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Macroscopic model for multi-anticipation self-driving cars

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Abstract. Self-driving cars technology and vehicle connectivity enable self-driving cars to precisely receive information of many cars leading them. By multi-anticipation, a flow of self-driving car is proved to be more stable than without multi-anticipation. The instability of traffic flow can cause a traffic jam as a little velocity fluctuation is amplified. In this work, we discuss the macroscopic effect of parameters in the microscopic model for a self-driving car. Those parameters are the number of the vehicle the car anticipate (field size) and weight for anticipation with the leadings (strength factor). The macroscopic model is derived from the microscopic model. We obtained a partial derivative equation that describe how the velocity fluctuation change through time. According to our model, these two variables play a crucial role in the traffic flow harmonization. Comparison between each model was carried out and discussed.

1. Introduction

A small velocity fluctuation in the homogeneous traffic flow can become a traffic jam [1]. Self-driving cars could be used to wipe out those fluctuations. Sometimes, people lose focus and perform over- or under-acceleration, which leads to velocity fluctuation. Self-driving cars are different from humans. They can precisely receive the data from its sensors and drive the vehicle according to those data. Moreover, self-driving cars can perceive and handle more information than human. Furthermore, The technology like V2V (vehicle to vehicle) or V2I (vehicle to infrastructure) enable self-driving car to receive and communicate with the others to drive efficiently.

Stability of the flow is one of the goals for self-driving cars to achieve. With stability, velocity fluctuation cannot be amplified. Without stability, velocity fluctuation can be amplified. Principally, a microscopic behavior plays a crucial role in the stability of the flow. There are a lot of contributions from many scientists on designing the behavior of self-driving cars to make the traffic flow stable. For example, Horn [2] proposed that the vehicles should balance its distance to the leading vehicle and the following car. Zhu Wen-Xing and Zhang Li-Dong [3] suggest that the cars should drive according to the headway of itself and the following car, but not necessary to be balanced. They propose the model describing the microscopic behavior of the cars that obtain data from many vehicles leading them[4]. In this research, we derived the macroscopic model from the idea proposed by Zhu Wen-Xing and Zhang Li-Dong [4] to describe the role of microscopic behavior in macroscopic perspective.

2. Material and method

2.1. Optimal velocity model

This car-following model is developed from the optimal velocity model, which is a car-following model for human-driven cars. The optimal velocity model suggests that human trying to drive at the optimal speed ($V(\Delta x)$). The optimal speed is a function that depends on the vehicle's headway (Δx). However, the car has to take the time "$\tau = \alpha^{-1}$", which is relaxation time, to adjust itself to the optimal speed. The relation between the optimal velocity for a particular gap is a sigmoid curve which is expressed in equation (2). The parameters b and Δs are for adjusting the function to the observation data.

Zhu Wen-Xing and Zhang Li-Dong [3] propose the self-driving car model. They proposed an idea that takes advantage of vehicular connectivity. In their idea, self-driving cars take a leading cars' gap into account. Therefore, instead of the car trying to adjust itself to the optimal velocity of itself, it trying to adjust its velocity to the mean optimal velocity of itself and those leading them. The parameter γ (also called "field size") will determine how many cars does the self-driving car take its gap to account. The mean between its optimal velocity and those of leading cars is weighted with the parameter β (also called "field strength"). The equation (1) describe the car-following model for the self-driving car.

$$\ddot{x}_j(t) = \alpha[(1 - \beta)V(\Delta x_j(t)) + \frac{\beta}{\gamma} \sum_{i=1}^{\gamma} V(\Delta x_{j+i}) - \dot{x}_j(t)] \tag{1}$$

The optimal velocity ($V(\Delta x)$) is

$$V(\Delta x_j) = \frac{v_{max}}{1 + \tanh(b)}(\tanh(\frac{\Delta x_j}{\Delta s} - b) + \tanh(b)) \tag{2}$$

