Machine Learning-Enhanced Feasibility Studies on the Production of New Particles with Preferential Couplings to Third Generation Fermions at the LHC

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Introduction

Standard Model of Particle Physics: A Successful Framework

Core Theoretical Structure

- **Fermion Sector:**
  - 3 generations
  - Left-handed doublets:

$$Q_L = \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L ; \quad L_L = \begin{pmatrix} \ell \\ \nu_\ell \end{pmatrix}_L$$

  - Right-handed singlets: $q_{uR}, q_{dR}; \ell_R$
  - Flavor structure from CKM mixing
  - Initially, ν_R not required (no mass)

- **Gauge Group:**
  - $SU(3)_C \times SU(2)_L \times U(1)_Y$
  - Strong force: $SU(3)_C$ (QCD)
  - Electroweak: $SU(2)_L \times U(1)_Y$

- **Higgs Mechanism:**
  - $SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$
  - Masses to $W^\pm, Z^0$ bosons
  - Massless γ
  - Fermion masses via Yukawa couplings
  - Higgs boson h discovered (LHC 2012)

Key Features

- **Particle spectrum:** 12 fermions + 5 bosons
- **QFT:** Renormalizable, gauge invariant, anomaly-free
- **Tested experimentally:** Exceptional agreement with data
- **Predictive power:** Successfully tested at LHC

Deficiencies of the SM

The SM cannot be complete:

- Unexplained ν masses
- No explanation for DM
- Matter-antimatter asymmetry unsolved
- Gravity not included

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Experimental hints of New Physics:

Recent measurements of the $R(D)$ and $R(D^*)$ ratios show a $\sim 3\sigma$ deviation from SM predictions, suggesting possible lepton universality violation.

B-Meson Anomalies

R_K Anomalies

$$R_K = \frac{\mathcal{B}(B \to K\mu^+\mu^-)}{\mathcal{B}(B \to Ke^+e^-)}$$

R_D Anomalies

$$R_D = \frac{\mathcal{B}(B \to D\tau\nu)}{\mathcal{B}(B \to D\ell\nu)}$$

- The B-meson anomalies hint at lepton flavor universality violation.
- NP coupled mainly to 3G-fermions address these anomalies.
- Currently, R_K is SM-compatible, tensions in R_D still persist.

The Large Hadron Collider (LHC)

LHC Physics Program

- SM parameter determinations
- Precision Tests
- Direct searches for BSM physics
- Cross sections & Decay rates measurements
- Angular Correlations, among others

World's largest particle collider:

- Located at CERN, at the France-Switzerland border
- 27 km circumference ring
- Designed to probe the TeV scale
- Four main experiments: ATLAS, CMS, LHCb, ALICE
- pp collisions at $\sqrt{s} = 13.6$ TeV
- Bunches of $\sim 10^{11}$ Particles
- 40 MHz collision rate
- Small beam spot size
- Run-II Luminosity ~ 150 fb^{-1}
- RUN-III currently on analysis with ~ 300 fb^{-1}
- High luminosity upgrade (HL-LHC) planned for 2029

Collider Observables

Collider Kinematics

$$\begin{cases} \text{Pseudorapidity: } \eta = -\ln\tan(\theta/2) \\ \text{Transverse momentum: } p_T = p\sin(\theta) \\ \text{Azimuthal angle: } \phi \\ \text{Deposited energy: } E \end{cases}$$

Phenomenological Framework

From Theory to Simulation: FeynRules

Input: `.fr` Model File

- Lagrangian $\mathcal{L}$ terms
- Particle definitions: F, V, S fields
- Gauge symmetries: $SU(N)$, $U(1)$
- Parameters: masses, couplings
- Mixing matrices, constraints
- Output: UFO model files

Madgraph-Pythia-Delphes

Ecosystem for Event Simulation

Goal: Produce realistic simulated data comparable to experimental observations
Input: BSM model in UFO format, collider setup, process definitions, parameters
Pipeline: UFO model → LHE (parton) → HEPMC (hadron) → ROOT (detector)
Output: Simulated events in ROOT format, ready for physics analysis
Automated: Cross-sections, widths, branching ratios calculated automatically

Statistical Significance

Statistical Significance

It is the parametric test

$$\kappa = \frac{\langle t \rangle_B - \langle t \rangle_{S+B}}{\sigma_{S+B}}$$

where $t \sim \chi^2$ is the optimal test.

Event Yield

The event yield in each bin is $N_i = \sigma_i \cdot \mathcal{L} \cdot \epsilon_i$, where:

- σ_i: cross section
- $\mathcal{L}$: integrated luminosity
- ϵ_i: selection efficiency

The significance becomes:

$$\boxed{\kappa = \frac{\sum_i \sigma_{s_i} \epsilon_{s_i} w_i}{\sqrt{\sum_i (\sigma_{s_i} \epsilon_{s_i} + \sigma_{b_i} \epsilon_{b_i} + \delta_{\text{sys}}^2) {w_i}^2}} \sqrt{\mathcal{L}}} \tag{1}$$

Strategies to improve κ:

- Increase luminosity $\mathcal{L}$
- Optimize cuts to enhance ϵ_s / ϵ_b
- Increase σ_s via higher $\sqrt{s}$ for heavier states
- Use multivariate discriminants (BDT, DNN)
- Reduce systematic uncertainties δ_{sys}
- Exploit correlations between bins

Machine Learning Revolution in HEP

Why ML in Particle Physics?

