MidTermAssignmentwith8Facts

November 14, 2024

1 Python assignment

Installing yahoofinance

```
[87]: #pip install yfinance
```

Installing statsmodels

```
[88]: #pip install statsmodels
```

2 First pandas dataframe of Amazon stocks

```
[90]: # Importing Amazon stock from yahoo finance
     Amazon = yf.download("AMZN", start="1999-01-21", end="2024-10-16")
     Amazon.head()
     1 of 1 completed
[90]:
                   Open
                            High
                                      Low
                                              Close Adj Close
                                                                 Volume
     Date
     1999-01-21 2.612500 2.759375 2.314063 2.650000
                                                     2.650000 940964000
                                                     3.075000 875316000
     1999-01-22 2.487500 3.146875 2.468750 3.075000
     1999-01-25 3.037500 3.084375 2.750000 2.809375
                                                     2.809375 546476000
     1999-01-26 2.815625 3.031250 2.765625 2.877344
                                                     2.877344 490696000
     1999-01-27 3.353125 3.493750 3.000000 3.140625
                                                     3.140625 700452000
[91]: #pip install perfplot
[92]: latex_table = Amazon.head().to_latex(index=True)
     with open("Latex/table.tex", "w") as file:
         file.write(latex_table)
```

/var/folders/5r/ft807c7n1ngd3fpt2_gwsg0m0000gn/T/ipykernel_78356/3304008134.py:1 : FutureWarning: In future versions `DataFrame.to_latex` is expected to utilise the base implementation of `Styler.to_latex` for formatting and rendering. The arguments signature may therefore change. It is recommended instead to use `DataFrame.style.to_latex` which also contains additional functionality. latex_table = Amazon.head().to_latex(index=True)

3 Cheking if timestamp is 25 years

```
[93]: print('Amazon data range is: ',Amazon.index[0],Amazon.index[-1])

#trying to find gaps

#First create a dataframe for a fullrange of our index, without any gap withuthe following formula:
full_range = pd.date_range(start=Amazon.index.min(), end=Amazon.index.max(),uther compare to our dataframe:

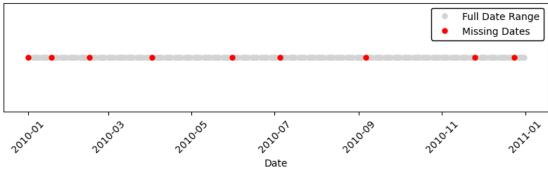
MissingDays=full_range.difference(Amazon.index)

#Print the count and the detail preview:
print('Missing Days count is: ',len(MissingDays))
print("missing dates",MissingDays)
```

```
print("total size of the 25 years range",len(Amazon.index),"the ratio of ⊔
      ⇒index))
     #We can see that data have ponctual gaps, no issue here we can still use it
     Amazon data range is: 1999-01-21 00:00:00 2024-10-15 00:00:00
     Missing Days count is: 238
     missing dates DatetimeIndex(['1999-02-15', '1999-04-02', '1999-05-31',
     '1999-07-05',
                   '1999-09-06', '1999-11-25', '1999-12-24', '2000-01-17',
                   '2000-02-21', '2000-04-21',
                   '2023-11-23', '2023-12-25', '2024-01-01', '2024-01-15',
                   '2024-02-19', '2024-03-29', '2024-05-27', '2024-06-19',
                   '2024-07-04', '2024-09-02'],
                  dtype='datetime64[ns]', length=238, freq=None)
     total size of the 25 years range 6476 the ratio of missing inputs/ total size of
     the data = 3.675108091414453
[94]: Amazon.index
     #extracting adjusted
     Amzn adj=Amazon['Adj Close']
     Amzn_adj.index = Amazon.index
     #display first 5 rows, now it is a pandas series instead of a dataframe
     Amzn_adj.head()
[94]: Date
     1999-01-21
                  2.650000
     1999-01-22 3.075000
     1999-01-25 2.809375
     1999-01-26 2.877344
     1999-01-27
                  3.140625
     Name: Adj Close, dtype: float64
     For the possible gaps in data, we plot them here
[95]: #plot the missing dates
     full data = Amazon.reindex(full range)
     #zoom in over one year
     start_date = "2010-01-01"
     end_date = "2011-01-01"
     filtered_full_range = full_range[(full_range >= start_date) & (full_range <=_u
      ⊶end date)]
     filtered_missing_dates = MissingDays[(MissingDays >= start_date) & (MissingDays_
      = end_date)]
```

```
plt.figure(figsize=(10, 2))
\# Plot all dates in the filtered range with gray dots (showing the full \sqcup
⇔timeline for this period)
plt.plot(filtered_full_range, [1] * len(filtered_full_range), 'o', __
 ⇔color='lightgray', markersize=5, label="Full Date Range")
# Overlay red dots only on the missing dates within the filtered range
plt.plot(filtered_missing_dates, [1] * len(filtered_missing_dates), 'ro', u
 →markersize=5, label="Missing Dates")
# Customize plot
plt.title("Missing Dates in Full Date Range (2010 to 2011)",color='black')
plt.xlabel("Date",color='black')
plt.yticks([]) # Hide y-axis labels for clarity
plt.xticks(rotation=45,color='black')
plt.legend(facecolor='white', edgecolor='black', framealpha=1, fontsize=10)
#Saving the plot in pdf format
plt.savefig('Latex/Img/MissingDates(2010_to_2011).pdf', format='pdf', u
 ⇔bbox_inches='tight')
plt.show()
```





4 PRICES

```
[96]: # extract the closing prices of the Amazon stok (as in lecture)
Pt_d_all = Amazon["Adj Close"]
Pt_d_all.name = 'Pt.d'
# mutate the Index into a DatetimeIndex
Pt_d_all.index = pd.to_datetime(Pt_d_all.index)
Pt_d_all.head()
```

```
[96]: Date
                    2.650000
     1999-01-21
      1999-01-22
                   3.075000
      1999-01-25
                    2.809375
      1999-01-26
                    2.877344
      1999-01-27
                    3.140625
     Name: Pt.d, dtype: float64
     Compute log price
[97]: pt_d_all = np.log(Pt_d_all)
      pt d all.name = 'pt.d'
      pt_d_all.head()
[97]: Date
      1999-01-21
                    0.974560
      1999-01-22
                  1.123305
      1999-01-25 1.032962
      1999-01-26 1.056868
      1999-01-27
                    1.144422
     Name: pt.d, dtype: float64
     Compute weekly monthly and yearly
[98]: pt_w_all = pt_d_all.resample('W').last()
      pt_m_all = pt_d_all.resample('M').last()
      pt_y_all = pt_d_all.resample('Y').last()
      # and rename them:
      pt_w_all.name = 'pt.w.all'
      pt_m_all.name = 'pt.m.all'
      pt_y_all.name = 'pt.y.all'
      #idem for simply prices
      Pt_w_all = Pt_d_all.resample('W').last()
      Pt_m_all = Pt_d_all.resample('M').last()
      Pt_y_all = Pt_d_all.resample('Y').last()
      # and rename them:
      Pt_w_all.name = 'Pt_w_all'
      Pt_m_all.name = 'Pt_m_all'
      Pt_y_all.name = 'Pt_y_all'
     Plot the simple prices
[99]: # set the 1x4 windows layout
      fig, axs = plt.subplots(1, 4, figsize=(15, 5))
      # Daily Price
      axs[0].plot(Pt_d_all.index, Pt_d_all, color='blue')
      axs[0].set_title('Daily price: $P_{d,t}$')
      axs[0].grid(True)
```

```
# Weekly price
axs[1].plot(Pt_w_all.index, Pt_w_all, color='blue')
axs[1].set_title('Weekly price: $P_{w,t}$')
axs[1].grid(True)
# Monthly price
axs[2].plot(Pt_m_all.index, Pt_m_all, color='blue')
axs[2].set_title('Monthly price: $P_{m,t}$')
axs[2].grid(True)
#Yearly price
axs[3].plot(Pt_y_all.index, Pt_y_all, color='blue')
axs[3].set_title('Yearly price: $P_{y,t}$')
axs[3].grid(True)
# Manage margings and plot
plt.tight_layout()
plt.savefig('Latex/Img/prices_time.pdf', format='pdf', bbox_inches='tight')
plt.show()
```



Adding python code to the latex document in the appendix part

```
return mean, std_dev

data = [1, 2, 3, 4, 5]
mean, std_dev = analyze_data(data)
print(f"Mean: {mean}, Standard Deviation: {std_dev}")
\end{lstlisting}
"""

# Write to the 'code_appendix.tex' file
with open("Latex/code_appendix.tex", "w") as file:
    file.write(code_content)
```

5 Calculating returns

```
[101]: #calculating return
      #log returns VS simple returns
      Rt_d_all_temp = Pt_d_all.pct_change()
      rt_d_all_temp = pt_d_all.diff()
      rt_d_all_temp, Rt_d_all_temp
[101]: (Date
       1999-01-21
                          NaN
       1999-01-22
                   0.148745
       1999-01-25 -0.090343
       1999-01-26
                     0.023906
       1999-01-27
                     0.087554
       2024-10-09
                     0.013319
       2024-10-10
                     0.007961
       2024-10-11
                     0.011559
       2024-10-14 -0.006802
       2024-10-15
                     0.000800
       Name: pt.d, Length: 6476, dtype: float64,
       Date
       1999-01-21
                          {\tt NaN}
       1999-01-22
                   0.160377
       1999-01-25
                   -0.086382
       1999-01-26
                   0.024194
       1999-01-27
                     0.091501
       2024-10-09
                     0.013408
       2024-10-10
                     0.007993
       2024-10-11
                  0.011626
       2024-10-14 -0.006779
       2024-10-15
                     0.000800
```

Name: Pt.d, Length: 6476, dtype: float64)

Compute daily, weekly, and monthly

```
[102]: rt_d_all = pt_d_all.diff().dropna() #dropna remove the first NaN
       rt_w_all = pt_w_all.diff().dropna()
       rt_m_all = pt_m_all.diff().dropna()
       rt_y_all = pt_y_all.diff().dropna()
       Rt_d_all = Pt_d_all.pct_change().dropna() #dropna remove the first NaN
       Rt_w_all = Pt_w_all.pct_change().dropna()
       Rt_m_all = Pt_m_all.pct_change().dropna()
       Rt_y_all = Pt_y_all.pct_change().dropna()
       # and rename them:
       rt d all.name = 'rt d all'
       rt_w_all.name = 'rt_w_all'
       rt m all.name = 'rt m all'
       rt_y_all.name = 'rt_y_all'
       Rt_d_all.name = 'Rt_d_all'
       Rt_w_all.name = 'Rt_w_all'
       Rt_m_all.name = 'Rt_m_all'
       Rt_y_all.name = 'Rt_y_all'
       rt_d_all.head()
       Rt_d_all.head()
```

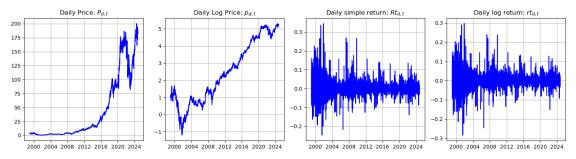
The first returns are correctly computed, we have to be careful to the dropna

