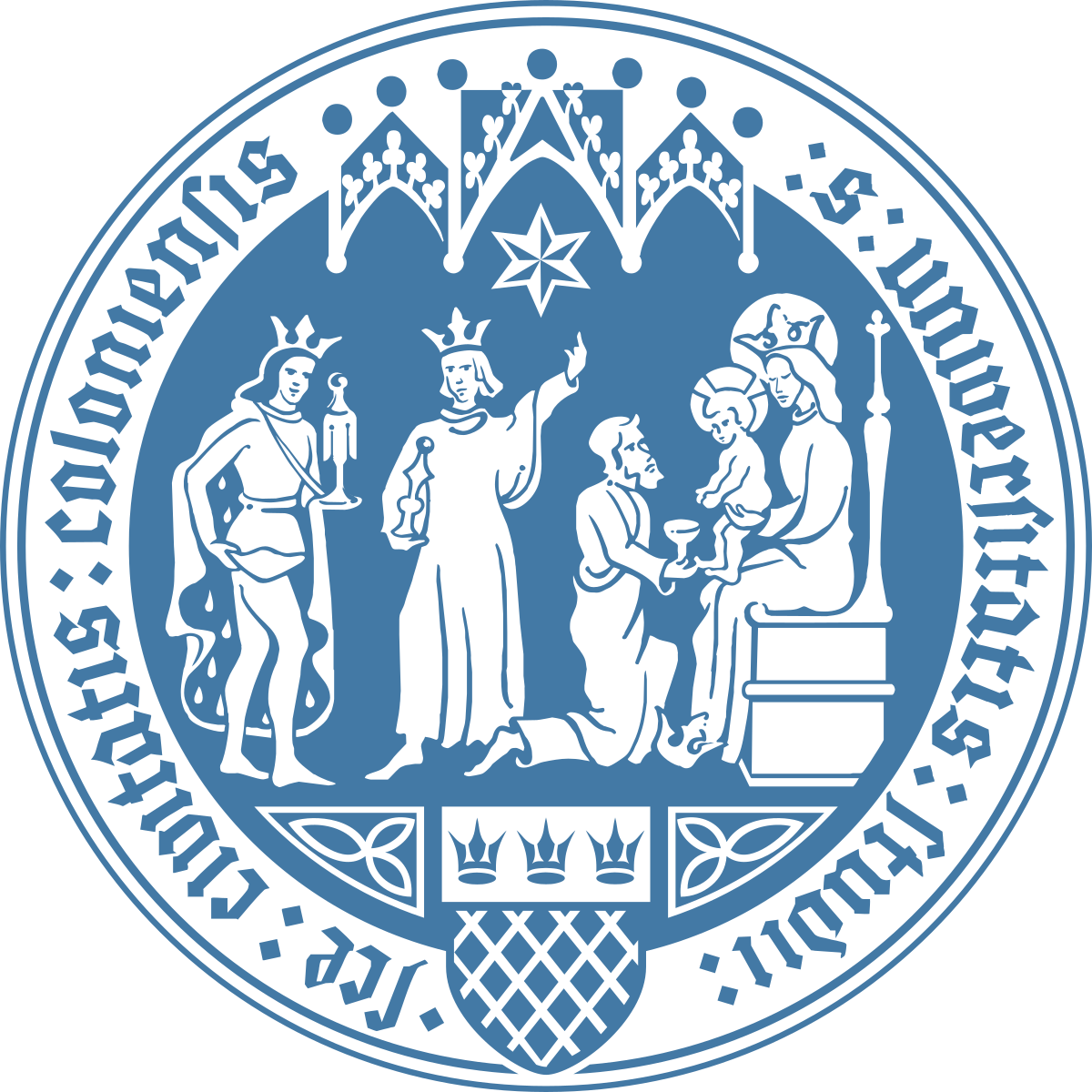
**Project Report Applied Mathematical Optimization**



**Solving the Pool Strategy Problem of a Wind Power Producer with Bilevel Programming Methods and Decomposition**

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Code can be found in the Git repository under:

<https://github.com/cgabler/AMO_Project2>

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**Table of Abbreviations**

k Index for up-regulation block at balancing market, from 1 to *NK*  
j Index for down-regulation block at balancing market, from 1 to *NJ*   
⍵ Index for scenario, from 1 to *NΩ*

ck Offered cost for up-regulation block *k*  
bj Offered benefit for down-regulation block *j*  
Ck Production limit for up-regulation block *k*  
Cj Consumption limit for down-regulation block *j*  
⍵⍵ Own wind power production in scenario *⍵*δ⍵ Residual system deviation in scenario *⍵*  
λDA⍵ Day-ahead market price in scenario *⍵*CW Installed capacity for wind power producer

ρk⍵ Up-regulation from block *k* in scenario *⍵*  
ρj⍵ Down-regulation from block *j* in scenario *⍵*   
λB⍵ Balancing market price in scenario *⍵*μSk⍵ Dual variable for capacity constraint at balancing market for *k* in scenario *⍵*μDj⍵ Dual variable for capacity constraint at balancing market for *j* in scenario *⍵*

*x⍵* Wind power producers offer in scenario *⍵*

# Abstract

*This project solves and compares the bilevel problem for an adequate pool strategy of a wind power producer, not only as price-taker, but as a price-maker in the balancing market. First the formulation of the Karush-Kuhn-Tucker (KKT) conditions is used, second the linearized formulation fitting the big-M method is implemented and compared to the previous results. At last, the problem is solved through Benders decomposition. The results of all three methods are compared.*

# Introduction

As the energy market is steadily changing and transforming over the last decade and probably also over the next decades to tackle different problems, the possibilities and challenges also rise. The deployment of renewable energy sources into the power system challenges planning with more uncertainty, for both producers and the system. Wind power (on- and off-shore) nowadays provides the biggest part of renewable energies. It differs from most other sources, that it is stochastic and energy production can only be forecast with a certain degree of accuracy. Planning for wind generation plants is a challenge for system operators but also the producers.

