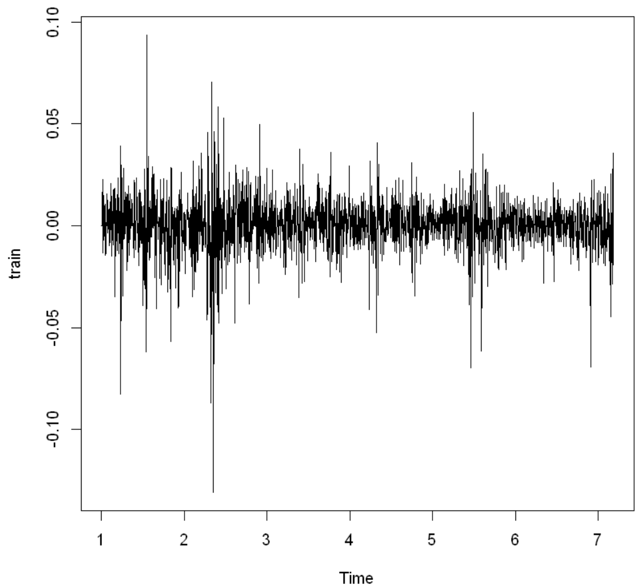
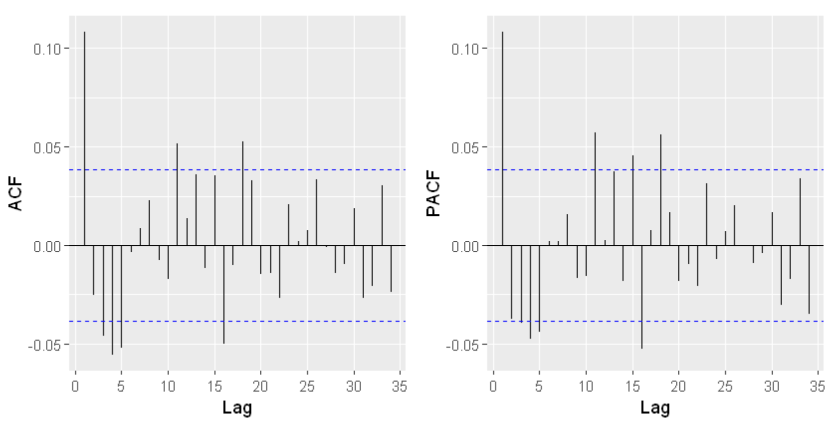
```r
# init
library(dplyr)
library(fpp2)
library(ggplot2)
library(nortest)
library(xts)
library(rugarch)
library(fpp2)
psei <-
read.csv("C:/Users/Administrator/Documents/PSEI.PS_AdjClose.csv")
psei <- stats::na.omit(psei)

psei <- xts(psei$PSEI, as.Date(psei$Date, format = "%Y-%m-%d"))
psei_ret <- na.omit(diff(log(psei)))*100


#functions to divide the dataset into training and test datasets
tail.ts <- function(data,n) {
  data <- as.ts(data)
  window(data,start = tsp(data)[2]-(n-1)/frequency(data))
}
head.ts <- function(data,n) {
  data <- as.ts(data)
  window(data, end = tsp(data)[2]-(n-1)/frequency(data)-1)
}

#training and test datasets
train <- head.ts(psei_ret,250)
test <- tail.ts(psei_ret,250)

#preliminary
plot(train, type="l")
train %>% ur.kpss() %>% summary()
```

```r
ggtsdisplay(train)

#ARMA GARCH
spec2 <- ugarchspec(
  variance.model = list(model = "eGARCH",
                        garchOrder = c(1, 1),
                        external.regressors = NULL),
  mean.model = list(armaOrder = c(2, 0),
                    include.mean = TRUE,
                    archm = FALSE, archpow = 1,
                    external.regressors = NULL),
  distribution.model = "sstd")

(fit2 <- ugarchfit(spec2,
                   psei_ret,
                   out.sample = 250, solver = "hybrid"))

fit2 %>% residuals(standardize = TRUE) %>% ggtsdisplay()
residuals(fit2, standardize = TRUE)^2 %>% ggtsdisplay()

plot(fit2, which="all")

#forecast mean and variance
fcast2 <- ugarchforecast(fit2, n.ahead = 250, n.roll = 250)
plot(fcast2, which = "all")


#forecast accuracy
fcast2 <- ugarchforecast(fit2, n.ahead = 250)
fpm(fcast2)
```

Below is the plot of daily log returns of PSEI, which seems to be stationary. A Kwiatkowski-Phillips-Schmidt-Shin (KPSS) unit root test was conducted to test for the stationarity. As shown below, the value of the test-statistic is 0.0912 which is smaller than the 10% critical value. This indicates that the null hypothesis is not rejected, and the PSEI daily log returns data are stationary.