- **High-dimensional data**: Complex detector outputs
- **Complex patterns**: Jet and event classification
- **Rare signals**: Need optimal background rejection
- **Automation**: Reduce manual feature engineering
- **Computational efficiency**: Speed up workflows

Traditional vs ML Approaches

Traditional:
- Manual cut-based analysis
- Simple kinematic variables
- Univariate methods
- Limited feature engineering

Machine Learning:
- Multivariate classifiers (BDT, NN)
- Deep learning on raw data
- Anomaly detection for BSM searches

Typical Current Applications

- **Event classification**: Signal/background separation
- **Jet tagging**: $b/c/\tau$-jet identification
- **Object reconstruction**: Particle flow, tracking
- **Anomaly detection**: Model-agnostic BSM searches
- **Simulation acceleration**: Fast calorimeter simulation

How does the binary classifier work?

Be aware: The real performance is the same for both algorithms in terms of classification accuracy. The difference lies in interpretability and probabilistic output.

A New Distribution Emerges

Feasibility Studies Workflow

$U(1)_{T^3_R}$ Model

(a) (b) (c)

Deeper Origin of Hypercharge?

SM Electric Charge Formula

In the SM, electric charge emerges from electroweak symmetry breaking:

$$Q_{\text{EM}} = T_L^3 + \frac{Y}{2}$$

where T_L^3 is weak isospin ($SU(2)_L$) and Y is hypercharge ($U(1)_Y$).

Could Y emerge from additional gauge symmetries?
Pati-Salam trys to restore left-right symmetry:

$$Y = 2T_R^3 + \frac{B-L}{2}$$

$$Q_{\text{EM}} = T_L^3 + T_R^3 + \frac{B-L}{4}$$

This induce challenges: Anomaly cancellation, Higgs sector, mass scales, fermion masses, among others.

Key Question

Can hypercharge be explained by a more fundamental gauge structure?
This could provide a deeper understanding of the SM's chiral fermion assignments.

Need of modify Higgs Mechanism

SM fermion masses from Yukawa couplings

The SM yukawa couplings are not invariant under the new symmetry,

$$\mathcal{L}_{\text{Yukawa}} = \underbrace{-y_e \bar{L}_L \Phi e_R}_{\text{Lepton Yukawa}} + \underbrace{-y_d \bar{Q}_L \Phi d_R}_{\text{Down quark Yukawa}} + \underbrace{-y_u \bar{Q}_L \tilde{\Phi} u_R}_{\text{Up quark Yukawa}} + \text{h.c.} \tag{2}$$

The SM masses arise after EWSB when $\langle \Phi \rangle = v_h/\sqrt{2} \implies \boxed{m_f = \frac{1}{\sqrt{2}} y_f v_h}$

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However, if we add the $U(1)_{T_R^3}$ charges to the SM fields as:

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{T_R^3}$
q_L	**3**	**2**	1/6	0
ℓ_L	**1**	**2**	-1/2	0
u_R^c	**3**	**1**	-2/3	-1/2
d_R^c	**3**	**1**	1/3	1/2
ℓ_R^c	**1**	**1**	1	1/2

We get $y_u = y_d = y_e = 0$ is mandatory $\implies$ **No Masses**. **Chiral anomalies** also arise.

Mass mechanism in the $U(1)_{T^3_R}$ Model

Modifications to the Scalar and Fermion Sectors

- We need two scalar fields:
  - The usual Higgs doublet H, with $\langle H \rangle = v_h/\sqrt{2}$.
  - A new complex scalar singlet ϕ, with $\langle \phi \rangle = v_\phi/\sqrt{2}$.

  in order to decouple the energy scales of $U(1)_{T^3_R}$ and $SU(2)_L \times U(1)_Y$ breaking.
- Fermions acquire mass from the mixture with a new vector-like fermion χ_f.
- Right-handed neutrinos ν_R are added to cancel anomalies in the lepton sector.

The terms in the Lagrangian density that contribute to the mass of physical fermions are,

$$-\mathcal{L} \supset Y_{f_L} \bar{f}'_L \chi'_{fR} H + Y_{f_R} \bar{\chi}'_{fL} f'_R \phi^* + m_{\chi'_f} \bar{\chi}'_{fL} \chi'_{fR} + \text{ h.c.} \tag{3}$$

Therefore, in the vacuum, the mass matrix for each fermion f is given by

$$M_f = \begin{pmatrix} 0 & Y_{f_L} v_h/\sqrt{2} \\ Y_{f_R} v_\phi/\sqrt{2} & m_{\chi'_f} \end{pmatrix}, \quad \Longrightarrow \begin{cases} m_f m_{\chi_f} = \frac{Y_{f_L} v_h Y_{f_R} v_\phi}{2} \\ m_f^2 + m_{\chi_f}^2 = m_{\chi'_f}^2 + \frac{1}{2}\left(Y_{f_L}^2 v_h^2 + Y_{f_R}^2 v_\phi^2\right), \end{cases} \tag{4}$$

The masses will rise from diagonalizing this matrix.