Wind power producers (WPP) typically are price-takers in the day-ahead market and balancing market, meaning they accept given market prices and deviate only slightly if at all, depending on market. This optimization problem shall be for WPPs that are price takers only in the day-ahead market, but price makers in the balancing market. For this we will follow Zugno et al. (2013).

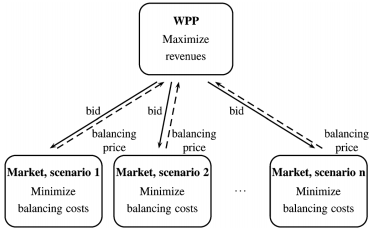
This bilevel setup consists of the maximization of revenue from day-ahead and balancing market at the upper level and a minimization of cost at the balancing market. The setup will be modeled according to the formulated conditions and constraints. In the next step it will be served with two different methods, whose outcomes are compared. At last we try to solve the problem with benders decomposition and compare the resulting outcome with the bilevel programming approaches.

*Personal note:* As I had problems with the implementation of this project due to inexperience with Julia and a lack of mathematical understanding with which it is a struggle to comprehend these problems a real result is missing. Therefore comparisons of different approaches are also missing. With a lack of time to choose another topic, this project paper describes the theoretical approach, as well as expected outcomes.

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# Problem

The problem is simply described in Figure 1, the wind power producer at the upper level wanting to maximize revenues from day-ahead and balancing markets, but simultaneously wanting to minimize costs as a price-maker in the balancing market.



**Figure 1**: Sketch of the problem - Zugno et al. (2013)

# Lower-Level Problem

The problem for the lower level problem, clearing the balancing market, is defined as follows.

We get presented with decision variables ρk⍵ and ρj⍵ as dispatch representation of up- and down-regulation power, Ck as price offer and bj as per unit benefit of power production decrease. In this formula we are given the objective for the problem, the balancing cost of each scenario. Constraints are formulated for the balance of supply and demand, as well as equations for the dispatch of regulating power depending on Ck and Cj  and it is ensured that power cannot be negative.

## KKT Conditions

Provided by Zugno et al. (2013) is a translation of the original formulation set into Karush-Kuhn-Tucker (KKT) conditions. Following are the KKT conditions for the lower level problem.

## Big-M Method

The ⊥ operator separating inequalities means that the KKT conditions discussed in section 3.1 still include nonlinearities, this can be solved by employing binary variables to reformulate the optimality conditions. Also adding a large positive constant to the right-hand side of a constraint – the *M* constant. This serves the purpose that the constraint is always satisfied. The included *z* variable is the before mentioned binary variable only taking the value 1 if the constraint is active or 0 if otherwise.

Big-M is a useful method to formulate linear programming problems, M should be chosen large (hence the name), as a small M can result in incorrect solutions. The typical value of one million shall be used.

# Upper-Level Problem

## KKT Conditions

On the upper level we look at the WPffP as a price-taker, in a one-price system, where all deviations from day-ahead are at the balancing market, in the formulation the dual λB⍵.

From Zugo et al. (2013) we can again take the final optimization problem with linearization of bilinear terms as:

(formulation from section 3.2)

# Decomposition

Decomposition is used to break a complex problem into multiple smaller sub-problems that are then solved independently. Afterwards all solutions are combined to obtain a solution for the original, complete problem.

For this present bilevel problem Benders decomposition was chosen, as it is the popular choice to solve mixed-integer linear programming (MILP) and for example already mentioned in Dantzig & Thapa (2003) - Section 10 that Benders decomposition is the Dantzig-Wolfe decomposition applied to the dual. Which then should be fitting for this problem.

Benders decomposition used for a originally bilevel problem typically uses the lower level problem as subproblem and the upper level problem as master problem,

# Conclusion

Reviewing and forming thoughts and critique on non-existent results is not possible, so again everything has to be of theoretical nature. The big-M method was chosen mainly because it is a popular method for solving linear bilevel optimization, but upon further research into this topic it can be found that much critique is made on this. (Kleinert & Schmidt, 2023) As big-M completely depends on the choice of a correct value for M, which is mostly described as it should be “not small, but also not too big”. The result is a recommendation to use something in the range of one million for M, but not many directions given upon what parameters this should be adjusted. Additionally it is proven that checking validity of big-Ms is very difficult to achieve. So in conclusion: this method gets mainly used because it is easy to use and popular.

As to Benders decomposition, the big advantage of decomposition methods is scalability and therefore the ability to solve large-scale problems with a big computational burden. With this each sub-problem can be computed by multiple efficient solvers. An advantage (or ability) in my opinion is not necessary for this problem as it is still of small scale. Another big advantage is the separation of problems for more easily formulated ones, again not needed with this problem.

In my opinion, solving the problem of this pool strategy of WPPs is still on a small scale and easy to use with run-of-the mill bilevel programming approaches.

# Future Work

For a comparison of these methods an extensive benchmarking could be interesting, also maybe with more methods than the three discussed in this report. Running all methods separately and up to a hundred times, to generate a meaningful and informative average not largely influenced by fluctuations of the processing power and other processes run at the same time.

Another interesting topic would be more research into big-M, an extensive evaluation of how different Ms influence the problems solution and to what result it changes the final outcome.

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