Two Higgs Doublet Model(s)

PhD(c). Cristian Fernando Rodríguez Cruz[1]
c.rodriguez45@uniandes.edu.co

Research advisors:

Prof. Andrés Florez[1] (ca.florez@uniandes.edu.co)

Prof. J. Jones-Pérez[2] (jones.j@pucp.edu.pe)

[1]Universidad de los Andes

[2]Pontificia Universidad Católica del Perú

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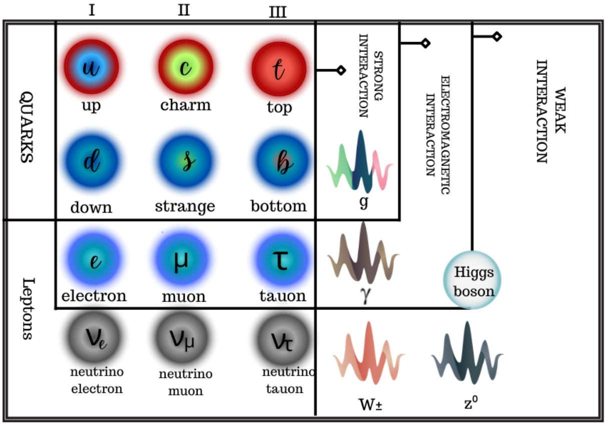
Outline

The Standard Model Higgs Mechanism



- One Higgs doublet provides EWSB
- Single physical Higgs boson discovered in 2012
- But is this the complete story?

Why Consider Extended Higgs Sectors?

BSM Motivations

- **Hierarchy Problem**
  Discrepancy between Higgs mass (∼125 GeV) and Planck scale ($\sim 10^{19}$ GeV)
- **Dark Matter Puzzle**
  No viable DM candidate in SM (WIMP miracle suggests ∼100 GeV scale)
- **Baryogenesis Requirements**
  Need strong 1st-order EW phase transition ($v_c/T_c > 1$)
- **Flavor Anomalies**
  Additional CP violation sources needed for B-physics observations
- **Theoretical Naturalness**
  SUSY, composite Higgs, etc. often require extended sectors

The Minimal Extension: Two Higgs Doublet Model (2HDM)

Simplest framework addressing some of these challenges
(Different types: Type I, II, Lepton-specific, Flipped with different phenomenology)

Purely Scalar Extension

- With Two Higgs doublets under $SU(2)_L \times U(1)_Y$, the most general potential is

$$
\begin{aligned}
V_{\text{tree}} = \; & m_{11}^2 \left(\Phi_1^\dagger \Phi_1\right) + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1\right)^2 + m_{22}^2 \left(\Phi_2^\dagger \Phi_2\right) + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2\right)^2 && (1) \\
& + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right) \left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right) \left(\Phi_2^\dagger \Phi_1\right) + \frac{1}{2} \left[\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right] && (2) \\
& + \left[-m_{12}^2 \Phi_1^\dagger \Phi_2 + \lambda_6 \left(\Phi_1^\dagger \Phi_1\right) \left(\Phi_1^\dagger \Phi_2\right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2\right) \left(\Phi_1^\dagger \Phi_2\right) + \text{h.c.}\right], && (3)
\end{aligned}
$$

1. Typical mass-like terms and quartic self-interactions
2. Mixed terms between doublets ($\mathbb{Z}_2$-Even)
3. Mixed terms between doublets (often $\mathbb{Z}_2$-Odd and then forbidden, but m_{12}^2 still allowed as soft $\mathbb{Z}_2$-symmetry-breaking term)

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&+ \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \frac{1}{2}\left[\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right] && (2)\\
&+ \left[-m_{12}^2\Phi_1^\dagger\Phi_2 + \lambda_6\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_1^\dagger\Phi_2\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right], && (3)
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- Written in Kibble parametrization, the two complex scalar doublets are:

$$
\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}\left(v_1 + \phi_1 + ia_1\right) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}\left(v_2 + \phi_2 + ia_2\right) \end{pmatrix}.
$$

Both doublets have a neutral part with a scalar component ϕ_i that acquire a VEV $v_i/\sqrt{2}$, a pseudoscalar component a_i, and have a charged part ϕ_i^+.

- Electroweak constraint: $v_1^2 + v_2^2 = v^2 = (246\text{ GeV})^2$

Minimization conditions

The minimization conditions around the vacuum Ω come from requiring $\left.\frac{\partial V}{\partial \Phi_1}\right|_{\Omega} = \left.\frac{\partial V}{\partial \Phi_2}\right|_{\Omega} = 0$:

$$\left.\frac{\partial V}{\partial \Phi_1}\right|_{\Omega} = v_1 \left[m_{11}^2 + \frac{\lambda_1}{2} v_1^2 + \frac{\lambda_{345}}{2} v_2^2 \right] - m_{12}^2 v_2 + \frac{3\lambda_6}{2} v_1^2 v_2 + \frac{\lambda_7}{2} v_2^3 = 0 \tag{4}$$

$$\left.\frac{\partial V}{\partial \Phi_2}\right|_{\Omega} = v_2 \left[m_{22}^2 + \frac{\lambda_2}{2} v_2^2 + \frac{\lambda_{345}}{2} v_1^2 \right] - m_{12}^2 v_1 + \frac{\lambda_6}{2} v_1^3 + \frac{3\lambda_7}{2} v_1 v_2^2 = 0 \tag{5}$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$.

