EE2703 : Applied Programming Lab
Assignment 5
The Resistor Problem

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1 Introduction

We wish to solve for the currents in a resistor. The currents depend on the shape of the resistor and we also want to know which part of the resistor is likely to get hottest. The assignment is to visualize the Laplace equation form of potential, solve it and see the current flow in a wire, soldered to the middle of a copper plate and its voltage is held at 1 Volt. One side of the plate is grounded, while the remaining are floating. The plate is 1 cm by 1 cm in size.

2 Derivation of Equations

Raw form of Ohm's law

$$j = \sigma E$$

Now the Electric field is the gradient of the potential,

$$E = -\nabla \phi$$

And continuity of charge yields

$$\nabla j = d\rho/dt$$

Assuming that our resistor contains a material of constant conductivity, the equation becomes,

$$\nabla^2 \phi = \sigma d\rho/dt$$

For DC currents, the right side is zero, and we obtain

$$\nabla^2 \phi = 0$$

Finally this equation can be solved to obtain the potential 2D array and current can be calculated.

3 Solution in the 2D plate

$$d^2\phi/dx^2 + d^2\phi/dy^2 = 0$$

The Laplace form of the equation can easily be converted to a difference equation. Assumingis available at points(xi,yj) after solving we can write,

$$\phi_{i,j} = (\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1})/4$$

Thus, if the solution holds, the potential at any point should be the average of its neighbours. This is a very general result and the above calculation is just a special case of it.So the solution process is obvious. Guess anything you like for the solution. At each point, replace the potential by the average of its neighbours. Keep iterating till the solution converges (i.e., the maximum change in elements of ϕ is less than some tolerance).But what do we do at the boundaries? At boundaries where the electrode is present, just put the value of potential itself. At boundaries where there is no electrode, the current should be tangential because charge can't leap out of the material into air. Since current is proportional to the Electric Field, what this means is the gradient of ϕ should be tangential. This is implemented by requiring that ϕ should not vary in the normal direction.

4 Code

The default parameters of the code are defined as

```
Nx = 25
Ny = 25
radius = 8
Niter = 1500
```

After defining ***phi*** array with no.of rows and columns as Nx and Ny respectively, we mark the potential of the area of central lead as 1.

4.1 Potential and Central lead

```
Y,X = np.meshgrid(y,x)
ii = np.where(X**2+Y**2 <= radius*radius)
phi[ii] =1.0
```

Y and X are 2D representation of phi and are used to plot contour and much more. The contour plot of the potential at initial condition is provided,

