

# Two Higgs Doublet Model(s)

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# Outline

## 1 Introduction

## 2 On the Scalar Potential

## 3 Phenomenological Constraints

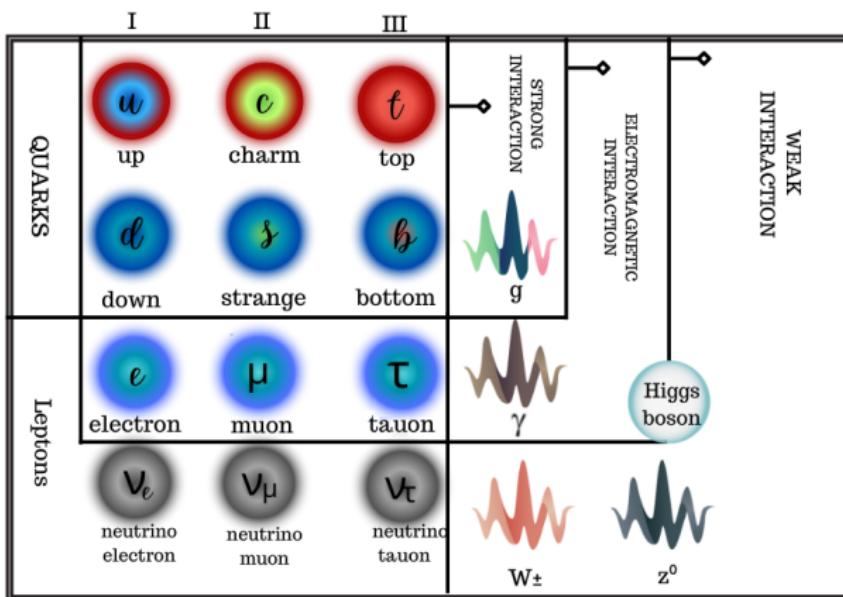
- Theoretical Constraints
- Experimental Constraints

## 4 Yukawa Sector

- FCNC-free Models
- Phenomenology of the Type II THDM

## 5 Final Remarks and Next Steps

# The Standard Model Higgs Mechanism



- One Higgs doublet provides EWSB
  - Single physical Higgs boson discovered in 2012
  - But is this the complete story?

## Why Consider Extended Higgs Sectors?

BSM Motivations

- ### ■ Hierarchy Problem

Discrepancy between Higgs mass ( $\sim 125$  GeV) and Planck scale ( $\sim 10^{19}$  GeV)

- ### ■ Dark Matter Puzzle

No viable DM candidate in SM (WIMP miracle suggests  $\sim 100$  GeV scale)

- ### ■ Baryogenesis Requirements

Need strong 1st-order EW phase transition ( $v_c/T_c > 1$ )

- ## ■ Flavor Anomalies

Additional CP violation sources needed for  $B$ -physics observations

- ### ■ Theoretical Naturalness

SUSY, composite Higgs, etc. often require extended sectors

The Minimal Extension: Two Higgs Doublet Model (2HDM)

**Simplest framework** addressing some of these challenges

(Different types: Type I, II, Lepton-specific, Flipped with different phenomenology)

# Purely Scalar Extension (arXiv:1106.0034)

- With Two Higgs doublets under  $SU(2)_L \times U(1)_Y$ , the most general potential is

$$V_{\text{tree}} = m_{11}^2 (\Phi_1^\dagger \Phi_1) + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + m_{22}^2 (\Phi_2^\dagger \Phi_2) + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 \quad (1)$$

$$+ \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} [\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}] \quad (2)$$

$$+ [-m_{12}^2 \Phi_1^\dagger \Phi_2 + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{h.c.}], \quad (3)$$

- 1 Typical mass-like terms and quartic self-interactions
- 2 Mixed terms between doublets ( $\mathbb{Z}_2$ -Even)
- 3 Mixed terms between doublets (often  $\mathbb{Z}_2$ -Odd and then forbidden, but  $m_{12}^2$  still allowed as soft  $\mathbb{Z}_2$ -symmetry-breaking term)

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- Written in Kibble parametrization, the two complex scalar doublets are:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}} (v_1 + \phi_1 + i a_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}} (v_2 + \phi_2 + i a_2) \end{pmatrix}.$$

Both doublets have a neutral part with a scalar component  $\phi_i$  that acquire a VEV  $v_i/\sqrt{2}$ , a pseudoscalar component  $a_i$ , and have a charged part  $\phi_i^+$ .

- Electroweak constraint:  $v_1^2 + v_2^2 = v^2 = (246 \text{ GeV})^2$

## Minimization conditions

The minimization conditions around the vacuum  $\Omega$  come from requiring

$$\left. \frac{\partial V}{\partial \Phi_1} \right|_{\Omega} = \left. \frac{\partial V}{\partial \Phi_2} \right|_{\Omega} = 0:$$

$$\left. \frac{\partial V}{\partial \Phi_1} \right|_{\Omega} = v_1 \left[ m_{11}^2 + \frac{\lambda_1}{2} v_1^2 + \frac{\lambda_{345}}{2} v_2^2 \right] - m_{12}^2 v_2 + \frac{3\lambda_6}{2} v_1^2 v_2 + \frac{\lambda_7}{2} v_2^3 = 0 \quad (4)$$

$$\left. \frac{\partial V}{\partial \Phi_2} \right|_{\Omega} = v_2 \left[ m_{22}^2 + \frac{\lambda_2}{2} v_2^2 + \frac{\lambda_{345}}{2} v_1^2 \right] - m_{12}^2 v_1 + \frac{\lambda_6}{2} v_1^3 + \frac{3\lambda_7}{2} v_1 v_2^2 = 0 \quad (5)$$

where  $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$ .

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where  $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$ .

In the cases where  $\lambda_6 = \lambda_7 = 0$  with the electroweak constraint,  
 $v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$  ( $\implies \tan \beta = v_2/v_1, \cos \beta = v_1/v, \sin \beta = v_2/v$ ),

$$m_{11}^2 = m_{12}^2 \tan \beta - \frac{1}{2} v^2 \left( \lambda_1 \cos^2 \beta + \lambda_{345} \sin^2 \beta \right) \quad (6)$$

$$m_{22}^2 = m_{12}^2 \cot \beta - \frac{1}{2} v^2 \left( \lambda_2 \sin^2 \beta + \lambda_{345} \cos^2 \beta \right) \quad (7)$$

Usually, you could see the short notation:  $c_\beta = \cos \beta, s_\beta = \sin \beta, t_\beta = \tan \beta$ .