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where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$.

In the cases where $\lambda_6 = \lambda_7 = 0$ with the electroweak constraint, $v^2 = v_1^2 + v_2^2 = (246\text{ GeV})^2$ ($\implies \tan\beta = v_2/v_1, \cos\beta = v_1/v, \sin\beta = v_2/v$),

$$m_{11}^2 = m_{12}^2 \tan\beta - \frac{1}{2}v^2\left(\lambda_1\cos^2\beta + \lambda_{345}\sin^2\beta\right) \tag{6}$$

$$m_{22}^2 = m_{12}^2 \cot\beta - \frac{1}{2}v^2\left(\lambda_2\sin^2\beta + \lambda_{345}\cos^2\beta\right) \tag{7}$$

Usually, you could see the short notation: $c_\beta = \cos\beta, s_\beta = \sin\beta, t_\beta = \tan\beta$.

Scalar Mass Matrices

Expanding at second order in the fields around the vacuum, setting $\lambda_6 = \lambda_7 = 0$, and using the minimization conditions, the mass matrices for scalar components become:

CP-even scalars (ϕ_1, ϕ_2):

$$\begin{pmatrix} \phi_1 & \phi_2 \end{pmatrix} \begin{pmatrix} m_{12}^2 t_\beta + \lambda_1 v^2 c_\beta^2 & -m_{12}^2 + \frac{\lambda_{345}}{2} v^2 s_{2\beta} \\ -m_{12}^2 + \frac{\lambda_{345}}{2} v^2 s_{2\beta} & m_{12}^2/t_\beta + \lambda_2 v^2 s_\beta^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

CP-odd scalars (a_1, a_2):

$$\begin{pmatrix} a_1 & a_2 \end{pmatrix} \left[m_{12}^2 - \frac{1}{2}\lambda_5 v^2 s_{2\beta} \right] \begin{pmatrix} t_\beta & -1 \\ -1 & 1/t_\beta \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Charged scalars ($\phi_1^\pm, \phi_2^\pm$):

$$\begin{pmatrix} \phi_1^+ & \phi_2^+ \end{pmatrix} \left[m_{12}^2 - \frac{1}{4}(\lambda_4 + \lambda_5) v^2 s_{2\beta} \right] \begin{pmatrix} t_\beta & -1 \\ -1 & 1/t_\beta \end{pmatrix} \begin{pmatrix} \phi_1^- \\ \phi_2^- \end{pmatrix}$$

Diagonalization implies two rotation matrices with angles α and β:

$$\begin{pmatrix} h \\ H \end{pmatrix} = \mathcal{R}(\alpha) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \begin{pmatrix} A \\ G^0 \end{pmatrix} = \mathcal{R}(\beta) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \mathcal{R}(\beta) \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

Scalar Mass Spectrum

Diagonalization implies two rotation matrices with angles α and β:

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After diagonalization, we obtain physical states:

- **CP-even Higgs**: h (SM-like, 125 GeV), H (heavy scalar)
- **CP-odd Higgs**: A (pseudoscalar)
- **Charged Higgs**: $H^\pm$
- **Goldstone bosons**: $G^\pm$, G^0 (absorbed by $W^\pm$, Z^0)

with eigenvalues:

$$m_{H,h}^2 = \frac{1}{2}\left[M_{P,11}^2 + M_{P,22}^2 \pm \sqrt{(M_{P,11}^2 - M_{P,22}^2)^2 + 4(M_{P,12}^2)^2}\right]$$

$$m_A^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \lambda_5 v^2$$

$$m_{H^\pm}^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \frac{1}{2}(\lambda_4 + \lambda_5)v^2$$

where M_P^2 is the CP-even scalar mass matrix.

Gauge Interactions and Decoupling Limit

The gauge-kinetic Lagrangian is given as

$$\mathcal{L}_{\mathrm{g}} = \left(D^{\mu}\Phi_1\right)^{\dagger}\left(D_{\mu}\Phi_1\right) + \left(D^{\mu}\Phi_2\right)^{\dagger}\left(D_{\mu}\Phi_2\right)$$

We obtain the neutral Higgs couplings to $VV (VV \equiv ZZ, WW)$

$$\begin{aligned}\mathcal{L}_{\mathrm{g}} \supset & \frac{g^2+g'^2}{8}v^2 ZZ\left(1+2\frac{h}{v}y_h^V+2\frac{H}{v}y_H^V\right)\\ &+\frac{g^2}{4}v^2W^+W^-\left(1+2\frac{h}{v}y_h^V+2\frac{H}{v}y_H^V\right)\end{aligned}$$

where $y_h^V = \sin(\beta-\alpha)$ and $y_H^V = \cos(\beta-\alpha)$.

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where $y_h^V = \sin(\beta - \alpha)$ and $y_H^V = \cos(\beta - \alpha)$.

Decoupling Limit

The SM has been frustratingly successful in describing the Higgs boson properties. In order to get a SM-like Higgs boson, we require $\cos(\beta - \alpha) \approx 0$, which implies $\beta - \alpha \approx \pi/2$.
This is known as the **decoupling limit**.