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After spontaneous symmetry breaking

$$H = \begin{pmatrix} G_+ \\ \frac{1}{\sqrt{2}}\left(v_h + \rho_0 + iG_0\right) \end{pmatrix}, \quad \phi = \frac{1}{\sqrt{2}}\left(v_\phi + \rho_\phi + iG_\phi\right), \tag{3}$$

where $G^\pm$, G^0 are the Goldstone bosons eaten by $W^\pm$ and Z^0, and G_ϕ is the Goldstone boson eaten by the new Z'/dark photon.
While the scalar parts ρ_0 and ρ_ϕ mix to get the physical mass eigenstates h and ϕ'

$$M^2_{\text{scalar}} = \begin{pmatrix} 2\lambda_H v_h^2 & \lambda_{H\phi} v_h v_\phi \\ \lambda_{H\phi} v_h v_\phi & 2\lambda_\phi v_\phi^2 \end{pmatrix} \implies \begin{cases} m_h^2 - m_{\phi'}^2 = \sqrt{(4\lambda_{H\phi}^2 v_h^2 v_\phi^2) + (\lambda_H v_h^2 - \lambda_\phi v_\phi^2)^2} \\ m_h^2 + m_{\phi'}^2 = \lambda_H v_h^2 + \lambda_\phi v_\phi^2 \end{cases} \tag{4}$$

New Yukawa Interactions

The Yukawa interactions of the physical fermions with the scalar bosons have the form

$$-\mathcal{L}_{\text{yuk}} = h\bar{\psi}_{f_L}\mathcal{Y}_h\psi_{f_R} + \phi'\bar{\psi}_{f_L}\mathcal{Y}_\phi\psi_{f_R}, \tag{5}$$

with $\psi_f = (f, \chi_f)^T$, and $\mathcal{Y}_{f_{L,R}}$ are 2×2 yukawa masses after rotation to the mass basis.

So, you have new vertex like

that converts a SM fermion to a heavy vector-like fermion via the emission of a ϕ' boson...

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So, you have new vertex like

that converts a SM fermion to a heavy vector-like fermion via the emission of a ϕ' boson...

... in addition to the vertex

that involves a top-like quark decay via the emission of a W boson.

Production Channel

Production Cross Section

Fig.: Projected cross section (fb) plot for $pp \to t\chi_u\phi'$ and subsequent decay as a function of $m(\chi_u)$ and $m(\phi')$.

Feasible Experimental Signatures

Search Channel

Representative Feynman diagram for the production of a ϕ' boson in association with a χ_u vector-like quark through the fusion of a top quark and χ_u vector-like quark.

The ϕ' decays to a pair of muons, the top quark decays fully hadronically, and the χ_u decays semi-leptonically to muons, neutrinos and b-jets.

Feasible Experimental Signatures

Search Channel

Representative Feynman diagram for the production of a ϕ' boson in association with a χ_u vector-like quark through the fusion of a gluon pair from incoming protons.

The ϕ' decays to a pair of muons, the top quark that decays fully hadronically, and the χ_u decay semi-leptonically to muons, neutrinos and jets.

Feasible Experimental Signatures

Background

Representative Feynman diagram for a background event. A Z boson is produced in association with a top quark through the fusion of a top, anti top pair from incoming protons.

The Z boson subsequently decays to a pair of muons and the two spectator top quarks decay semi-leptonically and purely hadronically to muons, neutrinos and jets, resulting in the same final states as the signal event.

Feasible Experimental Signatures

Kinematic Variables

Invariant mass distribution of the muon pair with the highest and second highest transverse momentum. The distributions are shown for the two main SM background processes and two signal benchmark points.

Feature Importance

- Benchmark signal scenario with $m(\phi') = 325\,\text{GeV}$ and $m(\chi_u) = 2000\,\text{GeV}$.
- Main Features: boosted μ_1 and b_1 jet, angular correlated muons and resonant ϕ' decay.

Optimal Discriminantor Output

Fig.: Output of the gradient boosting algorithm for a benchmark $m(\phi') = 100$ GeV and $m(\chi_u) = 500$ GeV signal, and dominant backgrounds. The distributions are normalized to unity.

Signal Significance

Fig.: Signal significance for the high luminosity LHC era, considering with 3000 fb^{-1} of collected data.

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Regular Article - Theoretical Physics

Probing light scalars and vector-like quarks at the high-luminosity LHC

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- Extends the SM with a $U(1)_{T_R^3}$ gauge symmetry that addressed the potential flavour anomalies, explains fermion mass hierarchies, and provides a thermal dark matter candidate.
- Introduces a light scalar ϕ' and a heavy vector-like quark χ_u, produced via gg and $t\chi_u$ fusion. A machine learning analysis at $\sqrt{s} = 13.6$ TeV and 3000 fb^{-1} probes challenging low-mass regions through decays to b-quarks, muons, and neutrinos.

U_1 Leptoquark Model

$R(D)$ and $R(D^*)$ anomalies

Flavor Observables

$$R(D) = \frac{\mathrm{BR}(B \to D\tau\nu)}{\mathrm{BR}\left(B \to D\ell_{(e,\mu)}\nu\right)}, \quad R(D^{(*)}) = \frac{\mathrm{BR}\left(B \to D^{(*)}\tau\nu\right)}{\mathrm{BR}\left(B \to D^{(*)}\ell_{(e,\mu)}\nu\right)}$$

Standard Model (Tree-level)

$B^- \to D^0 \ell^- \bar{\nu}_\ell$ via W^- exchange

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Leptoquark Mediated

$B^- \to D^0\ell^-\bar{\nu}_\ell$ via leptoquark exchange

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Standard Model (Tree-level)

$B^- \to D^0\ell^-\bar{\nu}_\ell$ via W^- exchange

Leptoquark Mediated

$B^- \to D^0\ell^-\bar{\nu}_\ell$ via leptoquark exchange

How can we test this hypothesis?

The vector leptoquark U_1 model

A leptoquark is defined as a particle with a vertex that mix vectors and quarks.

