Fast Lane Changing Computations using Polynomials

Iakovos Papadimitriou and Masayoshi Tomizuka
iakpap@newton.berkeley.edu tomizuka@me.berkeley.edu
Department of Mechanical Engineering
University of California at Berkeley

Abstract. This paper presents a fast lane changing algorithm for Intelligent Vehicle Highway Systems. The algorithm is based on the use of polynomials for trajectory planning and the use of simplified s-topes for obstacle representation. Dynamic constraints are also taken into account. The resulting criterion for the selection of the desired and achievable trajectory involves the selection of a single coefficient. Illustrative examples show the main advantages of the algorithm especially in the case of lane changing with multiple obstacles.

I. INTRODUCTION

Intelligent Vehicle Highway Systems (IVHS) have lately been subject to extensive research. Lane following and platooning algorithms have been proposed and tested rather extensively, yet automated lane changing has not been fully studied. The main reason for this is the complexity of the lane change maneuver, since it incorporates lateral and longitudinal control in the presence of obstacles. Furthermore, the whole procedure should be completed quickly, smoothly and safely. The most commonly accepted approach to the problem consists of, first, detecting the surrounding vehicles (vehicle detection), next, treating them as obstacles and representing them mathematically (obstacle modeling) and finally finding a dynamic trajectory planning algorithm that will guide the vehicle safely to the neighboring lane (trajectory planning). This paper deals with the latter two parts of the problem.

In terms of obstacle modeling, the simplest, yet less accurate, solution is to represent vehicles as point masses, [5]. Enclosing an entire vehicle in a circle or an ellipse is a more accurate, but rather conservative approach, since these shapes also enclose areas that are not part of the vehicle and could be used by other vehicles, especially in an emergency maneuver. On the other hand, checking collision between all combinations of line segments, which constitute a rectangular shape, proves time consuming and inappropriate for dynamic trajectory planning, [10]. Fast obstacle modeling, even in the 3-dimensional space, can be achieved if any real object is approximated by an infinite number of spheres (circles in the 2-D case), [12]. Variations of this concept include the use of polytopes and spherically extended polytopes, (s-topes), [1,4]. These techniques are relatively fast, but they introduce a significant number of new parameters, thus increasing the number of degrees of freedom in trajectory planning problems. Moreover, since vehicles can be represented as rectangular shapes and since lane changing involves small yaw angles, it seems wiser to simplify these techniques in order to produce faster results.

As far as trajectory planning algorithms are concerned, they can be categorized in search-based, probabilistic and geometric methods. The first two methods are not advisable for use in IVHS since they can prove to be highly inefficient in time-critical situations. A simple geometric approach is to concatenate line and arc segments, [6,9]. The disadvantage of this technique lies on the fact that the resulting curvature profile is discontinuous. Introducing canonical trajectories, i.e. trajectories having piecewise constant acceleration, also creates a discontinuous curvature profile, [3]. The problem can be overcome by the use of elementary paths, which are pairs of symmetric clothoid arcs, and bi-elementary paths, which are sequences of elementary paths, [11]. Moreover, the use of polynomials in non-holonomic mobile platforms proves very efficient, provided that the environment is not highly cluttered, [10].

IVHS-specific trajectory planners have also been studied. Some researchers have focused on the lane-changing strategies and propose a two-step lane changing procedure consisting of, first, gap selection and alignment by use of a trapezoidal longitudinal acceleration profile, and then moveover to the neighboring lane, [5,7,8]. In addition, various trajectory profiles have been studied under the assumption of constant longitudinal velocity, [2].

This paper presents a fast lane changing algorithm using a simplified s-tope method for obstacle representation and polynomials for trajectory planning. The thrust of this method is that the obstacles are mapped in a 2-dimensional space where the choice of the admissible trajectory can be made by the selection of a single coefficient. Dynamic constraints are mapped on the same space, allowing for feasibility evaluation. The method yields smooth trajectories, where lateral and longitudinal position adjustments are made simultaneously, thus providing more admissible trajectories than two-step methods.

II. VEHICLE REPRESENTATION AND TRAJECTORY PLANNING

As discussed earlier, vehicle representation and trajectory planning are separate procedures that lead to a lane changing algorithm. However, for reasons explained later, they are studied together in this section.

