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Financial Sensitivity Definition

◆ Financial sensitivity is the measure of the value reaction of a financial instrument to changes in underlying factors.

◆ The value of a financial instrument is impacted by many factors, such as interest rate, stock price, implied volatility, time, etc.

◆ Financial sensitivities are also called Greeks, such as Delta, Gamma, Vega and Theta.

◆ Financial sensitivities are risk measures that are more important than fair values.

◆ They are vital for risk management: isolating risk, hedging risk, explaining profit and loss, etc.

Delta Definition

◆ Delta is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying asset price.

◆ Interest rate Delta:

$$IrDelta = \frac{\partial V}{\partial r} = \frac{V(r + 0.0001) - V(r)}{0.0001}$$

where $V(r)$ is the instrument value and r is the underlying interest rate.

◆ PV01, or dollar duration, is analogous to interest rate Delta but has the change value of a one-dollar annuity given by

$$PV01 = V(r + 0.0001) - V(r)$$

Delta Definition (Cont)

◆ Credit Delta applicable to fixed income and credit product is given by

$$CreditDelta = \frac{\partial V}{\partial c} = \frac{V(c + 0.0001) - V(c)}{0.0001}$$

where c is the underlying credit spread.

◆ CR01 is analogous to credit Delta but has the change value of a one-dollar annuity given by

$$PV01 = V(r + 0.0001) - V(r)$$

◆ Equity/FX/Commodity Delta

$$Delta = \frac{\partial V}{\partial S} = \frac{V(1.01S) - V(S)}{0.01 * S}$$

where S is the underlying equity price or FX rate or commodity price

Vega Definition

◆ Vega is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying implied volatility.

$$Vega = \frac{\partial V}{\partial \sigma} = \frac{V(\sigma + \Delta\sigma) - V(\sigma)}{\Delta\sigma}$$

where σ is the implied volatility.

◆ Only non-linear products, such as options, have Vegas.

Gamma Definition

◆ Gamma is a second order Greek that measures the value change of a financial instrument with respect to changes in the underlying price.

$$Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{V(S + 0.5 * \Delta S) + V(S - 0.5 * \Delta S) - 2V(S)}{\Delta S^2}$$

Theta Definition

◆ Theta is a first order Greek that measures the value change of a financial instrument with respect to time.

$$Theta = \frac{\partial V}{\partial t} = \frac{V(t + \Delta t) - V(t)}{\Delta t}$$

Curvature Definition

◆ Curvature is a new risk measure for options introduced by Basel FRTB.

◆ It is a risk measure that captures the incremental risk not captured by the delta risk of price changes in the value of an option.
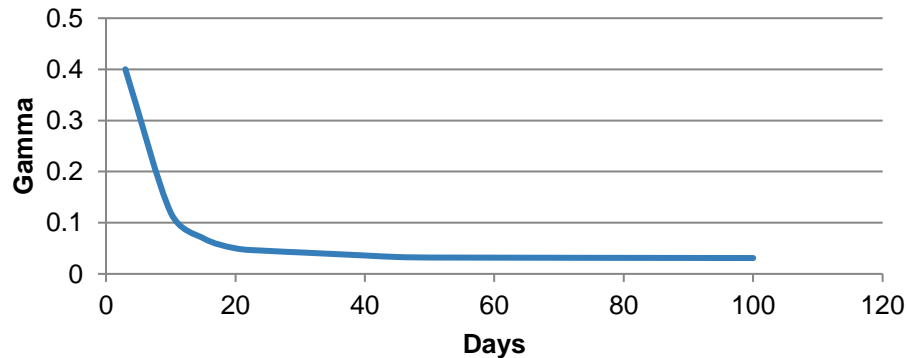
$$Curvature = min\{V(S + \Delta W) - V(S) - \Delta W * Delta, V(S - \Delta W) - V(S) - \Delta W * Delta\}$$

where ΔW is the risk weight.

Option Sensitivity Pattern
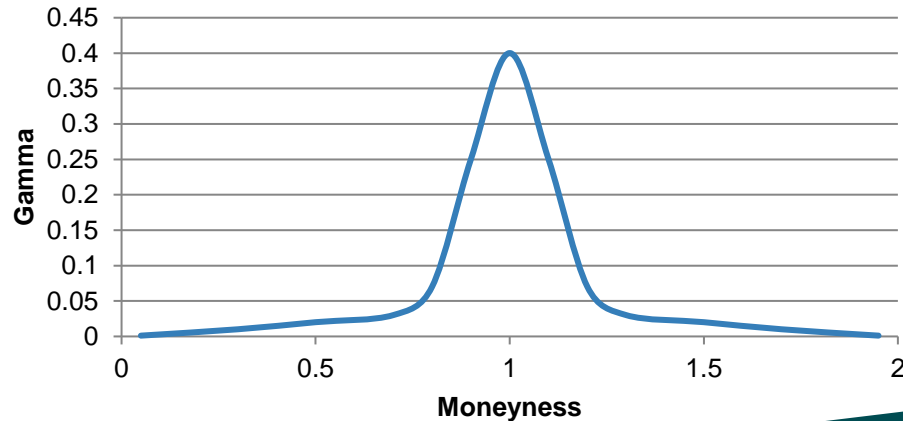
- Sensitivity behaviors are critical for managing risk.

