E-Hall Evacuation - Algebraic Model

SETS

T - time

N - nodes

- S1 - pathway nodes
  - RH - room-hall intersections
  - HH - hall-hall intersections
  - SH - stairway intersections
- S2 - source nodes
  - B - bathrooms
  - C - classrooms
  - F - conference rooms
  - H - machine shops
  - L - labs
  - M - computer lab's
  - O - offices
  - P - penthouse
  - R - research rooms
  - U - solo office
- S3 - sink nodes
  - E - exits
- NE - all nodes that are not exits

ARCS - the set of arcs connecting nodes

PARAMETERS

area - the area of floorspace occupiable by a person (ft^2)
$start_I$ - initial source node, I, population (# people)
period - time period length (seconds)
rate - rate at which people walk
$cap_{I,J}$ - capacity of arc IJ (# people)
$dist_{I,J}$ - length of arc IJ (ft)
$tau_{I,J}$ - number of time periods required to cross an arc (# period)
$width_{I,J}$ - width of arc I,J (ft)

PARAMETER ASSUMPTIONS

area = $9ft^2$

$start_I = \{ 2$ if $I=B$, 30 if $I=C$, 0 if $I=F$, 2 if $I=H$, 4 if $I=L$, 10 if $I=M$, 5 if $I=O$, 0 if $I=P$, 2 if $I=R$, 1 if $I=U \}$

period = 1 second

rate = 5 ft/sec

$cap_{I,J} = \dfrac{width_{IJ} * dist_{IJ}}{area} \;\Big|\; cap_{IJ} \in Z,\ cap_{IJ} \geq 0,\ cap_{IJ} \bmod tau_{IJ} = 0 \ \forall I$

- cap_{IJ} must be greater than 0 and it must of course be an integer to accurately represent the number of people
- cap_{IJ} must be an integer multiple of tau_{IJ} so that there is a "level" flow of people traversing an arc when flow is at max cap.

$dist_{IJ}$ and $width_{IJ}$ are drawn from E-Hall floor plans

period $\in Z^+$, $1 \leq$ period < 5

- period can be only so large relative to $dist_{IJ}$ and rate before it leads to a wildly innacurate (fractional) tau_{IJ}

$tau_{IJ} = \left\lceil \dfrac{dist_{IJ}}{rate * period} \right\rceil \;\Big|\; tau_{IJ} \in Z^+$

- tau is a necessary parameter for representing arc capacities from a discrete-time-oriented perspective
- rounding tau_{IJ} "cleans up" its time-distance formulation to give us a integer value that is more compatible with a discrete-time model

VARIABLES

X_{IJT} = number of people leaving node I and entering arc IJ towards node J at time T (positive variable)

Y_{IT} = number of people queued at node I at time T (pos. var)

$Z_T = \begin{cases} 1 & \text{if network has more than 0 people in it at time T} \\ 0 & \text{if network is empty} \end{cases}$

totalTime = total time spent evacuating network (free var)

OBJECTIVE

$$\min \ \text{totalTime} = \sum_T Z_T$$

CONSTRAINTS

flow balance: $$Y_{J,T} = Y_{J,T-1} + \sum_{I,S:\, S=T-tau_{IJ} \,\wedge\, \exists\, ARC_{IJ}} X_{IJS} - \sum_{K:\, \exists\, ARC_{JK}} X_{J,K,T-1}$$

$$\forall\, J,T \mid T>1$$

- Accounts for the number of people that will be arriving at node J in time T based on the time they left their precursor node I, which is in turn based on the number of periods tau_{IJ} required to traverse the arc

arc capacity: $$cap_{IJ} \geq \sum_{S:\, S \geq T-tau_{IJ}+1 \,\wedge\, S \leq T} X_{IJS} + X_{JIS} \quad \forall\, I,J,T$$

- At any time T, the volume of people on an arc can be no greater than its capacity cap_{IJ}

system population indication: $$\text{totalPop} \leq \sum_E Y_{E,T} + \text{totalPop} * Z_T \quad \forall\, T>1$$

- If the number of people that have reached the exits is less than the total number of people initially in the building, then the network has not been completely evacuated

X-constraint: $X_{IJT} \leq cap_{IJ} / tau_{IJ} \quad \forall\, I,J,T \mid \exists\, ARC_{IJ}$

- the number of people allowed to leave a node at a time; restricts model from fully populating an arc in just a single time period