The Derivative, Algebraically

Definition 2.5.1 (Derivative (Algebraic Definition))

Let $f(x)$ be a function defined on the open interval (a, b), and $x \in (a, b)$. Then, the derivative of f at point x is given by the formula

Question 2.5.2 Why is this definition the same as the one in §2.4?

It is useful to remember a few things from algebra when doing these calculations:

- ______________________________
- ______________________________
- ______________________________

When we algebraically find the derivative of a function, there is a four-step process which makes the algebra much simpler, and the derivative easier to find. The steps are:

1. ____
2. ____
3. ____
4. ____

This is demonstrated below.

Example 2.5.3. Algebraically find the derivative of the function $f(x) = x^2$ using the four-step process.

Example 2.5.4. Algebraically find the derivative of the function $f(x) = 5x - 2$ using the four-step process.

Example 2.5.5. The time it takes an average athlete to swim 100 meters freestyle at age x years can be modeled as

$$t(x) = 0.181x^2 - 8.463x + 147.376 \text{ seconds}$$

(a) Calculate the swim time at age 13 to the nearest second.

(b) Use the algebraic method to develop a formula for the derivative of t (ie, find $t'(x)$).

(c) How quickly is the time to swim 100 meters freestyle changing for an average 13-year-old athlete? Round to the nearest hundredth and interpret the result.

(d) Compute the percent rate of change of swimmers' time at age 13, to the nearest tenth.

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Example 2.5.6. Algebraically determine the derivative of $f(t) = \frac{1}{2}t^2 - \frac{1}{3}$, and evaluate $\left.\frac{df}{dt}\right|_{t=1}$

Example 2.5.7. Algebraically determine the derivative of $k(r) = r^2 - 2r^3$, and evaluate $k'(0)$