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Issued: January 1977

NAP: A Computer Program for the Computation of Two-Dimensional, Time-Dependent, Inviscid Nozzle Flow

by

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UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
CONTRACT W-7405-ENG. 36

Work supported by the US Energy Research and Development Administration, Magnetic Fusion Energy Division.

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161
Price: Printed Copy $5.00 Microfiche $3.00

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COMPUTATION OF TWO-DIMENSIONAL,

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ABSTRACT

A computer program, NAP, is presented for calculating inviscid, steady, and unsteady flow in two-dimensional and axisymmetric nozzles. Interior mesh points are computed using the MacCormack finite-difference scheme, while a characteristic scheme is used to calculate the boundary mesh points. An explicit artificial viscosity term is included for shock computations. The fluid is assumed to be a perfect gas. This method was used to compute the steady flow in a 45°-15° conical, converging-diverging nozzle, a 15° conical, converging nozzle, and a 10° conical, plug nozzle. The numerical solution agreed well with the experimental data. In contrast to previous time-dependent methods for calculating steady flows, the computational times were < 1 min on a CDC 6600 computer.

I. BASIC DESCRIPTION OF THE METHOD

A. Introduction

The equations of motion governing steady, inviscid flow are of a mixed type: hyperbolic in the supersonic region and elliptic in the subsonic region. These mathematical difficulties may be removed by using the "time-dependent" method, in which the flow is assumed to be unsteady or time-dependent. Then the governing equations are hyperbolic in both subsonic and supersonic regions. The steady-state solution may be obtained as the asymptotic solution for large time. This time-dependent technique has been used to compute steady converging-diverging nozzle flows (reported in Refs. 1-6), and it has also been used to compute steady converging nozzle flows (see Refs. 4 and 7). The results of those calculations are mainly good, but the computational times are rather large. In addition, although the computer program of Ref. 6 included a centerbody and those of Refs. 4 and 7 included the exhaust jet, none of the above codes is able to calculate both, that is, plug nozzles.

The object of this research was to develop a production-type computer program capable of solving steady converging, converging-diverging, and plug two-dimensional nozzle flows in computational times of ≤ 1 min on a CDC 6600 computer. Such a program would be able to solve unsteady flows as well.

B. Literature Review

The following is a discussion of the methods used in Refs. 1 through 7. The first paragraph deals with the computation of the interior mesh points; the next three paragraphs are concerned with the boundary mesh points.

Prozan (see Ref. 1), Wehofer and Moger,[4] and Laval[5] used variations of the two-step Lax-Wendroff scheme to compute the interior mesh points. Migdal et al.[3] and Brown and Ozcan[7] employed the original one-step Lax-Wendroff scheme, but with the equations of motion in nonconservation form. Serra[6] applied the original Lax-Wendroff scheme with the equations of motion in conservation form. To stabilize their schemes, Laval and Serra used artificial viscosity terms in their difference equations. Wehofer and

Moger reset the stagnation conditions along each streamline, reset the mass flow at each axial location, and smoothed the subsonic portion of the flow after each time step.

To compute the nozzle inlet mesh points, Prozan (in Ref. 1) assumed the inlet flow to be uniform. Wehofer and Moger assumed only that the pressure was radially uniform at the inlet. Migdal et al. and Brown and Ozcan mapped the inlet to minus infinity after Moretti,[8] thus allowing the static conditions to be set equal to the stagnation conditions. Laval used extrapolation of the interior mesh points to determine the inlet mesh points, while Serra employed a characteristic scheme.

Prozan (in Ref. 1), Wehofer and Moger, Laval, and Brown and Ozcan used an extrapolation technique to compute the wall mesh points. Migdal et al. employed a characteristic scheme after Moretti[8] to compute the wall mesh points, while Serra applied a reflection technique. For the converging nozzle problem to be properly posed, an exhaust jet calculation must be included. Wehofer and Moger used an extrapolation procedure to compute the exhaust jet boundary mesh points, while Brown and Ozcan employed a characteristic scheme after Moretti.[8]

All of the above authors used extrapolation to compute the exit mesh points when the flow was supersonic, since any errors incurred would be swept out of the mesh. Serra employed a characteristic scheme when the exit flow was subsonic.

C. Choice of a Method

The lengthy computational times associated with time-dependent calculations are usually caused by inefficient numerical schemes or poor treatment of boundaries, resulting in the requirement for excessively fine computational meshes (see Refs. 8 and 9). A technique for a much more efficient calculation of the interior and boundary mesh points will be discussed here.

The computation of steady flows by a time-dependent method differs from ordinary initial-value problems in that the initial data and much of the transient solution have a negligible effect on the final or steady solution. Therefore, accuracy is important only for the asymptotic state, and special attention to intermediate efficiency will result in reasonable computational times. For this reason, interior mesh points can be computed by using a very efficient finite-difference scheme, as opposed to those less efficient finite-difference or characteristic schemes that achieve high accuracy at every step.

In the class of finite-difference schemes, the two-step methods such as the MacCormack[10] and the two-step Lax-Wendroff schemes[11] are more efficient than the original Lax-Wendroff scheme,[11] especially if the governing equations are in conservation form. Moretti[12] showed that using the equations of motion in conservation form decreased efficiency and ease of programming while only slightly increasing the accuracy of shock calculations. The use of an explicit artificial viscosity term for shock-free flows also decreases efficiency and was shown to be physically unjustified.[12] In addition, such increases in the numerical dissipation can often destroy the weak shock structure of transonic flows. Therefore, the MacCormack scheme with the equations of motion in nonconservation form is used to calculate the interior mesh points. An explicit artificial viscosity term was included for shock computations only. Remember that the implicit dissipation always present as an effect of truncation terms assures numerical stability for the shock-free flow results.

The boundary mesh points, while making up only a small part of the total mesh points, must be handled most accurately,[8] because of the flowfield's sensitivity to precise boundary geometry. Moretti[8] and Abbett[9] showed that reflection, extrapolation, and one-sided difference techniques for computing solid wall boundaries give poor results and should be avoided. Therefore, the wall and centerbody mesh points are computed using a characteristic scheme. A characteristic scheme is also used to calculate the exhaust jet boundary mesh points.

In the case of the nozzle inlet mesh points for subsonic flow, the use of extrapolation techniques and the assumption of one-dimensional flow presume the form of the solution and in many cases are physically unjustified. On the other hand, a characteristic scheme could be used to calculate the inlet mesh points. While the stagnation pressure and temperature are assumed to remain constant at the inlet in a characteristic scheme (not necessarily the case for unsteady flow), this assumption would appear to be valid for the time-dependent calculation of steady flows. Moretti[8] recommends mapping the inlet to minus infinity, thus allowing the static conditions to be set equal to the stagnation conditions. In theory, this appears to be the best approach, but it should be kept in mind that the infinite physical plane must be replaced by a finite computational plane. Also, this technique requires additional mesh points upstream of the nozzle inlet. It is not presently resolved as to whether the characteristic scheme approach used by Serra or the mapping-to-minus-infinity approach suggested by Moretti[8] and employed by Migdal et al. and Brown and Ozcan is the best technique. To reduce the total number of mesh points to be computed, a characteristic scheme is used to compute the inlet mesh points. For supersonic flow, the inlet mesh points are set equal to specified values of velocity, pressure, and density, because in a supersonic stream the downstream conditions do not propagate upstream.

Extrapolation is used to compute the exit mesh points when the flow is supersonic, since any errors incurred will be swept out of the mesh, and a characteristic scheme is employed when the flow is subsonic.

D. Equations of Motion

The appropriate nonconservation form of equations for two-dimensional, inviscid, isentropic, rotational flow are

$$\rho_t + u\rho_x + v\rho_y + \rho u_x + \rho v_y + \varepsilon\rho v/y = 0, \quad (1)$$

$$u_t + uu_x + vu_y + p_x/\rho = 0, \quad (2)$$

$$v_t + uv_x + vv_y + p_y/\rho = 0, \quad (3)$$

$$p_t + up_x + vp_y - a^2(\rho_t + u\rho_x + v\rho_y) = 0, \quad (4)$$

where ρ is the density, u is the axial velocity, v is the radial velocity, p is the pressure, a is the local speed of sound, t is the time, x and y are the axial and radial coordinates, and the subscripts denote partial differentiation. The symbol ε is 0 for planar flow and 1 for axisymmetric flow.

The physical (x,y) plane is mapped into a rectangular computational plane (ζ,η) by the following coordinate transformation:

$$\zeta = x; \quad \eta = \frac{y - y_c(x)}{y_w(x,t) - y_c(x)}; \quad \tau = t , \quad (5)$$

where $y_w(x,t)$ denotes the nozzle wall and exhaust jet boundary radius as a function of x and t and $y_c(x)$ denotes the nozzle centerbody radius as a function of x. These mapping functions must be single-valued functions of the x coordinate. In the (ζ,η,τ) coordinate system Eqs. (1) through (4) become

$$\rho_\tau + u\rho_\zeta + \bar{v}\rho_\eta + \rho u_\zeta + \rho\alpha u_\eta + \rho\beta v_\eta + \varepsilon\rho v/(y_c + \eta/\beta) = 0 , \quad (6)$$

$$u_\tau + uu_\zeta + \bar{v}u_\eta + p_\zeta/\rho + \alpha p_\eta/\rho = 0 , \quad (7)$$

$$v_\tau + uv_\zeta + \bar{v}v_\eta + \beta p_\eta/\rho = 0 , \quad (8)$$

$$p_\tau + up_\zeta + \bar{v}p_\eta - a^2(\rho_\tau + u\rho_\zeta + \bar{v}\rho_\eta) = 0 , \quad (9)$$

where

$$\beta = \frac{1}{y_w - y_c} ; \quad \alpha = -\beta\frac{\partial y_c}{\partial x} - \eta\beta\left(\frac{\partial y_w}{\partial x} - \frac{\partial y_c}{\partial x}\right) ;$$

$$\delta = -\eta\beta\frac{\partial y_w}{\partial t} , \quad (10)$$

$$\bar{v} = \alpha u + \beta v + \delta . \quad (11)$$

The fluid is assumed to be thermally and calorically perfect; that is, a constant ratio of specific heats is assumed.

For shock computations, an artificial viscosity model of the form suggested by von Neumann-Richtmyer[11] is used. This model, which has a term corresponding to all the viscous and thermal conduction terms in the Navier-Stokes equations, is shown below.

$$[\text{RHS Eq. (21)}] = (\lambda + 2\mu)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \lambda\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial y}\right) + \mu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\varepsilon}{y}\left[(\lambda + \mu)\frac{\partial v}{\partial x} + \mu\frac{\partial u}{\partial y}\right] , \quad (12)$$

$$[\text{RHS Eq. (3)}] = (\lambda + 2\mu)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) + \lambda\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y}\right) + \mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\varepsilon(\lambda + 2\mu)}{y}\left(\frac{\partial v}{\partial y} - \frac{v}{y}\right), \tag{13}$$

$$[\text{RHS Eq. (4)}] = \rho(\gamma - 1)\left\{(\lambda + 2\mu)\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right] + 2\lambda\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + 2\mu\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + k\left[\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) + \frac{\varepsilon v}{y}\left[(\lambda + 2\mu)\frac{v}{y} + 2\lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{k}{v}\frac{\partial T}{\partial y}\right]\right\}, \tag{14}$$

$$\lambda = 2c\,c_\lambda\,\Delta x\,\Delta y\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

$$\mu = 2c\,c_\mu\,\Delta x\,\Delta y\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right),$$

$$k = \frac{\mu\gamma R}{(\gamma - 1)Pr},$$

where c, c_λ, and c_μ are nondimensional quantities that specify the distribution and amount of smoothing, γ is the ratio of specific heats, R is the gas constant, Δx and Δy are the axial and radial mesh spacing, and Pr is the Prandtl number.

In the (ζ,η,τ) coordinate system Eqs. (12) through (14) become

$$[\text{RHS Eq. (7)}] = (\lambda + 2\mu)\left(\frac{\partial}{\partial\zeta} + \alpha\frac{\partial}{\partial\eta}\right)\left(\frac{\partial u}{\partial\zeta} + \alpha\frac{\partial u}{\partial\eta}\right) + \lambda\left(\frac{\partial}{\partial\zeta} + \alpha\frac{\partial}{\partial\eta}\right)\left(\beta\frac{\partial v}{\partial\eta}\right) + \mu\beta\frac{\partial}{\partial\eta}\left(\beta\frac{\partial u}{\partial\eta} + \frac{\partial v}{\partial\zeta} + \alpha\frac{\partial v}{\partial\eta}\right) + \frac{\varepsilon}{\bar{\eta}}\left[(\lambda + \mu)\left(\frac{\partial v}{\partial\zeta} + \alpha\frac{\partial v}{\partial\eta}\right) + \mu\beta\frac{\partial u}{\partial\eta}\right], \tag{15}$$

$$[\text{RHS Eq. (8)}] = (\lambda + 2\mu)\beta\frac{\partial}{\partial\eta}\left(\frac{\partial u}{\partial\zeta} + \alpha\frac{\partial u}{\partial\eta}\right) + \lambda\frac{\beta\partial}{\partial\eta}\left(\beta\frac{\partial v}{\partial\eta}\right) + \mu\left(\frac{\partial}{\partial\zeta} + \alpha\frac{\partial}{\partial\eta}\right)\left(\beta\frac{\partial u}{\partial\eta} + \frac{\partial v}{\partial\zeta} + \alpha\frac{\partial v}{\partial\eta}\right) + \frac{\varepsilon(\lambda + 2\mu)}{\bar{\eta}}\left(\beta\frac{\partial v}{\partial\eta} - \frac{v}{\bar{\eta}}\right), \tag{16}$$

$$[\text{RHS Eq. (9)}] = \rho(\gamma - 1)\left\{(\lambda + 2\mu)\left[\left(\frac{\partial u}{\partial\zeta} + \alpha\frac{\partial u}{\partial\eta}\right)^2 + \left(\beta\frac{\partial v}{\partial\eta}\right)^2\right] + \mu\left[\left(\beta\frac{\partial u}{\partial\eta}\right)^2 + \left(\frac{\partial v}{\partial\zeta} + \alpha\frac{\partial v}{\partial\eta}\right)^2\right] + 2\lambda\left(\frac{\partial u}{\partial\zeta} + \alpha\frac{\partial u}{\partial\eta}\right)\left(\beta\frac{\partial v}{\partial\eta}\right) + 2\mu\left(\beta\frac{\partial u}{\partial\eta}\right)\left(\frac{\partial v}{\partial\zeta} + \alpha\frac{\partial v}{\partial\eta}\right) + k\left[\left(\frac{\partial}{\partial\zeta} + \alpha\frac{\partial}{\partial\eta}\right)\left(\frac{\partial T}{\partial\zeta} + \alpha\frac{\partial T}{\partial\eta}\right) + \beta\frac{\partial}{\partial\eta}\left(\beta\frac{\partial T}{\partial\eta}\right)\right] + \frac{\varepsilon v}{\bar{\eta}}\left[(\lambda + 2\mu)\frac{v}{\bar{\eta}} + 2\lambda\left(\frac{\partial u}{\partial\zeta} + \alpha\frac{\partial u}{\partial\eta} + \beta\frac{\partial v}{\partial\eta}\right) + \frac{k}{v}\beta\frac{\partial T}{\partial\eta}\right]\right\}, \tag{17}$$

$$\lambda = 2c\,c_\lambda\,\Delta\zeta\,\frac{\Delta\eta}{\beta}\left(\frac{\partial u}{\partial\zeta} + \alpha\frac{\partial u}{\partial\eta} + \beta\frac{\partial v}{\partial\eta}\right),$$

$$\mu = 2c\,c_\mu\,\Delta\zeta\,\frac{\Delta\eta}{\beta}\left(\frac{\partial v}{\partial\zeta} + \alpha\frac{\partial v}{\partial\eta} + \beta\frac{\partial u}{\partial\eta}\right),$$

where $\bar{\eta} = y_c + \eta/\beta$, and y_c = centerbody radius. These terms are nonzero only when the divergence of the velocity is negative.

E. Numerical Method

The computational plane is divided into five sets of mesh points: interior, inlet, exit, wall and centerbody, and exhaust jet boundary.

1. Interior Mesh Points. The interior mesh points are computed using the MacCormack scheme, a second-order, noncentered, two-step, finite-difference scheme. Backward differences are used on the first step; forward differences are used on the second. The governing equations are left in nonconservation form. An explicit artificial viscosity term is used for shock computations. Centerline mesh points are computed by enforcing symmetry of the flow. For example, the finite-difference equations for Eq. (1) for planar flow ($\varepsilon = 0$) and no artificial viscosity are

$$\bar{\rho}^{N+1}_{L,M} = \rho^N_{L,M} - \left[u^N_{L,M}\left(\frac{\rho^N_{L,M} - \rho^N_{L-1,M}}{\Delta x}\right) + v^N_{L,M}\left(\frac{\rho^N_{L,M} - \rho^N_{L,M-1}}{\Delta y}\right) + \rho^N_{L,M}\left(\frac{u^N_{L,M} - u^N_{L-1,M}}{\Delta x}\right) + \rho^N_{L,M}\left(\frac{v^N_{L,M} - v^N_{L,M-1}}{\Delta y}\right)\right]\Delta t \ , \tag{18}$$

$$\rho^{N+1}_{L,M} = 0.5\left\{\rho^N_{L,M} + \bar{\rho}^{N+1}_{L,M} - \left[\bar{u}^{N+1}_{L,M}\left(\frac{\bar{\rho}^{N+1}_{L+1,M} - \bar{\rho}^{N+1}_{L,M}}{\Delta x}\right) + \bar{v}^{N+1}_{L,M}\left(\frac{\bar{\rho}^{N+1}_{L,M+1} - \bar{\rho}^{N+1}_{L,M}}{\Delta y}\right) + \bar{\rho}^{N+1}_{L,M}\left(\frac{\bar{u}^{N+1}_{L+1,M} - \bar{u}^{N+1}_{L,M}}{\Delta x}\right) + \bar{\rho}^{N+1}_{L,M}\left(\frac{\bar{v}^{N+1}_{L,M+1} - \bar{v}^{N+1}_{L,M}}{\Delta y}\right)\right]\Delta t\right\} \ , \tag{19}$$

where L and M denote axial and radial mesh points, respectively, N denotes the time step, and the bar denotes values calculated on the first step. A complete description of the method is given in Ref. 10.

2. Inlet Mesh Points. The inlet mesh points for subsonic flow are computed using a second-order, reference-plane characteristic scheme. In this scheme, the partial derivatives with respect to η are computed in the initial-value and solution surfaces using noncentered differencing as in the MacCormack scheme. These approximations to the derivatives with respect to η are then treated as forcing terms and the resulting system of equations is solved in the η = constant reference planes using a two-independent-variable, characteristic scheme. The characteristic relations for the η = constant reference planes are derived in Appendix A. The boundary condition is the specification of the stagnation temperature and stagnation pressure. The use of a reference-plane characteristic scheme requires the specification of inlet flow angle as an additional boundary condition. The inlet flow angle can be approximately determined from the nozzle geometry. The equations relating the total and static conditions are

$$p_T/p = [1 + (\gamma - 1)\ M^2/2]^{\gamma/(\gamma-1)} \ , \tag{20}$$

$$T_T/T = 1 + (\gamma - 1)\ M^2/2 \ , \tag{21}$$

where γ is the ratio of specific heats, M is the Mach number, T is the temperature, and the subscript T denotes the total conditions.

The characteristic relations relating the interior flow to the nozzle inlet flow are Eq. (A-43) in Appendix A and can be written as

$$dp - \rho a du = (\psi_4 + a^2\psi_1 - \rho a\psi_2)\ d\tau$$
$$\text{for } d\zeta = (u-a)\ d\tau \ , \tag{22}$$

where the top equation is called the compatibility equation and the bottom equation is called the characteristic curve equation. The ψ terms (see Appendix A) represent the derivatives in the η direction. Equation (22) may be written in finite-difference form by first replacing the differentials by differences along the characteristic curve. Next, the coefficients are either evaluated in the initial-value plane (first step) or considered to be the average of the coefficients evaluated in both the initial-value and solution planes (second step). Finally, the ψ terms are treated as follows: on the first step the coefficients and derivatives, using backward differences, are evaluated in the initial-value plane; on the second and final step the coefficients and derivatives, now using forward differences, are evaluated in the solution plane and then averaged with the ψ terms from the first step. Equations (20), (21), and (22), along with the inlet flow angle and the equation of state $p = \rho RT$, where R is the gas constant, form a system of five equations for the five variables u, v, p, ρ, and T.

A brief description of the unit processes of this scheme is given below. The intersection of the characteristic curve through the solution point with the initial-value line in the η = constant plane is determined by solving the characteristic curve equation. The coefficient u-a is evaluated in the initial-value plane. The dependent variables and derivatives in the ψ terms are calculated at the intersection point using linear interpolation. Next, the compatibility equation, along with Eqs. (20) and (21) and the equation of state, are used to calculate the variables at the solution point. An iterative solution of these equations is required. Thus the first step has been used to compute all inlet mesh points. In the second step, the characteristic curve equation is solved again. Now the coefficient u-a is the average of the values in the initial-value plane and the first-step solution plane. Again, linear interpolation is used to obtain the variables at the intersection point. Finally, the compatibility equation, now with averaged coefficients and ψ terms, is used along with Eqs. (20) and (21) and the equation of state to determine the final solution.