Lambda Parameters in the Decoupling Limit

In the Decoupling limit the scalar mass spectrum simplifies and we can invert the relations to express the λ_i in terms of the physical masses:

$$\begin{aligned}
v^2\lambda_1 &= m_h^2 - \frac{t_\beta\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{c_\beta^2} \\
v^2\lambda_2 &= m_h^2 - \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{t_\beta s_\beta^2} \\
v^2\lambda_3 &= m_h^2 + 2m_{H^\pm}^2 - 2m_H^2 - \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta} \\
v^2\lambda_4 &= m_A^2 - 2m_{H^\pm}^2 + m_H^2 + \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta} \\
v^2\lambda_5 &= m_H^2 - m_A^2 + \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta}
\end{aligned} \tag{8}$$

where $m_h = 125$ GeV is the SM-like Higgs boson mass.

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v^2\lambda_2 &= m_h^2 - \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{t_\beta s_\beta^2} \\
v^2\lambda_3 &= m_h^2 + 2m_{H\pm}^2 - 2m_H^2 - \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta} \\
v^2\lambda_4 &= m_A^2 - 2m_{H\pm}^2 + m_H^2 + \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta} \\
v^2\lambda_5 &= m_H^2 - m_A^2 + \frac{\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{s_\beta c_\beta}
\end{aligned}
\tag{8}
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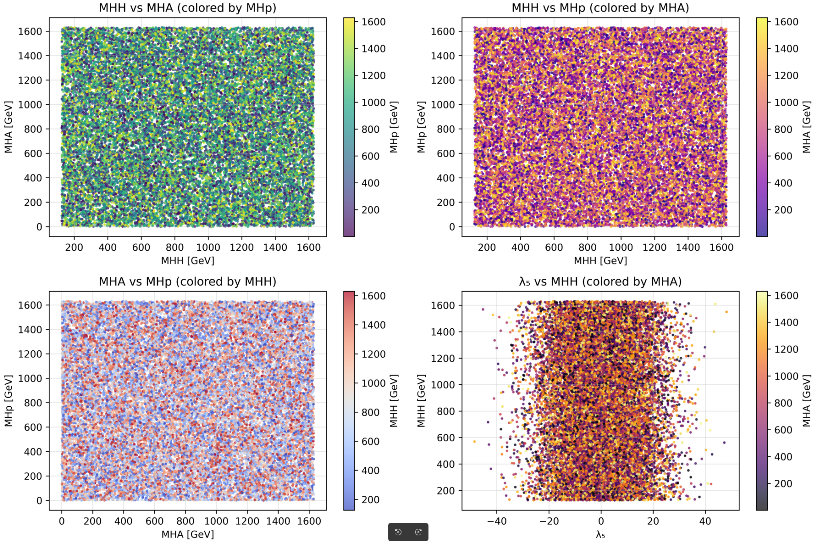
where $m_h = 125$ GeV is the SM-like Higgs boson mass.
So we have the following free parameters:

- Physical masses: $m_H, m_A, m_{H\pm}$
- Mixing angle: β (or $\tan\beta$).
- Soft $\mathbb{Z}_2$-breaking term: m_{12}^2 (I prefeer λ_5)

Initial State of the Scan ($\tan\beta = 10$ fixed)



Vacuum Stability Constraints:

The potential must be bounded from below at large field values.

The dominant quartic terms in the potential are:

$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + \lambda_3 X_1^2 X_2^2 + \lambda_4 X_1^2 X_2^2 \rho^2 + \lambda_5 X_1^2 X_2^2 \rho^2 \cos 2\theta.$$

where the fields have been parametrized as

$$\Phi_1^\dagger \Phi_1 = X_1^2, \quad \Phi_2^\dagger \Phi_2 = X_2^2, \quad \Phi_1^\dagger \Phi_2 = X_1 X_2 \rho \, \mathrm{e}^{i\theta} \quad \text{with} \quad 0 \leq \rho \leq 1.$$

Here, ρ is the normalized magnitude of the inner product between Φ_1 and Φ_2, obtained from the Cauchy–Schwarz inequality:

$$|\Phi_1^\dagger \Phi_2| \leq \sqrt{(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)} \Rightarrow \rho = \frac{|\Phi_1^\dagger \Phi_2|}{X_1 X_2} \in [0, 1].$$

Geometrically, ρ plays the role of the cosine of the angle between the two Higgs doublets in field space: $\rho = 1$ means they are aligned, while $\rho = 0$ means they are orthogonal.

V_4 must have a minimum for all directions in field space, i.e., for all $X_1, X_2 \geq 0$ and $\rho \in [0, 1]$.

The term with θ is $\lambda_5 X_1^2 X_2^2 \rho^2 \cos 2\theta$.

- If $\lambda_5 > 0$, the minimum occurs at $\cos\theta = -1$,
- else if $\lambda_5 < 0$, the minimum occurs at $\cos\theta = 1$.
- In any case, the term becomes $-|\lambda_5| X_1^2 X_2^2 \rho^2$ at the minimum.

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At the minimum, the term with ρ is $(\lambda_4 - |\lambda_5|)\, X_1^2 X_2^2 \rho^2$.

- If $\lambda_4 - |\lambda_5| > 0$, the minimum occurs at $\rho = 0$, then

$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + \lambda_3 X_1^2 X_2^2 \tag{9}$$

and we require $\lambda_1 > 0$, $\lambda_2 > 0$, and $\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0$.