Let's plot returns

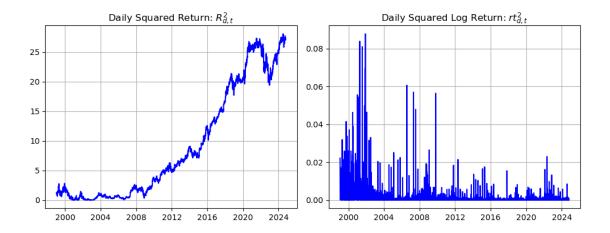
```
[103]: # set the 1x3 windows layout
fig, axs = plt.subplots(1, 4, figsize=(15, 4))
# Daily Price
axs[0].plot(Pt_d_all.index, Pt_d_all, color='blue')
axs[0].set_title('Daily Price: $P_{d,t}$')
axs[0].grid(True)
# Daily log price
axs[1].plot(pt_d_all.index, pt_d_all, color='blue')
axs[1].set_title('Daily Log Price: $p_{d,t}$')
```

```
axs[1].grid(True)
# Daily simple returns
axs[2].plot(Rt_d_all.index, Rt_d_all, color='blue')
axs[2].set_title('Daily simple return: $Rt_{d,t}$')
axs[2].grid(True)
# Daily log returns
axs[3].plot(rt_d_all.index, rt_d_all, color='blue')
axs[3].set_title('Daily log return: $rt_{d,t}$')
axs[3].grid(True)

plt.tight_layout()
plt.savefig('Latex/Img/log_returns.pdf', format='pdf', bbox_inches='tight')
plt.show()
```



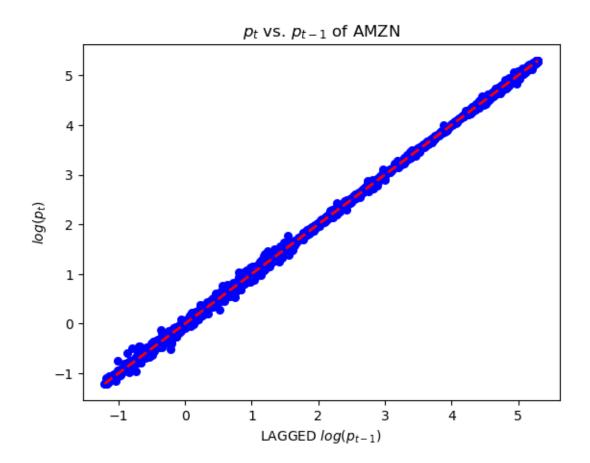
Squared returns



6 Scatterplot of $p_t - p_{t-1}$

First define the function for plotting a scatterplot

```
[106]: lag1_scatterplot(pt_d_all,"LAGGED $log(p_{t-1})$","$log(p_t)$","$p_t$ vs._ \Rightarrowp_{t-1}$ of AMZN")
```



7 Autocorrelation

8 4.1/ Prices are non-stationary

1. Profile of Log Prices with Time

- 2. Pt VS P(t-1)
- 3. Autocorrelation of Daily Prices

```
[109]: import matplotlib.pyplot as plt

# Set the layout for 1x3 subplots (though you may not need all subplots)
fig, axs = plt.subplots(1, 1, figsize=(15, 4))

# Plot Daily Price
axs.plot(pt_d_all.index, pt_d_all, color='blue')
axs.set_title('Log Daily Price: $p_{d,t}$')
axs.grid(True)

# Show the plot
plt.show()
```

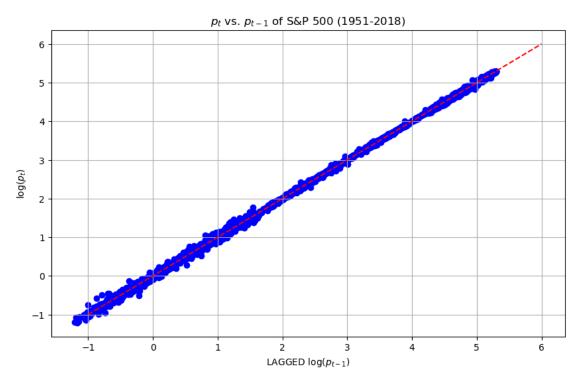


```
import yfinance as yf
import matplotlib.pyplot as plt
import numpy as np

# Estrai i log-prezzi giornalieri
log_price_daily = pt_d_all

# Calcola il log-prezzo al giorno precedente
log_price_previous = log_price_daily.shift(1)

# Creazione dello scatter plot
plt.figure(figsize=(10, 6))
plt.scatter(log_price_previous, log_price_daily, color='blue')
plt.plot([-1, 6], [-1, 6], color='red', linestyle='--')
plt.title('$p_t$ vs. $p_{t-1}$ of S&P 500 (1951-2018)')
plt.xlabel(r'LAGGED $\log(p_{t-1})$')
plt.ylabel(r'$\log(p_t)$')
plt.grid(True)
```



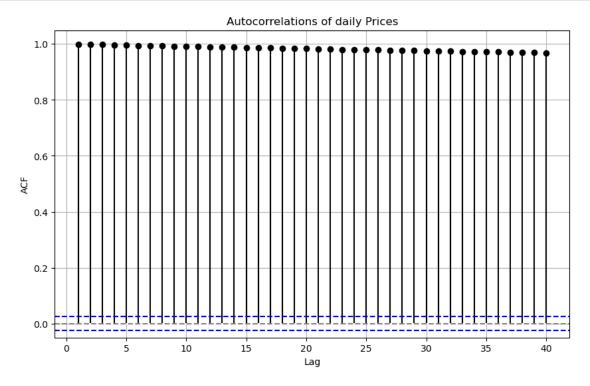
```
[111]: from statsmodels.tsa.stattools import acf

# Calculate empirical autocorrelation
lags = 40
acf_values = acf(Pt_d_all, nlags=lags)

# Calculate Bartlett intervals
Bart_Int = 1.96 / np.sqrt(len(Pt_d_all))

# Create the autocorrelation plot with Bartlett intervals
plt.figure(figsize=(10, 6))
plt.stem(np.arange(1, lags + 1), acf_values[1:], linefmt='k-', markerfmt='ko', upasefmt='w-')
plt.axhline(y=0, color='gray', linestyle='--')
plt.axhline(y=Bart_Int, color='blue', linestyle='--')
plt.axhline(y=Bart_Int, color='blue', linestyle='--')
```

```
plt.title('Autocorrelations of daily Prices')
plt.xlabel('Lag')
plt.ylabel('ACF')
plt.grid(True)
#plt.savefig('Latex/Autocorrel_daily.pdf', format='pdf', bbox_inches='tight')
plt.show()
```



9 4.2/ Log Returns are Stationary

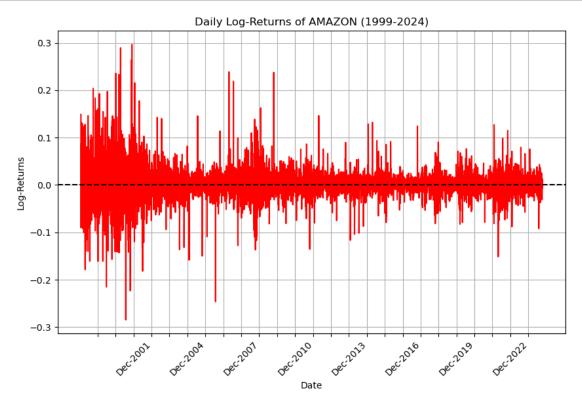
- 1. Profile of Log Returns with Time
- 2. Rt VS R(t-1)
- 3. Autocorrelation of Daily Returns

```
[112]: import yfinance as yf
import matplotlib.pyplot as plt
import numpy as np

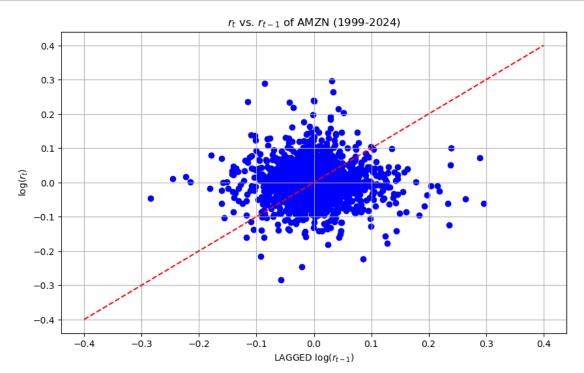
# Calculate daily log returns
log_returns_daily = rt_d_all

# Create the plot of daily log returns with a black horizontal line
plt.figure(figsize=(10, 6))
```

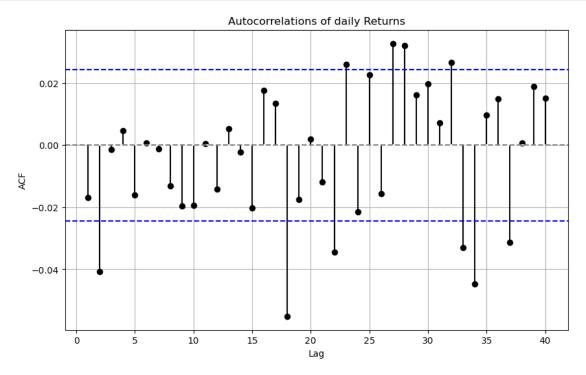
```
plt.plot(log_returns_daily.index, log_returns_daily, color='red')
plt.axhline(y=0, color='black', linestyle='--')
plt.title('Daily Log-Returns of AMAZON (1999-2024)')
plt.xlabel('Date')
plt.ylabel('Log-Returns')
plt.grid(True)
# Customizing x-axis labels for December 31 of each year
date_labels = pd.date_range(start='1999-12-31', end='2023-12-31', freq='A-DEC')
# Show 1 tick every 3 years
formatted_labels = [f'Dec-{date.year}' if date.year % 3 == 0 else '' for date__
→in date_labels]
# Add labels and rotate them
plt.xticks(date_labels, formatted_labels, rotation=45)
# Save the plot in png format
plt.savefig('Latex/Img/Daily Log Returns.pdf', format='pdf',
 ⇔bbox_inches='tight')
plt.show()
```



```
[113]: import yfinance as yf
       import matplotlib.pyplot as plt
       import numpy as np
       # Get the Daily Log Returns
       log_return_daily = rt_d_all
       # Calculation of the Lagged log returns
       log_return_previous = log_return_daily.shift(1)
       # Creation of the Scatter Plot
       plt.figure(figsize=(10, 6))
       plt.scatter(log_return_previous, log_return_daily, color='blue')
       plt.plot([-0.4, 0.4], [-0.4, 0.4], color='red', linestyle='--')
       plt.title('$r_t$ vs. $r_{t-1}$ of AMZN (1999-2024)')
       plt.xlabel(r'LAGGED $\log(r_{t-1})$')
       plt.ylabel(r'$\lceil (r_t)^* \rceil)
       plt.grid(True)
       # Saving the Image
       plt.savefig('Latex/Img/LogReturns_vs_LaggedLogReturns.pdf', format='png', | )
        ⇔bbox_inches='tight')
       plt.show()
```



```
[114]: from statsmodels.tsa.stattools import acf
       # Calculate empirical autocorrelation
       lags = 40
       acf_values = acf(Rt_d_all, nlags=lags)
       # Calculate Bartlett intervals
       Bart_Int = 1.96 / np.sqrt(len(Rt_d_all))
       # Create the autocorrelation plot with Bartlett intervals
       plt.figure(figsize=(10, 6))
       plt.stem(np.arange(1, lags + 1), acf_values[1:], linefmt='k-', markerfmt='ko', u
        ⇒basefmt='w-')
       plt.axhline(y=0, color='gray', linestyle='--')
       plt.axhline(y=Bart_Int, color='blue', linestyle='--')
      plt.axhline(y=-Bart_Int, color='blue', linestyle='--')
       plt.title('Autocorrelations of daily Returns')
       plt.xlabel('Lag')
       plt.ylabel('ACF')
       plt.grid(True)
       #plt.savefig('Latex/Autocorrel_Returns_daily.pdf', format='pdf',__
        ⇔bbox_inches='tight')
       plt.show()
```

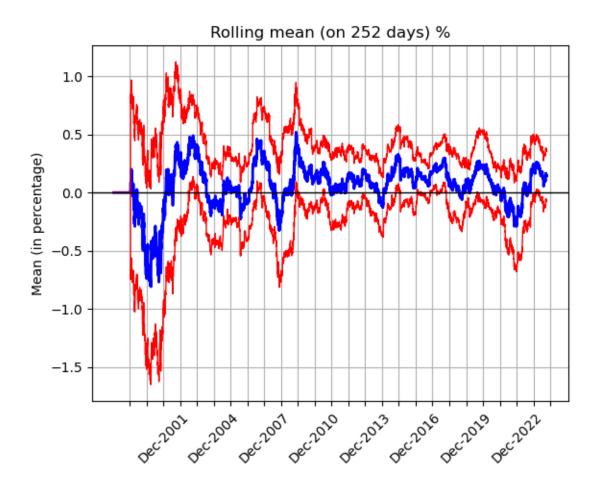


10 4.3/ Are Log Returns Asymmetric?

