SMFD

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Assignment-2

1) a) $Q = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0 & 0.25 & 0.75 \\ 0 & 0 & 0.75 & 0.25 \end{bmatrix}$

b) Recurrent → 1, 2, 3, 4

No transient state

c) $(Q - \lambda I)v = 0$

$\lambda = 1$

$(Q - I)v = 0$

$$\left(\begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0 & 0.25 & 0.75 \\ 0 & 0 & 0.75 & 0.25 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right) v = 0$$

$$v \begin{bmatrix} -0.5 & 0.5 & 0 & 0 \\ 0.25 & -0.25 & 0 & 0 \\ 0 & 0 & -0.75 & 0.75 \\ 0 & 0 & 0.75 & -0.75 \end{bmatrix} \cancel{v} = 0$$

$$[a\ b\ c\ d] \begin{bmatrix} -2 & 2 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -3 & 3 \\ 0 & 0 & 3 & -3 \end{bmatrix} \cancel{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}} = 0$$

$-2a + b = 0$ $\quad 2a - b = 0$ $\quad -3c + 3d = 0$ $\quad 3c - 3d = 0$

$2a = b$ $\quad 2a = b$ $\quad c = d$ $\quad c = d$

$c = 0, d = 0, a + b = 1$ $\quad$ Two classes $\quad c + d = 1, a = 0, b = 0$

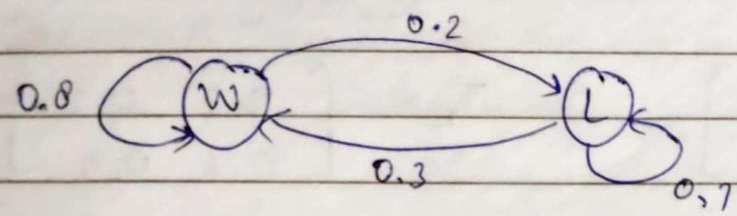
$a = 1/3, \ b = 2/3$ $\quad c = 1/2, \ d = 1/2$

$v = \left[\frac{1}{3}, \frac{2}{3}, 0, 0\right]$ & $\left[0, 0, \frac{1}{2}, \frac{1}{2}\right]$

2)

$W \xrightarrow{0.8} W$ $\qquad$ $L \xrightarrow{0.3} W$

$W \xrightarrow{0.7} D$ $\qquad$ $L \xrightarrow{0.2} D$



$T = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$

a) for long run
stationary state $\rightarrow v = [a, b]$

$v(T - I) = 0 \Rightarrow v \begin{bmatrix} -0.2 & +0.2 \\ +0.3 & -0.3 \end{bmatrix} = 0$

$[a, b] \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix} = 0$

$-2a + 3b = 0$ $\qquad$ $2a - 3b = 0$

$2a = 3b$ ✓ $\qquad$ $a + b = 1$

$\frac{3b}{2} + b = 1$ $\qquad$ $\frac{5b}{2} = 1$

$\boxed{a = 3/5}$ $\qquad$ $\Rightarrow \boxed{b = 2/5}$

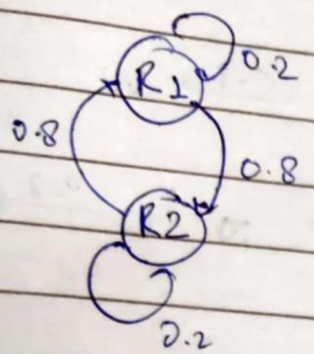
$\boxed{W = 3/5, \quad L = 2/5}$ Ans.

b) $P = 0.7\,W + 0.2 \times L = \frac{3}{5} \times \frac{7}{10} + \frac{2}{10} \times \frac{2}{5} = \frac{1}{2}$

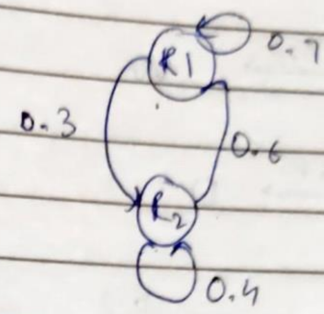
c) expected games $= \frac{1}{0.5} = 2$

3)