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  and we require $\lambda_1 > 0$, $\lambda_2 > 0$, and $\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0$.
- else if $\lambda_4 - |\lambda_5| < 0$, the minimum occurs at $\rho = 1$, then

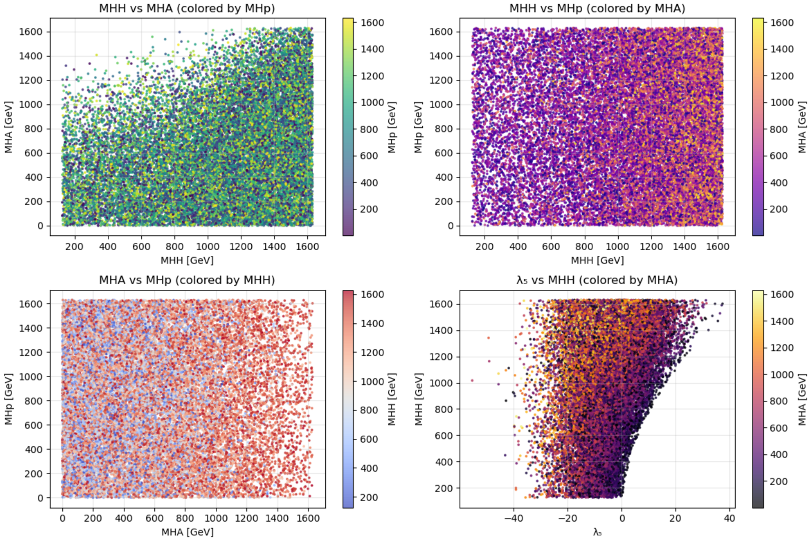
$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + (\lambda_3 + \lambda_4 - |\lambda_5|)\, X_1^2 X_2^2 \tag{10}$$

  and we require $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3' + \sqrt{\lambda_1 \lambda_2} > 0$, with $\lambda_3' = \lambda_3 + \lambda_4 - |\lambda_5|$.

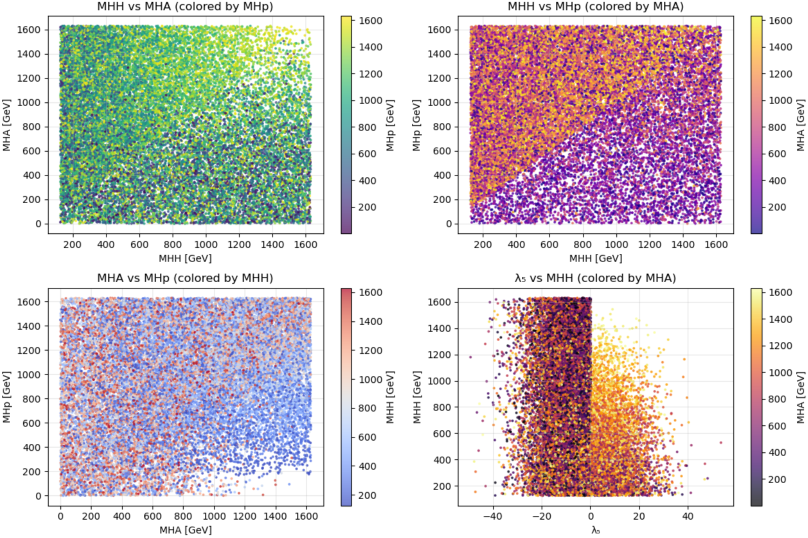
In Summary, the bounded from below conditions are:

$$\boxed{\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0, \quad \lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} > 0.}$$

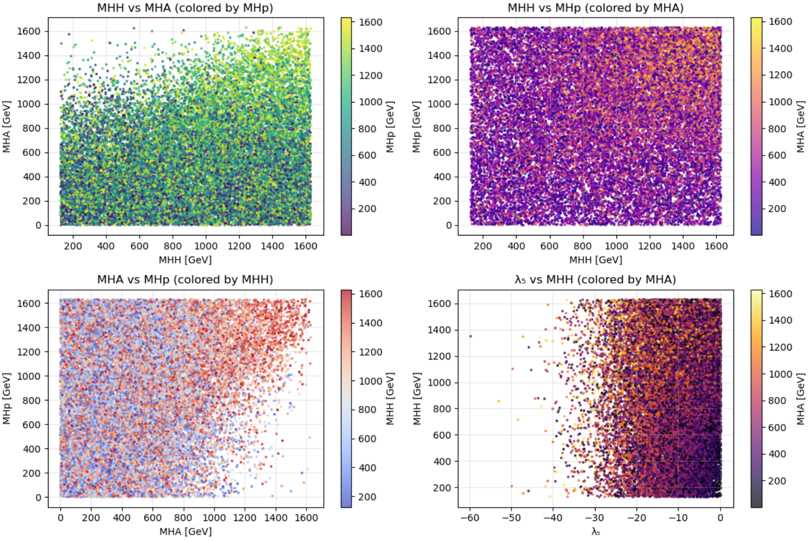
$\lambda_1 > 0, \quad \lambda_2 > 0$ constraints



$\sqrt{\lambda_1\lambda_2} + \lambda_3 + \lambda_4 > \text{máx}(\lambda_4, |\lambda_5|)$



