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Longitudinal and lateral control for automated lane change maneuvers

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***Abstract*— This paper considers the trajectory planning problem of a vehicle system for automated lane change maneuvers. By considering a lane change maneuver as primarily a longitudinal planning problem, the proposed trajectory planning algorithm determines whether there exists a longitudinal trajectory which allows the ego vehicle to safely position itself in a gap between surrounding vehicles in the target lane. If such a longitudinal trajectory exists, the algorithm plans the corresponding lateral trajectory. The lane change trajectory planning problem is thereby reduced to solving low-complexity model predictive control problems resulting in loosely coupled longitudinal and lateral motion trajectories. Simulation results demonstrate the ability of the proposed algorithm to generate smooth collision-free trajectories for lane change maneuvers.**

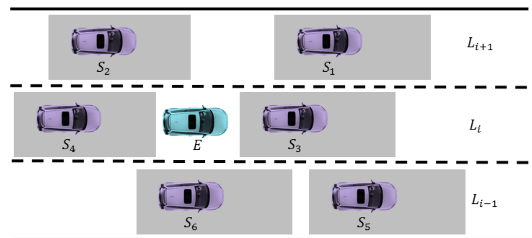

Fig. 1: Vehicles traveling on a road with three lanes, L_i, L_{i+1} and L_{i-1}. The ego vehicle (E) is shown in blue and the surrounding vehicles ($S_q,\ q = 1, \ldots, 6,$) are displayed in purple. The grey boxes around $S_q,\ (q = 1, \ldots, 6)$, indicate safety critical regions which E should not enter.

I. Introduction

Due to its potential to increase traffic safety, transport efficiency, and convenience for the everyday driver, the field of intelligent vehicles has developed rapidly over the last decades. One area where a high level of autonomy is expected to be both realizable and desirable is highway driving, where a substantial percentage of traffic accidents and fatalities are related to human errors when performing maneuvers such as lane change [1]-[3].

This paper concerns the problem of trajectory planning for automated lane change maneuvers. In the proposed approach, a lane change maneuver is considered as primarily a longitudinal motion. The trajectory planning algorithm thereby determines whether there exists a longitudinal trajectory which allows the ego vehicle to safely adapt its behavior in order to position itself in a lane change gap between two vehicles in the target lane. If such a longitudinal trajectory exists, the algorithm plans the corresponding lateral trajectory which allows the ego vehicle to move into the target lane. The longitudinal and lateral trajectories can thereby be attained by solving two loosely coupled low-complexity Model Predictive Control (MPC) problems. The proposed trajectory planning algorithm thus fulfills the requirements of commercial autonomous vehicle technology i.e. ability to generate verifiable, smooth, collision-free trajectories with low computational resources.

In the MPC planning framework, a trajectory is found as the solution of a constrained optimal control problem over a finite time horizon. In particular, a cost function is minimized subject to a set of constraints including the vehicle dynamics, system limitations, and constraints introduced to avoid collisions with surrounding vehicles. The constrained optimal control problem is solved in receding horizon, i.e. at every time step the problem is formulated over a shifted time horizon based on new available sensor measurement information. The main advantage of resorting to such a formulation is that collision avoidance is guaranteed, provided that the optimization problem is feasible. However, vehicle dynamics and collision avoidance constraints generally result in nonlinear and/or mixed-integer inequalities, which may lead to prohibitive computational complexity that prevents the real-time execution of a MPC trajectory planning algorithm. To reduce the computational burden a particular optimal control trajectory planning algorithm is therefore generally tailored to a certain application or considers only a short prediction horizon [4]-[8]. In order to overcome this limitation, the presented approach formulates the trajectory planning problem as loosely coupled longitudinal and lateral MPC problems in form of low-complexity Quadratic Programs (QPs) which can be efficiently solved [9].

The remainder of the paper is organized as follows: Section II describes the considered trajectory planning problem, while Section III presents the trajectory planning algorithm for which simulation results are presented in Section IV. Finally, conclusions are stated in Section V.