The figures below are the ACF and PACF plots of the log returns which will be used as a guide to determine the candidate ARIMA models for the training data. The ACF plot determines the order of q for an ARMA(p,q) model. The ACF cuts off after lag 1, then shows a sinusoidal pattern with decreasing intensity. Thus, the ACF plot suggests an MA(1). The PACF plot determines the order of p for an ARMA(p,q) model. The PACF cuts off after lag 1, then shows a sinusoidal pattern with decreasing intensity. Thus, the ACF and PACF plots suggest an ARMA (1,1).



Different ARMA specifications have also been used in conducting determining the best ARMA-GARCH model. The table below shows the Akaike information criterion values of the each ARMA-GARCH model tested before choosing the model which has the best possible distribution fit for the PSEI log return series:

ARMA-GARCH model	AIC	Remarks
eGARCH(1,1) ARMA(2,1) - jsu	3.0024	Autocorrelation problem at lags 1, 8, and 14 of the standardized residuals
eGARCH(1,1) ARMA(2,1) - sstd	3.0030	Autocorrelation problem at lags 1, 8, and 14 of the standardized residuals
gjrGARCH(1,1) ARMA(2,1) - jsu	3.0035	Autocorrelation problem at lags 1, 8, and 14 of the standardized residuals
gjrGARCH(1,1) ARMA(2,1) - sstd	3.0035	Autocorrelation problem at lags 1, 8, and 14 of the standardized residuals
eGARCH(1,1) ARMA(2,0) - jsu	3.0049	Autocorrelation problem at lag 5 of the standardized residuals
eGARCH(1,1) ARMA(2,0) - sstd	**3.0054**	**None**
eGARCH(1,1) ARMA(0,1) - jsu	3.0057	Insignificant mu and omega, but good enough
eGARCH(1,1) ARMA(1,1) - jsu	3.0058	Insignificant mu and omega, but good enough
eGARCH(1,1) ARMA(0,1) - sstd	3.0062	Insignificant mu and omega, but good enough
eGARCH(1,1) ARMA(1,1) - sstd	3.0063	None

Upon exploring the different specifications of GARCH models, it seems that eGARCH(1,1) with Skewed student-t as distribution model and ARMA (2,0) gives the lowest Akaike information criterion (AIC) value which is 3.0054. From the results below, the table of optimal parameters shows that all parameters are significant, which means that all parameters seem to be useful in this model setting.

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.042108    0.017519   2.4035 0.016238
ar1     0.089226    0.019792   4.5081 0.000007
ar2    -0.047538    0.018549  -2.5629 0.010381
omega   0.009307    0.005522   1.6855 0.091885
alpha1 -0.087899    0.016003  -5.4925 0.000000
beta1   0.952237    0.011357  83.8427 0.000000
gamma1  0.233208    0.028740   8.1145 0.000000
skew    0.898811    0.024989  35.9681 0.000000
shape   7.618762    1.042247   7.3099 0.000000
```

```
Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.042108    0.015809   2.6636 0.007731
ar1     0.089226    0.019053   4.6831 0.000003
ar2    -0.047538    0.015842  -3.0007 0.002694
omega   0.009307    0.006240   1.4915 0.135823
alpha1 -0.087899    0.020290  -4.3322 0.000015
beta1   0.952237    0.014813  64.2831 0.000000
gamma1  0.233208    0.032204   7.2416 0.000000
skew    0.898811    0.025330  35.4836 0.000000
shape   7.618762    1.104447   6.8983 0.000000

