Math for Critical Thinking

Math 1473
University of Oklahoma
Fall 2022

Contents

Qualitative Data 1
Quantitative Data 10
How Can Data Be Misrepresented? 17
Measurement & Units 21
Rates, Ratios, Proportions 27

Qualitative Data

Qualitative and Quantitative Data

Data is all around us, and informs much of how our lives run. Let's focus on two specific types of data, **qualitative data** and **qualitative data**.

Definition 0.1.1 (Qualitative/Quantitative Data)

Qualitative data is data which describes qualities or characteristics. Sometimes qualitative data is called **categorical data**.
Quantitative data is data which is represented using numbers.

Example 0.1.2. Classify the following data from members of our class as qualitative or quantitative. Give a brief explanation for the answer.

(a) Color of hair

(b) Political affiliation

(c) Height

(d) Classification by credit hours

(e) Number of hours taken this semester

Example 0.1.3. Come up with three sets of quantitative data and three sets of qualitative data. These should be different than the previous example!

Representing Qualitative Data

When data is collected, we need a way to communicate the information effectively. One way of communicating qualitative information is by using a **frequency distribution**.

Definition 0.1.4 (Frequency Distribution)

A **frequency distribution** is a two-column table which collects the number of times a category appears in a set of data.

Example 0.1.5. A store employee is doing an inventory of items near the register. They found 51 bags of chips, 62 candy bars, 47 bags of candy, 121 packs of gum, 16 bags of nuts, 20 soft drinks, and 20 bottles of water. Create a frequency table to display the result of the inventory.

Example 0.1.6. Find out the state (or country) of birth of the portion of the class your instructor specifies. Once you've collected the data, create a frequency table for the data.

We can also graphically represent qualitative data. This is often done using a **bar graph**.

Definition 0.1.7 (Bar Graph)

A **bar graph** is a graph that represents category frequencies with a bar whose length is equal to the frequency.

Example 0.1.8. Create a bar graph for the data from Example 1.1.2. For convenience, here is the data again: A store employee is doing an inventory of items near the register. They found 51 bags of chips, 62 candy bars, 47 bags of candy, 121 packs of gum, 16 bags of nuts, 20 soft drinks, and 20 bottles of water.

Bar Graph Features

Bar graphs should have the following features:

- Vertical axis with label
- Horizontal axis with labels
- A consistent scale on the vertical axis
- A descriptive title

Note

Bar graphs can be oriented vertically (like we just did) or horizontally. If they are oriented horizontally, the vertical and horizontal information flips, but all other features will remain the same.

Example 0.1.9. Now create a bar graph for the data you collected about home states/countries. Be sure to include the features listed above!

A special type of bar graph is called a **Pareto chart**.

Definition 0.1.10 (Pareto Chart)

A **Pareto chart** is a bar graph whose bars are ordered from highest to lowest frequency.

Example 0.1.11. A recent Math 1473 class had 15 freshmen, 3 sophomores, 10 juniors, and 8 seniors. Create the Pareto chart for the data.

Sometimes it is helpful to more than one qualtitative variable on a graph. This can be accomplished by creating a **side-by-side bar graph** or a **stacked bar graph**.

Definition 0.1.12 (Side-by-Side Bar Graph)

A **side-by-side bar graph** is a bar graph which shows two or more bars next to each other

Example 0.1.13. The 2020 US Census gave the following data about racial identities in the top three most populous cities in Oklahoma:

Race	Oklahoma City	Tulsa	Norman
White only	364,706	214,012	89,283
Black/African American only	95,634	61,526	6,398
Asian only	31,510	14,352	5,083

Create a side-by-side bar graph to display the data.

Note that in this example, we needed to include a **legend** in order to identify what each bar was telling us.

Definition 0.1.14 (Stacked Bar Graph)

A **stacked bar graph** is a bar graph in which bars are stacked on top of each other to show relative size.

Example 0.1.15. Use a stacked bar graph to display the Census data from Example 1.1.13. Here it is again:

The 2020 US Census gave the following data about racial identities in the top three most populous cities in Oklahoma:

Race	Oklahoma City	Tulsa	Norman
White only	364,706	214,012	89,283
Black/African American only	95,634	61,526	6,398
Asian only	31,510	14,352	5,083

Question 0.1.16 What are some pros and cons of using a side-by-side bar graph or a stacked bar graph for this data?

