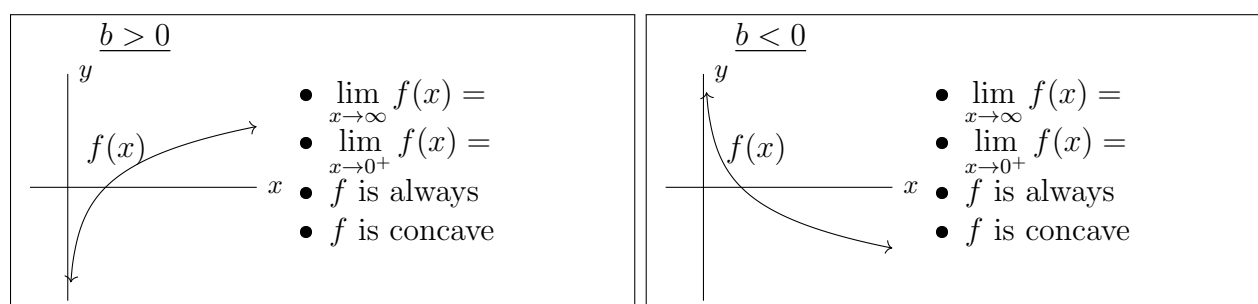# Logarithmic Functions & Models

Logarithmic Functions

We have the following descriptions for a logarithmic function:

- Algebraically: A logarithmic model has an equation of the form _____ , where $a, b \neq 0$ are constants, and $x > 0$.

- Verbally: A log function has a _____ at $x = 0$, and continues to grow (or decay) as x increases without bound.

- Graphically: The graph of a log model takes a form as below.

Logarithmic Models

$b > 0$		$b < 0$	
	• $\lim\limits_{x \to \infty} f(x) =$ • $\lim\limits_{x \to 0^+} f(x) =$ • f is always • f is concave		• $\lim\limits_{x \to \infty} f(x) =$ • $\lim\limits_{x \to 0^+} f(x) =$ • f is always • f is concave

Logarithmic Behavior

Example 1.8.1.   The percentage of viewers that have watched a DVR'd show before a certain number of days have passed is give in the table below.

Time (in days)	1	2	3	4	5	6	7	8
Viewers (in %)	46	62	76	84	91	95	98	100

(a) Why is a logarithmic model best here? Use a scatterplot to help you develop your reasons.

(b) Find the model corresponding to your answer in part (a). Write the complete model.

(c) Explain why the exponential model does not work.

Example 1.8.2. The average length of the ears of men after a certain age is given in the table below.

Age (in years)	0	20	70
Ear Length (in inches)	2.04	2.55	3.07

(a) Find the complete logarithmic model for the data. Do you encounter any problems?

(b) Align the data so that age 0 corresponds to an input of 10, and find the complete logarithmic model for the data.

(c) Use the model to find the ear length of a 45 year old man.

Example 1.8.3. The table below shows the life expectancy for women in Ireland between 1945 and 2011

Year	1945	1955	1965	1975	1985	1995	2005	2011
Expectancy (years)	65.2	71.1	73.1	74.7	77.4	78.8	79.3	80.2

(a) Align the data so that 1940 corresponds to an input of zero.

(b) What type of concavity does the scatter plot suggest?

(c) Describe the end behavior suggested by the scatterplot as the input increases without bound.

(d) Find the complete logarithmic model for the data.

(e) Using your model, find the year in which the life expectancy for Irish women was exactly 76.3 years.

(f) What was the life expectancy of an Irish woman in the year 1979?