II. Problem description

Consider the highway traffic scenario depicted in Fig. 1. The ego vehicle, E, drives in a lane L_i, with a preceding vehicle S_3 and a trailing vehicle S_4 traveling in the same lane, and two surrounding vehicles S_1 and S_2 driving in the left adjacent lane L_{i+1}, and two surrounding vehicles S_5 and S_6 driving in the right adjacent lane L_{i-1}.

For E to change lane e.g. from L_i to L_{i+1}, E has the

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options of positioning itself ahead of S_1, in the gap between S_1 and S_2, or behind S_2. However, by respectively considering the options of E positioning itself ahead of S_1 or behind S_2 as positioning E in the gap between S_1 or S_2 and some other surrounding vehicle, these three options are actually only different versions of the same problem i.e. positioning E in a gap between two surrounding vehicles. Further, when considering traffic scenarios involving highway entry or exit, merging, or lane drops, it becomes apparent that these scenarios can be considered as special types of lane change maneuvers.

For the lane change maneuver to be feasible it should not significantly disturb the surrounding vehicles and be planned such that E remains at its desired velocity, $v_{x_{des}}$, while

- avoiding collision conflicts with all surrounding vehicles,
- retaining E within the road boundaries, and
- respecting E's system limitations.

The trajectory planning problem for automated lane change maneuvers can thereby be formulated as following: *given a highway traffic environment, determine a feasible maneuver (if such exists) in terms of a longitudinal and a lateral trajectory i.e. control signals, which allow E to position itself in a gap between two surrounding vehicles in the target lane.*

III. Trajectory planning algorithm

The general idea of the trajectory planning algorithm is to divide the trajectory planning problem into one longitudinal and one lateral trajectory planning module. The trajectory planning algorithm determines the longitudinal and lateral safety corridor which E must be positioned in to avoid collisions with surrounding vehicles, and solves the corresponding trajectory planning problems for longitudinal and lateral control.

The trajectory planning algorithm is formulated based on the following set of assumptions:

a1 E is equipped with sensor systems which measure its own position within the road and the respective relative positions and velocities of surrounding vehicles i.e. all required sensor measurements are available.

a2 The behavior of surrounding vehicles is predictable over a limited time horizon.

a3 E is equipped with low-level control systems which are capable of following the planned longitudinal and lateral trajectory.

A simplified schematic architecture of an automated lane change system is illustrated in Fig. 2. Note that this paper is only concerned with the trajectory planning algorithm, in accordance with Assumptions a1, a2, and a3.

The structure of the proposed trajectory planning algorithm is schematically illustrated in Fig. 3. As can be seen in the figure, the trajectory planning algorithm consists of four main steps:

A) *Determine the longitudinal safety corridor*
A lane change trajectory is considered feasible for a given target gap, if E maintains a safe distance to its surrounding vehicles during the maneuver. This corresponds to upper and lower bounds on E's longitudinal position which determine a longitudinal safety corridor in which E must be positioned during the maneuver.

B) *Determine the longitudinal trajectory*
Once the longitudinal safety corridor has been determined, the longitudinal trajectory in terms of control signals can be formulated and solved as a QP optimization problem.

C) *Determine the lateral safety corridor*
The longitudinal trajectory is used to determine the upper and lower bounds on E's lateral position which corresponds to a lateral safety corridor which E must be positioned in during the maneuver.

D) *Determine the lateral trajectory*
Respecting the bounds given by the lateral safety corridor, the optimal lateral control signals are computed by QP optimization.

The following sections III-A-III-D, provides further details regarding how the longitudinal safety corridor, longitudinal trajectory, lateral safety corridor, and lateral trajectory are determined. Note that this paper only presents the four main

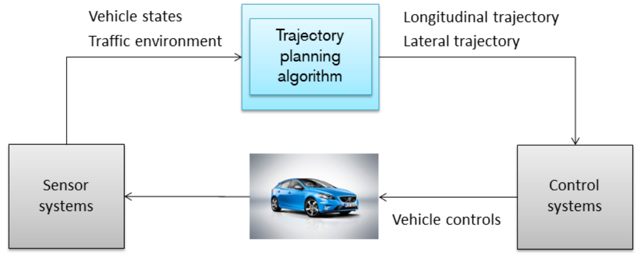

Fig. 2: Schematic architecture of the proposed automated lane change system.

