Rate of Change of Product Functions

Product Rule

The *product rule* allows us to take derivatives of product functions, i.e. functions that look like $h(x) = f(x) \cdot g(x)$. The product rule is

Example 3.5.1. Find the derivative of $h(x)$, where $h(x) = [4(3^x)] \cdot [5x^2]$.

Example 3.5.2. Find the derivative of $g(y) = \dfrac{\ln(2y)}{1 + y^2}$.

Example 3.5.3. Compute the derivative of the function $g(x) = 3x^{-0.7} \cdot 5^x$

Example 3.5.4. Let $f(x) = 3\ln(2 + 5x)$, and $h(x) = \dfrac{1}{\ln x}$. Find $(f \cdot h)'(x)$.

Example 3.5.5. The profit from the supply of a certain commodity is modeled by $P(q) = 72qe^{-0.2q}$ dollars, when q units are produced.

(a) Write the complete rate of change model for profit.

(b) At what production level(s) is the rate of change of profit zero? If necessary, round your answer to two decimal places.

(c) What is profit at the production level for the value in part (b)?

Quotient Rule

Alternately, for a quotient function, we may employ the *quotient rule*. If $k(x) = \dfrac{f(x)}{g(x)}$, then

This can be remembered as

Example 3.5.6. Use the quotient rule to find the derivative of $g(y) = \dfrac{\ln(2y)}{1+y^2}$

Examples

Example 3.5.7. The production level at a plant manufacturing radios can be modeled as $f(x) = 10.54x^{0.5}(2 - 0.13x)^{0.3}$ thousand radios, where x thousand dollars has been spent on modernizing plant technology.

(a) Identify two functions g and h that, when multiplied, form the production model- one of the functions will need the chain rule.

(b) Using function notation, write the notation for the production model and for the rate of change of the production model.

(c) Write a model for the rate of change of production

Example 3.5.8. Write the product function and rate-of-change function for the given functions.

(a) $g(x) = 5x^2 - 3$ and $h(x) = 1.2^x$

(b) $g(x) = 6e^{-x} + \ln x$ and $h(x) = 4x^{2.1}$

Example 3.5.9. Find the derivatives of the following functions.

(a) $f(x) = (\ln x)e^x$

(b) $g(x) = (x+5)e^x$

(c) $t(x) = (5.7x^2 + 3.5x + 2.9)^3(3.8x^2 + 5.2x + 7)^{-2}$

(d) $f(x) = \dfrac{2x^3 + 3}{2.7x + 15}$

(e) $f(x) = (8x^2 + 13)\left(\dfrac{39}{1 + 15e^{-0.09x}}\right)$

(f) $f(x) = [\ln(15.7x^3)] \cdot [e^{15.7x^3}]$

(g) $f(x) = \dfrac{4(3^x)}{\sqrt{x}}$

(h) $f(x) = \dfrac{14x}{1 + 12.6e^{-0.73x}}$

Example 3.5.10. A store has determined that the number of cookies sold monthly is approximately $c(x) = 6250(0.929^x)$ cookies, where x is the average price of a cookie (in dollars).

(a) Write a model for revenue as a function of price.

(b) If each cookie costs the store \$1, write a model for profit as a function of price.

(c) Write the complete rate of change model for revenue as a function of price.

Example 3.5.11. Find the derivative of $t(x) = (6x^3)(\ln x)(e^{2x})$.