# Scalar Mass Matrices

Expanding at second order in the fields around the vacuum, setting  $\lambda_6 = \lambda_7 = 0$ , and using the minimization conditions, the mass matrices for scalar components become:

**CP-even scalars ( $\phi_1, \phi_2$ ):**

$$\begin{pmatrix} \phi_1 & \phi_2 \end{pmatrix} \begin{pmatrix} m_{12}^2 t_\beta + \lambda_1 v^2 c_\beta^2 & -m_{12}^2 + \frac{\lambda_{345}}{2} v^2 s_{2\beta} \\ -m_{12}^2 + \frac{\lambda_{345}}{2} v^2 s_{2\beta} & m_{12}^2 / t_\beta + \lambda_2 v^2 s_\beta^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

**CP-odd scalars ( $a_1, a_2$ ):**

$$\begin{pmatrix} a_1 & a_2 \end{pmatrix} \left[ m_{12}^2 - \frac{1}{2} \lambda_5 v^2 s_{2\beta} \right] \begin{pmatrix} t_\beta & -1 \\ -1 & 1/t_\beta \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

**Charged scalars ( $\phi_1^\pm, \phi_2^\pm$ ):**

$$\begin{pmatrix} \phi_1^+ & \phi_2^+ \end{pmatrix} \left[ m_{12}^2 - \frac{1}{4} (\lambda_4 + \lambda_5) v^2 s_{2\beta} \right] \begin{pmatrix} t_\beta & -1 \\ -1 & 1/t_\beta \end{pmatrix} \begin{pmatrix} \phi_1^- \\ \phi_2^- \end{pmatrix}$$

Diagonalization implies two rotation matrices with angles  $\alpha$  and  $\beta$ :

$$\begin{pmatrix} h \\ H \end{pmatrix} = \mathcal{R}(\alpha) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \begin{pmatrix} A \\ G^0 \end{pmatrix} = \mathcal{R}(\beta) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \mathcal{R}(\beta) \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

## Scalar Mass Spectrum

Diagonalization implies two rotation matrices with angles  $\alpha$  and  $\beta$ :

$$\begin{pmatrix} h \\ H \end{pmatrix} = \mathcal{R}(\alpha) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \begin{pmatrix} A \\ G^0 \end{pmatrix} = \mathcal{R}(\beta) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \mathcal{R}(\beta) \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

After diagonalization, we obtain physical states:

- **CP-even Higgs:**  $h$  (SM-like, 125 GeV),  $H$  (heavy scalar)
  - **CP-odd Higgs:**  $A$  (pseudoscalar)
  - **Charged Higgs:**  $H^\pm$
  - **Goldstone bosons:**  $G^\pm, G^0$  (absorbed by  $W^\pm, Z^0$ )

with eigenvalues:

$$m_{H,h}^2 = \frac{1}{2} \left[ M_{P,11}^2 + M_{P,22}^2 \pm \sqrt{(M_{P,11}^2 - M_{P,22}^2)^2 + 4(M_{P,12}^2)^2} \right]$$

$$m_A^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \lambda_5 v^2$$

$$m_{H^\pm}^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \frac{1}{2}(\lambda_4 + \lambda_5)v^2$$

where  $M_P^2$  is the CP-even scalar mass matrix.

# Gauge Interactions and Decoupling Limit

The gauge-kinetic Lagrangian is given as

$$\mathcal{L}_g = (D^\mu \Phi_1)^\dagger (D_\mu \Phi_1) + (D^\mu \Phi_2)^\dagger (D_\mu \Phi_2)$$

We obtain the neutral Higgs couplings to  $VV$  ( $VV \equiv ZZ, WW$ )

$$\begin{aligned} \mathcal{L}_g \supset & \frac{g^2 + g'^2}{8} v^2 ZZ \left( 1 + 2 \frac{h}{v} y_h^V + 2 \frac{H}{v} y_H^V \right) \\ & + \frac{g^2}{4} v^2 W^+ W^- \left( 1 + 2 \frac{h}{v} y_h^V + 2 \frac{H}{v} y_H^V \right) \end{aligned}$$

where  $y_h^V = \sin(\beta - \alpha)$  and  $y_H^V = \cos(\beta - \alpha)$ .

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## Decoupling Limit

The SM has been frustratingly successful in describing the Higgs boson properties. In order to get a SM-like Higgs boson, we require  $\cos(\beta - \alpha) \approx 0$ , which implies  $\beta - \alpha \approx \pi/2$ .

This is known as the **decoupling limit**.

# Lambda Parameters in the Decoupling Limit (arXiv:1604.01406)

In the Decoupling limit the scalar mass spectrum simplifies and we can invert the relations to express the  $\lambda_i$  in terms of the physical masses:

$$\begin{aligned} v^2 \lambda_1 &= m_h^2 - \frac{t_\beta (m_{12}^2 - m_H^2 s_\beta c_\beta)}{c_\beta^2} \\ v^2 \lambda_2 &= m_h^2 - \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{t_\beta s_\beta^2} \\ v^2 \lambda_3 &= m_h^2 + 2m_{H^\pm}^2 - 2m_H^2 - \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta c_\beta} \\ v^2 \lambda_4 &= m_A^2 - 2m_{H^\pm}^2 + m_H^2 + \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta c_\beta} \\ v^2 \lambda_5 &= m_H^2 - m_A^2 + \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta c_\beta} \end{aligned} \tag{8}$$

where  $m_h = 125$  GeV is the SM-like Higgs boson mass.

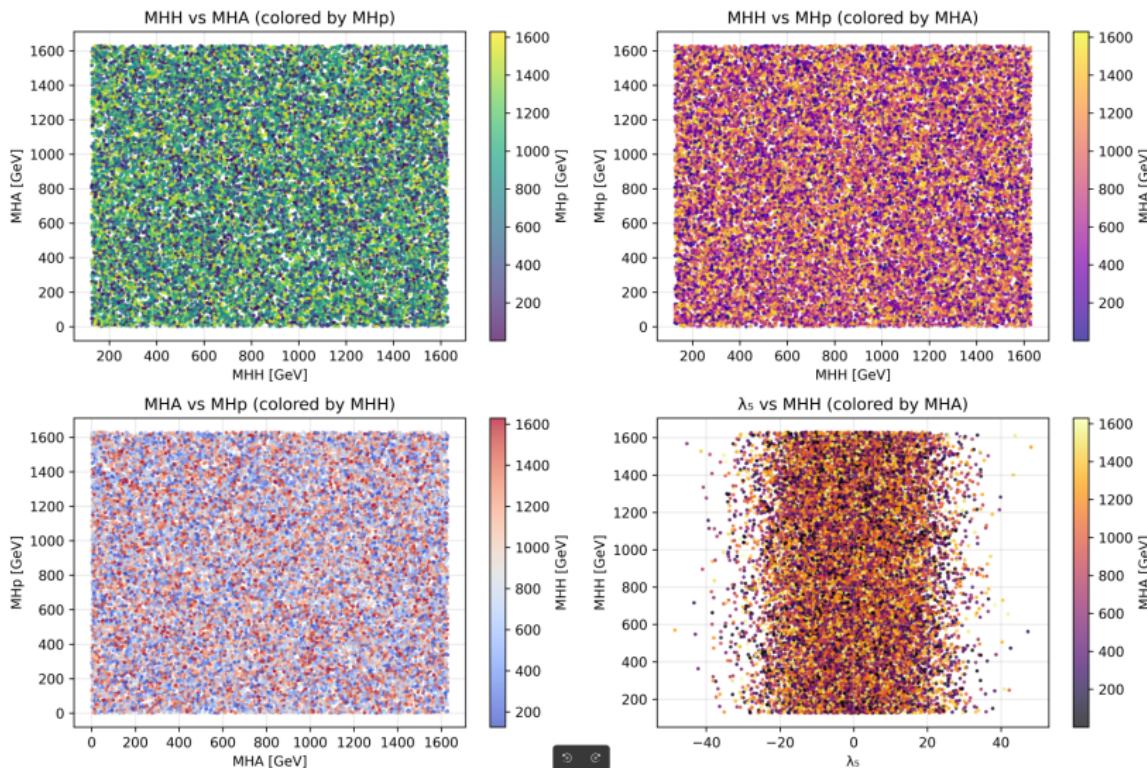
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where  $m_h = 125$  GeV is the SM-like Higgs boson mass.  
So we have the following free parameters:

- Physical masses:  $m_H, m_A, m_{H^\pm}$
- Mixing angle:  $\beta$  (or  $\tan \beta$ ).
- Soft  $\mathbb{Z}_2$ -breaking term:  $m_{12}^2$  (I prefer  $\lambda_5$ )

Initial State of the Scan ( $\tan \beta = 10$  fixed)

# Vacuum Stability Constraints (arXiv:2203.07244)

The potential must be bounded from below at large field values.