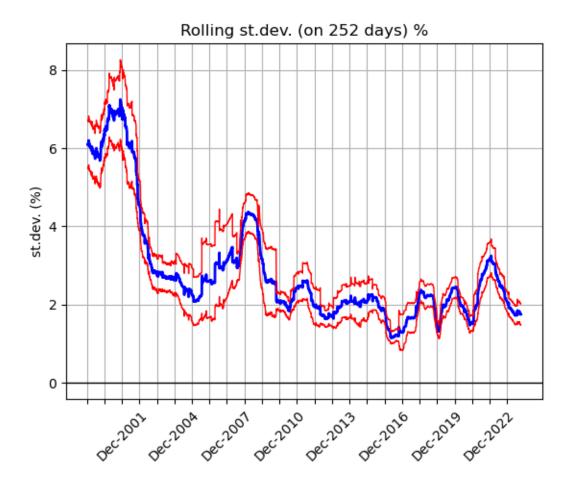
- 1. Rolling Mean
- 2. Rolling Standard Deviation
- 3. Rolling Skewness
- 4. Current Skewness and Interpretation

```
[161]: import yfinance as yf
       import numpy as np
       import matplotlib.pyplot as plt
       from scipy.stats import skew, kurtosis
       import pandas as pd
       # Compute daily log-returns
       log_returns_daily = rt_d_all
       # set the rolling window equal to 252 days
       window_length = 252
       T = log_returns_daily.shape[0]
       # Create an empty matrix to store data
       roll_mom_manual = np.zeros((T, 5))
       # Run a for loop to fill the matrix with moments
       for i in range(window_length, T):
           est_window = np.arange(i - window_length + 1, i + 1)
           # Use .iloc to select rows by integer positions, not labels
           y = log_returns_daily.iloc[est_window]
           # Compute the moments for each
           roll_mom_manual[i, 0] = np.mean(y)
           roll_mom_manual[i, 1] = np.std(y, ddof=1)
           roll_mom_manual[i, 2] = skew(y)
           roll_mom_manual[i, 3] = kurtosis(y)
           roll_mom_manual[i, 4] = np.mean((y - np.mean(y))**4)
       # Plot results of manually computed rolling mean
       mean_plot_man = roll_mom_manual[:, 0]
       mean_plot_man_ub = mean_plot_man + 1.96 * roll_mom_manual[:, 1] / np.
       ⇔sqrt(window_length)
       mean_plot_man_lb = mean_plot_man - 1.96 * roll_mom_manual[:, 1] / np.
        ⇔sqrt(window_length)
```

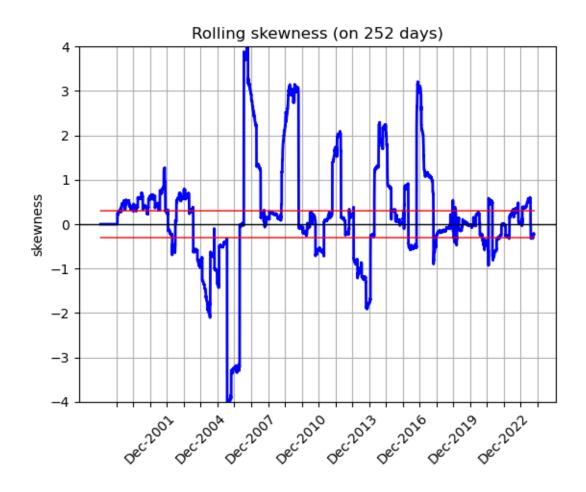
```
data2plot_na = np.column_stack((mean_plot_man, mean_plot_man_lb,__
 →mean_plot_man_ub))
data_index = log_returns_daily.index
data2plot_na = pd.DataFrame({'Mean': mean_plot_man, 'LowerBound':
 →mean_plot_man_lb, 'UpperBound': mean_plot_man_ub},
                               index=data_index)
# Select only rows without missing values
data2plot = data2plot na.dropna()
# retrieve the data index
data2plot
# Customizing x-axis labels for December 31 of each year
date_labels = pd.date_range(start='1999-12-31', end='2024-12-31', freq='A-DEC')
# Show 1 tick every 3 years
formatted_labels = [f'Dec-{date.year}' if date.year % 3 == 0 else '' for date__
 →in date_labels]
# Add labels and rotate them
plt.xticks(date_labels, formatted_labels, rotation=45)
# Plot the data
plt.plot(data2plot.index, data2plot["Mean"] * 100, color='blue', linestyle='-', u
 →linewidth=2)
plt.plot(data2plot.index, data2plot["LowerBound"] * 100, color='red', __
 ⇒linestyle='-', linewidth=1)
plt.plot(data2plot.index, data2plot["UpperBound"] * 100, color='red',__
 →linestyle='-', linewidth=1)
plt.grid(True)
plt.xlabel('')
plt.ylabel('Mean (in percentage)')
plt.title('Rolling mean (on 252 days) %')
plt.axhline(0, linestyle='-', color='black', linewidth=1) # Add a zero line
plt.savefig('Latex/Img/AMZN_MEAN_rolling_1999_2024.pdf', format='pdf',u
 ⇔bbox_inches='tight')
plt.show()
```



```
# retrieve the data index
data2plot
# Customizing x-axis labels for December 31 of each year
date_labels = pd.date_range(start='1999-01-01', end='2024-10-01', freq='A-DEC')
# Show 1 tick every 3 years
formatted_labels = [f'Dec-{date.year}' if date.year % 3 == 0 else '' for date__
 →in date_labels]
# Add labels and rotate them
plt.xticks(date_labels, formatted_labels, rotation=45)
# Plot the data
plt.plot(data2plot.index, data2plot["StD"] * 100, color='blue', linestyle='-', u
 ⇒linewidth=2)
plt.plot(data2plot.index, data2plot["LowerBound"] * 100, color='red', u
 ⇔linestyle='-', linewidth=1)
plt.plot(data2plot.index, data2plot["UpperBound"] * 100, color='red',__
 →linestyle='-', linewidth=1)
plt.xlabel('')
plt.grid(True)
plt.ylabel('st.dev. (%)')
plt.title('Rolling st.dev. (on 252 days) %')
plt.axhline(0, linestyle='-', color='black', linewidth=1) # Add a zero line
plt.savefig('Latex/Img/Fact7_AMZN_rolling_stdev.pdf', format='pdf', __
  ⇔bbox_inches='tight')
plt.show()
/var/folders/5r/ft807c7n1ngd3fpt2_gwsg0m0000gn/T/ipykernel_78356/1880708915.py:4
: RuntimeWarning: divide by zero encountered in true_divide
  sd_plot_ub = roll_mom_manual[:,1]+1.96*(1/(2*sd_plot)*np.sqrt(mu4-
sd_plot**4))/np.sqrt(window_length)
/var/folders/5r/ft807c7n1ngd3fpt2 gwsg0m0000gn/T/ipykernel 78356/1880708915.py:4
: RuntimeWarning: invalid value encountered in multiply
  sd plot_ub = roll_mom_manual[:,1]+1.96*(1/(2*sd_plot)*np.sqrt(mu4-
sd_plot**4))/np.sqrt(window_length)
/var/folders/5r/ft807c7n1ngd3fpt2 gwsg0m0000gn/T/ipykernel 78356/1880708915.py:5
: RuntimeWarning: divide by zero encountered in true_divide
  sd_plot_lb = roll_mom_manual[:,1]-1.96*(1/(2*sd_plot)*np.sqrt(mu4-
sd plot**4))/np.sqrt(window length)
/var/folders/5r/ft807c7n1ngd3fpt2_gwsg0m0000gn/T/ipykernel_78356/1880708915.py:5
: RuntimeWarning: invalid value encountered in multiply
  sd_plot_lb = roll_mom_manual[:,1]-1.96*(1/(2*sd_plot)*np.sqrt(mu4-
sd_plot**4))/np.sqrt(window_length)
```



```
data2plot
# Customizing x-axis labels for December 31 of each year
date_labels = pd.date_range(start='1999-12-31', end='2024-12-31', freq='A-DEC')
# Show 1 tick every 3 years
formatted_labels = [f'Dec-{date.year}' if date.year % 3 == 0 else '' for date__
 →in date_labels]
# Add labels and rotate them
plt.xticks(date_labels, formatted_labels, rotation=45)
# Plot the data
plt.plot(data2plot.index, data2plot["Skewness"], color='blue', linestyle='-',__
 →linewidth=2)
plt.plot(data2plot.index, data2plot["LowerBound"], color='red', linestyle='-',u
 →linewidth=1)
plt.plot(data2plot.index, data2plot["UpperBound"], color='red', linestyle='-', u
 →linewidth=1)
plt.ylim(-4,4)
plt.grid(True)
plt.xlabel('')
plt.ylabel('skewness')
plt.title('Rolling skewness (on 252 days)')
plt.axhline(0, linestyle='-', color='black', linewidth=1) # Add a zero line
plt.savefig('Latex/Img/AMZN_skew_rolling_1999_2024.pdf', format='pdf',__
 ⇔bbox_inches='tight')
plt.show()
```



11 4.4/ Heavy Tailed Distribution for the Daily Log Returns?

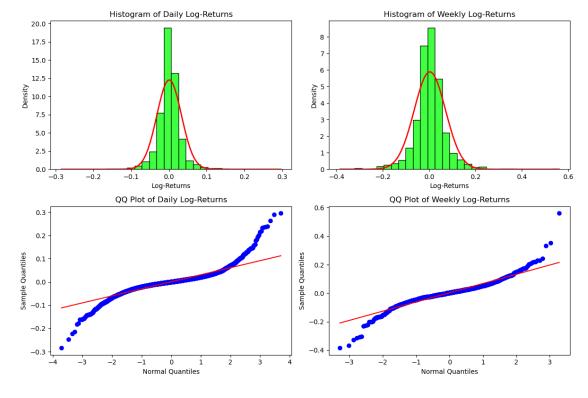
- 1. Comparison of the Normal Distribution vs our actual Values
- 2. Excess Kurtosis of our data
- 3. Interpretation

```
[117]: import yfinance as yf
import matplotlib.pyplot as plt
import numpy as np
import scipy.stats as stats
import seaborn as sns

# Extract daily log-returns
log_price_daily = pt_d_all # Ensure this is a pandas DataFrame or Series
log_returns_daily = rt_d_all # Ensure this is a pandas DataFrame or Series
```

