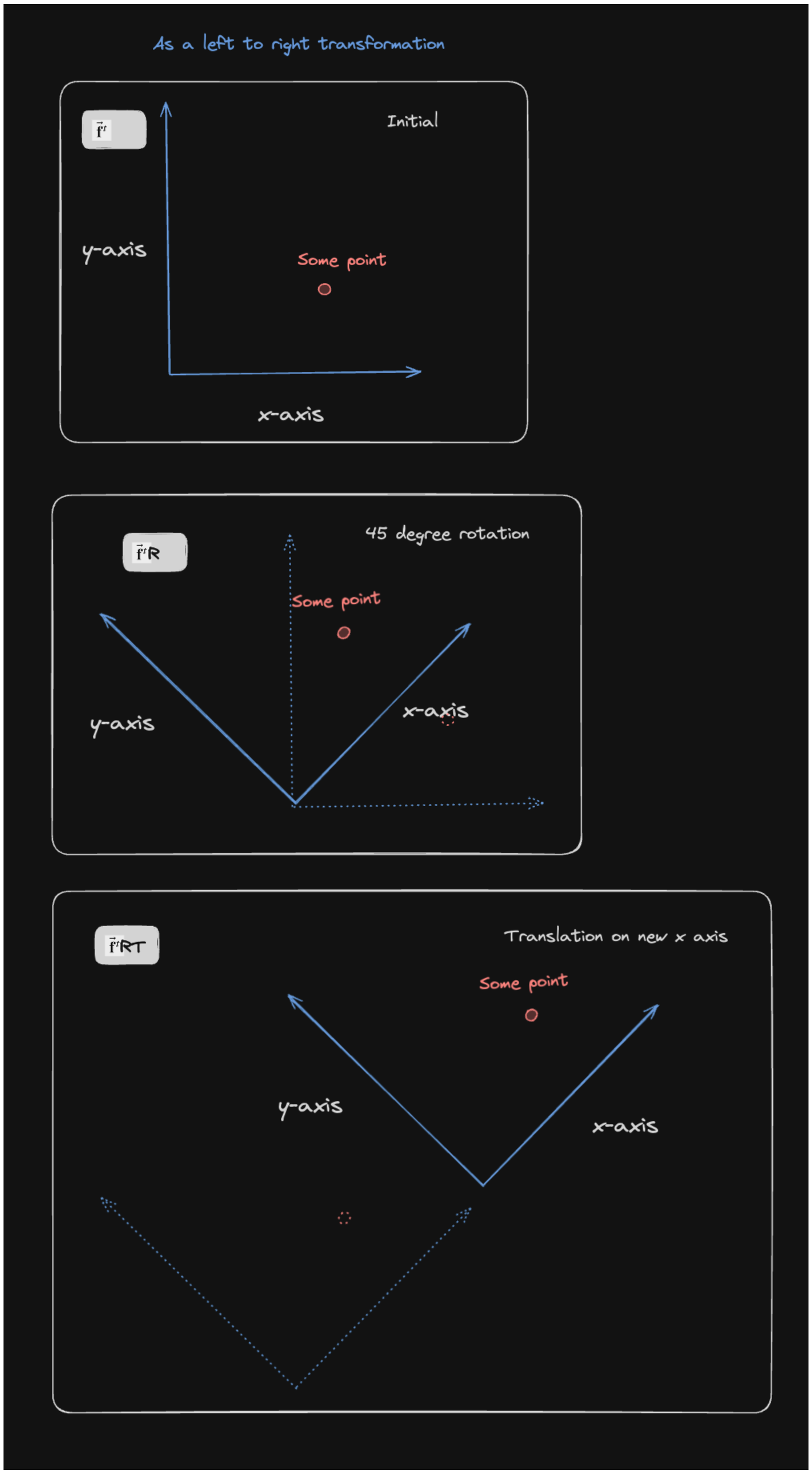
