# Linear Functions & Models

## Linear Functions

Remember that a linear function requires two pieces of information- a starting value ($b$, the $y$-intercept), and an amount of incremental change in the independent variable ($m$, the slope of the function). This gives us three ways to describe a linear function:

- Verbally:

- Graphically:

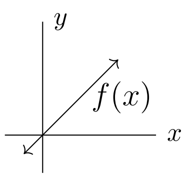
- Algebraically:

**Question 1.4.1** Given two points $(x_1, y_1)$ and $(x_2, y_2)$, how can we find the slope of the line between them?

## Linear Models

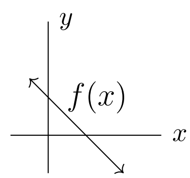
For our general model, $f(x) = ax + b$, we have the following characteristics:

$a > 0$



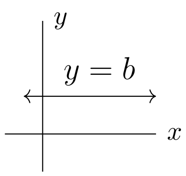

- $\lim_{x\to\infty} f(x) =$
- $\lim_{x\to-\infty} f(x) =$
- $f$ is always
- $f$ has no concavity

$a < 0$



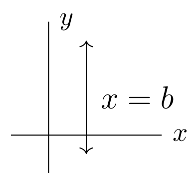

- $\lim_{x\to\infty} f(x) =$
- $\lim_{x\to-\infty} f(x) =$
- $f$ is always
- $f$ has no concavity

$a = 0$




- $\lim_{x\to\infty} f(x) =$
- $\lim_{x\to-\infty} f(x) =$
- $f$ is always
- $f$ has no concavity

$a$ undefined




- $\lim_{x\to\infty} f(x) =$
- $\lim_{x\to-\infty} f(x) =$
- Neither inc. nor dec.
- No concavity

For any given graph, the scales ***will*** change; use algebra, don't trust your eyes.

## Elements of a Model

From now on, when we refer to a model, we are referring to a specific collection of information. These pieces are listed below; *memorize them*!

(1) Proper and consistent function notation

(2) Model coefficients rounded to **three** decimal places

(3) Output units

(4) Output description

(5) Input units

(6) Input description

**Example 1.4.2.** The following table gives the percentage of new companies which remained open $t$ years after beginning business.

| **Years After Opening** | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|
| **Companies Still Open** (in %) | 50 | 47 | 44 | 41 | 38 | 35 |

(a) Fill in the new inputs if we align the data so that the fifth year corresponds to an input of zero.

| **Years After Opening** | | | | | | |
|---|---|---|---|---|---|---|
| **Companies Still Open** (in %) | 50 | 47 | 44 | 41 | 38 | 35 |

(b) Use the aligned data to create a **complete** model.

**Definition 1.4.3** (Extrapolation)

When using a model, we say that data is **extrapolated** if we find an output value

_______________________________________________.

**Definition 1.4.4** (Interpolation)

When using a model, we say that data is **interpolated** if we find an output value

_______________________________________________.

**Example 1.4.5.** In the example above, predict the number of companies open in the twelfth year of operation. Is this extrapolation or interpolation?

**Example 1.4.6.** Do the same, but after 8.5 years after opening. Is this extrapolation or interpolation?

**Example 1.4.7.** The amount of electricity sold by a power company in year $x$ is given below.

| **Year** | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 |
|---|---|---|---|---|---|---|
| **Retail Sales** (in quadrillion kWh) | 1.2 | 1.23 | 1.27 | 1.3 | 1.33 | 1.35 |

(a) Find a **complete** linear model to fit the data.

(b) Write an interpretation the slope of the linear model.

(c) When did retail sales first exceed 1.4 quadrillion kWh? Is this interpolation or extrapolation?

## Data Alignment

When using an input value of years, alignment should (usually) happen so that the first year given corresponds to an input of zero.

**Example 1.4.8.** Find the **complete** linear model to fit the data of the previous example, aligning the input so that the year 2003 corresponds to an input of zero.

## Numerical Considerations

Since numerical approximations can vary, we will use the following guidelines:

(1) Use common sense; if a model outputs something like "2.5 people", we would round to 3 people.

(2) The accuracy of the output **must** be the same as the original model's accuracy.

(3) All answers **must** have proper units; answers without labels are useless.

(4) If arriving at your answer requires multiple steps, **do not** round until the *final* answer.

**Example 1.4.9.** The world's daily demand of oil was recorded in various years, and is listed below.

| **Year** | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
|---|---|---|---|---|---|---|
| **Oil Demand** (in million barrels) | 82.327 | 83.652 | 84.622 | 85.385 | 86.384 | 87.698 |

(a) Based on the scatterplot, why is a linear model best?

(b) Align the data so that the year 2000 corresponds to an input of 0, and find the **complete** linear model.

(c) Estimate the demand in the year 2015.

**Example 1.4.10.** Expenditure on pets in the United States was recorded over the span of several years, and is recorded in the table below.

| **Year** | 1994 | 1996 | 1998 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Expenditure** (billion USD) | 17 | 21 | 23 | 28.5 | 29.5 | 32.4 | 34.4 | 36.3 | 38.5 | 41.2 | 43.4 |

(a) Align the data so that the year 1994 corresponds to an input of zero, and find the **complete** linear model.

(b) Use the model to estimate the expenditure in the year 2013.

**Example 1.4.11.** The number of successful tax audits performed by a company between 2000 and 2006 can be modeled by $A(t) = -83.9t + 1063$ audits, where $t$ is the number of years since 2000.

(a) Give the rate of change of $A$. Include units.

(b) Evaluate $A(0)$. Write a sentence interpreting your answer.

(c) Find the number of successful audits in 2005. Is this interpolation or extrapolation?

(d) Find the number of successful audits in 2010. is this interpolation or extrapolation?

**Example 1.4.12.** The population of a town in selected years is given below.

| **Year** | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|
| **Population** (in thousands) | 125.2 | 128.7 | 132.4 | 136.0 | 139.8 | 143.6 |

(a) Find a **complete** model for the population $P$ of the town in year $y$.

(b) According to your model, what is the constant rate of change of the population of the town?

(c) Use your model to predict the population of the town in 2015.

**Example 1.4.13.** Honda engineers are designing a new car, and are measuring the distance it takes the car to come to a complete stop on dry pavement. Their measurements are given below.

| **Speed** (mph) | 55 | 60 | 65 | 70 | 75 |
|---|---|---|---|---|---|
| **Distance** (ft) | 77.6 | 131.4 | 186.3 | 236.7 | 289.3 |

(a) Find a **complete** model for the braking distance of the car.

(b) Use your model to find the braking distance needed when the car is traveling at 77 miles per hour; write your answer using function notation.

(c) Find another **complete** model, aligning the data so that a speed of 50 mph corresponds to an input of 0.

(d) Repeat part (b).

(e) How fast is the car traveling if it requires 156 ft to come to a complete stop?