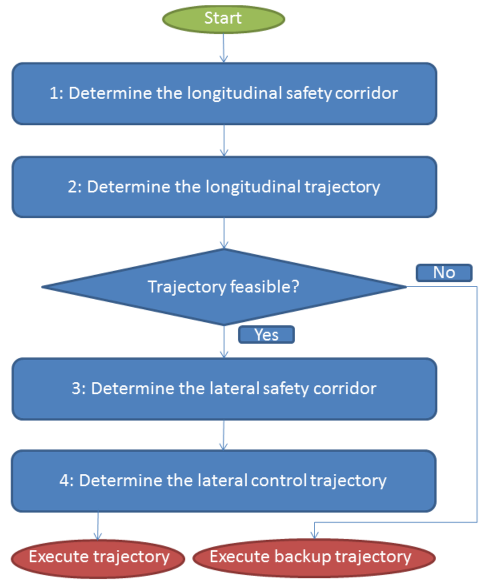

Fig. 3: Schematic structure of the trajectory planning algorithm.

steps of the trajectory planning algorithm and does not consider the generation of backup trajectories as safe default.

A. Longitudinal safety corridor

For E to avoid collision conflicts with surrounding vehicles it must be able to maintain each surrounding vehicle at a safe distance,

$$s_d(v_{S_{qk}}) = \min(\epsilon,\ \tau v_{S_{qk}})\ [\mathrm{m}],\ \forall k = 0, \dots, N, \quad (1)$$

where N is the prediction horizon, v_{S_q} is the longitudinal velocity of S_q, and ϵ and τ respectively denotes the minimum distance and time gap which E must maintain to its surrounding vehicles i.e. S_q.

For the scenario in Fig. 1, assume E has a desire to change lane in the gap between S_1 and S_2. Before E has initiated the lane change maneuver it must be able to follow at a safe distance a potential preceding vehicle S_3 traveling in the same lane. Therefore, prior to the time instance when the lane change maneuver is initiated, $N_{LC_{in}}$ the upper bound on E's longitudinal position is determined by S_3 as

$$x_{\max_k} = x_{S_{3k}} - s_d(v_{S_{3k}}),\ \forall k = 0, \dots, N_{LC_{in}} - 1, \quad (2)$$

where x_{S_3} denotes the longitudinal position of S_3's rear axle.

Once the lane change maneuver is initiated, the upper boundary on E's longitudinal position is determined by

$$\begin{aligned} x_{\max_k} = \min\left(x_{S_{1k}} - s_d(v_{S_{1k}}), x_{S_{3k}} - s_d(v_{S_{3k}})\right), \\ \forall k = N_{LC_{in}}, \dots, N_{LC_e} - 1, \end{aligned} \quad (3)$$

where x_{S_1} denotes the longitudinal position of S_1's rear axle and N_{LC_e} denotes the time when E has left its original lane L_i and moved into its target lane L_{i+1} i.e.

$$N_{LC_e} = N_{LC_{in}} + n_{\min}, \quad (4)$$

and $n_{\min}$ is the discrete time version of the minimum time it takes to laterally move into the target lane, $t_{\min}$ [s].

When E has moved into its target lane L_{i+1}, it must be able to follow S_1 at a safe distance. Hence, when E has left its original lane, the upper boundary on E's longitudinal position is determined by

$$x_{\max_k} = x_{S_{1k}} - s_d(v_{S_{1k}}),\ \forall k = N_{LC_e}, \dots, N. \quad (5)$$

Similarly, the lower boundary on the longitudinal position of E is determined by

$$x_{\min_k} = x_{S_{4k}} + s_d(v_{S_{4k}}),\ \forall k = 0, \dots, N_{LC_{in}} - 1, \quad (6a)$$

$$\begin{aligned} x_{\min_k} = \max\left(x_{S_{2k}} + s_d(v_{S_{2k}}), x_{S_{4k}} + s_d(v_{S_{4k}})\right), \\ \forall k = N_{LC_{in}}, \dots, N_{LC_e} - 1, \end{aligned} \quad (6b)$$

$$x_{\min_k} = x_{S_{2k}} + s_d(v_{S_{2k}}),\ \forall k = N_{LC_e}, \dots, N, \quad (6c)$$

where x_{S_2} and x_{S_4} respectively denotes the longitudinal position of the front axle of S_2 and S_4.

