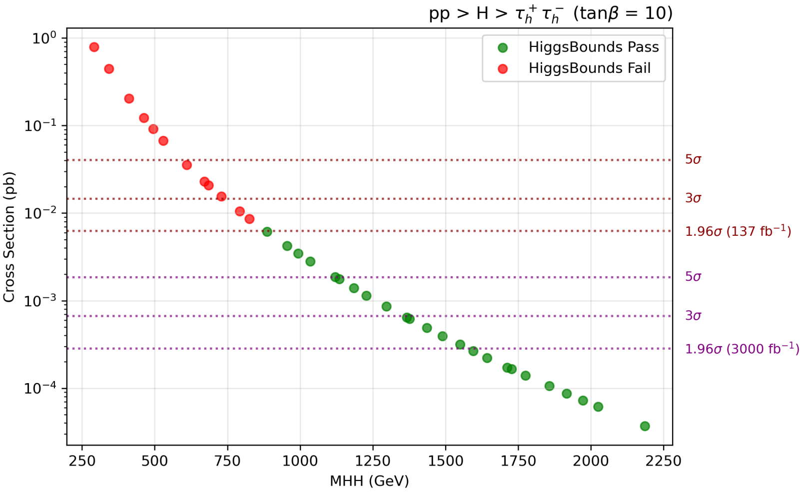
# $H^{\pm}$ Constraints (ar$\chi$iv:2412.04572)



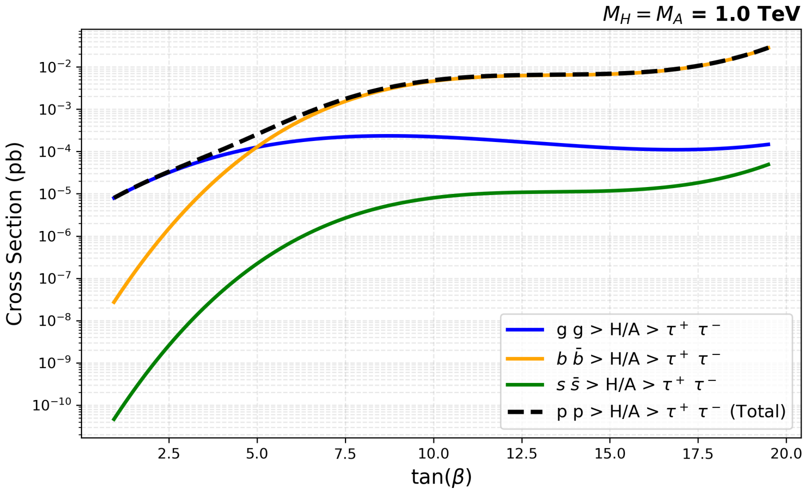
# $m_{H^\pm} \gtrsim 750$ GeV + HB constraints (Work in progress)



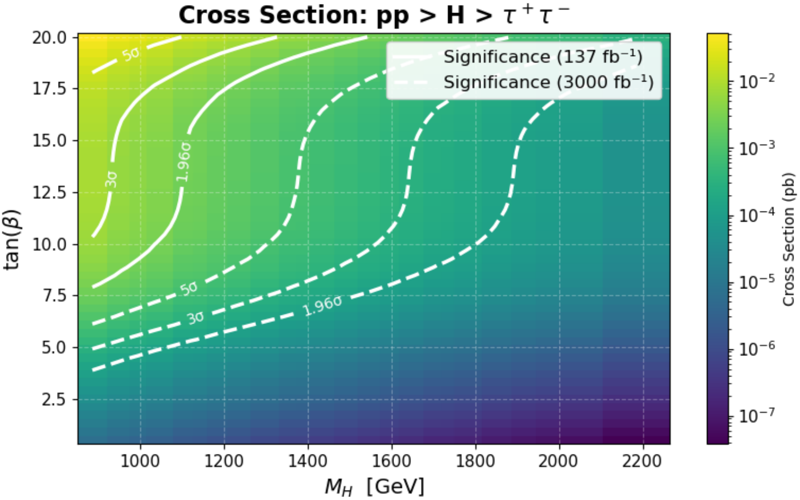
# LHC production cross-section (hadronic channel) (work in progress)



# $\tan\beta$ effects on the cross-section (work in progress)



# High-Lumi Projections (work in progress)



# Summary

## Key Parts from THDM Analysis

- **Theoretical Framework:** Extended SM with two Higgs doublets $\Phi_1, \Phi_2$
  - $\rightarrow$ 5 physical scalars: $h$ (125 GeV), $H, A, H^{\pm}$ + 3 Goldstones
  - $\rightarrow$ Decoupling limit: $\cos(\beta - \alpha) \approx 0$ for SM-like behavior
- **Constraints to the Scalar Potential:**
  - $\rightarrow$ Vacuum stability: $\lambda_1, \lambda_2 > 0$; $\sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 > \text{máx}(\lambda_4, |\lambda_5|)$ bounded-from-below conditions
  - $\rightarrow$ Unitarity: $|a_{\pm}|, |b_{\pm}|, |c_{\pm}|, |e_{\pm}|, |f_{\pm}|, |g_{\pm}| \leq 8\pi$
  - $\rightarrow$ EW precision: Oblique parameters $S, T, U$ within $1\sigma$
- **Type II THDM Phenomenology:**
  - $\rightarrow$ Enhanced $\tau, b$ couplings: $\kappa_d = \kappa_\ell = \tan\beta$
  - $\rightarrow$ Suppressed top couplings: $\kappa_u = 1/\tan\beta$
  - $\rightarrow$ Strong $H^{\pm}$ constraints: $m_{H^\pm} \gtrsim 750$ GeV, $\tan\beta \in [1, 15]$
- **Experimental Constraints On the Yukawa Sector:**
  - $\rightarrow$ LHC searches exclude regions via $H/A \rightarrow \tau\tau, H \rightarrow WW/ZZ$
  - $\rightarrow$ Allowed parameter space identified for future studies

## Main Achievement

Systematic parameter scan establishing **viable THDM Type II regions** consistent with all theoretical and experimental constraints

# Thank you for your attention!

Questions?

*Details to be added based on specific analysis requirements*