The dominant quartic terms in the potential are:

$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + \lambda_3 X_1^2 X_2^2 + \lambda_4 X_1^2 X_2^2 \rho^2 + \lambda_5 X_1^2 X_2^2 \rho^2 \cos 2\theta.$$

where the fields have been parametrized as

$$\Phi_1^\dagger \Phi_1 = X_1^2, \quad \Phi_2^\dagger \Phi_2 = X_2^2, \quad \Phi_1^\dagger \Phi_2 = X_1 X_2 \rho e^{i\theta} \quad \text{with} \quad 0 \leq \rho \leq 1.$$

Here,  $\rho$  is the normalized magnitude of the inner product between  $\Phi_1$  and  $\Phi_2$ , obtained from the Cauchy-Schwarz inequality:

$$|\Phi_1^\dagger \Phi_2| \leq \sqrt{(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)} \Rightarrow \rho = \frac{|\Phi_1^\dagger \Phi_2|}{X_1 X_2} \in [0, 1].$$

Geometrically,  $\rho$  plays the role of the cosine of the angle between the two Higgs doublets in field space:  $\rho = 1$  means they are aligned, while  $\rho = 0$  means they are orthogonal.

## Theoretical Constraints

$V_4$  must have a minimum for all directions in field space, i.e., for all  $X_1, X_2 \geq 0$  and  $\rho \in [0, 1]$ .

The term with  $\theta$  is  $\lambda_5 X_1^2 X_2^2 \rho^2 \cos 2\theta$ .

- If  $\lambda_5 > 0$ , the minimum occurs at  $\cos \theta = -1$ ,
- else if  $\lambda_5 < 0$ , the minimum occurs at  $\cos \theta = 1$ .
- In any case, the term becomes  $-|\lambda_5| X_1^2 X_2^2 \rho^2$  at the minimum.

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At the minimum, the term with  $\rho$  is  $(\lambda_4 - |\lambda_5|) X_1^2 X_2^2 \rho^2$ .

- If  $\lambda_4 - |\lambda_5| > 0$ , the minimum occurs at  $\rho = 0$ , then

$$V_4 = \frac{\lambda_1}{2} X_1^4 + \frac{\lambda_2}{2} X_2^4 + \lambda_3 X_1^2 X_2^2 \quad (9)$$

and we require  $\lambda_1 > 0$ ,  $\lambda_2 > 0$ , and  $\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0$ .

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and we require  $\lambda_1 > 0$ ,  $\lambda_2 > 0$ , and  $\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0$ .

- else if  $\lambda_4 - |\lambda_5| < 0$ , the minimum occurs at  $\rho = 1$ , then

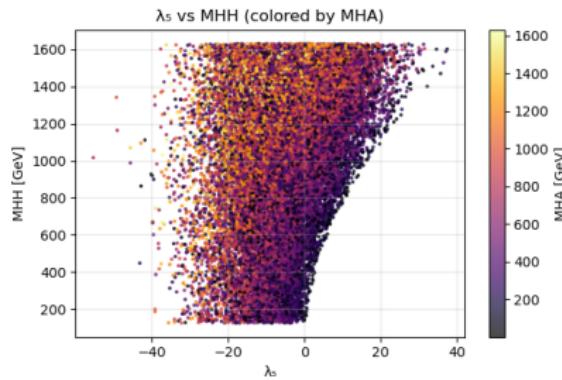
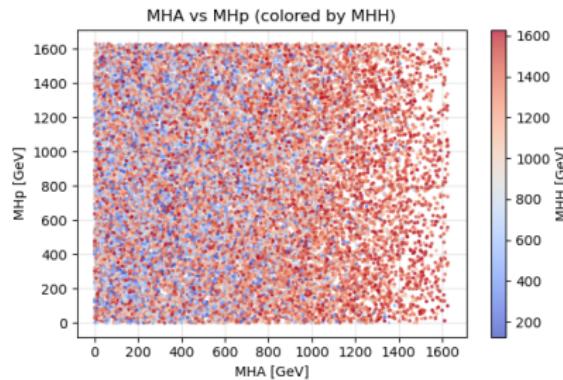
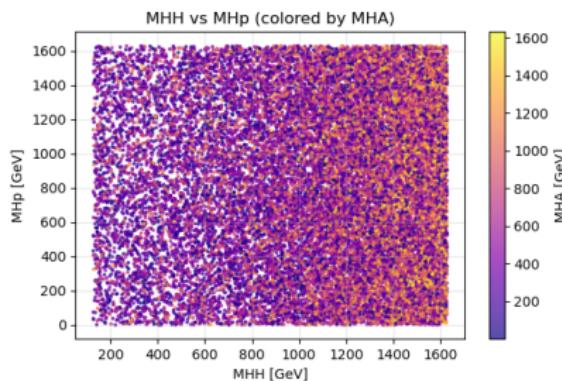
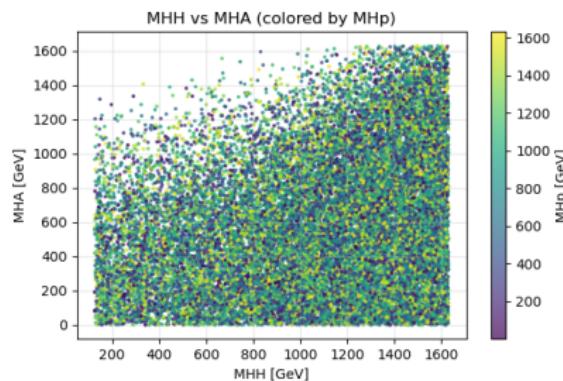
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and we require  $\lambda_1 > 0$ ,  $\lambda_2 > 0$ ,  $\lambda'_3 + \sqrt{\lambda_1 \lambda_2} > 0$ , with  $\lambda'_3 = \lambda_3 + \lambda_4 - |\lambda_5|$ .

