Rate of Change of Composite Functions

Review: Composite Functions

Two functions $f(x)$ and $x(t)$ can be composed *if and only if* the output of $x(t)$ is the input of $f(x)$. Notice how the notation is suggestive; f inputs x, which is exactly what $x(t)$ outputs. We write the composition either as $(f \circ x)(t)$ or $f(x(t))$. The new input is now the input of x (ie, t), and the new output is the output of f (namely, f).

Example 3.3.1. Identify the functions which make up the composite functions given below.

(a) $f(x) = \dfrac{1}{x+2}$

(b) $g(x) = \ln(x^2)$

(c) $h(t) = e^{5t}$

(d) $q(x) = (2x+1)^5$

(e) $n(f) = \left(3 + \dfrac{1}{f}\right)^3$

(f) $s(h) = \ln\left(5h^2 + \frac{1}{h}\right)$

(g) $y(r) = \frac{5.317}{(2r^5 + 1.7)^2}$

(h) $w(c) = \sqrt[3]{\frac{c}{1+c}}$

(i) $f(x) = 1 - \sqrt{e^x + 5x}$

The Chain Rule

The *chain rule* is a rule for finding the derivative of composite functions. Let $h(x) = f(g(x))$, where the output of g is the input of f. Then,

The best way to learn the chain rule is with practice **inside and outside of class**.

Examples

Example 3.3.2. For $f(t) = 3t^2$ and $t(x) = 4 + 7\ln x$, find the rate of change function $(f \circ t)'(x)$ with respect to x.

Example 3.3.3. Let $c(x) = 3x^2 - 2$ and $x(t) = 4 - 6t$. Find $c'(t)$

Example 3.3.4. Consider the following functions:

$$f(g) = \ln g \qquad g(h) = 5h + 2 \qquad h(j) = e^j \qquad j(x) = 4x^{-1}$$

Find $f(x)$ and $f'(x)$.

Example 3.3.5. Find the derivative of $f(x) = \dfrac{1}{x+2}$

Example 3.3.6. Find the derivative of $f(x) = \ln(x^2)$

Example 3.3.7. Find the derivative of $f(x) = (\ln x)^3$

Example 3.3.8. Find the derivative of $f(x) = e^{5x}$

Example 3.3.9. Find the derivative of $f(x) = (e^x)^4$

Example 3.3.10. Find the derivative of $f(x) = 7 + 5\ln(4x^2 + 3)$

Example 3.3.11. If $s(t) = 3e^{5t}$, find $s'(t)$

Example 3.3.12. Find the derivative of $k(x) = 3e^{4x^2}$

Example 3.3.13. Find the derivative of $p(t) = (5 + 6e^{2t})^3$

Example 3.3.14. $f(x) = 6(4x^2 + 3)^5$

Example 3.3.15. $f(x) = -12\ln(6x^2 + 3^x)$

Example 3.3.16. $f(x) = 2e^{0.5x} - 2x$

Example 3.3.17. $f(x) = \dfrac{7.2}{(4x^3+1)^4}$

Example 3.3.18. $f(x) = 3\sqrt{x^3 + 2\ln x}$

Example 3.3.19. Find the derivative of $f(x) = e^{kx}$

Example 3.3.20. Compute the derivative of $e^{f(x)}$

Example 3.3.21. Find the derivative of the function $\dfrac{1.356}{1 + 20.5e^{-4.6t}}$

Example 3.3.22. Compute the derivative of $j(x) = \ln(\ln(\ln(x^2 - e^{3x})))$

Example 3.3.23. The number of children under 18 living in households headed by a grandparent can be modeled as

$$p(t) = 2.111e^{0.04t} \quad \text{million children}$$

where t is the number of years since 1980.

(a) Write the rate-of-change formula for p.

(b) How rapidly was the number of children living with their grandparents growing in 2010?

Example 3.3.24. The tuition x years from now at OU is projected to be $t(x) = 24072e^{0.056x}$ dollars.

(a) Write the rate-of-change formula for tuition.

(b) What is the rate of change in tuition four years from now?