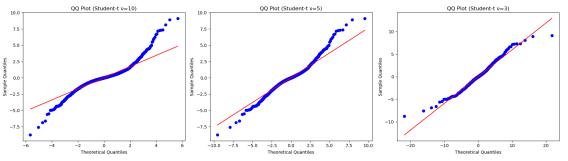
```
# If log_returns_daily is a DataFrame, convert it to a 1D array (assuming_
→'column_name' is the name of the column)
log_returns_daily = log_returns_daily.values.flatten() # Ensure it's 1D array
# Calculate monthly log-returns
log price weekly = pt w all # Ensure this is a pandas DataFrame or Series
log_returns_weekly = rt_w_all # Ensure this is a pandas DataFrame or Series
# If log_returns_monthly is a DataFrame, convert it to a 1D array (assuming_
⇔'column_name' is the name of the column)
log_returns_weekly = log_returns_weekly.values.flatten() # Ensure it's 1D array
# Create the figure with four subplots
fig, axs = plt.subplots(2, 2, figsize=(12, 8))
# Plot histogram of daily log-returns
sns.histplot(log_returns_daily, bins=30, color='lime', edgecolor='black',_
skde_kws={'color': 'red'}, ax=axs[0, 0], stat='density')
axs[0, 0].plot(np.linspace(log returns daily.min(), log_returns_daily.max(),__
 →100),
              stats.norm.pdf(np.linspace(log_returns_daily.min(),__
 ⇒log returns daily.max(), 100),
                             log_returns_daily.mean(), log_returns_daily.
⇔std()), color='red', linewidth=2)
axs[0, 0].set_title('Histogram of Daily Log-Returns')
axs[0, 0].set xlabel('Log-Returns')
axs[0, 0].set_ylabel('Density')
# Plot histogram of monthly log-returns
sns.histplot(log_returns_weekly, bins=30, color='lime', edgecolor='black',
 axs[0, 1].plot(np.linspace(log_returns_weekly.min(), log_returns_weekly.max(),
 →100).
              stats.norm.pdf(np.linspace(log_returns_weekly.min(),_
 ⇒log_returns_weekly.max(), 100),
                             log_returns_weekly.mean(), log_returns_weekly.

std()), color='red', linewidth=2)
axs[0, 1].set_title('Histogram of Weekly Log-Returns')
axs[0, 1].set_xlabel('Log-Returns')
axs[0, 1].set_ylabel('Density')
# QQ plot of daily log-returns
stats.probplot(log_returns_daily, dist="norm", plot=axs[1, 0])
axs[1, 0].set_title('QQ Plot of Daily Log-Returns')
axs[1, 0].set_xlabel('Normal Quantiles')
axs[1, 0].set_ylabel('Sample Quantiles')
```



```
[118]: import yfinance as yf
import matplotlib.pyplot as plt
import numpy as np
import scipy.stats as stats
# Extract daily log-returns
```

```
log_returns_daily = rt_d_all.values.flatten()
# Create three side-by-side QQ plots
fig, axs = plt.subplots(1, 3, figsize=(18, 5))
# Normalize the data to have zero mean and unit variance
log_returns_daily_normalized = log_returns_daily / np.std(log_returns_daily)
# QQ plot against Student-t distribution with = 10
stats.probplot(log_returns_daily_normalized, dist=stats.t, sparams=(10,),_u
→plot=axs[0])
axs[0].set_title('QQ Plot (Student-t =10)')
axs[0].set_xlabel('Theoretical Quantiles')
axs[0].set_ylabel('Sample Quantiles')
# QQ plot against Student-t distribution with
stats.probplot(log_returns_daily_normalized, dist=stats.t, sparams=(5,),_u
→plot=axs[1])
axs[1].set_title('QQ Plot (Student-t =5)')
axs[1].set_xlabel('Theoretical Quantiles')
axs[1].set_ylabel('Sample Quantiles')
# QQ plot against Student-t distribution with
stats.probplot(log_returns_daily_normalized, dist=stats.t, sparams=(3,),_u
→plot=axs[2])
axs[2].set title('QQ Plot (Student-t =3)')
axs[2].set xlabel('Theoretical Quantiles')
axs[2].set_ylabel('Sample Quantiles')
# Adjust spacing between QQ plots
plt.tight_layout()
# Save the plot in png format
plt.savefig('Latex/Img/qqplt_tstudents_AMZNdaily.pdf', format='pdf',u
 ⇔bbox_inches='tight')
plt.show()
```



Kurtosis of the sample

```
[119]: from scipy.stats import kurtosis
  exc_kurt = kurtosis(Rt_d_all) - 3
  print("Excess Kurtosis = ", exc_kurt)
```

Excess Kurtosis = 10.51221657872248

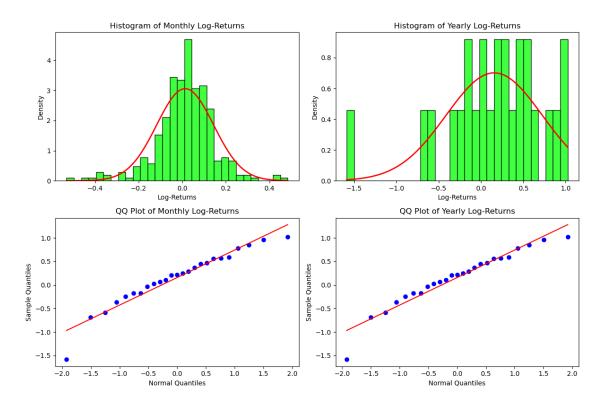
The sample of data defined by the simple returns of AMZN stock is HEAVY TAILED

12 4.5/High Frequecy non-Gaussianity

- 1. Overall Shapes
- 2. Lilliefors Test

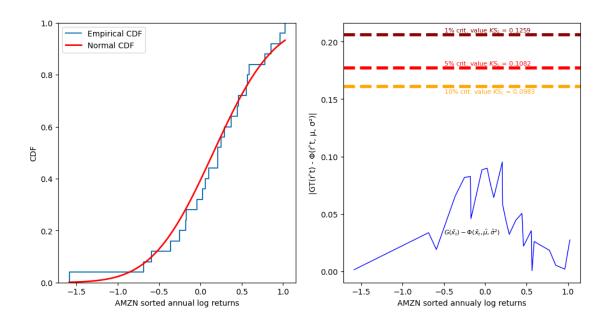
```
[120]: import yfinance as yf
       import matplotlib.pyplot as plt
       import numpy as np
       import scipy.stats as stats
       import seaborn as sns
       # Extract daily log-returns
       log_price_mothly = pt_m_all # Ensure this is a pandas DataFrame or Series
       log_returns_monthly = rt_m_all # Ensure this is a pandas DataFrame or Series
       # If log_returns_daily is a DataFrame, convert it to a 1D array (assuming_
        → 'column_name' is the name of the column)
       log_returns_monthly = log_returns_monthly.values.flatten() # Ensure it's 1D_
        \hookrightarrow array
       # Calculate monthly log-returns
       log_price_yearly = pt_y_all # Ensure this is a pandas DataFrame or Series
       log_returns_yearly = rt_y_all # Ensure this is a pandas DataFrame or Series
       # If log_returns_monthly is a DataFrame, convert it to a 1D array (assuming_
       →'column_name' is the name of the column)
       log_returns_yearly = log_returns_yearly.values.flatten() # Ensure it's 1D array
       # Create the figure with four subplots
       fig, axs = plt.subplots(2, 2, figsize=(12, 8))
       # Plot histogram of daily log-returns
       sns.histplot(log_returns_monthly, bins=30, color='lime', edgecolor='black',u
        ⇔kde_kws={'color': 'red'}, ax=axs[0, 0], stat='density')
```

```
axs[0, 0].plot(np.linspace(log returns monthly.min(), log returns monthly.
 \rightarrowmax(), 100),
              stats.norm.pdf(np.linspace(log_returns_monthly.min(),__
 ⇒log returns monthly.max(), 100),
                             log_returns_monthly.mean(), log_returns_monthly.
⇔std()), color='red', linewidth=2)
axs[0, 0].set_title('Histogram of Monthly Log-Returns')
axs[0, 0].set xlabel('Log-Returns')
axs[0, 0].set_ylabel('Density')
# Plot histogram of monthly log-returns
sns.histplot(log_returns_yearly, bins=30, color='lime', edgecolor='black',__
 axs[0, 1].plot(np.linspace(log returns yearly.min(), log returns yearly.max(),
 →100),
              stats.norm.pdf(np.linspace(log_returns_yearly.min(),__
 ⇒log_returns_yearly.max(), 100),
                             log_returns_yearly.mean(), log_returns_yearly.
⇔std()), color='red', linewidth=2)
axs[0, 1].set title('Histogram of Yearly Log-Returns')
axs[0, 1].set xlabel('Log-Returns')
axs[0, 1].set_ylabel('Density')
# QQ plot of daily log-returns
stats.probplot(log_returns_yearly, dist="norm", plot=axs[1, 0])
axs[1, 0].set_title('QQ Plot of Monthly Log-Returns')
axs[1, 0].set_xlabel('Normal Quantiles')
axs[1, 0].set_ylabel('Sample Quantiles')
# QQ plot of monthly log-returns
stats.probplot(log_returns_yearly, dist="norm", plot=axs[1, 1])
axs[1, 1].set title('QQ Plot of Yearly Log-Returns')
axs[1, 1].set_xlabel('Normal Quantiles')
axs[1, 1].set ylabel('Sample Quantiles')
# Adjust spacing between plots
plt.tight_layout()
# Save the plot in pdf format
plt.savefig('Latex/Img/QQplot_monthly_yearly_AMZN.pdf', format='pdf',__
 ⇔bbox_inches='tight')
# Show the plot
plt.show()
```



```
[121]: import yfinance as yf
       import matplotlib.pyplot as plt
       import numpy as np
       import scipy.stats as stats
       import seaborn as sns
       # Compute annual log-returns
       log_returns_weekly = rt_w_all
       log_returns_yearly = rt_y_all
       # Compute mean and std
       mean_data = log_returns_yearly.mean()
       sd_data = log_returns_yearly.std()
       samp_size = len(log_returns_yearly)
       seq_ind = np.arange(1, samp_size + 1, 1)
       emp_cdf = seq_ind / samp_size
       emp_cdf_2 = (seq_ind - 1) / samp_size
       my_data_ordered = np.sort(log_returns_yearly)
       theor_cdf = stats.norm.cdf(my_data_ordered, mean_data, sd_data)
       # Set the layout
       fig, axs = plt.subplots(1, 2, figsize=(12, 6))
```

```
# Left panel: empirical and Normal cdf's
sns.ecdfplot(log_returns_yearly, ax=axs[0], label='Empirical CDF')
axs[0].plot(np.linspace(log_returns_yearly.min(), log_returns_yearly.max(),_u
 →100),
            stats.norm.cdf(np.linspace(log_returns_yearly.min(),__
 ⇒log returns yearly.max(), 100),
                           mean_data, sd_data),
            color='red', linewidth=2, label='Normal CDF')
axs[0].set_xlabel('AMZN sorted annual log returns')
axs[0].set_ylabel('CDF')
axs[0].set_title('')
axs[0].legend()
# Right panel: Lilliefors test
KS_L_stat1 = np.max(np.abs(emp_cdf - theor_cdf))
KS_L_stat2 = np.max(np.abs(emp_cdf_2 - theor_cdf))
KS_L_stat = max(KS_L_stat1, KS_L_stat2)
axs[1].plot(my_data_ordered, np.abs(emp_cdf_2 - theor_cdf), color='blue',_
 →linewidth=1)
axs[1].axhline(y=0.805/np.sqrt(samp_size), color='orange', linewidth=4,__
→linestyle='--')
axs[1].axhline(y=0.886/np.sqrt(samp size), color='red', linewidth=4,,,
 ⇔linestyle='--')
axs[1].axhline(y=1.031/np.sqrt(samp_size), color='darkred', linewidth=4,__
 ⇔linestyle='--')
axs[1].text(-0.5, 0.805/np.sqrt(samp_size)-0.006, '10% crit. value $KS_L$ = 0.
⇔0983', fontsize=8, color='orange')
axs[1].text(-0.5, 0.886/np.sqrt(samp_size)+0.002, '5% crit. value $KS_L$ = 0.
⇔1082', fontsize=8, color='red')
axs[1].text(-0.5, 1.031/np.sqrt(samp_size)+0.002, '1% crit. value $KS_L$ = 0.
 ⇔1259', fontsize=8, color='darkred')
axs[1].text(-0.5, 0.032, '$G(\tilde{x}_t)-\Phi(\tilde{x}_t, \hat{\mu},_{\sqcup})
axs[1].set xlabel('AMZN sorted annualy log returns')
axs[1].set ylabel('|GT(r^*t) - \Phi(r^*t, , ^2)|')
axs[1].set_title('')
# Set the space within plots
#plt.tight_layout()
# Save the figure in png format
plt.savefig('Latex/Img/lillie_test_AMZNannualy.pdf', format='pdf', __
 ⇔bbox_inches='tight')
plt.show()
```

