# Advanced Data Analysis with Python

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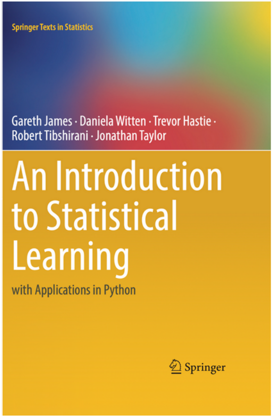
## Other recommended resources

- Think Python by Allen Downey offers electronic resources here that will allow you to directly code there
- Kaggle - Learn Python is a more interactive guide
- Invent your Own Computer Games with Python by Al Sweigart

# Resources for this class




An Introduction to Statistical Learning

- This class follows **Chapter 3** of this book
- Refer to the book if you want to deepen today's topic
- It is a bit more mathematically intensive than this class!

# Homework Correction

# Statistical Modeling Foundations

# What is a statistical model?

**Definition (Statistical Model)**

A statistical model is a mathematical model that embodies a set of statistical assumptions concerning the generation of sample data (and similar data from a larger population).

Source: Wikipedia

# Example: Regression

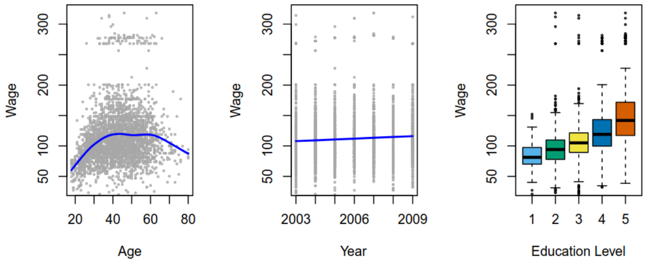



**FIGURE 1.1.** Wage *data, which contains income survey information for men from the central Atlantic region of the United States.* Left: wage *as a function of* age. *On average,* wage *increases with* age *until about* 60 *years of age, at which point it begins to decline.* Center: wage *as a function of* year. *There is a slow but steady increase of approximately* \$10,000 *in the average* wage *between* 2003 *and* 2009. Right: *Boxplots displaying* wage *as a function of* education, *with* 1 *indicating the lowest level (no high school diploma) and* 5 *the highest level (an advanced graduate degree). On average,* wage *increases with the level of education.*

Source: *An Introduction to Statistical Learning with Applications in Python.*

# Example: Classification

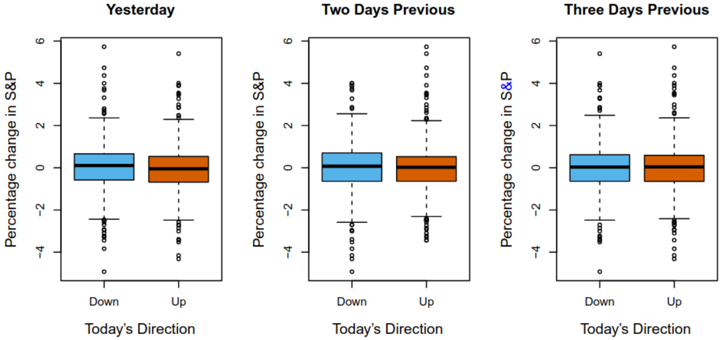



**FIGURE 1.2.** Left: *Boxplots of the previous day's percentage change in the S&P index for the days for which the market increased or decreased, obtained from the* `Smarket` *data.* Center and Right: *Same as left panel, but the percentage changes for 2 and 3 days previous are shown.*

Source: *An Introduction to Statistical Learning with Applications in Python.*

# Example: Classification

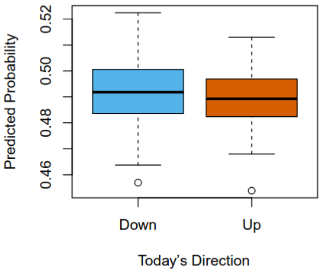



**FIGURE 1.3.** *We fit a quadratic discriminant analysis model to the subset of the* `Smarket` *data corresponding to the 2001–2004 time period, and predicted the probability of a stock market decrease using the 2005 data. On average, the predicted probability of decrease is higher for the days in which the market does decrease. Based on these results, we are able to correctly predict the direction of movement in the market 60% of the time.*