- Gamma

  - Gamma behavior in relation to time to maturity shown below.
  - Gamma has a greater effect on shorter dated options.

Option Sensitivity Pattern (Cont)

◆ Gamma behavior in relation to moneyness shown below.
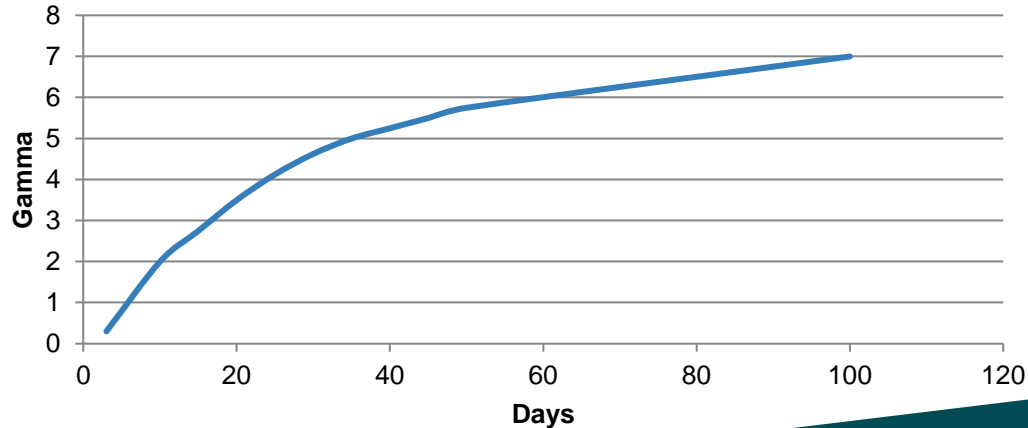
◆ Gamma has the greatest impact on at-the-money options.

Option Sensitivity Pattern (Cont)

◆ Vega
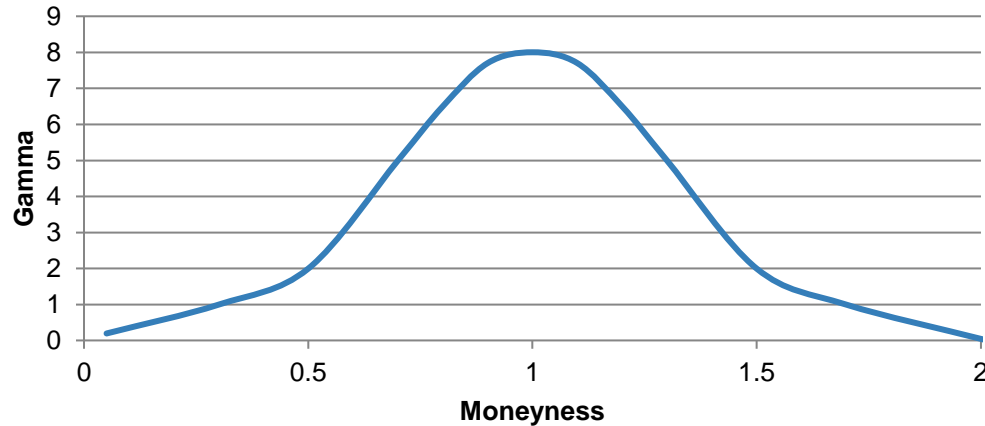
  ◆ Vega behavior in relation to time to maturity shown below.
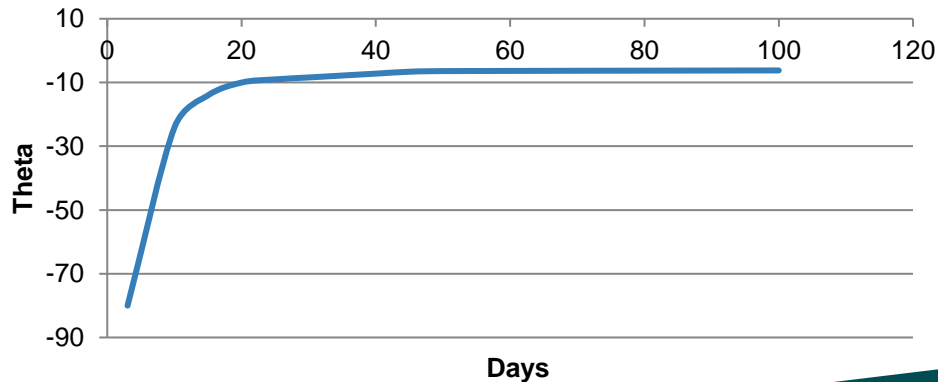 ◆ Vega has a greater effect on longer dated options.

Option Sensitivity Pattern (Cont)

- Vega behavior in relation to moneyness shown below.
- Vega has the greatest impact on at-the-money options.