2.2. Linear car-following model

Before the development of optimal velocity model, a linear model of human-driven car was proposed in 1950 by Chandler et at. [5]. This model suggests that the human try to drive the car at the leading car's velocity, instead of driving at the optimal speed like the optimal velocity model. The equation describes this model is

$$\ddot{x} = \alpha(\dot{x}_{n+1} - \dot{x}_n) \tag{3}$$

2.3. Deriving the macroscopic model

The role of leading car's speed in the linear model is the same as the role of optimal velocity in the optimal velocity model. Thus, if the optimal velocity in (1) is substituted by the leading's velocity, it will become the linear self-driving car model as

$$\ddot{x}_n = \alpha((1 - \beta)\dot{x}_{n+1} + \frac{\beta}{\gamma} \sum_{i=1}^{\gamma} \dot{x}_{n+1+i} - \dot{x}_n) \tag{4}$$

Although the gap can be a function of time and space, the gap between each vehicle is approximately small and constant in case of high density flow with small velocity fluctuation. Thus, we can approximate the velocity of the leading car with Taylor expansion.

$$v((n + \xi)\Delta x, t) = v(n\Delta x, t) + \xi \Delta x \frac{\partial v}{\partial x} + \frac{\xi^2 \Delta x^2}{2} \frac{\partial^2 v}{\partial x^2} + O(\Delta x^3) \tag{5}$$

By using this relation to approximate the every leading vehicles' velocities around the car n^{th}, this equation become partial differential equation in (6). For the sum of $i = 1$ to γ, this summation will be simplified to sum of i and i^2.

$$\frac{\partial v}{\partial t} = \alpha \Delta x \left(1 + \frac{\beta(\gamma + 1)}{2}\right) \frac{\partial v}{\partial x} + \alpha \Delta x^2 \left(\frac{1}{2} + \frac{\beta(2\gamma^2 + 9\gamma + 7)}{12}\right) \frac{\partial^2 v}{\partial x^2} \tag{6}$$

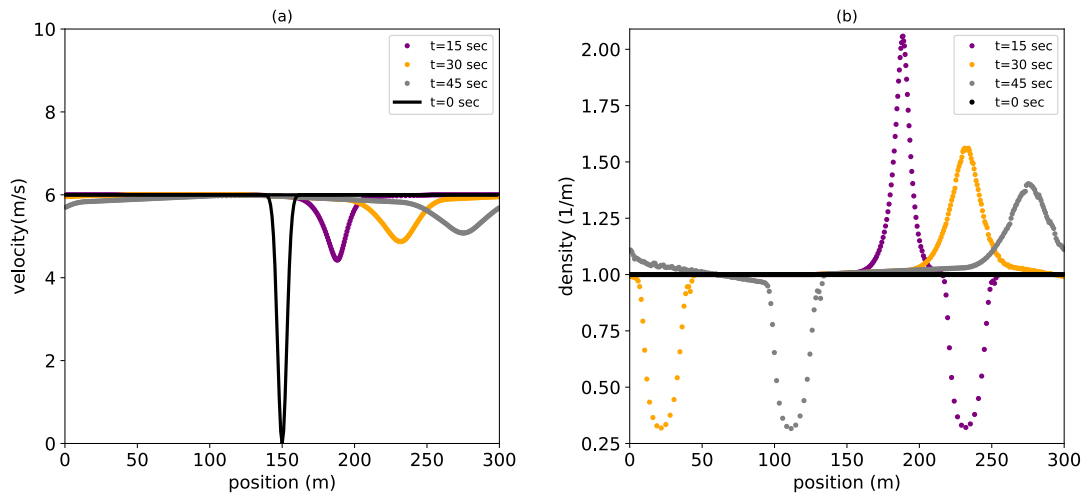We call the equation (6) as the macroscopic model.

Figure 1. Velocity (a) and density (b) at each point on the road is shown in this figure. The initial condition in equation (7) is shown in the black dots. As the time past, the fluctuation (both in velocity and density profile) travel backward and dissipate continually. In density profile, there are both hill and valley which is evolve from a uniform density profile.