A. Trajectory planning with no obstacles

Suppose that a vehicle, running at longitudinal speed $\dot{x}$, decides to make a lane change. If there are no obstacles in the vicinity, the time t_{lc} to complete the lane change depends on the passengers' comfort, and it can be considered independent of the vehicle longitudinal speed, and thus prespecified. Hence, t_{lc} is known in advance, $\dot{x}$ is measured by the vehicle speedometer, and the width of the current and target lanes is measured using a vision system (or can be known in advance); therefore the lane changing problem can be formulated as a simple boundary condition problem as follows: find a smooth path that guides the vehicle from the initial state $(x_{in}, \dot{x}_{in}, \ddot{x}_{in}, y_{in}, \dot{y}_{in}, \ddot{y}_{in})$ to the final state $(x_{fin}, \dot{x}_{fin}, \ddot{x}_{fin}, y_{fin}, \dot{y}_{fin}, \ddot{y}_{fin})$, as shown in Fig. 1.

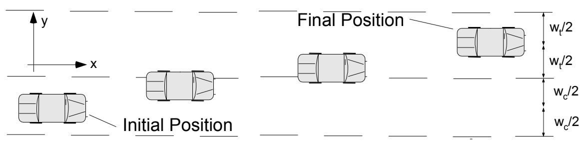

Fig. 1. Lane changing without obstacles.

where x_{fin} corresponds to the longitudinal distance traveled if the speed was constant and y_{fin} corresponds to the sum of the current and target lane width w_c, w_t divided by 2. Selecting 5th order polynomials for x and y position, that is:

$$y(t) = a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \quad (1)$$

$$x(t) = b_5 t^5 + b_4 t^4 + b_3 t^3 + b_2 t^2 + b_1 t + b_0 \quad (2)$$

the problem can be solved trivially by solving the following systems of linear equations:

$$(a_0 \; a_1 \; a_2 \; a_3 \; a_4 \; a_5)^T = A_{6\times 6} \cdot (y_{in} \; y_{fin} \; \dot{y}_{in} \; \dot{y}_{fin} \; \ddot{y}_{in} \; \ddot{y}_{fin})^T \quad (3)$$

$$(b_0 \; b_1 \; b_2 \; b_3 \; b_4 \; b_5)^T = B_{6\times 6} \cdot (x_{in} \; x_{fin} \; \dot{x}_{in} \; \dot{x}_{fin} \; \ddot{x}_{in} \; \ddot{x}_{fin})^T \quad (4)$$

where A and B are functions of the initial and final time.

Example 1

Consider a vehicle running at $\dot{x} = 20ms^{-1}$ that starts a lane change at time $t_0 = 0s$ and completes it at $t_{lc} = 5s$. The coordinate system is placed on the vehicle's geometric center at time t_0 and remains fixed. The lane change is considered complete when both longitudinal and lateral accelerations are *0*. The width of each lane is *4m*. Thus, the initial and final state of the vehicle is:

$$(x_{in}, \dot{x}_{in}, \ddot{x}_{in}, y_{in}, \dot{y}_{in}, \ddot{y}_{in}) = (0,20,0,0,0,0) \quad (5)$$

$$(x_{fin}, \dot{x}_{fin}, \ddot{x}_{fin}, y_{fin}, \dot{y}_{fin}, \ddot{y}_{fin}) = (100,20,0,4,0,0) \quad (6)$$

Solving Eqs. (3) and (4) yields the path shown in Fig. 2. It is clear that, unlike other methodologies, the constructed path results in continuous curvature profile.

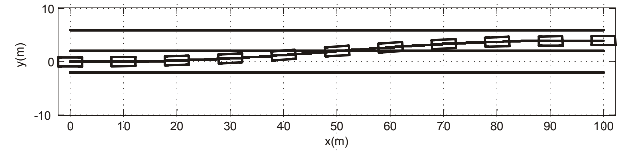

Fig. 2. Calculated path without obstacles.

B. Trajectory planning with 1 obstacle

If an obstacle is encountered during the lane change, the previous algorithm cannot take it into account and a collision will occur. However, if the order of one of the polynomials is increased, and the boundary conditions are still respected, then the shape of the path changes, thus providing the vehicle with more freedom to avoid the obstacle. The problem now is what polynomial should the order be increased of and what are the possible values of the extra coefficient that yield admissible trajectories.