LogLikelihood : -3944.626
```

The weighted Ljung-Box test tests for the serial correlation of the residuals and squared residuals. Since the *p*-values of the different lags of the standardized residuals are higher than .05, there is not enough evidence to reject the null hypothesis. Thus, there is no serial correlation at lags 1, 5, and 9. Similarly, since the *p*-values of the different lags of the squared residuals are also higher than .05, there is also not enough evidence to reject the null hypothesis for the autocorrelation among the squared residuals. That is, there is no indication of serial correlation in lags 1, 5, and 9. Based on these Ljung-Box tests, where there are no serial correlation among the residuals and squared residuals, it may be inferred that the residuals behave like white noise. Further, this means that the eGARCH (1,1) with ARMA (2,0) specification is good enough in capturing the autocorrelation and volatility in the time series, and it seems that there is no need to modify this specification. The weighted ARCH LM tests test the adequacy of the ARCH process, and since the *p*-values for each of the lags are above .05, the ARCH process is an adequate fit for this data.

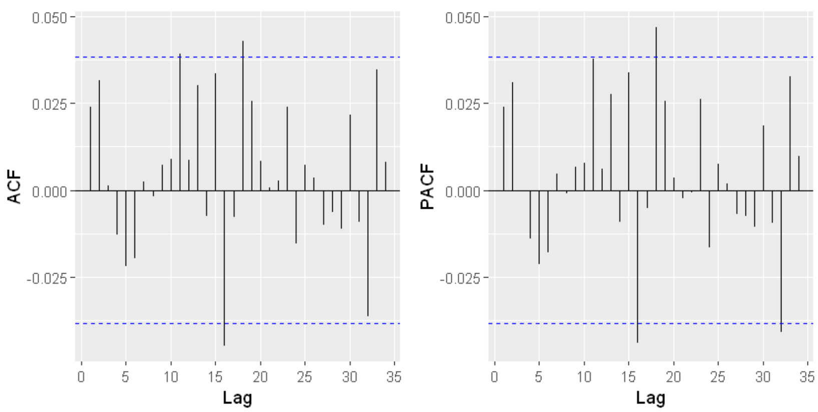
```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.518 0.21791
Lag[2*(p+q)+(p+q)-1][5]     4.029 0.06174
Lag[4*(p+q)+(p+q)-1][9]     5.299 0.38821
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.02346  0.8783
Lag[2*(p+q)+(p+q)-1][5]   1.13093  0.8294
Lag[4*(p+q)+(p+q)-1][9]   2.34702  0.8598
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2341 0.500 2.000  0.6285
ARCH Lag[5]    1.2948 1.440 1.667  0.6479
ARCH Lag[7]    2.0031 2.315 1.543  0.7167
```

The figure below contains the results of the Adjusted Pearson Goodness-of-Fit Test, which tests if the residuals follow the assigned distribution, skew student-t distribution. Since the *p*-values are greater than .05, there is not enough evidence to reject the null hypothesis that the residuals fit well in the skew student-t distribution distribution.

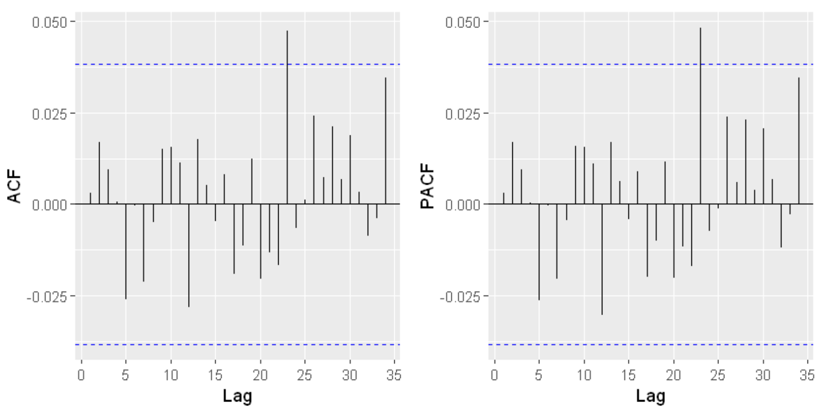
Adjusted Pearson Goodness-of-Fit Test:

	group	statistic	p-value(g-1)
1	20	14.24	0.7697
2	30	29.44	0.4421
3	40	33.80	0.7058
4	50	36.83	0.8998

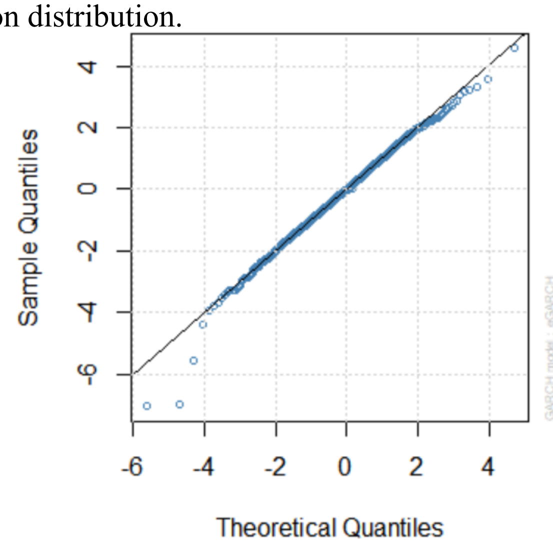
The figures below show the ACF and PACF plots of the eGARCH(1,1) with ARMA(2,0) model and Skew Student-t distribution model standardized residuals. Clearly, no lags among the first five lags of both plots are significant. For ACF, some lags are significant, specifically, lags 11, 16, and 18. For PACF, some lags are also significant, specifically, lags 16, 18, and 32. Despite all of these, these plots seem to be good enough as the autocorrelation is not that large. That is, the specified lags are unlikely to have huge impacts on the forecasts, and the plots resemble white noise.