3. Discussion

The equation (6) shows a relation between the parameter γ and β in macroscopic perspective. The term $\frac{\partial v}{\partial x}$ imply the transportation property of the "fluctuation" in velocity. The term $\frac{\partial^2 v}{\partial x^2}$ imply the diffusion property of the fluctuation in velocity. The macroscopic model we derived shows the nature of the flow of vehicle. The velocity fluctuation diffuses while it moves backward. As expressed in the macroscopic model, the traveling speed of the velocity fluctuation and dissipation speed depends on the parameters β and γ. Thus, anticipating the gaps of more cars or adding more weight to those gaps results in the faster move and diffusion of fluctuation.

However, this macroscopic model of the self-driving car cannot explain the traffic jam emergence because it only has transportation and dissipation terms. The traffic jam emergence is possible in the traffic flow system [1]. Hence, the macroscopic model for this kind of self-driving car that can describe the jam emergence is needed.

3.1. Comparison with car-following model

As the velocity of leading vehicles is approximated by Taylor series up to second order, there has to be a certain amount of difference between the linear microscopic and macroscopic model. We carried out simulations on the dynamics of the flow and compared the results of both models. For the simulation, the initial condition is set according to equation (7), and the gap between each vehicle is set at the constant ($\Delta x = 1$). (see figure 1) The macroscopic model can be solved numerically. For the microscopic, the simulation is carried out numerically by the Euler method on the periodic boundary condition consist of 300 cars.

$$v(x,0) = 6 - 6e^{-0.05x^2} \tag{7}$$

As shown in figure 2, the minimum velocities of the system in both models increase through time. The minimum velocity from the microscopic model seems to be smaller than the velocity from the macroscopic model principally. Moreover, the higher value of γ (field size) results in the more error between both models. However, this comparison only compares the dissipation property between each model. Traveling property of velocity fluctuation can also be compared.
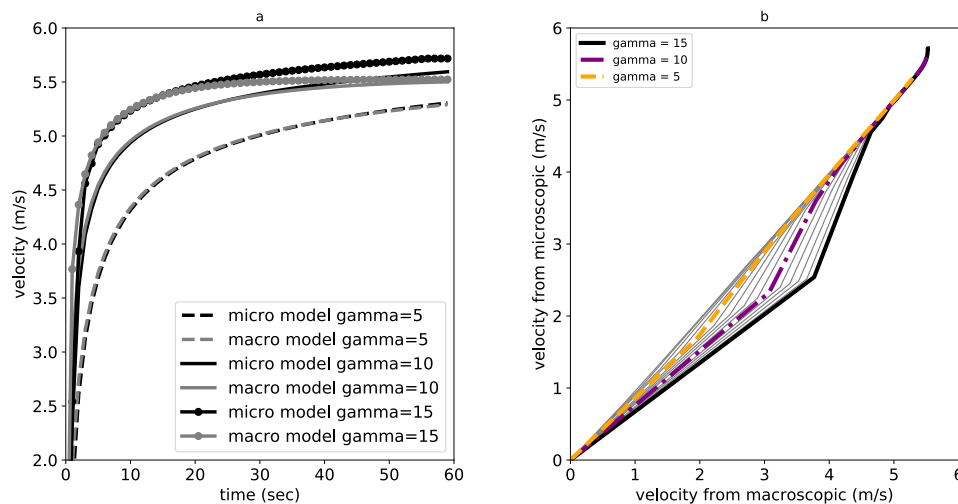
Figure 2. The left figure show the time series of minimum velocity of the system with different value of γ parameter, which is simulate from microscopic and macroscopic model. The right figure show the comparison between the value from microscopic and macroscopic with different γ parameter (from $\gamma = 1$ to 15).

4. Conclusion

In this article, we show our approach to obtain the macroscopic model. This macroscopic model describes the role of microscopic parameters on the macroscopic behavior. One is the number of cars the self-driving car anticipate (γ: field size). Another parameter is the weight parameter determines how much the car anticipate with the leading car gap (β: field strength). The model shows these two parameters influence on the speed of the velocity fluctuation transportation and diffusion. In other words, the more γ and β, the faster the velocity fluctuation travels and diffuses.

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