As far as the former issue is concerned, increasing the order of *y(t)* only, is not a wise choice. Indeed, shaping the lateral trajectory can yield only few admissible paths, because the vehicle is restricted to maneuvers in the current and target lane, which is not the case in the longitudinal direction. Moreover, the maximum achievable lateral accelerations are relatively small compared to the longitudinal ones (4-5 times smaller). If the order of both *x(t)* and *y(t)* is increased, then the complexity of the problem increases substantially because there are two extra coefficients to be determined. Thus, increasing the order of *x(t)* only, is an acceptable choice. Then, *x(t)* is:

$$x(t) = b_6 t^6 + b_5 t^5 + b_4 t^4 + b_3 t^3 + b_2 t^2 + b_1 t + b_0 \quad (7)$$

Substituting the boundary conditions in Eq. (7) and in its 1st and 2nd derivatives, coefficients b_i, $i = 0,1,...5$ are written as:

$$b_i = m_i + n_i b_6, \quad i = 0,1,...5 \quad (8)$$

where m_i and n_i are functions of the boundary conditions. Substituting Eqs. (8) into Eq. (7) yields *x(t)* as an affine function in b_6:

$$x(t) = c_1 + c_2 b_6 \quad (9)$$

where c_1 and c_2 are functions of time and the boundary conditions. The simplicity of the form of Eq. (9) makes it very attractive to use in finding an analytical way to determine the admissible values for b_6. To this end, the appropriate vehicle representation has to be achieved.

As discussed in Section I, a fast and accurate way of representing objects in *2-D* space is approximating them by an infinite number of circles. In particular, a rectangular shape, which roughly corresponds to that of a vehicle, can be represented by the area swept by a dynamic circle of radius *R*, whose center obeys a linear function in $\lambda \in [0,1]$.

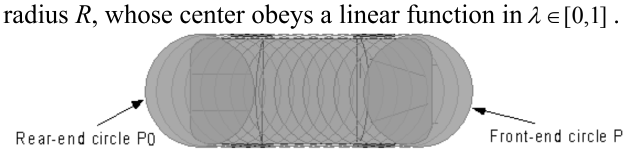

Fig. 3. Vehicle representation.

$$P = P_0 + \lambda(P_n - P_0) \quad (10)$$

where P_0 and P_n are the vectors of the end circle centers. This shape will hereafter be called the oval shape. Eq. (10) can also be written in Cartesian coordinates as follows:

$$x = x_0 + \lambda(x_n - x_0) \quad (11)$$

$$y = y_0 + \lambda(y_n - y_0) \quad (12)$$

where x and y represent the line segment that connects P_0 and P_n. Hence, if two vehicles are enclosed in oval shapes, the criterion for collision avoidance is:

$$\begin{gathered}(x_c - x_o)^2 + (y_c - y_o)^2 > (R_c + R_o)^2 \rightarrow \\ (x_{c0} + \lambda_c(x_{cn} - x_{c0}) - x_{o0} - \lambda_o(x_{on} - x_{o0}))^2 + \\ (y_{c0} + \lambda_c(y_{cn} - y_{c0}) - y_{o0} - \lambda_o(y_{on} - y_{o0}))^2 > (R_c + R_o)^2 \end{gathered} \quad (13)$$

where subscript *"c"* corresponds to the controlled vehicle and *"o"* corresponds to the obstacle. Assuming that the yaw angle of the controlled vehicle remains *0*, taking into account the geometric characteristics of the vehicle and making use of Eq. (9), we obtain:

$$\begin{gathered}(c_1 + c_2 b_6 - l_c/2 + \lambda_c l_c - x_{o0} - \lambda_o(x_{on} - x_{o0}))^2 + \\ (y_{c0} - y_{o0} - \lambda_o(y_{on} - y_{o0}))^2 > (R_c + R_o)^2 \quad \forall \lambda_c, \lambda_o \in [0,1]\end{gathered} \quad (14)$$

The above assumption, although not true in practice, is a fair approximation of reality. Indeed, the yaw angle remains very small even during emergency lane changing. Further aspects on how this approximation can be taken into account during implementation are discussed later. After some algebraic manipulations, Eq. (14) becomes:

$$a^2 b_6^2 + \beta b_6 + \gamma > 0 \quad (15)$$

where a, β and γ are functions of time, boundary conditions, and parameters. Since the coefficient of b_6^2 is larger than *0*, Eq. (15), i.e. collision avoidance, is satisfied when b_6 lies outside the roots of this 2nd order polynomial, that is:

$$b_6 < \frac{-\beta - \sqrt{\Delta}}{2\alpha^2} \quad \cup \quad b_6 > \frac{-\beta + \sqrt{\Delta}}{2\alpha^2}, \quad \Delta = \beta^2 - 4\alpha^2\gamma \quad (16)$$