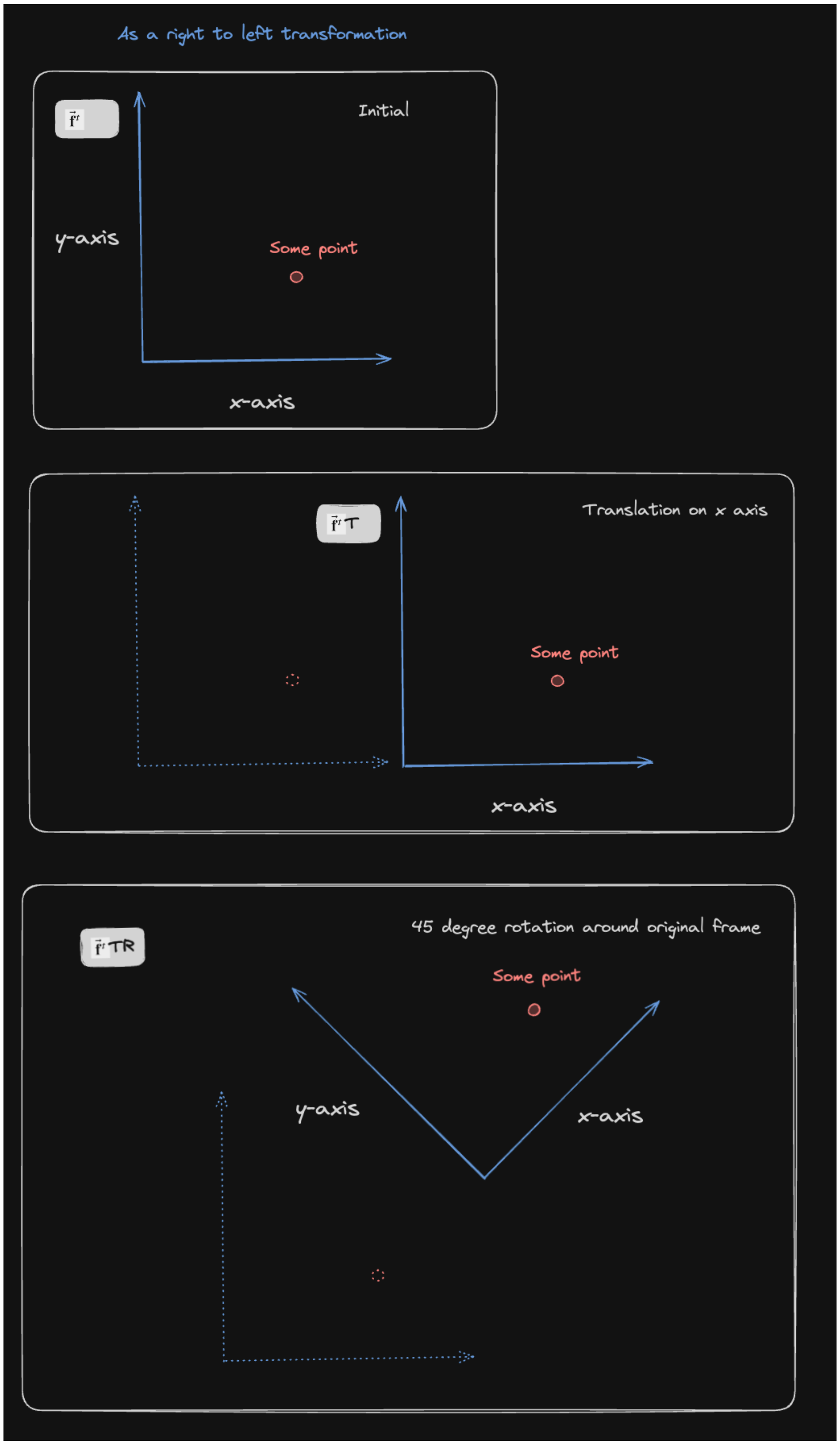
Cory Zimmerman

