Models in Finance

Definition 1.6.1 (Future Value/Present Value)

The **future value** of an investment/loan at time t is the sum of the prevent value and all accumulated interest; this is denoted F or FV. The **present value**, denoted $F(0) = P$ (principal) is the value "today", or at $t = 0$.

Simple Interest

Definition 1.6.2 (Simple Interest)

Simple interest is interest earned on _____ the rate (as a decimal) is

called the _____, or nominal rate.

We have two formulas for simple interest:

$$I(t) = Prt \text{ dollars}$$

$$F_s(t) = P(1 + rt) \text{ dollars}$$

where P is the _____, r is the _____, and t is the

_____.

Example 1.6.3. A family friend offers to loan you \$10,000 to cover your outlandishly high tuition this year. She wants to earn 5.75% interest on the loan.

(a) If you pay the loan back in 1 year, how much interest does the friend make?

(b) What about if you pay the loan back in 3 years?

(c) What about 4 months?

Example 1.6.4. I invest \$500 at 8.5%. How much is the investment worth in 5 years?

Discretely Compounding Interest

Definition 1.6.5 (Discretely Compounding Interest)

Discretely compounding interest is interest earned on the balance at discrete time intervals.

We have two formulas for discretely compounding interest:

$$I = \frac{r}{n}$$

$$F_d(t) = P\left(1+\frac{r}{n}\right)^{nt} \text{ dollars}$$

where P is the _____, r is the _____, t is the

_____, and n is the _____.

Example 1.6.6. You take out a \$16,750 loan for a new car. Find the value of the loan (assuming no payments were made) with:

(a) $r = 12.5\%$, monthly

(b) $r = 6.2\%$, $n = 12$

(c) $r = 12.5\%$, yearly

(d) $r = 3.79\%$, quarterly

(e) $r = 3.79\%$, $n = 6$

(f) $r = 7.2\%$, daily

Definition 1.6.7 (Annual Percentage Yield)

The **annual percentage yield** of an investment (also called the _____) gives the return on investment in one year. APY for discretely compounding interest is calculated with the formula

$$APY_{\text{D}} = \left[\left(1 + \frac{r}{n}\right)^n - 1\right] \cdot 100\%$$

Example 1.6.8. Calculate the APY for each of the situations from the last example. Round each to the nearest tenth:

(a) $r = 12.5\%$, monthly

(b) $r = 6.2\%$, $n = 12$

(c) $r = 12.5\%$, yearly

(d) $r = 3.79\%$, quarterly

(e) $r = 3.79\%$, $n = 6$

(f) $r = 7.2\%$, daily

Example 1.6.9. OU Federal Credit Union offers an APR of 6.35% (compounded monthly) for an investment opportunity, while First Fidelity offers you an APY of 5.95%. Which option will give the highest return after one year?

Continuously Compounding Interest

Definition 1.6.10 (Continuously Compounding Interest)

Interest earned on the balance at any given time t is called **continuously compounding interest**, and has the future value formula given by

$$F_c(t) = Pe^{rt} \text{ dollars}$$

where P is the principal, r is the rate, and t is the time.

We also have a formula for the APY of continuously compounding interest:

$$APY_{\text{C}} = (e^r - 1) \cdot 100\%$$

Example 1.6.11. Determine the amount that must be invested in the following situations to get \$7000 payable in 4 years:

(a) 3% APR, compounded continuously

(b) 3.9% APR, compounded monthly

(c) 15.1% APR, simple interest

(d) 10% APR, compounded weekly.

Example 1.6.12. Find the APY for the examples above, rounding to the nearest hundredth.

(a) 3% APR, compounded continuously

(b) 3.9% APR, compounded monthly

(c) 15.1% APR, simple interest

(d) 10% APR, compounded weekly.