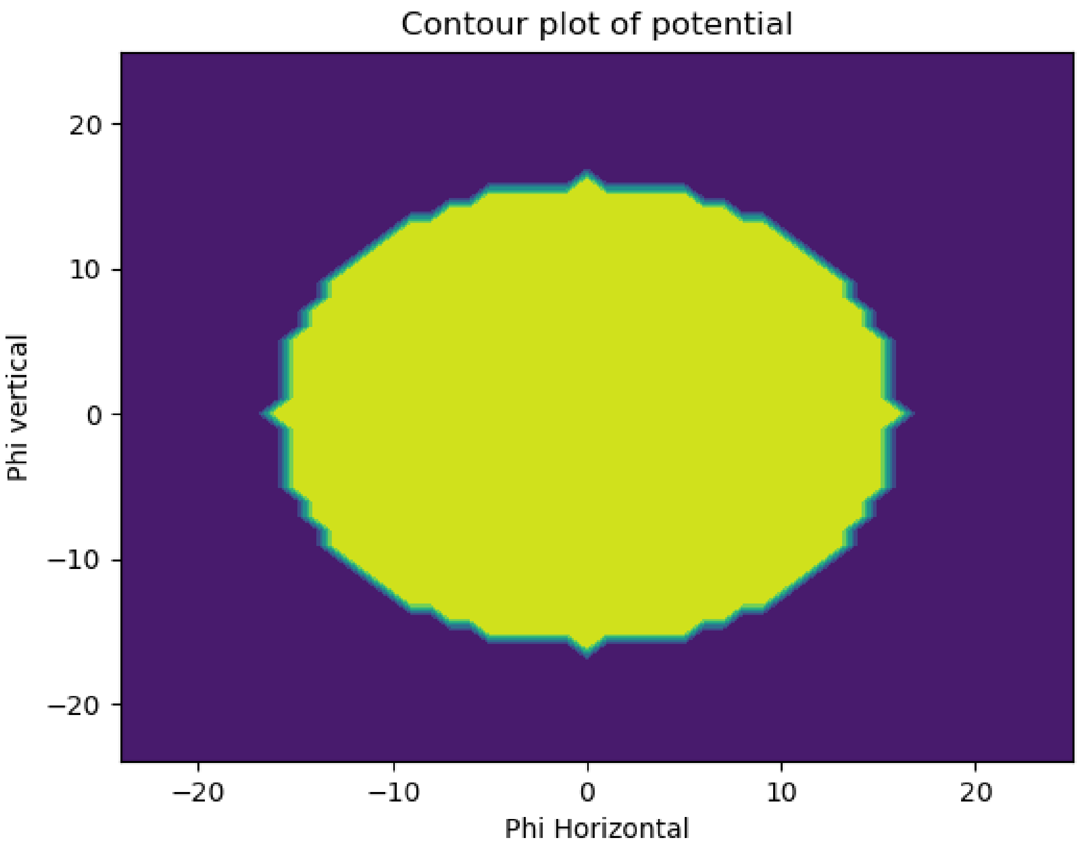

Figure 1

4.2 Iteration loop DC saturated stage of phi

There are 4 parts in the iteration loop

- Save a copy of phi using ***.copy()*** function as using a temporary variable to update gets updated through the same pointer and changes the value.

```
oldphi=phi.copy()
```

- Solving the Laplace difference function and updating phi. Using python sub arrays to do this to finish in one line of code

```
phi[1:-1,1:-1]=0.25*(phi[1:-1,0:-2]+ phi[1:-1,2:]+phi[0:-2,1:-1]+phi[2:,1:-1])
```

- Boundary conditions at boundaries where there is no electrode, the gradient of phi should be tangential. This is implemented by requiring that phi should not vary in the normal direction.

```
    phi[1:-1,0] = phi[1:-1,1]
phi[1:-1,-1] = phi[1:-1,-2]
phi[0,1:-1] = phi[1,1:-1]
phi[ii] = 1.0
```

- Calculating error before and after updation of phi

```
error[k] = (abs(phi - oldphi)).max()
```

4.3 Error visualization and fitting

Now we can fit the error in an arbitrary exponential function to see how the error decreases after 500 iterations. Using a semilog plot, the original error is plotter initially.

