You are given the random vector $\boldsymbol{X}' = [X_1, X_2, X_3, X_4]$with mean vector $\boldsymbol{\mu}'_{\boldsymbol{X}} = [3, 2, -2, 0]$ and variance-covariance matrix

$$\Sigma_X = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Let

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & -3 \end{bmatrix}$$

(a) Find E (**AX**), the mean of **AX**.

(b) Find Cov (**AX**), the variances and covariances of **AX**.

(c) Which pairs of linear combinations have zero covariances?

a.

The required calculation is given as below:

$$E(\mathrm{AX}) = \mathrm{A}\begin{bmatrix} E(X_1) \\ E(X_2) \\ E(X_3) \\ E(X_3) \end{bmatrix}$$

$$= \mathrm{A}\mu_{\mathrm{X}}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ -2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 9 \\ 3 \end{bmatrix}$$

b.

The required calculation is given as below:

$$\mathrm{Cov}(AX) = A\Sigma_X A'$$

Now use the given variance-covariance matrix, $\mathrm{Cov}(AX)$ is given below:

$$\mathrm{Cov}(AX) = A\Sigma_X A'$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 3 & 3 & 3 \\ -3 & 3 & 3 \\ 0 & -6 & 3 \\ 0 & 0 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 36 \end{bmatrix}$$

c.

According to the result obtained in the part **(b)**, the linear combination of all non-diagonal pairs is zero. Thus all pairs of linear combination have zero covariance.