

VaR Introduction I: Parametric VaR

Parametric VaR

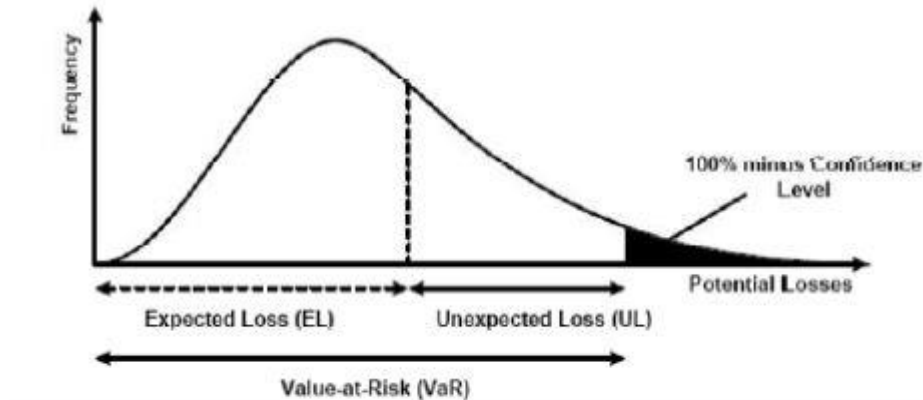
Summary

- ◆ VaR Definition
- ◆ VaR Roles
- ◆ VaR Pros and Cons
- ◆ VaR Approaches
- ◆ Parametric VaR
- ◆ Parametric VaR Methodology
- ◆ Parametric VaR Implementation
- ◆ VaR Scaling
- ◆ VaR Backtest

Parametric VaR

Value at Risk (VaR) Definition

- ◆ The maximum likely loss on a portfolio for a given probability defined as x% confidence level over N days
- ◆ $\Pr(\text{Loss} > \text{VaR}(x\%)) < 1 - x\%$



Parametric VaR

VaR Roles

- ◆ Risk measurement
- ◆ Risk management
- ◆ Risk control
- ◆ Financial reporting
- ◆ Regulatory and economic capital

Parametric VaR

VaR Pros & Cons

- ◆ Pros
 - ◆ Regulatory measurement for market risk
 - ◆ Objective assessment
 - ◆ Intuition and clear interpretation
 - ◆ Consistent and flexible measurement
- ◆ Cons
 - ◆ Doesn't measure risk beyond the confidence level: tail risk
 - ◆ Non sub-additive

Three VaR Approaches

- ◆ Parametric VaR
- ◆ Historical VaR
- ◆ Monte Carlo VaR

The presentation focuses on parametric VaR.

Parametric VaR

Parametric VaR

- ◆ Assumption

 - Asset returns follow normal distribution

- ◆ Pros

 - Fast and simple calculation

 - Intuitive

- ◆ Cons

 - Poor accuracy for non-linear products

 - Second order approximation

 - Hard to incorporate stress test

Parametric VaR

Parametric VaR Methodology

- ◆ Assuming an asset return/valueChange follows normal distribution, the quantile of 99% confidence level is 2.326σ where σ is standard derivation
- ◆ If absolute return $X_1 - X_0$ is normally distributed, the 99% worse change of X is $X_1 - X_0 = 2.326\sigma$
- ◆ The VaR is given by $VaR = \frac{\partial F}{\partial X} \Delta X = \frac{\partial F}{\partial X} \times 2.326 \times \sigma$ where $\frac{\partial F}{\partial X}$ is the delta
- ◆ Similarly for a relative return $\frac{X_1 - X_0}{X_0}$, the VaR can be expressed as

$$VaR = \frac{\partial F}{\partial X} \Delta X = \frac{\partial F}{\partial X} (X_1 - X_0) = \frac{\partial F}{\partial X} \times X_0 \times 2.326 \sigma$$

Parametric VaR

Parametric VaR Implementation

- ◆ For each asset/instrument/riskFactor, calibrate volatility σ_i based on daily return
- ◆ For each risk factor pair, calibrate correlation ρ_{ij}
- ◆ Calculate the variance of a portfolio value change

$$V_p^2 = [\Delta(P_1)\sigma_1 \quad \dots \quad \Delta(P_n)\sigma_n] \begin{bmatrix} \rho_{11} & \dots & \rho_{1n} \\ \vdots & & \vdots \\ \rho_{n1} & \dots & \rho_{nn} \end{bmatrix} \begin{bmatrix} \Delta(P_1)\sigma_1 \\ \vdots \\ \Delta(P_n)\sigma_n \end{bmatrix}$$

- ◆ The portfolio VaR is $2.326 \sqrt{V_p^2}$

VaR Scaling

- ◆ Normally firms compute 1-day 99% VaR
- ◆ Regulators require 10-day 99% VaR
- ◆ Under IID assumption, 10-day VaR = $\sqrt{10} * VaR_{1-day}$

VaR Backtest

- ◆ The only way to verify a VaR system is to backtest
- ◆ At a certain day, compute hypothetical P&L. If (hypothetical P&L > VaR) → breach, otherwise, ok
- ◆ Hypothetic P&L is computed by holding valuation date and portfolio unchanged
- ◆ In one year period,
 - ◆ If number of breaches is 0-4, the VaR system is in Green zone
 - ◆ If number of breaches is 5-9, the VaR system is in Yellow zone
 - ◆ If number of breaches is 10 or more, the VaR system is in Red zone



Thanks!



You can find more online presentations at
<https://finpricing.com/lib/EqVariance.html>

