Monte Carlo Value At Risk Introduction

Value at Risk (VaR) is the regulatory measurement for assessing market risk. It reports the maximum likely loss on a portfolio for a given probability defined as x% confidence level over N days. VaR is vital in market risk management and control. Also regulatory and economic capital computation is based on VaR results. Although VaR measure is objective and intuitive, it doesn't capture tail risk. There are three commonly used methodologies to calculate VaR – parametric, historical simulation and Monte Carlo simulation. This presentation focuses on Monte Carlo VaR.
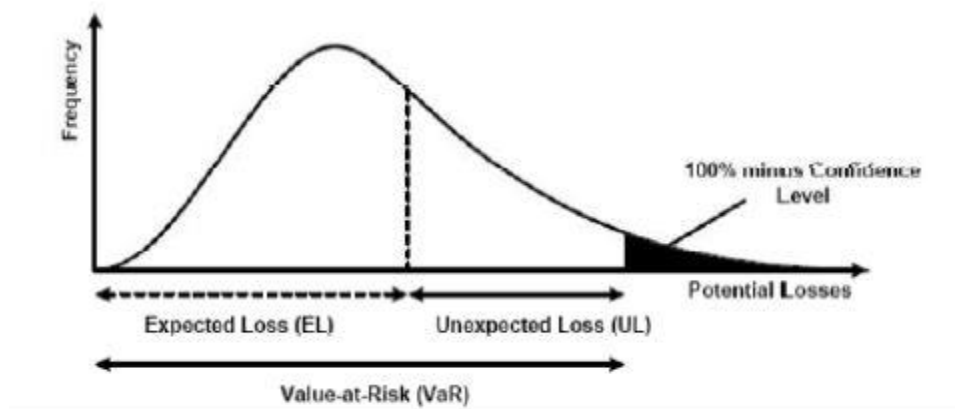
1. Monte Carlo VaR
1.1   Definition
Value at Risk (VaR) is the regulatory measurement for assessing market risk. It reports the maximum likely loss on a portfolio for a given probability defined as x% confidence level over N days. VaR is vital in market risk management and control.

1.2   VaR Roles
• Risk measurement
• Risk management
• Risk control

- Financial reporting
- Regulatory and economic capital

1.3 VaR Pros & Cons
- Regulatory measurement for market risk
- Objective assessment
- Intuition and clear interpretation
- Consistent, flexible and stable measurement
- Doesn't measure risk beyond the confidence level: tail risk
- Non sub-additive

1.4 VaR Approaches
- Parametric VaR
- Historical VaR
- Monte Carlo VaR

1.5 Monte Carlo Simulation
- Assumption
  Assuming market factors follow certain stochastic processes.
- Pros
  Easy back and stress test
  Good for high confidence level and tail risk
- Cons
  Dependent on distribution assumption
  Calibration required
  Extensive computation

1.6 Monte Carlo VaR Methodology
- Assume each market factor follows certain stochastic process: $\vartheta(\sigma_i W_i)$ where W is a Wiener process
- Calibrate each volatility σ_i and pair-wise correlation ρ_{ij} for any two market factors
- Simulate market factor changes δ_i based on the stochastic processes and correlated random variables.
- Generate market scenarios $x_i = x_0 \delta_i$
- Compute scenario PVs: $P(x_i)$
- Compute scenario P&L: $P(x_i) - P(x_0)$
- Sort all scenario P&Ls. The VaR is the number at 1% lowest level

1.7 VaR Scaling

- Normally firms compute 1-day 99% VaR
- Regulators require 10-day 99% VaR
- Under IID assumption, 10-day VaR = $\sqrt{10} * VaR_{1-day}$

1.8   VaR Backtest
- The only way to verify a VaR system is backtest
- At a certain day, compute hypothetic P&L (valuation date and portfolio unchanged)
  If (hypothetic P&L > VaR) ➜ breach
- For one year
  If number of breaches is 0-4, the VaR system is in Green zone
  If number of breaches is 5-9, the VaR system is in Yellow zone
  If number of breaches is 10 or more, the VaR system is in Red zone

You can find more details at

https://finpricing.com/lib/EqRangeAccrual.html