Example 3.4 (Calculating $\mathbf{S}_n$ and R from deviation vectors) Given the deviation vectors in Example 3.3, let us compute the sample variance–covariance matrix $\mathbf{S}_n$ and sample correlation matrix $\mathbf{R}$ using the geometrical concepts just introduced.

From Example 3.3,

$$\mathbf{d}_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{d}_2 = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$$

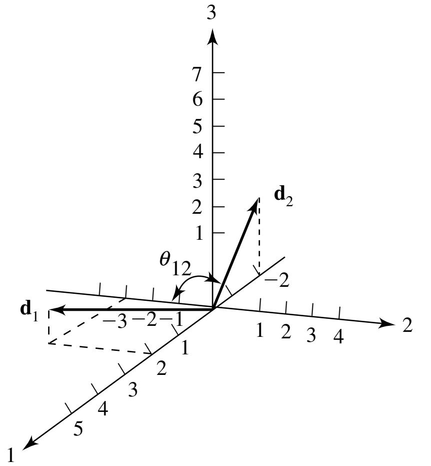

Figure 3.5 The deviation vectors $\mathbf{d}_1$ and $\mathbf{d}_2$.

These vectors, translated to the origin, are shown in Figure 3.5. Now,

$$\mathbf{d}_1'\mathbf{d}_1 = [2 \quad -3 \quad 1]\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = 14 = 3s_{11}$$

or $s_{11} = \frac{14}{3}$. Also,

$$\mathbf{d}_2'\mathbf{d}_2 = [-2 \quad 0 \quad 2]\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} = 8 = 3s_{22}$$

or $s_{22} = \frac{8}{3}$. Finally,

$$\mathbf{d}_1'\mathbf{d}_2 = [2 \quad -3 \quad 1]\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} = -2 = 3s_{12}$$

or $s_{12} = -\frac{2}{3}$. Consequently,

$$r_{12} = \frac{s_{12}}{\sqrt{s_{11}}\sqrt{s_{22}}} = \frac{-\frac{2}{3}}{\sqrt{\frac{14}{3}}\sqrt{\frac{8}{3}}} = -.189$$

and

$$\mathbf{S}_n = \begin{bmatrix} \frac{14}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{8}{3} \end{bmatrix}, \qquad \mathbf{R} = \begin{bmatrix} 1 & -.189 \\ -.189 & 1 \end{bmatrix} \qquad \blacksquare$$