In Summary, the bounded from below conditions are:

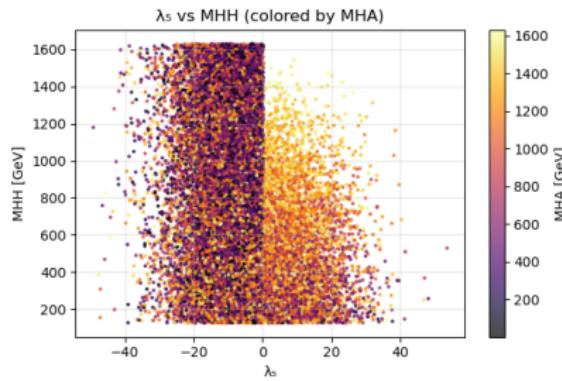
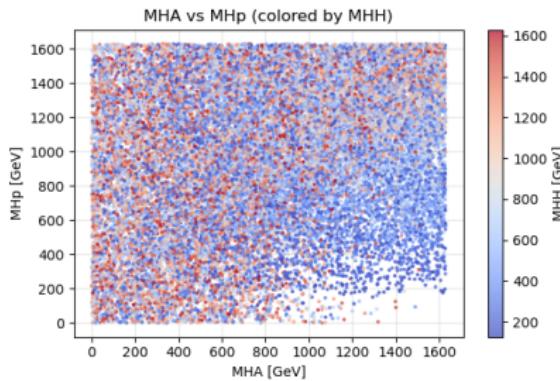
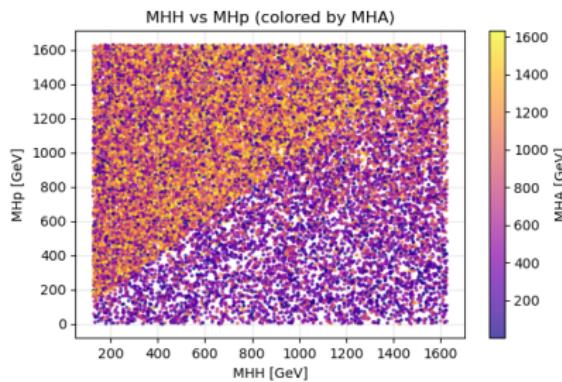
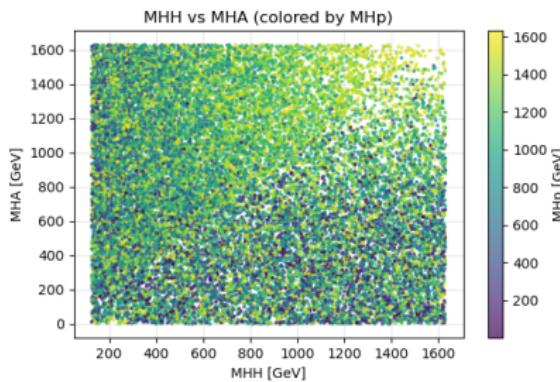
$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0, \quad \lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} > 0.$$

## Theoretical Constraints

 $\lambda_1 > 0, \quad \lambda_2 > 0$  constraints

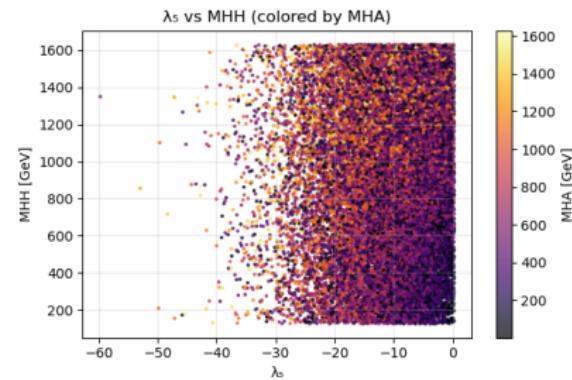
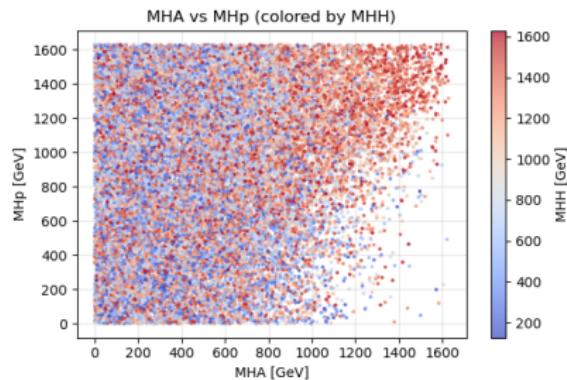
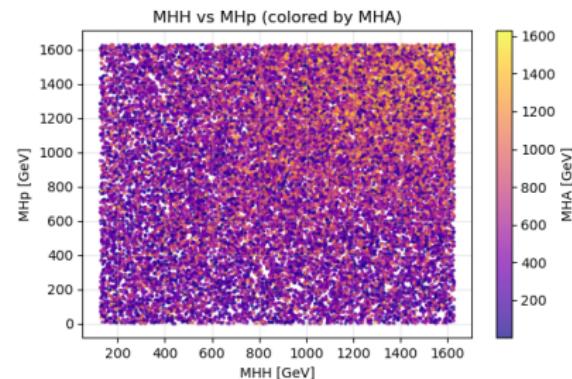
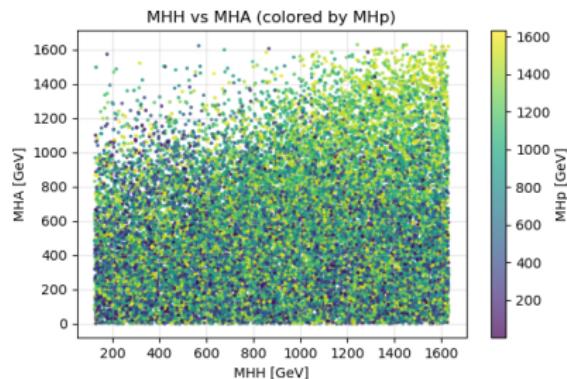
## Theoretical Constraints

$$\sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 > \max(\lambda_4, |\lambda_5|)$$



## Theoretical Constraints

## Bounded from Below Conditions



## Theoretical Constraints

## Electroweak Vacuum Stability (arXiv:1303.5098)

- The scalar potential can have multiple minima - we must ensure our EW-breaking vacuum is the **global minimum**
- Metastability could lead to dangerous vacuum decay via quantum tunneling.
- To guarantee the selected vacuum is truly the lowest-energy state, we require:

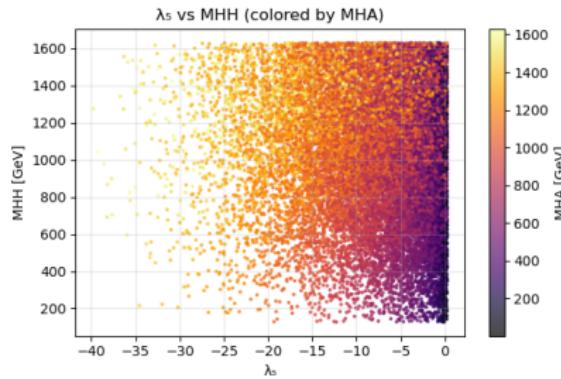
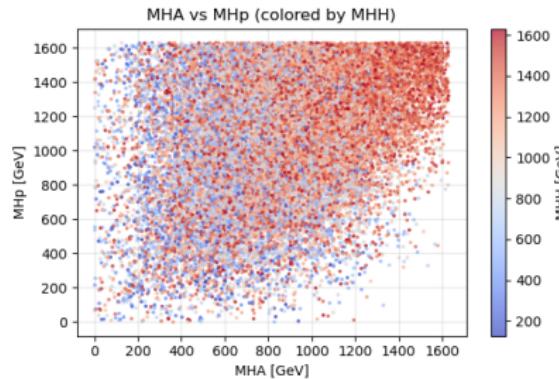
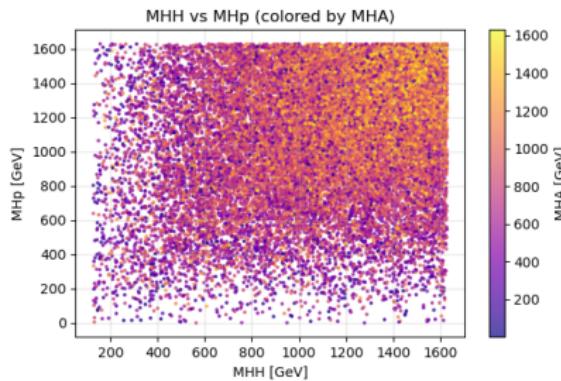
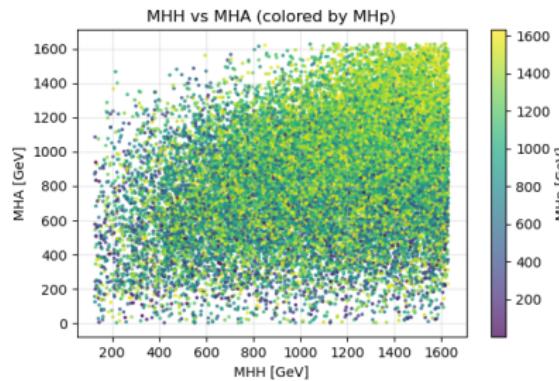
$$m_{12}^2 \left( m_{11}^2 - k^2 m_{22}^2 \right) (\tan \beta - k) > 0$$

where  $k \equiv (\lambda_1/\lambda_2)^{1/4}$  compares the quartic couplings.

- The details of this condition, requires a mapping of the potential as a Minkowskian manifold and stability of the Landau-Ginzburg effective potential, are given in arXiv:1303.5098.

## Theoretical Constraints

## Vacuum Stability Conditions



# Unitarity Constraints (arXiv:1805.07306)

From quantum mechanics, the S-matrix must be unitary:

$$S = 1 + iT \quad \Rightarrow \quad T^\dagger T = -i(T - T^\dagger)$$

For scattering processes  $s_1 s_2 \rightarrow s_3 s_4$ , this implies bounds on amplitudes.

The matrix element for the process is defined as:

$$\langle \{s_1, s_2\} | iT | \{s_3, s_4\} \rangle \equiv i\mathcal{M}\delta^4(k_3 + k_4 - p_1 - p_2)(2\pi)^4$$

From the scattering theory, the amplitude  $\mathcal{M}$  can be expanded in partial waves:

$$\mathcal{M} = \sum_{J=0}^{\infty} (2J+1) a_J P_J(\cos \theta) \implies a_J \equiv \frac{1}{32\pi} \sqrt{\frac{4|\mathbf{p}^{\text{in}}| |\mathbf{p}^{\text{out}}|}{2^{\delta_{12}} 2^{\delta_{34}} S}} \int_{-1}^1 d(x) \mathcal{M} P_J(x)$$

where  $P_J$  are Legendre polynomials and  $a_J$  are the partial wave amplitudes.  
So, we have

$$-\frac{i}{2} (a_J - a_J^\dagger) \geq a_J a_J^\dagger \implies \text{Re}(a_J) \leq \frac{1}{2} \quad (11)$$

# Unitarity Constraints (arXiv:1805.07306)

At large energies, the dominant contributions are the spherically symmetric,  $J = 0$ , partial wave amplitudes and  $s \approx |\mathbf{p}|^2$ . Thus,

$$\text{Re}\{a_0\} \sim \frac{1}{16\pi} \sqrt{2^{-\delta_{12}-\delta_{34}}} Q_{1234} \leq \frac{1}{2} \quad (12)$$

where  $Q_{1234}$  is the quartic coupling of the process  $s_1 s_2 \rightarrow s_3 s_4$ . Considering all the possible processes,

$$|a_{\pm}|, |b_{\pm}|, |c_{\pm}|, |e_{\pm}|, |f_{\pm}|, |g_{\pm}| \leq 8\pi$$

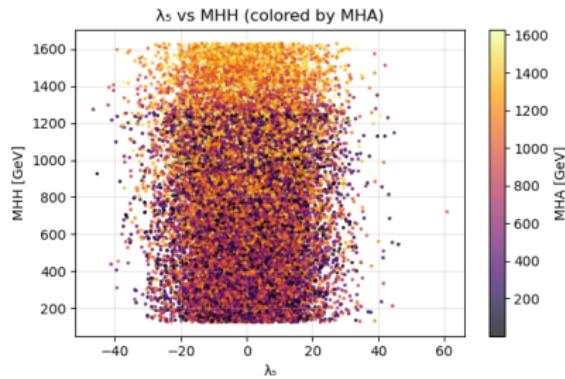
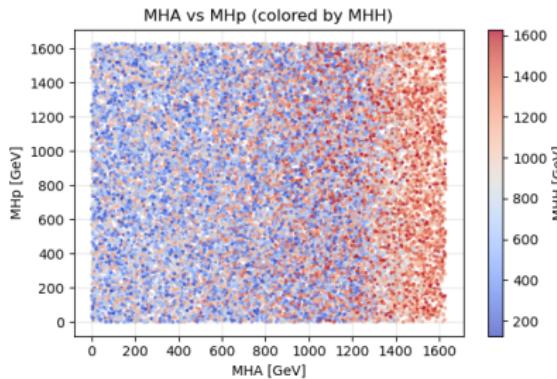
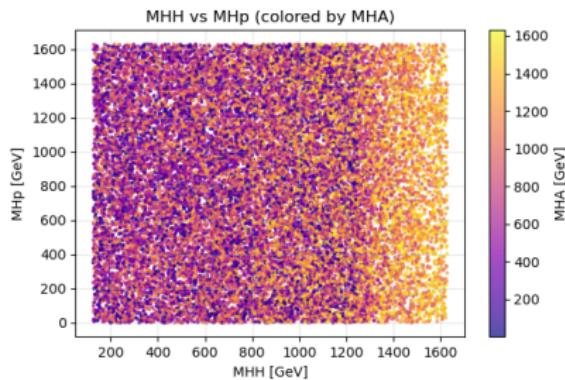
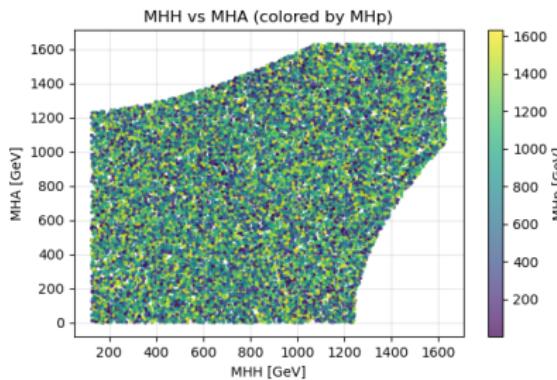
where the eigenvalues are:

$$\begin{aligned} a_{\pm} &= \frac{3}{2} (\lambda_1 + \lambda_2) \pm \sqrt{\frac{9}{4} (\lambda_1 - \lambda_2)^2 + (2\lambda_3 + \lambda_4)^2} & e_{\pm} &= \lambda_3 + 2\lambda_4 \pm 3\lambda_5 \\ b_{\pm} &= \frac{1}{2} (\lambda_1 + \lambda_2) \pm \sqrt{\frac{1}{4} (\lambda_1 - \lambda_2)^2 + \lambda_4^2} & f_{\pm} &= \lambda_3 \pm \lambda_4 \\ c_{\pm} &= \frac{1}{2} (\lambda_1 + \lambda_2) \pm \sqrt{\frac{1}{4} (\lambda_1 - \lambda_2)^2 + \lambda_5^2} & g_{\pm} &= \lambda_3 \pm \lambda_5 \end{aligned} \quad (13)$$