Hence plotting, at every time instant, the solutions of

$$a^2 b_6^2 + \beta b_6 + \gamma = 0 \quad (17)$$

and selecting a value for b_6 outside these roots ensures a collision-free path. It has to be noted, that Eq. (17) depends on λ_c and λ_o, which means that, in the general case, for every $\lambda_c \in [0,1]$ and $\lambda_o \in [0,1]$ there is a specific pair of roots, hence there is a specific area of admissible b_6's. The number of combinations of λ_c and λ_o that are checked depends on the trade-off between accuracy and fast computation that exists in almost all trajectory planning methodologies. However, Eq. (16) is an affine function of λ_c, thus the minimum and maximum acceptable values for b_6 are achieved at the minimum and maximum values of λ_c, hence, without loss of accuracy, the root plotting can be limited to the combinations corresponding to the possible collisions of the front and rear end circles of the controlled vehicle with the oval shape. To illustrate the methodology described above, the following example is presented.

Example 2

Consider a vehicle running at $20ms^{-1}$ that starts a lane change at time $t_0 = 0s$ and a second vehicle running at $20ms^{-1}$ in the target lane exactly beside the controlled vehicle. It is assumed that the controlled vehicle will eventually adjust its speed to that of the obstacle, which is one of the principles in cooperative driving. Thus, the lane change is considered complete when both longitudinal and lateral accelerations of the controlled vehicle are *0*, the longitudinal velocity is $20ms^{-1}$ and the x-position is *10m* behind the obstacle. The total time of lane change is again $t_{lc} = 5s$. The initial and final state of the vehicle is:

$$(x_{in}, \dot{x}_{in}, \ddot{x}_{in}, y_{in}, \dot{y}_{in}, \ddot{y}_{in}) = (0,20,0,0,0,0) \quad (18)$$

$$(x_{fin}, \dot{x}_{fin}, \ddot{x}_{fin}, y_{fin}, \dot{y}_{fin}, \ddot{y}_{fin}) = (90,20,0,4,0,0) \quad (19)$$

Applying the obstacle avoidance methodology results in the plot of the solutions of Eq. (16), corresponding to the rear and front-end circle of the controlled vehicle with the oval shape that represents the obstacle, see Fig. 4.

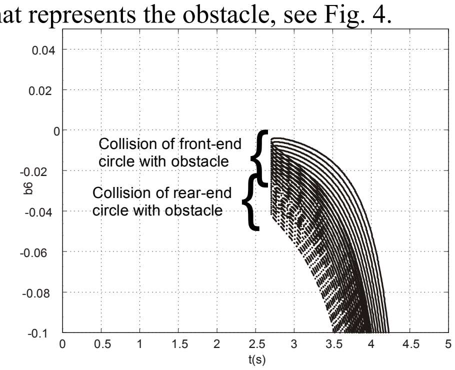

Fig. 4. Map for selection of admissible trajectory.

It is clear that selecting $b_6 = -0.055$ results in a collision at time $t = 3s$ of the rear end circle with the second vehicle. Selecting b_6 outside the highlighted areas ensures a collision free trajectory. For example, selecting $b_6 = 0.01$ yields the path shown in Fig. 5.

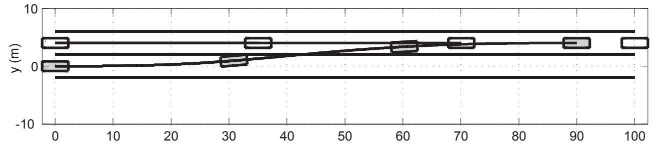

Fig. 5. Collision-free path.

In Fig. 6, the positions, velocities and accelerations of the controlled vehicle versus time are shown. As expected the proposed methodology yields smooth trajectories.