If several lane change maneuvers are required in order to reach E's desired target lane, the upper and lower bounds on E's longitudinal position can be determined by repeating the above described min max operations (2), (3), (5), (6), over each lane with the corresponding surrounding vehicles. The same methodology applies when determining the longitudinal safety corridor of an overtake maneuver which by its nature constitutes two subsequent lane change maneuvers.

Remark 1: If a mandatory lane change due to e.g. a lane drop, highway entry or exit should be planned, the longitudinal position where a lane terminates can be modeled as a stationary preceding vehicle. As such, the longitudinal safety corridor will ensure that E will remain on the road.

Remark 2: Since the time when to initiate the lane change maneuver affects how the upper and lower boundaries on E's longitudinal position is defined in (2)-(6), $N_{LC_{in}}$ can be set repeatedly over a time interval. The corresponding upper and lower boundaries on the longitudinal position of E is then determined and the longitudinal control signals are computed as described in Section III-B. The most appropriate time for when to initiate the lane change maneuver is then set as the time which corresponds to the longitudinal optimization problem which has the best i.e. lowest cost function.

B. Longitudinal control problem

The longitudinal dynamics of E is modeled by a simple double integrator. The motion of E can thus be linearly expressed as

$$x_{k+1} = x_k + v_{x_k} t_s + a_{x_k}\frac{t_s^2}{2}, \qquad \forall k = 0, \dots, N, \quad (7a)$$

$$v_{x_{k+1}} = v_{x_k} + a_{x_k} t_s, \qquad \forall k = 0, \dots, N, \quad (7b)$$

where x, v_x, and a_x respectively denotes E's longitudinal position, velocity, and acceleration, and t_s denotes the discrete sampling time. Further, the system described by (7) is subjected to the following set of constraints

$$x_{\min_k} \le x_k \le x_{\max_k},\ \forall k = 0, \dots, N, \quad (8a)$$

$$v_{x_{\min}} \le v_{x_k} \le v_{x_{\max}},\ \forall k = 0, \dots, N, \quad (8b)$$

$$a_{x_{\min}} \le a_{x_k} \le a_{x_{\max}},\ \forall k = 0, \dots, N, \quad (8c)$$

$$\Delta a_{x_{\min}} \le \Delta a_{x_k} \le \Delta a_{x_{\max}},\ \forall k = 1, \dots, N, \quad (8d)$$

where $\Delta a_{x_k} = a_{x_k} - a_{x_{k-1}}$. Constraint (8a) ensures that E remains within the longitudinal safety corridor while inequality (8b) limits the longitudinal velocity. Conditions (8c)-(8d) respectively limit the longitudinal acceleration and jerk in order to allow for smooth and comfortable maneuvers. Further, (8c)-(8d) ensure that the planned trajectory is within the capability of the assumed low-level control systems i.e. Assumption a3.

Since the sets $\mathcal{X}$,$\mathcal{U}$ of feasible states and control inputs i.e. $x,\ v_x \in \mathcal{X}$, and $a_x \in \mathcal{U}$ are convex, the longitudinal control problem can be written as the following standard QP,

$$\min_w J = \frac{1}{2} w^T H w \quad (9a)$$

subject to

$$H_{eq} w = K_{eq}, \quad (9b)$$

$$H_{in} w \le K_{in}, \quad (9c)$$

where $w \triangleq [x_k, v_{x_k}, a_{x_k}],\ k = 0, \dots, N$, condition (9b) corresponds to the system dynamics (7) and the constraint (9c) corresponds to the set of design constraints (8). The cost function J_x for the longitudinal control problem is defined as

$$J_x = \sum_{k=0}^{N} \vartheta(v_{x_k} - v_{x_{des}})^2 + \kappa a_{x_k}^2 \qquad (10)$$

where ϑ and κ are positive scalar weights.

