NewtonVerfahren

September 29, 2025

Beispielfunktion: $f(x) = x^2 - x - 1$, $x_0 = 0$

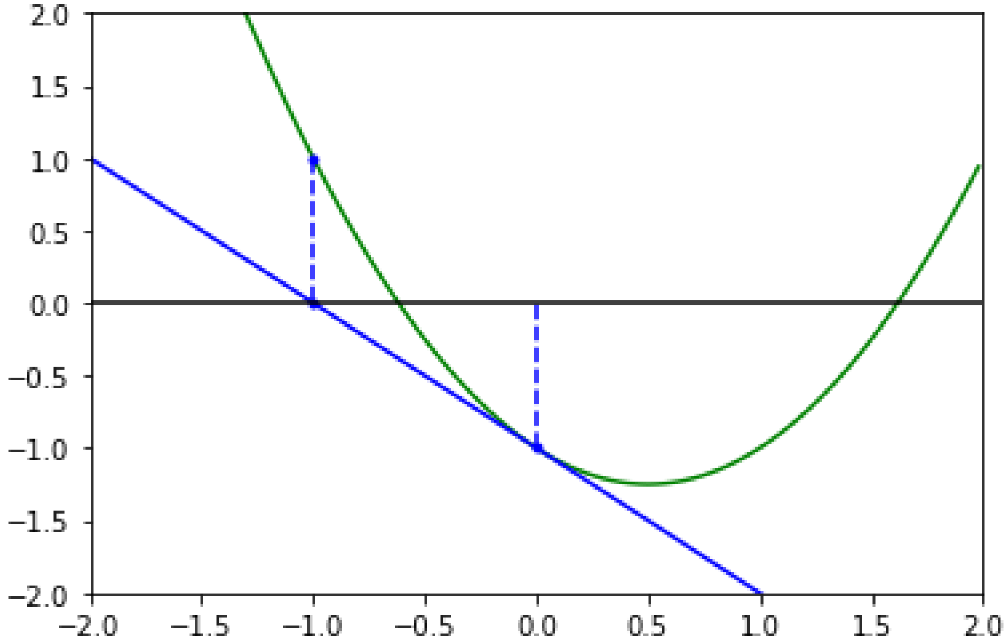
Newton Verfahren

[55]:
```python
import math
import numpy as np              # array Funktionen
import matplotlib.pyplot as plt # plot Funktionen

x = np.arange(-2, 2, 0.02) # x-Werte
y = x**2-x-1               # Zugehörige y-Werte der Wertetabelle
dy= 2*x-1
plt.plot(x, y, 'g-')       # Plot der Wertetabelle

x0=0
x1=x0-(x0**2-x0-1)/(2*x0-1)
dy0 = -1
yt  = -1*(x)-1
plt.plot(x, yt, 'b-')      # Plot der Wertetabelle
plt.plot([x0], [x0**2-x0-1], 'b.')
plt.plot([x0, x0], [x0**2-x0-1, 0], 'b--')

plt.plot([x1], [0], 'b.')
plt.plot([x1], [x1**2-x1-1], 'b.')
plt.plot([x1, x1], [x1**2-x1-1, 0], 'b--')
#plt.plot( [0, 0], [-2, 2], 'k-')
plt.plot([-2, 2], [0,  0], 'k-')
plt.axis([-2, 2, -2, 2])
plt.show()
```