Bounded from Below Conditions



Electroweak Vacuum Stability

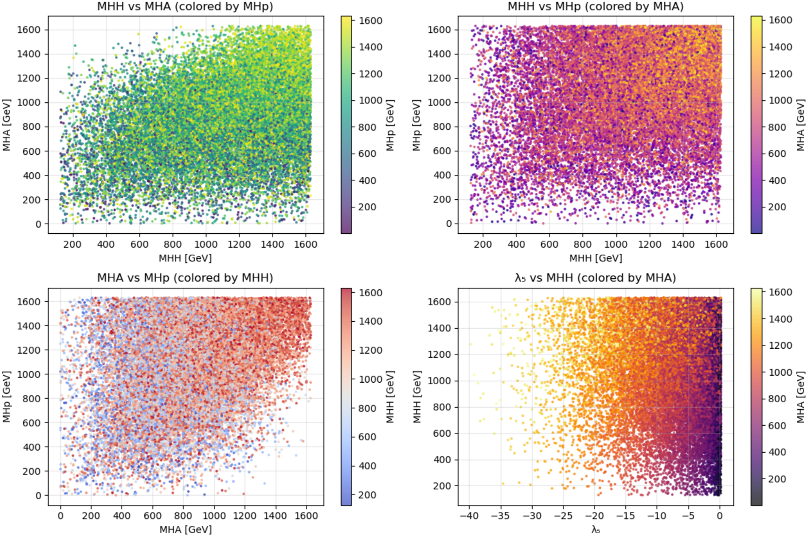
- The scalar potential can have multiple minima - we must ensure our EW-breaking vacuum is the **global minimum**
- Metastability could lead to dangerous vacuum decay via quantum tunneling.
- To guarantee the selected vacuum is truly the lowest-energy state, we require:

$$m_{12}^2 \left(m_{11}^2 - k^2 m_{22}^2 \right) (\tan\beta - k) > 0$$

where $k \equiv (\lambda_1/\lambda_2)^{1/4}$ compares the quartic couplings.

- The details of this condition requires a mapping of the potential as a Minkowskian manifold and stability of the Landau-Ginzburg effective potential is given in Barroso et al., "Metastability bounds on the two Higgs doublet model" (2013).

Vacuum Stability Conditions



Unitarity Constraints

From quantum mechanics, the S-matrix must be unitary:

$$S = 1 + iT \quad \Rightarrow \quad T^\dagger T = -i(T - T^\dagger)$$

For scattering processes $s_1 s_2 \to s_3 s_4$, this implies bounds on amplitudes.

The matrix element for the process is defined as:

$$\langle \{s_1, s_2\} | iT | \{s_3, s_4\} \rangle \equiv i\mathcal{M}\delta^4(k_3 + k_4 - p_1 - p_2)(2\pi)^4$$

From the scattering theory, the amplitude $\mathcal{M}$ can be expaded in partial waves:

$$\mathcal{M} = \sum_{J=0}^{\infty}(2J+1)a_J P_J(\cos\theta) \implies a_J \equiv \frac{1}{32\pi}\sqrt{\frac{4\,|\mathbf{p}^{\text{in}}|\,|\mathbf{p}^{\text{out}}|}{2^{\delta_{12}}2^{\delta_{34}}s}}\int_{-1}^{1} d(x)\mathcal{M}P_J(x)$$

where P_J are Legendre polynomials and a_J are the partial wave amplitudes.
So, we have

$$-\frac{i}{2}\left(a_J - a_J^\dagger\right) \geq a_J a_J^\dagger \implies \text{Re}(a_J) \leq \frac{1}{2} \tag{11}$$

Unitarity Constraints

At large energies, the dominant contributions are the spherically symmetric, $J = 0$, partial wave amplitudes and $s \approx |\mathbf{p}|^2$. Thus,

$$\mathrm{Re}\{a_0\} \sim \frac{1}{16\pi}\sqrt{2^{-\delta_{12}-\delta_{34}}}Q_{1234} \leq \frac{1}{2} \tag{12}$$

where Q_{1234} is the quartic coupling of the process $s_1 s_2 \to s_3 s_4$.
Considering all the possible processes,

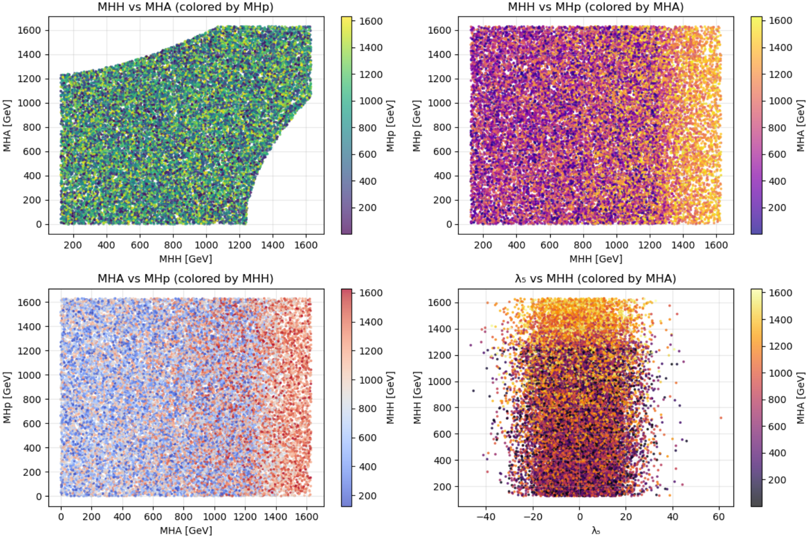
$$|a_\pm|, |b_\pm|, |c_\pm|, |e_\pm|, |f_\pm|, |g_\pm| \leq 8\pi$$

where the eigenvalues are:

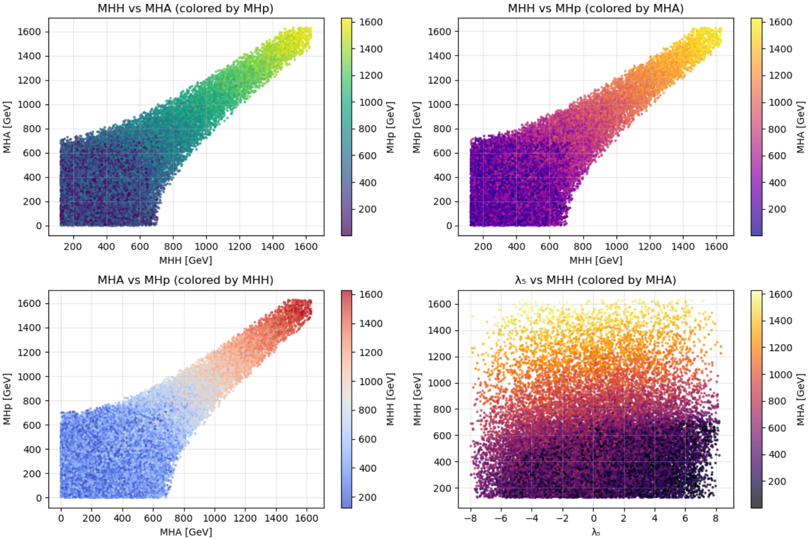
$$\begin{aligned}
a_\pm &= \frac{3}{2}(\lambda_1+\lambda_2) \pm \sqrt{\frac{9}{4}(\lambda_1-\lambda_2)^2 + (2\lambda_3+\lambda_4)^2} & \mathrm{e}_\pm &= \lambda_3 + 2\lambda_4 \pm 3\lambda_5 \\
b_\pm &= \frac{1}{2}(\lambda_1+\lambda_2) \pm \sqrt{\frac{1}{4}(\lambda_1-\lambda_2)^2 + \lambda_4^2} & \mathrm{f}_\pm &= \lambda_3 \pm \lambda_4 \\
c_\pm &= \frac{1}{2}(\lambda_1+\lambda_2) \pm \sqrt{\frac{1}{4}(\lambda_1-\lambda_2)^2 + \lambda_5^2} & \mathrm{g}_\pm &= \lambda_3 \pm \lambda_5
\end{aligned} \tag{13}$$

It is just a safety check, not the strict theory limitation.

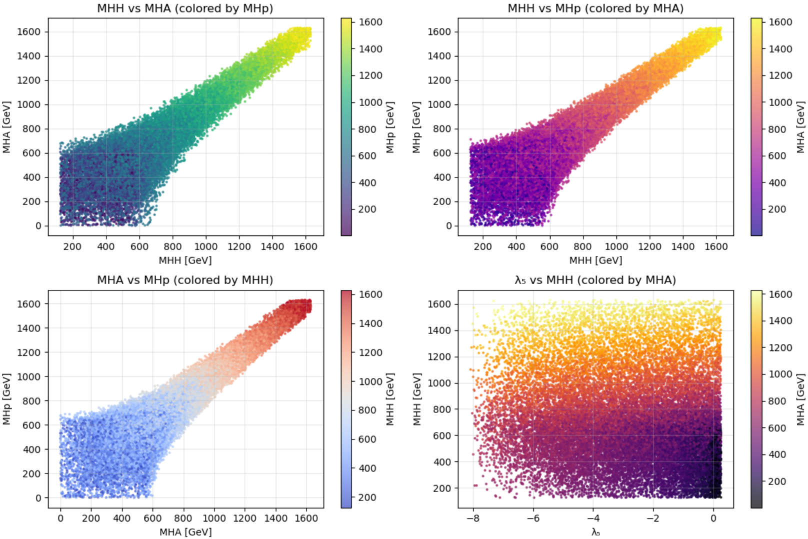
Example: $|f_+| = |\lambda_3 + \lambda_4| \approx \left|m_h^2 + m_A^2 - m_H^2\right| / v^2 < 8\pi$



All Unitarity Constraints



All the theoretical constraints



Oblique Parameters

W-boson mass at one-loop level could have small corrections to the mass:

$$m_W^2 = m_W^2(\mathrm{SM}) + \frac{\alpha c_W^2}{c_W^2 - s_W^2} m_Z^2 \left(-\frac{1}{2} S + c_W^2 T + \frac{c_W^2 - s_W^2}{4 s_W^2} U \right)$$

