# Exponential Functions & Models

## Exponential Functions

As with the linear model, we have three descriptions of an exponential model:

- Algebraic: An exponential model has an equation of the form ____________________________

  The percentage change over one unit input is __________________________, and _____ is the *initial value*, the output corresponding to an input of zero.

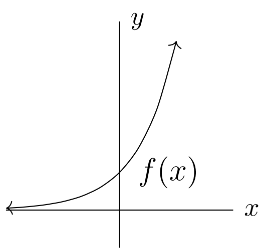
- Verbally: An exponential model has _____________________________________________________.

- Graphically: An exponential model will look like the pictures below.

## Exponential Models

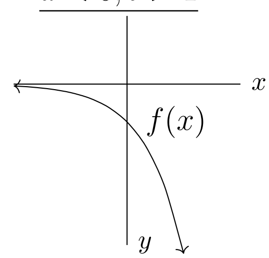
For exponential models, we have the following information:

$a > 0, b > 1$



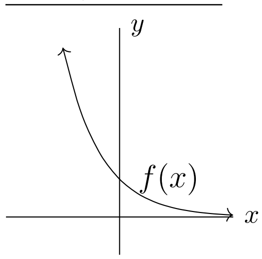

- $\lim_{x\to\infty} f(x) =$
- $\lim_{x\to-\infty} f(x) =$
- $f$ is always
- $f$ is concave

$a < 0, b > 1$




- $\lim_{x\to\infty} f(x) =$
- $\lim_{x\to-\infty} f(x) =$
- $f$ is always
- $f$ is concave

$a > 0, 0 < b < 1$




- $\lim_{x\to\infty} f(x) =$
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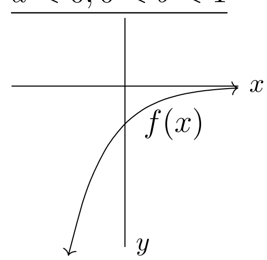
$a < 0, 0 < b < 1$




- $\lim_{x\to\infty} f(x) =$
- $\lim_{x\to-\infty} f(x) =$
- $f$ is always
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For us, an exponential model will always have an asymptote at ____________________.

## Formulas and Examples

There are two formulas which will be useful to memorize. For exponential models, we have a constant percent change; this is given above as

**Percent Change (Exponential)**

For *every other model*, we calculate the percent change between two input values $x_1, x_2$ as

**Percent Change (Other Models)**

**Example 1.5.1.** iPod sales were 7.68 million units in 2006, and increased by 9.1% each year between 2006 and 2008.

(a) Write an exponential model for this situation.

(b) Explain why the exponential model is best.

(c) Use the model to predict the number of iPods sold in 2010.

**Example 1.5.2.** The population of Northern cod in a certain body of water is given in the table below.

| **Decade** (since 1963) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Population** (in billions) | 1.72 | 0.63 | 0.24 | 0.085 | 0.032 |

(a) Identify which model (linear/exponential) is best for this data.

(b) Find the **complete** model.

(c) Find the percent change of the model.

**Example 1.5.3.** Early in the millennium, it was predicted that United States imports of petroleum products would be 4.81 quadrillion Btu, and increase by 5.47% each year through 2020.

(a) Find the associated exponential model.

(b) When will imports exceed 10 quadrillion Btu?

(c) Describe the end behavior of your model.

**Example 1.5.4.** According to the Social Security Advisory Board, the number of workers per beneficiary of the Social Security program was 3.3 in 1995 and is projected to decline by 1.46% each year until 2030.

(a) Write a model for the number of workers per beneficiary from 1995 through 2030.

(b) What does the model predict the number of workers per beneficiary will be in 2030?

**Example 1.5.5.** A social media website collected data on its users. Below are the users of a certain age and gender, as a percentage of total users.

| **Age** (years) | 27 | 29 | 31 | 33 | 35 | 37 | 39 | 41 | 43 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Females** (as %) | 9.6 | 7.8 | 6.1 | 5.1 | 4.3 | 3.8 | 2.4 | 2.1 | 1.2 | 1.1 |
| **Males** (as %) | 8.8 | 7.6 | 6.0 | 4.6 | 4.0 | 4.4 | 2.7 | 1.9 | 1.5 | 1.3 |

(a) Align the input data to the number of years after 27. Write an exponential model for the female user data.

(b) According to the model in part (a), what is the percentage change in your model? Write a sentence interpreting your answer.

(c) What percentage of female users are 30 years old? What about 48 years old? Are these interpolation or extrapolation?

(d) Write the exponential model for the male user data.

(e) According to your model in part (d), what is the percentage change in your model? Write a sentence interpreting your answer.

(f) What percentage of male users are 30 years old? What about 48 years old?

## Doubling Time and Half Life

**Definition 1.5.6** (Doubling Time)

For an exponential function $f$, the **doubling time** is defined to be the amount of time it takes an initial quantity to double (or grow by 100%).

**Definition 1.5.7** (Half Life)

For an exponential function $f$, the **half life** is defined to be the amount of time it takes an initial quantity to decay to half of its original size (or decrease by 50%).

**Example 1.5.8.** Albuterol is used to calm bronchospasm. It has a biological half-life of 7 hours and is normally inhaled as a 1.25 mg dose.

(a) Find a model for the amount of albuterol left in the body after an initial dose 1.25 mg.

(b) How much albuterol is left in the body after 24 hours?

**Example 1.5.9.** The amount of money Frank has in a particular investment is given by $f(t) = Pe^{.06t}$, where $P$ is the principal invested and $t$ is the amount of time (in years) the investment has been active.

(a) If Frank began the investment 15 years ago, and currently has \$25,500 in the account, what was the principal that he invested?

(b) If Frank currently has \$14,250 in the account and invested \$2,500 to start, how long as the investment been active?

(c) Compute the doubling time for an investment of \$1000.

(d) How long will it take an investment to *triple* instead of double?