**It is just a safety check, not the strict theory limitation.**

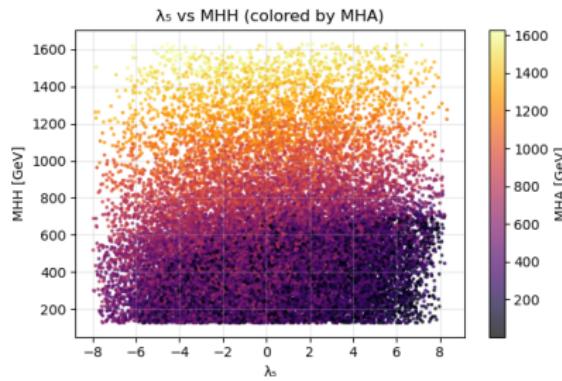
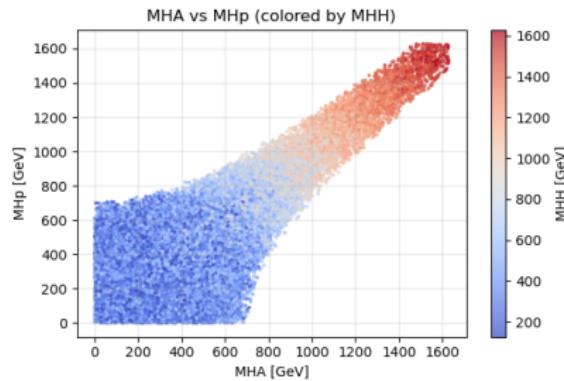
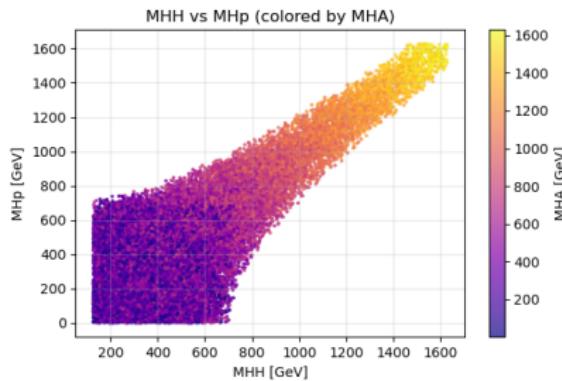
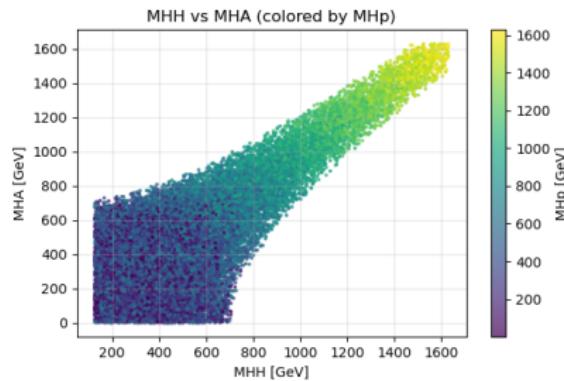
## Theoretical Constraints

$$\text{Example: } |f_+| = |\lambda_3 + \lambda_4| \approx |m_h^2 + m_A^2 - m_H^2| / v^2 < 8\pi$$

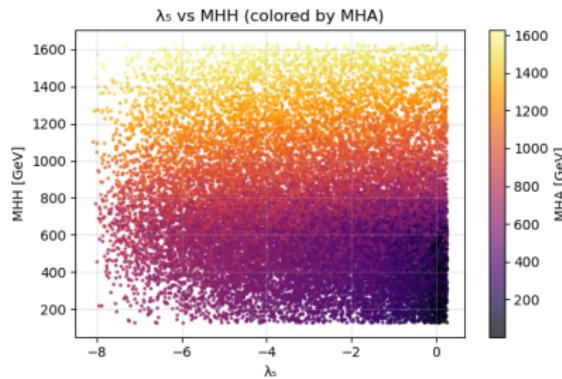
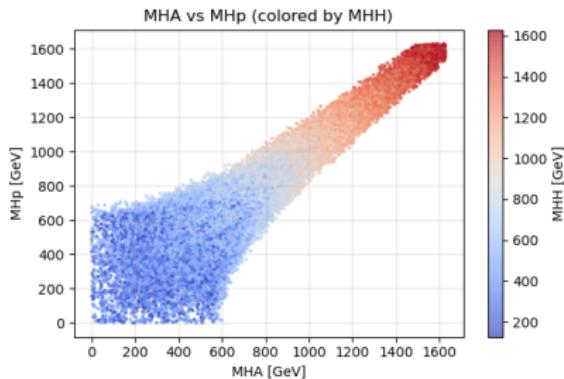
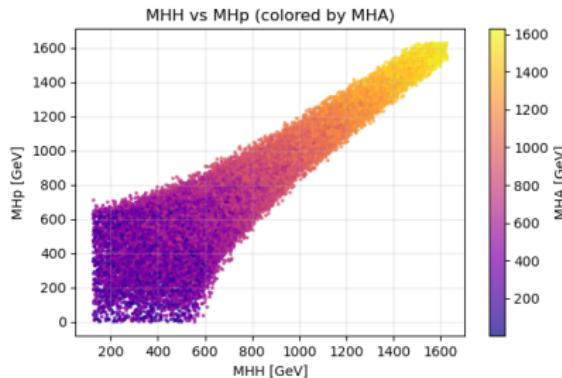
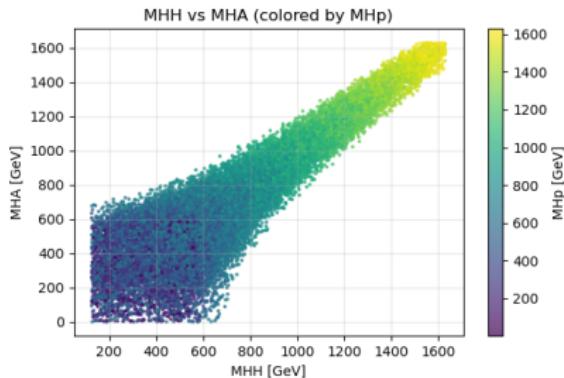


## Theoretical Constraints

## All Unitarity Constraints



# All the theoretical constraints



# Oblique Parameters (DOI: 10.1093/ptep/ptaa104)

$W$ -boson mass at one-loop level could have small corrections to the mass:

$$m_W^2 = m_W^2(\text{SM}) + \frac{\alpha c_W^2}{c_W^2 - s_W^2} m_Z^2 \left( -\frac{1}{2}S + c_W^2 T + \frac{c_W^2 - s_W^2}{4s_W^2} U \right)$$

This corrections are parametrized by the oblique parameters  $S$ ,  $T$ , and  $U$  which are complicated loop-function that we calculated in Spheno.