Another way of visualizing the data is by using a **pie chart**.

Definition 0.1.17 (Pie Chart)

A **pie chart** is a circle with wedges for each category whose size corresponds to the relative frequency of the category.

Example 0.1.18. Data for the 2022 Democratic Primary for US Senator in Oklahoma is given below:

Candidate	Bollinger	Wade	Horn	Azma	Glenn	Baker
Votes	23,367	19,986	60,691	11,478	21,198	22,467

(a) Create the relative frequency distribution for the data set

(b) Use the relative frequency distribution to create a pie chart for the data.

Example 0.1.19.

(a) Take a poll to determine the make of cars that your classmates drive (if they drive). Use the poll data to create a frequency table.

(b) Create the relative frequency table by dividing each frequency by the total responses. Write the relative frequency as a percentage.

(c) Use the relative frequency table to create a pie chart.

Quantitative Data

Quantitative Data

In order to represent quantitative data, we can use a frequency table just as we could for qualitative data.

Example 0.2.1. The results for a math quiz are given below.

7.5	7	9.5	9	10	9	7.5	5	9.5
10	9.5	9	7	7.5	10	8	7.5	9

Construct a frequency table for the data above, then construct a bar graph from the frequency table.

Question 0.2.2 What are some graphical concerns with the bar graph from the previous example? What are some ways we could improve the graph?

Definition 0.2.3 (Histogram)

A **histogram** is a graphical representation of numerical data that is similar to a bar graph, but uses a number line as the horizontal axis.

Example 0.2.4. Create the histogram for the previous example.

With quantitative data, often we have to work with large input ranges. We can handle large ranges by using **class intervals**.

Definition 0.2.5 (Class Intervals)

Class intervals are groupings of input data used to make histograms are easier to read

Class Interval Rules

When creating class intervals, we have several rules.

- Each piece of data must fall into one of the classes
- Classes must not overlap
- Classes must be of equal width
- There must be no gaps between classes, even if a class has no data

Example 0.2.6. The height (in inches) of the members of a high school basketball team are collected in the table below:

65	70	68	72	71
70	77	72	69	71

(a) Create four class intervals using the data.

(b) Create the relative frequency distribution for the class intervals.

(c) Use part (b) to create the histogram for the data.

Example 0.2.7. Get into groups (your instructor will tell you the size).

(a) Within your group, figure out how many countries each person has been to.

(b) Your instructor will gather the information from each group and write it down for the class. With your group, create the histogram for the data.

Example 0.2.8. The following table presents the number of hours worked last week by employees at a local drug store.

52	18	2	20	9	9	11	6
4	12	9	16	10	37	15	18
4	3	17	19	12	20	11	14
21	36	17	3	23	28	19	20

Create the hisogram by breaking the data into 5 classes.

Data can also be visualized by using another kind of graphic, called a **stem-and-leaf plot**.

Definition 0.2.9 (Stem-and-Leaf Plot)

A **stem-and-leaf plot** is a graphic which separates numerical data into two pieces: the **stem** (such as the left-most digits) and the **leaf** (such as the right-most digit).

Example 0.2.10. Create the stem-and-leaf plot for the previous example. Here's the data again:

The following table presents the number of hours worked last week by employees at a local drug store.

52	18	2	20	9	9	11	6
4	12	9	16	10	37	15	18
4	3	17	19	12	20	11	14
21	36	17	3	23	28	19	20

Example 0.2.11. A Spotify playlist purports to collect the 50 top songs of the week. For the week of July 25 - July 29, the BPM (beats per minute) of those top 50 songs were collected and are listed below.

101	140	109	174	160	107	107	141	109	107
132	169	103	142	126	147	154	166	115	103
81	115	156	102	129	102	108	165	170	78
122	95	170	84	112	186	118	180	125	176
94	120	135	113	101	93	158	117	81	161

(a) Create a histogram for the data by dividing into classes of your choosing.

(b) Create a stem-and-leaf plot for the data.