Option Sensitivity Pattern (Cont)
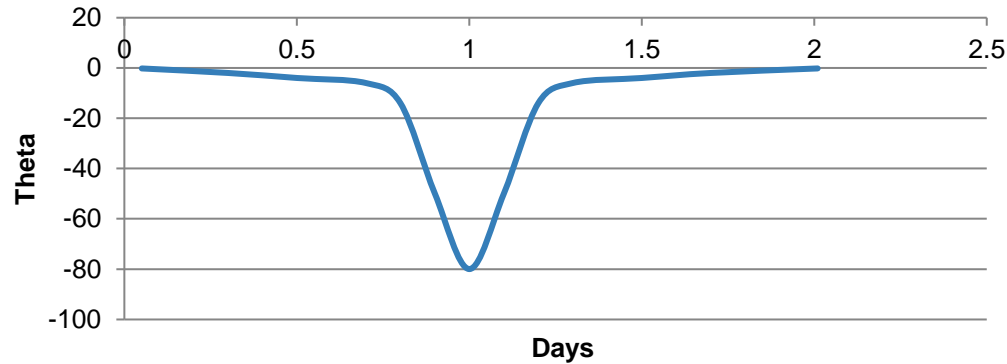
◆ Theta or time decay

  ◆ Theta is normally negative except some deeply in-the-money deals.
  ◆ Theta behavior in relation to time to maturity shown below.
  ◆ Theta has a greater effect on shorter dated options.

Option Sensitivity Pattern (Cont)

◆ Theta behavior in relation to moneyness shown below.

◆ Theta has the biggest impact on at-the-money options.

Sensitivity Hedging

◆ The objective of hedging is to have a lower price volatility that eliminates both downside risk (loss) and upside profit.

◆ Hedging is a double-edged sword.

◆ The profit of a broker or an investment bank comes from spread rather than market movement. Thus it is better to hedge all risks.

◆ Delta is normally hedged.

◆ Vega can be hedged by using options.

◆ Gamma is hardly hedged in real world.

Sensitivity Profit & Loss (P&L)

◆ Hypothetic P&L is the P&L that is purely driven by market movement.

◆ Hypothetic P&L is calculated by revaluing a position held at the end of the previous day using the market data at the end of the current day, i.e.,

$$Hypothetical P\&L = V(t-1, P_{t-1}, M_t) - V(t-1, P_{t-1}, M_{t-1})$$

where t-1 is yesterday; t is today; P_{t-1} is the position at yesterday; M_{t-1} is yesterday's market and M_t is today's market.

◆ Sensitivity P&L is the sum of Delta P&L, Vega P&L and Gamma P&L.

◆ Unexplained P&L = Hypothetical P&L − Sensitivity P&L.

Sensitivity Profit & Loss (Cont)

◆ Delta P&L:

$$DeltaP\&L = Delta * (S_t - S_{t-1})$$

where S_t is today's underlying price and S_{t-1} is yesterday's underlying price.

◆ Vega P&L:

$$VegaP\&L = Vega * (\sigma_t - \sigma_{t-1})$$

where σ_t is today's implied volatility and σ_{t-1} is yesterday's implied volatility.

◆ Gamma P&L:

$$GammaP\&L = 0.5 * Gamma * (S_t - S_{t-1})^2$$

Backbone Adjustment

◆ Backbone adjustment is an advanced topic in sensitivity P&L.

◆ It can be best explained mathematically.

◆ Assume the value of an option is a function of the underlying price S and implied volatility σ, i.e., $V = F(S, \sigma)$.

◆ If the implied volatility is a function of the ATM volatility and strike (sticky strike assumption), i.e., $\sigma = \sigma_A + f(K)$, the first order approximation of the option value is

$$\Delta V = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial \sigma_A} d\sigma_A = DeltaP\&L + VegaP\&L$$

where $DeltaP\&L = \frac{\partial F}{\partial S} dS$ and $VegaP\&L = \frac{\partial F}{\partial \sigma_A} d\sigma_A$

Backbone Adjustment (Cont)

◆ If the implied volatility is a function of the ATM volatility and moneyness K/S (sticky moneyness or stricky Delta assumption), i.e., $\sigma = \sigma_A + f(S, K)$, the first order approximation of the option value is

$$\Delta V = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial \sigma_A} d\sigma_A + \frac{\partial F}{\partial \sigma} \frac{\partial \sigma}{\partial S} dS = DeltaP\&L + VegaP\&L$$

where $DeltaP\&L = (\frac{\partial F}{\partial S} + \frac{\partial F}{\partial \sigma} \frac{\partial \sigma}{\partial S})dS$ and $VegaP\&L = \frac{\partial F}{\partial \sigma_A} d\sigma_A$

◆ Under sticky moneyness/Delta assumption, the DeltaP&L above has one more item, i.e., $\frac{\partial F}{\partial \sigma} \frac{\partial \sigma}{\partial S} dS$ that is the backbone adjustment.

Thanks!

You can find more details at
https://finpricing.com/lib/EqConvertible.html