A reference-plane characteristic scheme was chosen over a bicharacteristic scheme because the increased accuracy of a bicharacteristic scheme seemed not to be worth the increased computational time for time-dependent flows.

For supersonic flow, the inlet mesh points are set equal to specified values of velocity, pressure, and density.

3. Exit Mesh Points. For subsonic flow, a reference-plane characteristic scheme similar to the inlet scheme is used. The exit pressure is specified. The characterisitic relations relating the interior flow to the nozzle exit flow are Eqs. (A-41), (A-42), and (A-44). These equations can be written as

$$\left.\begin{aligned} dp - a^2 d\rho &= \psi_4 d\tau \qquad (23)\\ dv &= \psi_3 d\tau \qquad (24)\end{aligned}\right\} \text{ for } d\zeta = u d\tau \ ,$$

$$dp + \rho a du = (\psi_4 + a^2\psi_1 + \rho a \psi_2)\, d\tau \quad \text{for } d\zeta = (u+a)\, d\tau \ . \qquad (25)$$

These equations are written in finite-difference form in the same manner as was done for the nozzle inlet scheme. Equations (23), (24), and (25), along with the exit pressure condition, form a system of four equations for the variables u, v, p, and ρ.

For supersonic flow, the exit mesh points are computed using linear extrapolation.

4. Wall and Centerbody Mesh Points. The wall and centerbody mesh points are also computed using a reference-plane characteristic scheme. In this scheme, the derivatives with respect to ζ are approximated, and the resulting system of equations is solved in the ζ = constant reference planes. The characteristic relations for the ζ = constant reference planes are given in Appendix B. The wall and centerbody contours and therefore their slopes are specified. The boundary condition is given by

$$v = u \tan\theta + \partial y_w/\partial t \ , \qquad (26)$$

where θ is the local wall or centerbody angle.

The characteristic relations relating the interior flow to the flow at the nozzle wall are Eqs. (B-15), (B-16), and (B-18) in Appendix B. These equations are

$$\left.\begin{aligned} \beta du - \alpha dv &= (\beta\psi_2 - \alpha\psi_3) d\tau \qquad (27)\\ dp - a^2 d\rho &= \psi_4 d\tau \qquad (28)\end{aligned}\right\} \text{ for } d\eta = \bar{v} d\tau \ ,$$

$$dp + \rho\alpha a du/\alpha^* + \rho\beta a dv/\alpha^* = (\psi_4 + a^2\psi_1 + \rho\alpha a\psi_2/\alpha^* + \rho\beta a\psi_3/\alpha^* \text{ for } d\eta = (\bar{v} + \alpha^* a) d\tau . \qquad (29)$$

These equations are written in finite-difference form in the same manner as was done for the nozzle inlet scheme. Equations (26), (27), (28), and (29) form a system of four equations for the four variables u, v, p, and ρ.

5. Exhaust Jet Boundary Mesh Points. The exhaust jet boundary mesh points are computed by the wall routine such that the pressure boundary condition

$$p = p_{ambient} \tag{30}$$

is satisfied. This is accomplished by first assuming the shape of the jet boundary and then using the wall routine to calculate the pressure. Next, the jet boundary location is slightly changed and a second pressure is computed. By use of an interpolation procedure, a new jet boundary location is determined. This interpolation-extrapolation procedure is then repeated at each point until the jet boundary pressure and the ambient pressure agree within some specified tolerance.

When an exhaust jet calculation is made, the nozzle wall exit lip mesh point becomes a singularity and, therefore, is treated by a special procedure. First, an upstream solution is computed at the exit mesh point, using the flow tangency condition as the boundary condition and backward ζ differences in both the initial-value and solution planes. Next, a downstream solution is calculated, using Eq. (30) as the boundary condition and the total conditions calculated from the upstream mesh point. The upstream solution is used when computing wall mesh points upstream of the exit mesh point, whereas the downstream solution is used when computing downstream wall mesh points. A third exit mesh point solution to be used for interior mesh point calculation is determined as follows. When the upstream solution is subsonic, the two solution Mach numbers are averaged such that the averaged Mach number is less than or equal to one. This Mach number is then used to calculate the exit mesh point solution to be used to compute the interior mesh points. When the upstream solution is supersonic, the upstream solution is used to calculate the interior mesh points.

6. Step Size. The step size Δt is controlled by the well-known Courant or C-F-L condition which can be expressed as

$$\Delta t \leqslant 1/[(V + a)\ (1/\Delta x^2 + 1/\Delta y^2)^{1/2}] \quad , \tag{31}$$

where V is the velocity magnitude. Using Eqs. (5) and (10), Eq. (31) can be written as

$$\Delta \tau \leqslant A/[(V + a)\ (1/\Delta\zeta^2 + \beta^2/\Delta\eta^2)^{1/2}] \quad , \tag{32}$$

where the coefficient A was determined from actual calculations and varied between 0.4 and 1.6 depending on the geometry of the flow in question.

F. Overall Program

The nozzle inlet flow, as well as the flow leaving the nozzle, may be either subsonic or supersonic. The flow may contain variations in stagnation temperature and stagnation pressure from streamline to streamline. The nozzle wall and centerbody geometries may be either one of two analytical contours or a completely general tabular contour. The program is capable of calculating the exhaust jet boundary for subsonic or supersonic flow. The initial data may be read in or calculated internally by the program. The internally computed data are calculated assuming one-dimensional, steady, isentropic flow with area change. The program output includes the coordinates, velocities, pressure, density, Mach number, temperature, mass flow, and axial thrust in both English and metric units.

G. Results and Discussion

The results presented here have been published in Ref. 13. The CDC 6600 computational times represent the central processor time not including compilation. So that these results can be compared with those of other investigators, the following table of relative machine speeds is given.

Computer	Relative Machine Speed
IBM 7094	0.1
IBM 360/50	0.1
IBM 360/65	0.3
IBM 360/75	0.5
Univac 1108	0.5
CDC 6600	1.0

These relative speeds were obtained from Refs. 14 and 15 and are only rough estimates because values may vary considerably depending on the compiler and machine configuration. In each case, the one-dimensional values computed internally by the program were the initial data. When the relative change in axial velocity in the throat and downstream regions was less than a prescribed convergence tolerance, the flow was assumed to have reached steady state. The convergence tolerance was found to be a function of the mesh spacing, flow speed, and nozzle geometry. For the results presented here, a convergence tolerance of 0.003% was used for flows without exhaust jet calculations; 0.005% for flows with exhaust jet calculations.

The present method was used to compute the steady-state solution for flow in the 45°-15° conical, converging-diverging nozzle shown in Fig. 1a. The Mach number contours and wall pressure ratio are shown in Fig. 2. Although the code works with English and metric units, the units in the original publication of the experimental data (English) were used here. The experimental data are those of Cuffel et al.[2] The computed discharge coefficient is 0.983, compared with the experimental value of 0.985. The 21x8 computational mesh required 301 time planes and a computational time of 35 s. There is good agreement with the experimental data. This case was also

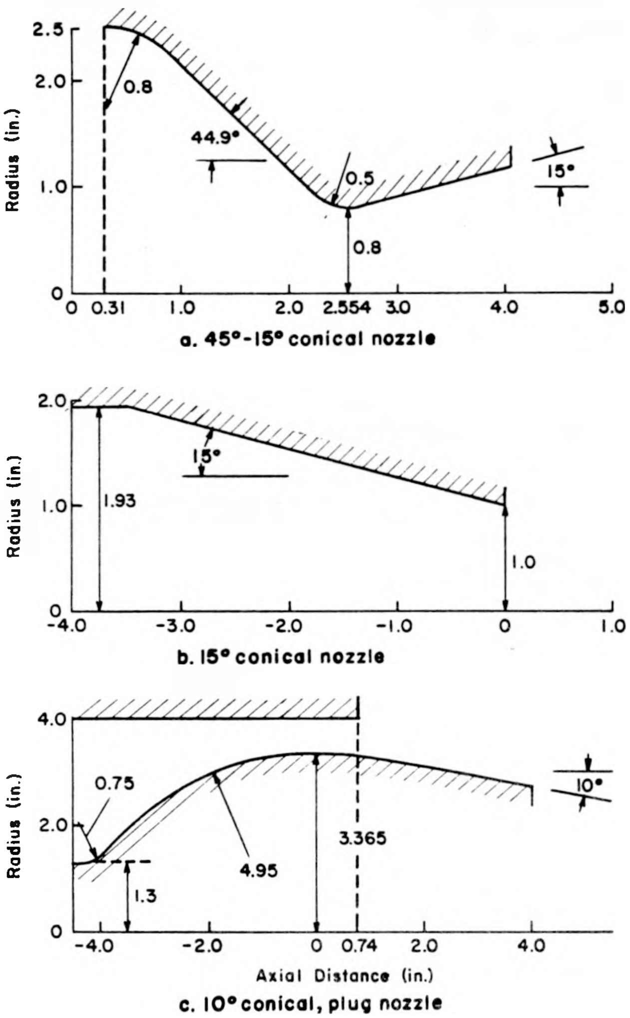

Fig. 1. Nozzle geometries.

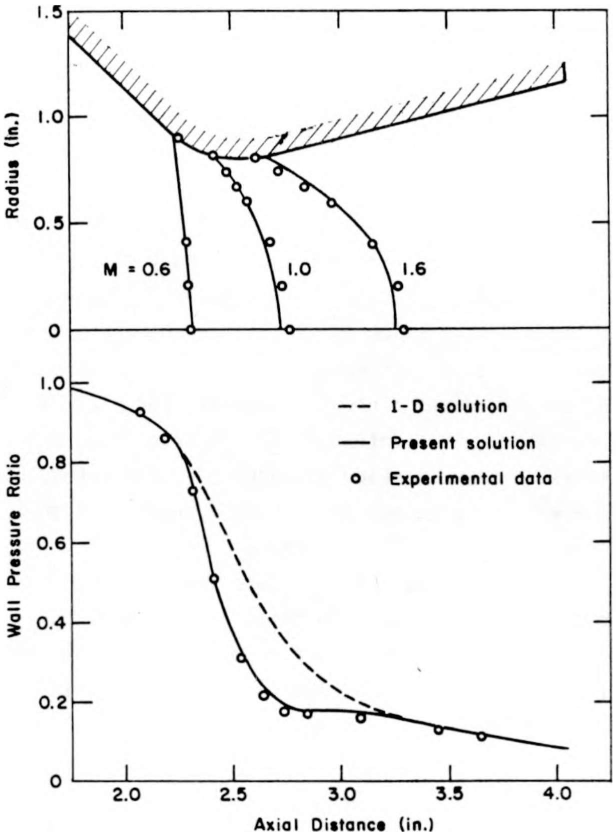

Fig. 2. Mach number contours (above) and wall pressure ratio for 45°-15° conical nozzle.

solved by Prozan (see Ref. 2), Migdal, Laval, and Serra. The details of Prozan's computation were not reported by Cuffel et al., but Saunders reported a time of 45 min on a CDC 3200 (23x11 mesh) for computing the flow in a nozzle with a large radius of curvature. Migdal et al. reported a computational time of less than 5 min on an IBM 350/75; Laval reported a computational time on the order of 2 h on an IBM 360/50 (61x21 mesh); and Serra reported a computational time of 80 min on a Univac 1108 (3000 mesh points). This case was also solved by Prozan and Kooker,[16] using a relaxation scheme to solve the steady, irrotational equations of motion. Their computational time was 5 to 10 min on an IBM 7094 (21x11 mesh).

The present method was also used to compute the steady-state flow in a 15° conical, converging nozzle. The nozzle geometry is shown in Fig. 1b. The Mach number contours and wall pressure ratio for a nozzle pressure ratio of 2.0 are shown in Fig. 3. The experimental data are those of Thornock.[17] The computed discharge coefficient is 0.957, compared

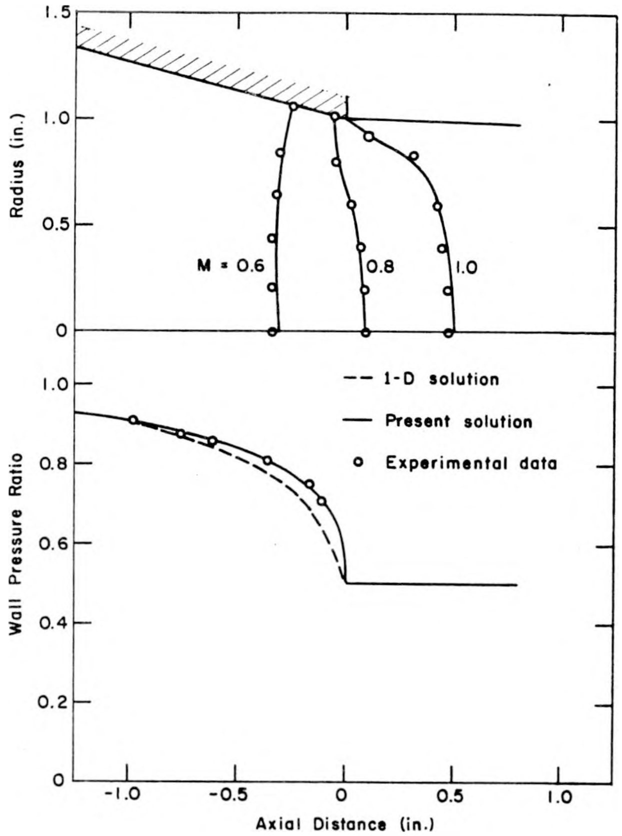

Fig. 3. Mach number contours (above) and wall pressure ratio for 15° conical nozzle.

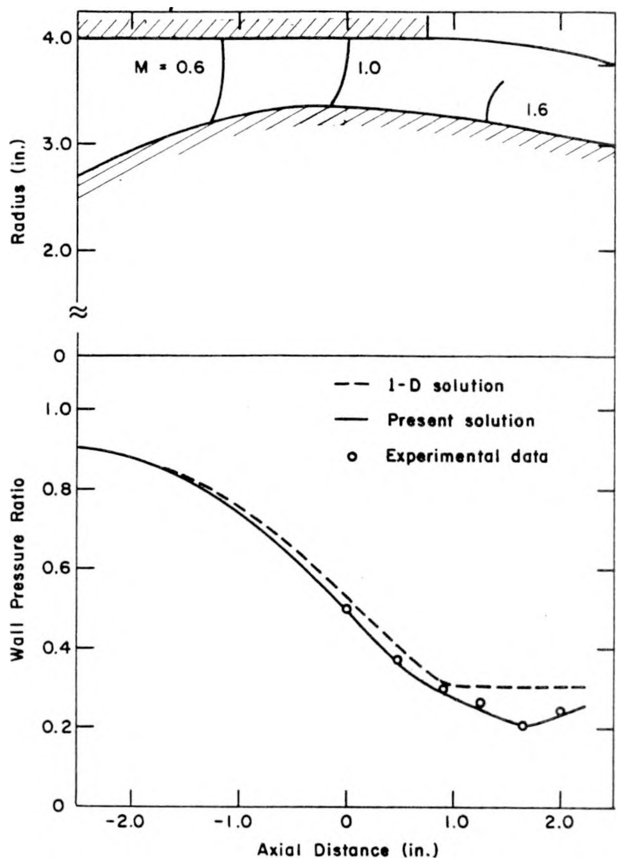

Fig. 4. Mach number contours (above) and plug pressure ratio for 10° conical plug nozzle.

with the experimental value of 0.960. The 23x7 computational mesh required 249 time planes and a computational time of 29 s. There is good agreement with the experimental data. This case was also solved by Wehofer and Moger and Brown and Ozcan. Wehofer and Moger's solution for a pressure ratio of 2 required over 2 h on an IBM 360/50 (47x11 mesh); Brown and Ozcan's results required 17 min on an IBM 360/65 (20x6 mesh).

Finally, the present method was used to calculate the flow in a 10° conical, plug nozzle. The nozzle geometry is shown in Fig. 1c. The Mach number contours and plug pressure ratio for a nozzle pressure ratio of 3.29 are shown in Fig. 4. The experimental data are those of Bresnahan and Johns.[18] The 31x6 computational mesh required 327 time planes and a computational time of 52 s. Again, there is good agreement with the experimental data. The author is unaware of any other time-dependent analyses of plug nozzles.

H. Concluding Remarks

A method of computing nozzle flows has been presented. A production-type computer program capable of solving a wide variety of nozzle flows has been developed. The program's accuracy was demonstrated by computing the steady flow in a 45°-15° conical, converging-diverging nozzle, a 15° conical, converging nozzle, and a 10° conical, plug nozzle. The < 1-min computational time for these steady flows is considerably faster than for any of the earlier time-dependent techniques.

II. DESCRIPTION AND USE OF THE NAP PROGRAM

A. Subroutine Description

The computer program consists of one program, one function, and twelve subroutines.

1. Program MAIN. Program MAIN initiates a run by reading in the input data. Next, the program title, abstract, and input data descriptions are printed. The program then calls subroutines GEØM and GEØMCB to calculate the nozzle geometry. The

input data are then converted to the internal units. If requested, program MAIN calls subroutine ØNEDIM to calculate the one-dimensional, initial-value surface. Program MAIN then prints the initial-value surface, which includes a mass flow and thrust calculation made by subroutine MASFLØ. Next, subroutine PLØT is called and it plots the data on film. The final part of MAIN consists of the time-step loop, which performs the following operations: calculates the next time-step size; calls subroutine INTER to compute the interior mesh points; calls subroutine WALL to compute the wall mesh points; calls subroutine INLET to compute the inlet mesh points for subsonic flow; calls subroutine EXITT to compute the exit mesh points for subsonic flow; calls subroutine MASFLØ to compute the mass flow and thrust; prints the solution surface; calls subroutine PLØT to plot the data on film; checks the solution for its convergence to the steady-state solution; and punches the last solution plane on cards for restart.

2. Subroutine GEØM. Subroutine GEØM calculates the nozzle-wall coordinates and slopes for four different wall geometries: a constant area duct, a circular-arc, conical nozzle, and two tabular input nozzles. In the case of the first tabular nozzle, a completely general set of wall coordinates is input. Subroutine GEØM then calls subroutine MTLUP, which interpolates for equally spaced coordinates. Next, subroutine GEØM calls function DIF, which calculates the slopes of the equally spaced coordinates. For the second tabular nozzle, equally spaced coordinates and slopes are read in.

3. Subroutine GEØMCB. Subroutine GEØMCB calculates the nozzle centerbody coordinates and slopes for four different centerbody geometries and is the same as subroutine GEØM.

4. Subroutine MTLUP. Subroutine MTLUP was taken from the NASA Langley program library. The data of this version is 9-12-69. This subroutine is called by subroutines GEØM and GEØMCB to interpolate the nozzle-wall and centerbody coordinates for equally spaced coordinates.

5. Function DIF. Function DIF was also taken from the NASA Langley program library. The date of this version is 8-1-68. This function is called by subroutines GEØM and GEØMCB to calculate the slopes of the nozzle-wall and centerbody coordinates.

6. Subroutine ØNEDIM. Subroutine ØNEDIM is called by program MAIN to compute the one-dimensional, isentropic, initial-value surface. A Newton-Raphson scheme is used to calculate the Mach number for the area ratios, which are determined from the nozzle geometry.

7. Subroutine MAP. Subroutine MAP calculates the mapping functions that map the physical plane to a rectangular computational plane. Therefore, this subroutine is called before each mesh point is calculated.

8. Subroutine MASFLØ. Subroutine MASFLØ is called by program MAIN to calculate the mass flow and thrust for the initial-value and solution surfaces. This subroutine uses the trapezoidal rule to evaluate the mass flow and thrust integrals.

9. Subroutine PLØT. Subroutine PLØT produces velocity vector plots, and contours of density, pressure, temperature, and Mach number plots using the SC-4020 microfilm recorder. English units are used for the contour values. This subroutine uses five LASL system routines: GETQ, ADV, LINCNT, PLT, and DRV. GETQ obtains the job identification label. ADV advances the film. LINCNT indexes a specified horizontal line on each frame. PLT and DRV plot a plus sign at a specified point and draw a vector between two specified points, respectively. If this program is used on non-LASL computers, these five routines will have to be replaced by their respective counterparts on another computing system.

10. Subroutine SHØCK. Subroutine SHØCK calculates the artificial viscosity for shock computations. The artificial viscosity model is a quadratic viscosity after von Neumann-Richtmyer and has a term corresponding to all the viscous and thermal-conduction terms in the Navier-Stokes equations. In addition, this subroutine adds numerical smoothing to stabilize the calculations for very nonuniform initial-data surfaces or to accelerate the convergence to steady state.