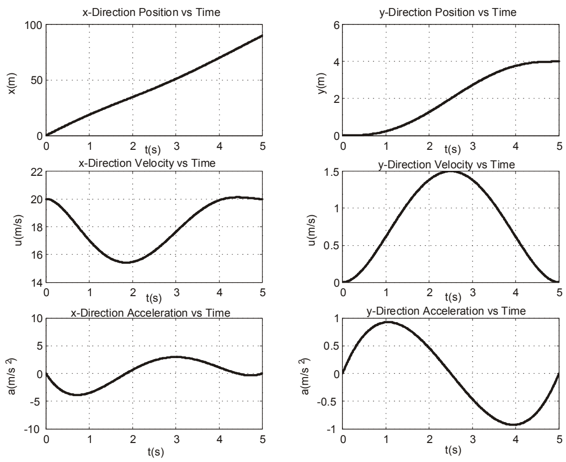

Fig. 6. Positions, velocities and accelerations of the vehicle's geometric center.

It is clear that the whole procedure, in fact, maps the obstacle in the $b_6 - t$ space and thus the selection of an admissible trajectory becomes a simple task. Further simplification, can be achieved if it is assumed that the obstacle is parallel to the controlled vehicle. This is a valid simplification, since, in most lane changing cases, the angle between the vehicles is very small. In this case, following the procedure described above, Eq. (16) becomes an affine function of l_o, too, thus the obstacle avoidance criterion can be further limited to checking possible collisions among the vehicles' end circles. In the previous example, the map of the obstacle under these assumptions is shown in Fig 7.

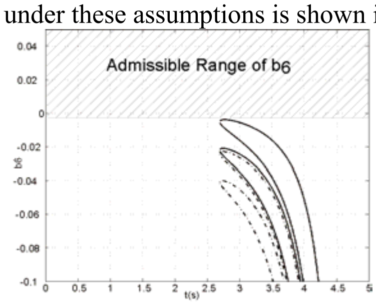

Fig. 7. Simplified map for trajectory selection.

If the selection of an admissible value for b_6 is the only goal, the only mapping that has to be done is that between the front-end circle of the controlled vehicle and the rear-end circle of the obstacle. This is in total agreement with the intuitive fact that when a vehicle decides to follow another one in the neighboring lane, and its lateral position is monotonously increasing, then collision avoidance between the front end of the controlled vehicle with the rear end of the obstacle ensures complete collision avoidance. Conversely, when deciding to stay in front of the obstacle the collision avoidance criterion should be applied between the rear-end circle of the controlled vehicle and the front-end circle of the obstacle. This further reduces the number of computations resulting in an even faster algorithm. During implementation, a small increase in the radius of these circles is suggested in order to take into account the simplifications made and increase safety.

III. DYNAMIC CONSTRAINTS

Until now, it has been shown that the choice of admissible trajectories can be a simple task since the obstacles are mapped in the $b_6 - t$ space. In this section, the same concept is applied for the constraints imposed by vehicle dynamics.

In practice, the maximum lateral and longitudinal accelerations depend on each other, [8] yet, in this paper, a more conservative approach is adopted. Specifically, it is assumed that a vehicle can achieve, at most, the following constant threshold accelerations:

$$-|a_l| \le \ddot{y} \le |a_l| \tag{20}$$

$$-|a_{dec}| \le \ddot{x} \le |a_{ac}| \tag{21}$$

In order to relate the above threshold accelerations with the choice of b_6, the following procedure is proposed. Recall that there is no freedom in *y(t)*, which depends only on the boundary conditions. Thus, checking if the boundary conditions result in lateral acceleration within the bounds specified above, involves the trivial task of differentiating twice Eq. (1) and checking its minimum and maximum. If at least one of these values is beyond the bounds, then the boundary conditions (in fact the final conditions) have to change. In practice, this can be implemented by a rough lookup table, which relates the time of lane change completion with the final lateral position.

Having ensured that the desired lateral acceleration is feasible, the next step is the selection of the value for b_6 in order to calculate *x(t)*. Differentiating Eq. (7), the longitudinal acceleration is:

$$\ddot{x}(t) = 30b_6t^4 + 20b_5t^3 + 12b_4t^2 + 6b_3t + 2b_2 \tag{22}$$

Substituting Eq. (8) into Eq. (22), yields:

$$\ddot{x}(t) = c_3 + c_4 b_6 \tag{23}$$

where c_3 and c_4 are functions of time and the boundary conditions. This is again an affine function of b_6, thus, for each time step, the dynamic constraints are satisfied when:

$$-|a_{dec}| < c_3 + c_4 b_6 < |a_{ac}| \tag{24}$$

Hence, plotting the solutions of the following equations:

$$b_6 = \frac{\pm|a_{dec}| - c_3}{c_4} \tag{25}$$

and selecting a b_6 outside the unacceptable areas ensure that the dynamic constraints are not violated. To illustrate the procedure the following example is presented.