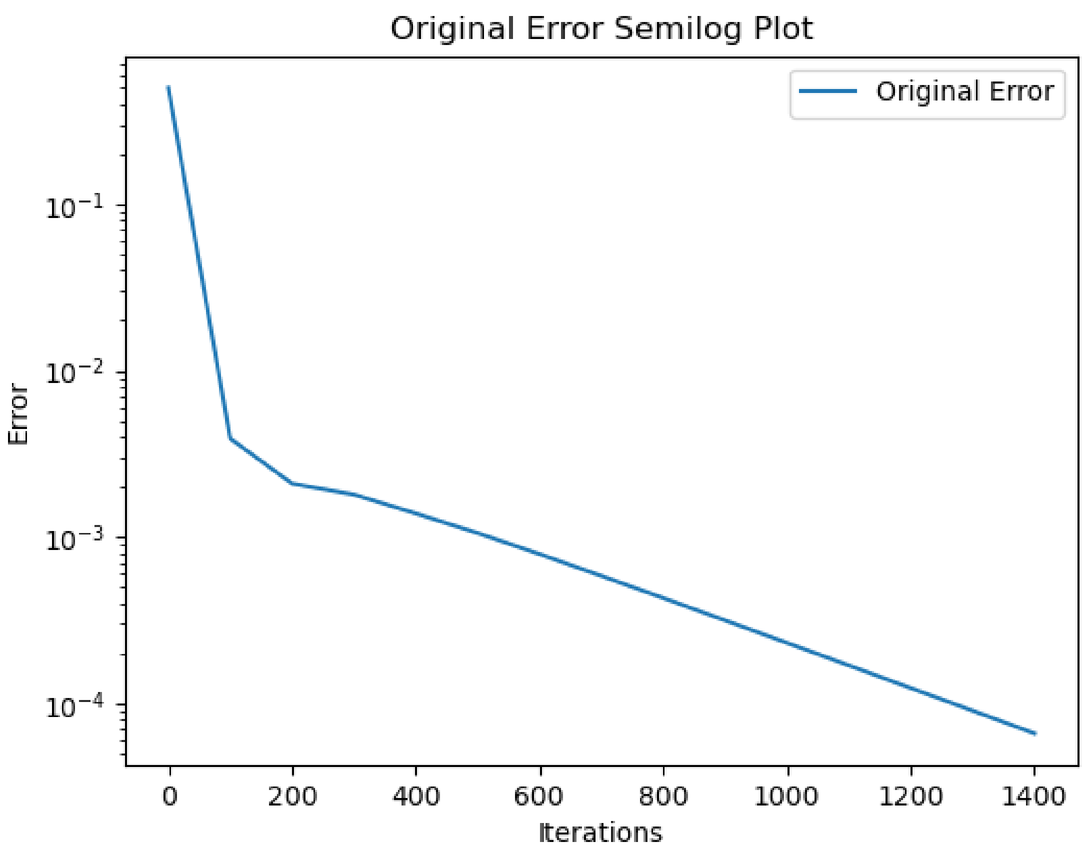

Figure 2

$$y = Aexp(Bx)$$

$$log(y) = log(A) + Bx \tag{1}$$

Thus the exponential function becomes a linear function in the semilog plot and is easily visualized and compared. Also the fitting design is plotted whose estimate is obtained by $\boldsymbol{Leastsquares Method}$. The estimated function plots are also given.

```
Estimated_matrix = np.linalg.lstsq(Variable_matrix,Error_matrix,rcond=-1)[0]
```

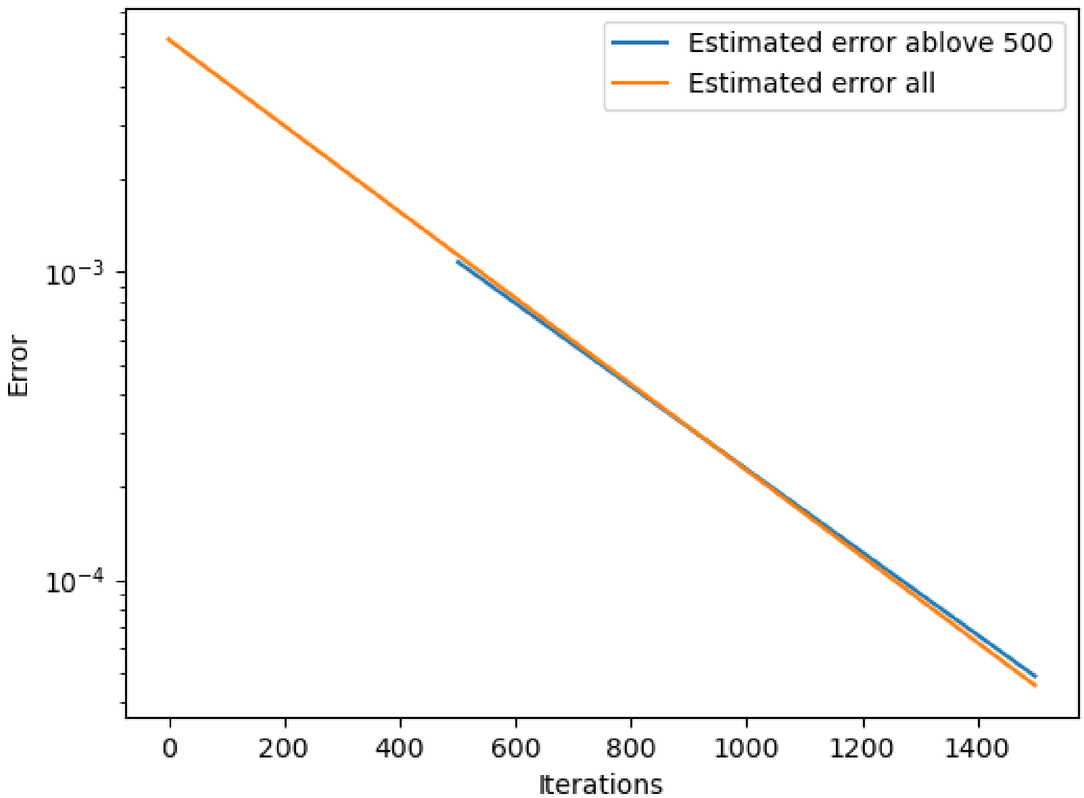

Figure 3

4.4 Surface Plot

Now the 3D mpl toolkit can be used to plot the surface plot of the potential. This plot clearly shows the varying potential throughout the plate area.

```
fig4=figure(4)     # open a new figure
ax=p3.Axes3D(fig4) # Axes3D is the means to do a surface plot
plt.title("The 3-D surface plot of the potential")
surf = ax.plot_surface(Y,X,phi.T,rstride = 1,cstride = 1,cmap = cm.jet,linewidth=0, antialiased=Fal
```

