

# Linear Regression with multiple variables

- Mutilple Features
  - Twp Practice
- Method
  - Normal equation

## Mutliple Features

### Notation:

$n$  = number of features

$x^{(i)}$  = input(features) of  $i^{th}$  training example.

$x_j^{(i)}$  = value of feature  $j$  in  $i^{th}$  training example.

### Hypothesis:

$$h_{\theta}(x) = \sum_{i=0}^n \theta_i x_i = \theta^T x$$

$$\text{CostFunction } J = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta) = \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m [(h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}]$$

## Twp Practice

1. Feature Scaling
2. Learning Rate

Further, Feature Scaling can be devided into Feature Scaling & Mean Normalization.

### Feature Scaling:

Idea: Make sure features are on a similar scale.

Get every feature into approximately  $-1 \leq x_i \leq 1$  range.

### Mean Normalization:

Replace  $x_i$  with  $x_i - \mu_i$  to make features have approximately zero mean.(Do not apply to  $x_0 = 1$ ).

Learning Rate:

If  $\alpha$  is too small: slow convergence.

If  $\alpha$  is too large:  $J(\theta)$  may not decrease on every iteration; may not converge.

# Method

Here are two methods to fitting the target:

1. Gradient descent
2. Normal equation

## Normal equation

$$\frac{\partial}{\partial \theta_j} J(\theta) = 0 \text{ for every } j$$

Solve for  $\theta_i$

$$\text{which means } 0 = \frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^m [(h_{\theta}(x^{(i)}) - y^{(i)})x_j^{(i)}]$$

$X^T(X\theta - Y) = 0 \Rightarrow \theta = (X^T X)^{-1} X^T Y$ , while it's often substitute  $\text{pinv}(X^T X)$  for  $(X^T X)^{-1}$ .

Mehtod	Character
Gradient Descent	Need to choose $\alpha$ Need many Operations. Work well even when $n$ is large.
Normal Equation	No need to choose $\alpha$ . Don't need to iterate. Need to compute $(X^T X)^{-1}$ , whose time complexity is $O(n^3)$ Slow if $n$ is very large.