Source: *An Introduction to Statistical Learning with Applications in Python.*

# Regression: Example

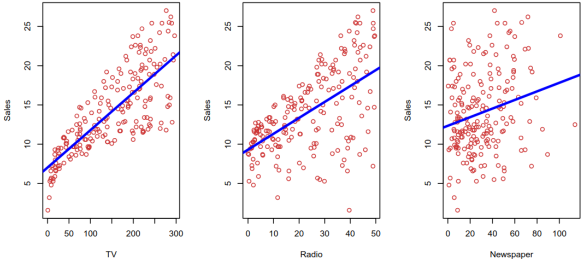



**FIGURE 2.1.** *The* Advertising *data set. The plot displays* sales*, in thousands of units, as a function of* TV*,* radio*, and* newspaper *budgets, in thousands of dollars, for* 200 *different markets. In each plot we show the simple least squares fit of* sales *to that variable, as described in Chapter 3. In other words, each blue line represents a simple model that can be used to predict* sales *using* TV*,* radio*, and* newspaper*, respectively.*

Source: An Introduction to Statistical Learning with Application in Python

# Regression: Research Questions

1. Is there a relationship between advertising budget and sales?
2. How strong is the relationship between advertising budget and sales?
3. Which media are associated with sales?
4. How large is the association between each medium and sales?
5. How accurately can we predict future sales?
6. Is the relationship linear?
7. Is there synergy among the advertising media?

Source: An Introduction to Statistical Learning with Application in Python

# Predictors and dependent variables

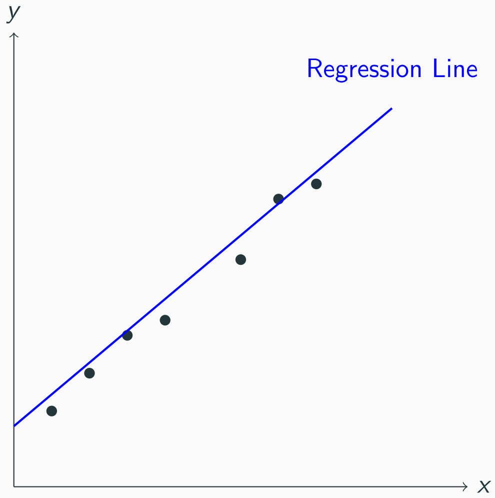
- **Dependent variable**: the output you are trying to predict.
- **Predictor(s)**: the variable(s) used to forecast the output.

# What is Simple Linear Regression?

- **Linear Regression** predicts a quantitative response assuming a **linear relationship** with the predictor(s).
- Single linear regression predicts the quantitative output $Y$ on the basis of a **single** predictor variable $X$.
- Assumes linear relationship:

$$y = \beta_0 + \beta_1 x + \epsilon$$

- Parameters or coefficients:
  - $\beta_0$: intercept (value of $y$ when $x = 0$)
  - $\beta_1$: slope (how much $y$ changes for 1 unit increase in $x$)
- $\epsilon$: error term


y
Regression Line
x

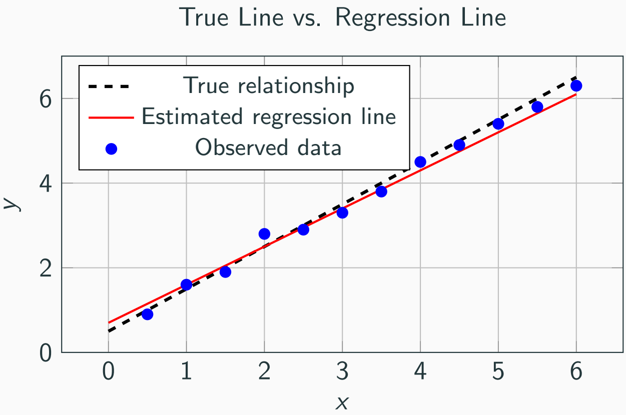



**Figure 1:**

Comparison of the true relationship (black dashed line) and the estimated regression line (red). The regression line approximates the true line but is not perfectly identical due to random noise in the data. Readapted from An introduction to Statistical Learning with Python.

# From Abstract X and Y to Real Data

**Substitute the example values:**

$$Y \longrightarrow \mathit{sales}$$

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**Substitute the example values:**

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**Apply the regression formula:**

$$\mathit{sales} = \beta_0 + \beta_1 \times \mathit{TV}$$

**Problem**

You cannot compute the above example without **knowing the values of the coefficients**!