The figures below show the ACF and PACF plots of the eGARCH(1,1) with ARMA(2,0) model and Skew Student-t distribution model standardized squared residuals. In both plots, no lags among are significant, except for lag 23. Despite this, these plots seem to be good enough as the autocorrelation is not that large. That is, the specified lags are unlikely to have huge impacts on the forecasts, and both plots resemble white noise.

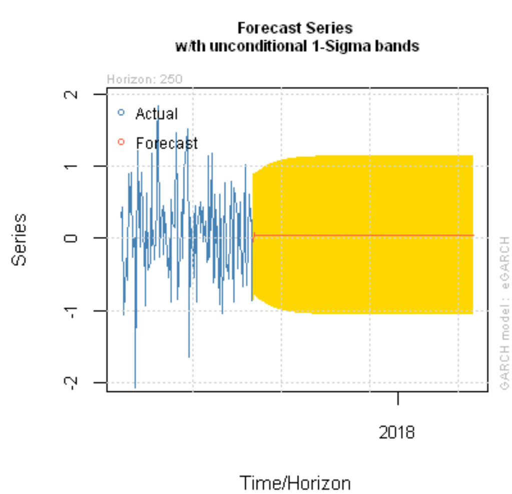


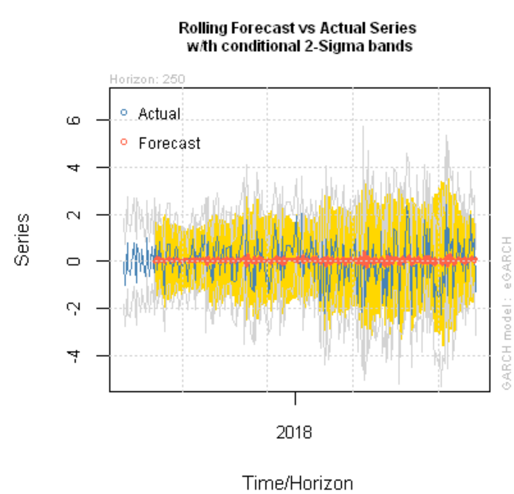
The figure below shows the QQ-plot for the eGARCH(1,1) with ARMA(2,0) model and Skew Student-t distribution. Since the blue points indicated follow the straight line, it can be stated that this illustrates a good model fit with few deviations from the theoretical quantiles in the tails. The findings in this plot supports the conclusion from the Adjusted Pearson GOF test that there is not enough evidence to reject the null hypothesis. That is, the residuals fit well in the skew student-t distribution distribution.

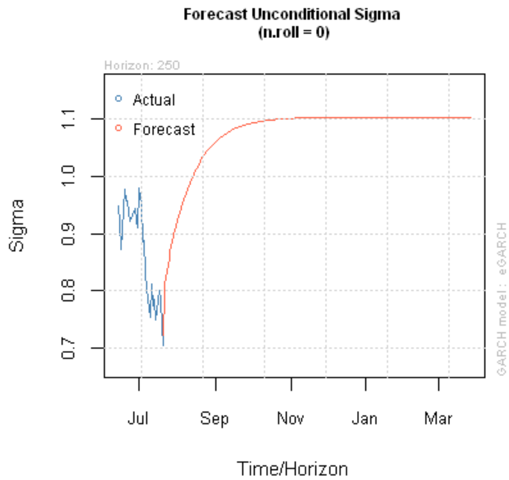


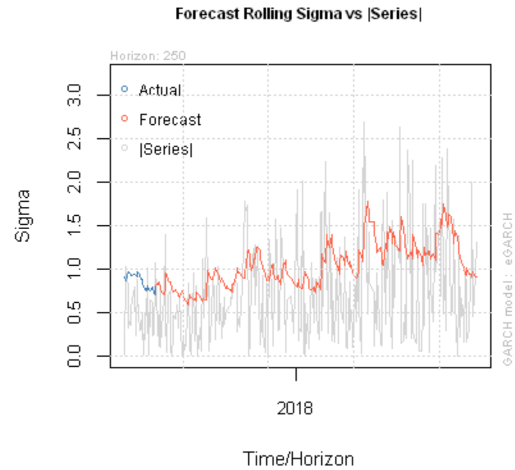
In EGARCH, there are no restrictions on the parameters ω, α, *and* γ, which means to say that it doesn't matter whether their values are positive or negative unlike in sGARCH. On the other hand, β must be positive and less than 1 to maintain stationarity (Ezzat, 2012). From the analysis conducted, it appears that the eGARCH(1,1) with ARMA(2,0) model and Skew Student-t distribution is a good fit for the PSEI daily log returns training dataset as it fulfills the GARCH assumptions and residual diagnostic tests.

Based on this model, plots for the forecasted mean and variance of PSEI log returns were generated using the ugarchforecast() function and shown below. The forecasting horizon was set to 250. The forecasted mean plot shows that the PSEI daily log returns eventually go back to the long term mean similar to the actual time series. The forecast for the sigma, standard deviation of the time series, is more precise as it is observed to follow the observed daily log returns which are shown in gray. However, it still shows some deviations from the actual data. It can also be observed that the dynamics of unconditional sigma seems to go upwards and converges at a value of ~1.1023.