Remark 3: If no feasible solution to the control problem (9) is obtained when planning a mandatory lane change maneuver, the set of control inputs, $\mathcal{U}$, can be enlarged. For instance, by only accounting for actuator limitations rather than consider trajectory smoothness in (8c)-(8d), the bounds on the control inputs can be increased.

Remark 4: The optimization problem has N optimization variables i.e. control input a_x, and $11N$ linear constraints corresponding to vehicle dynamics (7), system limitations, and design constraints (8).

Remark 5: For the trajectory planning algorithm to be able to produce feasible trajectories, the planning i.e. prediction horizon must be sufficiently long. However, according to Remark 4 increasing the prediction horizon, increases the algorithm's computational complexity, which can be a limitation for real-time vehicle implementation. Hence, if necessary, the prediction horizon can be prolonged, not by increasing N but rather by allowing t_s to vary over the prediction horizon. Since the timing of the lane change maneuver is fixed when determining the longitudinal safety corridor, the time interval prior to, and after, the lane change maneuver can be discretized with a larger sampling time than the time interval when performing the maneuver. Thus the prediction horizon can be extended without increasing the computational complexity nor compromising the solution fidelity.

C. Lateral safety corridor

For a left lane change maneuver the upper bound of the lateral safety corridor is given by

$$y_{\max_k} = L_{l_i k},\ \forall k = 0, \ldots, N_{LC_{in}} - 1, \qquad (11a)$$
$$y_{\max_k} = L_{l_{i+1} k},\ \forall k = N_{LC_{in}}, \ldots, N, \qquad (11b)$$

where L_{l_i} and L_{r_i} respectively denotes the left and right lane boundary of the i:th lane. The lower bound on the lateral safety corridor for a left lane change is given by

$$y_{\min_k} = L_{r_i k},\ \forall k = 0, \ldots, N_{LC_e} - 1, \qquad (12a)$$
$$y_{\min_k} = L_{r_{i+1} k},\ \forall k = N_{LC_e}, \ldots, N. \qquad (12b)$$

Similarly, for a right lane change, the corresponding upper and lower bounds of the lateral safety corridor is determined by

$$y_{\max_k} = L_{l_i k},\ \forall k = 0, \ldots, N_{LC_e} - 1, \qquad (13a)$$
$$y_{\max_k} = L_{l_{i-1} k},\ \forall k = N_{LC_e}, \ldots, N, \qquad (13b)$$

and

$$y_{\min_k} = L_{r_i k},\ \forall k = 0, \ldots, N_{LC_{in}} - 1, \qquad (14a)$$
$$y_{\min_k} = L_{r_{i-1} k},\ \forall k = N_{LC_{in}}, \ldots, N. \qquad (14b)$$

Remark 6: When computing the longitudinal safety corridor as described in Section III-A, it is assumed that the lane change maneuver is performed during $t_{\min}$ [s] (4). However, although the lower and upper boundaries on the longitudinal position of E are set with respect to the assumed time to perform the lane change i.e. $t_{\min}$ [s], the solution to the longitudinal control problem in Section III-B, might entail E being positioned in the lane change region during a longer time interval. Therefore, when computing the lateral safety corridor, the constraints on the lateral position of E can be set with respect to the actual time E is positioned in the lane change region rather than $t_{\min}$.

D. Lateral control problem

The lateral trajectory is determined in similar manner as described in Section III-B, i.e. by modeling the lateral dynamics of E by a double integrator

$$y_{k+1} = y_k + v_{y_k} t_s + a_{y_k} \frac{t_s^2}{2}, \quad \forall k = 0, \ldots, N, \qquad (15a)$$
$$v_{y_{k+1}} = v_{y_k} + a_{y_k} t_s, \quad \forall k = 0, \ldots, N, \qquad (15b)$$

where y, v_y, and a_y respectively denotes E's lateral position, velocity, and acceleration. The system (15) is subjected to a set of constraints

