Chapter 10

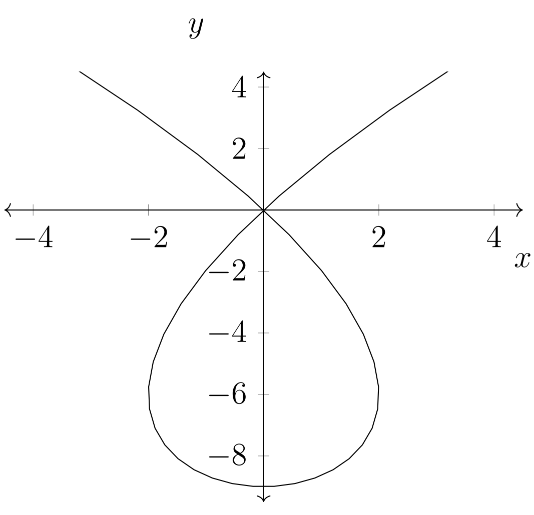
Parametric Equations and Polar Coordinates

Curves Defined by Parametric Equations

Before Class

Parametric Equations

Question 10.1.1 The picture below does *not* describe a function? Why?



Definition 10.1.2 (Parameter/Parametric Equation)

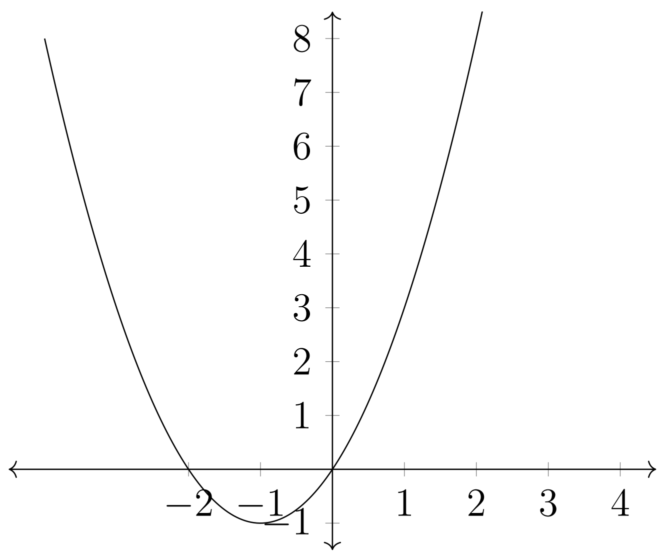
Let C be a curve, and let the functions $x = f(t)$ and $y = g(t)$ describe the $x-$ and $y-$coordinate of C. The variable t is called the **parameter**, and the function $x = f(t)$ and $y = g(t)$ are called **parametric equations**.

Example 10.1.3. Consider the parametric equations $x(t) = t - 2$ and $y(t) = t^2 - 2t$.

(a) Fill out the table below.

t	x	y	t	x	y
-2			2		
-1			3		
0			4		
1					

(b) Plot the points from the table on the graph below. Connect the plotted points to see the parametric curve, and draw arrows to indicate its direction of travel. What kind of function do you see?



Example 10.1.4. Rather than plotting points, we could have directly seen the function. We can do this by a process called *eliminating the parameter.*

(a) Given the parametric equations $x = t - 2$ and $y = t^2 - 2t$, eliminate the parameter to show that the parametric curve is given by the function $y = x^2 - 2x$

(b) If we wanted to only include the *right* portion of the graph shown, what sort of restriction would we need to put on t?

Example 10.1.5. Eliminate the parameter to find the curve represented by the parametric equations $x = \sqrt{t}$, $y = t^2 + 1$. There is a natural restriction on the time interval; what is it?

Example 10.1.6.

(a) What curve is represented by the parametric equations $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$?

(b) The parametric equations $x = \cos 2t$, $y = \sin 2t$, $0 \le t \le 2\pi$ represent the same curve as in part (a). What is the difference between the parametrizations in (a) and (b)?

(c) What if in (b) we instead took the time interval $0 \le t \le \pi$? What would be the difference between (a) and (b) in that case?

Pre-Class Activities

Example 10.1.7. Use this space to write any questions you might have from the videos.

Example 10.1.8. For each of the following parametric curves, sketch the curve and indicate the direction in which the curve is traced as t increases. Then, eliminate the parameter to find a Cartesian equation for the curve.