Example 3

Consider the vehicle described in Example 2, whose initial and final state are given by Eqs. (18) and (19). For simplicity, the vehicle in the target lane is ignored, however the increased order polynomial is used for *x(t)*. The threshold accelerations, that the vehicle can achieve, are:

$$-2ms^{-2} \le \ddot{y} \le 2ms^{-2}, \quad -10ms^{-2} \le \ddot{x} \le 2.5ms^{-2} \tag{26}$$

After solving the boundary condition problem for *y(t)* and differentiating Eq. (1) twice, the lateral acceleration is compared to the maximum. Since it is within the bounds, (see lower right plot of Fig. 6), the procedure for obtaining admissible values for b_6 is followed. Plotting the solutions of Eq. (25) yields the admissible range of b_6, as shown in Fig. 8. In that sense, the dynamic constraints are mapped on the same space as the obstacles, and thus they can be considered as virtual obstacles. It is evident that the selection of a feasible trajectory is now a simple task.

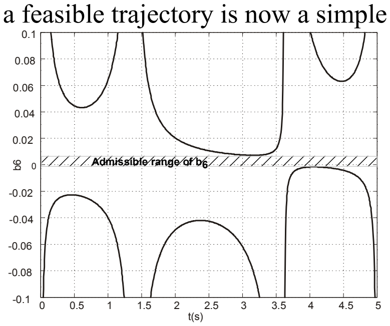

Fig. 8. Dynamic constraints mapped on $b_6 - t$ space.

IV. GENERAL LANE CHANGING PROBLEM

Having explained how obstacles and dynamic constraints are mapped on the $b_6 - t$ space, it is now possible to analyze the general lane changing problem and demonstrate how it can be solved using the methodology described above. In the general case, lane changing involves the controlled vehicle (vehicle "*c*"), a vehicle in the current lane (vehicle "o_1") and two vehicles in the target lane (vehicles "o_2" and "o_3"), as shown in Fig. 9. Under these circumstances, there are four ways that the controlled vehicle can complete the lane change, depending on the speed of the vehicles in the target lane and their position relative to that of the controlled vehicle. These are shown in Fig. 10.

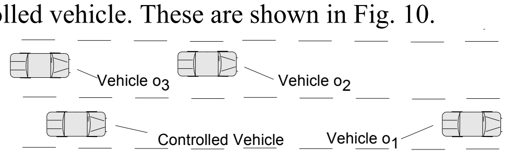

Fig. 9. General lane changing problem.

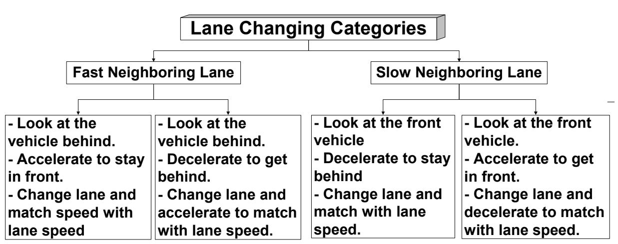

Fig. 10. Lane changing strategies.

Thus, as soon as the controlled vehicle decides to make the lane change, it measures the velocity of the vehicles in the target lane (through its on board sensors and possibly an inter-vehicle communication system), compares it with its own and chooses a lane changing strategy. Next, the boundary conditions are formed, the coefficients of *y(t)* are calculated and the obstacles (real and virtual ones) are mapped on the $b_6 - t$ space. If there is an admissible b_6, lane change is performed, while the speed of the obstacles is continuously monitored and the admissible trajectory is recalculated in case a significant change is detected. Otherwise, the same procedure, but this time with the second strategy, is conducted. It has to be noted that it is possible that none of the strategies will be able to yield an admissible trajectory. In that case, a simple brake algorithm is proposed, which checks the distance and relative velocity between the controlled vehicle and vehicle "o_1" and computes the deceleration required to avoid collision. The procedure is demonstrated by the following example.