The accuracy of the  $W$  boson mass requires :

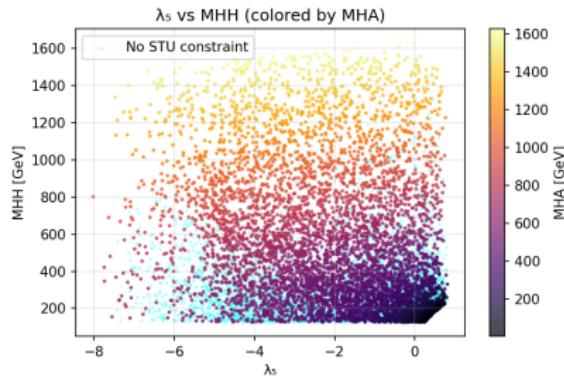
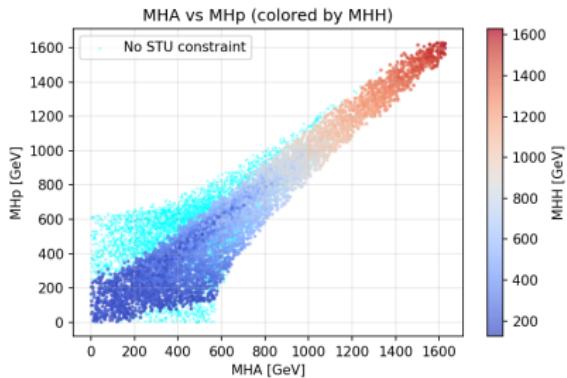
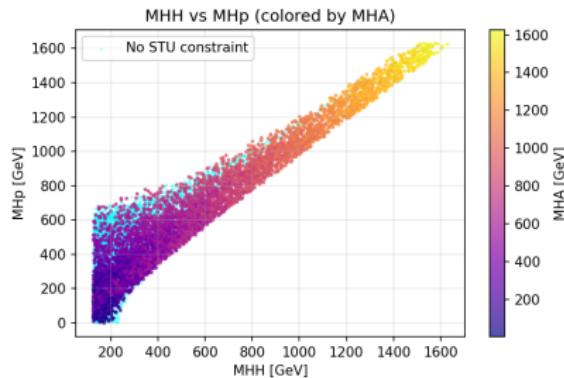
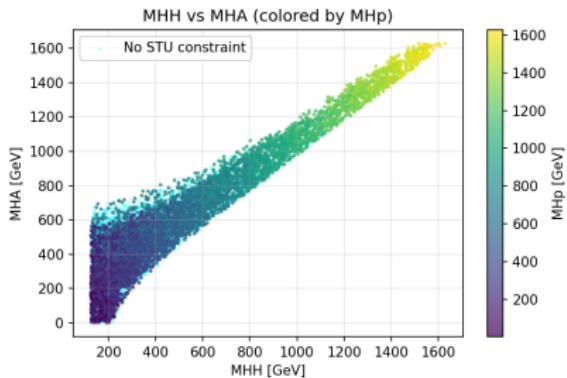
$$S = 0.02 \pm 0.10, \tag{14}$$

$$T = 0.07 \pm 0.12, \tag{15}$$

$$U = 0.00 \pm 0.09 \tag{16}$$

These parameters are crucial for precision tests of the Standard Model and for constraining new physics scenarios.

# Oblique Parameters Constraints



Yukawa Sector (arXiv:1106.0034)

In order to suppress flavor-changing neutral currents (FCNCs), we impose a  $\mathbb{Z}_2$  symmetry on the Yukawa sector resulting in four types of Yukawa interactions:

Model	$\Phi_2$	$\Phi_1$	$u_R^i$	$d_R^i$	$e_R^i$
Type I	+	-	+	+	+
Type II	+	-	+	-	-
Lepton-specific	+	-	+	+	-
Flipped	+	-	+	-	+

According to different charge assignments, there are four different models with Yukawa interactions:

- $- \mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2u_R + Y_{d2}\bar{Q}_L\Phi_2d_R + Y_{\ell 2}\bar{L}_L\Phi_2e_R + \text{h.c.}$  ( type I ),
  - $- \mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2u_R + Y_{d1}\bar{Q}_L\Phi_1d_R + Y_{\ell 1}\bar{L}_L\Phi_1e_R + \text{h.c.}$  ( type II ),
  - $- \mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2u_R + Y_{d1}\bar{Q}_L\Phi_2d_R + Y_{\ell 1}\bar{L}_L\Phi_1e_R + \text{h.c.}$  ( lepton specific ),
  - $- \mathcal{L} = Y_{u2}\bar{Q}_L\tilde{\Phi}_2u_R + Y_{d1}\bar{Q}_L\Phi_1d_R + Y_{\ell 1}\bar{L}_L\Phi_2e_R + \text{h.c.}$  ( flipped model ),

where  $Q_L^T = (u_L, d_L)$ ,  $L_L^T = (\nu_L, l_L)$ ,  $\tilde{\Phi}_{1,2} = i\tau_2 \Phi_{1,2}^*$ , and  $Y_{u2}$ ,  $Y_{d1,2}$  and  $Y_{\ell 1,2}$  are  $3 \times 3$  matrices in family space.

Yukawa sector After EWSB

We can obtain the Yukawa couplings

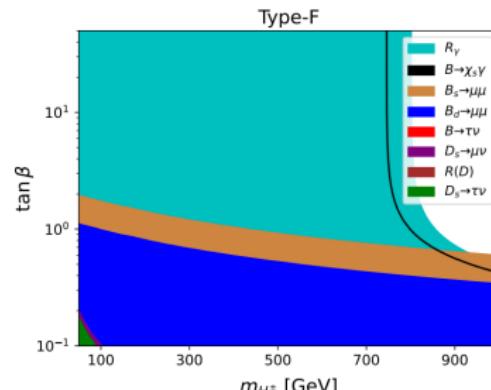
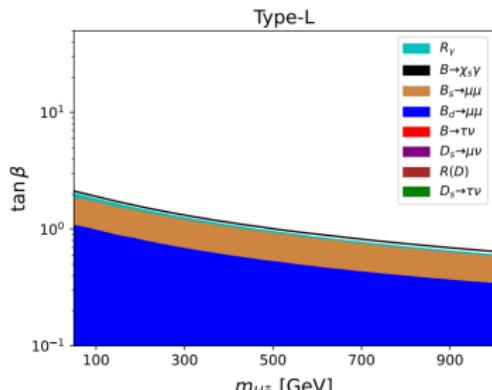
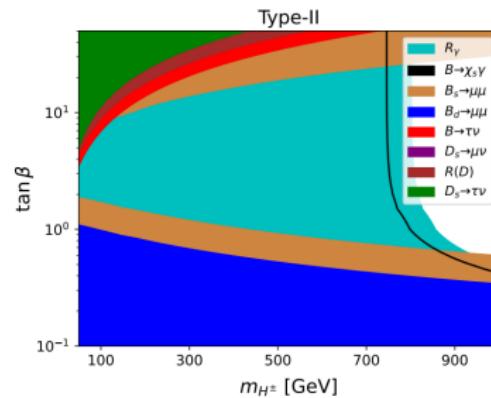
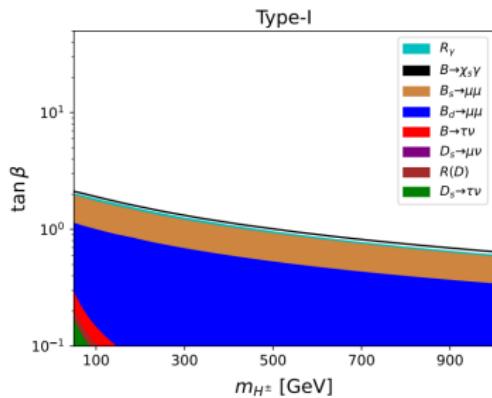
$$\begin{aligned}
-\mathcal{L}_Y = & \frac{m_f}{v} y_h^f h \bar{f} f + \frac{m_f}{v} y_H^f H \bar{f} f \\
& - i \frac{m_u}{v} \kappa_u A \bar{u} \gamma_5 u + i \frac{m_d}{v} \kappa_d A \bar{d} \gamma_5 d + i \frac{m_\ell}{v} \kappa_\ell A \bar{\ell} \gamma_5 \ell \\
& + H^+ \bar{u} V_{CKM} \left( \frac{\sqrt{2} m_d}{v} \kappa_d P_R - \frac{\sqrt{2} m_u}{v} \kappa_u P_L \right) d + h.c. \\
& + \frac{\sqrt{2} m_\ell}{v} \kappa_\ell H^+ \bar{\nu} P_R e + h.c.
\end{aligned}$$