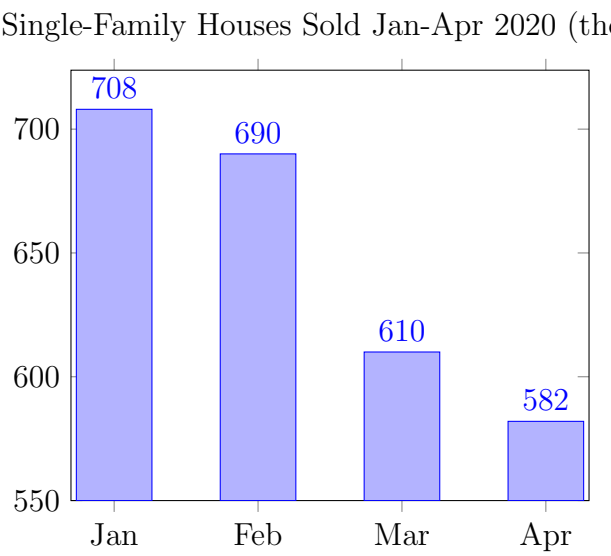
(c) Which graphical representation (if either) do you think best displays the data? What do you think makes it better than the other? Or, why do you think both representations are good representations?

How Can Data Be Misrepresented?

Qualitative Data

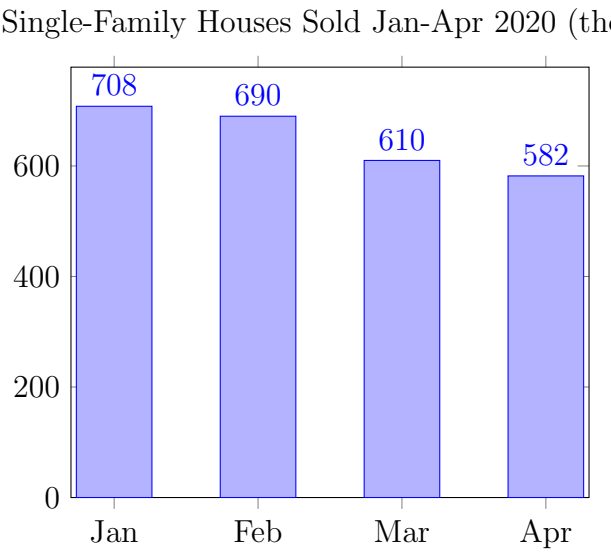
It is not uncommon for data to be used to misinform or mislead. We'll explore some ways graphs can be used to misrepresent data and some ways to identify when that happens.

Example 0.3.1. The seasonally adjusted annual rate for new single-family houses sold in the United States (in thousands) between January 2020 and April 2020 is shown in the bar graph below.



Discuss why this bar graph might be misleading.

Example 0.3.2. The same data from the previous example is graphed again below.



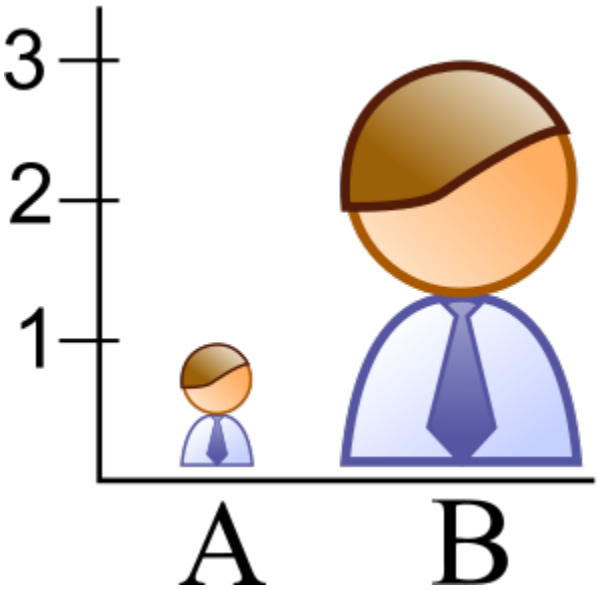
What is different about this graph than the previous one? In what ways might it be a better representation of the data?