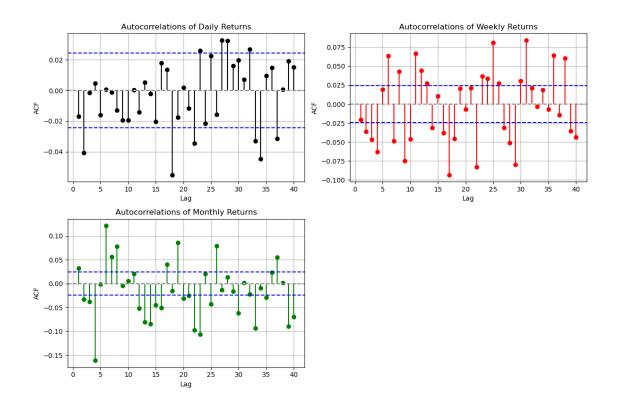


13 4.6/ Returns are not autocorrelated

1. Daily, Weekly and Monthly Autocorrelations

```
[122]: import numpy as np
       import matplotlib.pyplot as plt
       from statsmodels.tsa.stattools import acf
       # Calculate empirical autocorrelations for daily, weekly, monthly, and yearly_
        \hookrightarrow returns
       lags = 40
       # Daily ACF
       acf_daily_values = acf(Rt_d_all, nlags=lags)
       # Weekly ACF
       acf_weekly_values = acf(Rt_w_all, nlags=lags)
       # Monthly ACF
       acf_monthly_values = acf(Rt_m_all, nlags=lags)
       # Calculate Bartlett intervals
       Bart_Int = 1.96 / np.sqrt(len(Rt_d_all))
       # Create the autocorrelation plot with Bartlett intervals for each time frame
       plt.figure(figsize=(12, 8))
```

```
# Plot daily autocorrelations
plt.subplot(2, 2, 1)
plt.stem(np.arange(1, lags + 1), acf_daily_values[1:], linefmt='k-',_
 →markerfmt='ko', basefmt='w-')
plt.axhline(y=0, color='gray', linestyle='--')
plt.axhline(y=Bart Int, color='blue', linestyle='--')
plt.axhline(y=-Bart_Int, color='blue', linestyle='--')
plt.title('Autocorrelations of Daily Returns')
plt.xlabel('Lag')
plt.ylabel('ACF')
plt.grid(True)
# Plot weekly autocorrelations
plt.subplot(2, 2, 2)
plt.stem(np.arange(1, lags + 1), acf_weekly_values[1:], linefmt='r-',_
 →markerfmt='ro', basefmt='w-')
plt.axhline(y=0, color='gray', linestyle='--')
plt.axhline(y=Bart_Int, color='blue', linestyle='--')
plt.axhline(y=-Bart_Int, color='blue', linestyle='--')
plt.title('Autocorrelations of Weekly Returns')
plt.xlabel('Lag')
plt.ylabel('ACF')
plt.grid(True)
# Plot monthly autocorrelations
plt.subplot(2, 2, 3)
plt.stem(np.arange(1, lags + 1), acf monthly values[1:], linefmt='g-',,
 →markerfmt='go', basefmt='w-')
plt.axhline(y=0, color='gray', linestyle='--')
plt.axhline(y=Bart_Int, color='blue', linestyle='--')
plt.axhline(y=-Bart_Int, color='blue', linestyle='--')
plt.title('Autocorrelations of Monthly Returns')
plt.xlabel('Lag')
plt.ylabel('ACF')
plt.grid(True)
# Adjust layout and show plot
plt.tight_layout()
plt.show()
```



```
[123]: """autocorrelate = pt_d_all.shift(1).corrwith(pt_d_all, method='pearson')
print(autocorrelate.round(4))"""
```

14 4.7/ Returns feature volatility clustering—long run range dependence of squared returns

```
[124]: # Extract daily log-returns
log_returns_daily = rt_d_all

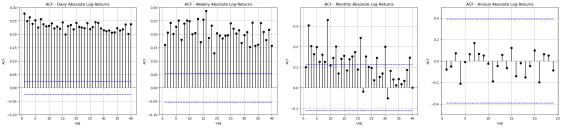
# Parameter for the empirical autocorrelation
lags = 40

# Creation of the three side-by-side graphs
fig, axs = plt.subplots(1, 4, figsize=(30, 6))

# ACF of daily log-returns with confidence bands
acf_values_daily = acf(abs(log_returns_daily), nlags=lags)
confint = 1.96 / np.sqrt(len(log_returns_daily))
confint_upper = np.full(lags, confint)
```

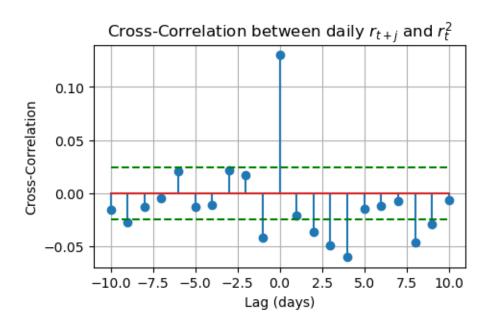
```
confint_lower = -np.full(lags, confint)
axs[0].stem(np.arange(1, lags + 1), acf_values_daily[1:], linefmt='k-',__
 →markerfmt='ko', basefmt='w-')
axs[0].axhline(y=0, color='gray', linestyle='--')
axs[0].plot(np.arange(1, lags + 1), confint upper, color='blue', ...
 →linestyle='dashed')
axs[0].plot(np.arange(1, lags + 1), confint_lower, color='blue',__
 →linestyle='dashed')
axs[0].set ylim(-0.1, 0.3)
axs[0].set_title('ACF - Daily Absolute Log-Returns')
axs[0].set xlabel('Lag')
axs[0].set_ylabel('ACF')
axs[0].grid(True)
# ACF of weekly log-returns with confidence bands
acf_values_weekly = acf(abs(log_returns_weekly), nlags=lags)
confint_weekly = 1.96 / np.sqrt(len(log_returns_weekly))
confint_weekly_upper = np.full(lags, confint_weekly)
confint_weekly_lower = -np.full(lags, confint_weekly)
axs[1].stem(np.arange(1, lags + 1), acf_values_weekly[1:], linefmt='k-',__
 →markerfmt='ko', basefmt='w-')
axs[1].axhline(y=0, color='gray', linestyle='--')
axs[1].plot(np.arange(1, lags + 1), confint_weekly_upper, color='blue',_
 →linestyle='dashed')
axs[1].plot(np.arange(1, lags + 1), confint_weekly_lower, color='blue',_
 ⇔linestyle='dashed')
axs[1].set_ylim(-0.1, 0.3)
axs[1].set_title('ACF - Weekly Absolute Log-Returns')
axs[1].set_xlabel('Lag')
axs[1].set_ylabel('ACF')
axs[1].grid(True)
# ACF of monthly log-returns with confidence bands
acf_values_monthly = acf(abs(log_returns_monthly), nlags=lags)
confint_monthly = 1.96 / np.sqrt(len(log_returns_monthly))
confint_monthly_upper = np.full(lags, confint_monthly)
confint_monthly_lower = -np.full(lags, confint_monthly)
axs[2].stem(np.arange(1, lags + 1), acf_values_monthly[1:], linefmt='k-',u
 →markerfmt='ko', basefmt='w-')
axs[2].axhline(y=0, color='gray', linestyle='--')
axs[2].plot(np.arange(1, lags + 1), confint_monthly_upper, color='blue',_
 ⇔linestyle='dashed')
```

```
axs[2].plot(np.arange(1, lags + 1), confint_monthly_lower, color='blue',_
 ⇔linestyle='dashed')
axs[2].set_ylim(-0.13, 0.39)
axs[2].set title('ACF - Monthly Absolute Log-Returns')
axs[2].set_xlabel('Lag')
axs[2].set ylabel('ACF')
axs[2].grid(True)
# ACF of annual log-returns with confidence bands
lags = 24
acf_values_yearly = acf(abs(log_returns_yearly), nlags=lags)
confint = 1.96 / np.sqrt(len(log_returns_yearly))
confint_upper = np.full(lags, confint)
confint_lower = -np.full(lags, confint)
axs[3].stem(np.arange(1, lags + 1), acf_values_yearly[1:], linefmt='k-',u
axs[3].axhline(y=0, color='gray', linestyle='--')
axs[3].plot(np.arange(1, lags + 1), confint_upper, color='blue',_
→linestyle='dashed')
axs[3].plot(np.arange(1, lags + 1), confint_lower, color='blue',__
 →linestyle='dashed')
axs[3].set_ylim(-0.5, 0.5)
axs[3].set_title('ACF - Annual Absolute Log-Returns')
axs[3].set_xlabel('Lag')
axs[3].set_ylabel('ACF')
axs[3].grid(True)
# Adjusting the spacing between graphs
#plt.tight_layout()
# Save the graphic in png format
plt.savefig('Latex/Img/Fact7_AbsoluteLogReturns.pdf', format='pdf',u
 ⇔bbox inches='tight')
plt.show()
```



15 4.8/ Leverage Effect

```
[]: # Define a function
     def ccf(x, y, lag max = 100):
         # Compute correlation
         result = ss.correlate(y - np.mean(y), x - np.mean(x), method='direct') / ___
      \rightarrow (np.std(y) * np.std(x) * len(y))
         # Define the length
         length = (len(result) - 1) // 2
         lo = length - lag_max
         hi = length + (lag_max + 1)
         return result[lo:hi]
     # Choose the max lag and execute the function
     lag max = 10
     log_returns_daily = np.array(log_returns_daily)
     cross_corr = ccf(log_returns_daily,log_returns_daily**2,lag_max=lag_max)
     # Plot results
     lags = np.arange(-lag_max, lag_max + 1)
     # ACF of monthly log-returns with confidence bands
     confint_daily = 1.96 / np.sqrt(len(log_returns_daily))
     confint_daily_upper = np.full(len(lags), confint_daily)
     confint_daily_lower = -np.full(len(lags), confint_daily)
     plt.figure(figsize=(5, 3))
     plt.stem(lags, cross_corr)
     plt.plot(lags, confint_daily_upper, color='green', linestyle='dashed')
     plt.plot(lags, confint_daily_lower, color='green', linestyle='dashed')
     plt.xlabel('Lag (days)')
     plt.ylabel('Cross-Correlation')
     plt.title('Cross-Correlation between daily $r_{t+j}$ and $r_t^2$')
    plt.grid(True)
     # Add the bartlet intervals
     plt.savefig('Latex/Img/Fact8_CrossCorr_r_r2.pdf', format='pdf',
      ⇔bbox_inches='tight')
     plt.show()
```