# Estimating the coefficients

- Let's consider a sequence of data points

$$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$$

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$$y_i \approx \hat{\beta}_0 + \hat{\beta}_1 x_i$$

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- The most common approach is minimizing the **least squares criterion**.

# Residual Sum of Squares (RSS) in Linear Regression

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- The **Residual Sum of Squares (RSS)** is:

$$RSS = e_1^2 + e_2^2 + e_3^2 + ... + e_n^2$$

- Equivalent to:

$$RSS = (y_1 - \hat{\beta}_0 - \hat{\beta}_1 x_1)^2 + (y_2 - \hat{\beta}_0 - \hat{\beta}_1 x_2)^2 + \cdots + (y_n - \hat{\beta}_0 - \hat{\beta}_1 x_n)^2$$

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- To sum up:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

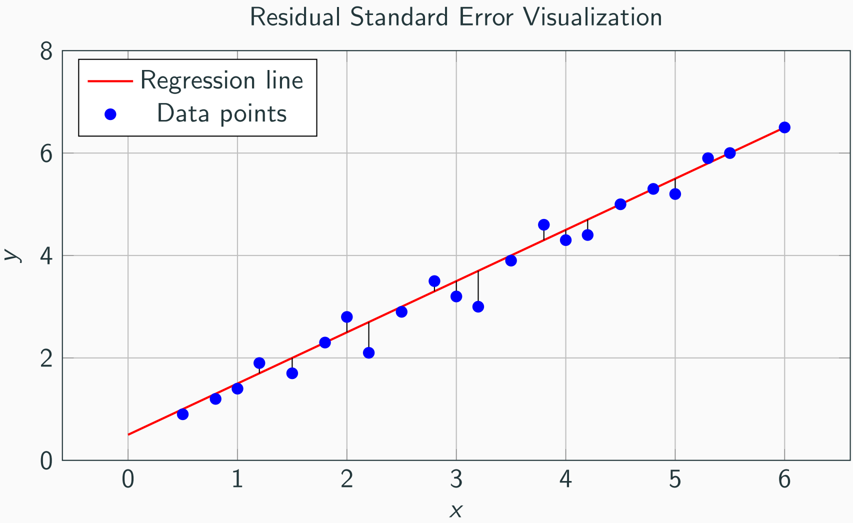


**Figure 2:** Residual standard error: vertical distances from observed data points to the fitted regression line. Readapted from Statistical Learning with Applications in Python.

**It's not over yet!**

Next step is **assessing the accuracy of the coefficients** that you estimated!

# Assessing the accuracy of the coefficients

- Let's recall the formula:

$$y = \beta_0 + \beta_1 x + \epsilon$$

- The error term $\epsilon$ captures what we missed with this model
- Therefore, **it is important to know how well the estimated coefficients are estimated**.

# Mean Standard Error (SE)

How accurate is the sample mean $\bar{\mu}$ as an estimate of $\mu$?

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- Example: Let $\mu$ be the population mean, and $\bar{\mu}$ be mean of a sample:

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- Key insight: Larger samples $\rightarrow$ smaller SE $\rightarrow$ more precise estimates

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**Formulas:**

$$\mathrm{SE}(\beta_0)^2 = s^2 \left( \frac{1}{n} + \frac{\bar{x}^2}{\sum (x_i - \bar{x})^2} \right)$$

$$\mathrm{SE}(\beta_1)^2 = \frac{s^2}{\sum (x_i - \bar{x})^2}$$

Where:

- $s^2 = \frac{\sum (y_i - \hat{y}_i)^2}{n-2}$ is the residual variance ($\mathrm{Var}(\epsilon)$)
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**Keep in mind:** Smaller standard error = more precise coefficient estimates

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- Formula:

$$\text{CI} = \bar{x} \pm Z \cdot SE$$

where

  - $\bar{x}$ = sample mean
  - $SE$ = standard error
  - $Z$ = critical value from standard normal distribution (1.96 for 95%)

# Example of a 95% Confidence Interval

- Sample parameter value: 150
- Standard error: 20
- 95% CI:

$$150 \pm 1.96 \cdot 20 = 150 \pm 39.2$$

- Interval: [110.8, 189.2]

We are 95% confident that the true value lies within this range.

