$$\hat{H} = \hat{T} + \hat{V}$$

$$\{\phi_i\}=\{\phi_1, \phi_2, \phi_3, \ldots\}$$

$$\boldsymbol{H} = \langle i | \hat{H} | j \rangle$$

$$\hat{H} | j \rangle = \sum_k h_k^{(j)} | k \rangle$$

$$\langle i | \hat{H} | j \rangle = \sum_k h_k^{(j)} \langle i | k \rangle$$

$$\hat{H} \psi_n = E_n \psi_n$$

$$\boldsymbol{H} = \begin{pmatrix} \langle 0 | \hat{H} | 0 \rangle & \langle 0 | \hat{H} | 1 \rangle & \square & \square \\ \langle 1 | \hat{H} | 0 \rangle & \cdots & \square & \square \\ \cdots & \square & \square & \square \\ \square & \square & \square & \langle N | \hat{H} | N \rangle \end{pmatrix}$$

$$\begin{aligned} \boldsymbol{H\Psi} &= \boldsymbol{E\Psi} \\ \boldsymbol{H\psi}_0 &= E_0 \boldsymbol{\psi}_0 \\ \boldsymbol{H\psi}_1 &= E_1 \boldsymbol{\psi}_1 \\ &\cdots \\ \boldsymbol{H\psi}_N &= E_N \boldsymbol{\psi}_N \end{aligned}$$

```
a = RandomReal[{}, {100, 100}];
H = a + Transpose[a];
{engs, wfns} = Eigensystem[H];
```

$$\psi_n = \sum_k c_k^{(n)} | k \rangle$$

In[122]:=
```
engs
```

Out[122]=
```
{101.179, -8.0043, 7.92349, 7.59313, -7.58848, 7.32018, -7.06523, 7.00818,
 6.76508, -6.65089, 6.43429, -6.4266, 6.39575, 6.27105, -6.10927, 6.10148,
 -6.09574, 5.93028, -5.92146, -5.72323, 5.70458, 5.61429, -5.49158,
 5.3708, -5.28453, -5.24087, 5.18209, -5.08459, 5.05129, -4.97488, 4.91607,
 4.81369, -4.67693, 4.64648, -4.4636, -4.36362, 4.35003, -4.28172, 4.22549,
 4.12278, -4.03745, 3.88197, -3.87642, 3.83535, -3.74428, -3.65675, 3.58449,
 -3.58195, 3.37729, 3.3429, -3.24621, 3.22717, -3.20706, 3.10843, -3.0553,
 -2.98217, 2.92626, -2.88503, 2.79667, -2.70177, 2.67188, -2.50373, 2.4846,
 -2.47168, -2.33024, -2.27519, 2.24029, 2.08236, 1.9281, -1.86432, -1.81028,
 1.78309, 1.70112, -1.63967, 1.61751, -1.50347, 1.42892, -1.41947, 1.32968,
 -1.25449, 1.16168, 1.09375, -1.05284, 0.88105, -0.880825, -0.753754,
 0.73197, -0.724547, 0.578093, -0.567645, 0.50635, 0.499356, 0.428357,
 -0.40143, -0.272378, 0.26503, -0.172787, -0.131066, -0.117469, 0.020895}
```

