NewtonVerfahren

October 8, 2025

Beispielfunktion: $f(x) = x^2 - x - 1$, $x_0 = 0$

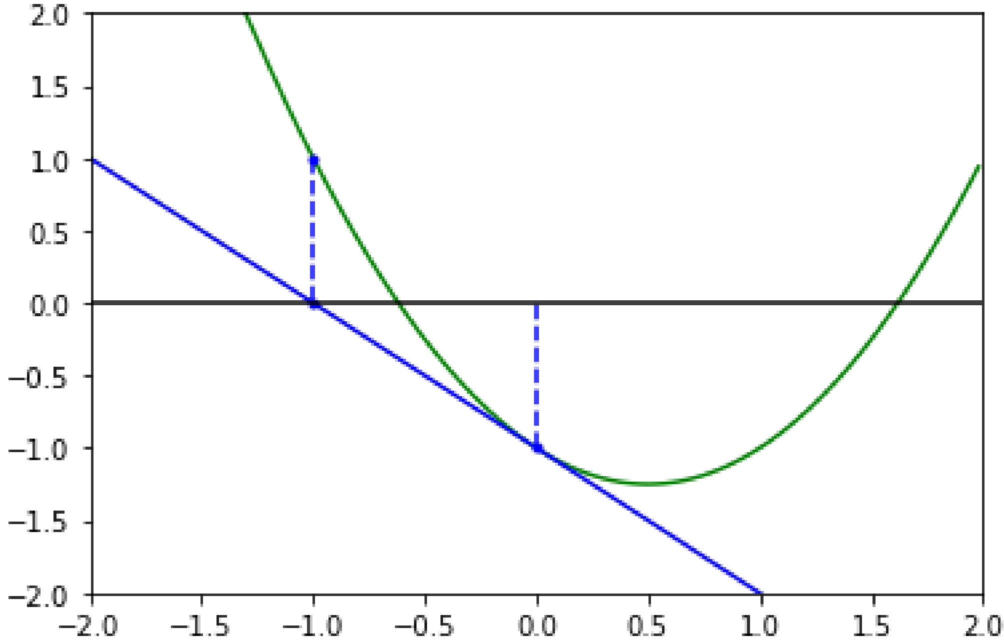
Newton Verfahren

[1]:
```
import math
import numpy as np               # array Funktionen
import matplotlib.pyplot as plt # plot Funktionen

x = np.arange(-2, 2, 0.02) # x-Werte
y = x**2-x-1               # Zugehörige y-Werte der Wertetabelle
dy= 2*x-1
plt.plot(x, y, 'g-')       # Plot der Wertetabelle

x0=0
x1=x0-(x0**2-x0-1)/(2*x0-1)
dy0 = -1
yt  = -1*(x)-1
plt.plot(x, yt, 'b-')      # Plot der Wertetabelle
plt.plot([x0], [x0**2-x0-1], 'b.')
plt.plot([x0, x0], [x0**2-x0-1, 0], 'b--')

plt.plot([x1], [0], 'b.')
plt.plot([x1], [x1**2-x1-1], 'b.')
plt.plot([x1, x1], [x1**2-x1-1, 0], 'b--')
#plt.plot( [0, 0], [-2, 2], 'k-')
plt.plot([-2, 2], [0,  0], 'k-')
plt.axis([-2, 2, -2, 2])
plt.show()
```



[2]:
```
import math

i    = 0
x_i  = 0.0
here = math.inf
a   = 0.5*(1-5**0.5) # Nullstelle
f   = lambda x: x**2-x-1
r   = list()
print("i=", "\tx_i=", "\t\t\tf_i=", "\t\t\tr_i=")
#while (math.fabs(f(x_i)) > 1e-6):
while (here > 1e-6):
    df   = 2*x_i-1
    x_ii = x_i - f(x_i)/df
    r.append(math.fabs((x_ii-a)/(x_i-a)) if math.fabs(x_i-a) > 1e-16 else math.
 ↪inf)
    print(("%d" % (i)), ("\t%0.16f" % (x_i)), ("\t%0.16f" % (f(x_i))), ("\t%0.
 ↪16f" % (r[-1])))
    last = here
    here = math.fabs(x_ii-x_i)
    x_i  = x_ii
    i   += 1

    if (here >= last): # (x_i) divergiert
        break
```

```
o = [math.log(r[i])/math.log(r[i-1]) if r[i-1]>0 and math.log(r[i-1])!=0.0 else␣
 ↪0 for i in range(1, len(r))]
print(o)
```

```
i=      x_i=                    f_i=                    r_i=
0       0.0000000000000000      -1.0000000000000000     0.6180339887498947
1       -1.0000000000000000     1.0000000000000000      0.1273220037500351
2       -0.6666666666666667     0.1111111111111112      0.0208425762500444
3       -0.6190476190476191     0.0022675736961451      0.0004528986436768
4       -0.6180344478216819     0.0000010265159331      0.0000002053228655
[4.283011828589276, 1.878064024235918, 1.9892339685514349, 1.9998698335259109]
```

Abbruchbedingung

Abstand der Iterierten

$$|x_{i+1} - x_i| \leq 1\text{e-}6$$

Messung der Konvergenzrate

Fehler gleich Abstand zur Nullstelle: $e_i := |x_i - a|$. Vergleichen aufeinanderfolgende Fehler e_i, $e_i = r_i \cdot e_{i-1}$.