If U_1 is a vector leptoquark that preserves the chirality on the vertex, we expect an interaction term like

$$\sim \beta_L U_1^\mu \bar{q}_L \gamma_\mu \ell_L,$$

and these allows a similar interaction term for the right handed currents

$$\sim \beta_R U_1^\mu \bar{d}_R \gamma_\mu e_R.$$

Non-resonant

Single LQ

Double LQ

Where the SM charges for the leptoquark, in the $Y = 2(Q - T_3)$ convention, are

	$\bar{q}_L$	ℓ_L^j	$\bar{q}_L \gamma_\mu \ell_L$	U_1^μ
$U(1)$	$-1/3$	-1	$-4/3$	$+4/3$
SU(2)	$\bar{\mathbf{2}}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
SU(3)	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\bar{\mathbf{3}}$	$\mathbf{3}$

Then, the leptoquark $U_1 \sim (\mathbf{3}_C, \mathbf{1}_I, 4/3_Y)$.

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Then, the leptoquark $U_1 \sim (\mathbf{3}_C, \mathbf{1}_I, 4/3_Y)$.

The full Lagrangian for the vector leptoquark is

$$\mathcal{L}_U = -\frac{1}{2} U^\dagger_{\mu\nu} U^{\mu\nu} + M_U^2 U^\dagger_\mu U^\mu - i g_s U^\dagger_\mu T^a U_\nu G_a^{\mu\nu} - \frac{2i}{3} g' U^\dagger_\mu U_\nu B^{\mu\nu}$$
$$+ \frac{g_U}{\sqrt{2}} \left[U_1^\mu \left(\beta_L^{ij} \bar{q}_L^i \gamma_\mu e_L^j + \beta_R^{ij} \bar{d}_R^i \gamma_\mu e_R^j \right) + \text{ h.c. } \right]$$

where $U_{\mu\nu} = \mathcal{D}_\mu U_\nu - \mathcal{D}_\nu U_\mu$, $\mathcal{D}_\mu = \partial_\mu - i g_s G_\mu^a T^a - i \frac{2}{3} g_Y B_\mu$, and the couplings β_L and β_R are complex 3×3 matrices in flavor space.

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$$+ \frac{g_U}{\sqrt{2}} \left[U_1^\mu \left(\beta_L^{ij} \bar{q}_L^i \gamma_\mu e_L^j + \beta_R^{ij} \bar{d}_R^i \gamma_\mu e_R^j \right) + \text{ h.c. } \right]$$

where $U_{\mu\nu} = \mathcal{D}_\mu U_\nu - \mathcal{D}_\nu U_\mu$, $\mathcal{D}_\mu = \partial_\mu - i g_s G_\mu^a T^a - i \frac{2}{3} g_Y B_\mu$, and the couplings β_L and β_R are complex 3×3 matrices in flavor space.

Constraints from $\Delta F = 2$ and lepton flavor violation impose a hierarchy with dominant third generation couplings:

$$\left|\beta_L^{11}\right|, \left|\beta_L^{12}\right|, \left|\beta_L^{21}\right|, \left|\beta_L^{22}\right|, \left|\beta_L^{31}\right| \ll \left|\beta_L^{13}\right| \ll \left|\beta_L^{23}\right|, \left|\beta_L^{32}\right| \ll \left|\beta_R^{33}\right|, \left|\beta_L^{33}\right| = \mathcal{O}(1), \quad (6)$$

where β_R is diagonal.

Leptoquark Production Mechanisms at 13 TeV

Production Channels

- **dLQ**: Pair production
  QCD-mediated
  β_R independent
- **sLQ**: Single production
  β_R sensitive
  $\times 2$ enhancement
- **non-res**: Non-resonant
  β_R sensitive
  $\times 10$ enhancement

Cross-over Point

$\sigma_{\text{sLQ}} > \sigma_{\text{dLQ}}$ for $M_{\text{LQ}} \gtrsim 1.5$ TeV
(exact value depends on g_U)

Leptoquark Branching Fractions

Branching Ratios vs. β_R Parameter

Main Decay Channels

- LQ $\to b\tau$: Dominant
  Primary for $R(D^{(*)})$ anomalies
- LQ $\to t\nu_\tau$: Competing
  Affects total width
- LQ $\to c\tau, s\nu_\tau$: Subdominant

β_R Dependence

Controls right-handed couplings:

- $\beta_R = 0$: Pure left-handed
- $\beta_R > 0$: Mixed chirality
- Affects collider signatures

S_T^{MeT} observable

$$S_T^{\mathrm{MeT}} = \mathrm{MeT} + \sum_i \left| p_T^i \right|$$

ΔR Separation

$$\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$$

- Separation of sLQ and dLQ with the backgrounds enhanced via S_T^{MeT}, a measure of the boostedness of the event.
- Separation of non-resonant production with the backgrounds enhanced via ΔR between the τ candidates.

XGB-output

- We can evaluate a score for the signal and background events using the discriminator algorithm.
- Each production channel has contributions in each one of the six detection channels.

Double Leptoquark Production

The Sensitivity Reach / only left-handed currents

- dLQ production depends only on α_{QCD} and the available energy.
- So, the potential reach is independent of the global coupling with fermions g_U.

Single leptoquark production

The Sensitivity Reach / only left-handed currents

- Single leptoquark production is sensitive to both, mass and couplings.
- It contributes to the regions of high coupling constants at higher masses than double leptoquark production.