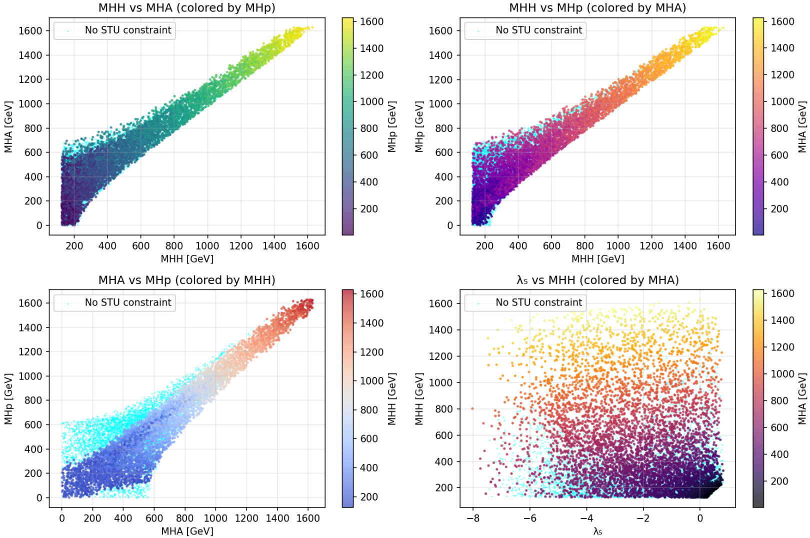
This corrections are parametrized by the oblique parameters S, T, and U which are complicated loop-function that we calculated in Spheno.

The accuracy of the W boson mass requires:

$$S = 0.02 \pm 0.10, \tag{14}$$
$$T = 0.07 \pm 0.12, \tag{15}$$
$$U = 0.00 \pm 0.09 \tag{16}$$

These parameters are crucial for precision tests of the Standard Model and for constraining new physics scenarios.

Oblique Parameters Constraints



Yukawa Sector

In order to suppress flavor-changing neutral currents (FCNCs), we impose a $\mathbb{Z}_2$ symmetry on the Yukawa sector resulting in four types of Yukawa interactions:

Model	Φ_2	Φ_1	u_R^i	d_R^i	e_R^i
Type I	$+$	$-$	$+$	$+$	$+$
Type II	$+$	$-$	$+$	$-$	$-$
Lepton-specific	$+$	$-$	$+$	$+$	$-$
Flipped	$+$	$-$	$+$	$-$	$+$

According to different charge assignments, there are four different models with Yukawa interactions:

$$-\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2 u_R + Y_{d2}\bar{Q}_L\Phi_2 d_R + Y_{\ell 2}\bar{L}_L\Phi_2 e_R + \text{ h.c. (type I)},$$
$$-\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2 u_R + Y_{d1}\bar{Q}_L\Phi_1 d_R + Y_{\ell 1}\bar{L}_L\Phi_1 e_R + \text{ h.c. (type II)},$$
$$-\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2 u_R + Y_{d1}\bar{Q}_L\Phi_2 d_R + Y_{\ell 1}\bar{L}_L\Phi_1 e_R + \text{ h.c. (lepton specific)},$$
$$-\mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2 u_R + Y_{d1}\bar{Q}_L\Phi_1 d_R + Y_{\ell 1}\bar{L}_L\Phi_2 e_R + \text{ h.c. (flipped model) },$$

where $Q_L^T = (u_L, d_L)$, $L_L^T = (\nu_L, l_L)$, $\widetilde{\Phi}_{1,2} = i\tau_2\Phi^*_{1,2}$, and $Y_{u2}, Y_{d1,2}$ and $Y_{\ell 1,2}$ are 3×3 matrices in family space.

Yukawa sector After EWSB

We can obtain the Yukawa couplings

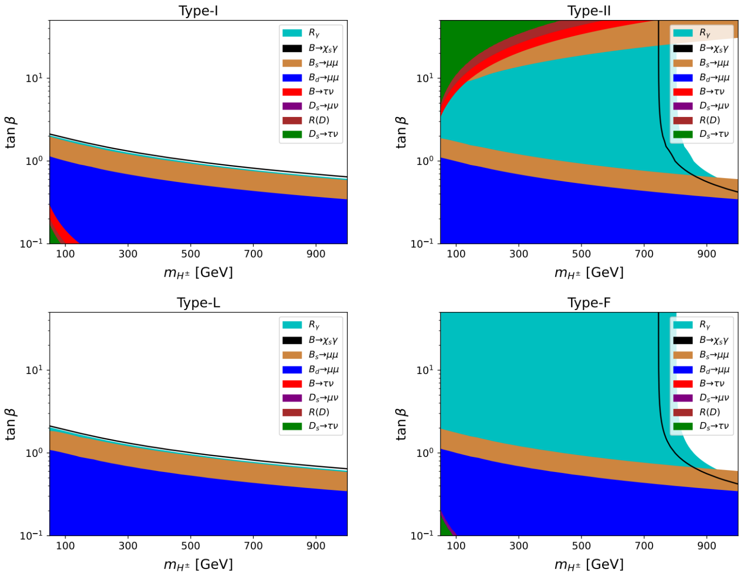
$$
\begin{aligned}
-\mathcal{L}_Y = & \frac{m_f}{v} y_h^f h\bar{f}f + \frac{m_f}{v} y_H^f H\bar{f}f \\
& - i\frac{m_u}{v}\kappa_u A\bar{u}\gamma_5 u + i\frac{m_d}{v}\kappa_d A\bar{d}\gamma_5 d + i\frac{m_\ell}{v}\kappa_\ell A\bar{\ell}\gamma_5\ell \\
& + H^+\bar{u}V_{\text{CKM}}\left(\frac{\sqrt{2}m_d}{v}\kappa_d P_R - \frac{\sqrt{2}m_u}{v}\kappa_u P_L\right) d + h.c. \\
& + \frac{\sqrt{2}m_\ell}{v}\kappa_\ell H^+\bar{\nu}P_R e + h.c.
\end{aligned}
$$

where $y_h^f = \sin(\beta-\alpha) + \cos(\beta-\alpha)\kappa_f$ and $y_H^f = \cos(\beta-\alpha) - \sin(\beta-\alpha)\kappa_f$. The values of κ_u, κ_d and κ_ℓ for the four models are

	type-I	type-II	lepton-specific	flipped
κ_u	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$
κ_d	$1/t_\beta$	$-t_\beta$	$1/t_\beta$	$-t_\beta$
κ_ℓ	$1/t_\beta$	$-t_\beta$	$-t_\beta$	$1/t_\beta$