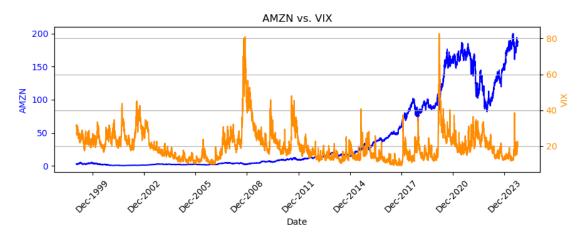


```
[126]: print(Pt_d_all, type)
      Date
      1999-01-21
                      2.650000
      1999-01-22
                      3.075000
      1999-01-25
                      2.809375
      1999-01-26
                      2.877344
      1999-01-27
                      3.140625
      2024-10-09
                    185.169998
      2024-10-10
                    186.649994
      2024-10-11
                    188.820007
      2024-10-14
                    187.539993
      2024-10-15
                    187.690002
      Name: Pt.d, Length: 6476, dtype: float64 <class 'type'>
[127]: #Get the starting and ending date of our stock
       start_date = Pt_d_all.index.min()
       end_date = Pt_d_all.index.max()
       # Get VIX data
       VIX = yf.download("^VIX", start=start_date, end=end_date)
       # Extract and Rename the adjusted closing prices
       VIX_d = VIX["Adj Close"]
       VIX_d.name = 'VIX.d'
```

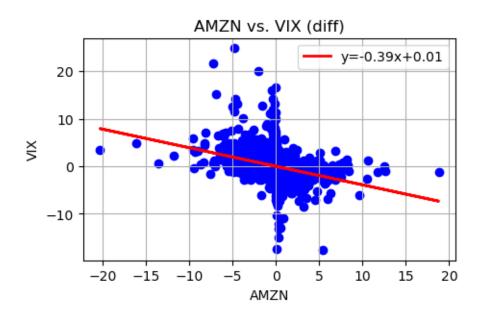
```
# Mutate the Index into a DatetimeIndex
VIX_d.index = pd.to_datetime(VIX_d.index)
# Merge the two datasets and rename columns
merged_df = pd.merge(Pt_d_all, VIX_d, on='Date', how='outer') # outer: only__
⇔commond indexes (dates)
merged df.head()
# Compute changes in pt and VIX compared to previous period (NaN are kept)
diff_df = merged_df.diff()
diff_df.head()
# Remove from the price dataframe
merged_df = merged_df.dropna()
# And from the second one
diff_df = diff_df.dropna()
# Define the figure parameters
fig, ax1 = plt.subplots(figsize=(10, 3))
# Customizing x-axis labels for December of each year
date_labels = pd.date_range(start=start_date, end=end_date, freq='3Y')
formatted_labels = [f'Dec-{date.year}' for date in date_labels]
# Add label and rotate them
plt.xticks(date_labels, formatted_labels, rotation=45)
# Work on the first y-axis: SEP
ax1.plot(merged_df.index, merged_df['Pt.d'], label="AMZN" + ' Prices', |

color='blue')
ax1.set xlabel('Date')
ax1.set_ylabel("AMZN", color='blue')
ax1.tick_params(axis='y', labelcolor='blue')
# Work on the second y-axis: VIX
ax2 = ax1.twinx()
ax2.plot(merged_df.index, merged_df['VIX.d'], label='VIX', color='darkorange')
ax2.set_ylabel('VIX', color='darkorange')
ax2.tick_params(axis='y', labelcolor='darkorange')
# Adjust the figure
plt.title("AMZN" + ' vs. VIX')
plt.grid(True)
# Save the figure
plt.savefig('Latex/Img/Fact8.pdf', format='png', bbox_inches='tight')
plt.show()
```

[********* 100%********** 1 of 1 completed



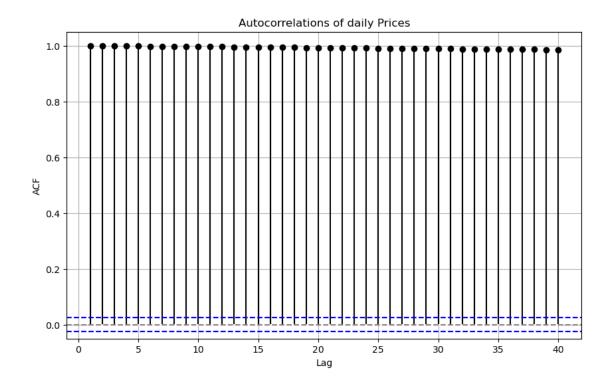
```
[128]: plt.figure(figsize=(5, 3))
       plt.scatter(diff_df['Pt.d'], diff_df['VIX.d'], color='blue', marker='o')
       # Add labels and title
       plt.xlabel("AMZN")
       plt.ylabel('VIX')
       plt.title("AMZN" + ' vs. VIX (diff)')
       plt.grid(True)
       # Add regression line
       coefficients = np.polyfit(diff_df['Pt.d'], diff_df['VIX.d'], 1)
       regression_line = np.polyval(coefficients, diff_df['Pt.d'])
       plt.plot(diff_df['Pt.d'], regression_line, color='red', linewidth=2,__
       ⇒label='y='+str(round(coefficients[0],2))+'x+'+str(round(coefficients[1],2)))
       plt.legend()
       plt.savefig('Latex/Img/Fact_8_3'+"AMZN"+'_.pdf', format='pdf',__
        ⇔bbox_inches='tight')
       # Show plot
       plt.show()
```



15.0.1 Other Material

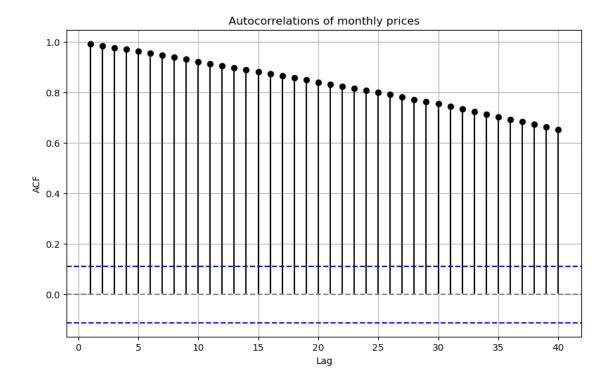
took from stylized facts 1

```
[129]: from statsmodels.tsa.stattools import acf
       # Calculate empirical autocorrelation
       lags = 40
       acf_values = acf(pt_d_all, nlags=lags)
       # Calculate Bartlett intervals
       Bart_Int = 1.96 / np.sqrt(len(pt_d_all))
       # Create the autocorrelation plot with Bartlett intervals
       plt.figure(figsize=(10, 6))
       plt.stem(np.arange(1, lags + 1), acf_values[1:], linefmt='k-', markerfmt='ko', u
        ⇒basefmt='w-')
       plt.axhline(y=0, color='gray', linestyle='--')
       plt.axhline(y=Bart_Int, color='blue', linestyle='--')
       plt.axhline(y=-Bart_Int, color='blue', linestyle='--')
       plt.title('Autocorrelations of daily Prices')
       plt.xlabel('Lag')
       plt.ylabel('ACF')
       plt.grid(True)
       #plt.savefig('Latex/Autocorrel_daily.pdf', format='pdf', bbox_inches='tight')
       plt.show()
```



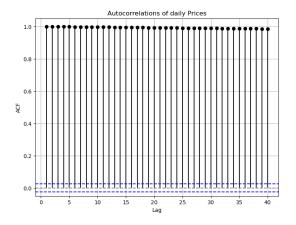
ACF with monthly data

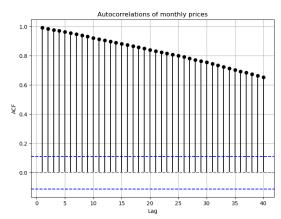
```
[130]: from statsmodels.tsa.stattools import acf
       # Calculate empirical autocorrelation
       lags = 40
       acf_values = acf(pt_m_all, nlags=lags)
       # Calculate Bartlett intervals
       Bart_Int = 1.96 / np.sqrt(len(pt_m_all))
       # Create the autocorrelation plot with Bartlett intervals
       plt.figure(figsize=(10, 6))
       plt.stem(np.arange(1, lags + 1), acf_values[1:], linefmt='k-', markerfmt='ko',__
        ⇔basefmt='w-')
       plt.axhline(y=0, color='gray', linestyle='--')
       plt.axhline(y=Bart_Int, color='blue', linestyle='--')
       plt.axhline(y=-Bart_Int, color='blue', linestyle='--')
       plt.title('Autocorrelations of monthly prices')
       plt.xlabel('Lag')
       plt.ylabel('ACF')
       plt.grid(True)
       #plt.savefig('Latex/Autocorrel_monthly.pdf', format='pdf', bbox_inches='tight')
       plt.show()
```



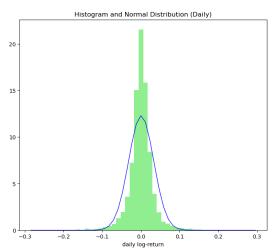
15.0.2 The two figures subplotted

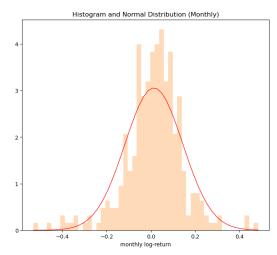
```
[131]: fig, axs = plt.subplots(1, 2, figsize=(18, 6))
       #first fig
       # Calculate empirical autocorrelation
       lags = 40
       acf_values = acf(pt_d_all, nlags=lags)
       # Calculate Bartlett intervals
       Bart_Int = 1.96 / np.sqrt(len(pt_d_all))
       axs[0].stem(np.arange(1, lags + 1), acf_values[1:], linefmt='k-',__
        →markerfmt='ko', basefmt='w-')
       axs[0].axhline(y=0, color='gray', linestyle='--')
       axs[0].axhline(y=Bart_Int, color='blue', linestyle='--')
       axs[0].axhline(y=-Bart_Int, color='blue', linestyle='--')
       axs[0].set_title('Autocorrelations of daily Prices')
       axs[0].set_xlabel('Lag')
       axs[0].set_ylabel('ACF')
       axs[0].grid(True)
       #second fig
       # Calculate empirical autocorrelation
       lags = 40
       acf_values = acf(pt_m_all, nlags=lags)
```





Histogram of daily prices and normal density





15.0.3 QQ-plot (Normal distribution)

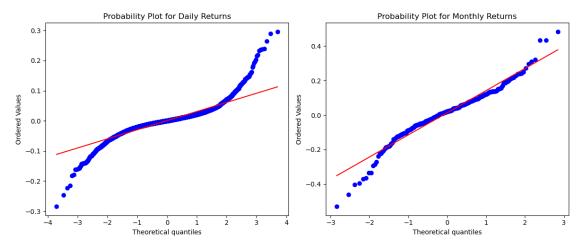
```
[133]: # Set up the subplots
fig, axs = plt.subplots(1, 2, figsize=(12, 5))

# Probability Plot for Daily Returns
stats.probplot(rt_d_all, dist="norm", plot=axs[0])
axs[0].set_title("Probability Plot for Daily Returns")

# Probability Plot for Monthly Returns
stats.probplot(rt_m_all, dist="norm", plot=axs[1])
axs[1].set_title("Probability Plot for Monthly Returns")

# Adjust layout and display the plot
```

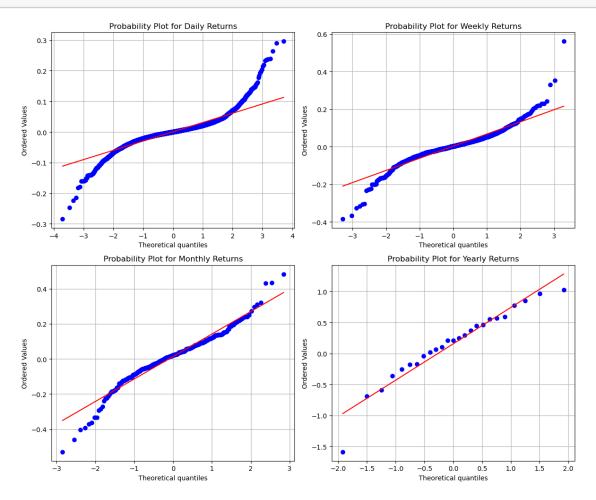
```
plt.tight_layout()
#plt.savefig('Latex/QQ-plot.pdf', format='pdf', bbox_inches='tight')
plt.show()
```



