Exponential & Logarithmic Rate of Change Formulas

The Formulas

For exponential and logarithmic functions, we have the following formulas:

Name	Function	Derivative
General Exponential Rule	$f(x) = b^x$	$f'(x) =$
Exponential Rule	$f(x) = e^x$	$f'(x) =$
Logarithm Rule	$f(x) = \ln x$	$f'(x) =$

Examples

Example 3.2.1. Write the formula for the derivative of the function.

(a) $h(x) = 3 - 7e^x$

(b) $f(x) = 6(0.8)^x$

(c) $f(a) = 10\left(1 + \dfrac{0.05}{4}\right)^{4a}$

(d) $g(x) = 4\ln x - e^{\pi}$

(e) $f(x) = 3.7e^x - 2\ln x$

(f) $y(x) = -\ln x + 2e^x$

(g) $f(g) = 4\sqrt{g} + 5(1.2)^g$

(h) $k(t) = P\left(1 + \frac{r}{n}\right)^{nt}$

Example 3.2.2. For the first two hours after yeast dough has been kneaded, it doubles in volume approximately every 42 minutes. If 1 quart of yeast dough is left to rise in a warm room, its growth can be modeled as $v(h) = e^h$ quarts, where h is the number of hours the dough has been allowed to rise.

(a) How many minutes will it take the dough to attain a volume of 2.5 quarts?

(b) Write a model for the rate of growth of the yeast dough.

Example 3.2.3. The weight of a laboratory mouse between 3 and 11 weeks of age can be modeled as $w(t) = 11.3 + 7.37 \ln t$ grams, where the age of the mouse is $t + 2$ weeks.

(a) What is the weight of a 9-week-old mouse? Round to the nearest hundredth.

(b) Write a rate of change model for the weight of the mouse, and determine how rapidly its weight is changing at 9 weeks.

(c) What is the average rate of change in the weight of the mouse between ages 7 and 11 weeks? Round to the nearest hundredth.

(d) Does the rate at which the mouse is growing increase or decrease as the mouse gets older? Why?

Example 3.2.4. Suppose the managers of a dairy company have modeled weekly production costs as $c(u) = 3250 + 75 \ln u$ dollars for u units of dairy products. Weekly shipping cost for u units is given by $s(u) = 50u + 1500$ dollars.

(a) Write the formula for the total weekly cost of production and shipping of u units.

(b) Write the rate of change model of the total weekly cost of producing and shipping u units.

(c) Calculate the total cost to produce and ship 5000 units in 1 week.

(d) Calculate and interpret the rate of change in the total cost to produce and ship 5000 units in 1 week.

Example 3.2.5. An individual has \$45,000 to invest. \$32,000 will be put into a low-risk mutual fund averaging 6.2% interest compounded monthly, and the remainder will be invested in a high-yield bond fund averaging 9.7% interest, compounded continuously.

(a) Write an equation for the total amount in the two investments, using $I(t)$ as your function.

(b) Write the rate of change model for the low-risk fund, using $L(t)$ as your function.

(c) Write the rate of change model for the high-yield fund, using $H(t)$ as your function.

(d) Write the rate of change model for the combined investment.

(e) Calculate and interpret $\frac{dI}{dt}$ after 8 months, and after 18 months.