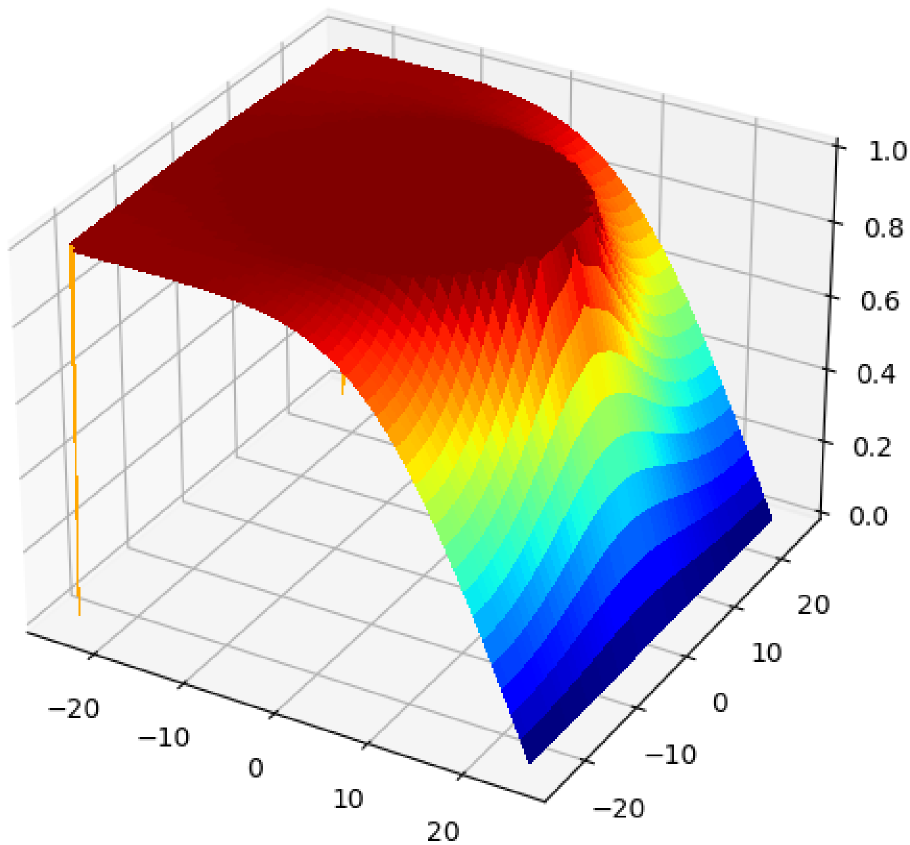

Figure 4

4.5 Contour(Dotted at 1 volt) and Current flow(quiver) plot

In the same figure the contour plot of the potential where the potential is 1 volt and also the current flow is plotted through the quiver plot.

```
Jx = np.zeros((Ny,Nx))
Jy = np.zeros((Ny,Nx))

Jx[1:-1, 1:-1] = (phi[1:-1, 0:-2] - phi[1:-1, 2:])*0.5
Jy[1:-1, 1:-1] = (phi[2:, 1:-1] - phi[0:-2, 1:-1])*0.5

plt.scatter(x[ii[0]],y[ii[1]],color='r')
plt.quiver(-Y,-X,Jx,Jy)
```