Non-resonant Production

The Sensitivity Reach / only left-handed currents

- Non-resonant production is highly dependent on the couplings, so it dominates the regions of high coupling constants at all masses.

Combined Sensitivity Reach

- A combined analysis of the three production channels suggests better sensitivity for high coupling constants.
- Depending on the benchmark β_L, β_R values, the branching ratio to $b\tau$ can vary, affecting the sensitivity reach.

ATLAS Results

Left-handed currents only

- In parallel ATLAS relased a search following a similar approach [ArXiv:2305.15962].

- Our ML-based analysis suggests better sensitivity for high coupling constants.

Sensitivity Reach for only Right-Handed Current coupling

CMS excess in non-resonant channel [ArXiv 2308.07826]

Comparison with CMS and ATLAS searches [ArXiv 2308.07826, 2303.01294]

Key Results for $BR(LQ \to b\tau) = 1, \quad g_u\beta_R = \sqrt{2}\lambda$

- Assumes pure right-handed currents
- CMS excess can be explained at $M_{LQ} \approx 1.8$ TeV, $\lambda \approx 2.5$
- Our analysis shows better sensitivity at high coupling constants
- **Note: In contrast to the claims made by CMS, a pure $b\tau$ decay channel cannot arise from left-handed couplings and then cannot explain the B-anomalies.**

Z' Interferences

The need of a Z' boson in gauge U_1 models

If U_1 has a gauge origin, we could rewrite the interaction term in the covariant derivative as

$$\psi_L^{\rm SM} = \begin{pmatrix} q_{Lr} \\ q_{Lg} \\ q_{Lb} \\ \ell_L \end{pmatrix} \Longrightarrow \quad \mathcal{L}_{\rm int} \sim U_{1\alpha}^{\mu} \bar{\psi}_L^{\rm SM} \gamma_\mu T_+^\alpha \psi_L^{\rm SM} + \text{h.c.}, \quad T_+^\alpha = \begin{pmatrix} 0 & 0 & 0 & \delta_{r\alpha} \\ 0 & 0 & 0 & \delta_{g\alpha} \\ 0 & 0 & 0 & \delta_{b\alpha} \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

we have six generators $T_\pm^\alpha$ with closure relation and projecting into a color singlet operator:

$$\sum_\alpha \left[T_+^\alpha, T_-^\alpha\right] = 3T_{B-L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}.$$

So, the gauge group with this leptoquark must include a $U(1)_{B-L}$ symmetry (The right-handed currents also have a similar interaction term).

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So, the gauge group with this leptoquark must include a $U(1)_{B-L}$ symmetry (The right-handed currents also have a similar interaction term).

The interaction terms for the Z' boson have the form

$$\begin{aligned} \mathcal{L}_{\rm int} &\sim Z'_\mu \left(\bar{\psi}_L^{\rm SM} \gamma^\mu (3T_{B-L}) \psi_L^{\rm SM}\right) \\ &\sim Z'_\mu \left(\bar{q}_L \gamma^\mu q_L - 3\bar{\ell}_L \gamma^\mu \ell_L\right). \end{aligned}$$

Interference with a Z' vector boson

Non-Resonant Production (leptoquarks)

$$\mathcal{M}_{U_1} \sim \frac{1}{t - m_{U_1}^2 + i m_{U_1} \Gamma_{U_1}}, \quad (7)$$

Resonant Production (neutral bosons)

$$\mathcal{M}_{Z'} \sim \frac{1}{s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'}}, \quad (8)$$

so the interference has the form

$$\frac{\mathrm{d}}{\mathrm{d}m}\left[\sigma_{LQ+Z'} - \left(\sigma_{LQ} + \sigma_{Z'}\right)\right] \sim \frac{g_{z'} g_U}{s} \frac{m_{LQ} m_{Z'} \Gamma_{LQ} \Gamma_{Z'} - (t - m_{LQ}^2)(s - m_{Z'}^2)}{\left[(t - m_{LQ}^2)^2 + m_{LQ}^2 \Gamma_{LQ}^2\right]\left[(s - m_{Z'}^2)^2 + m_{Z'}^2 \Gamma_{Z'}^2\right]}.$$

Similary for Polarized final states.

Interference with a Z' vector boson

Effects on the Sensitivity reach

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THE EUROPEAN PHYSICAL JOURNAL C

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Regular Article - Theoretical Physics

On the sensitivity reach of LQ production with preferential couplings to third generation fermions at the LHC

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- Third-generation coupled vector leptoquark explain R_D anomalies.
- We study LQ production at $\sqrt{s} = 13$ TeV, considering decays to τb final states, interference with Z' processes, and chiral structure effects.
- Using machine learning techniques, we extend discovery reach up to $M_{LQ} \sim 5$ TeV at the RUN-II Luminosity, probing the parameter space that addresses the B-meson anomalies.

