Function Behavior and End Behavior Limits

Descriptions of Function Behavior

Especially as we consider longer-term models, we are concerned about what happens to a function as time advances.

Definition 1.2.1 (Increasing, Decreasing, Constant)

Let f be a function defined over some input interval. The function is said to be

- **increasing** if the output values ______________________________
- **decreasing** if the output values ______________________________
- **constant** if the output values ______________________________

Example 1.2.2. On what intervals is $g(x) = -x^3 + 16x - 5$ increasing, decreasing, or constant? Use the calculator to help you find the answer.

Example 1.2.3. Is the function given in the table below increasing, decreasing, or constant? Why?

x	2	4	6	8	10
$h(x)$	5	6	8	12	20

Example 1.2.4. The function $f(x) = c$ is constant. Look at the graph and/or table and explain why.

Direction and Curvature

Definition 1.2.5 (Concavity)

A function f defined over an input interval is said to be

- **concave up** if a graph of the function appears to be ______________________________

- **concave down** if a graph of the function appears to be ______________________________

The curvature of a function is called _________________________.

Example 1.2.6. Describe the concavity of $k(t) = -t^2 + 8t - 13$. Does it appear to ever change? If so, where? Draw a picture.

Example 1.2.7. Describe the concavity of the function $f(z) = \ln z$. Does it ever appear to change? If so, where? Draw a picture.

Example 1.2.8. Describe the concavity of the function $g(x) = -2x$. Draw a picture.

Example 1.2.9. Describe the concavity of $h(d) = d^3 - 11d^2 + 38d - 37$. Does it ever appear to change? Draw a picture.

Definition 1.2.10 (Inflection Point)

A point on a continuous function where the concavity of the function changes is called an **inflection point**.

Example 1.2.11. The function $P(t) = 2t^3 - 10t^2 - 3t + 275$ describes the profit (in hundred dollars) made by a small business after a rash of bad Yelp reviews where t is the number of weeks since the reviews were put online. Let $0 \leq t \leq 5$.

(a) Sketch a picture of the graph.

(b) Estimate the input and output values at the inflection point(s).

(c) Identify the intervals where P is increasing, decreasing, and constant.

(d) Identify the intervals where P is concave up, concave down, or neither.

(e) Use the information from (a)-(c) to describe what was happening to the profit made by the business between the first and sixth week.

Limits and End Behavior

Definition 1.2.12 (End Behavior)

The **end behavior** of a function refers to the behavior of the output values of the function as the input values become larger and larger, or smaller and smaller.

As the input values become larger and larger (more and more positive), we say that the input ______________________________________. As they become smaller and smaller (more and more negative), the input ______________________________________.

Example 1.2.13. Consider $h(d) = d^3 - 11d^2 + 38d - 37$.

(a) Sketch the function on the interval $[0, 6]$.

(b) In a sentence, describe the end behavior of h as the input increases without bound.

(c) In a sentence, describe the end behavior of h as the input decreases without bound.

There are **three possibilities** when we consider the end behavior of a function:

- The output values may ____
- The output values may ____
- The output values may ____

Example 1.2.14. Draw three functions that have will have each of these three end behaviors.

Example 1.2.15. Determine the end behavior/limit of the function $f(x) = \dfrac{2x}{x-1}$ as the input increases without bound, using numerical estimation. Record your approximations with **full decimal accuracy**, and round the final answer to the hundredths.

x	$f(x) = \dfrac{2x}{x-1}$
10	
100	
1000	
10000	
100000	
End Behavior/ Limit	

Note

When creating a table, you need to stop when the digit **after** the one you're rounding to repeats twice.

Definition 1.2.16 (Limit)

A function $f(x)$ is said to have a **limit** L if the __________________ of f approaches ________

as the __________________ approaches some (possibly infinite) value a. We write this using the following notation:

Example 1.2.17. Rewrite the end behavior/limit from the previous example using limit notation.

Definition 1.2.18 (Horizontal Asymptote)

A horizontal line with the equation ____________________ is called a **horizontal asymptote**.

Example 1.2.19. Let $f(x) = x^2$ and $g(x) = x^3$.

(a) Write the statement "The limit of $f(x)$ as x approaches ∞ is ∞" in limit notation.

(b) Find $\lim_{x \to -\infty} f(x)$, and write the notation in words (like in (a))

(c) Find the end behaviors of $g(x)$, and write them in limit notation.

Example 1.2.20. Sketch the following functions, and use the sketches to find the limit as the input increases without bound and decreases without bound:

(a) $f(x) = \ln x$

(b) $g(x) = e^x$

(c) $h(x) = \dfrac{1}{1+e^x}$

Example 1.2.21. Use numerical estimation to find $\lim_{x\to\infty}(1-0.6^x)$. Make a table showing at least five inputs and the corresponding outputs; write *all* decimals in the table, and round your final answer to two decimal places. Start your input at 2 and double.

Example 1.2.22. Use numerical estimation to find $\lim_{t\to-\infty}\left(1+t^{-2}\right)$. Make a table showing at least five inputs, and the corresponding outputs; write *all* decimals in the table, and round your final answer to the nearest integer. Start your input at -10 and double.