11. Subroutine INTER. Subroutine INTER is called by program MAIN to calculate the interior mesh points. The technique used by this subroutine is the MacCormack, second-order, finite-difference scheme.

12. Subroutine WALL. Subroutine WALL is called by program MAIN to compute the wall, centerbody, and exhaust jet boundary mesh points. This subroutine uses a second-order, reference-plane characteristic scheme and also controls the inter-

polation process for locating the exhaust jet boundary.

13. Subroutine INLET. Subroutine INLET is called by program MAIN to compute the inlet mesh points for subsonic flow. A second-order, reference-plane characteristic scheme is employed by subroutine INLET.

14. Subroutine EXITT. Subroutine EXITT is called by program MAIN to calculate the exit mesh points when the flow is subsonic. It uses a second-order, reference-plane characteristic scheme.

B. Input Data Description

The program input data is entered by a title card and five namelists. The namelists are CNTRL, IVS, GEMTRY, GCBL, and BC. The title card and each namelist are discussed below. The program will continue reading in data decks and executing them until a file mark is encountered. After each data deck is executed, the default values for the input data are restored before the next data deck is read in.

1. Title Card. The first card of each data deck is a title card consisting of 80 alphanumeric characters which identify the job. This card must always be the first card of the data deck, even if no information is specified on the card. The five namelists must appear in the order in which they are discussed below.

2. Namelist CNTRL. This namelist inputs the parameters that control the overall logic of the program.

LMAX — An integer specifying the number of mesh points in the X or axial direction (81 maximum). No default value is specified.

MMAX — An integer specifying the number of mesh points in the Y or radial direction (21 maximum). No default value is specified.

NMAX — An integer specifying the maximum number of time steps. For NMAX = 0, only the initial-data surface is computed and printed (provided NPRINT > 0). The default value is 0.

NPRINT — An integer specifying the amount of output desired. For NPRINT = N, every Nth solution plane, plus the initial-data and final solution planes, are printed. For NPRINT = -N, every Nth solution plane, plus the final solution plane, are printed. For NPRINT = 0, only the final solution plane is printed. The default value is 0.

TCØNV — Specifies the axial velocity steady-state convergence tolerance in percent. If less than or equal to zero, the convergence is not checked. The default is 0.0.

FDT — A parameter that premultiplies the allowable C-F-L time step. It is desirable to use as large a value of FDT as possible without causing the computation to become unstable. Values as large as 1.6 have been used successfully for shock-free flows, while values of 0.4 to 0.6 are required for flows with shocks. The default value is 1.0.

GAMMA — Denotes the ratio of specific heats. The default value is 1.4.

RGAS — Denotes the gas constant in lbf-ft/lbm-°R if English units are used, or J/kg-°K if metric units are used. The default value is 53.35.

TSTØP — Specifies the physical time, in seconds, at which the computations will be stopped. The default value is 1.0.

IAV — An integer which, if nonzero, requests the addition of a local artificial viscosity to stabilize calculations in the vicinity of a shock. The value of FDT must be reduced to ~0.6. The default value for IAV is 0.

IUI — An integer specifying the type of units to be used for the input quantities. If IUI equals 1, English units are assumed; if equal to 2, metric units are assumed. In using any default values, make sure the values correspond to the proper units. The default value is 1.

IUØ — Same as IUI except for output quantities. If IUØ equals 3, both English and metric units are printed. The default value is 1.

IPUNCH — An integer which, if nonzero, punches the last solution plane on cards for restart. The default value is 0.

NPLOT — An integer which, if greater than or equal to zero, plots both velocity vectors and contours of density, pressure, temperature, and Mach number on a SC-4020 microfilm recorder. For NPLOT = N, all Nth solution planes, plus the initial-data and final solution plane, are plotted. For NPLØT = 0, only the final solution plane is plotted. The default value is -1.

NST An integer denoting the time step at which a small amount of numerical smoothing is stopped. This smoothing may be required to stabilize the calculations for very nonuniform initial-data surfaces. Some initial smoothing caused subsonic flows to reach steady state faster, but this was not the case for transonic and supersonic flows. The default value is 0 (no smoothing). When using the restart option, make sure NST is set equal to zero.

The remaining parameters in namelist CNTRL are less important than the parameters given above. For most nozzle flows, these remaining parameters can be left at their default values.

NASM An integer specifying which part of the flowfield is tested for steady-state convergence. For NASM = 0, the entire flowfield is tested. For NASM = 1, the transonic and supersonic (throat region to exit) regions are tested. The default value is 1.

NAME An integer which, when nonzero, causes the five namelists to be printed in addition to the regular output. The default value is 0.

NCØNVI An integer specifying how many times the convergence tolerance TCØNV must be satisfied on consecutive time steps before the solution is considered to have converged. The default value is 1.

IUNIT An integer which, when equal to zero, causes the program to use either English or metric units (see IUI and IUO). For IUNIT equal to 1, a nondimensional set of units is used. The default value is 0.

CAV Denotes the artificial viscosity premultiplier c in the equations for λ and μ. The default value is 4.0.

XMU Denotes the coefficient c_μ in the equation for μ in the artificial viscosity model. A nondimensional value is used. The default value is 0.2.

XLA Denotes the coefficient c_λ in the equation for λ in the artificial viscosity model. A nondimensional value is used. The default value is 1.0.

RKMU Denotes the Prandtl number for the fluid used in the artificial viscosity model. The default value is 0.7.

CTA Denotes the amount of time-averaging for the artificial viscosity. For CTA = 1.0, the values at the current time step are used. For CTA = 0.0, the values at the previous time step are used. For CTA between 0.0 and 1.0, a linear average is used. The default value is 0.5.

LSS An integer specifying the axial mesh point at which the addition of the artificial viscosity will begin. The default value is 2.

PLØW If the pressure becomes negative during a calculation, it is set equal to PLØW (psia). The default value is 0.01 psia.

RØLØW If the density becomes negative during a calculation, it is set equal to RØLØW (lbm/ft^3). The default value is 0.0001 lbm/ft^3.

SMP A parameter controlling the amount of smoothing (provided NST $\neq$ 0). The dependent variables are smoothed by the following formula:

$$u_{L,M} = SMP*u_{L,M} + (1.0-SMP)*$$
$$(u_{L+1,M} + u_{L,M+1} + u_{L-1,M} + u_{L,M-1})/4.0$$

The default value is 0.95.

3. Namelist IVS. This namelist specifies the flow variables for the initial-data surface.

NID An integer specifying the type of initial-data surface desired. For NID = 0, a two-dimensional, initial-data surface is read in. A value of U, V, P, and RØ (discussed below) must be read in for all L = 1 to LMAX and M = 1 to MMAX mesh points. For NID $\neq$ 0, a one-dimensional data surface is computed internally.

The following combinations are possible:

N1D = -2 subsonic } see RSTAR and RSTARS
N1D = -1 supersonic }

N1D = 1 subsonic-sonic-supersonic
N1D = 2 subsonic-sonic-subsonic
N1D = 3 supersonic-sonic-supersonic
N1D = 4 supersonic-sonic-subsonic
} No additional data is needed

The default value is 1.

U(L,M,1) An array denoting the X or axial direction velocity component in ft/s or m/s. For N1D = 0, U(L,M,1) must be input for L = 1 to LMAX and M = 1 to MMAX. For N1D ≠ 0, U(L,M,1) is not input. No default values are specified.

V(L,M,1) An array denoting the Y or radial direction velocity component in ft/s or m/s. See U(L,M,1) for additional information. No default values are specified.

P(L,M,1) An array denoting the pressure in psia or kPa. See U(L,M,1) for additional information. No default values are specified.

RØ(L,M,1) An array denoting the density in lbm/ft^3 or kg/m^3. See U(L,M,1) for additional information. No default values are specified.

RSTAR, RSTARS If N1D = -1 or -2, either RSTAR for planar or RSTARS for axisymmetric flow must be input. RSTAR is the area per unit depth, in inches or cm, where the Mach number is unity. RSTARS is the area divided by π, in $in.^2$ or cm^2, where the Mach number is unity. No default values are specified.

If the restart option is to be used, the initial run must have been made with IPUNCH ≠ 0 in CNTRL, thereby causing a new IVS deck to be punched. The new IVS replaces the one used in the initial run and includes two additional parameters, NSTART and TSTART, which denote, respectively, the time step and physical time where the solution was restarted.

When N1D ≠ 0, the initial data is calculated using one-dimensional, isentropic theory. However, the axial and radial velocity components are adjusted while keeping the magnitude constant and satisfying the flow angle. The flow angles are linearly interpolated between the slope of the wall and centerbody.

4. Namelist GEMTRY. This namelist specifies the parameters that define the nozzle-wall contour.

NDIM An integer denoting the flow geometry. For NDIM = 0, two-dimensional, planar flow is assumed, and for NDIM = 1, axisymmetric flow is assumed. The default value is 1.

NGEØM An integer specifying one of four different nozzle-wall geometries. A discussion of these four cases follows the definitions of the additional parameters in this namelist. No default value is specified.

XI The axial coordinate, in inches or cm, of the nozzle-wall inlet. No default value is specified.

RI The radial coordinate, in inches or cm, of the nozzle-wall inlet. No default value is specified.

RT The radial coordinate, in inches or cm, of the nozzle-wall throat. No default value is specified.

XE The axial coordinate, in inches or cm, of the nozzle-wall exit. No default value is specified.

RCI The radius of curvature, in inches or cm, of the nozzle-wall inlet. No default value is specified.

RCT The radius of curvature, in inches or cm, of the nozzle-wall throat. No default value is specified.

ANGI The angle, in degrees, of the converging section. No default value is specified.

ANGE The angle, in degrees, of the diverging section. No default value is specified.

XWI A one-dimensional array of nonequally spaced axial coordinates in inches or cm. No default values are specified.

YWI A one-dimensional array of radial coordinates, in inches or cm, corresponding to the axial coordinates in array XWI. No default values are specified.

NWPTS An integer specifying the number of entries in arrays XWI and YWI. The maximum value is 81. No default value is specified.

IINT An integer specifying the order of interpolation used. The maximum value is 2. The default value is 1.

IDIF An integer specifying the order of differentiation used. The maximum value is 5. The default value is 1.

YW A one-dimensional array of radial coordinates, in inches or cm, which correspond to LMAX equally spaced axial coordinates. No default values are specified.

NXNY A one-dimensional array (floating point) of the negative of the wall slopes corresponding to the elements of YW. No default values are specified.

JFLAG An integer which, when nonzero, denotes that an exhaust jet calculation is to be carried out. The default value is 0.

LJET An integer specifying the first mesh point of the exhaust jet boundary. For example, if the nozzle wall ends at the 10th axial mesh point, LJET = 11. The program assumes that the nozzle always ends at a mesh point. No default value is specified.

The following is a discussion of the four different wall geometries considered by this program.

a. Constant area duct (NGEØM = 1). The parameters XI, RI (radius of the duct), and XE must be specified.

b. Circular-arc, conical nozzle wall (NGEØM = 2). The geometry for this case is shown in Fig. 5. The parameters XI, RI, RT, XE, RCI, RCT, ANGI, and ANGE are specified. The axial coordinate of the throat and the radius of the exit are computed internally.

c. General nozzle wall (NGEØM = 3). An arbitrary nozzle-wall contour is specified by tabular input. NWPTS axial and radial coordinate pairs are specified by the arrays XWI and YWI, respectively. For good accuracy, one coordinate pair should be the nozzle throat. The tabular data need not be equally

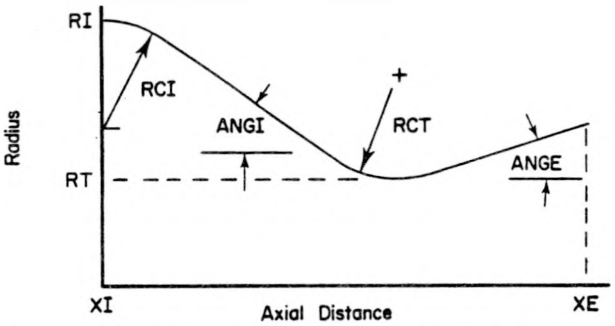

Fig. 5. Circular-arc, conical nozzle-wall geometry.

spaced. From the specified values of NWPTS, XWI, YWI, IINT, and IDIF, the program uses IINT-order interpolation to obtain LMAX equally spaced nozzle-wall contour points. Next, IDIF-order differentiation is used to obtain the nozzle-wall slope at these LMAX points.

d. General nozzle wall (NGEØM = 4). An arbitrary nozzle-wall contour is specified by tabular input. LMAX radial coordinates and the negative of their slopes are specified by the arrays YW and NXNY, respectively. These radial coordinates correspond to the LMAX equally spaced, axial mesh points. Therefore, XI and XE are input instead of each axial coordinate. For good accuracy, one pair of coordinates should be the nozzle throat.

5. Namelist GCBL. This namelist specifies the parameters that define the nozzle centerbody geometry. If no centerbody is present, this namelist is left blank but must still be present in the data deck.

NGCB An integer which, when nonzero, specifies one of four different centerbody geometries. A discussion of these four cases will follow the definitions of the additional parameters in this namelist. The default value is 0.

RICB The radial coordinate, in inches or cm, of the centerbody inlet. No default value is specified.

RTCB The radial coordinate, in inches or cm, of the centerbody maximum radius. No default value is specified.

RCICB The radius of curvature, in inches or cm, of the centerbody inlet. No default value is specified.

RCTCB The radius of curvature, in inches or cm, of the centerbody maximum radius. No default value is specified.

ANGICB The angle, in degrees, of the converging section. No default value is specified.

ANGECB The angle, in degrees, of the diverging section. No default value is specified.

XCBI A one-dimensional array of nonequally spaced axial coordinates in inches or cm. No default values are specified.

YCBI A one-dimensional array of radial coordinates, in inches or cm, corresponding to the axial coordinates in array XCBI. No default value are specified.

NCBPTS An integer specifying the number of entries in arrays XCBI and YCBI. The maximum value is 81. No default value is specified.

IINTCB An integer specifying the order of interpolation used. The maximum value is 2. The default value is 1.

IDIFCB An integer specifying the order of differentiation used. The maximum value is 5. The default value is 1.

YCB A one-dimensional array of radial coordinates, in inches or cm, which correspond to LMAX equally spaced axial coordinates. No default values are specified.

NXNYCB A one-dimensional array (floating point) of the negative of the centerbody slopes corresponding to the elements of YCB. No default values are specified.

The following is a discussion of the four different centerbody geometries considered by this program.

a. Cylindrical centerbody (NGCB = 1). The parameter RICB (radius of the centerbody) must be specified.

b. Circular-arc, conical centerbody (NGCB = 2). The geometry for this case is shown in Fig. 6. The parameters RICB, RTCB, RCICB, RCTCB, ANGICB, and ANGECB are specified. The axial coordinate of the maximum radius and the radius of the exit are computed internally.

c. General centerbody (NGCB = 3). An arbitrary centerbody contour is specified by tabular input. NCBPTS axial and radial coordinate pairs are specified by the arrays XCBI and YCBI, respectively. The tabular data need not be equally spaced. From the specified values of NCBPTS, XCBI, YCBI, IINTCB, and IDIFCB, the program uses IINTCB-order interpolation to obtain LMAX equally spaced centerbody contour points. Next, IDIFCB-order differentiation is used to obtain the centerbody slope at these LMAX points.

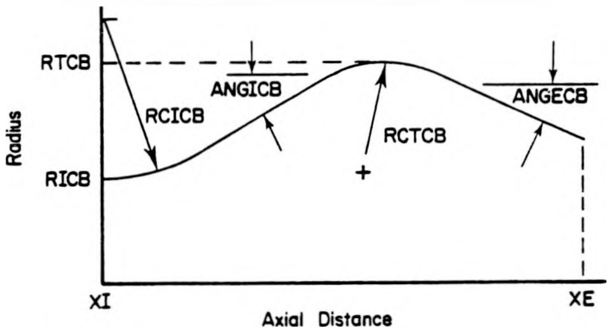

Fig. 6. Circular-arc, conical nozzle centerbody geometry.

d. General centerbody (NGCB = 4). An arbitrary centerbody contour is specified by tabular input. LMAX radial coordinates and the negative of their slopes are specified by the arrays YCB and NXNYCB, respectively. These radial coordinates correspond to the LMAX equally spaced, axial mesh points.

6. Namelist BC. This namelist specifies the flow variables for the nozzle inlet and exit computational boundaries.

NSTAG An integer which, when nonzero, denotes that variable total pressure PT, variable total temperature TT, and variable flow angle THETA (all discussed below) across the nozzle inlet have been specified. If NSTAG $\neq$ 0, then a value for PT, TT, and THETA must be specified at the M = 1 to MMAX points even if one or two of the variables are constant. If NSTAG = 0, only the first value for each of the three arrays needs to be specified. The default value is 0.

PT(M) A one-dimensional array denoting the stagnation pressure, in psia or kPa, across the nozzle inlet. No default values are specified.

TT(M) A one-dimensional array denoting the stagnation temperature, in °F or °C, across the nozzle inlet. No default values are specified.

THETA(M) A one-dimensional array denoting the flow angle, in degrees, across the nozzle inlet. The default value is THETA(1) = 0.0, which is meaningful only when NSTAG = 0.

PE The pressure, in psia or kPa to which the nozzle is exiting. This pressure is used to compute the nozzle exit conditions when the flow is subsonic and the exhaust jet boundary location when an exhaust jet calculation is requested. The default value is 14.7.

ISUPER An integer which, when nonzero, specifies that the inlet flow is supersonic. For this case the boundary condition is the specification of the variables UI, VI, PI, and RØI discussed below. The default value is 0.

UI(M) A one-dimensional array denoting the axial velocity, in ft/s or m/s, across the nozzle inlet. This array, as well as the arrays VI, PI, and RØI below, start with the centerline or centerbody value and end with the wall value. No default values are specified.

VI(M) Same as UI, except radial velocity.

PI(M) Same as UI, except pressure in psia or kPa.

RØI(M) Same as UI, except density in lbm/ft^3 or kg/m^3.

The present version of this program allows for a maximum of 81 axial and 21 radial mesh points. To increase or decrease the maximum number of mesh points, the dimensions of the arrays QUT, QVT, and QPT in common AV, the arrays U,V,P, and RØ in common SØLUTN, the arrays XW, YW, NXNY, XWI, and YWI in common GEMTRY, the arrays XCB, YCB, NXNYCB, XCBI, and YCBI in common GCB, the arrays PT, TT, and THETA in common BCC, the arrays UI, VI, PI and RØI in program MAIN, the arrays YW and NXNY in subroutine GEØMCB, and the array CQ in subroutine PLØT must be changed to the desired values. In addition, card MAI 2950 must be changed such that LD and MD correspond to the new maximum axial and radial mesh points respectively.

C. Output Description

Program output consists of printed output, film plots, and punched cards for restart. The program has no options to output any results on magnetic tapes. For all computer-printed figures, the number zero has a slash through it, while the typed text has a slash through the letter O.

The first page (or first two pages in the tabular input nozzle case), of output includes the program title, abstract, list of control parameters, fluid model, flow geometry, nozzle geometry, and boundary conditions.

Following the title page is the initial-data surface. This data is either data that has been input or a one-dimensional solution that has been computed by the program. All units are given. At the bottom of the initial-data surface is the mass flow at the minimum cross section (MASS), the thrust (THRUST) due to the exit momentum only, the inlet mass flow (MASSI), and the exit mass flow (MASSE). For planar flow the mass flow units are lbm/in-s or kg/in-s and the thrust units are lb_f/in. or newtons/in. When the initial-data surface is the one-dimensional solution calculated by the program, the mass flow and thrust values are also the one-dimensional values, even though the velocity components are not.

After the initial-data surface has been printed, the solution surfaces are printed. These surfaces have the same format as the initial-data surface. Each solution surface gives the flowfield for a certain value of time. As many solution planes as desired are printed by varying the input data. If the artifical viscosity option is used, a page of artifical viscosity parameters is printed before each solution plane. Also, film plots are made for each requested time step. When the computation is stopped because the flow has satisfied the convergence tolerance, the physical time equals TSTØP, or the maximum number of time steps has been reached, the final solution plane is always printed and plotted. As in the case of the initial-data surface, the mass flow and thrust are printed below the solution surface. The thrust calculation includes only the exit momentum.