Dann *Konvergenzfaktor*, *Konvergenzrate* r_i:

$$r_i = \frac{e_i}{e_{i-1}}$$

Dann *Konvergenzordnung* o_i:

$$e_i = e_{i-1}^{o_i}, o_i = \frac{\log e_i}{\log e_{i-1}}$$

Regula Falsi

[3]:
```
import math

x_ii = -1.0
x_i  = 0.0
a  = 0.5*(1-5**0.5) # Nullstelle
print(a)
f  = lambda x: x**2-x-1
r  = list()
print("i=", "\tx_i=", "\t\t\tf_i=", "\t\t\tr_i=")
i = 0
#while (math.fabs(f(x_i)) > 1e-6):
while (math.fabs(x_ii-x_i) > 1e-6):
    assert(f(x_i)*f(x_ii) <= 0.0) # f(x_i), f(x_ii) verschiedenes Vorzeichen,␣
 ↪also Intervall enthält Null
    df = (f(x_ii)-f(x_i))/(x_ii-x_i)
    # f(x_i)+df*(x-x_i) == 0
    x_new = x_i-f(x_i)/df
```

```
    print(x_i, x_new, x_ii)
    if f(x_new)*f(x_i) > 0.0: # f(x_new), f(x_i) gleiches Vorzeichen
        x_i = x_new
    else:                     # f(x_new), f(x_ii) gleiches Vorzeichen oder Null!
        x_ii = x_new

    #print(math.fabs(x_ii-x_i))
    r.append(math.fabs((x_ii-a)/(x_i-a)) if math.fabs(x_i-a) > 1e-16 else math.
 ↪inf)
    print(("%d" % (i)), ("\t%0.16f" % (x_i)), ("\t%0.16f" % (f(x_i))), ("\t%0.
 ↪16f" % (r[-1])))
    i += 1

o = [math.log(r[i])/math.log(r[i-1]) if r[i-1]>0 and r[i]>0 else 0 for i in␣
 ↪range(1, len(r))]
print(o)
```

```
-0.6180339887498949
i=      x_i=                    f_i=                    r_i=
0.0 -0.5 -1.0
0       -0.5000000000000000     -0.2500000000000000     3.2360679774997876
-0.5 -0.6 -1.0
1       -0.6000000000000000     -0.0400000000000000     21.1803398874988567
-0.6 -0.6153846153846154 -1.0
2       -0.6153846153846154     -0.0059171597633135     144.1722092687420513
-0.6153846153846154 -0.6176470588235294 -1.0
3       -0.6176470588235294     -0.0008650519031141     987.1710230273814659
-0.6176470588235294 -0.6179775280898876 -1.0
4       -0.6179775280898876     -0.0001262466860246     6765.1708499452906835
-0.6179775280898876 -0.6180257510729613 -1.0
5       -0.6180257510729613     -0.0000184199377408     46368.1708240762673086
-0.6180257510729613 -0.6180327868852459 -1.0
6       -0.6180327868852459     -0.0000026874496104     317811.1708002091618255
-0.6180327868852459 -0.6180338134001252 -1.0
7       -0.6180338134001252     -0.0000003920939740     2178309.1696905475109816
-0.6180338134001252 -0.6180339631667064 -1.0
8       -0.6180339631667064     -0.0000000572057477
14930352.0903342626988888
-0.6180339631667064 -0.618033985017358 -1.0
9       -0.6180339850173580     -0.0000000083462062
102334154.1575837731361389
-0.618033985017358 -0.6180339882053251 -1.0
10      -0.6180339882053251     -0.0000000012176951
701408685.5948505401611328
-0.6180339882053251 -0.6180339886704432 -1.0
11      -0.6180339886704432     -0.0000000001776592
```

```
4807526681.6393451690673828
-0.6180339886704432 -0.6180339887383031 -1.0
12      -0.6180339887383031     -0.0000000000259200
32951287921.3517494201660156
-0.6180339887383031 -0.6180339887482036 -1.0
13      -0.6180339887482036     -0.0000000000037818
225839830108.2011413574218750
-0.6180339887482036 -0.6180339887496481 -1.0
14      -0.6180339887496481     -0.0000000000005518
1547658107003.3000488281250000
-0.6180339887496481 -0.6180339887498588 -1.0
15      -0.6180339887498588     -0.0000000000000806
10585981451902.5722656250000000
-0.6180339887498588 -0.6180339887498896 -1.0
16      -0.6180339887498896     -0.0000000000000118
71675916080590.3281250000000000
-0.6180339887498896 -0.618033988749894 -1.0
17      -0.6180339887498940     -0.0000000000000019
430055496483542.0000000000000000
-0.618033988749894 -0.6180339887498947 -1.0
18      -0.6180339887498947     -0.0000000000000004
1720221985934168.0000000000000000
-0.6180339887498947 -0.6180339887498949 -1.0
19      -0.6180339887498947     -0.0000000000000004     0.0000000000000000
[2.5997785796364874, 1.628198163505728, 1.387010970499449, 1.2791505784075181,
1.2182455234498017, 1.179149118762673, 1.1519310543515613, 1.1318925159430326,
1.1165238907668884, 1.1043630942445761, 1.0945007111802252, 1.0863414016671362,
1.07947692396302, 1.073620677563329, 1.0685051980361264, 1.0637742242350163,
1.0561624170069617, 1.0411425143147786, 0]
```

