Plotting Z distributions

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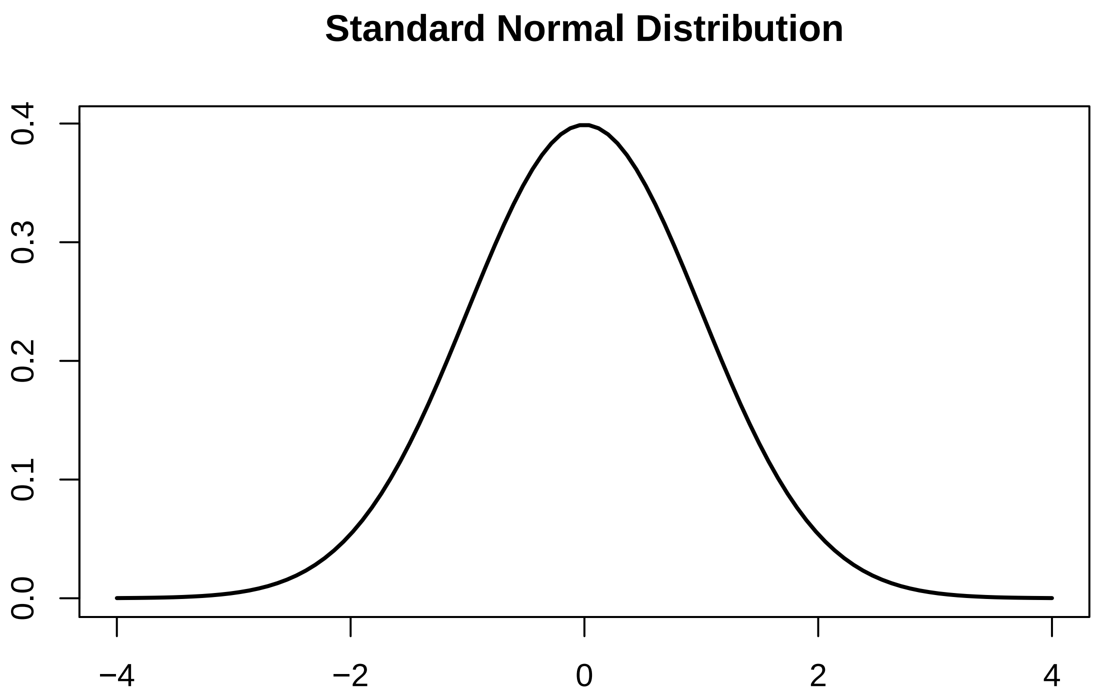
Graphing a Z-distribution

A z-distribution is a Gaussian distribution, which has a mean of 0 and standard deviation of 1. To see why this is, see the section below called Demonstrations of $\mu_z = 0$ and $\sigma_z = 1$.

```
mu = 0; sd = 1
x = seq(-4, 4, length = 100)
y = dnorm(x, mu, sd)
plot(x, y, type="l", lwd = 2, ylab = "", xlab = "", main="Standard Normal Distribution")
```



Demonstrations of $\mu_z = 0$ and $\sigma_z = 1$

To show that the mean of the Z-distribution is 0 and the standard deviation of the Z-distribution is 1, we can take two approaches. First we simply use algebra.

First Approach

Mean

To show that $\mu_z = 0$ it is sufficient to show that $\sum z = 0$.

$$\sum z = \sum \frac{z - \mu_z}{\sigma_z} = \frac{1}{\sigma_z} \sum (z - \mu_z) = \frac{1}{\sigma_z}((\sum z) - N\mu_z) = \frac{1}{\sigma_z}(N\mu_z - N\mu_z) = 0$$

Standard Deviation

We know that $\sum z = 0$, and next we need to show that $N\sigma_z^2 = \sum z^2$ and $\sum z^2 = N$.

$$\sigma_z = \sqrt{\frac{\sum(z-\mu_z)^2}{N}} = \sqrt{\frac{\sum z^2}{N}} \Longrightarrow \sigma_z^2 = \frac{\sum z^2}{N} \Longrightarrow N\sigma_z^2 = \sum z^2$$

$$\sum z^2 = \sum \frac{(z-\mu_z)^2}{\sigma_z^2} = \frac{1}{\sigma_z^2}\sum z^2 = \frac{1}{\sigma_z^2}(N\sigma_z^2) = N$$

Now we can see the solution:

$$\sigma_z = \sqrt{\frac{\sum(z^2) - \frac{(\sum z)^2}{N}}{N}} = \sqrt{\frac{N - \frac{0}{N}}{N}} = \sqrt{\frac{N}{N}} = \pm 1$$

Second approach

Mean A Gaussian distribution is defined by the function $f(x) = \frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}x^2)$ where $x \in X \subseteq \mathbb{R}$.

$$\mu_z = \int_{-\infty}^{\infty} x f(x) dx = (2\pi)^{-1/2}(-1+0) + (2\pi)^{-1/2}(0+1) = 0$$

Standard Deviation

Note that $\sigma_z^2 = Var(Z) = E(Z^2) - E(Z)^2 = E(Z^2) - (\mu_z)^2 = E(Z^2)$. So we solve for $E(Z^2)$, which gives us $Var(Z)$, then solve for $SD(Z)$ or σ_z.

$$E(Z^2) = \int_{-\infty}^{\infty} x^2 f(x) dx = (2\pi)^{-1/2} \int_{-\infty}^{\infty} exp(-\frac{1}{2}x^2) dx = \int_{-\infty}^{\infty} f(x) dx = 1$$

(The integral of a PDF over its support is always 1)

$$Var(Z) = E(Z^2) - E(Z)^2 = 1 - 0 = 1 \Longrightarrow \sigma_z^2 = 1$$
$$\Longrightarrow \sigma_z = 1$$