

Monte Carlo Value At Risk

Value at Risk is the regulatory measurement for assessing market risk. It reports the maximum likely loss on a portfolio for a given probability defined as x% confidence level over N days.

Value at Risk is vital in market risk management and control. Also, regulatory and economic capital computation is based on Value at Risk results. Although Value at Risk measure is objective and intuitive, it doesn't capture tail risk.

There are three commonly used methodologies to calculate Value at Risk – parametric, historical simulation and Monte Carlo simulation. This presentation focuses on Monte Carlo Value at Risk.

Monte Carlo Simulation

- Assumption
Assuming market factors follow certain stochastic processes.
- Pros
Easy back and stress test
Good for high confidence level and tail risk
- Cons
Dependent on distribution assumption
Calibration required
Extensive computation

Monte Carlo Value at Risk Methodology

- Assume each market factor follows certain stochastic process: $\vartheta(\sigma_i W_i)$ where W is a Wiener process
- Calibrate each volatility σ_i and pair-wise correlation ρ_{ij} for any two market factors

- Simulate market factor changes δ_i based on the stochastic processes and correlated random Value at Risk.
- Generate market scenarios $x_i = x_0 \delta_i$
- Compute scenario PVs: $P(x_i)$
- Compute scenario P&L: $P(x_i) - P(x_0)$
- Sort all scenario P&Ls. The Value at Risk is the number at 1% lowest level

1.1 Value at Risk Scaling

- Normally firms compute 1-day 99% Value at Risk
- Regulators require 10-day 99% Value at Risk
- Under IID assumption, 10-day Value at Risk = $\sqrt{10} * VaR_{1-day}$

Reference:

<https://finpricing.com/lib/EqRangeAccrual.html>