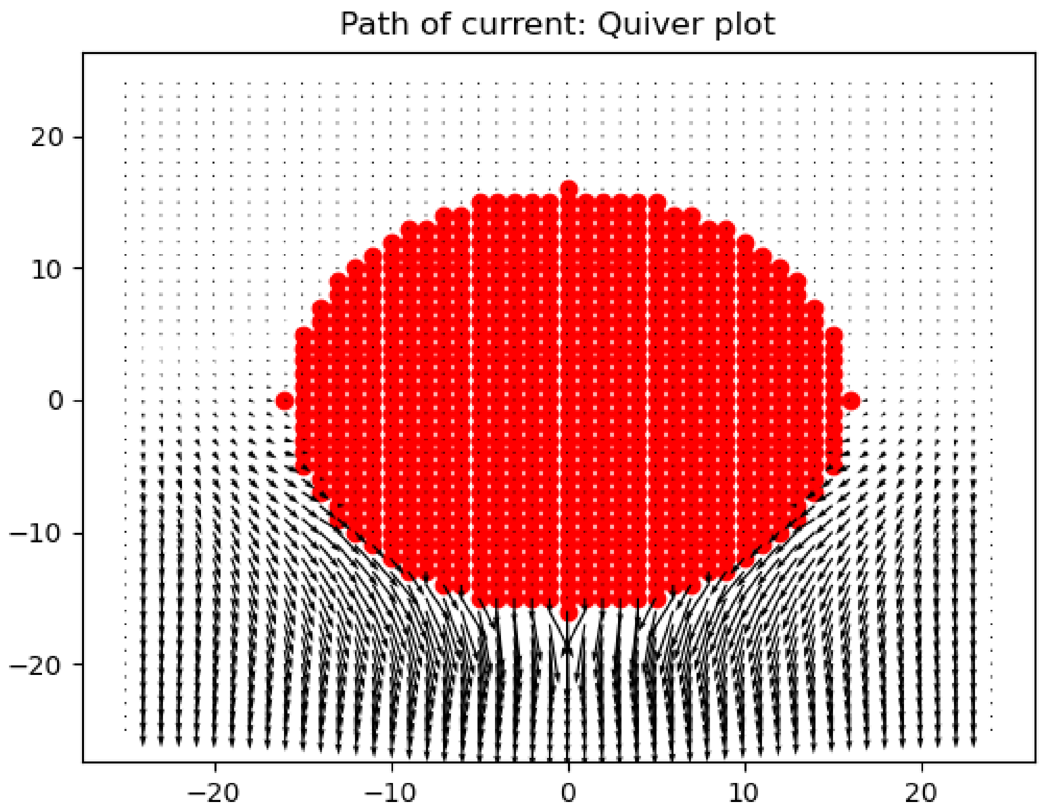

Figure 5

5 Heated regions

The heat is calculated by the formula given by,

$$Q = \sigma^{-1} * mod(J)^2$$

Where The current flow is max, those regions are heated maximum and the plot is also given. This heat is due to ohmic losses from the resistance featured in the plate. The boundary condition is T=300 at the wire and the ground while dT/dn=0 at the other three edges.

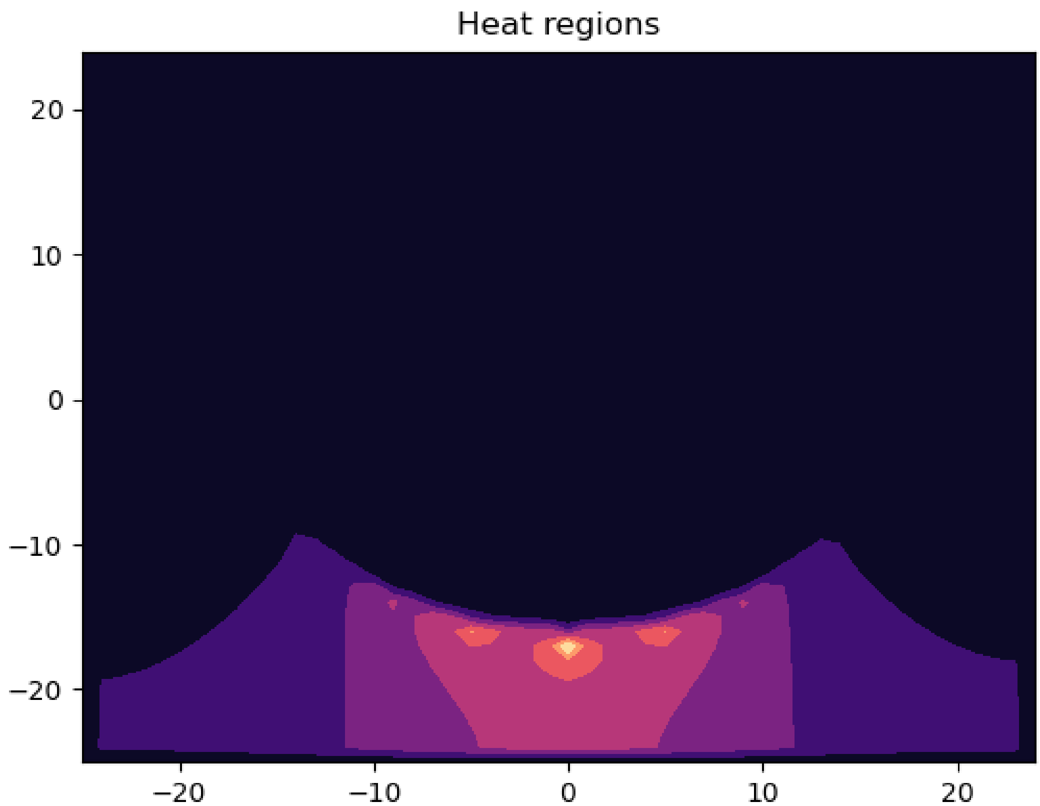

Figure 6