Example 4

Without loss of generality, the following scenario is examined: Vehicle "*c*", running at $25ms^{-1}$ starts a lane change, because vehicle "o_1" is stationary (due to an accident, for instance). Vehicles "o_2" and "o_3" form a platoon running at the speed of $20ms^{-1}$, which will remain constant along the lane changing procedure. The distances between the vehicles are shown in Fig. 11.

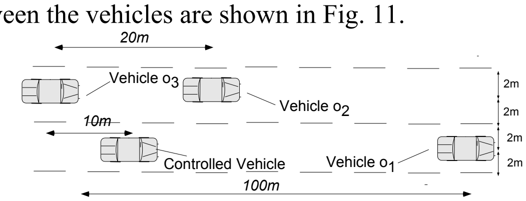

Fig. 11. Lane changing scenario at initial state.

The algorithm determines that the target lane is a slower lane and decides to follow the strategy of staying behind vehicle "o_2". Its final position will be exactly in the middle of the two obstacles and parallel to the stationary obstacle. After simple calculations (recall that the speed of "o_2" and "o_3" remains constant), the boundary conditions are determined. Next *y(t)* is calculated using Eqs. (1) and (3) and then the obstacles and the dynamic constraints are mapped using Eqs. (16) and (25), see Fig. 12.

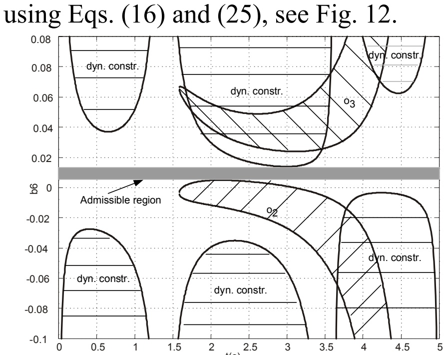

Fig. 12. Region of admissible values for b_6.

It has to be noted that the obstacles are represented using the simplified method, that is, the distance criterion is applied three times: between the front-end circle of vehicle *"c"* and the rear-end circle of vehicle "o_1" and that of vehicle "o_2", and between the rear-end circle of vehicle *"c"* and the front-end circle of vehicle "o_3". The fact that there is no curve corresponding to obstacle "o_1" is due to the scaling of the plot. The highlighted area is the area of admissible values for b_6 and selecting a value within this area ensures a smooth, feasible, collision-free trajectory. Fig. 13 depicts the snapshots taken while the vehicle follows the designed trajectory and Fig. 14 shows positions, velocities and accelerations of the controlled vehicle.

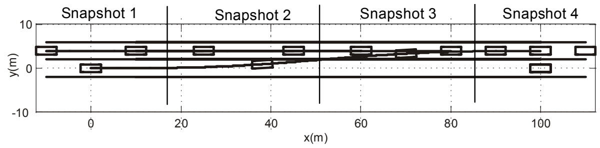

Fig. 13. Snapshots of collision-free path.

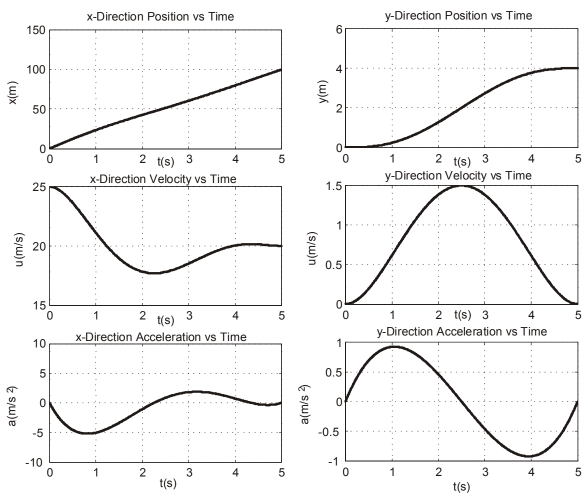

Fig. 14. Positions, velocities and accelerations of the vehicle's geometric center.

It is obvious that the selection of b_6 can be part of an optimization problem, with an objective function incorporating passenger comfort and safety. In addition, if more obstacles are in the vicinity then the distance criterion is applied again. This means that the number of computations increase linearly with the number of obstacles, which is a very attractive property of the algorithm. Finally, during implementation, vehicles "o_2" and "o_3" are expected to increase their distance during the lane change, either due to their own automated vehicle spacing algorithm or because information will be exchanged with vehicle "c". This issue is discussed in the existing literature and it exceeds the scope of this paper, [7].