Common Ways to Mislead Using Graphs

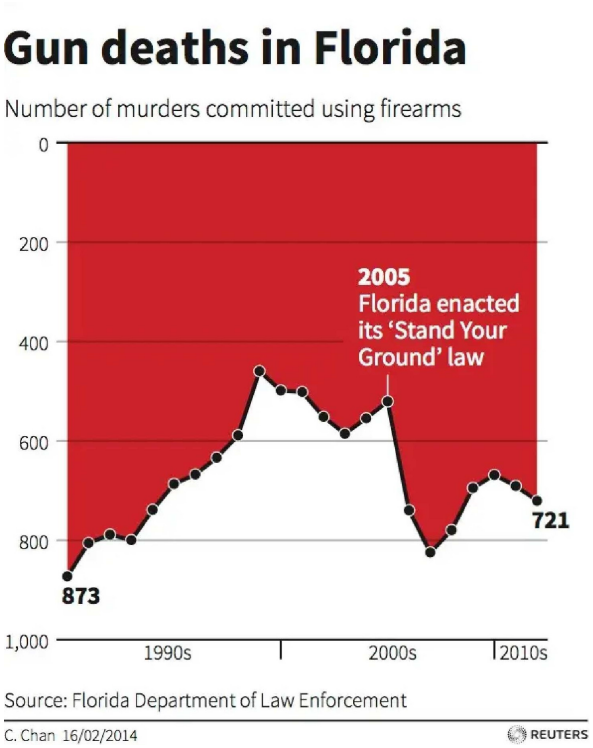
- Adjusting $y-$axis
- Using the wrong graphic
- Poor scaling or labeling
- Cherry picking

Example 0.3.3. Why would a pie chart be inappropriate for displaying the new home sales data?

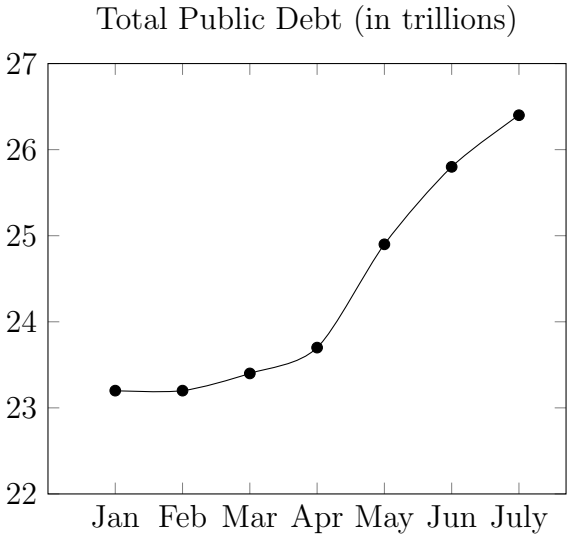
Example 0.3.4. Describe why the graph below might be considered misleading.



Example 0.3.5. Discuss your thoughts about the graphic below with one or two people near you. What do you notice? What do you believe is missing?



Example 0.3.6. The graph below shows the total amount of debt (in trillions) held by the US government between January and July 2020.



Do you think that the graph gives an accurate picture of the rise of national debt? Why or why not?

Measurement & Units

Measuring Distance

There are many ways of measuring distance. Here are a few common units of measurement for distance.

- Inches (in)
- Feet (ft)
- Yards (yd)
- Miles (mi)
- Centimeters (cm)
- Meters (m)
- Kilometers (km)

Example 0.4.1. Give an example of something that can be measured using each of the units above.

Relationships Between Units

- There are 12 inches in 1 foot
- There are 3 feet in 1 yard
- There are 5280 feet in 1 mile
- There are 100 centimeters in 1 meter
- There are 100 meters in 1 kilometer

Example 0.4.2. Determine an appropriate unit to measure the distance given below.

(a) Distance between Norman and Oklahoma City

(b) Height of a tall building

(c) Diameter of a cookie

(d) Length of a bug

Converting between units is very important to help us conceptualize and compute information.

Example 0.4.3. If there are 3.28 feet in 1 meter, convert 1 mile to 1 kilometer

Example 0.4.4. Convert your height from imperial units (feet and inches) to centimeters. Use the fact that there are 2.54 centimeters in 1 inch.

Example 0.4.5. The average distance between the Earth and the Sun is called an astronomical unit. If there are 149,600,000 km in 1 AU, how many miles is in 1 AU?

Measuring Area

While length measures one dimensional objects, area is a measure of two-dimensional objects.

Area Formulas for Common Shapes

- Rectangles: Length$\cdot$ Width
- Triangles: $\frac{1}{2}\cdot$ Length $\cdot$ Width
- Circles: $\pi\cdot$Radius^2

The units of area are similar to the units of length; we find the units for area by squaring the component length unit. Some common area units are:

- square inches (in^2)
- square miles (mi^2)
- square meters (m^2)

Example 0.4.6. Give an example of something that could be measured using each of the units above

Example 0.4.7. A new house has a 2.5 acre lot. What is the square footage of the lot? Use the fact that there are 3 feet in 1 yard, and 4,840 square yards in 1 acre.

