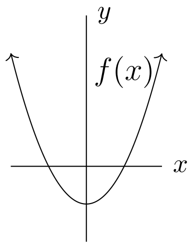
Quadratic Functions & Models

Quadratic Models

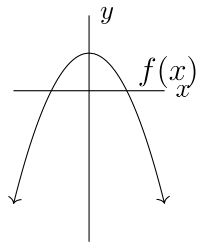
The model is given by $f(x) = ax^2 + bx + c$, where a, b, c are constants ($a \neq 0$). The function has an absolute maximum if $a < 0$, and absolute minimum if $a > 0$.

$\underline{a > 0}$



- $\lim_{x\to\infty} f(x) =$
- $\lim_{x\to-\infty} f(x) =$
- f is decreasing then increasing
- f is concave up

$\underline{a < 0}$



- $\lim_{x\to\infty} f(x) =$
- $\lim_{x\to-\infty} f(x) =$
- f is increasing then decreasing
- f is concave up then down

Choosing Models

At this point, we have five models to choose from when analyzing a data set. The process of choosing a model should go as follows:

- Does the scatterplot show any sort of concavity? If yes, then go to the next step. If not, try a **linear** model.
- If the scatterplot shows concavity, does it appear to *change concavity*? If yes, then the model could be **logistic** or **cubic**. If not, then the model could be **exponential**, **logarithmic**, or **quadratic**.
  1. If the scatterplot changes concavity, then does it have an asymptote? If yes, then the model is **logistic**. If no, then the model is **cubic**.
  2. If the scatterplot does not change concavity, then look at the end behavior and for asymptotes. If there is an asymptote at $x = 0$, then the model is **logarithmic**; if it is at $y = 0$, then the model is **exponential**; if there is no asymptote, then it is **quadratic**.
- If it is still difficult to determine between exponential and quadratic, then use the method of second differences (described below). If second differences gives roughly constant values, then the model is **quadratic**; if it does not, then, it is **exponential**.
- If in doubt, one can develop multiple models and compare the fit of each model against the data.
- It is never a bad idea to apply common sense to models.

Example 1.9.1. Draw a decision tree/diagram for choosing a model.

Example 1.9.2. The table below shows the profit (in millions of dollars) that American Airlines makes on tickets between Dallas and Chicago when tickets are set at a certain price:

Ticket Price (dollars)	200	250	300	350	400	450
Profit (million dollars)	3.08	3.52	3.76	3.82	3.7	3.38

(a) Give two reasons why a quadratic model is more appropriate than a log or exponential model.

(b) Find a quadratic model for the data.

(c) Why doesn't the airline profit increase as the ticket price increases?

(d) At what price does the airline begin posting a loss?

Example 1.9.3. The table below gives the braking distance required for a vehicle to come to a complete stop, given the initial velocity of the vehicle.

Speed (mph)	10	20	30	40	50	60	70	80	90
Distance (feet)	27	63	109	164	229	304	388	481	584

(a) Find the second differences of the data above.

(b) Find a quadratic model for stopping distance.

(c) What other factors besides the initial speed would impact the stopping distance?

(d) What speed is the vehicle moving if its braking distance is exactly 412 feet? Round your answer to two decimal places, if needed.

Example 1.9.4. The ratios of public school students to instructional computers with Internet access for years between 1998 and 2004 are given below:

Year	1998	1999	2000	2001	2002	2003	2004
Ratio	9.1	6.1	3.6	2.4	1.8	1.4	1.8

(a) Align the input so that 1998 corresponds to an input of 0.

(b) Write the complete quadratic model for the data.

(c) Write the complete exponential model for the data.

(d) Which model best fits the data: (b) or (c)?

(e) Give two reasons why an exponential model might be best for this data.