(a) $x = 2t - 1$, $y = \frac{1}{2}t + 1$

(b) $x = t^2 - 3$, $y = t + 2$, $-3 \leq t \leq 3$

(c) $x = \sin t$, $y = 1 - \cos t$, $0 \leq t \leq 2\pi$

In Class

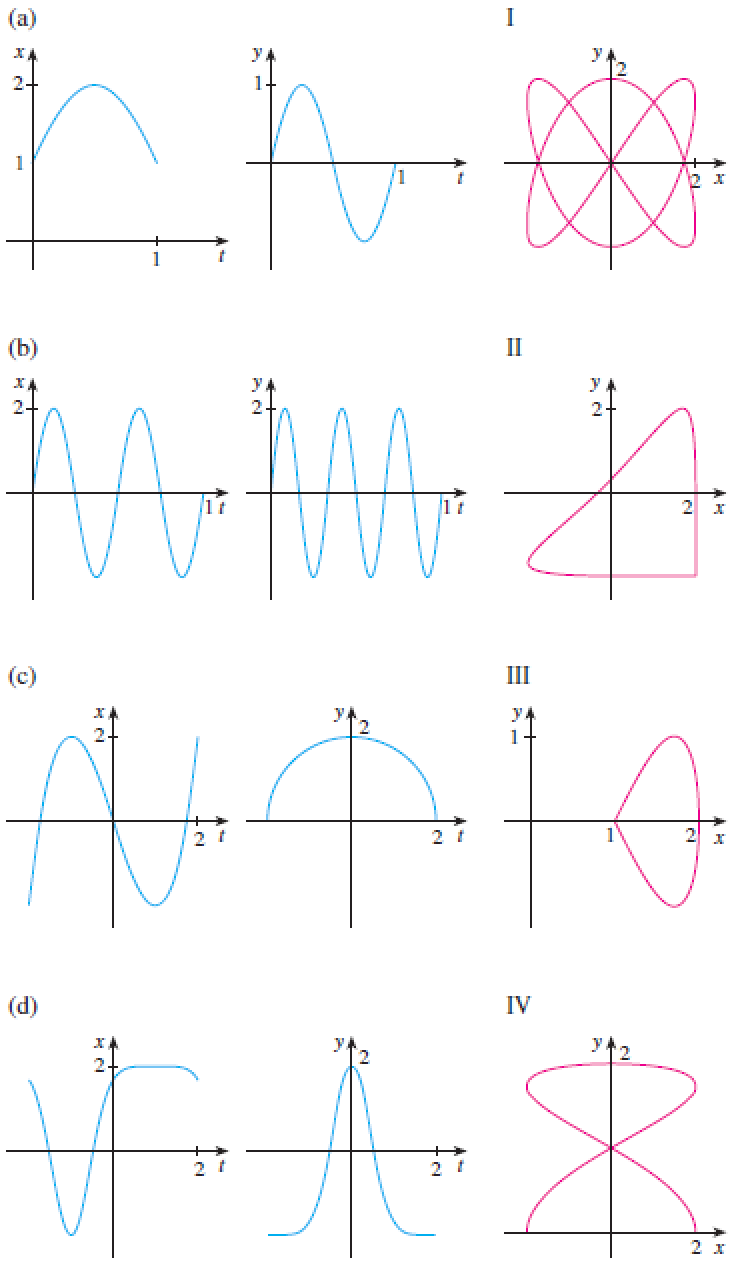
Examples

Example 10.1.9. Sketch the curve $x = \sin\left(\frac{1}{2}\theta\right)$, $y = \cos\left(\frac{1}{2}\theta\right)$, $-\pi \leq \theta \leq \pi$. Then, eliminate the parameter to find a Cartesian expression for the curve.

Example 10.1.10. Do the same thing for the parametric equations $x = e^t$, $y = e^{-2t}$.

Example 10.1.11. Describe the motion of a particle with position $x = 5 + 2\cos(\pi t)$, $y = 3 + 2\sin(\pi t)$ as t varies in the interval $[1, 2]$.

Example 10.1.12. Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)-(d) with the parametric curves labeled I - IV. Justify your response.



Example 10.1.13. Develop a parametrization which traces out a circle three times in the time interval $0 \leq t \leq \pi$.

Example 10.1.14. Compare the curves represented by the parametric equations given. How do they differ?

(a) $x = t^3$, $y = t^2$

(b) $x = e^{-3t}$, $y = e^{-2t}$

(c) $x = t^6$, $y = t^4$

After Class Activities

Example 10.1.15. Sketch the parametric curve $x = t^2$, $y = t^3$, and eliminate the parameter to find a Cartesian equation for the curve.

Example 10.1.16. Do the same thing for the curve $x = t^2$, $y = \ln t$

Example 10.1.17. Describe the motion of a particle with position $x = 2 + \sin t$, $y = 1 + 3\cos t$ on the interval $\pi/2 \leq t \leq 2\pi$.

Example 10.1.18. Compare the curves represented by the parametric equations below. How do they differ?

(a) $x = t$, $y = t^{-2}$

(b) $x = \cos t$, $y = \sec t$

(c) $x = e^t$, $y = e^{-2t}$

Example 10.1.19. The graphs of $x = f(t)$ and $y = g(t)$ are given below. Use the graphs to sketch the corresponding parametric curve; use arrows to indicate the direction in which the curve travels.