Constructed Functions

Definitions

Definition 1.7.1 (Fixed Cost)

A **fixed cost** is a cost which remains the same, no matter how much of a product is produced.

Definition 1.7.2 (Variable Cost)

A **variable cost** is a cost which changes depending on the number of units produced.

Definition 1.7.3 (Total Cost)

The **total cost** is the sum of the fixed cost and variable cost.

Definition 1.7.4 (Revenue)

Revenue is the _____

_____.

Definition 1.7.5 (Profit)

Profit is _____.

Definition 1.7.6 (Break-Even Point)

The **break-even point** is the point when _____

_____.

Function Operations

There are five operations which we will need to be familiar with in order to move on.

- **Addition**: _____, *if the output units of f and g are*

  _____.

- **Subtraction**: _____, *if the output units of f and g are*

  _____.

- **Multiplication**: _____, *if the output units of f and g are*

  _____.

- **Division**: _____, *if the output units of f and g are*

  _____.

- **Composition**: _____, *if* _____

  _____.

Addition creates total cost from fixed and variable costs by adding the two; profit is created using subtraction. Variable cost (and revenue) are created by multiplication, and division gives us average cost $\overline{C}$.

Examples

Example 1.7.7. The number of student tickets sold for a home basketball game at OU is represented by $S(w)$ hundred tickets when w is the winning percentage of the team. The number of non student tickets sold for the same game is represented by $N(w)$ hundred tickets where w is the winning percentage of the team. Combine the functions to construct a new function giving the total number of tickets sold for a home basketball game at OU.

Example 1.7.8. Sales of 12-ounce bottles of sparkling water are modeled as $D(x) = 287.411(0.266^x)$ million bottles, when the price is x dollars per bottle. Write a model for the revenue from the sale of 12-ounce bottles of sparkling water.

Example 1.7.9. The profit from the supply of a certain commodity is modeled as $P(q) = 30 + 60 \ln q$ thousand dollars, where q is the number of units produced in millions. Write a model for the average profit when q units are produced.

Example 1.7.10. A travel agency offers spring break cruise packages. The agency advertises a cruise to Cancun for \$1200 per person. To promote the cruise among student organizations on campus, the agency offers a discount for student groups selling the cruise to over 50 of their members. The price per student will be discounted by \$10 for each student in excess of 50 (for example, if an organization had 55 members go on the cruise, each of those students would pay \$1150). Write a model for the travel agency's revenue that depends on the number of students from a student organization.

Example 1.7.11. The sales of a certain brand of backpack is modeled by $f(s) = 1.56s + 4.3$ million dollars, when s is the number of stores that sell the brand of backpack. The number of stores that sell the brand of backpack is modeled by $s(t) = 3t + 5.4$ stores, t months since the beginning of 2000. Write a model for the sales of a certain brand of backpack with respect to time.

Example 1.7.12.   The level of contamination in groundsoil is $f(p) = \sqrt{p}$ parts per million when the population of the surrounding community if p people. The population of the surrounding community in year t is modeled as $p(t) = 400t^2 + 2500$ people, t years since 2000.

(a) Why can we use function composition?

(b) Find a model for the contamination of the groundsoil.

Example 1.7.13.   It costs a company \$19.50 to produce 150 glass bottles. Write a model for $\overline{C}(q)$, the average cost of producing a bottle when q units are produced.

Example 1.7.14. Write the following functions as composite functions, and then evaluate the composite at an input of 2.

(a) $f(t) = 3e^t$, $t(p) = 4p^2$

(b) $h(p) = \dfrac{4}{p}$, $p(t) = 1 + 3e^{-0.5t}$

(c) $g(x) = \sqrt{7x^2}$, $x(w) = 4e^w$

(d) $c(x) = 3x^2 - 2x + 5$, $x(t) = 2e^t$