$$y_{\min_k} \le y_k \le y_{\max_k},\ \forall k = 0, \ldots, N, \qquad (16a)$$
$$v_{y_{\min}} \le v_{y_k} \le v_{y_{\max}},\ \forall k = 0, \ldots, N, \qquad (16b)$$
$$a_{y_{\min}} \le a_{y_k} \le a_{y_{\max}},\ \forall k = 0, \ldots, N, \qquad (16c)$$
$$\Delta a_{y_{\min}} \le \Delta a_{y_k} \le \Delta a_{y_{\max}},\ \forall k = 1, \ldots, N, \qquad (16d)$$

where $\Delta a_{y_k} = a_{y_k} - a_{y_{k-1}}$. Finally, the corresponding QP is formulated with the cost function, J_y, defined as

$$J_y = \sum_{k=0}^{N} \phi v_{y_k}^2 + \psi a_{y_k}^2 \qquad (17)$$

where ϕ and ψ are positive scalar weights.

Remark 7: The optimization problem has N optimization variables i.e. control input a_y, and $11N$ linear constraints corresponding to vehicle dynamics (15), system limitations, and design constraints (16).

IV. Simulation results

In this section the proposed trajectory planning algorithm is evaluated in simulated lane change traffic situations. The algorithm has been implemented in Matlab using `quadprog` and the general design parameters for the longitudinal and lateral control optimization problems are given in Table I and Table II respectively.

The lane change traffic situation considers a one-way, two-lane highway where E initially drives in the right lane with a preceding vehicle S_3 driving in the same lane, and the surrounding vehicles S_1 and S_2 driving in the adjacent left lane, as illustrated in Fig. 4. The aim of the trajectory planning algorithm for the traffic situation is to plan a lane change maneuver which allows E to change lane in the gap between S_1 and S_2.

TABLE I: General design parameters for the longitudinal control problem.

$v_x \in \{0, 30\}$ [m/s]	$\tau = 0.5$ [s]	$\epsilon = 1$ [m]
$a_x \in \{-4, 2\}$ [m/s^2]	$N = 20$	$\vartheta = 1$
$\Delta a_x \in \{-3t_s, 1.5t_s\}$ [m/s^2]	$t_s = 0.5$ [s]	$\kappa = 1$

TABLE II: General design parameters for the lateral control problem.

$v_y \in \{-5, 5\}$ [m/s]	$N = 20$	$\phi = 1$
$a_y \in \{-2, 2\}$ [m/s^2]	$t_s = 0.5$ [s]	$\psi = 10$
$\Delta a_y \in \{-0.5t_s, 0.5t_s\}$ [m/s^2]		

Two scenarios of the lane change traffic situation are considered, referred to as *Scenario 1* and *Scenario 2*. The initial conditions for the two considered scenarios are given in Table III. In all scenarios it is assumed that S_1, S_2, and S_3, drive at constant longitudinal velocity without performing any lane change maneuvers over the prediction horizon according to Assumption a2. This is a simple assumption purely in order to easily illustrate the trajectory planning algorithm. However, any predicted behavior i.e. trajectory, of the surrounding vehicles can be incorporated into the algorithm when determining the longitudinal safety corridor (2), (3), (5), (6). Further, uncertainties resulting from e.g. sensor measurements and behavior prediction can be taken into account by for instance increasing the safety distance which E must maintain to its surrounding vehicles (1).

In Scenario 1, all vehicles i.e. E, S_1, S_2, and S_3, initially drive at the same longitudinal velocity. From Fig. 5 it can be seen that for E to change lane in the gap between S_1 and S_2 its velocity must be reduced in order to align itself with the gap. Once E is positioned in the gap its velocity will increase in order to maintain safe distance to S_1 and S_2. Hence, Scenario 1 illustrates the ability of the trajectory planning algorithm to plan lane change maneuvers which require E to decelerate into the lane change gap.

Scenario 2 considers a lane change traffic situation with a lane drop, where E must move into its target lane before a certain longitudinal position. In the scenario, E initially drives at a longitudinal velocity which is lower than the velocity of S_1 and S_2, while S_3 is used to model the lane drop as described in Remark 1.

TABLE III: Initial conditions [x_0 [m], v_{x_0} [m/s], a_{x_0} [m/s^2], y_0 [m], v_{y_0} [m/s], a_{y_0} [m/s^2]] for the two scenarios of the lane change traffic situation.

