

Rates of Change: Notation & Interpretation

Average ROC vs. Instantaneous ROC

Average Rate of Change

- _____
- _____

(Instantaneous) Rate of Change

- _____
- _____
- _____

Notation and Terminology

Rate of change at a specific point a is often referred to as any of the following (given a function f):

- _____
- _____
- _____
- _____

We also have two different notations for the derivative of f at point a :

- $f'(a)$. This is read “ f prime of a ”.
- $\left. \frac{df}{dx} \right|_{x=a}$. This is read “d-f d-x, evaluated at a ”, or as “the derivative of f with respect to x , evaluated at a ”.

NOTE: We will freely interchange between *any* of the above terminology or notations.

Examples

Example 2.3.1. The function f gives weekly profit, in thousands of dollars, that an airline makes on flights from Boston to Washington, D.C. when the ticket price is p dollars. Write a sentence interpreting the following:

(a) $f(65) = 15$

(b) $f'(65) = 1.5$

(c) $f'(90) = -2$

Example 2.3.2. The function C gives the number of bushels of corn produced on a tract of farmland that is treated with f pounds of nitrogen per acre.

(a) Is it possible for $C(90)$ to be negative? Why?

(b) What are the units of $\left. \frac{dC}{df} \right|_{f=90}$?

(c) Is it possible for $\left. \frac{dC}{df} \right|_{f=90}$ to be negative? Why?

(d) Give an alternate notation for the statement $\left. \frac{dC}{df} \right|_{f=90}$.

Example 2.3.3. Sketch a possible graph of $t(x)$, given that:

- $t(3) = 7$
- $t(4.4) = t(8) = 0$
- $t'(6.2) = 0$
- t has no change in concavity

Example 2.3.4. The function w gives a certain Business Calculus instructor's weight (in pounds) t weeks after he begins a diet. Write a sentence of interpretation for each of the following statements:

(a) $w(0) = 180$ and $w(12) = 165$

(b) $w'(1) = -2$ and $w'(9) = -1$

(c) $\left. \frac{dw}{dt} \right|_{t=12} = 0$ and $\left. \frac{dw}{dt} \right|_{t=15} = 0.25$

Example 2.3.5. Sketch a possible graph of the function m with input t , given that

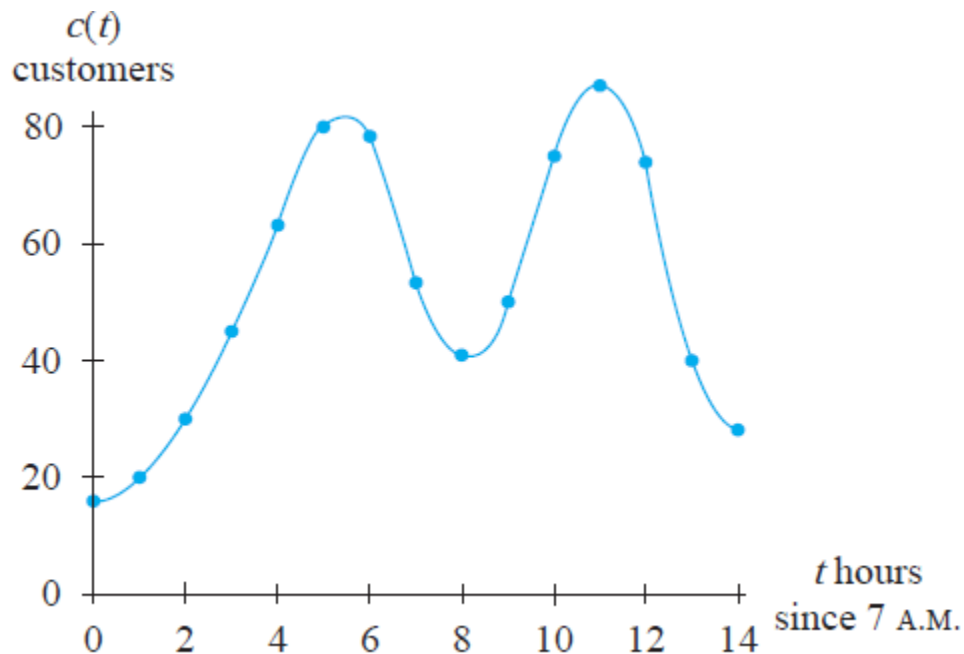
- $m(4) = 8$
- $m'(4)$ is greater than any other slope.
- $m'(0) = m'(6) = 0$
- The graph of m has no direction changes.

Example 2.3.6. The function g gives the fuel efficiency in miles per gallon of a car traveling v miles per hour. Write a sentence of interpretation for each of the following:

(a) $g(55) = 32$ and $g'(55) = -0.25$

(b) $g'(45) = 0.15$ and $g'(51) = 0$

Example 2.3.7. The figure below depicts the number of customers that a fast-food restaurant serves each hour on a typical weekday:



- (a) Estimate the average rate of change of the number of customers between 7am and 11am. Interpret your answer.
- (b) Estimate the instantaneous rate of change and percentage rate of change of the number of customers at 4pm. Interpret your answers.