```
*---------------------------------*
*       GARCH Model Forecast      *
*---------------------------------*
Model: eGARCH
Horizon: 250
Roll Steps: 0
Out of Sample: 250

0-roll forecast [T0=2017-07-20]:
        Series  Sigma
T+1   -0.04863 0.8111
T+2    0.07707 0.8230
T+3    0.04954 0.8346
T+4    0.04111 0.8458
T+5    0.04167 0.8565
T+6    0.04212 0.8669
T+7    0.04213 0.8769
T+8    0.04211 0.8866
T+9    0.04211 0.8958
T+10   0.04211 0.9048
T+11   0.04211 0.9133
T+12   0.04211 0.9216
T+13   0.04211 0.9295
T+14   0.04211 0.9371
T+15   0.04211 0.9444
T+16   0.04211 0.9514
T+17   0.04211 0.9581
T+18   0.04211 0.9645
T+19   0.04211 0.9707
T+20   0.04211 0.9766
```

The table below contains the accuracy measures of the eGARCH(1,1) with ARMA(2,0) model and Skew Student-t distribution with respect to the testing dataset. The model's mean absolute error (MAE) is 0.7451, which means that, on average, the log returns forecast is expected to be 0.7451 units away from the actual PSEI log returns. The MSE 0.9195 is close enough to 0, which means that the model is accurate.

MSE	MAE
0.9195	0.7451

REFERENCES

Ezzat, H. (2012, August 1). *The application of GARCH and EGARCH in modeling the volatility of daily stock returns during massive shocks: The empirical case of Egypt*. Munich Personal RePEc Archive. Retrieved December 22, 2022, from https://mpra.ub.uni-muenchen.de/50530/