Regula Falsi konvergiert hier langsamer als das Newton Verfahren. Insbesondere in der Nähe der Nullstelle ist die Konvergenzordnung nur noch linear. Aber Vorsicht mit der Abbruchbedingung!

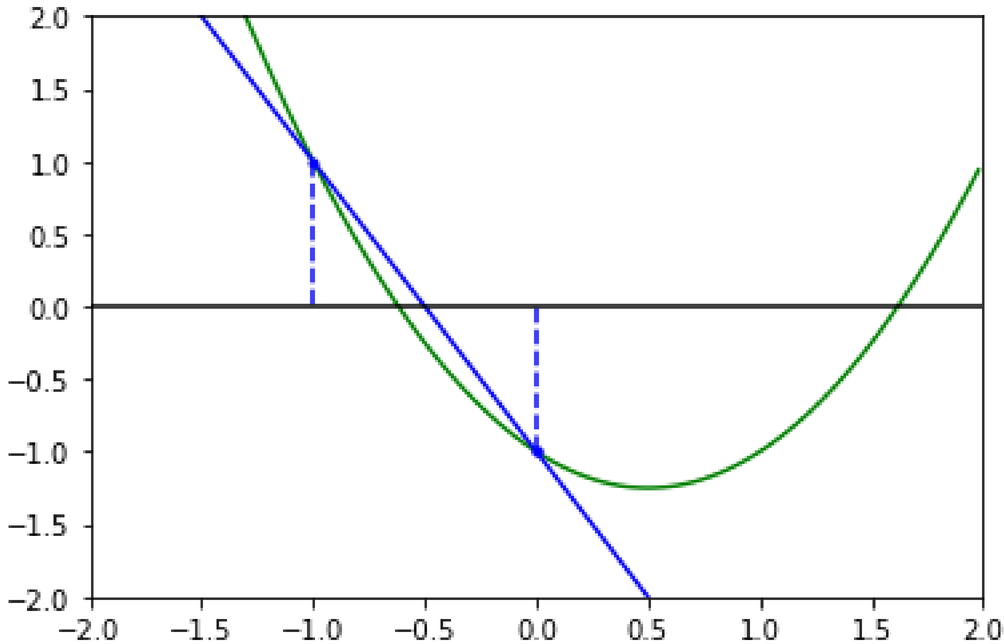
[4]:
```
import math
import numpy as np              # array Funktionen
import matplotlib.pyplot as plt # plot Funktionen

x = np.arange(-2, 2, 0.02) # x-Werte
y = x**2-x-1               # Zugehörige y-Werte der Wertetabelle
dy= 2*x-1                  # Zugehörige Ableitungen
plt.plot(x, y, 'g-')       # Plot der Wertetabelle

x0  = 0
y0  = x0**2-x0-1
x1  = -1
y1  = x1**2-x1-1
dy0 = (y1-y0)/(x1-x0)
# x0 + ()*x
```

```
yt  = y0+dy0*(x-x0)
plt.plot(x, yt, 'b-')       # Plot der Wertetabelle
plt.plot([x0], [y0], 'b.')
plt.plot([x0, x0], [y0, 0], 'b--')

plt.plot([x0],     [y0], 'b.')
plt.plot([x1],     [y1], 'b.')
plt.plot([x1, x1], [y1, 0], 'b--')
#plt.plot( [0, 0], [-2, 2], 'k-')
plt.plot([-2, 2], [0,  0], 'k-')
plt.axis([-2, 2, -2, 2])
plt.show()
```

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[ ]:
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