# Example

**Table 1:** Linear Regression of Wage on Age: Key Statistics

| **Measure** | **Value** | **Meaning** |
|---|---|---|
| Mean Standard Error (SEM) | 50 | Average variation of sample mean wage if we resampled |
| Standard deviation of $b_0$ (SE $b_0$) | 300 | Intercept (2000) varies $\pm$300 due to sample randomness |
| Standard deviation of $b_1$ (SE $b_1$) | 20 | Slope (150) varies $\pm$20; uncertainty in wage increase per age |
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# Hypothesis testing

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**Alternative Hypothesis**

$H_a$: There is a relationship between $X$ and $Y$.

Mathematically, this equals to:

$$\beta_1 \neq 0$$

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- Intuition:
  - Large $|t| \rightarrow$ coefficient far from 0 $\rightarrow$ likely real effect
  - Small $|t| \rightarrow$ coefficient close to 0 $\rightarrow$ effect might be random

# P-value (for a coefficient)

- **P-value** $=$ probability of observing a t-statistic at least as extreme as the one obtained if $H_0$ were true.
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- Translated: tells you how unusual the result would be if $H_0$ was true ($=$ no effect of $x$ on $y$)
- Interpretation:
  - Small p-value (e.g., $< 0.05$) $\rightarrow$ reject $H_0$, coefficient likely significant
  - Large p-value (e.g., $> 0.05$) $\rightarrow$ fail to reject $H_0$, no strong evidence

# Example

Regression of wage on age: $wage = \beta_0 + \beta_1 \times age + \epsilon$ Random example values: $wage = 2000 + 150 \times age + \epsilon$

**Table 2:** Linear Regression of Wage on Age: Key Statistics

| **Measure** | **Value** | **Meaning** |
|---|---|---|
| Mean Standard Error (SEM) | 50 | Average variation of sample mean wage if we resampled |
| Standard deviation of $b_0$ (SE $b_0$) | 300 | Intercept (2000) varies $\pm$300 due to sample randomness |
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| Confidence Interval (95%) | [110, 190] | We are 95% confident true slope lies between 110 and 190 |
| T-test (for $b_1$) | 7.5 | Tests if age significantly affects wage; higher t $\longrightarrow$ more evidence |
| P-value (for $b_1$) | 0.0001 | Very low $\longrightarrow$ strong evidence that age significantly affects wage |

These values are not real, they just deem as an example. We will see real values in the practical session.

We talked about how to evaluate the accuracy of the coefficients. What about the **accuracy of the model?**

- Coefficient accuracy: how well is the relationship between X and Y modeled?
- Model accuracy: how well does the model predict?

# Assessing the accuracy of the model

The **error** of an observation is the deviation of the observed value from the true value of a quantity of interest (for example, a population mean).

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The **error** of an observation is the deviation of the observed value from the true value of a quantity of interest (for example, a population mean).

The **residual** is the difference between the observed value and the estimated value of the quantity of interest (for example, a sample mean).

Source: Wikipedia

# Assessing the accuracy of the model

- **Residual Standard Error (RSE)**
- **Coefficient of Determination (R-Squared)**

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- The **smaller** your RSE, the better your model.

# R-squared

The **coefficient of determination**, denoted as $r^2$, is the proportion of the variation in the dependent variable that is predictable from the independent variable(s).

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$$1 - \frac{\text{Residual Sum of Squares}}{\text{Total Sum of Squares}}$$

- $R^2 = 1$: the regression predictions perfectly fit the data
- $R^2 = 0$: the regression does not fit the data

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| P-value (for $b_1$) | 0.0001 | Very low $\longrightarrow$ strong evidence that age s |
| Residual Standard Error (RSS) | 12,000 | Difference between observed and predic |
| 0.85 | 85% of wage variation explained by age; higher $\longrightarrow$ better model fit | |

What happens when we have more than one predictor?

## Regression: Example

# What is multiple linear regression?

- Models the relationship between a dependent variable $Y$ and multiple independent variables $X_1, X_2, \ldots, X_p$:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \varepsilon$$

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where:

- $Y$ = dependent variable (outcome)
- $X_1, X_2, \ldots, X_p$ = independent variables (predictors)
- $\beta_0$ = intercept
- $\beta_1, \ldots, \beta_p$ = regression coefficients
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**Remember**

The coefficient $\beta_i$ represents the change in $Y$ for a one-unit increase in $X_i$ *(remember single linear regression!)*, **holding other variables constant**.