where  $y_h^f = \sin(\beta - \alpha) + \cos(\beta - \alpha)\kappa_f$  and  $y_H^f = \cos(\beta - \alpha) - \sin(\beta - \alpha)\kappa_f$ . The values of  $\kappa_u$ ,  $\kappa_d$  and  $\kappa_\ell$  for the four models are

	type-I	type-II	lepton-specific	flipped
$\kappa_u$	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$
$\kappa_d$	$1/t_\beta$	$-t_\beta$	$1/t_\beta$	$-t_\beta$
$\kappa_\ell$	$1/t_\beta$	$-t_\beta$	$-t_\beta$	$1/t_\beta$

These models naturally have preferential couplings to the third generation fermions, due to the Higgs Mechanism.

# $H^\pm$ Constraints (arXiv:2412.04572)



# Phenomenology of the Type II THDM (arXiv:2203.07244)

We are interested in channels  $\tau\tau$ . As the heavy Higgs and the pseudoscalar have the structure

$$\kappa_u = \frac{1}{\tan \beta}, \quad \kappa_d = \kappa_\ell = \tan \beta,$$

in the Yukawa sector for the Type II THDM, we have an enhancement of the  $b$  and  $\tau$  Yukawa couplings for large  $\tan \beta$  values.

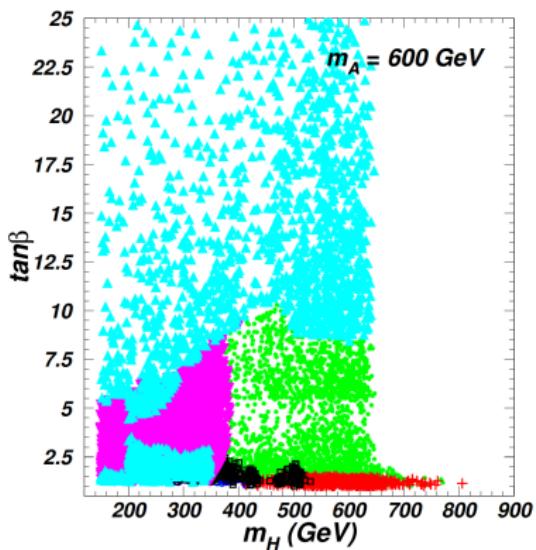
That is why the  $H^\pm$  Constraints are more stringent for the Type II THDM, leaving an allowed region starting at  $m_{H^\pm} \gtrsim 800$  GeV, and  $\tan \beta \in [1, 15]$ .

The contribution of the top-loop diagram of the gluon-gluon fusion process  $gg \rightarrow H/A$  will be suppressed, meanwhile the contribution of the  $b$ -loop diagram will be enhanced, and the  $b\bar{b}$  fusion become relevant for the production of the heavy Higgs boson  $H$ .

In the same way, the branching fraction of the  $H/A \rightarrow \tau\tau$  decay channel will be enhanced for large  $\tan \beta$  values, while the decay to top quarks will be suppressed.

# H Constraints For Type II THDM (arXiv:2010.03730)

Using the software Higgs-Bounds 5, we can check the constraints on the heavy Higgs boson  $H$ .



- Triangles (sky blue): excluded by  $H/A \rightarrow \tau^+\tau^-$
- Circles (royal blue): excluded by  $H \rightarrow WW, ZZ, \gamma\gamma$
- Squares (black): excluded by  $H \rightarrow hh$
- Inverted triangles (purple): excluded by  $A \rightarrow Hz$
- Pluses (red): excluded by  $A \rightarrow hZ$
- Bullets (green): allowed by various LHC searches

# Summary

## Key Results from THDM Analysis

- **Theoretical Framework:** Extended SM with two Higgs doublets  $\Phi_1, \Phi_2$ 
  - 5 physical scalars:  $h$  (125 GeV),  $H, A, H^\pm + 3$  Goldstones
  - Decoupling limit:  $\cos(\beta - \alpha) \approx 0$  for SM-like behavior
- **Constraints to the Scalar Potential:**
  - Vacuum stability:  $\lambda_1, \lambda_2 > 0; \sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 > \max(\lambda_4, |\lambda_5|)$  bounded-from-below conditions
  - Unitarity:  $|a_\pm|, |b_\pm|, |c_\pm|, |e_\pm|, |f_\pm|, |g_\pm| \leq 8\pi$
  - EW precision: Oblique parameters  $S, T, U$  within  $1\sigma$
- **Type II THDM Phenomenology:**
  - Enhanced  $\tau, b$  couplings:  $\kappa_d = \kappa_\ell = \tan \beta$
  - Suppressed top couplings:  $\kappa_u = 1/\tan \beta$
  - Strong  $H^\pm$  constraints:  $m_{H^\pm} \gtrsim 800 \text{ GeV}, \tan \beta \in [1, 15]$
- **Experimental Constraints On the Yukawa Sector:**
  - LHC searches exclude regions via  $H/A \rightarrow \tau\tau, H \rightarrow WW/ZZ$
  - Allowed parameter space identified for future studies

## Main Achievement

Systematic parameter scan establishing **viable THDM Type II regions**  
consistent with all theoretical and experimental constraints

# Next Steps

- **Parameter Scan:** Perform a scan over the parameter space of the THDM, focusing on the Type II model with  $\tan \beta$  values between 1 and 15 and  $m_{H^\pm}$  starting from 800 GeV.
- **Performance improves:** Define a compressed mass region using a single mass parameter.
- **Collider Phenomenology:** check with higgs bounds the allowed points and constraints on the heavy Higgs boson  $H$  and the pseudoscalar  $A$  on the entire points that pass the preliminar steps from the scan.
- **Good Parameter Space:** Identify the good parameter space for the Type II THDM, to add further analysis.

# Thank you for your attention!

Questions?

*Details to be added based on specific analysis requirements*