Assignment Set for Laboratory 3

Hand in your assignments on Canvas by uploading a single pdf.

- ATSC 409: Hand-in answers to all parts of Problem 1from the Lab.

- EOSC 511/ATSC 506: Hand-in an answer to the Problem (#5) below

5. Consider a long hallway in an office building. If we assume that any cigarette smoke mixes across the width of the hallway and vertically through the depth of the hallway much faster than it mixes along the hallway, we can write the diffusion of cigarette smoke as an equation

$$\frac{\partial S}{\partial t} = \kappa \frac{\partial^2 S}{\partial x^2} - \gamma S + \alpha(x)$$

where S is the concentration of smoke, κ is the rate of diffusion of smoke, γ is the rate at which the smoke sticks to the walls or otherwise leaves the system, $\alpha(x)$ is the sources of smoke, t is the time and x is distance along the hallway.

(a) Write the appropriate equation for the steady state.

(b) Discretize the hall into N segments and write the equation for the steady state as a matrix equation.

(c) Taking $\kappa = 0.05$ m^2 s^{-1}, $\gamma = (3600$ s$)^{-1}$, and

$$\alpha(x) = \begin{cases} \alpha_o, & x = x_* \\ 0, & x \neq x_* \end{cases} \tag{1}$$

where you can choose the point x_* and $\alpha_o = 0.005$ kg m^{-1} s^{-1}, find the solution (using python) for your choice of N between 5 and 15. Take the length of the hall as 20 m.

(d) What is the condition number of the matrix? (using python)

(e) If γ is 0 what is the condition number of the matrix? (using python) Physically why is there no single solution?

(f) If γ is 0 and α is 0, why physically is there no single solution?