# From Abstract X and Y to Real Data

**Substitute the example values:**

$$Y \longrightarrow \textit{sales}$$

$$X_1 \longrightarrow \textit{TV}$$

$$X_2 \longrightarrow \textit{radio}$$

$$X_3 \longrightarrow \textit{newspaper}$$

# From Abstract X and Y to Real Data

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$$

**Apply the regression formula:**

$$\textit{sales} = \beta_0 + \beta_1 \times \textit{TV} + \beta_2 \times \textit{radio} + \beta_3 \times \textit{newspaper}$$

# Frame Title

**Remember**

Correlation among predictors can be problematic: low correlation is desired.

# Estimating the coefficients

- **Minimizing least squares** as in simple linear regression

$$\text{RSS} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

where:

- $y_i$ = actual value of the $i$-th observation
- $\hat{y}_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}$ = predicted value
- $n$ = number of observations

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$$\sum_{i=1}^{n} \left( y_i - \hat{\beta}_0 - \hat{\beta}_1 \times x_{i1} - \hat{\beta}_2 \times x_{i2} - \ldots - \hat{\beta}_n \times x_{in} \right)$$

# Some assumptions of Linear Regression

# Linearity

- **Linear relationship** between the dependent and independent variable(s)

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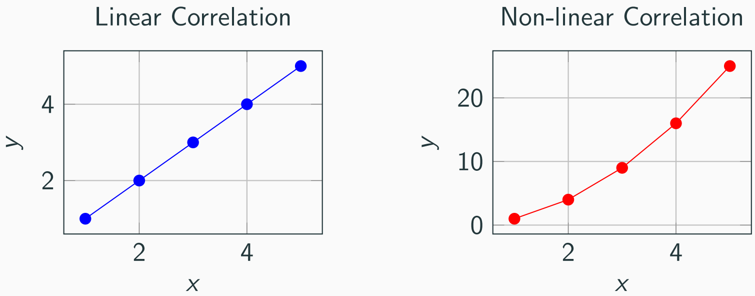


**Figure 3:** Left: Linear correlation. Right: Non-linear correlation.

# Linearity

- **Linear relationship** between the dependent and independent variable(s)

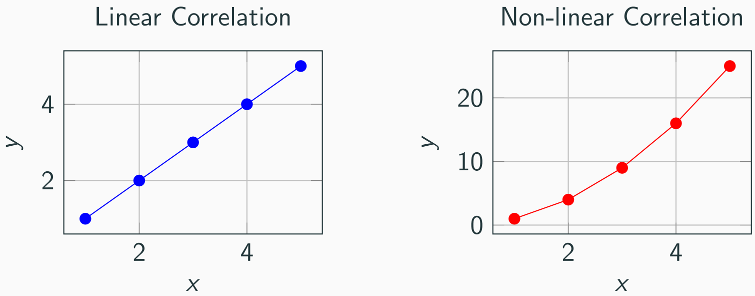


**Figure 3:** Left: Linear correlation. Right: Non-linear correlation.

- **How to check**: With a scatterplot.

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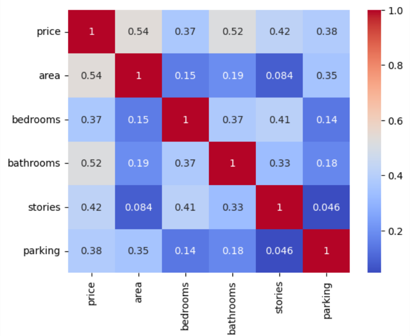


**Figure 4:** From last week's lab: a heatmap representing correlation values.

# Homoscedasticy

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- **Homoscedasticy** means that the variance of the errors is roughly the same across all values of the predictor(s) in your regression model.
- **How to check** (intuitively, without formal tests): Plot the residuals and look for (approximately) even variance.

# Other limitations

- Linear regression involves other limitations and assumptions, **not treated in this course due to its more limited scope**.

How can you elaborate a meaningful data analysis pipeline that involves linear regression?

## Adapt to your own research question

- Elaborate a **public policy research question** related to your own studies and knowledge
- This way, **errors in your model will still be meaningful.**
- Example:
    - I evaluate **wage** in relation to **age**: this is meaningful!
    - I evaluate **wage** in relation to **height**: I might detect some patterns (for example, due to biases in my samples), but this research question makes no sense!
- This seems obvious, yes; but I am sure you can think of several field-specific examples, and of correlation that people who do not study public policies do not know!