Cat



Mouse



a) $T_c = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$ $\quad T_M = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$

cat → $[R_1, R_2]\begin{bmatrix} -0.8 & 0.8 \\ 0.8 & -0.8 \end{bmatrix} = 0$ $\quad [R_1, R_2]\begin{bmatrix} -0.3 & 0.3 \\ 0.6 & -0.6 \end{bmatrix} = 0$

$R_1 = R_2$ & $R_1 + R_2 = 1$ $\quad R_1 = 2R_2$ $\quad R_1 + R_2 = 1$

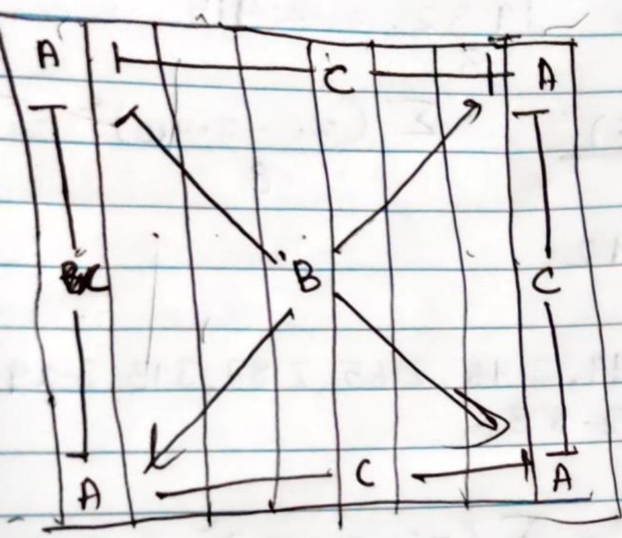
$\rightarrow \boxed{R_1 = R_2 = 0.5}$ $\quad \boxed{R_1 = 2/3, R_2 = 1/3}$

$[R_1, R_2] = [1/2, 1/2]$ $\quad [R_1, R_2] = [2/3, 1/3]$

b) yes can be treated as ~~2 chains of a markov~~ ~~chain~~ markov chain as current state only depends on previous state for both cat & ~~dog~~ mouse and hence for both combined.

apsara

Date:



A – 4
C – 4×6 = 24
B = 36

N → no. of available loops

N(A) = 3
N(B) = 8
N(C) = ~~8~~ 5

Total stationary prob = 3×4 + 36×8 + 24×5

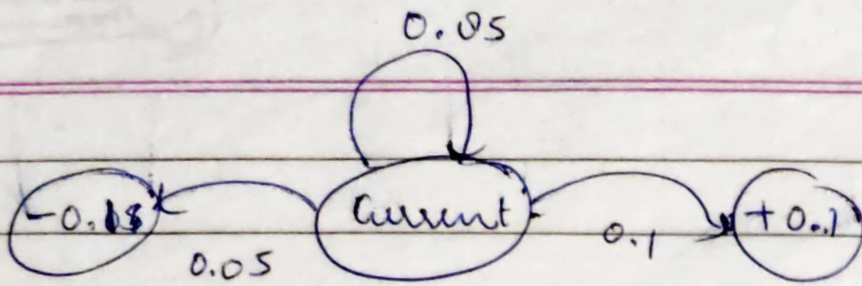
Stationary prob = SP

$$SP(A) = \frac{3\times4}{420} = \frac{1}{35}$$

$$SP(B) = \frac{24\times5}{420} = \frac{2}{7}$$

$$SP(C) = \frac{8\times5}{420} = \frac{2}{21}$$

5)



$X_n = X_{n-1} + 0.1 \rightarrow P = 0.1$

$X_n = X_n \rightarrow P = 0.85$

$X_n = X_{n-1} - 0.1 \rightarrow P = 0.05$

a) ~~Transient~~ Transient as it is boundless and can go up indefinitely

b) If there is no limit → no ~~ste~~ stationary state

c) Probability of earning ₹5 before 1 pm is 0
0% chance stock will reach 130 before 1 P.M.