Summary and Conclusions

Related Research Projects

Feasibility to probe the dynamical scotogenic model at the LHC

Gustavo Ardila-Tafurth,* Andrés Flórez,† Cristian Rodríguez,‡ and Maud Sarazin§
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Óscar Zapata¶
Instituto de Física, Universidad de Antioquia, Calle 70 # 52-21, Apartado Aéreo 1226, Medellín, Colombia
(Dated: December 22, 2025)

Related Research Projects

Feasibility to probe the dynamical scotogenic model at the LHC

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On BSM Signatures in Ditau Channels at the LHC

A. Flórez,* C. Rodriguez,† J. Reyes,‡ and AC. Parra§
Departamento de Física, Universidad de Los Andes, Cra. 1 # 18a-12, Bogotá, Colombia

J. Jones-Pérez¶
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(Dated: Upcoming)

Future explorations

- Extended study of the $R(D)$, $R(D^*)$ anomalies in the context of the $U(1)_{T_R^3}$ model including right-handed neutrinos.
- We will explore the neutrino sector in the $U(1)_{T_R^3}$ model, including possible connections to dark matter, through a intensive study of the parameter space.
- We has been exploring models with new scalar doublets and singlets that exposes an alternative hypothesis for toponium-like pseudoscalar resonance observed at the LHC.
- Extend feasibility studies for DM motivated models at the LHC using machine learning techniques.

Backup Slides

$U(1)_{T_R^3}$ Field Content

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{T_R^3}$
q_L	**3**	**2**	1/6	0
ℓ_L	**1**	**2**	-1/2	0
H	**1**	**2**	1/2	0
u_R^c	**3**	**1**	-2/3	-1
d_R^c	**3**	**1**	1/3	1
ℓ_R^c	**1**	**1**	1	1
ν_R^c	**1**	**1**	0	-1
ϕ	**1**	**1**	0	1
χ'_{u_L}	**3**	**1**	2/3	0
$\chi^c_{u_R}$	**3**	**1**	-2/3	0
χ'_{d_L}	**3**	**1**	-1/3	0
$\chi^c_{d_R}$	**3**	**1**	1/3	0
χ'_{ℓ_L}	**1**	**1**	-1	0
$\chi^c_{\ell_R}$	**1**	**1**	1	0
χ'_{ν_L}	**1**	**1**	0	0
$\chi^c_{\nu_R}$	**1**	**1**	0	0

The Universal Seesaw Mechanism

The Yukawa interactions of the physical fermions with the scalar bosons have the form

$$-\mathcal{L}_{\text{yuk}} = h\bar{\psi}_{f_L}\mathcal{Y}_h\psi_{f_R} + \phi'\bar{\psi}_{f_L}\mathcal{Y}_\phi\psi_{f_R}, \tag{9}$$

with $\psi_f = (f, \chi_f)^T$, and the matrices $\mathcal{Y}_{f_{L,R}}$ given by

$$\mathcal{Y}_h = \frac{1}{\sqrt{2}}\mathcal{R}(\theta_{f_L})\left(Y_{f_L}\sigma_+\cos\alpha - Y_{f_R}\sigma_-\sin\alpha\right)\mathcal{R}^{-1}(\theta_{f_R}) \tag{10}$$

$$\mathcal{Y}_\phi = \frac{1}{\sqrt{2}}\mathcal{R}(\theta_{f_L})\left(Y_{f_L}\sigma_+\sin\alpha + Y_{f_R}\sigma_-\cos\alpha\right)\mathcal{R}^{-1}(\theta_{f_R}), \tag{11}$$

where $\sigma_\pm = (\sigma_1 \pm i\sigma_2)/2$ are the ladder Pauli matrices.
So, you have new vertex like

Preselection Criteria Leptoquark Search

Variable	**Threshold**					
	$\tau_h\tau_h$	$b\tau_h\tau_h$	$bb\tau_h\tau_h$	$\tau_h\tau_\ell$	$b\tau_h\tau_\ell$	$bb\tau_h\tau_\ell$
$N(b)$	$= 0$	$= 1$	≥ 2	$= 0$	$= 1$	≥ 2
$p_T(b)$	-	$\geq$ 30GeV		-	$\geq$ 30GeV	
$\lvert\eta(b)\rvert$	-	≤ 2.4		-	≤ 2.4	
$N(\ell)$	$= 0$			$= 1$		
$N(\tau_h)$	$= 2$			$= 1$		
$p_T(\ell)$	-			$\geq$ 35GeV		
$p_T(\mu)$	-			$\geq$ 30GeV		
$\lvert\eta(\ell)\rvert$	-			≤ 2.4		
$p_T(\tau_h)$	$\geq$ 50 GeV					
$\lvert\eta(\tau_h)\rvert$	≤ 2.3					
$\Delta R(p_i, p_j)$	≥ 0.3					

What Do We Look For at the LHC?

Three Complementary Approaches

- **SM Parameter Determination**
  - Precisely measure fundamental SM parameters
  - Test consistency of the SM framework
  - Reduce theoretical uncertainties
- **Indirect Searches (Precision Tests)**
  - Measure SM processes with high precision
  - Look for deviations in predicted distributions
  - Constrain new physics through virtual effects
- **Direct Searches**
  - Look for new particles in final states (resonances, excesses)
  - Examples: SUSY particles, Z', leptoquarks, dark matter mediators

Key Types of Measurements

Direct Signatures:

- Resonant mass peaks
- Excess events over SM background
- Missing transverse energy (MET)
- Unusual kinematic features

Precision Observables:

- Differential cross sections
- Angular correlations
- Rare decay rates
- Lepton flavor universality ratios
- Charge-parity (CP) asymmetries

The Quark-Gluon Sea

Partons are the fundamental constituents inside protons: valence quarks (uud) and a **sea** of virtual quark-antiquark pairs and gluons.

The interacting partons are typically **sea quarks or gluons**, which carry only a fraction of the proton's momentum but dominate the cross-section at high energies.

Jet Clustering & Missing Transverse Momentum

Jet clustering groups collimated particle showers into jets to reconstruct the original quarks/gluons from the hard scatter.

