Inflection Points & Second Derivatives

Definitions

Definition 4.4.2 (Inflection Point)

An **inflection point** is the point on the graph of a function where _____

_____.

An inflection point gives the point of _____ change of the function.

Theorem 4.4.3 (Properties of the Second Derivative)

Let $f(x)$ be a twice-differentiable function defined on input interval $[a, b]$, with $a < c < b$.

- f has an inflection point at $x = c$ if and only if _____.
- If $f''(c)$ is positive, then _____.
- If $f''(c)$ is positive, then _____.
- If $f''(c)$ is negative, then _____.
- If $f''(c)$ is negative, then _____.

Theorem 4.4.4 (Second Derivative Test)

Let $f(x)$ be a twice-differentiable function defined on input interval $[a, b]$, with $a < c < b$.

- _____.
- _____.

Examples

Example 4.4.5. The percentage of people living in California in 2007 who were born in the state can be modeled as

$$P(x) = -0.0016x^3 + 0.224x^2 - 10.577x + 204.8 \text{ percent}$$

where x is the age of the resident.

(a) Find the inflection point of the function P.

(b) Give a sentence of interpretation for the age between 20 and 70 at which the percentage of California residents who were born in the state was decreasing least rapidly.

Example 4.4.6. For the function

$$f(t) = -2.1t^2 + 7t$$

(a) Write the first and second derivative.

(b) Identify any inflection points, and label them as the point of *least rapid* or *most rapid* change.

Example 4.4.7. The percentage of new material that an average college student will retain after studying for t hours without a break can be modeled as

$$p(t) = \frac{83}{1 + 5.94e^{-0.969t}} \text{ percent}$$

(a) Find when the retention rate is increasing most rapidly.

(b) Determine the rate of change of retention as well as the percentage of retention at the input found in item (a).

(c) Describe the difference between the direction of p and p' to the right of the input found in item (a).

(d) Explain what happens to the student's retention rate after the input found in item (a).

Example 4.4.8. For the function

$$g(s) = 32s^3 + 2.1s^2 + 7s$$

(a) Write the first and second derivative.

(b) Identify any inflection points, and label them as the point of *least rapid* or *most rapid* change.

Example 4.4.9. For the function

$$h(x) = e^{3x} - \ln 3x$$

(a) Write the first and second derivative.

(b) Identify any inflection points, and label them as the point of *least rapid* or *most rapid* change.

Example 4.4.10. For the function

$$k(t) = \frac{16}{1 + 2.1e^{3.9t}}$$

(a) Write the first and second derivative.

(b) Identify any inflection points, and label them as the point of *least rapid* or *most rapid* change.

Example 4.4.11. For the function

$$f(x) = -x^3 + 12x^2 + 36x + 45$$

(a) Write the first and second derivative.

(b) Identify any inflection points, and label them as the point of *least rapid* or *most rapid* change.

Example 4.4.12. The table below shows the monthly revenue levels associated with various monthly levels of advertising by a furniture store.

Advertising (in hundreds of dollars)	1	4	7	10	13	16	19
Revenue (in thousands of dollars)	114	210	265	299	338	449	632

(a) Find the **complete** cubic model $R(x)$ for the data (do not align the input).

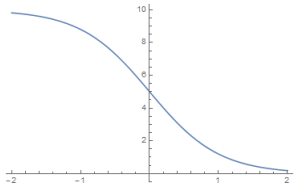
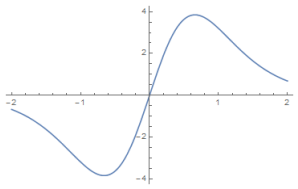
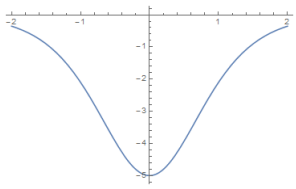
(b) Write the **complete** rate of change model for $R(x)$.

(c) Find $R''(x)$.

(d) Find the inflection point of $R(x)$ on the interval $[0, 19]$. Round both coordinates to the hundredths place and be sure to label them with units.

(e) Find the rate of change at the inflection point. Round to the hundredths place and include units in your answer.

Example 4.4.13. Consider the following graphs:

A  **B**  **C** 

(a) Write "True" or "False" to the left of the following statements:

_____ The graph of the derivative of **A** will have no $x-$intercepts.

_____ The graph of the derivative of **B** will have exactly one $x-$intercept.

_____ The graph of the second derivative of **C** will have exactly two $x-$intercepts.

_____ The graph of the second derivative of **A** will always be negative.

(b) Which of the following describes the relationship between these three graphs? *Mark an X to the left of your choice.*

_____ **B** is $f(x)$, **C** is $f'(x)$, and **A** is $f''(x)$.

_____ **A** is $f(x)$, **C** is $f'(x)$, and **B** is $f''(x)$.

_____ **C** is $f(x)$, **A** is $f'(x)$, and **B** is $f''(x)$.

_____ **B** is $f(x)$, **A** is $f'(x)$, and **C** is $f''(x)$.