D. Sample Calculations

1. Case No. 1 - Converging-Diverging Nozzle. The nozzle geometry for this case is shown in Fig. 1a. and results are shown in Fig. 2. The data deck and printed output are presented in Figs. 7 and 8, respectively.

a. Namelist CNTRL. This case used a 21x8 mesh; therefore, LMAX = 21 and MMAX = 8. The maximum number of time steps NMAX is set equal to 400. The convergence tolerance TCØNV is set equal to 0.003. The step-size premultiplier FDT is set equal to 1.6. The additional parameters are left equal to their default values.

b. Namelist IVS. A one-dimensional, subsonic-sonic-supersonic, initial-data surface was computed by the program; therefore, no input is required.

c. Namelist GEMTRY. For this case, the nozzle wall is a conical, converging-diverging nozzle; therefore, NGEØM = 2. The axial location of the inlet XI equals 0.31 in., the radius of the inlet RI equals 2.5 in., the radius of the throat RT equals 0.8 in., and the axial location of the exit XE equals 4.05 in. The radius of curvature of the inlet RCI

```
CASE NO. 1 - CONVERGING-DIVERGING NOZZLE (45 DEG INLET, 15 DEG EXIT)
 $CNTRL LMAX=21,MMAX=8,NMAX=400,TCONV=0.003,FDT=1.6   $
 $IVS   $
 $GEMTRY NGEOM=2,XI=0.31,RI=2.5,RT=0.8,XE=4.05,RCI=0.8,RCT=0.5,ANGI=44.88,
 ANGE=15.0   $
 $GCBL   $
 $BC PT=70.0,TT=80.0   $
```

Fig. 7. Case No. 1 data deck.

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NAP, A COMPUTER PROGRAM FOR THE COMPUTATION OF TWO-DIMENSIONAL, TIME-DEPENDENT, INVISCID NOZZLE FLOW

BY MICHAEL C, CLINE, T-3 - LOS ALAMOS SCIENTIFIC LABORATORY


PROGRAM ABSTRACT -

          THE EQUATIONS OF MOTION FOR TWO-DIMENSIONAL, TIME DEPENDENT, INVISCID FLOW IN A NOZZLE
     ARE SOLVED USING THE SECOND-ORDER, MACCORMACK, FINITE-DIFFERENCE SCHEME, THE FLUID IS ASSUMED
     TO BE A PERFECT GAS, ALL BOUNDARY CONDITIONS ARE COMPUTED USING A SECOND-ORDER, REFERENCE PLANE
     CHARACTERISTIC SCHEME, THE STEADY STATE SOLUTION IS OBTAINED AS THE ASYMPTOTIC SOLUTION FOR
     LARGE TIME, THE NOZZLES MAY BE EITHER CONVERGING, CONVERGING-DIVERGING, OR PLUG GEOMETRIES,


JOB TITLE -

     CASE NO, 1 - CONVERGING-DIVERGING NOZZLE (45 DEG INLET, 15 DEG EXIT)


CONTROL PARAMETERS -

     LMAX=21   MMAX= 8    NMAX= 400  NPRINT=   0  TCONV=  ,003    FDT=1,60   NSTAG=0     NASM=1    IUNIT=0
     IUI=1     IUO=1      IEX=1      NCONVI= 1    TSTOP=1,00000   NID= 1     NPLOT=  -1  IPUNCH=0  ISUPER=0
     IAV=0     CAV= 5,0   XMU=1,00   XLA=0,00     RKMU=  ,50      CTA= ,50   LSS= 2      SMP= ,95  NST=   0


FLUID MODEL -

     THE RATIO OF SPECIFIC HEATS, GAMMA =1,4000 AND THE GAS CONSTANT, R =  53,3500 (FT-LBF/LBM-R)


FLOW GEOMETRY -

     AXISYMMETRIC FLOW HAS BEEN SPECIFIED


NOZZLE GEOMETRY -

     A CIRCULAR-ARC, CONICAL NOZZLE HAS BEEN SPECIFIED BY XI=   ,3100 (IN), RI=  2,5000 (IN),
     RT=   ,8000 (IN), XE=  4,0500 (IN), RCI=   ,8000 (IN), RCT=   ,5000 (IN), ANGI= 44,88 (DEG),
     AND ANGE= 15,00 (DEG), THE COMPUTED VALUES ARE XT=  2,5540 (IN) AND RE=  1,1832 (IN),


BOUNDARY CONDITIONS -

     PT=  70,0000 (PSIA)     TT=  80,0000 (F)     THETA=   0,0000 (DEG)     PE=  14,7000 (PSIA)
N=  10
N=  20
N=  30
N=  40
N=  50
N=  60
N=  70
N=  80
N=  90
N= 100
N= 110
N= 120
N= 130
```

Fig. 8. Case No. 1 output.

N= 140
N= 150
N= 160
N= 170
N= 180
N= 190
N= 200
N= 210
N= 220
N= 230
N= 240
N= 250
N= 260
N= 270
N= 280
N= 290
N= 300

Fig. 8 (cont)

SOLUTION SURFACE NO. 301 - TIME = .00162916 SECONDS (DELTA T = .00000541)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
1	1	.3100	0.0000	144.3320	-0.0000	69.2160	.347087	144.3320	.1269	78.2651
1	2	.3100	.3571	141.7219	-0.0000	69.2440	.347187	141.7219	.1246	78.3272
1	3	.3100	.7143	128.2883	-0.0000	69.3801	.347675	128.2883	.1128	78.6293
1	4	.3100	1.0714	109.3612	-0.0000	69.5491	.348279	109.3612	.0961	79.0040
1	5	.3100	1.4286	86.4350	-0.0000	69.7181	.348884	86.4350	.0759	79.3778
1	6	.3100	1.7857	62.2730	-0.0000	69.8536	.349368	62.2730	.0547	79.6771
1	7	.3100	2.1429	39.9999	-0.0000	69.9396	.349675	39.9999	.0351	79.8668
1	8	.3100	2.5000	19.8533	-0.0000	69.9851	.349838	19.8533	.0174	79.9672
2	1	.4970	0.0000	146.0691	0.0000	69.2904	.347288	146.0691	.1284	78.5319
2	2	.4970	.3540	143.4827	-4.4481	69.3196	.347388	143.5516	.1262	78.6038
2	3	.4970	.7080	130.0260	-8.4555	69.4507	.347845	130.3006	.1145	78.9123
2	4	.4970	1.0619	110.9310	-10.9369	69.6135	.348395	111.4688	.0979	79.3234
2	5	.4970	1.4159	87.8029	-11.7973	69.7741	.348920	88.5919	.0778	79.7545
2	6	.4970	1.7699	63.3199	-10.8059	69.8959	.349328	64.2353	.0564	80.0651
2	7	.4970	2.1239	40.7250	-8.3105	69.9730	.349708	41.5643	.0365	80.0739
2	8	.4970	2.4778	20.2377	-4.8653	70.0063	.349850	20.8143	.0183	80.1108
3	1	.6840	0.0000	153.2228	0.0000	69.1441	.346842	153.2228	.1347	78.0858
3	2	.6840	.3439	150.4176	-8.0795	69.1721	.346941	150.6344	.1325	78.1501
3	3	.6840	.6878	136.5817	-15.2234	69.2963	.347379	137.4275	.1208	78.4365
3	4	.6840	1.0317	116.4617	-19.6710	69.4572	.347946	118.1113	.1038	78.8078
3	5	.6840	1.3755	92.1280	-21.2109	69.6187	.348556	94.5382	.0831	79.1154
3	6	.6840	1.7194	66.2422	-19.7190	69.7494	.349177	69.1149	.0607	79.1672
3	7	.6840	2.0633	42.2740	-15.8931	69.8388	.349479	45.1628	.0397	79.3908
3	8	.6840	2.4072	21.3548	-11.2935	69.8902	.349784	24.1572	.0212	79.3180
4	1	.8710	0.0000	165.0230	0.0000	69.0375	.346366	165.0230	.1451	77.9945
4	2	.8710	.3243	161.8471	-12.7268	69.0718	.346482	162.3467	.1428	78.0821
4	3	.8710	.6487	148.1388	-24.0734	69.2124	.346970	150.0821	.1319	78.4187
4	4	.8710	.9739	127.6638	-31.8727	69.4034	.347638	131.5824	.1156	78.8667
4	5	.8710	1.2973	102.9220	-35.8430	69.6042	.348388	108.9846	.0957	79.2632
4	6	.8710	1.6217	76.1942	-35.2481	69.7817	.348968	83.9523	.0737	79.7398
4	7	.8710	1.9460	51.5053	-32.0512	69.9154	.349415	60.6636	.0532	80.0812
4	8	.8710	2.2703	29.4211	-28.9397	70.0166	.349004	41.2688	.0362	81.4995
5	1	1.0580	0.0000	184.0563	0.0000	68.7930	.345583	184.0563	.1620	77.3041
5	2	1.0580	.2977	180.4888	-16.5732	68.8270	.345704	181.2481	.1595	77.3811
5	3	1.0580	.5955	167.0840	-31.6930	68.9608	.346189	170.0632	.1496	77.6716
5	4	1.0580	.8932	146.2731	-42.9213	69.1497	.346899	152.4403	.1341	78.0406
5	5	1.0580	1.1909	120.6574	-49.3831	69.3592	.347728	130.3722	.1146	78.3850
5	6	1.0580	1.4887	92.4029	-49.9827	69.5637	.348495	105.0170	.0923	78.7833
5	7	1.0580	1.7864	66.3097	-46.6519	69.7342	.349073	81.0763	.0712	79.2094
5	8	1.0580	2.0841	45.6459	-45.4552	69.8646	.351093	64.4183	.0567	77.2645
6	1	1.2450	0.0000	208.6485	0.0000	68.3644	.343931	208.6485	.1838	76.5209
6	2	1.2450	.2711	204.5468	-20.3330	68.4076	.344081	205.5549	.1810	76.6265
6	3	1.2450	.5423	191.1932	-39.2038	68.5551	.344612	195.1712	.1718	76.9548
6	4	1.2450	.8134	169.6380	-54.0883	68.7762	.345419	178.0522	.1567	77.4270
6	5	1.2450	1.0845	142.5697	-63.7640	69.0343	.346336	156.1792	.1374	78.0158
6	6	1.2450	1.3557	111.8738	-65.9052	69.3100	.347214	129.8432	.1141	78.7985
6	7	1.2450	1.6268	83.9892	-64.4311	69.5343	.348057	105.8563	.0930	79.2339
6	8	1.2450	1.8979	60.8833	-60.6288	69.8047	.348123	85.9223	.0753	81.2287
7	1	1.4320	0.0000	244.4324	0.0000	67.8755	.342319	244.4324	.2155	75.1934
7	2	1.4320	.2445	239.7789	-25.3265	67.9268	.342507	241.1127	.2126	75.3031
7	3	1.4320	.4891	226.5992	-49.2547	68.0787	.343078	231.8906	.2044	75.6076
7	4	1.4320	.7336	204.3100	-69.2211	68.3138	.343971	215.7177	.1901	76.0607
7	5	1.4320	.9781	174.9460	-82.8765	68.6031	.345044	193.5836	.1705	76.6578
7	6	1.4320	1.2226	140.4528	-87.7474	68.9166	.346251	165.6098	.1458	77.2313

Fig. 8 (cont)

SOLUTION SURFACE NO. 301 - TIME = .00162916 SECONDS (DELTA T = .00000541)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
7	7	1.4320	1.4672	107.5888	-85.8497	69.2024	.347372	137.6427	.1211	77.7176
7	8	1.4320	1.7117	79.2457	-78.9145	69.5794	.349485	111.8364	.0984	77.3792
8	1	1.6190	0.0000	289.8529	0.0000	66.7858	.338237	289.8529	.2561	72.9549
8	2	1.6190	.2179	284.6910	-28.5977	66.9547	.338482	286.1237	.2528	73.1195
8	3	1.6190	.4359	271.8205	-56.1340	67.0230	.339096	277.5561	.2451	73.4932
8	4	1.6190	.6538	248.8184	-88.3206	67.2999	.340096	261.4613	.2308	74.1227
8	5	1.6190	.8717	217.0973	-98.7450	67.6666	.341368	238.4991	.2103	75.0325
8	6	1.6190	1.0896	178.2156	-108.4309	68.1022	.342872	208.6098	.1838	76.1139
8	7	1.6190	1.3076	139.1469	-110.3320	68.5385	.344469	177.5810	.1563	77.0468
8	8	1.6190	1.5255	103.3920	-102.9598	69.3680	.345942	145.9131	.1279	81.2318
9	1	1.8060	0.0000	353.4364	0.0000	65.5800	.334082	353.4364	.3132	69.8416
9	2	1.8060	.1913	348.2441	-34.5414	65.6636	.334394	349.9529	.3101	70.0227
9	3	1.8060	.3826	336.6731	-68.5580	65.8359	.335051	343.5825	.3043	70.3717
9	4	1.8060	.5740	314.9155	-100.6910	66.1349	.336185	330.6213	.2927	70.9827
9	5	1.8060	.7653	282.8929	-128.3042	66.5481	.337729	310.6290	.2748	71.8578
9	6	1.8060	.9566	240.3598	-147.2737	67.0482	.339698	281.8907	.2491	72.7483
9	7	1.8060	1.1479	192.3554	-154.1167	67.6139	.341812	246.2463	.2174	73.9216
9	8	1.8060	1.3393	133.7156	-133.1567	68.7510	.346700	188.7076	.1664	75.2460
10	1	1.9930	0.0000	436.8635	0.0000	62.9419	.324223	436.8635	.3893	63.9908
10	2	1.9930	.1647	432.1713	-35.7768	63.0217	.324508	433.6496	.3864	64.1953
10	3	1.9930	.3294	423.9175	-71.8588	63.1559	.324995	429.0775	.3822	64.5243
10	4	1.9930	.4942	404.2725	-107.9751	63.4042	.325878	418.4434	.3725	65.1589
10	5	1.9930	.6589	373.7684	-142.2246	63.7957	.327237	399.9133	.3556	66.2076
10	6	1.9930	.8236	328.4187	-171.8434	64.2934	.329012	370.6602	.3292	67.4514
10	7	1.9930	.9883	264.4466	-185.0507	65.0844	.331875	322.7626	.2862	69.3341
10	8	1.9930	1.1530	173.9843	-172.3608	65.8445	.333145	244.2672	.2157	73.4752
11	1	2.1800	0.0000	549.3454	0.0000	59.7185	.312647	549.3454	.4935	55.5636
11	2	2.1800	.1381	546.9007	-38.8068	59.7792	.312892	548.2758	.4925	55.6838
11	3	2.1800	.2762	543.1126	-79.2687	59.8393	.313169	548.8669	.4930	55.7461
11	4	2.1800	.4144	534.6434	-124.1219	59.9824	.313805	548.8623	.4929	55.9318
11	5	2.1800	.5525	518.7335	-174.8851	60.2570	.314978	547.4206	.4914	56.3641
11	6	2.1800	.6906	491.7599	-234.3386	60.7072	.316985	544.7407	.4888	56.9273
11	7	2.1800	.8287	448.5535	-301.6778	61.5756	.320739	540.5643	.4844	58.1852
11	8	2.1800	.9668	392.7455	-391.1038	61.5951	.321452	554.2664	.4972	57.1991
12	1	2.3670	0.0000	703.1835	0.0000	53.3954	.288589	703.1835	.6419	39.4042
12	2	2.3670	.1195	705.4581	-30.2450	53.2708	.288114	706.1062	.6448	39.0606
12	3	2.3670	.2389	711.9606	-62.2785	52.9060	.286719	714.6793	.6533	38.0537
12	4	2.3670	.3584	724.0761	-98.6945	52.2435	.284195	730.7714	.6692	36.1859
12	5	2.3670	.4779	742.4833	-142.4451	51.1951	.280222	756.0238	.6945	33.1228
12	6	2.3670	.5974	769.8530	-199.6792	49.5354	.274167	795.3273	.7347	27.6732
12	7	2.3670	.7168	806.1904	-274.0574	47.0968	.266002	851.4989	.7946	17.8985
12	8	2.3670	.8363	926.4913	-373.6663	40.8628	.242460	999.0058	.9555	-5.1008
13	1	2.5540	0.0000	867.9594	0.0000	45.7225	.258957	867.9594	.8111	16.5731
13	2	2.5540	.1143	873.9191	-18.6820	45.4031	.257720	874.1188	.8177	15.5161
13	3	2.5540	.2266	887.6345	-37.8442	44.6459	.254760	888.4409	.8333	13.0183
13	4	2.5540	.3429	914.6763	-56.5003	43.2240	.249189	916.4197	.8640	8.1917
13	5	2.5540	.4571	958.2606	-72.0883	40.8916	.239920	961.0287	.9140	.0397
13	6	2.5540	.5714	1029.4131	-84.1867	37.0759	.224347	1032.8498	.9976	-13.9336
13	7	2.5540	.6857	1148.3074	-75.4358	30.8928	.197504	1150.7826	1.1425	-37.8082
13	8	2.5540	.8000	1368.1926	-.0522	22.7786	.157750	1368.1926	1.4137	-70.2496
14	1	2.7410	0.0000	1057.9041	0.0000	36.0264	.218543	1057.9041	1.0231	-15.0489
14	2	2.7410	.1189	1067.8182	4.0614	35.4940	.216255	1067.8292	1.0349	-16.9857
14	3	2.7410	.2370	1089.2563	11.4080	34.3251	.211179	1089.3160	1.0609	-21.2789
14	4	2.7410	.3568	1130.3124	26.9803	32.2010	.201817	1130.6344	1.1114	-29.3362

Fig. 8 (cont)

SOLUTION SURFACE NO. 301 - TIME = .00162916 SECONDS (DELTA T = .00000541)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
14	5	2.7410	.4757	1193.1032	59.7158	28.9781	.187128	1194.5967	1.1919	-42.0154
14	6	2.7410	.5946	1286.8774	125.7866	24.3779	.165031	1293.0103	1.3210	-61.2883
14	7	2.7410	.7135	1418.7214	259.2371	18.2527	.133085	1442.2116	1.5291	-89.8075
14	8	2.7410	.8325	1505.1920	403.3150	13.9588	.108703	1558.2894	1.7074	-113.3946
15	1	2.9280	0.0000	1229.8892	0.0000	27.4288	.179803	1229.8892	1.2364	-48.2460
15	2	2.9280	.1261	1240.8800	32.2297	26.8607	.177122	1241.2985	1.2516	-50.6718
15	3	2.9280	.2522	1263.7031	67.2349	25.6518	.171356	1265.4904	1.2843	-55.9399
15	4	2.9280	.3782	1305.5670	116.1644	23.5457	.161098	1310.7247	1.3462	-65.4968
15	5	2.9280	.5043	1363.5225	186.1417	20.6584	.146475	1376.1695	1.4388	-79.3179
15	6	2.9280	.6304	1434.6074	283.8963	17.2610	.128325	1462.4280	1.5657	-96.9372
15	7	2.9280	.7565	1495.9417	379.2887	14.3497	.111752	1543.2763	1.6910	-113.4100
15	8	2.9280	.8826	1523.0676	408.1040	13.2341	.104698	1576.7956	1.7414	-118.8190
16	1	3.1150	0.0000	1379.6124	0.0000	20.5366	.146067	1379.6124	1.4447	-80.5066
16	2	3.1150	.1332	1389.4608	57.2871	20.0520	.143567	1390.6493	1.4611	-83.0081
16	3	3.1150	.2665	1409.1488	116.9047	19.0318	.138250	1413.9898	1.4964	-88.4294
16	4	3.1150	.3997	1443.4828	190.1083	17.3330	.129183	1455.9477	1.5607	-97.8448
16	5	3.1150	.5330	1485.3850	276.8617	15.2534	.117647	1510.9669	1.6476	-110.0416
16	6	3.1150	.6662	1525.9339	367.4337	13.2773	.106148	1569.5483	1.7425	-122.3815
16	7	3.1150	.7994	1549.1994	421.6142	12.1092	.098925	1605.5458	1.8019	-129.6003
16	8	3.1150	.9327	1556.6257	417.0966	11.9876	.097596	1611.5375	1.8055	-128.4656
17	1	3.3020	0.0000	1507.1046	0.0000	15.3430	.118416	1507.1046	1.6440	-110.2754
17	2	3.3020	.1404	1514.6529	77.2857	14.9837	.116398	1516.6234	1.6597	-112.5417
17	3	3.3020	.2808	1529.1232	155.3746	14.2210	.112077	1536.9967	1.6942	-117.5164
17	4	3.3020	.4212	1552.9446	242.7868	13.0110	.105050	1571.8086	1.7537	-125.6938
17	5	3.3020	.5616	1577.6060	331.4854	11.7060	.097193	1612.0556	1.8239	-134.9123
17	6	3.3020	.7020	1595.3473	403.6915	10.7335	.091068	1645.6305	1.8821	-141.8704
17	7	3.3020	.8424	1597.4021	431.4081	10.4723	.089155	1654.6318	1.8956	-142.9533
17	8	3.3020	.9828	1597.3052	427.9967	10.5899	.089340	1653.6520	1.8859	-140.0558
18	1	3.4890	0.0000	1612.0423	0.0000	11.5688	.096652	1612.0423	1.8295	-136.9232
18	2	3.4890	.1476	1616.9281	91.6926	11.3290	.095191	1619.5259	1.8433	-138.7643
18	3	3.4890	.2951	1625.8324	182.2494	10.8049	.091980	1636.0152	1.8742	-142.9296
18	4	3.4890	.4427	1639.4093	276.0081	10.0197	.087043	1662.4810	1.9240	-149.2947
18	5	3.4890	.5902	1649.7370	358.9217	9.3066	.082402	1688.3296	1.9726	-155.1532
18	6	3.4890	.7378	1651.7627	412.3465	8.9842	.080145	1702.4540	1.9965	-157.4273
18	7	3.4890	.8853	1641.7622	427.9160	9.1585	.081014	1696.6129	1.9813	-154.8662
18	8	3.4890	1.0329	1638.9946	439.1673	9.2411	.081065	1696.8121	1.9733	-152.3077
19	1	3.6760	0.0000	1704.0260	0.0000	8.8149	.079430	1704.0260	2.0085	-160.4557
19	2	3.6760	.1547	1706.2877	101.0366	8.6716	.078480	1709.2764	2.0191	-161.7878
19	3	3.6760	.3094	1709.9690	198.9763	8.3402	.076304	1721.5868	2.0445	-164.9760
19	4	3.6760	.4641	1714.2779	293.1834	7.8884	.073245	1739.1680	2.0808	-169.3052
19	5	3.6760	.6189	1712.3564	365.8767	7.6060	.071250	1751.0084	2.1043	-171.8626
19	6	3.6760	.7736	1701.0635	403.9740	7.6934	.071703	1749.1524	2.0967	-170.3935
19	7	3.6760	.9283	1685.4753	420.4454	8.0075	.073602	1737.1245	2.0679	-166.3468
19	8	3.6760	1.0830	1682.1090	450.7198	7.9878	.073040	1741.4474	2.0677	-164.8157
20	1	3.8630	0.0000	1766.8661	0.0000	6.9239	.066761	1766.8661	2.1542	-180.0643
20	2	3.8630	.1619	1767.1843	105.9080	6.8519	.066250	1770.3558	2.1615	-180.8404
20	3	3.8630	.3237	1767.0745	206.8870	6.6633	.064921	1779.1443	2.1805	-182.9639
20	4	3.8630	.4856	1765.0631	297.6374	6.4524	.063380	1789.9821	2.2028	-185.2113
20	5	3.8630	.6475	1755.9466	360.1684	6.4495	.063276	1792.5037	2.2045	-184.8822
20	6	3.8630	.8094	1740.1859	391.2931	6.7291	.065123	1783.6360	2.1787	-181.0971
20	7	3.8630	.9712	1723.3741	415.9469	7.0217	.066976	1772.8593	2.1499	-177.0232
20	8	3.8630	1.1331	1719.3424	460.6964	6.9351	.065991	1779.9942	2.1559	-176.3406
21	1	4.0500	0.0000	1829.7062	-0.0000	5.0329	.054092	1829.7062	2.3553	-208.8585
21	2	4.0500	.1690	1828.0810	110.7794	5.0322	.054012	1831.4345	2.3559	-208.5265

Fig. 8 (cont)

SOLUTION SURFACE NO. 301 - TIME = .00162916 SECONDS (DELTA T = .00000541)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
21	3	4.0500	.3381	1824.1799	214.7978	4.9865	.053538	1836.7827	2.3632	-208.6011
21	4	4.0500	.5071	1815.8484	302.0914	5.0164	.053515	1840.8054	2.3607	-206.9819
21	5	4.0500	.6761	1799.5367	354.4601	5.2930	.055301	1834.1141	2.3278	-201.6568
21	6	4.0500	.8451	1778.5083	378.6123	5.7648	.058542	1818.3617	2.2752	-194.2070
21	7	4.0500	1.0142	1761.2728	411.4485	6.0360	.060351	1808.6933	2.2456	-190.0439
21	8	4.0500	1.1832	1756.5757	470.6731	5.8824	.058942	1818.5410	2.2603	-190.6220

MASS= 3.1653 (LBM/SEC) THRUST= 172.3716 (LBF) MASSI= 3.3454 MASSE= 3.1829

Fig. 8 (cont)

equals 0.8 in., while the radius of curvature of the throat RCT equals 0.5 in. The angle of the converging section ANGI equals 44.88°, while the angle of the diverging section is 15°. No other input is required.

d. Namelist GCBL. This nozzle has no centerbody, and no input is required.

e. Namelist BC. The total pressure PT equals 70 psia and the total temperature TT equals 80°F. No other input is required.

2. Case No. 2 - Converging Nozzle. The nozzle geometry for this case is shown in Fig. 1b and results are shown in Fig. 3. The data deck and printed output are presented in Figs. 9 and 10, respectively.

a. Namelist CNTRL. This case uses a 23x7 mesh; therefore, LMAX = 23 and MMAX = 7. The maximum number of time steps NMAX is set equal to 400. The convergence tolerance TCØNV is set equal to 0.005. The step-size premultiplier FDT is set equal to 1.4. The additional parameters are left equal to their default values.

b. Namelist IVS. No input is required because a one-dimensional, subsonic-sonic-supersonic, initial-data surface was computed by the program.

c. Namelist GEMTRY. For this case, the nozzle is a conical converging nozzle and either the NGEØM = 3 or 4 option could be used. For this case, the NGEØM = 4 option was chosen; therefore, the YW and NXNY arrays must be input. The axial location of the inlet XI equals -3.6 in. and the axial location of the exit XE equals 0.8 in. Since an exhaust jet calculation is required for convergent sonic nozzles, JFLAG is set equal to 1. The nozzle ends at the 19th axial mesh point; therefore, LJET is set equal to 20. The values of YW and NXNY for L = 20 to 23 are an initial guess of the shape of the exhaust jet. No other input is required.

d. Namelist GCBL. This nozzle has no centerbody, and no input is required.

e. Namelist BC. The total pressure PT equals 25.0 psia, the total temperature TT equals 180.0°F, and the ambient pressure to which the jet is exiting PE is 12.5 psia. No other input is required.

3. Case No. 3 - Converging-Diverging, Plug Nozzle. The nozzle geometry for this case is shown in Fig. 1c and results are shown in Fig. 4. The data deck and printed output are presented in Figs. 11 and 12, respectively.

a. Namelist CNTRL. This case used a 31 by 6 mesh; therefore, LMAX = 31 and MMAX = 6. The maximum number of time steps NMAX is set equal to 400. The convergence tolerance TCØNV is set equal to 0.005. The step-size premultiplier FDT is set equal to 1.6. The additional parameters are left equal to their default values.

b. Namelist IVS. A one-dimensional subsonic-sonic-supersonic, initial-data surface was computed by the program; therefore, no input is required.

c. Namelist GEMTRY. For this case, the nozzle wall is a constant area duct; therefore, NGEØM is set equal to 1. The axial location of the inlet XI and exit XE are equal to -4.44 and 2.96 in., respectively. The duct radius RI is 4.0 in. Since an exhaust jet calculation is required for plug nozzles, JFLAG is set equal to 1. The duct ends at the 22nd mesh point; therefore, LJET is set equal to 23. The NGEØM = 1 option specifies a constant radius as the initial guess of the shape of the exhaust jet. No other input is required.

d. Namelist GCBL. For this case, the nozzle centerbody is a conical, converging-diverging nozzle; therefore, NGCB = 2. The radii of the inlet RICB and throat RTCB sections are 1.3 and 3.365 in., respectively. The radii of curvature of the inlet RCICB and throat RCTCB, sections are 0.75 and 4.95 in., respectively. The angles of the inlet ANGICB and exit ANGECB sections are 45.0° and 10.0°, respectively. No other input is required.

e. Namelist BC. The total pressure PT equals 100.0 psia, the total temperature TT equals 70.0°F, and the ambient pressure to which the nozzle is exiting PE is 30.4 psia. No other input is required.

ACKNOWLEDGMENT

The initial phase of this research was accomplished while the author held a National Research Council Postdoctoral Resident Research Associateship supported by the NASA Langley Research Center, Hampton, VA.

REFERENCES

1. L. M. Saunders, "Numerical Solution of the Flow Field in the Throat Region of a Nozzle," Brown Engineering Co. report BSVD-P-66-TN-001 (NASA CR 82601) (August 1966).

2. R. F. Cuffel, L. H. Back, and P. F. Massier, "Transonic Flow-Field in a Supersonic Nozzle with Small Throat Radius of Curvature," AIAA J. 7, 1364-1366 (July 1969).

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CASE NO. 2 - CONVERGING NOZZLE (15 DEG INLET, PT/PE=2.0)
 $CNTRL LMAX=23,MMAX=7,NMAX=400,TCONV=0.005,FDT=1.4   $
 $IVS   $
 $GEMTRY NGEOM=4,XI=-3.6,XE=0.8,JFLAG=1,LJET=20,
 YW=1.93,1.91103,1.85744,1.80385,1.75026,1.69667,1.64308,1.58949,1.5359,
 1.48231,1.42872,1.37513,1.32154,1.26795,1.21436,1.16077,1.10718,1.05359,
 1.0,1.01,1.02,1.03,1.04,
 NXNY=0.0,18*0.26795,4*-0.05   $
 $GCBL   $
 $BC PT=25.0,TT=180.0,PE=12.5   $
```