Missing p_T appears as transverse momentum imbalance, signaling undetected particles like neutrinos:

$$\vec{p}_T^{\,\text{miss}} = -\sum_{\text{visible prods}} \vec{p}_T$$

From Theory to Simulation: FeynRules

Input: `.fr` Model File

- Lagrangian $\mathcal{L}$ terms
- Particle definitions: F, V, S fields
- Gauge symmetries: $SU(N)$, $U(1)$
- Parameters: masses, couplings
- Mixing matrices, constraints

Output: UFO Format

- Complete Feynman rules (vertices)
- Lorentz and color structures
- Parameter definitions
- Ready to simulate

Why Simulate New Physics Models?

- **Predict signals** for experimental searches
- **Test theoretical consistency** (unitarity, constraints)
- **Optimize analyses** before data collection
- **Interpret potential discoveries** from LHC data
- **Compare predictions** across different models

Features

↗ **Standardized** UFO format
↗ **Flexible** for BSM models
↗ **Community-driven**
↘ **NLO complexity**
↘ **Poor debugging**
↘ **Performance issues**

Statistical Significance

The statistical significance for discovery is a parametric test defined as:

$$\kappa = \frac{\langle t \rangle_B - \langle t \rangle_{S+B}}{\sigma_{S+B}} \tag{12}$$

where $t = -2\ln[\mathcal{L}(n|S+B)/\mathcal{L}(n|B)] \sim \chi^2$ is the optimal test statistic.

We denote $\mathcal{L}$ as the likelihood for Poisson-distributed event counts n in each bin and gaussian systematic uncertainties.

For binned data analysis, this simplifies to:

$$\kappa = \frac{\sum_i s_i w_i}{\sqrt{\sum_i (s_i + b_i + \delta_{\text{sys}}^2) w_i^2}} \tag{13}$$

where s_i, b_i are signal/background events in bin i, $w_i \sim \ln(1 + s_i/b_i)$ are optimal weights, and δ_{sys} is the systematic uncertainty.

Boosted Decision Trees vs Neural Networks

BDTs

Strengths:

- **Interpretable**: Feature importance rankings
- **Robust**: Handle missing data well
- **Fast training**: Efficient on tabular data
- **Low hyperparameter tuning**: Reasonable defaults work

Limitations:

- **Extrapolation**: Poor outside training range
- **High-dimensional**: Can struggle with many features
- **Discontinuous**: Piecewise constant predictions

Deep NNs

Strengths:

- **Flexible**:
  Can learn complex non-linearities
- **High-dimensional**:
  Excel with many features
- **Continuous**:
  Smooth function approximations
- **Transfer learning**:
  Pretrained models possible

Limitations:

- **Black box**:
  Difficult to interpret
- **Data hungry**:
  Need large training sets
- **Computational**:
  Require GPUs for large networks
- **Sensitive**:
  Require careful hyperparameter tuning

HEP Use Cases

BDTs: Preferred for $\sim 10 - 100$ features
NNs: Preferred for low-level data (images, graphs)

Feasible Experimental Signatures

Kinematic Variables

Fig.: Transverse momentum distribution of the leading b-quark jet candidate. The distributions are shown for the two main SM background processes and two signal benchmark points.

Feasible Experimental Signatures

Kinematic Variables

Fig.: Transverse momentum distribution of the leading muon candidate. The distributions are shown for the two main SM background processes and two signal benchmark points.

Where the SM charges for the leptoquark, in the $Y = 2(Q - T_3)$ convention, are

	$\bar{q}_L$	ℓ_L^j	$\bar{q}_L \gamma_\mu \ell_L$	U_1^μ
$U(1)$	$-1/3$	-1	$-4/3$	$+4/3$
SU(2)	$\bar{\mathbf{2}}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
SU(3)	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\bar{\mathbf{3}}$	$\mathbf{3}$

Then, the leptoquark $U_1 \sim (\mathbf{3}_C, \mathbf{1}_I, 4/3_Y)$.

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The full Lagrangian for the vector leptoquark is

$$\mathcal{L}_U = -\frac{1}{2} U^\dagger_{\mu\nu} U^{\mu\nu} + M_U^2 U^\dagger_\mu U^\mu - i g_s U^\dagger_\mu T^a U_\nu G_a^{\mu\nu} - \frac{2i}{3} g' U^\dagger_\mu U_\nu B^{\mu\nu}$$
$$+ \frac{g_U}{\sqrt{2}} \left[U_1^\mu \left(\beta_L^{ij} \bar{q}_L^i \gamma_\mu e_L^j + \beta_R^{ij} \bar{d}_R^i \gamma_\mu e_R^j \right) + \text{ h.c. } \right]$$

where $U_{\mu\nu} = \mathcal{D}_\mu U_\nu - \mathcal{D}_\nu U_\mu$, $\mathcal{D}_\mu = \partial_\mu - i g_s G_\mu^a T^a - i \frac{2}{3} g_Y B_\mu$, and the couplings β_L and β_R are complex 3×3 matrices in flavor space.

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where $U_{\mu\nu} = \mathcal{D}_\mu U_\nu - \mathcal{D}_\nu U_\mu$, $\mathcal{D}_\mu = \partial_\mu - i g_s G_\mu^a T^a - i \frac{2}{3} g_Y B_\mu$, and the couplings β_L and β_R are complex 3×3 matrices in flavor space.