V. CONCLUSIONS

This paper focused on the lane changing problem and a simple, yet accurate, way of representing obstacles was proposed. This method simplifies the s-tope representation and, under certain assumptions, reduces it to finding the distance between circles. It has been shown that by use of polynomials for both the lateral and longitudinal position, the problem of obstacle avoidance can be solved using fast algebraic computations and eventually be reduced to a problem of selecting the value of a single coefficient with the possible tradeoff of missing some admissible paths. Dynamic constraints were also taken into account by mapping them on the same space with the obstacles. The computational burden increases linearly with the number of obstacles and the resulting trajectories are smooth. Finally, the general lane changing problem, along with simple lane changing strategies, were presented and solved. Illustrative examples demonstrated the use of the algorithm.

ACKNOWLEDGMENTS

Support of this work by the California PATH (Partners for Advanced Transit and Highways) program and the A.S. Onassis Public Benefit Foundation is acknowledged.

REFERENCES

[1] Bernabeu, E.J., Tornero, J., Tomizuka, M., "Collision Prediction and Avoidance Amidst Moving Objects for Trajectory Planning Applications. Proc. of the 2001 IEEE Int. Conf. on Robotics and Automation, Korea, May 2001, pp.3801-3806.

[2] Chee, W., Tomizuka, M., "Vehicle Lane Change Maneuver in Automated Highway Systems", California PATH Research Report, UCB-ITS-PRR-94-22, October 1994.

[3] Fraichard, T., "Dynamic Trajectory Planning with Dynamic Constraints: a 'State-Time Space' Approach", Proc. of the 1993 IEEE/RSJ Int. Conf. On Intellingent Robots and Systems, Yokohama, Japan, July 1993, pp. 1393- 1400.

[4] Gilbert, E.G., Johnson, D.W., Keerthi, S.S., "A Fast Procedure for Computing the Distance Between Complex Objects in Three-Dimensional Space", IEEE Journal of Robotics and Autom., vol. 4, No. 2, April 1988, pp. 193-203.

[5] Godbole, D. N., Hagenmeyer, V., Sengupta, R., Swaroop, D., "Design of Emergency Maneuvers for Automated Highway System: Obstacle Avoidance Problem", Proc. of the 36th Conf. on Decision and Control, San Diego, CA, December 1997, pp. 4474-4779.

[6] Jacobs, P., and Canny, J., "Planning Smooth Paths for Mobile Robots", Proc. of the IEEE Int. Conf. on Robotics and Automation, April 1989, pp. 2-7.

[7] Jula, H., Kosmatopoulos, E.B., Ioannou, P.A., "Collision Avoidance Analysis for Lane Changing and Merging",IEEE Trans. on Vehicular Technology, vol. 49, No. 6, November 2000, pp. 2295-2308.

[8] Kanaris, A., Kosmatopoulos, E.B., Ioannou, P.A., "Strategies and Spacing Requirements for Lane Changing and Merging in Automated Highway Systems", IEEE Trans. on Vehicular Technology, vol. 50, No. 6, November 2001, pp. 1568-1581.

[9] Laumond, J.P., Jacobs, J., Taix, M., Murray, R. M., "A Motion Planner for Nonholonomic Mobile Robots", IEEE Trans. on Robotics and Automation, Vol. 10, No. 5, October 1994, pp. 577-593.

[10] Papadopoulos, E., Poulakakis, I., Papadimitriou, I., "On Path Planning and Obstacle Avoidance for Nonholonomic Platforms with Manipulators: A Polynomial Approach", Int. Journal of Robotics Res., Vol. 21, No. 4, 2002, pp.367-383.

[11] Scheuer, A., Fraichard, T., "Collision-Free and Continuous-Curvature Path Palnning for Car-Like Robots", Proc. of the 1997 IEEE Int. Conf. On Robotics and Automation, Albuquerque, New Mexico, April 1997, pp. 867-873.

[12] Tornero, J., Hamlin, J., Kelley, R. B., "Spherical-Object Representation and Fast Distance Computation for Robotic Applications", Proc. of 1991 IEEE Int. Conf. on Robotics and Automation, Sacramento, CA, April 1991, pp. 1602-1608.