Fig. 9. Case No. 2 data deck.

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NAP, A COMPUTER PROGRAM FOR THE COMPUTATION OF TWO-DIMENSIONAL, TIME-DEPENDENT, INVISCID NOZZLE FLOW

BY MICHAEL C. CLINE, T-3 - LOS ALAMOS SCIENTIFIC LABORATORY


PROGRAM ABSTRACT -

     THE EQUATIONS OF MOTION FOR TWO-DIMENSIONAL, TIME DEPENDENT, INVISCID FLOW IN A NOZZLE
ARE SOLVED USING THE SECOND-ORDER, MACCORMACK, FINITE-DIFFERENCE SCHEME. THE FLUID IS ASSUMED
TO BE A PERFECT GAS. ALL BOUNDARY CONDITIONS ARE COMPUTED USING A SECOND-ORDER, REFERENCE PLANE
CHARACTERISTIC SCHEME. THE STEADY STATE SOLUTION IS OBTAINED AS THE ASYMPTOTIC SOLUTION FOR
LARGE TIME. THE NOZZLES MAY BE EITHER CONVERGING, CONVERGING-DIVERGING, OR PLUG GEOMETRIES.


JOB TITLE -

CASE NO. 2 - CONVERGING NOZZLE (15 DEG INLET, PT/PE=2.0)


CONTROL PARAMETERS -

LMAX=23   MMAX= 7    NMAX= 400  NPRINT=   0  TCONV=  .005    FDT=1.40  NSTAG=0     NASM=1     IUNIT=0
IUI=1     IUO=1      IEX=1      NCONVI= 1    TSTOP=1.00000   NID= 1    NPLOT=  -1  IPUNCH=0   ISUPER=0
IAV=0     CAV= 5.0   XMU=1.00   XLA=0.00     RKMU=  .50      CTA= .50  LSS= 2      SMP= .95   NST=   0


FLUID MODEL -

THE RATIO OF SPECIFIC HEATS, GAMMA =1.4000 AND THE GAS CONSTANT, R =  53.3500 (FT-LBF/LBM-R)