Check how the QQ-plots change aggregating data

```
[134]: # Set up the subplots
       fig, axs = plt.subplots(2, 2, figsize=(12, 10))
       # Probability Plot for Daily Returns
       stats.probplot(rt_d_all, dist="norm", plot=axs[0, 0])
       axs[0, 0].set_title("Probability Plot for Daily Returns")
       axs[0, 0].grid(True)
       # Probability Plot for Weekly Returns
       stats.probplot(rt_w_all, dist="norm", plot=axs[0, 1])
       axs[0, 1].set_title("Probability Plot for Weekly Returns")
       axs[0, 1].grid(True)
       # Probability Plot for Monthly Returns
       stats.probplot(rt_m_all, dist="norm", plot=axs[1, 0])
       axs[1, 0].set_title("Probability Plot for Monthly Returns")
       axs[1, 0].grid(True)
       # Probability Plot for Yearly Returns
       stats.probplot(rt_y_all, dist="norm", plot=axs[1, 1])
       axs[1, 1].set_title("Probability Plot for Yearly Returns")
       axs[1, 1].grid(True)
       # Adjust layout and display the plot
       plt.tight_layout()
```

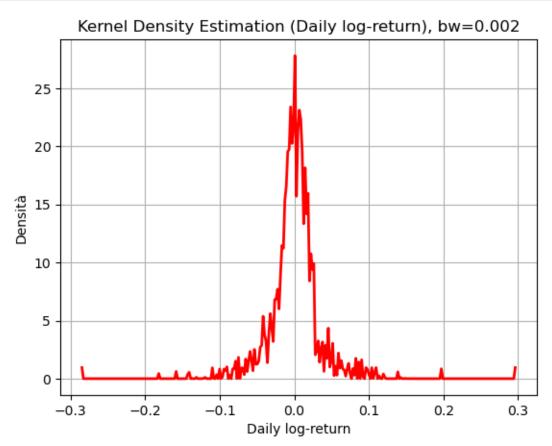
plt.show()



15.0.4 Kernel density

It is similar to a smooth histogram!

```
plt.ylabel("Densità")
plt.title("Kernel Density Estimation (Daily log-return), bw=0.002")
plt.grid(True)
plt.show()
```



 bw_method defines the bandwidth parameter:

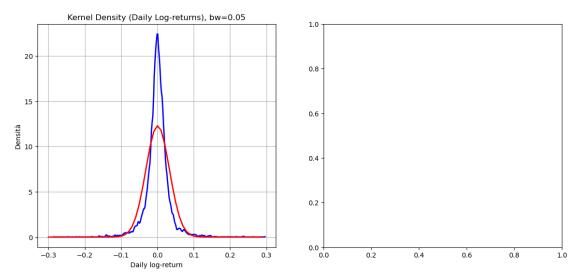
 \Rightarrow the larger the bandwidth, the smoother the histogram:

```
[136]: ## Compute the kernel density: daily returns
# divide the interval between the min and max returns into 300 segments
density_eval_points = np.linspace(rt_d_all.min(), rt_d_all.max(), num=300)
# estimate the kernel density of our returns
kde = gaussian_kde(rt_d_all, bw_method=0.05)
# and evaluate in the interval defined above
density_estimation = kde(density_eval_points)

# Empirical mean and std
mean_empirical= log_returns_daily.mean()
std_empirical= log_returns_daily.std()
```

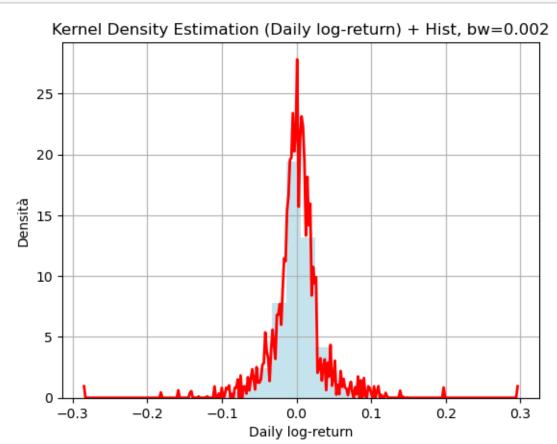
```
x=np.arange(-0.3,0.3,0.01)
fig,axs=plt.subplots(1,2,figsize=(14,6))
# Plotting
#sns.kdeplot(log_returns_daily, color='blue', ax=axs[0])
# 1rst plot is kernel density daily log returns, compared to the standard,
 \hookrightarrownormal
axs[0].plot(density_eval_points, density_estimation, color='blue', lw=2,__
 ⇔label='Kernel density')
axs[0].plot(x, stats.norm.pdf(x, mean_empirical, std_empirical), color='red',__
 →linewidth=2)
axs[0].set_xlabel("Daily log-return")
axs[0].set ylabel("Densità")
axs[0].set_title("Kernel Density (Daily Log-returns), bw=0.05")
axs[0].grid(True)
"""sns.histplot(log_returns_daily, bins=60, color='lime', edgecolor='black',\Box

→kde_kws={'color': 'red'}, stat='density', ax=axs[1])
axs[1].plot(stats.norm.pdf(np.linspace(log returns daily.min(), ), )
 → log_returns_daily.max(), 100), log_returns_daily.mean(), log_returns_daily.
 ⇔std()),color='red', linewidth=2)
n n n
plt.show()
```

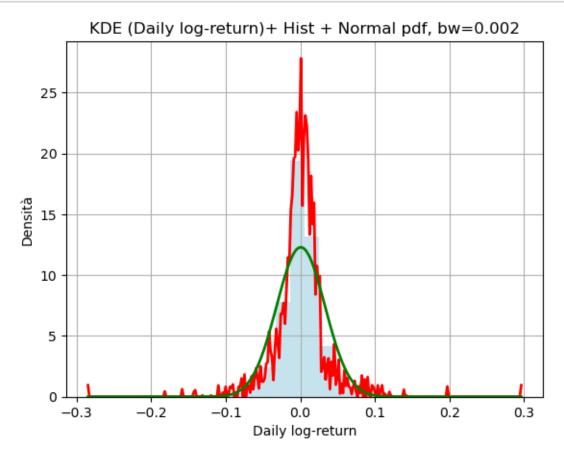


Above, there was an issue with a histogramm, but we did not used it in the interpretation

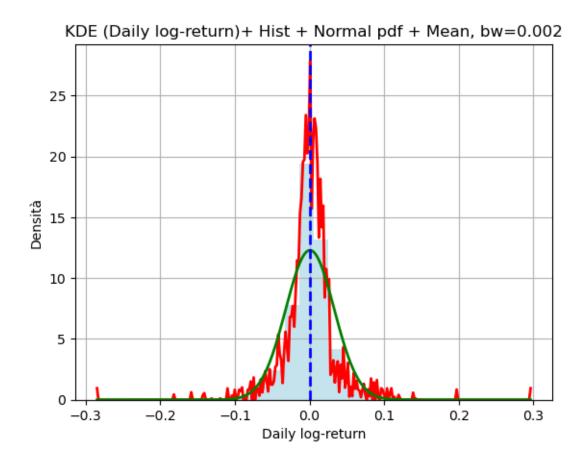
```
[137]: ## Compute the kernel density: daily returns
       # divide the interval between the min and max returns into 300 segments
       density_eval_points = np.linspace(rt_d_all.min(), rt_d_all.max(), num=300)
       # estimate the kernel density of our returns
       kde = gaussian_kde(rt_d_all, bw_method=0.002)
       # and evaluate in the interval defined above
       density_estimation = kde(density_eval_points)
       # Plotting
       plt.hist(rt_d_all, bins=30, density=True, alpha=0.7, color='lightblue')
       plt.plot(density eval points, density estimation, color='red', lw=2,,,
        ⇔label='Kernel density')
       plt.xlabel("Daily log-return")
       plt.ylabel("Densità")
       plt.title("Kernel Density Estimation (Daily log-return) + Hist, bw=0.002")
       plt.grid(True)
       plt.show()
```



```
[138]: ## Compute the kernel density: daily returns
       # divide the interval between the min and max returns into 300 segments
       density_eval_points = np.linspace(rt_d_all.min(), rt_d_all.max(), num=300)
       # estimate the kernel density of our returns
       kde = gaussian_kde(rt_d_all, bw_method=0.002)
       # and evaluate in the interval defined above
       density_estimation = kde(density_eval_points)
       # on the same interval, we evaluate a Normal pdf
       pdf_theoretical = norm.pdf(density_eval_points, np.mean(rt_d_all), np.
        ⇔std(rt_d_all))
       # Plotting
       plt.hist(rt d all, bins=30, density=True, alpha=0.7, color='lightblue')
       plt.plot(density_eval_points, density_estimation, color='red', lw=2,__
        ⇔label='Kernel density')
       plt.plot(density_eval_points, pdf_theoretical, color='green', lw=2, label='PDF_u
        →Teorica (Normale)')
       plt.xlabel("Daily log-return")
       plt.ylabel("Densità")
       plt.title("KDE (Daily log-return)+ Hist + Normal pdf, bw=0.002")
       plt.grid(True)
       plt.show()
```



```
[139]: ## Compute the kernel density: daily returns
       # divide the interval between the min and max returns into 300 segments
       density_eval_points = np.linspace(rt_d_all.min(), rt_d_all.max(), num=300)
       # estimate the kernel density of our returns
       kde = gaussian_kde(rt_d_all, bw_method=0.002)
       # and evaluate in the interval defined above
       density_estimation = kde(density_eval_points)
       # on the same interval, we evaluate a Normal pdf
       pdf_theoretical = norm.pdf(density_eval_points, np.mean(rt_d_all), np.
       ⇔std(rt_d_all))
       # compute the mean
       mean_data = np.mean(rt_d_all)
       # Plotting
       plt.hist(rt_d_all, bins=30, density=True, alpha=0.7, color='lightblue')
       plt.plot(density_eval_points, density_estimation, color='red', lw=2,_u
        ⇔label='Kernel density')
       plt.plot(density_eval_points, pdf_theoretical, color='green', lw=2, label='PDF_u
        →Teorica (Normale)')
       plt.axvline(mean_data, color='blue', linestyle='dashed', linewidth=2,__
        ⇔label='Media')
       plt.xlabel("Daily log-return")
      plt.ylabel("Densità")
       plt.title("KDE (Daily log-return)+ Hist + Normal pdf + Mean, bw=0.002")
       plt.grid(True)
       plt.show()
```



15.1 Summary Statistics

There is a function which compute some summary statistics...not really the ones we want called describe:

```
[140]: rt_d_all.describe()
[140]: count
                6475.000000
                   0.000658
       mean
       std
                   0.032465
                  -0.284568
       min
       25%
                  -0.012598
       50%
                   0.000413
       75%
                   0.013969
                   0.296181
       max
       Name: rt_d_all, dtype: float64
```

15.1.1 Skewness & Kurtosis

We use the fucntions which came from scipy.stats:

from scipy.stats import gaussian_kde, norm, iqr, skew, kurtosis, jarque_bera, kstest, anderson These functions replicate the formulas you find on slides.

```
[141]: rt_d_skew = skew(rt_d_all, nan_policy='omit')
rt_d_kurt = kurtosis(rt_d_all, nan_policy='omit')

print("The skewness is:", rt_d_skew)
print("The kurtosis is:", rt_d_kurt)

# NOTE: There are several formulas to compute skewness and kurtosis.
# These functions both divide the summations of the estimators by 1/T
```

