Problem Set 1

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October 10th 2024

(a) The gravitational acceleration enacted on our test object is

$$\vec{a} = -\frac{GM}{r^2}\hat{r}, \tag{1}$$

where G is the gravitational constant, M is the mass of the deflector, r is the separation between the particles and $\hat{r}$ is unit vector pointing towards the point mass from the deflector. Using the impulse approximation

$$\Delta v = a\Delta t, \tag{2}$$

where Δv is the change in velocity, a is the acceleration at closest approach and Δt is the relevant timescale near the deflector. Assuming that minimal deflection, the distance between the deflector and the point mass will be $\sim b$, and assuming we change the velocity of order v_{rel} we can plug into eq. 2 to get

$$v_{rel} = \frac{GM}{b^2}\Delta t, \tag{3}$$

Since for our situation the relavent timescale is

$$\Delta t \sim \frac{2b}{v_{rel}}, \tag{4}$$

we have

$$b = 2\frac{GM}{v_{rel}^2}, \tag{5}$$

Finally, we can substitute

$$v_{esc} = \left(\frac{2GM}{R}\right)^2 \tag{6}$$

to get

$$b = R\left(\frac{v_{esc}}{v_{rel}}\right)^2 \tag{7}$$

(b) Conservation of energy gives

$$(1/2)mv_{rel}^2 = (1/2)mv_{surf}^2 - \frac{GMm}{R}, \tag{8}$$

Where v_{surf} is the velocity of the particle at the surface. Conservation of angular momentum gives

$$v_{rel}b = v_{surf}R \tag{9}$$

Combining these two equations we have

$$b = \sqrt{1 + \frac{2GM}{Rv_rel^2}}R = \sqrt{1 + \left(\frac{v_{esc}}{v_{rel}}\right)^2}R \tag{10}$$

(c) See jupyter notebook