(4.1)
The first transformation considers left to right. So, $\vec{f}^t$is rotated, and that rotated frame is translated around its new x-axis.

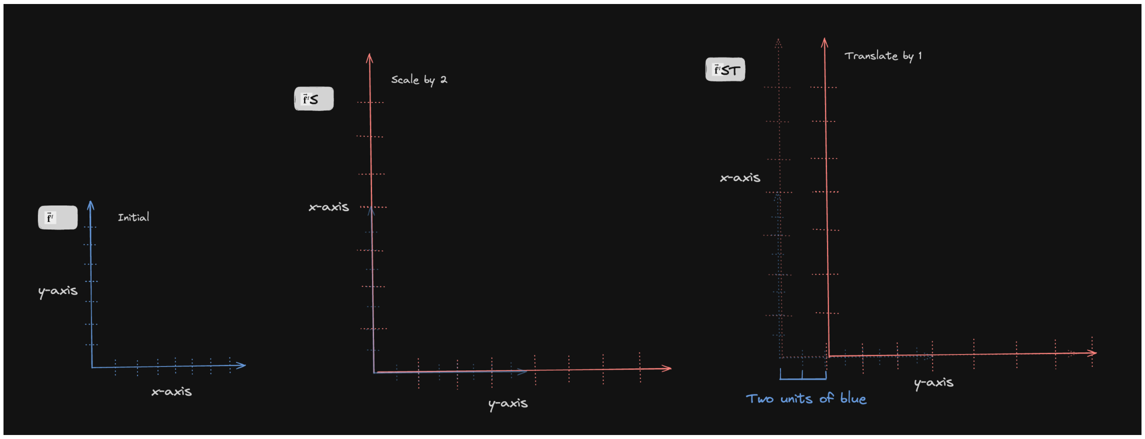


The second transformation considers right to left. Here, $\vec{f}^t$is translated along its original x-axis. But then it's rotated around the original frame's origin. This produces the same result.



(4.2)

The origin is now 2 $\vec{f}^t$units from where it started. Once S is applied uniformly to $\vec{f}^t$, one unit of $\vec{f}^t S$ is now two units of $\vec{f}^t$. Thus, translating $\vec{f}^t S$ by one unit translates by two units of $\vec{f}^t$. Because there is no rotation, this is the only movement of the origin. The picture below represents this. A right to left perspective yields the same result. An initial translation makes $\vec{f}^t T$ one unit away from the origin of $\vec{f}^t$. Scaling that by 2 makes $\vec{f}^t ST$ now 2 units away from the origin of $\vec{f}^t$.



(4.3)
$\vec{b}^t = \vec{a}^t TR$:
Because $\vec{a}^t$ is a frame, it's possible to apply an affine translation. From $\vec{a}^t$, the transformation moves by d_1 in a positive x direction and by d_2 in a positive y direction. Encoding this into a 2d affine matrix T yields the following:

$$T = \begin{bmatrix} 1 & 0 & d_1 \\ 0 & 1 & d_2 \\ 0 & 0 & 1 \end{bmatrix}$$

Assert from the appearance of the drawing that d_3 is parallel to the x-axis of $\vec{b}^t$. Thus, θ is the degree of rotation in $\vec{b}^t$. This yields an affine rotation matrix R defined like this:

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\vec{b}^t = \vec{a}^t RT$:
Again, $\vec{b}^t$ is rotated by θ degrees, so the rotation matrix R is the same:

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, the line marked d_3 is along the x axis, and the line marked d_4 is along the y axis. So, T is the affine matrix with those x and y transformations:

$$T = \begin{bmatrix} 1 & 0 & d_3 \\ 0 & 1 & d_4 \\ 0 & 0 & 1 \end{bmatrix}$$

(4.4)
Let R_θ be a matrix expressing a clockwise rotation by θ degrees. From $\vec{a}^t$, the transformation to $\vec{b}^t$ involves a counterclockwise rotation by ϕ, an x translation by d, and a clockwise rotation by some κ. After rotating $\vec{b}^t$ by θ, it also becomes apparent due to the angle ϕ and distance d that $\vec{c}^t$ is at the N^{-1} transformation of this rotated $\vec{b}^t$. This yields the relation $\vec{c}^t = \vec{b}^t R_\theta N^{-1}$. From this relation, derive the following:

$$
\begin{aligned}
\vec{a}^t M &= \vec{c}^t && \text{Initial} \\
\vec{a}^t M &= \vec{b}^t R_\theta N^{-1} && \text{Defined above} \\
\vec{a}^t M &= \vec{a}^t N R_\theta N^{-1} && \text{Definition of } \vec{b}^t \\
M &= N R_\theta N^{-1} && \text{Multiply by inverse of } \vec{a}^t
\end{aligned}
$$

Thus, $M = N R_\theta N^{-1}$.