The skewness is: 0.4304836875303971 The kurtosis is: 11.128625063290052

Aggregational Kurtosis We compute the kurtosis of the daily, weekly, monthly, and annual returns:

```
[142]: rt_d_kurt = kurtosis(rt_d_all, nan_policy='omit')
rt_w_kurt = kurtosis(rt_w_all, nan_policy='omit')
rt_m_kurt = kurtosis(rt_m_all, nan_policy='omit')
rt_y_kurt = kurtosis(rt_y_all, nan_policy='omit')

print("Daily: ", round(rt_d_kurt,3))
print("Weekly: ", round(rt_w_kurt,3))
print("Monthly: ", round(rt_m_kurt,3))
print("Annual: ", round(rt_y_kurt,3))
```

Daily: 11.129
Weekly: 7.605
Monthly: 2.604
Annual: 1.461

15.1.2 Normality Tests

Compute Normality tests and sample summary statistics

Jarque-Bera Test

```
[143]: JB_rt_d = jarque_bera(rt_d_all)
# first position (0): statistic
print("JB Stat: ", round(JB_rt_d[0],3))
# second position (1): p-value
print("JB p-value: ", JB_rt_d[1])
```

JB Stat: 33612.686 JB p-value: 0.0

Check the Aggregational Normality:

```
[144]: print("JB p-value", "daily", "returns:", jarque_bera(rt_d_all)[1])
       print("JB p-value", "weekly", "returns:", jarque_bera(rt_w_all)[1])
       print("JB p-value", "monthly", "returns:", jarque_bera(rt_m_all)[1])
       print("JB p-value", "yearly", "returns:", jarque_bera(rt_y_all)[1])
      JB p-value daily returns: 0.0
      JB p-value weekly returns: 0.0
      JB p-value monthly returns: 0.0
      JB p-value yearly returns: 0.04262486815078237
      We can also compute the p-value. The JB Stats follows a \chi^2_2 distribution. So:
[145]: p_value = 1 - stats.chi2.cdf(STATISTIC, df=2)
       print("The associated p-value is:",p_value)
      The associated p-value is: 0.04262486815078237
      15.1.3 Other normality tests:
      Lilliefors test:
[146]: lill_rt_y = lilliefors(rt_y_all)
       print("Stat:",lill_rt_y[0])
       print("p-val:",lill_rt_y[1])
      Stat: 0.0952201438460884
      p-val: 0.7974329823750796
      Kolmogorov-Smirnov test:
[147]: ks_rt_y = kstest(rt_y_all, 'norm')
       print("Stat:",ks_rt_y[0])
       print("p-val:",ks_rt_y[1])
      Stat: 0.24100976414208733
      p-val: 0.0919870853397472
      Anderson-Darling test:
[148]: ad_rt_y = anderson(rt_y_all, 'norm')
       print("Stat:",ad_rt_y[0])
       print("critical val:",ad_rt_y[1])
       print("sign level:",ad_rt_y[2])
      Stat: 0.34577306424633036
      critical val: [0.514 0.586 0.703 0.82 0.975]
```

15.2 Generates table exactly equal to the one in slide n.91

2.5 1.]

5.

Personalized table of summary statistics.

sign level: [15. 10.

```
[149]: # X contains returns at different frequencies
       X = {
           'daily': rt_d_all,
           'weekly': rt_w_all,
           'monthly': rt_m_all,
           'annual': rt_y_all
       }
[150]: def multi fun(x):
           stat tab = {
               'Mean': round(np.mean(x) * 100,5),
               'St.Deviation': round(np.std(x) * 100,5),
               'Diameter.C.I.Mean': round(1.96 * np.sqrt(np.var(x) / len(x)) * 100,5),
               'Skewness': round(skew(x),5),
               'Kurtosis': round(kurtosis(x),5),
               'Excess.Kurtosis': round(kurtosis(x) - 3,5),
               'Min': round(np.min(x) * 100,5),
               'Quant5': round(np.quantile(x, 0.05) * 100,5),
               'Quant25': round(np.quantile(x, 0.25) * 100,5),
               'Median': round(np.quantile(x, 0.50) * 100,5),
               'Quant75': round(np.quantile(x, 0.75) * 100,5),
               'Quant95': round(np.quantile(x, 0.95) * 100,5),
               'Max': round(np.max(x) * 100,5),
               'Jarque.Bera.stat': round(jarque_bera(x)[0],5),
               'Jarque.Bera.pvalue.X100': round(jarque_bera(x)[1] *100,5),
```

- 1. Define a new dictionary to store the stats:
 - a. key will contains the key (i.e., daily, weekly, ...)

'Lillie.test.stat': round(lilliefors(x)[0],5),

'Lillie.test.pvalue.X100': round(lilliefors(x)[1] * 100,5),

- b. data will contains the returns
- 2. Apply *multi_fun* to each data series

'N.obs': len(x)

return stat_tab

- 3. Define a DataFrame with the stats results
- 4. Print the dictionary

```
[151]: # 1.
statistics_dict = {}

# 2.
statistics_dict = {
    key: multi_fun(data.iloc[1:])
```

```
for key, data in X.items()
}
# apply multi_fun to each returns ("series" in pandas)
# which is located in one of the four key of our dictionary X
# 3.
statistics_df = pd.DataFrame(statistics_dict)
# 4.
print(statistics_df)
```

	daily	weekly	monthly	annual
Mean	0.06351	0.31014	1.32164	22.85692
St.Deviation	3.24131	6.76830	13.06275	45.28052
Diameter.C.I.Mean	0.07896	0.36213	1.45887	18.11598
Skewness	0.41992	0.05008	-0.45895	-0.15137
Kurtosis	11.15158	7.60655	2.59462	-0.64793
Excess.Kurtosis	8.15158	4.60655	-0.40538	-3.64793
Min	-28.45678	-38.51804	-53.02674	-68.54809
Quant5	-4.61051	-9.74288	-20.16713	-55.72274
Quant25	-1.25994	-2.64062	-4.98163	-7.23999
Median	0.04108	0.30519	2.09626	23.07665
Quant75	1.39659	3.40897	8.45973	55.96192
Quant95	4.47118	10.67416	20.90661	94.77653
Max	29.61811	56.11507	48.35221	102.44636
Jarque.Bera.stat	33735.75720	3235.87866	97.20696	0.51147
Jarque.Bera.pvalue.X100	0.00000	0.00000	0.00000	77.43487
Lillie.test.stat	0.10194	0.09591	0.08194	0.06494
Lillie.test.pvalue.X100	0.10000	0.10000	0.10000	99.00000
N.obs	6474.00000	1342.00000	308.00000	24.00000

Export it as a latex table

```
[152]: latex_table = statistics_df.to_latex(index=True)
with open("Latex/8stylized.tex", "w") as file:
    file.write(latex_table)
```

/var/folders/5r/ft807c7n1ngd3fpt2_gwsg0m0000gn/T/ipykernel_78356/805359341.py:1: FutureWarning: In future versions `DataFrame.to_latex` is expected to utilise the base implementation of `Styler.to_latex` for formatting and rendering. The arguments signature may therefore change. It is recommended instead to use `DataFrame.style.to_latex` which also contains additional functionality. latex_table = statistics_df.to_latex(index=True)

```
[153]: #skewness & kurtosis dict

def skewness_dict(x):
    stat_tab = {
        'Skewness': round(skew(x),5),
        'Kurtosis': round(kurtosis(x),5),
```

```
}
return stat_tab
```

```
[154]: # Y contains returns at different frequencies
Y = {
    'daily': Rt_d_all,
    'weekly': Rt_w_all,
    'monthly': Rt_m_all,
    'annual': Rt_y_all
}
```

Creating a table with isolated skewness and kurtosis

```
statistics_dict_sk = {}

statistics_dict_sk_log = {
    key: skewness_dict(data.iloc[1:])
    for key, data in X.items()
}

statistics_dict_sk_simple = {
    key: skewness_dict(data.iloc[1:])
    for key, data in Y.items()
}

#printing results

print("Log returns",pd.DataFrame(statistics_dict_sk_log))
print("Simple returns",pd.DataFrame(statistics_dict_sk_simple))
```

```
Log returns daily weekly monthly annual Skewness 0.41992 0.05008 -0.45895 -0.15137  
Kurtosis 11.15158 7.60655 2.59462 -0.64793  
Simple returns daily weekly monthly annual Skewness 1.07310 1.16787 0.40997 0.69139  
Kurtosis 13.54753 13.70696 2.93127 -0.30325
```

Export it as a Latex table

```
[156]: latex_table = pd.DataFrame(statistics_dict_sk_log).to_latex(index=True)
with open("Latex/table_Skewness_kurtosis.tex", "w") as file:
    file.write(latex_table)
```

/var/folders/5r/ft807c7n1ngd3fpt2_gwsg0m0000gn/T/ipykernel_78356/3564160307.py:1 : FutureWarning: In future versions `DataFrame.to_latex` is expected to utilise the base implementation of `Styler.to_latex` for formatting and rendering. The arguments signature may therefore change. It is recommended instead to use `DataFrame.style.to_latex` which also contains additional functionality.

```
[157]: # Compute Box Pierce and Ljung Box tests
      for log_returns in [log_returns_daily, log_returns_monthly]:
        my_max_lag = 25
        lags_all = np.arange(1, my_max_lag + 1)
        my_acf = sm.tsa.acf(log_returns, nlags=my_max_lag)
        my acf diameter = 1.96 / np.sqrt(len(log returns))
        my_acf_tstat_0 = (my_acf[1:] - 0) / np.sqrt(1 / len(log_returns))
        my LjungBox = sm.stats.diagnostic.acorr ljungbox(log returns, lags=lags all,
        ⇒boxpierce=False)
        my_BoxPierce = sm.stats.diagnostic.acorr_ljungbox(log_returns, lags=lags_all,_
        ⇒boxpierce=True)
        crit_value_5_BP = stats.chi2.ppf(0.95,lags_all)
      my_table = np.column_stack((
          lags_all,
          my acf[1:],
          np.full(my_max_lag, my_acf_diameter),
          my acf tstat 0,
          my_BoxPierce['bp_stat'],
          my_BoxPierce['bp_pvalue'],
          my_LjungBox['lb_stat'],
          my_LjungBox['lb_pvalue'],
          np.full(my_max_lag, crit_value_5_BP)
      ))
      column_names = ["lag", "acf", "acf diam.", "acf test", "B-P stat", "B-P pval",
       my_table_df = pd.DataFrame(data=my_table, columns=column_names)
      # Reducing the selection of lags
      my_table_df = my_table_df.iloc[[0,4,14,24]]
      # Print the rounded table
      my_table_df = my_table_df.round(3)
      my_table_df['lag'] = my_table_df['lag'].astype(int)
      print(my_table_df)
                acf acf diam. acf test B-P stat B-P pval L-B stat L-B pval \
          lag
                         0.112
                                                      0.352
                                                                0.874
                                                                          0.350
      0
            1 0.053
                                   0.930
                                             0.865
                         0.112
      4
           5 0.021
                                   0.376
                                             7.654
                                                      0.176
                                                                7.796
                                                                          0.168
      14 15 -0.049
                         0.112 -0.865
                                           22.975
                                                      0.085
                                                               23.673
                                                                          0.071
      24
          25 -0.048
                       0.112
                                -0.846
                                           36.101
                                                      0.070
                                                               37.859
                                                                          0.048
            crit
      0
          3.841
```

```
4 11.070
14 24.996
```

24 37.652

Export it as a Latex Table

```
[158]: latex_table = my_table_df.to_latex(index=True)
with open("Latex/LB_BP.tex", "w") as file:
    file.write(latex_table)
```

/var/folders/5r/ft807c7n1ngd3fpt2_gwsg0m0000gn/T/ipykernel_78356/4088390180.py:1 : FutureWarning: In future versions `DataFrame.to_latex` is expected to utilise the base implementation of `Styler.to_latex` for formatting and rendering. The arguments signature may therefore change. It is recommended instead to use `DataFrame.style.to_latex` which also contains additional functionality. latex_table = my_table_df.to_latex(index=True)