In[123]:=
```
wfns // Dimensions
```

Out[123]=
```
{100, 100}
```

In[124]:=
```
wfns[[1]]
```

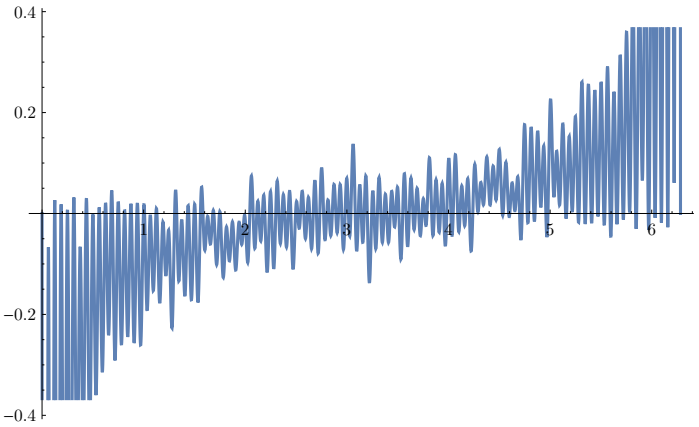
Out[124]=
```
{-0.104087, -0.102147, -0.0904153, -0.0982184, -0.0967095, -0.102407, -0.10094,
 -0.104582, -0.0963196, -0.100479, -0.101242, -0.0949961, -0.0913744,
 -0.104545, -0.103744, -0.101432, -0.101754, -0.107876, -0.0973557, -0.100665,
 -0.102276, -0.103714, -0.100987, -0.100313, -0.096739, -0.109312, -0.103588,
 -0.100071, -0.101338, -0.0982598, -0.0994516, -0.100375, -0.0965511,
 -0.103241, -0.0999661, -0.104298, -0.101629, -0.0996022, -0.102558,
 -0.0991022, -0.104669, -0.0997378, -0.101963, -0.0968098, -0.102322,
 -0.101642, -0.102079, -0.0935485, -0.0976752, -0.0989465, -0.106983,
 -0.0971894, -0.0985047, -0.106615, -0.105776, -0.105726, -0.102562,
 -0.0935267, -0.0986888, -0.0988274, -0.100832, -0.0901811, -0.0966173,
 -0.0943959, -0.0954381, -0.102123, -0.100358, -0.0883129, -0.0973674,
 -0.103334, -0.101425, -0.104078, -0.0973132, -0.101117, -0.104361, -0.0922964,
 -0.0955305, -0.0976166, -0.103185, -0.0946714, -0.0986926, -0.100092,
 -0.0976793, -0.0997751, -0.0985439, -0.108618, -0.0972537, -0.0966342,
 -0.0964978, -0.102026, -0.106624, -0.104135, -0.0925356, -0.102311,
 -0.0978287, -0.0945822, -0.0963842, -0.107477, -0.0950313, -0.0995666}
```

$$H\,\Psi = E\Psi$$

$$\psi_n = \sum_i c_i^{(n)}\,\phi_i$$

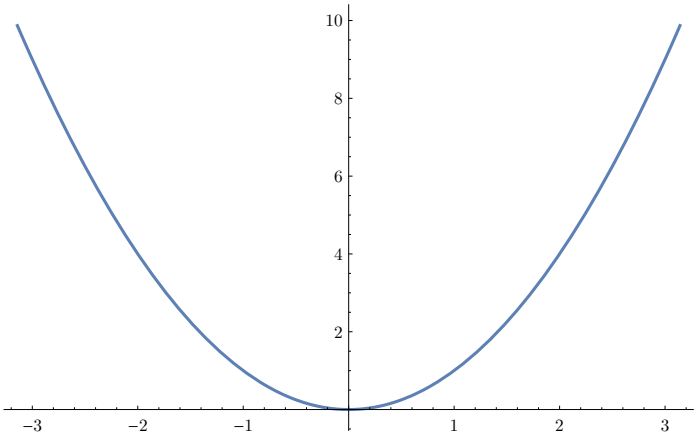
In[126]:=
```
Plot[Sum[wfns[[1, i]] * Sin[i * x], {i, 100}], {x, 0, 2 π}]
```

Out[126]=



In[128]:=
```
Plot[x^2, {x, -π, π}]
```

Out[128]=



In[134]:=
```
potRep[i_, j_] := Integrate[Sin[i * x] * x^2 * Sin[j * x], {-π, π}]
```

```
potRep[
```

In[132]:=
```
keRep[i_, j_] := Integrate[Sin[i * x] * D[Sin[j * x], {x, 2}], {x, -π, π}]
```

In[133]:=
```
keRep[i, j]
```

Out[133]=

$$\frac{2\,j^2\left(j\,\text{Cos}[j\,\pi]\,\text{Sin}[i\,\pi]-i\,\text{Cos}[i\,\pi]\,\text{Sin}[j\,\pi]\right)}{-i^2+j^2}$$

In[129]:=
```
D[Sin[j * x], {x, 2}]
```

Out[129]=
```
-j^2 Sin[j x]
```