Chapter 4

Analyzing Change: Applications of Derivatives

Relative Extreme Points

Definitions

Definition 4.2.1 (Relative Extrema)

Let $f(x)$ be a function defined on an input interval $[a, b]$. Let $a < c < b$.

We say that f has a ________________________ at c if the output $f(c)$ is ______________

______________ any other output in some interval around c. Likewise, f has a

______________________________ at c if the output $f(c)$ is ________________________ any other output in some interval around c.

Relative maxima/minima are also referred to as ____

Definition 4.2.2 (Critical Point)

A **critical point** of a continuous function f is a pair __________________at which f is

____. The input value c of a

critical point is called the ________________________.

Example 4.2.3. Find the critical points of $f(x) = 4x^3 + 8x^2 - 20x - 21$ on the interval $[-5, 5]$.

Theorem 4.2.4 (First Derivative Test)

Suppose c is a critical input of a continuous function f.

- If f' changes from positive to negative at c, then ______________________________ ______________________.
- If f' changes from negative to positive at c, then ______________________________ ______________________.
- If f' does not change its sign at c, then ____________________________________ ________________.

Theorem 4.2.5 (Conditions Where Extreme Points Exist)

For a function f with input x, a relative extremum can occur at $x = c$ only if $f(c)$ exists/is defined. Further,

- A relative extremum exists where ______________________ and the graph of $f'(x)$ ______________________ the input axis at $x = c$.
- A relative extremum *can* exist where $f(x)$ exists, but $f'(x)$ does not exist; further investigation will be needed.

Theorem 4.2.6 (Old Derivative Facts)

Let $f(x)$ be a function defined on an input interval $[a, b]$, and let $a < c < b$.

- If ________________________, then $f(x)$ is increasing at $x = c$.
- If ________________________, then $f(x)$ is decreasing at $x = c$.
- If ________________________, then the line tangent to $f(x)$ at $x = c$ is horizontal.

Methods of Finding Extrema

Let $f(x)$ be differentiable on an open interval (a, b).

Algebraic Method:

1. Calculate the derivative $f'(x)$
2. Set $f'(x) = 0$ and solve for x. All solutions will be horizontal asymptotes; individual analysis (corresponding to 1st Derivative Test) will determine if $f'(c)$ is a maximum, minimum, or neither.

Calculator, Method 1:

1. Input $f(x)$ into Y_1
2. Plot $f(x)$, and do `Zoom`→`0:ZoomFit`
3. If you are finding a **local maximum**, press `2nd`→`Trace`→`4:maximum`. If you are finding a **local minimum**, press `2nd`→`Trace`→`3:minimum`.
   (a) The calculator will prompt for a left bound. Input a number *slightly less than* where you expect the maximum/minimum to be.
   (b) The calculator will then prompt for a right bound. Input a number *slightly greater than* where you expect the maximum/minimum to be.
   (c) The calculator will then prompt for a guess. Input a guess, or press enter.
   (d) The max/min will be displayed as a coordinate pair. If the pair is needed, use appropriate rounding guidelines.
   (e) If you forget the value of the max/min, the calculator will store the $x-$coordinate in the variable X. In order to recall it, on the home screen, press X and the calculator will return the appropriate value.

Calculator, Method 2:

1. Input $f(x)$ into Y_1
2. In Y_2, use the `nDeriv` command by pressing `Math`→`nDeriv(`. The complete entry *must* look like
$$Y_2 = \texttt{nDeriv(Y1(X),X,X)}$$
This will have the calculator graph the derivative as well as the original function
3. The local extrema are given by wherever the derivative graph touches the $x-$axis. According to 1st Derivative Test, a local max occurs when the derivative crosses from positive to negative, and a local min occurs when the derivative crosses from negative to positive.

Examples

Example 4.2.7. The percentage of people in the United States (aged 15 and above) who are sleeping at a given time of night can be modeled as

$$s(t) = -2.63t^2 + 29.52t + 13.52 \text{ percent, } t \text{ number of hours after 9pm}$$

(a) Find the critical input values of s on the interval $0 \leq t \leq 11$, and calculate the output value for any critical point.

(b) Graph $s(t)$ and $s'(t)$. Label and interpret the critical inputs.

Example 4.2.8. Sketch a graph such that

- $f'(x) > 0$ for $x < -1$
- $f'(x) < 0$ for $x > -1$
- $f'(-1) = 0$

Example 4.2.9. Sketch a graph such that

- f has a relative maximum at $x = 3$
- f has a relative minimum at $x = -1$
- $f'(x) < 0$ for $x < -1$ and $x > 3$
- $f'(x) > 0$ for $-1 < x < 3$
- $f'(-1) = f'(3) = 0$

Example 4.2.10. Consider the function $f(x) = x^2 + 2.5x - 6$.

(a) Write the derivative formula

(b) Locate and classify each critical point.

Example 4.2.11. Consider the function $f(x) = 0.3x^3 + 1.2x^2 - 6x + 4$.

(a) Write the derivative formula

(b) Locate and classify each critical point.

Example 4.2.12. Consider the function $f(x) = 5e^{-x} + \ln x$ (for $x > 0$).

(a) Write the derivative formula

(b) Locate and classify each critical point.

Example 4.2.13. Consider the function $f(x) = \dfrac{10}{1 + 2e^{-0.5x}}$.

(a) Write the derivative formula

(b) Locate and classify each critical point.

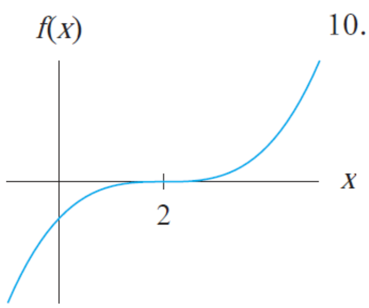
Example 4.2.14. For the following graphs, determine which of the following statements are true:

(1) $f'(x) > 0$ for $x < 2$

(2) $f'(x) > 0$ for $x > 2$

(3) $f'(x) = 0$ for $x = 2$

9.



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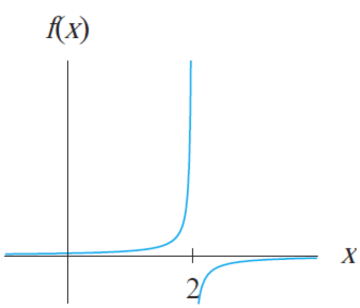

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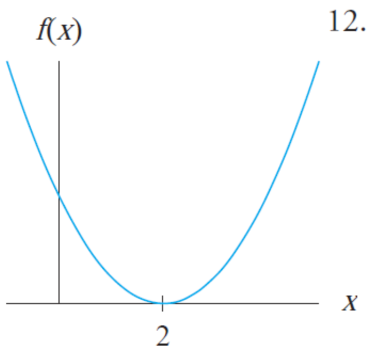

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