	Scenario 1	Scenario 2
E	[0, 15, 0, −1.75, 0, 0]	[0, 15, 0, −1.75, 0, 0]
S_1	[−20, 15, 0, 1.75, 0, 0]	[−15, 21, 0, 1.75, 0, 0]
S_2	[−45, 15, 0, 1.75, 0, 0]	[−35, 21, 0, 1.75, 0, 0]
S_3	[35, 15, 0, −1.75, 0, 0]	[80, 0, 0, −1.75, 0, 0]

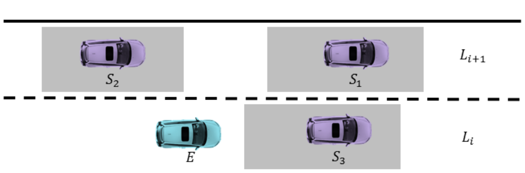

Fig. 4: Highway traffic situation with two lanes, L_i and L_{i+1}. The ego vehicle (E) is shown in blue and the surrounding vehicles (S_q, $q = 1, \ldots, 3$,) are displayed in purple. The grey boxes indicate safety critical regions which E should not enter.

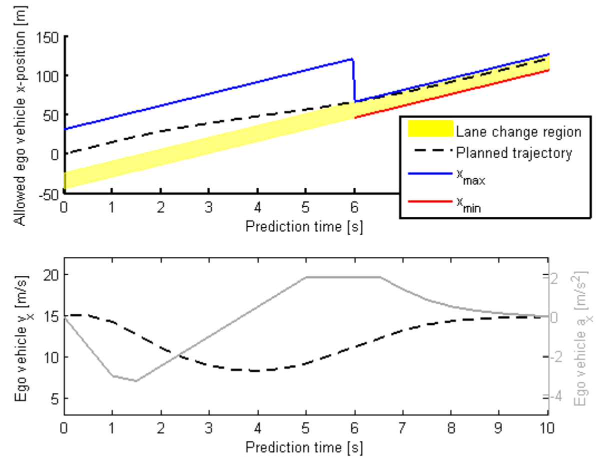

Fig. 5: *Top:* Longitudinal position trajectory for the ego vehicle's (E's) lane change maneuver, respecting the longitudinal safety corridor defined by $x_{\max}$ and $x_{\min}$ for Scenario 1 of the lane change traffic situation. *Bottom:* Longitudinal velocity and acceleration trajectory for E's lane change maneuver, for Scenario 1 of the lane change traffic situation.

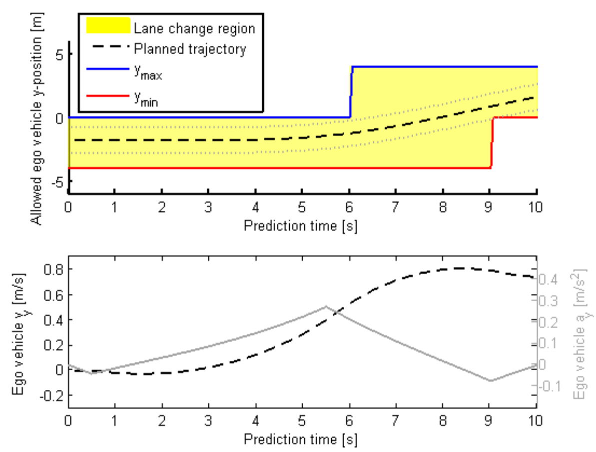

Fig. 6: *Top:* Lateral position trajectory for the ego vehicle's (E's) lane change maneuver, respecting the lateral safety corridor defined by $y_{\max}$ and $y_{\min}$ for Scenario 1 of the lane change traffic situation. The two dotted lines represent E's left and right wheels.
Bottom: Lateral velocity and acceleration trajectory for E's lane change maneuver, for Scenario 1 of the lane change traffic situation.

From Fig. 7 it can be seen that for E to change lane in the gap between S_1 and S_2 it must increase its velocity. Hence, Scenario 2 illustrates the ability of the trajectory planning algorithm to plan lane change maneuvers which require E to accelerate into the traffic gap, while accounting for a lane drop.