FLOW GEOMETRY -

AXISYMMETRIC FLOW HAS BEEN SPECIFIED
```

Fig. 10. Case No. 2 output.

NOZZLE GEOMETRY -

A GENERAL NOZZLE HAS BEEN SPECIFIED BY THE FOLLOWING PARAMETERS, XT= -,0000 (IN), RT= 1,0000 (IN),

L	XW(IN)	YW(IN)	SLOPE
1	-3.6000	1.9300	-0,0000
2	-3.4000	1.9110	-,2680
3	-3.2000	1.8574	-,2680
4	-3.0000	1.8038	-,2680
5	-2.8000	1.7503	-,2680
6	-2.6000	1.6967	-,2680
7	-2.4000	1.6431	-,2680
8	-2.2000	1.5895	-,2680
9	-2.0300	1,5359	-,2680
10	-1,8900	1.4823	-,2680
11	-1.6000	1.4287	-,2680
12	-1.4000	1,3751	-,2680
13	-1.2000	1.3215	-,2680
14	-1.0000	1.2679	-,2680
15	-.8000	1.2144	-,2680
16	-.6000	1.1608	-,2680
17	-.4000	1,1072	-,2680
18	-.2000	1,0536	-,2680
19	-.0000	1.0000	-,2680
20	.2000	1.9100	,0500
21	,4000	1,0200	,0500
22	.6000	1.0300	,0500
23	,8000	1,0400	,0500

AN EXHAUST JET CALCULATION HAS BEEN REQUESTED, THE NOZZLE ENDS AT X= -,0000 (IN), THE
MESH POINTS L= 20 TO L= 23 ARE AN INITIAL APPROXIMATION TO THE EXHAUST JET BOUNDARY,

BOUNDARY CONDITIONS -

PT= 25,0000 (PSIA)   TT= 180,0000 (F)   THETA= 0,0000 (DEG)   PE= 12,5000 (PSIA)

N= 10
N= 20
N= 30
N= 40
N= 50
N= 60
N= 70
N= 80
N= 90
N= 100
N= 110
N= 120
N= 130
N= 140
N= 150
N= 160
N= 170
N= 180
N= 190
N= 200
N= 210
N= 220
N= 230
N= 240
N= 250

Fig. 10 (cont)

SOLUTION SURFACE NO. 250 - TIME = .00158212 SECONDS (DELTA T = .00000638)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
1	1	-3.6000	0.0000	218.2506	-0.0000	24.4617	.103809	218.2506	.1765	176.0323
1	2	-3.6000	.3217	216.3467	-0.0000	24.4710	.103837	216.3467	.1750	176.1012
1	3	-3.6000	.6433	211.0650	-0.0000	24.4963	.103914	211.0650	.1707	176.2889
1	4	-3.6000	.9650	202.4249	-0.0000	24.5363	.104035	202.4249	.1637	176.5859
1	5	-3.6000	1.2867	189.9110	-0.0000	24.5914	.104202	189.9110	.1535	176.9940
1	6	-3.6000	1.6083	174.1168	-0.0000	24.6561	.104398	174.1168	.1407	177.4720
1	7	-3.6000	1.9300	164.5359	-0.0000	24.6934	.104510	164.5359	.1329	177.7474
2	1	-3.4000	0.0000	220.5038	0.0000	24.4990	.103923	220.5038	.1783	176.3022
2	2	-3.4000	.3185	218.7409	-4.0464	24.5109	.103960	218.7783	.1769	176.3904
2	3	-3.4000	.6370	213.8199	-8.2875	24.5401	.104048	213.9805	.1730	176.6059
2	4	-3.4000	.9555	205.9606	-13.4604	24.5896	.104198	206.4000	.1668	176.9704
2	5	-3.4000	1.2740	194.7161	-20.2002	24.6637	.104423	195.7611	.1582	177.5146
2	6	-3.4000	1.5925	188.9249	-29.6254	24.7622	.104722	183.3343	.1480	178.2341
2	7	-3.4000	1.9110	174.0626	-46.6401	24.8416	.104960	180.2029	.1454	178.8254
3	1	-3.2000	0.0000	227.1668	0.0000	24.4463	.103763	227.1668	.1838	175.9150
3	2	-3.2000	.3096	225.7720	-5.3792	24.4543	.103787	225.8360	.1827	175.9741
3	3	-3.2000	.6191	221.6900	-10.9482	24.4738	.103846	221.9601	.1795	176.1190
3	4	-3.2000	.9287	215.5644	-17.4423	24.5049	.103941	216.2689	.1749	176.3485
3	5	-3.2000	1.2383	207.0481	-25.0312	24.5548	.104092	208.5557	.1686	176.7162
3	6	-3.2000	1.5479	198.1668	-35.7006	24.6088	.104257	201.3569	.1627	177.1105
3	7	-3.2000	1.8574	198.2077	-53.1097	24.6476	.104374	205.1997	.1658	177.3994
4	1	-3.0000	0.0000	235.6639	0.0000	24.4175	.103676	235.6639	.1907	175.6968
4	2	-3.0000	.3006	234.4539	-7.3429	24.4250	.103699	234.5689	.1898	175.7526
4	3	-3.0000	.6013	230.8325	-14.8498	24.4423	.103752	231.3097	.1871	175.8806
4	4	-3.0000	.9019	225.5461	-23.0375	24.4701	.103836	226.7196	.1834	176.0856
4	5	-3.0000	1.2026	218.3708	-31.7397	24.5143	.103971	220.6654	.1784	176.4092
4	6	-3.0000	1.5032	211.9665	-43.1284	24.5537	.104092	216.3096	.1749	176.6889
4	7	-3.0000	1.8038	212.2437	-56.8707	24.6007	.104231	219.7309	.1776	177.0544
5	1	-2.8000	0.0000	247.5469	0.0000	24.3459	.103458	247.5469	.2004	175.1670
5	2	-2.8000	.2917	246.5466	-8.7738	24.3526	.103479	246.7027	.1997	175.2168
5	3	-2.8000	.5834	243.3623	-17.7079	24.3666	.103521	244.0057	.1975	175.3216
5	4	-2.8000	.8751	238.7445	-27.0514	24.3904	.103594	240.2721	.1944	175.4974
5	5	-2.8000	1.1668	232.8556	-37.0282	24.4250	.103699	235.7813	.1908	175.7524
5	6	-2.8000	1.4585	227.6809	-48.4235	24.4573	.103798	232.7812	.1883	175.9860
5	7	-2.8000	1.7503	228.6649	-61.2708	24.5005	.103929	236.7314	.1914	176.3066
6	1	-2.6000	0.0000	261.7128	0.0000	24.2648	.103213	261.7128	.2119	174.5560
6	2	-2.6000	.2828	260.8302	-10.1521	24.2709	.103232	261.0277	.2114	174.6021
6	3	-2.6000	.5656	257.8653	-20.3801	24.2848	.103274	258.6694	.2094	174.7047
6	4	-2.6000	.8483	253.6428	-30.8959	24.3073	.103343	255.5175	.2069	174.8705
6	5	-2.6000	1.1311	248.3621	-41.6794	24.3390	.103440	251.8351	.2038	175.1021
6	6	-2.6000	1.4139	243.7855	-53.3241	24.3676	.103529	249.5492	.2020	175.3015
6	7	-2.6000	1.6967	244.3368	-65.4700	24.4135	.103665	252.9561	.2047	175.6627
7	1	-2.4000	0.0000	278.6913	0.0000	24.1552	.102879	278.6913	.2258	173.7395
7	2	-2.4000	.2738	277.8506	-11.4203	24.1616	.102899	278.0932	.2253	173.7883
7	3	-2.4000	.5477	275.0155	-22.8667	24.1760	.102942	275.9645	.2236	173.8962
7	4	-2.4000	.8215	270.9450	-34.4379	24.1995	.103014	273.1248	.2213	174.0711
7	5	-2.4000	1.0954	265.9299	-46.0672	24.2306	.103109	269.8906	.2186	174.3033
7	6	-2.4000	1.3692	261.3584	-57.9952	24.2601	.103199	267.7156	.2168	174.5205
7	7	-2.4000	1.6431	261.2947	-70.0139	24.3087	.103348	270.5122	.2190	174.8742
8	1	-2.2000	0.0000	298.2684	0.0000	24.0233	.102479	298.2684	.2419	172.7423
8	2	-2.2000	.2649	297.4346	-12.6689	24.0303	.102500	297.7043	.2414	172.7947
8	3	-2.2000	.5298	294.5968	-25.3010	24.0464	.102549	295.6813	.2398	172.9156
8	4	-2.2000	.7947	290.4766	-37.9156	24.0724	.102629	292.9407	.2375	173.1089
8	5	-2.2000	1.0597	285.3708	-50.3370	24.1059	.102732	289.7842	.2349	173.3559

Fig. 10 (cont)

SOLUTION SURFACE NO. 250 - TIME = .00158212 SECONDS (DELTA T = .00000638)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
8	6	-2.2000	1.3246	280.4825	-62.7287	24.1385	.102833	287.4114	.2329	173.5864
8	7	-2.2000	1.5895	279.4577	-74.8807	24.1921	.102992	289.3160	.2344	174.0116
9	1	-2.0000	0.0000	320.7490	0.0000	23.8619	.101985	320.7490	.2604	171.5322
9	2	-2.0000	.2560	319.8654	-13.9317	23.8697	.102009	320.1687	.2599	171.5911
9	3	-2.0000	.5120	316.9519	-27.7697	23.8881	.102065	318.1661	.2582	171.7312
9	4	-2.0000	.7679	312.6118	-41.4462	23.9180	.102156	315.3473	.2559	171.9570
9	5	-2.0000	1.0239	307.1939	-54.7331	23.9557	.102271	312.0317	.2531	172.2427
9	6	-2.0000	1.2799	301.6978	-67.7192	23.9939	.102387	309.2046	.2508	172.5310
9	7	-2.0000	1.5359	299.5372	-80.2610	24.0541	.102574	310.1038	.2514	172.9667
10	1	-1.8000	0.0000	346.3661	0.0000	23.6682	.101395	346.3661	.2815	170.0524
10	2	-1.8000	.2471	345.3925	-15.2419	23.6768	.101421	345.7286	.2810	170.1177
10	3	-1.8000	.4941	342.3345	-30.3344	23.6978	.101486	343.6758	.2793	170.2769
10	4	-1.8000	.7412	337.6399	-45.1344	23.7321	.101591	340.6432	.2767	170.5347
10	5	-1.8000	.9882	331.7133	-59.3789	23.7754	.101724	336.9860	.2737	170.8565
10	6	-1.8000	1.2353	325.4397	-73.1379	23.8208	.101866	333.5568	.2708	171.1825
10	7	-1.8000	1.4823	321.9512	-86.2668	23.8898	.102071	333.3085	.2705	171.7416
11	1	-1.6000	0.0000	375.5134	0.0000	23.4324	.100671	375.5134	.3056	168.2648
11	2	-1.6000	.2381	374.4108	-16.6154	23.4421	.100700	374.7873	.3050	168.3393
11	3	-1.6000	.4762	371.1009	-33.0393	23.4659	.100773	372.6485	.3032	168.5224
11	4	-1.6000	.7144	366.0622	-49.0830	23.5053	.100894	369.3381	.3005	168.8245
11	5	-1.6000	.9525	359.5270	-64.4590	23.5554	.101047	365.2597	.2970	169.2101
11	6	-1.6000	1.1906	352.3245	-79.1683	23.6098	.101213	361.1097	.2936	169.6314
11	7	-1.6000	1.4287	347.3365	-93.0688	23.6904	.101465	359.5893	.2922	170.2123
12	1	-1.4000	0.0000	408.9079	0.0000	23.1479	.099799	408.9079	.3334	166.0546
12	2	-1.4000	.2292	407.6655	-18.1308	23.1585	.099832	408.0685	.3327	166.1365
12	3	-1.4000	.4584	404.2326	-36.0390	23.1845	.099912	405.8359	.3308	166.3361
12	4	-1.4000	.6876	398.6307	-53.4933	23.2283	.100047	402.2039	.3278	166.6697
12	5	-1.4000	.9168	391.3549	-70.1610	23.2850	.100224	397.5943	.3239	167.0972
12	6	-1.4000	1.1459	383.0706	-86.0187	23.3495	.100426	392.6096	.3197	167.5669
12	7	-1.4000	1.3751	376.3341	-100.8387	23.4443	.100707	389.6098	.3171	168.3548
13	1	-1.2000	0.0000	447.1313	0.0000	22.7906	.098693	447.1313	.3653	163.2986
13	2	-1.2000	.2203	445.7369	-19.6613	22.8027	.098730	446.1704	.3645	163.3940
13	3	-1.2000	.4405	442.1427	-39.1094	22.8313	.098819	443.8690	.3626	163.6186
13	4	-1.2000	.6608	436.0959	-58.1395	22.8806	.098971	439.9543	.3593	164.0057
13	5	-1.2000	.8810	428.1243	-76.4190	22.9464	.099173	434.8911	.3550	164.5264
13	6	-1.2000	1.1013	418.6672	-93.7621	23.0238	.099408	429.0379	.3500	165.1462
13	7	-1.2000	1.3215	409.9834	-109.8551	23.1400	.099777	424.4462	.3461	165.9803
14	1	-1.0000	0.0000	491.8170	0.0000	22.3572	.097356	491.8170	.4030	159.8453
14	2	-1.0000	.2113	490.2796	-21.5428	22.3701	.097396	490.7527	.4021	159.9470
14	3	-1.0000	.4226	486.6123	-42.8966	22.3991	.097487	488.4994	.4002	160.1742
14	4	-1.0000	.6340	480.2180	-63.8922	22.4504	.097647	484.4498	.3967	160.5728
14	5	-1.0000	.8453	471.4310	-84.0395	22.5213	.097870	478.8631	.3920	161.1150
14	6	-1.0000	1.0566	460.5844	-103.1474	22.6094	.098150	471.9930	.3861	161.7650
14	7	-1.0000	1.2679	449.1457	-120.3486	22.7455	.098553	464.9899	.3800	162.9506
15	1	-.8000	0.0000	543.6828	0.0000	21.7821	.095555	543.6828	.4471	155.2834
15	2	-.8000	.2024	542.0487	-22.9703	21.7965	.095600	542.5352	.4461	155.4009
15	3	-.0000	.4048	538.4690	-45.8802	21.8258	.095691	540.4200	.4443	155.6414
15	4	-.8000	.6072	531.9002	-68.7134	21.8802	.095860	536.3202	.4408	156.0874
15	5	-.8000	.8096	522.6432	-91.1922	21.9605	.096107	530.5392	.4358	156.7555
15	6	-.8000	1.0120	510.3910	-112.8464	22.0651	.096426	522.7172	.4291	157.6460
15	7	-.8000	1.2144	496.5545	-133.0518	22.2581	.097051	514.0711	.4215	159.0362
16	1	-.6000	0.0000	605.7994	0.0000	21.0913	.093395	605.7994	.5005	149.5466
16	2	-.6000	.1935	604.2591	-25.2777	21.1046	.093437	604.7876	.4997	149.6538
16	3	-.6000	.3869	601.2402	-50.6770	21.1282	.093513	603.3722	.4984	149.8407

Fig. 10 (cont)

SOLUTION SURFACE NO. 250 - TIME = .00158212 SECONDS (DELTA T = .00000638)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
16	4	-.6000	.5804	595.4506	-76.5530	21.1741	.093661	600.3513	.4958	150.2000
16	5	-.6000	.7738	586.3424	-102.2688	21.2470	.093898	595.1943	.4913	150.7570
16	6	-.6000	.9673	573.5778	-128.0177	21.3467	.094228	587.6905	.4848	151.4746
16	7	-.6000	1.1608	553.1442	-148.2150	21.5507	.094828	572.6572	.4717	153.4126
17	1	-.4000	0.0000	679.9293	0.0000	20.1120	.090272	679.9293	.5656	141.3566
17	2	-.4000	.1845	678.6968	-25.3110	20.1200	.090297	679.1686	.5649	141.4296
17	3	-.4000	.3691	676.8210	-51.0818	20.1274	.090319	678.7459	.5646	141.5005
17	4	-.4000	.5536	672.4041	-77.9199	20.1488	.090384	676.9038	.5629	141.7062
17	5	-.4000	.7381	664.7553	-106.3046	20.1967	.090527	673.2015	.5596	142.1826
17	6	-.4000	.9226	649.7730	-136.1958	20.2636	.090717	663.8933	.5516	142.9160
17	7	-.4000	1.1072	627.9869	-168.2691	20.7049	.092179	650.1400	.5386	146.2742
18	1	-.2000	0.0000	767.7202	0.0000	18.9623	.086583	767.7202	.6441	131.1364
18	2	-.2000	.1756	767.7318	-26.9237	18.9591	.086574	768.2037	.6446	131.0940
18	3	-.2000	.3512	768.7246	-55.0030	18.9365	.086506	770.6898	.6468	130.8548
18	4	-.2000	.5268	770.0819	-86.4261	18.9007	.086403	774.9165	.6506	130.4457
18	5	-.2000	.7024	770.7476	-121.8434	18.8823	.086372	780.3190	.6553	130.0779
18	6	-.2000	.8780	774.0238	-175.1882	18.7029	.085858	793.6018	.6676	127.9712
18	7	-.2000	1.0536	731.2056	-195.9265	19.2978	.087683	756.9999	.6336	134.0480
19	1	.0000	0.0000	881.7838	0.0000	17.2847	.081080	881.7838	.7499	115.4064
19	2	.0000	.1667	884.8922	-21.7872	17.2351	.080919	885.1603	.7531	114.8961
19	3	.0000	.3333	892.4690	-44.5059	17.1095	.080509	893.5780	.7611	113.6201
19	4	.0000	.5000	907.4413	-69.5616	16.8712	.079729	910.1036	.7768	111.1565
19	5	.0000	.6667	929.8310	-95.3880	16.5257	.078596	934.7109	.8004	107.5286
19	6	.0000	.8333	970.9132	-136.3303	15.7535	.076019	980.4337	.8457	99.3511
19	7	.0000	1.0000	1016.1021	-147.6067	15.2390	.074031	1026.7674	.8886	95.6104
20	1	.2000	0.0000	1001.4017	0.0000	15.3667	.074543	1001.4017	.8660	96.4174
20	2	.2000	.1659	1006.6100	-10.2280	15.2797	.074239	1006.6620	.8713	95.5332
20	3	.2000	.3318	1018.5216	-20.1734	15.0807	.073541	1018.7213	.8833	93.5037
20	4	.2000	.4977	1041.5218	-28.0663	14.7043	.072212	1041.8999	.9066	89.6199
20	5	.2000	.6637	1074.2768	-28.8371	14.1890	.070371	1074.6638	.9397	84.2341
20	6	.2000	.8296	1138.3593	-40.2369	13.1441	.066588	1139.0702	1.0067	72.7999
20	7	.2000	.9955	1193.2730	-26.9502	12.4972	.064252	1193.5773	1.0626	64.9985
21	1	.4000	0.0000	1099.4854	0.0000	13.7553	.068831	1099.4854	.9657	79.4089
21	2	.4000	.1659	1103.1248	-1.2130	13.6961	.068616	1103.1255	.9695	78.7672
21	3	.4000	.3317	1111.2973	-2.4079	13.5634	.068134	1111.2999	.9780	77.3249
21	4	.4000	.4976	1125.6585	-2.8971	13.3319	.067290	1125.6623	.9930	74.7738
21	5	.4000	.6634	1140.3312	.7380	13.1050	.066459	1140.3014	1.0083	72.2436
21	6	.4000	.8293	1163.6872	-.5522	12.7467	.065140	1163.6873	1.0329	68.1722
21	7	.4000	.9951	1190.9668	-2.0159	12.4996	.064260	1190.9685	1.0603	65.0239
22	1	.6000	0.0000	1161.1336	0.0000	12.7426	.065154	1161.1336	1.0309	67.8962
22	2	.6000	.1660	1162.4664	1.8530	12.7227	.065079	1162.4678	1.0323	67.6732
22	3	.6000	.3320	1165.5589	3.5348	12.6768	.064908	1165.5642	1.0356	67.1596
22	4	.6000	.4980	1170.6776	4.7717	12.5970	.064612	1170.6873	1.0410	66.2712
22	5	.6000	.6640	1173.8592	7.0631	12.5493	.064427	1173.8804	1.0444	65.7481
22	6	.6000	.8299	1178.9652	7.5425	12.5091	.064270	1178.9893	1.0493	65.3489
22	7	.6000	.9959	1192.3993	4.7125	12.4965	.064249	1192.4086	1.0616	64.9867
23	1	.8000	0.0000	1222.7818	-0.0000	11.7299	.061477	1222.7818	1.0992	55.0064
23	2	.8000	.1661	1221.8079	4.9190	11.7494	.061543	1221.8178	1.0980	55.3043
23	3	.8000	.3322	1219.8205	9.4775	11.7902	.061682	1219.8573	1.0955	55.9309
23	4	.8000	.4984	1215.6967	12.4405	11.8637	.061934	1215.7603	1.0907	57.0332
23	5	.8000	.6645	1207.4172	13.4183	11.9936	.062396	1207.4918	1.0814	58.8296
23	6	.8000	.8306	1194.2432	15.6371	12.2716	.063399	1194.3456	1.0659	62.4479
23	7	.8000	.9967	1193.8317	4.7182	12.4934	.064238	1193.8410	1.0629	64.9495

MASS= 1.5757 (LBM/SEC) THRUST= 44.7800 (LBF) MASSI= 1.5915 MASSE= 1.5757

Fig. 10 (cont)

```
CASE NO. 3 - CONVERGING-DIVERGING, PLUG NOZZLE (10 DEG CONE, PT/PE=3.29)
 $CNTRL LMAX=31,MMAX=6,NMAX=400,TCONV=0.005,FDT=1.6   $
 $IVS   $
 $GEMTRY NGEOM=1,XI=-4.440,XE=2.9600,RI=4.0,JFLAG=1,LJET=23   $
 $GCBL NGCB=2,RICB=1.3,RTCB=3.365,RCICB=0.75,RCTCB=4.95,
 ANGICB=45.0,ANGECB=10.0   $
 $BC PT=100.0,TT=70.0,PE=30.4   $
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Fig. 11. Case No. 3 data deck.

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             NAP, A COMPUTER PROGRAM FOR THE COMPUTATION OF TWO-DIMENSIONAL, TIME-DEPENDENT, INVISCID NOZZLE FLOW

                          BY MICHAEL C. CLINE, T-3 - LOS ALAMOS SCIENTIFIC LABORATORY


PROGRAM ABSTRACT -

              THE EQUATIONS OF MOTION FOR TWO-DIMENSIONAL, TIME DEPENDENT, INVISCID FLOW IN A NOZZLE
          ARE SOLVED USING THE SECOND-ORDER, MACCORMACK, FINITE-DIFFERENCE SCHEME. THE FLUID IS ASSUMED
          TO BE A PERFECT GAS. ALL BOUNDARY CONDITIONS ARE COMPUTED USING A SECOND-ORDER, REFERENCE PLANE
          CHARACTERISTIC SCHEME. THE STEADY STATE SOLUTION IS OBTAINED AS THE ASYMPTOTIC SOLUTION FOR
          LARGE TIME. THE NOZZLES MAY BE EITHER CONVERGING, CONVERGING-DIVERGING, OR PLUG GEOMETRIES.


JOB TITLE -

          CASE NO. 3 - CONVERGING-DIVERGING, PLUG NOZZLE (10 DEG CONE, PT/PE=3.29)


CONTROL PARAMETERS -

          LMAX=31  MMAX= 6    NMAX= 400  NPRINT=   0  TCONV=  .005     FDT=1.60  NSTAG=0      NASM=1     IUNIT=0
          IUI=1    IUO=1      IEX=1      NCONVI= 1    TSTOP=1.00000    NID= 1    NPLOT=  -1   IPUNCH=0   ISUPER=0
          IAV=0    CAV= 5.0   XMU=1.00   XLA=0.00     RKMU=  .50       CTA= .50  LSS= 2       SMP= .95   NST=   0


FLUID MODEL -

          THE RATIO OF SPECIFIC HEATS, GAMMA =1.4000 AND THE GAS CONSTANT, R =  53.3500 (FT-LBF/LBM-R)


FLOW GEOMETRY -

          AXISYMMETRIC FLOW HAS BEEN SPECIFIED


NOZZLE GEOMETRY -

          A CONSTANT AREA DUCT HAS BEEN SPECIFIED BY XI= -4.4400 (IN), RI=  4.0000 (IN), AND XE=  2.9600 (IN)

          AN EXHAUST JET CALCULATION HAS BEEN REQUESTED.  THE NOZZLE ENDS AT X=    .7400 (IN).  THE
          MESH POINTS L= 23 TO L= 31 ARE AN INITIAL APPROXIMATION TO THE EXHAUST JET BOUNDARY.

          A CIRCULAR-ARC, CONICAL CENTERBODY HAS BEEN SPECIFIED BY XICB= -4.4400 (IN), RICB=  1.3000 (IN),
          RTCB=  3.3650 (IN), XECB=  2.9600 (IN), RCICB=   .7500 (IN), RCTCB=  4.9500 (IN), ANGICB= 45.00 (DEG),
          AND ANGECB= 10.00 (DEG). THE COMPUTED VALUES ARE XTCB=  -.0140 (IN) AND RECB=  2.9170 (IN).


BOUNDARY CONDITIONS -

          PT= 100.0000 (PSIA)     TT=  70.0000 (F)     THETA=   0.0000 (DEG)     PE=  30.4000 (PSIA)
N=  10
N=  20
N=  30
N=  40
N=  50
N=  60
N=  70
N=  80
```

Fig. 12. Case No. 3 output.