Constraints from $\Delta F = 2$ and lepton flavor violation impose a hierarchy with dominant third generation couplings:

$$\left|\beta_L^{11}\right|, \left|\beta_L^{12}\right|, \left|\beta_L^{21}\right|, \left|\beta_L^{22}\right|, \left|\beta_L^{31}\right| \ll \left|\beta_L^{13}\right| \ll \left|\beta_L^{23}\right|, \left|\beta_L^{32}\right| \ll \left|\beta_R^{33}\right|, \left|\beta_L^{33}\right| = \mathcal{O}(1), \quad (14)$$

where β_R is diagonal.

Two body scattering

CM-Frame

Consider the process

$$A(\vec{p}_1) + B(\vec{p}_2) \longrightarrow C(\vec{p}_3) + D(\vec{p}_4), \tag{15}$$

From the Golden Rule, the cross section is given by

$$\sigma = \frac{n!(2\pi)^4}{4\sqrt{(\vec{p}_1 \cdot \vec{p}_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3} \frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}. \tag{16}$$

But, in the CM frame, $\vec{p}_1 + \vec{p}_2 = 0$, where

$$\sqrt{(\vec{p}_1 \cdot \vec{p}_2)^2 - (m_1 m_2)^2} = E_1 E_2 |\vec{p}_1|, \tag{17}$$

$$\delta^{(4)}(p_1 + p_2 - p_3 - p_4) = \delta\left(E_1 + E_2 - E_3 - E_4\right) \delta^{(3)}(\vec{p}_3 + \vec{p}_4). \tag{18}$$

Thus

$$\sigma = \left(\frac{1}{8\pi}\right)^2 \frac{n!}{(E_1 E_2)|\vec{p}_1|} \int |\mathcal{M}|^2 \frac{\delta\left(E_1 + E_2 - \sqrt{\vec{\mathbf{p}}_3^2 + m_3^2} - \sqrt{\vec{\mathbf{p}}_3^2 + m_4^2}\right)}{\sqrt{\vec{\mathbf{p}}_3^2 + m_3^2}\sqrt{\vec{\mathbf{p}}_3^2 + m_4^2}} \mathrm{d}\vec{\mathbf{p}}_3 \tag{19}$$

Integrating over the radial part $|\vec{\mathbf{p}}_3|$, we get

$$\sigma = \left(\frac{1}{8\pi}\right)^2 \frac{n!|\vec{p}_3|}{(E_1+E_2)^2|\vec{p}_1|} \int |\mathcal{M}|^2 \mathrm{d}\Omega, \tag{20}$$

with

$$|\vec{p}_3| = \frac{1}{2}\frac{\sqrt{((E_1+E_2)^2 - m_3^2 - m_4^2)^2 - 4m_3^2 m_4^2}}{E_1+E_2}, \tag{21}$$

the outgoing momentum in the CM frame.

We prefer work with differential cross section as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{n!}{(E_1+E_2)^2}\frac{|\vec{p}_3|}{|\vec{p}_1|}|\mathcal{M}|^2. \tag{22}$$

Defining $\sqrt{s} = E_1 + E_2$, we have

$$|\vec{p}_3| = \frac{1}{2}\frac{\sqrt{(s - m_3^2 - m_4^2)^2 - 4m_3^2 m_4^2}}{\sqrt{s}}, \quad |\vec{p}_1| = \frac{1}{2}\frac{\sqrt{(s - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2}}{\sqrt{s}}. \tag{23}$$

so the differential cross section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{n!}{s}\sqrt{\frac{(s-(m_3+m_4)^2)(s-(m_3-m_4)^2)}{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}|\mathcal{M}|^2. \tag{24}$$

Note that at this point, we don't need to know the explicit form of the matrix element $\mathcal{M}$, so it is a generic result.

Writting t in terms of s and θ

In general, there are three Lorentz-invariant useful kinematical variables to describe the scattering process, known as Mandelstam variables:

$$\hat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2 = m_1^2 + m_2^2 + 2p_1^\mu p_{2\mu} = m_3^2 + m_4^2 + 2p_3^\mu p_{4\mu}, \quad (25)$$

$$\hat{t} = (p_1 - p_3)^2 = (p_2 - p_4)^2 = m_1^2 + m_3^2 - 2p_1^\mu p_{3\mu} = m_2^2 + m_4^2 - 2p_2^\mu p_{4\mu}, \quad (26)$$

$$\hat{u} = (p_1 - p_4)^2 = (p_2 - p_3)^2 = m_1^2 + m_4^2 - 2p_1^\mu p_{4\mu} = m_2^2 + m_3^2 - 2p_2^\mu p_{3\mu}. \quad (27)$$

In the CM-frame, $\hat{s} = s = (E_1 + E_2)^2$. If, $m_3 = m_4$ and $m_1 = m_2$ with $E_1 = E_2 = E_3 = E_4 = E$, we have

$$t = -(\vec{\mathbf{p}}_1 - \vec{\mathbf{p}}_3)^2 = -\vec{\mathbf{p}}_1^2 - \vec{\mathbf{p}}_3^2 + 2\vec{\mathbf{p}}_1\vec{\mathbf{p}}_3 \quad (28)$$

where $\vec{\mathbf{p}}_1^2 = E^2 - m_1^2$ and $\vec{\mathbf{p}}_3^2 = E^2 - m_3^2$.

So, in terms of s, t could be written as

$$t = -2s + (m_1^2 + m_3^2) + 2\sqrt{(s/4 - m_1^2)(s/4 - m_3^2)}\cos\theta, \quad (29)$$

with θ the scattering angle in the CM-frame.