The lateral safety corridor and the corresponding lateral trajectories of the lane change maneuver for Scenario 1 and 2 are shown in Figs. 6 and 8 respectively. From the figures it can be seen that the time to laterally move into the target lane is longer in Scenario 1 than in Scenario 2. This is because although $t_{\min}(= 2)$ [s] is the same for both Scenario 1 and 2, the actual time E is in the lane change region (illustrated in Figs. 5, 7) can be taken into account when determining the lateral safety corridor, as mentioned in Remark 6. It is thereby possible to perform a smoother lane change maneuver in Scenario 1 than in Scenario 2.

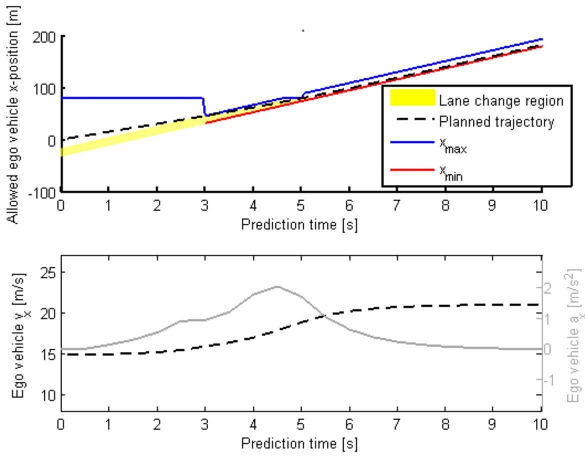

Fig. 7: *Top:* Longitudinal position trajectory for the ego vehicle's (E's) lane change maneuver, respecting the longitudinal safety corridor defined by $x_{\max}$ and $x_{\min}$ for Scenario 2 of the lane change traffic situation.
Bottom: Longitudinal velocity and acceleration trajectory for E's lane change maneuver, for Scenario 2 of the lane change traffic situation.

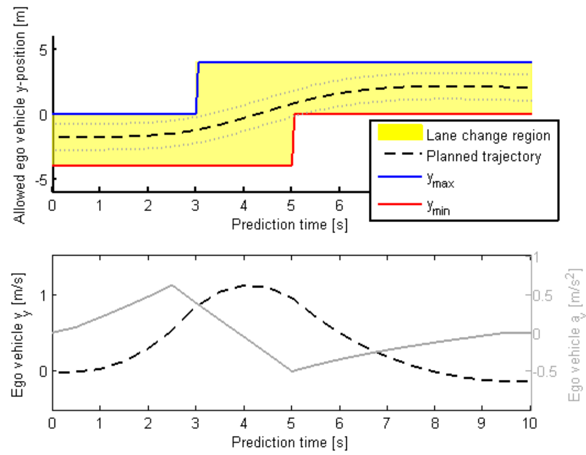

Fig. 8: *Top:* Lateral position trajectory for the ego vehicle's (E's) lane change maneuver, respecting the lateral safety corridor defined by $y_{\max}$ and $y_{\min}$ for Scenario 2 of the lane change traffic situation. The two dotted lines represent E's left and right wheels.
Bottom: Lateral velocity and acceleration trajectory for E's lane change maneuver, for Scenario 2 of the lane change traffic situation.

V. Conclusions

This paper presents a trajectory planning algorithm for automated lane change maneuvers. By considering a lane change maneuver as primarily a longitudinal motion, the proposed algorithm determines whether there exists a longitudinal trajectory which allows the ego vehicle to safely position itself in a gap between surrounding vehicles in the target lane, and if so, the algorithm plans the corresponding lateral trajectory. The lane change trajectory planning algorithm can thereby generate the longitudinal and lateral motion trajectory by solving two loosely coupled low-complexity quadratic programs.

Simulation results show the ability of the proposed trajectory planning algorithm to generate smooth collision-free trajectories which are appropriate for lane change maneuvers in various traffic situations. These results motivate future work in incorporating a dynamic prediction model of the traffic environment, which also includes sensor noise and uncertainty. Further, efforts should be made towards a real-time vehicle implementation for evaluating the proposed algorithm in traffic.

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