```
N=  90
N= 100
N= 110
N= 120
N= 130
N= 140
N= 150
N= 160
N= 170
N= 180
N= 190
N= 200
N= 210
N= 220
N= 230
N= 240
N= 250
N= 260
N= 270
N= 280
N= 290
N= 300
N= 310
N= 320
N= 330
N= 340
```

Fig. 12 (cont)

SOLUTION SURFACE NO. 345 - TIME = .00237634 SECONDS (DELTA T = .00000691)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
1	1	-4.4400	1.3000	109.4812	-0.0000	99.3430	.506883	109.4812	.0971	69.0027
1	2	-4.4400	1.8400	106.0169	-0.0000	99.3831	.507029	106.0169	.0940	69.0638
1	3	-4.4400	2.3800	212.9815	-0.0000	97.5259	.500243	212.9815	.1894	66.2199
1	4	-4.4400	2.9200	236.6861	-0.0000	96.9532	.498143	236.6861	.2107	65.3351
1	5	-4.4400	3.4600	276.8857	-0.0000	95.8470	.494076	276.8857	.2468	63.6156
1	6	-4.4400	4.0000	273.4428	-0.0000	95.9570	.494481	273.4428	.2437	63.7873
2	1	-4.1933	1.3417	107.4307	37.4142	99.2108	.506393	113.7593	.1009	68.8098
2	2	-4.1933	1.8734	104.0377	29.6151	99.2062	.506362	108.1707	.0960	68.8175
2	3	-4.1933	2.4050	214.2242	22.0820	97.4455	.499962	215.3593	.1915	66.0820
2	4	-4.1933	2.9367	237.1094	13.9520	96.9338	.498071	237.5195	.2114	65.3056
2	5	-4.1933	3.4683	277.6715	7.3298	95.8509	.494092	277.7683	.2476	63.6205
2	6	-4.1933	4.0000	274.0878	0.0000	95.9535	.494468	274.0878	.2443	63.7824
3	1	-3.9467	1.4851	112.9896	98.6735	98.7348	.504664	150.0103	.1332	68.0751
3	2	-3.9467	1.9881	119.2811	55.7369	99.1459	.506168	131.6609	.1168	68.6990
3	3	-3.9467	2.4911	230.3713	50.8948	97.1220	.498801	235.9263	.2099	65.5554
3	4	-3.9467	2.9940	245.1004	28.8405	96.7471	.497390	246.7914	.2197	65.0115
3	5	-3.9467	3.4970	284.0488	15.6501	95.6688	.493423	284.4796	.2537	63.3345
3	6	-3.9467	4.0000	279.0134	0.0000	95.8206	.493979	279.0134	.2487	63.5740
4	1	-3.7000	1.7293	133.0260	133.0260	97.9867	.501924	188.1272	.1672	66.9351
4	2	-3.7000	2.1835	152.2549	70.3057	98.2987	.503055	167.7034	.1490	67.4243
4	3	-3.7000	2.6376	251.8800	69.9268	96.4944	.496505	261.4141	.2328	64.5736
4	4	-3.7000	3.0917	258.5497	37.0252	96.2556	.495578	261.1873	.2327	64.2546
4	5	-3.7000	3.5459	293.1525	20.9197	95.3831	.492369	293.8980	.2622	62.8874
4	6	-3.7000	4.0000	287.4930	0.0000	95.5579	.493011	287.4930	.2564	63.1645
5	1	-3.4533	1.9750	165.0872	159.4938	97.2086	.499081	229.5475	.2042	65.7285
5	2	-3.4533	2.3800	189.7552	88.9515	97.4650	.500033	209.5695	.1864	66.1113
5	3	-3.4533	2.7850	274.0965	86.3530	95.8902	.494289	287.3773	.2562	63.6267
5	4	-3.4533	3.1900	275.5988	45.2297	95.7240	.493625	279.2855	.2490	63.4220
5	5	-3.4533	3.5950	305.6406	25.4583	95.0148	.491013	306.7070	.2738	62.3076
5	6	-3.4533	4.0000	300.2397	0.0000	95.1903	.491657	300.2397	.2679	62.5870
6	1	-3.2067	2.1978	205.2765	173.2552	96.0735	.494909	268.6183	.2394	63.9703
6	2	-3.2067	2.5582	229.6718	104.0684	96.2531	.495595	252.1495	.2247	64.2229
6	3	-3.2067	2.9187	297.7834	94.9364	95.0783	.491294	312.5506	.2790	62.3581
6	4	-3.2067	3.2791	296.9780	50.8083	94.9696	.490842	301.2929	.2689	62.2412
6	5	-3.2067	3.6396	321.7514	27.9020	94.4542	.488941	322.9590	.2885	61.4256
6	6	-3.2067	4.0000	317.4202	0.0000	94.6142	.489529	317.4202	.2835	61.6822
7	1	-2.9600	2.3929	246.1173	182.2747	94.9109	.490626	306.2642	.2734	62.1473
7	2	-2.9600	2.7143	267.6866	117.9731	95.0919	.491347	292.5299	.2611	62.3756
7	3	-2.9600	3.0357	323.2732	100.9577	94.2213	.488127	338.6709	.3027	61.0076
7	4	-2.9600	3.3571	321.5757	56.3837	94.1319	.487751	326.4813	.2918	60.9145
7	5	-2.9600	3.6786	341.9079	29.8608	93.7507	.486340	343.2094	.3069	60.3106
7	6	-2.9600	4.0000	338.8474	0.0000	93.8859	.486837	338.8474	.3030	60.5294
8	1	-2.7133	2.5642	289.9157	188.6100	93.5079	.485435	345.8682	.3094	59.9300
8	2	-2.7133	2.8514	307.9821	127.9301	93.7199	.486292	333.4952	.2983	60.1912
8	3	-2.7133	3.1385	353.1446	104.0948	93.1129	.484017	368.1668	.3296	59.2511
8	4	-2.7133	3.4257	350.7214	60.2814	93.0513	.483748	355.8642	.3186	59.1957
8	5	-2.7133	3.7128	367.0750	30.9629	92.7906	.482778	368.3786	.3299	58.7817
8	6	-2.7133	4.0000	365.0832	0.0000	92.9035	.483193	365.0832	.3269	58.9667
9	1	-2.4667	2.7146	335.9522	191.6407	91.9380	.479603	386.7687	.3469	57.4175
9	2	-2.4667	2.9717	350.4988	134.3668	92.1960	.480650	375.3716	.3365	57.7394
9	3	-2.4667	3.2288	387.1967	105.6010	91.7925	.479105	401.3389	.3600	57.1358
9	4	-2.4667	3.4859	384.1126	63.0205	91.7586	.478944	389.2481	.3492	57.1185
9	5	-2.4667	3.7429	397.2721	31.6727	91.5882	.478304	398.5327	.3576	56.8485
9	6	-2.4667	4.0000	396.0271	0.0000	91.6830	.478654	396.0271	.3553	57.0061

Fig. 12 (cont)

SOLUTION SURFACE NO. 345 - TIME = .00237634 SECONDS (DELTA T = .00000691)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
10	1	-2.2200	2.8463	385.7795	192.0533	90.0749	.472645	430.9412	.3876	54.3954
10	2	-2.2200	3.0770	396.9902	137.6490	90.3795	.473882	420.1767	.3778	54.7871
10	3	-2.2200	3.3078	426.5248	105.6109	90.1440	.472945	439.4053	.3952	54.4635
10	4	-2.2200	3.5385	422.6905	64.3804	90.1463	.472923	427.5654	.3845	54.5002
10	5	-2.2200	3.7693	433.1660	31.9255	90.0516	.472562	434.3409	.3907	54.3518
10	6	-2.2200	4.0000	432.3408	0.0000	90.1354	.472872	432.3408	.3888	54.4940
11	1	-1.9733	2.9607	440.1175	189.7053	87.8573	.464311	479.2614	.4326	50.7368
11	2	-1.9733	3.1686	448.0872	138.0339	88.2164	.465769	468.8662	.4230	51.2182
11	3	-1.9733	3.3764	471.3597	104.1647	88.1287	.465373	482.7320	.4356	51.1456
11	4	-1.9733	3.5843	466.6714	64.4745	88.1745	.465519	471.1042	.4250	51.2510
11	5	-1.9733	3.7921	474.8778	31.7414	88.1431	.465393	475.9374	.4294	51.2060
11	6	-1.9733	4.0000	474.2174	0.0000	88.2221	.465686	474.2174	.4278	51.3423
12	1	-1.7267	3.0593	500.0783	184.4158	85.1723	.454142	532.9986	.4833	46.2146
12	2	-1.7267	3.2474	504.7398	135.5893	85.6008	.455882	522.6344	.4736	46.8202
12	3	-1.7267	3.4356	522.3541	101.1329	85.6519	.456001	532.0542	.4820	46.9892
12	4	-1.7267	3.6237	516.6715	63.2783	85.7507	.456353	520.5320	.4715	47.1831
12	5	-1.7267	3.8119	522.8704	31.0492	85.7759	.456442	523.7915	.4744	47.2336
12	6	-1.7267	4.0000	522.1891	0.0000	85.8554	.456738	522.1891	.4729	47.3748
13	1	-1.4800	3.1429	566.2529	175.5815	81.9333	.441756	592.8501	.5405	40.6179
13	2	-1.4800	3.3143	567.4408	130.0717	82.4540	.443870	582.1578	.5304	41.4001
13	3	-1.4800	3.4858	579.7939	96.2310	82.6443	.444521	587.7255	.5352	41.8215
13	4	-1.4800	3.6572	572.9430	60.6967	82.8076	.445125	576.1491	.5245	42.1299
13	5	-1.4800	3.8286	577.3073	29.7520	82.8869	.445422	578.0735	.5262	42.2757
13	6	-1.4800	4.0000	576.4726	0.0000	82.9705	.445737	576.4726	.5246	42.4267
14	1	-1.2333	3.2125	639.0676	162.4290	78.0504	.426727	659.3865	.6054	33.6883
14	2	-1.2333	3.3700	636.5520	121.0465	78.6926	.429342	647.9589	.5943	34.7195
14	3	-1.2333	3.5275	643.9203	89.0877	79.0292	.430567	650.0539	.5958	35.4213
14	4	-1.2333	3.6850	635.7200	56.5499	79.2706	.431485	638.2302	.5847	35.8779
14	5	-1.2333	3.8425	638.3532	27.7308	79.4040	.431995	638.9553	.5852	36.1254
14	6	-1.2333	4.0000	637.2739	0.0000	79.4949	.432340	637.2739	.5835	36.2970
15	1	-.9867	3.2685	718.4278	143.9795	73.4700	.408725	732.7132	.6786	25.1849
15	2	-.9867	3.4148	711.9765	107.8905	74.2675	.411994	720.1048	.6660	26.5599
15	3	-.9867	3.5611	714.5835	79.2171	74.7626	.413865	718.9610	.6642	27.5894
15	4	-.9867	3.7074	704.8565	50.5751	75.0970	.415164	706.6686	.6524	28.2372
15	5	-.9867	3.8537	705.8355	24.8312	75.2871	.415905	706.2721	.6518	28.6006
15	6	-.9867	4.0000	704.4402	0.0000	75.3868	.416293	704.4402	.6500	28.7919
16	1	-.7400	3.3115	803.7732	119.1784	68.1821	.387547	812.5607	.7607	14.8693
16	2	-.7400	3.4492	793.2104	89.8965	69.1705	.391649	798.2882	.7458	16.7076
16	3	-.7400	3.5869	791.2734	66.0987	69.8382	.394253	794.0294	.7408	18.1298
16	4	-.7400	3.7246	779.9013	42.4700	70.2801	.396011	781.0568	.7280	19.0199
16	5	-.7400	3.8623	779.2786	20.8999	70.5298	.397005	779.5588	.7262	19.5177
16	6	-.7400	4.0000	777.5410	0.0000	70.6408	.397439	777.5410	.7242	19.7486
17	1	-.4933	3.3417	893.9432	86.9769	62.2513	.363220	898.1644	.8519	2.6020
17	2	-.4933	3.4734	879.1927	66.3566	63.4636	.368344	881.6933	.8340	5.0490
17	3	-.4933	3.6050	873.0032	49.2059	64.3169	.371780	874.3888	.8254	6.9467
17	4	-.4933	3.7367	859.9066	31.9340	64.8781	.374068	860.4994	.8113	8.1395
17	5	-.4933	3.8683	857.7794	15.7672	65.1907	.375342	857.9243	.8083	8.7992
17	6	-.4933	4.0000	855.6361	0.0000	65.3116	.375837	855.6361	.8059	9.0498
18	1	-.2467	3.3595	987.1818	46.4557	55.8187	.336086	988.2743	.9522	-11.7118
18	2	-.2467	3.4876	968.3785	36.6601	57.2806	.342422	969.0721	.9303	-8.4827
18	3	-.2467	3.6157	958.2832	28.1325	58.3275	.346782	958.6961	.9179	-6.0113
18	4	-.2467	3.7438	943.5326	18.7218	59.0161	.349673	943.7183	.9020	-4.4493
18	5	-.2467	3.8719	939.9736	9.3489	59.3931	.351253	940.0201	.8976	-3.6020
18	6	-.2467	4.0000	937.6150	0.0000	59.5321	.351811	937.6150	.8950	-3.2594

Fig. 12 (cont)

SOLUTION SURFACE NO. 345 - TIME = .00237634 SECONDS (DELTA T = .00000691)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
19	1	.0000	3.3650	1081.4350	-3.0549	49.1261	.306824	1081.4393	1.0612	-27.8336
19	2	.0000	3.4920	1058.8194	.3966	50.8497	.314533	1058.8195	1.0340	-23.6353
19	3	.0000	3.6190	1045.3491	2.5186	52.0884	.319889	1045.3522	1.0172	-20.4885
19	4	.0000	3.7460	1029.0488	2.6347	52.9016	.323416	1029.0521	.9990	-18.4954
19	5	.0000	3.8730	1024.3133	1.4601	53.3412	.325314	1024.3143	.9932	-17.4236
19	6	.0000	4.0000	1020.9992	0.0000	53.4719	.325911	1020.9992	.9897	-17.1525
20	1	.2467	3.3581	1173.1244	-61.8584	42.3494	.276131	1174.7541	1.1778	-46.0380
20	2	.2467	3.4865	1148.0302	-42.2793	44.3423	.285382	1148.8084	1.1443	-40.6080
20	3	.2467	3.6149	1132.0781	-26.9501	45.7752	.291831	1132.3988	1.1227	-36.6236
20	4	.2467	3.7433	1114.2692	-15.5124	46.7326	.296129	1114.3772	1.1014	-34.0414
20	5	.2467	3.8716	1107.7929	-6.9782	47.2613	.298489	1107.8149	1.0932	-32.6288
20	6	.2467	4.0000	1107.0192	0.0000	47.5041	.299451	1107.0192	1.0913	-31.8133
21	1	.4933	3.3389	1269.0032	-130.7462	35.7235	.244355	1275.7208	1.3101	-65.3967
21	2	.4933	3.4711	1238.3914	-93.2116	37.9196	.255030	1241.8944	1.2646	-58.6706
21	3	.4933	3.6034	1219.8855	-62.9528	39.5784	.262856	1221.5088	1.2360	-53.5867
21	4	.4933	3.7356	1200.0219	-39.0646	40.7549	.268366	1200.6576	1.2097	-50.0979
21	5	.4933	3.8678	1193.6524	-20.1323	41.3780	.271285	1193.8222	1.2002	-48.3097
21	6	.4933	4.0000	1179.4275	0.0000	41.3454	.271483	1179.4275	1.1867	-48.9338
22	1	.7400	3.3072	1331.6916	-205.2378	30.9496	.220443	1347.4142	1.4120	-81.0462
22	2	.7400	3.4458	1305.5170	-155.2045	32.8860	.230385	1314.7103	1.3663	-74.7132
22	3	.7400	3.5843	1288.8579	-111.5273	34.4362	.238202	1293.6743	1.3360	-69.7912
22	4	.7400	3.7229	1272.2609	-71.9720	35.4028	.243148	1274.2950	1.3113	-66.9984
22	5	.7400	3.8614	1267.2158	-32.7531	35.8114	.245264	1267.6390	1.3026	-65.8919
22	6	.7400	4.0000	1286.7778	0.0000	35.2207	.241585	1286.7778	1.3233	-66.4902
23	1	.9867	3.2649	1383.2720	-243.9082	27.2685	.201293	1404.6112	1.4984	-94.3544
23	2	.9867	3.4123	1374.5314	-187.5136	28.2471	.206581	1387.2628	1.4731	-90.9272
23	3	.9867	3.5596	1374.0887	-128.3918	28.8864	.209959	1380.0740	1.4609	-88.6462
23	4	.9867	3.7070	1372.0843	-67.7226	29.0913	.210905	1373.7546	1.4524	-87.6901
23	5	.9867	3.8543	1375.5143	-12.7883	29.2328	.211388	1375.5738	1.4524	-86.7353
23	6	.9867	4.0016	1355.3965	9.0228	30.4018	.217243	1355.4266	1.4227	-82.2707
24	1	1.2333	3.2214	1434.0378	-252.8596	24.1425	.184583	1456.1601	1.5809	-106.9657
24	2	1.2333	3.3752	1436.4195	-196.1252	24.5272	.186714	1449.7469	1.5706	-105.4328
24	3	1.2333	3.5291	1442.6305	-135.8278	24.8334	.188222	1449.0107	1.5664	-103.8817
24	4	1.2333	3.6829	1429.7078	-89.7732	25.6828	.192670	1432.5235	1.5406	-100.2039
24	5	1.2333	3.8367	1397.4981	-69.4626	27.8136	.203959	1399.2234	1.4878	-91.9200
24	6	1.2333	3.9905	1354.6985	-61.0675	30.3938	.217202	1356.0742	1.4234	-82.2982
25	1	1.4800	3.1779	1481.9167	-261.3019	21.3229	.168627	1504.7777	1.6616	-118.6924
25	2	1.4800	3.3359	1482.1809	-207.8316	21.7522	.171088	1496.6810	1.6481	-116.8278
25	3	1.4800	3.4939	1474.3214	-165.8729	22.8291	.177072	1483.6231	1.6224	-112.0096
25	4	1.4800	3.6519	1433.9630	-149.1645	25.1620	.189852	1441.7004	1.5549	-102.2670
25	5	1.4800	3.8098	1392.5720	-137.2785	27.8314	.204048	1399.3220	1.4877	-91.8457
25	6	1.4800	3.9678	1349.1683	-124.2698	30.4025	.217246	1354.8794	1.4221	-82.2663
26	1	1.7267	3.1344	1509.8281	-266.2235	20.0212	.161484	1533.1197	1.7096	-125.3505
26	2	1.7267	3.2942	1499.4577	-251.5096	20.6565	.165113	1520.4047	1.6878	-122.3211
26	3	1.7267	3.4540	1463.9559	-241.1145	22.8440	.177299	1483.6789	1.6230	-112.2279
26	4	1.7267	3.6137	1417.7407	-227.3063	25.5073	.191743	1435.8471	1.5457	-100.9358
26	5	1.7267	3.7735	1380.4098	-207.8969	27.9914	.204894	1395.9772	1.4830	-91.2585
26	6	1.7267	3.9333	1342.5048	-187.8490	30.4071	.217269	1355.5834	1.4228	-82.2483
27	1	1.9733	3.0909	1459.0269	-257.2658	22.5791	.175554	1481.5348	1.6221	-112.8446
27	2	1.9733	3.2505	1447.6522	-284.2170	22.7877	.176792	1475.2885	1.6135	-112.0919
27	3	1.9733	3.4101	1421.8776	-293.0628	24.3402	.185340	1451.7650	1.5730	-105.5272
27	4	1.9733	3.5696	1394.0753	-279.9659	26.1367	.195040	1421.9095	1.5251	-98.2955
27	5	1.9733	3.7292	1365.8397	-260.7981	28.2135	.206032	1390.5157	1.4754	-90.3841
27	6	1.9733	3.8888	1334.1585	-240.7433	30.4107	.217288	1355.7051	1.4229	-82.2369

Fig. 12 (cont)

SOLUTION SURFACE NO. 345 - TIME = .00237634 SECONDS (DELTA T = .00000691)

L	M	X (IN)	Y (IN)	U (FPS)	V (FPS)	P (PSIA)	RHO (LBM/FT3)	Q (FPS)	MACH NO	T (F)
28	1	2.2200	3.0475	1410.7945	-248.7612	25.4098	.192169	1432.5583	1.5469	-103.1005
28	2	2.2200	3.2047	1406.2072	-302.8893	25.0046	.189801	1438.4577	1.5561	-104.4101
28	3	2.2200	3.3620	1391.8446	-333.8577	25.5297	.192227	1431.3254	1.5421	-101.5240
28	4	2.2200	3.5193	1370.4248	-334.0730	26.7231	.198370	1410.5562	1.5090	-96.3879
28	5	2.2200	3.6766	1346.7431	-320.5242	28.5479	.207844	1384.3600	1.4667	-89.2631
28	6	2.2200	3.8339	1317.7933	-293.3750	30.3985	.217221	1350.0549	1.4170	-82.2726
29	1	2.4667	3.0040	1291.9779	-227.8106	32.9470	.231216	1311.9087	1.3646	-75.3852
29	2	2.4667	3.1572	1303.0941	-283.3728	31.6862	.224897	1333.5495	1.3950	-79.7104
29	3	2.4667	3.3104	1313.7524	-335.5893	30.4038	.218006	1355.9372	1.4256	-83.5673
29	4	2.4667	3.4637	1313.3000	-362.5924	29.8082	.214542	1362.4354	1.4352	-84.9830
29	5	2.4667	3.6169	1309.8925	-363.4159	30.1072	.215866	1359.3710	1.4292	-83.5434
29	6	2.4667	3.7702	1315.1981	-339.5192	30.3952	.217210	1358.3149	1.4257	-82.2946
30	1	2.7133	2.9605	1194.3369	-210.5938	40.0065	.264795	1212.7615	1.2251	-52.1976
30	2	2.7133	3.1101	1210.9114	-238.5102	38.6329	.258294	1234.1772	1.2530	-56.2885
30	3	2.7133	3.2598	1240.8253	-278.4410	36.0241	.245565	1271.6827	1.3037	-64.0377
30	4	2.7133	3.4095	1267.3081	-314.9771	33.3000	.232040	1305.8638	1.3535	-72.6441
30	5	2.7133	3.5591	1293.9764	-334.6486	31.3972	.222419	1336.5495	1.3968	-78.9811
30	6	2.7133	3.7088	1306.3885	-325.0857	30.3957	.217212	1346.2286	1.4130	-82.2909
31	1	2.9600	2.9170	1096.6960	-193.3771	47.0660	.298373	1113.6143	1.1009	-34.2290
31	2	2.9600	3.0631	1118.7287	-193.6476	45.5797	.291692	1135.3648	1.1278	-38.2301
31	3	2.9600	3.2092	1167.8982	-221.2927	41.6444	.273125	1188.6785	1.1953	-48.4494
31	4	2.9600	3.3552	1221.3161	-267.3618	36.7919	.249537	1250.2381	1.2785	-62.0356
31	5	2.9600	3.5013	1278.0603	-305.8814	32.6871	.228973	1314.1543	1.3657	-74.6800
31	6	2.9600	3.6474	1297.5788	-322.8935	30.3963	.217213	1337.1503	1.4035	-82.2872

MASS= 33.9930 (LBM/SEC) THRUST= 1353.5464 (LBF) MASSI= 34.5377 MASSE= 33.8257

$EJ

Fig. 12 (cont)

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APPENDIX A

ETA CONSTANT REFERENCE PLANE CHARACTERISTIC RELATIONS

I. EQUATIONS OF MOTION

The equations of motion, Eqs. (6) through (9), can be written as

$$\rho_\tau + u\rho_\zeta + \rho u_\zeta = -\bar{v}\rho_\eta - \rho\alpha u_\eta - \rho\beta v_\eta - \epsilon\rho v\beta/\eta, \quad \text{(A-1)}$$

$$u_\tau + uu_\zeta + p_\zeta/\rho = -\bar{v}u_\eta - \alpha p_\eta/\rho, \quad \text{(A-2)}$$

$$v_\tau + uv_\zeta = -\bar{v}v_\eta - \beta p_\eta/\rho, \quad \text{(A-3)}$$

$$p_\tau + up_\zeta - a^2(\rho_\tau + u\rho_\zeta) = -\bar{v}p_\eta + a^2\bar{v}\rho_\eta. \quad \text{(A-4)}$$

Letting

$$\psi_1 = -\bar{v}\rho_\eta - \rho\alpha u_\eta - \rho\beta v_\eta - \epsilon\rho v\beta/\eta, \quad \text{(A-5)}$$

$$\psi_2 = -\bar{v}u_\eta - \alpha p_\eta/\rho, \quad \text{(A-6)}$$

$$\psi_3 = -\bar{v}v_\eta - \beta p_\eta/\rho, \quad \text{(A-7)}$$

$$\psi_4 = -\bar{v}p_\eta + a^2\bar{v}\rho_\eta, \quad \text{(A-8)}$$

Eqs. (A-1) through (A-4) become

$$\rho_\tau + u\rho_\zeta + \rho u_\zeta = \psi_1, \quad \text{(A-9)}$$

$$u_\tau + uu_\zeta + p_\zeta/\rho = \psi_2, \quad \text{(A-10)}$$

$$v_\tau + uv_\zeta = \psi_3, \tag{A-11}$$

$$p_\tau + up_\zeta - a^2\rho_\tau - a^2u\rho_\zeta = \psi_4. \tag{A-12}$$

II. CHARACTERISTIC CURVES

A linear combination of the equations of motion can be formed by multiplying Eqs. (A-9) through (A-12) by ℓ_i, i = 1, 2, 3, 4, respectively, and then summing them. This linear combination can be written as

$$\ell_1 (\rho_\tau + u\rho_\zeta + \rho u_\zeta - \psi_1)$$

$$+ \ell_2 (u_\tau + uu_\zeta + p_\zeta/\rho - \psi_2) + \ell_3 (v_\tau + uv_\zeta - \psi_3)$$

$$+ \ell_4 (p_\tau + up_\zeta - a^2\rho_\tau - a^2u\rho_\zeta - \psi_4) = 0. \tag{A-13}$$

Rearrangement of Eq. (A-13) yields

$$(u\ell_1 - a^2u\ell_4)\rho_\zeta + (\ell_1 - a^2\ell_4)\rho_\tau + (\rho\ell_1 + u\ell_2)u_\zeta$$

$$+ \ell_2 u_\tau + u\ell_3 v_\zeta + \ell_3 v_\tau + (\ell_2/\rho + u\ell_4)p_\zeta + \ell_4 p_\tau$$

$$= \ell_1\psi_1 + \ell_2\psi_2 + \ell_3\psi_3 + \ell_4\psi_4 . \tag{A-14}$$

The following set of vectors can be defined, where the components are the coefficients of the partial derivatives in Eq. (A-14).

$$W_1 = (u\ell_1 - a^2u\ell_4, \ \ell_1 - a^2\ell_4), \tag{A-15}$$

$$W_2 = (\rho\ell_1 + u\ell_2, \ \ell_2), \tag{A-16}$$

$$W_3 = (u\ell_3, \ \ell_3), \tag{A-17}$$

$$W_4 = (\ell_2/\rho + u\ell_4, \ \ell_4). \tag{A-18}$$

Therefore, Eq. (A-14) can be written as

$$d_{W_1} \rho + d_{W_2} u + d_{W_3} v + d_{W_4} p$$

$$= \ell_1\psi_1 + \ell_2\psi_2 + \ell_3\psi_3 + \ell_4\psi_4 , \tag{A-19}$$

where $d_{W_1} \rho$ is defined as the derivative of ρ in the direction of the vector W_1, etc.

A question is now posed: Can the ℓ_i, i = 1, 2, 3, 4, be chosen such that the vectors W_j, j = 1, 2, 3, 4, would be linearly dependent or, in other words, would lie in one direction. If such ℓ_i do exist, the curve which contains the vectors W_j is called the characteristic curve, its normal N is called the characteristic normal, and Eq. (A-19) is called the compatibility equation. Therefore, if $N = (N_\zeta, N_\tau)$ is the characteristic normal in the ζ-τ plane, N and W_j are related by

$$N \cdot W_j = 0 \quad (j = 1, 2, 3, 4). \tag{A-20}$$

When Eq. (A-20) is expanded,

$$(u\ell_1 - a^2u\ell_4) N_\zeta + (\ell_1 - a^2\ell_4) N_\tau = 0 , \tag{A-21}$$

$$(\rho\ell_1 + u\ell_2) N_\zeta + \ell_2 N_\tau = 0 , \tag{A-22}$$

$$u\ell_3 N_\zeta + \ell_3 N_\tau = 0 , \tag{A-23}$$

$$(\ell_2/\rho + u\ell_4) N_\zeta + \ell_4 N_\tau = 0 . \tag{A-24}$$

In matrix form, Eqs. (A-21) through (A-24) become

$$\begin{vmatrix} uN_\zeta + N_\tau & 0 & 0 & -a^2(uN_\zeta + N_\tau) \\ \rho N_\zeta & uN_\zeta + N_\tau & 0 & 0 \\ 0 & 0 & uN_\zeta + N_\tau & 0 \\ 0 & N_\zeta/\rho & 0 & uN_\zeta + N_\tau \end{vmatrix} \begin{vmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \end{vmatrix} = 0 . \tag{A-25}$$

Equation (A-25) is a system of homogeneous equations. For Eq. (A-25) to have a nontrivial solution, the coefficient matrix must be singular; in other words, its determinant must equal zero. Setting the determinant equal to zero yields

$$(uN_\zeta + N_\tau)^2 \, [(uN_\zeta + N_\tau)^2 - a^2 N_\zeta^2] = 0 . \tag{A-26}$$

Setting the first factor of Eq. (A-26) equal to zero yields

$$uN_\zeta + N_\tau = 0 \ . \tag{A-27}$$

Setting the second factor of Eq. (A-26) equal to zero yields

$$uN_\zeta + N_t = \pm\, aN_\zeta \ . \tag{A-28}$$

Noting that $d\zeta/d\tau = -N_\tau/N_\zeta$, Eqs. (A-27) and (A-28) can be written as

$$d\zeta/d\tau = u \ , \tag{A-29}$$

$$d\zeta/d\tau = u \mp a \ . \tag{A-30}$$

Equation (A-29) represents the projection of the flow pathlines on the η = constant planes. Equation (A-30) represents the projection of the Mach cones on the η = constant planes.

III. SOLUTION FOR THE ℓ_i

If the compatibility equation (A-19) is to be used, the arbitrary parameters ℓ_i must be evaluated in the following manner. Consider first the characteristic curve given by Eq. (A-27). Substituting Eq. (A-27) into Eq. (A-25) yields

$$\left\| \begin{matrix} 0 & 0 & 0 & 0 \\ \rho N_\zeta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & N_\zeta/\rho & 0 & 0 \end{matrix} \right\| \left\| \begin{matrix} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \end{matrix} \right\| = 0 \ . \tag{A-31}$$

Since the rank of the coefficient matrix of Eq. (A-31) is two, there are two independent solutions for ℓ_i. From Eq. (A-31),

$$\ell_1 = \ell_2 = 0,\ \ell_3 \text{ and } \ell_4 \text{ are arbitrary.} \tag{A-32}$$

Therefore, two possible solutions are

$$\ell_1 = \ell_2 = \ell_3 = 0,\ \ell_4 = 1 \ , \tag{A-33}$$

$$\ell_1 = \ell_2 = \ell_4 = 0,\ \ell_3 = 1 \ . \tag{A-34}$$

Consider next the characteristic curve given by Eq. (A-28). Substituting Eq. (A-28) into Eq. (A-25) yields

$$\left\| \begin{matrix} \pm aN_\zeta & 0 & 0 & \mp a^3 N_\zeta \\ \rho N_\zeta & \pm aN_\zeta & 0 & 0 \\ 0 & 0 & \pm aN_\zeta & 0 \\ 0 & N_\zeta/\rho & 0 & \pm aN_\zeta \end{matrix} \right\| \left\| \begin{matrix} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \end{matrix} \right\| = 0 \ . \tag{A-35}$$

Since the rank of the coefficient matrix of Eq. (A-35) is three, there is only one independent solution for ℓ_i. From Eq. (A-35),

$$\ell_3 = 0,\ \ell_1 = a^2\ell_4,\ \ell_2 = \mp\, \rho a \ell_4 \ . \tag{A-36}$$

Therefore, one possible solution is

$$\ell_1 = a^2,\ \ell_2 = \mp\, \rho a,\ \ell_3 = 0,\ \ell_4 = 1 \ . \tag{A-37}$$

IV. COMPATIBILITY EQUATIONS

Substituting Eqs. (A-33) and (A-34) into Eq. (A-14) yields

$$p_\tau + up_\zeta - a^2 (\rho_\tau + u\rho_\zeta) = \psi_4 \ , \tag{A-38}$$

$$v_\tau + uv_\zeta = \psi_3 \ . \tag{A-39}$$

Substituting Eq. (A-37) into Eq. (A-14) yields

$$a^2(\rho_\tau + u\rho_\zeta + \rho u_\zeta - \psi_1) \mp \rho a\,(u_\tau + uu_\zeta + p_\zeta/\rho - \psi_2)$$

$$+\, p_\tau + up_\zeta - a^2 (\rho_\tau + u\rho_\zeta) - \psi_4 = 0 \ . \tag{A-40}$$

Equations (A-38) through (A-40) can be written as

$$\left.\begin{aligned} dp - a^2 d\rho &= \psi_4 d\tau \qquad \text{(A-41)} \\ dv &= \psi_3 d\tau \qquad \text{(A-42)} \end{aligned}\right\} \text{ for } d\zeta = u\,d\tau \ ,$$

$$dp - \rho a du = (\psi_4 + a^2\psi_1 - \rho a \psi_2) d\tau$$

$$\text{for } d\zeta = (u-a)d\tau \ , \tag{A-43}$$

$$dp + \rho a du = (\psi_4 + a^2\psi_1 + \rho a \psi_2) d\tau$$

$$\text{for } d\zeta = (u+a)d\tau \ . \tag{A-44}$$

APPENDIX B
ZETA CONSTANT REFERENCE PLANE CHARACTERISTIC RELATIONS

I. EQUATIONS OF MOTION

The equations of motion, Eqs. (6) through (9), can be written as

$$\rho_\tau + \bar{v}\rho_\eta + \rho\alpha u_\eta + \rho\beta v_\eta = - u\rho_\zeta - \rho u_\zeta - \epsilon\rho v\beta/\eta , \quad \text{(B-1)}$$

$$u_\tau + \bar{v}u_\eta + \alpha p_\eta/\rho = - uu_\zeta - p_\zeta/\rho , \quad \text{(B-2)}$$

$$v_\tau + \bar{v}v_\eta + \beta p_\eta/\rho = - uv_\zeta , \quad \text{(B-3)}$$

$$p_\tau + \bar{v}p_\eta - a^2(\rho_\tau + \bar{v}\rho_\eta) = - up_\zeta + a^2 u\rho_\zeta . \quad \text{(B-4)}$$

Letting

$$\psi_1 = -u\rho_\zeta - \rho u_\zeta - \epsilon\rho u\beta/\eta , \quad \text{(B-5)}$$

$$\psi_2 = -uu_\zeta - p_\zeta/\rho , \quad \text{(B-6)}$$

$$\psi_3 = -uv_\zeta , \quad \text{(B-7)}$$

$$\psi_4 = -up_\zeta + a^2 u\rho_\zeta , \quad \text{(B-8)}$$

Eqs. (B-1) through (B-4) become

$$\rho_\tau + \bar{v}\rho_\eta + \rho\alpha u_\eta + \rho\beta v_\eta = \psi_1 , \quad \text{(B-9)}$$

$$u_\tau + \bar{v}u_\eta + \alpha p_\eta/\rho = \psi_2 , \quad \text{(B-10)}$$

$$v_\tau + \bar{v}v_\eta + \beta p_\eta/\rho = \psi_3 , \quad \text{(B-11)}$$

$$p_\tau + \bar{v}p_\eta - a^2 (\rho_\tau + \bar{v}\rho_\eta) = \psi_4 . \quad \text{(B-12)}$$

II. CHARACTERISTIC CURVES

Following the development of Appendix A, the characteristic curves can be shown to be

$$d\eta/d\tau = \bar{v} , \quad \text{(B-13)}$$

$$d\eta/d\tau = \bar{v} \mp \alpha^* a , \quad \text{(B-14)}$$

where $\alpha^* = (\alpha^2 + \beta^2)^{1/2}$.

III. COMPATIBILITY EQUATIONS

Again, following the development of Appendix A, the compatibility equations can be shown to be

$$\left.\begin{aligned} \beta du - \alpha dv &= (\beta\psi_2 - \alpha\psi_3)d\tau \quad \text{(B-15)} \\ dp - a^2 d\rho &= \psi_4 d\tau \quad \text{(B-16)} \end{aligned}\right\} \text{ for } d\eta = \bar{v}d\tau ,$$

$$dp - \rho\alpha a du/\alpha^* - \rho\beta a dv/\alpha^* = (\psi_4 + a^2\psi_1 - \rho\alpha a\psi_2/\alpha^* - \rho\beta a\psi_3/\alpha^*)d\tau \text{ for } d\eta = (\bar{v} - \alpha^* a)d\tau , \quad \text{(B-17)}$$

$$dp + \rho\alpha a du/\alpha^* + \rho\beta a dv/\alpha^* = (\psi_4 + a^2\psi_1 + \rho\alpha a\psi_2/\alpha^* + \rho\beta a\psi_3/\alpha^*)d\tau \text{ for } d\eta = (\bar{v} + \alpha^* a)d\tau . \quad \text{(B-18)}$$

APPENDIX C

FORTRAN IV LISTING OF THE NAP PROGRAM

LASL Identification: LP-0537