Measuring Volume

Volume is a measure of three-dimensional objects, the three-dimensional analogue of area.

Volume Formulas for Common Shapes

- Rectangular prism: Length·Width·Height
- Sphere: $\frac{4\pi}{3}$·Radius^3

Example 0.4.8. A new piece of luggage has a carrying capacity of 5.2 cubic feet of storage space. How many cubic inches of space is there?

Liquid volume is often measured with special units. Some of these are:

- Milliliters (mL)
- Liters (L)
- Fluid Ounces (oz)
- Cups (c)
- Quarts (qt)
- Gallons (gal)

Relationships Between Units

- There are 1000 milliliters in 1 liter
- There are 8 fluid ounces in 1 cup
- There are 4 cups in 1 quart
- There are 4 quarts in 1 gallon

Example 0.4.9. How many fluid ounces are in 2 gallons?

Measuring Temperature

There are two primary units used to measure temperature: Fahrenheit and Celsius

Converting Between Fahrenheit and Celsius

To convert from Fahrenheit to Celsius, use the formula

$$^\circ C = (^\circ F - 32) \cdot \frac{5}{9}$$

To convert from Celsius to Fahrenheit, use the formula

$$^\circ F =^\circ C \cdot \frac{9}{5} + 32$$

A decent estimate to go from Celsius to Fahrenheit is to double the Celsius temperature, then add 32.

Example 0.4.10. The average high in Norman in October is $75^\circ F$. How high is that in Celsius?

Example 0.4.11. The 2022 Men's World Cup was moved from Summer 2022 to Winter 2022 because the average temperature in Qatar is about $41^\circ C$. About how hot is that in Fahrenheit?

Rates, Ratios, Proportions

Rates & Ratios

Definition 0.5.1 (Ratio)

A **ratio** is a comparison of amounts or quantities.

Example 0.5.2. The Dodge Family College of Arts and Sciences oversees 30 departments while the Gallogly College of Engineering has 9 departments. What is the ratio of Arts and Sciences departments to Engineering departments?

Ratios can be expressed several ways:

Ratio Expressions

- Simplified fractions in the form $\frac{a}{b}$
- With a colon in the form $a : b$
- Verbally in the form "a to b"

Example 0.5.3. Express the answer from the previous example in two other ways

Example 0.5.4. In Giada de Laurentiis' recipe for lasagna rolls, she calls for 15 ounces of ricotta and 1 cup of shredded mozzarella cheese. What is the ratio of mozzarella to ricotta? 1 dry cup is equal to 6.8 ounces.

Definition 0.5.5 (Rate)

A **rate** is a ratio comparing two items with different units

Example 0.5.6. Give five different examples of rates that you might see in the wild.

Example 0.5.7. On March 16, 2022, Shai Gilgeous-Alexander scored 14 field goals on 22 shot attempts against the San Antonio Spurs.

(a) What is the rate of field goals made to field goals attempted?

(b) When we perform the division represented by a rate, we find the **unit rate**. What was Gilgeous-Alexander's unit rate for shots attempted to shots made in this game? (In basketball, this unit rate is called the field goal percentage)

Example 0.5.8. According to Google Maps, the distance between Oklahoma State and the University of Oklahoma is 88.1 miles. Amy drove from OSU to OU in exactly 100 minutes. What the unit rate (in miles per hour) for her speed on the trip? (This unit rate is her average speed)

Example 0.5.9. At Walmart, a 12.4 oz box of Cheez-Its costs \$3.68 while a 21 oz box costs \$5.28. Which box is the most cost effective purchase? Why?

Proportions

Definition 0.5.10 (Proportion)

A **proportion** is an equation which relates two ratios.

Example 0.5.11. Write a proportion relating the number of centimeters in one meter to the number of centimeters in five meters.

Example 0.5.12. 25% of what number is 32?

Example 0.5.13. You have an irregularly sized photograph measuring 4 inches by 8 inches, but want to scale it up so that the larger side measures 20 inches. How long does the shorter side have to be to maintain the size ratio?