[59]:

```
import math

i    = 0
x_i  = 0.0
here = math.inf
a   = 0.5*(1-5**0.5) # Nullstelle
f   = lambda x: x**2-x-1
r   = list()
print("i=", "\tx_i=", "\t\t\tf_i=", "\t\t\tr_i=")
while (math.fabs(f(x_i)) > 1e-6):
    df   = 2*x_i-x_i-1
    x_ii = x_i - f(x_i)/df
    r.append(math.fabs((x_ii-a)/(x_i-a)) if math.fabs(x_i-a) > 1e-16 else math.
 ↪inf)
    print(("%d" % (i)), ("\t%0.16f" % (x_i)), ("\t%0.16f" % (f(x_i))), ("\t%0.
 ↪16f" % (r[-1])))
    last = here
    here = math.fabs(x_ii-x_i)
    x_i  = x_ii
    i   += 1

    if (here >= last): # (x_i) divergiert
        break
```

```
#o = [math.log(r[i])/math.log(r[i-1]) if r[i-1]>0 and math.log(r[i-1])!=0.0␣
 ↪else 0 for i in range(1, len(r))]
print(r)
```

```
i=      x_i=                    f_i=                    r_i=
0       0.0000000000000000      -1.0000000000000000     0.6180339887498947
1       -1.0000000000000000     1.0000000000000000      0.3090169943749476
2       -0.5000000000000000     -0.2500000000000000     0.4120226591665956
3       -0.6666666666666666     0.1111111111111112      0.3708203932499415
4       -0.5999999999999999     -0.0400000000000003     0.3862712429686773
5       -0.6250000000000000     0.0156250000000000      0.3803286084614796
6       -0.6153846153846154     -0.0059171597633135     0.3825924692261100
7       -0.6190476190476191     0.0022675736961451      0.3817268754043872
8       -0.6176470588235294     -0.0008650519031141     0.3820573748633614
9       -0.6181818181818182     0.0003305785123966      0.3819311166432957
10      -0.6179775280898877     -0.0001262466860243     0.3819793402685869
11      -0.6180555555555556     0.0000482253086420      0.3819609200863413
12      -0.6180257510729614     -0.0000184199377404     0.3819679559015968
13      -0.6180371352785146     0.0000070358617874      0.3819652684567305
14      -0.6180327868852460     -0.0000026874496102     0.3819662949007375
15      -0.6180344478216818     0.0000010265159329      0.3819659029158854
[0.6180339887498947, 0.3090169943749476, 0.4120226591665956, 0.3708203932499415,
0.38627124296867726, 0.3803286084614796, 0.38259246922610995,
0.38172687540438716, 0.3820573748633614, 0.3819311166432957, 0.3819793402685869,
0.3819609200863413, 0.3819679559015968, 0.3819652684567305, 0.3819662949007375,
0.38196590291588545]
```

Abbruchbedingung

Funktionswert

$$|f(x_i)| \leq \text{1e-6}$$

Messung der Konvergenzrate

Fehler gleich Abstand zur Nullstelle: $e_i := |x_i - a|$. Vergleichen aufeinanderfolgende Fehler e_i, $e_i = r_i \cdot e_{i-1}$.

Dann *Konvergenzfaktor*, *Konvergenzrate* r_i:

$$r_i = \frac{e_i}{e_{i-1}}$$

Dann *Konvergenzordnung* o_i:

$$e_i = e_{i-1}^{o_i}, o_i = \frac{\log e_i}{\log e_{i-1}}$$

Regula Falsi

[58]:
```python
import math

x_i  = 0.0
x_ii = -1.0
a  = 0.5*(1-5**0.5) # Nullstelle
f  = lambda x: x**2-x-1
r  = list()
print("i=", "\tx_i=", "\t\t\tf_i=", "\t\t\tr_i=")
i = 0
while (math.fabs(f(x_i)) > 1e-6):
    assert(f(x_i)*f(x_ii) <= 0.0) # f(x_i), f(x_ii) verschiedenes Vorzeichen,␣
↪also Intervall enthält Null
    df = (f(x_ii)-f(x_i))/(x_ii-x_i)
    # f(x_i)+df*(x-x_i) == 0
    x_new = x_i-f(x_i)/df
    if f(x_new)*f(x_i) >= 0.0: # f(x_new), f(x_i) gleiches Vorzeichen
        x_i = x_new
    else:                      # f(x_new), f(x_ii) gleiches Vorzeichen
        x_ii = x_new

    r.append(math.fabs((x_ii-a)/(x_i-a)) if math.fabs(x_i-a) > 1e-16 else math.
↪inf)
    print(("%d" % (i)), ("\t%0.16f" % (x_i)), ("\t%0.16f" % (f(x_i))), ("\t%0.
↪16f" % (r[-1])))
    i += 1

#o = [math.log(r[i])/math.log(r[i-1]) if r[i-1]>0 and math.log(r[i-1])!=0.0␣
↪else 0 for i in range(1, len(r))]
print(r)
```

```
i=      x_i=                    f_i=                    r_i=
0       -0.5000000000000000     -0.2500000000000000     3.2360679774997876
1       -0.6000000000000000     -0.0400000000000000     21.1803398874988567
2       -0.6153846153846154     -0.0059171597633135     144.1722092687420513
3       -0.6176470588235294     -0.0008650519031141     987.1710230273814659
4       -0.6179775280898876     -0.0001262466860246     6765.1708499452906835
5       -0.6180257510729613     -0.0000184199377408     46368.1708240762673086
6       -0.6180327868852459     -0.0000026874496104     317811.1708002091618255
7       -0.6180338134001252     -0.0000003920939740     2178309.1696905475109816
[3.2360679774997876, 21.180339887498857, 144.17220926874205, 987.1710230273815,
6765.170849945291, 46368.17082407627, 317811.17080020916, 2178309.1696905475]
```

Wir sehen:

Regula Falsi konvergiert hier sehr viel schneller. Es wird die Hälfte der Iterationen bis zu Funktionswert kleiner gleich 1e-6 benötigt.

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[ ]:
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