Rates of Change: Numerical Limits & Nonexistence

Derivative: Numerical Definition

Let a be fixed, and let x be some point on f other than a. Then, the slope of the secant line is given by

Taking limits, we have the definition for the derivative of f at a point a:

Example 2.4.1. Find the derivative of the function $f(x) = \sqrt{2x}$ at $x = 3$ using the numerical method. Round your final answer to the thousandths place, if necessary.

x	$\dfrac{f(x)-f(3)}{x-3}$	x	$\dfrac{f(x)-f(3)}{x-3}$
2.9		3.1	
2.99		3.01	
2.999		3.001	
2.9999		3.0001	
2.99999		3.00001	
$f'(3) \approx$		$f'(3) \approx$	

$f'(3) \approx$ _____

Example 2.4.2. A multinational corporation invests \$32 billion in assets, resulting in the future value $F(t) = 32(1.12^t)$ billion dollars after t years.

(a) By how much is the investment growing in the fourth year? Write a sentence interpreting your answer, and round to the nearest hundredth.

t	$\dfrac{F(t)-F(4)}{t-4}$	t	$\dfrac{F(t)-F(4)}{t-4}$
$\lim\limits_{x\to 4^-} \dfrac{F(t)-F(4)}{t-4}$		$\lim\limits_{x\to 4^+} \dfrac{F(t)-F(4)}{t-4}$	

$$F'(4) \approx \_\_\_\_\_$$

(b) Find the percent rate of change in the fourth year. Round to 2 decimal places.

Derivative: Existence

The derivative of a function does not always exist; the definition requires that the function be smooth and continuous. Formally, we say that a function is *differentiable* when the derivative exists for all x in some interval (a, b). We have three cases for nonexistence:

- _____
- _____
- _____

Exercises

Example 2.4.3. Numerically estimate the derivative of the function $f(x) = -x^2 + 4x$ at $x = -1$. Round your final answer to the nearest tenth.

x		x	

Example 2.4.4. Numerically estimate the derivative of the function $g(y) = 5 \ln y$ at $x = 5$. Round your final answer to the nearest hundredth.

x		x	

Example 2.4.5. The annual number of passengers going through the Atlanta airport between 2000 and 2008 can be modeled as $p(t) = -0.102t^3 + 1.39t^2 - 3.29t + 79.25$ million passengers, t years since 2000.

(a) Estimate $p'(6)$ numerically to the nearest thousandth.

t		t	

(b) Write an interpretation of $p'(6)$.

(c) Find the percent rate of change in 2006, to the nearest hundredth.

Example 2.4.6. The average weekly sales (in million dollars) for Abercrombie & Fitch between 2004 and 2008 is given in the table below.

Year	2004	2005	2006	2007	2008
Sales (in million dollars)	38.87	53.56	63.81	72.12	68.08

(a) Align the data so that the year 2000 corresponds to an input of 0. Determine and write the most appropriate model for the data using this alignment.

(b) Estimate the rate of change of average weekly sales in the year 2007 and interpret your answer.

x		x	