These models naturally have preferential couplings to the third generation fermions, due to the Higgs Mechanism.

$H^{\pm}$ Constraints



Phenomenology of the Type II THDM

We are interested in channels $\tau\tau$. As the heavy Higgs and the pseudoscalar have the structure

$$\kappa_u = \frac{1}{\tan\beta}, \qquad \kappa_d = \kappa_\ell = \tan\beta,$$

in the Yukawa sector for the Type II THDM, we have a enhancement of the b and τ Yukawa couplings for large $\tan\beta$ values.

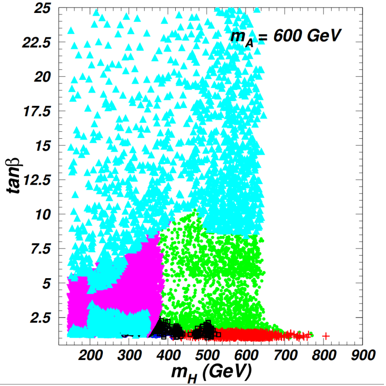
That is why the $H^\pm$ Constraints are more stringent for the Type II THDM, leaving a allowed region starting at $m_{H^\pm} \gtrsim 800$ GeV, and $\tan\beta \in [1, 15]$.

The contribution of the top-loop diagram of the gluon-gluon fusion process $gg \to H/A$ will be suppressed, meanwhile the contribution of the b-loop diagram will be enhanced, and the $b\bar{b}$ fusion become relevant for the production of the heavy Higgs boson H.

In the same way, the branching fraction of the $H/A \to \tau\tau$ decay channel will be enhanced for large $\tan\beta$ values, while the decay to top quarks will be suppressed.

H Constraints For Type II THDM

Using the software Higgs-Bounds 5, we can check the constraints on the heavy Higgs boson H.



The triangles (sky blue), circles (royal blue), squares (black), inverted triangles (purple), and pluses (red) are respectively excluded by the $H/A \to \tau^+\tau^-$, $H \to WW, ZZ, \gamma\gamma$, $H \to hh$, $A \to HZ$, and $A \to hZ$ channels at the LHC. The bullets (green) samples are allowed by various LHC direct searches.

Summary

Key Results from THDM Analysis

- **Theoretical Framework:** Extended SM with two Higgs doublets Φ_1, Φ_2
  - → 5 physical scalars: h (125 GeV), $H, A, H^{\pm}$ + 3 Goldstones
  - → Decoupling limit: $\cos(\beta - \alpha) \approx 0$ for SM-like behavior
- **Constraints to the Scalar Potential:**
  - → Vacuum stability: $\lambda_1, \lambda_2 > 0$; $\sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 > \text{máx}(\lambda_4, |\lambda_5|)$ bounded-from-below conditions
  - → Unitarity: $|a_{\pm}|, |b_{\pm}|, |c_{\pm}|, |e_{\pm}|, |f_{\pm}|, |g_{\pm}| \leq 8\pi$
  - → EW precision: Oblique parameters S, T, U within 1σ
- **Type II THDM Phenomenology:**
  - → Enhanced τ, b couplings: $\kappa_d = \kappa_\ell = \tan\beta$
  - → Suppressed top couplings: $\kappa_u = 1/\tan\beta$
  - → Strong $H^{\pm}$ constraints: $m_{H^{\pm}} \gtrsim 800$ GeV, $\tan\beta \in [1, 15]$
- **Experimental Constraints:**
  - → LHC searches exclude regions via $H/A \to \tau\tau, H \to WW/ZZ$
  - → Allowed parameter space identified for future studies

Main Achievement

Systematic parameter scan establishing **viable THDM Type II regions** consistent with all theoretical and experimental constraints

Next Steps

- **Parameter Scan:** Perform a scan over the parameter space of the THDM, focusing on the Type II model with $\tan\beta$ values between 1 and 15 and $m_{H^\pm}$ starting from 800 GeV.

- **Performance improves:** Define a compressed mass region using a single mass parameter.

- **Collider Phenomenology:** check with higgs bounds the allowed points and constraints on the heavy Higgs boson H and the pseudoscalar A on the entire points that pass the preliminar steps from the scan.

- **Good Parameter Space:** Identify the good parameter space for the Type II THDM, to add further analysis.

Thank you for your attention!

Questions?

Details to be added based on specific analysis requirements

References I

Barroso, A. et al. “Metastability bounds on the two Higgs doublet model”. In: **JHEP** 06 (2013), p. 045. DOI: 10.1007/JHEP06(2013)045. arXiv: 1303.5098 [hep-ph].