```
      PROGRAM MAIN(INPUT,OUTPUT,FILM,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,
     1TAPE7=FILM)
C
C     *****************************************************************
C
C     NAP, A COMPUTER PROGRAM FOR THE COMPUTATION OF TWO-DIMENSIONAL,
C                  TIME-DEPENDENT, INVISCID NOZZLE FLOW
C
C                        BY MICHAEL C. CLINE, T-3
C                   LOS ALAMOS SCIENTIFIC LABORATORY
C
C     *****************************************************************
C
C                             PROGRAM ABSTRACT
C
C          THE EQUATIONS OF MOTION FOR TWO-DIMENSIONAL, TIME DEPENDENT,
C     INVISCID FLOW IN A NOZZLE ARE SOLVED USING THE SECOND-ORDER,
C     MACCORMACK, FINITE-DIFFERENCE SCHEME. THE FLUID IS ASSUMED TO BE
C     A PERFECT GAS. ALL BOUNDARY CONDITIONS ARE COMPUTED USING A
C     SECOND-ORDER, REFERENCE PLANE CHARACTERISTIC SCHEME. THE STEADY
C     STATE SOLUTION IS OBTAINED AS THE ASYMPTOTIC SOLUTION FOR LARGE
C     TIME. THE NOZZLES MAY BE EITHER CONVERGING, CONVERGING-DIVERGING,
C     OR PLUG GEOMETRIES.
C
      DIMENSION TITLE(8), UI(21), VI(21), PI(21), ROI(21)
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
      NAMELIST /CNTRL/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,TSTOP,GAMMA,RGAS,
     1NASM,NAME,NCONVI,NST,IUI,IUO,SMP,IPUNCH,IAV,CAV,NPLOT,IEX,LSS,CTA,
     2XMU,XLA,RKMU,IUNIT,PLOW,ROLOW
      NAMELIST /IVS/ U,V,P,RO,N1D,NSTART,TSTART,RSTAR,RSTARS
      NAMELIST /GEMTRY/ NDIM,XI,RI,RT,XE,RCI,RCT,ANGI,ANGE,NGEOM,XWI,YWI
     1,NWPTS,IINT,IDIF,LJET,JFLAG,NXNY,YW
      NAMELIST /GCBL/ NGCB,RICB,RTCB,RCICB,RCTCB,ANGICB,ANGECB,YCB,NXNYC
     1B,XCBI,YCBI,NCBPTS,IINTCB,IDIFCB
      NAMELIST /BC/ PT,TT,THETA,PE,NSTAG,ISUPER,UI,VI,PI,ROI
C
C     READ IN DATA
C
10    TCONV=0.0 $ FDT=1.0 $ TSTOP=1.0 $ NASM=1 $ NSTAG=0 $ NAME=0
      IPUNCH=0 $ NGCB=0 $ IINTCB=1 $ IDIFCB=1 $ NSTART=0 $ TSTART=0.0
      IINT=1 $ IDIF=1 $ NMAX=0 $ NPRINT=0 $ GAMMA=1.4 $ RGAS=53.35
      N1D=1 $ NDIM=1 $ THETA(1)=0.0 $ PE=14.7 $ NST=0 $ N=0 $ IEX=1
      NCONVI=1 $ IERR=0 $ JFLAG=0 $ IUI=1 $ IUO=1 $ SMP=0.95 $ ISUPER=0
      IAV=0 $ CAV=4.0 $ NPLOT=-1 $ G=32.174 $ PC=144.0 $ TC=460.0
      LC=12.0 $ IUNIT=0 $ LSS=2 $ CTA=0.5 $ XMU=0.2 $ XLA=1.0
      RKMU=0.7 $ PLOW=0.01 $ ROLOW=0.0001 $ RSTAR=0.0 $ RSTARS=0.0
      READ 650, TITLE
      IF (EOF,5) 20,30
20    STOP
30    READ (5,CNTRL)
      READ (5,IVS)
      READ (5,GEMTRY)
      READ (5,GCBL)
      READ (5,BC)
      IF (NAME.EQ.0) GO TO 40
      WRITE (6,CNTRL)
      WRITE (6,IVS)
      WRITE (6,GEMTRY)
      WRITE (6,GCBL)
      WRITE (6,BC)
C
C     PRINT INPUT DATA
C
40    PRINT 660
      PRINT 690
      PRINT 680
      PRINT 700
      PRINT 670
```

```
      PRINT 710, TITLE
      PRINT 670
      PRINT 720
      NPRIND=ABS(FLOAT(NPRINT))
      PRINT 730, LMAX,MMAX,NMAX,NPRIND,TCONV,FDT,NSTAG,NASM,IUNIT,IUI,IU
     1O,IEX,NCONVI,TSTOP,N1D,NPLOT,IPUNCH,ISUPER,IAV,CAV,XMU,XLA,RKMU,CT
     2A,LSS,SMP,NST
      PRINT 670
      IF (IUI.EQ.1) PRINT 740, GAMMA,RGAS
      IF (IUI.EQ.2) PRINT 750, GAMMA,RGAS
      PRINT 670
      PRINT 780
      IF (NDIM.EQ.0) PRINT 790
      IF (NDIM.EQ.1) PRINT 800
C
C     CALCULATE THE NOZZLE RADIUS AND NORMAL
C
      PRINT 670
      CALL GEOM
      IF (IERR.NE.0) GO TO 10
      DY=1.0/FLOAT(MMAX-1)
      IF (NGCB.NE.0) GO TO 60
      RICB=0.0
      RTCB=0.0
      DO 50 L=1,LMAX
      YCB(L)=0.0
      NXNYCB(L)=0.0
50    CONTINUE
      GO TO 90
60    XICB=XI
      XECB=XE
      CALL GEOMCB
      LT=1 $ XI=XICB $ XE=XECB
      YO=0.0
      DO 80 L=1,LMAX
      IF (NDIM.EQ.0) Y=YW(L)-YCB(L)
      IF (NDIM.EQ.1) Y=YW(L)**2-YCB(L)**2
      IF (Y.GT.0.0) GO TO 70
      PRINT 920
      GO TO 10
70    IF (Y.LT.YO) LT=L
      YO=Y
80    CONTINUE
90    IF (NSTAG.NE.0) GO TO 110
      DO 100 M=2,MMAX
      PT(M)=PT(1)
      TT(M)=TT(1)
      THETA(M)=THETA(1)
100   CONTINUE
      PRINT 670
      IF (IUI.EQ.1) PRINT 760, PT(1),TT(1),THETA(1),PE
      IF (IUI.EQ.2) PRINT 770, PT(1),TT(1),THETA(1),PE
      GO TO 130
110   PRINT 660
      IF (IUI.EQ.1) PRINT 890, PE
      IF (IUI.EQ.2) PRINT 900, PE
      DO 120 M=1,MMAX
      PRINT 910, M,PT(M),TT(M),THETA(M)
120   CONTINUE
C
C     CONVERT METRIC UNITS TO ENGLISH UNITS
C
130   IF (IUI.EQ.1) GO TO 180
      RSTAR=RSTAR/2.54
      RSTARS=RSTARS/6.4516
      RGAS=RGAS/5.38032
      DO 140 M=1,MMAX
      PT(M)=PT(M)/6.8948
      TT(M)=(TT(M)+40.0)*9.0/5.0-40.0
140   CONTINUE
      PE=PE/6.8948
      IF (ISUPER.EQ.0) GO TO 160
      DO 150 M=1,MMAX
      UI(M)=UI(M)/0.3048
      VI(M)=VI(M)/0.3048
      PI(M)=PI(M)/6.8948
      ROI(M)=ROI(M)/16.02
150   CONTINUE
160   IF (N1D.NE.0) GO TO 180
      IF (NSTART.NE.0) GO TO 180
      DO 170 L=1,LMAX
      DO 170 M=1,MMAX
      U(L,M,1)=U(L,M,1)/0.3048
```

```
      V(L,M,1)=V(L,M,1)/0.3048
      P(L,M,1)=P(L,M,1)/6.8948
      RO(L,M,1)=RO(L,M,1)/16.02
170   CONTINUE
C
C     CONVERT INPUT DATA UNITS TO INTERNAL UNITS
C
180   IF (IUNIT.EQ.0) GO TO 190
      PC=LC=G=1.0
      TC=0.0
190   TCONV=TCONV/100.0
      T=TSTART*LC
      TSTOP=TSTOP*LC
      DO 200 L=1,LMAX
      XWI(L)=0.0
200   CONTINUE
      DO 210 M=1,MMAX
      PT(M)=PT(M)*PC
      TT(M)=TT(M)+TC
      THETA(M)=THETA(M)*0.0174533
210   CONTINUE
      PE=PE*PC
      IF (N1D.NE.0) GO TO 230
      DO 220 L=1,LMAX
      DO 220 M=1,MMAX
      P(L,M,1)=P(L,M,1)*PC
      RO(L,M,1)=RO(L,M,1)/G
220   CONTINUE
230   GAM1=GAMMA/(GAMMA-1.0)
      GAM2=(GAMMA-1.0)/2.0
      IF (ISUPER.EQ.0) GO TO 250
      DO 240 M=1,MMAX
      U(1,M,1)=UI(M)
      V(1,M,1)=VI(M)
      P(1,M,1)=PI(M)*PC
      RO(1,M,1)=ROI(M)/G
      U(1,M,2)=U(1,M,1)
      V(1,M,2)=V(1,M,1)
      P(1,M,2)=P(1,M,1)
      RO(1,M,2)=RO(1,M,1)
240   CONTINUE
250   L1=LMAX-1
      L2=LMAX-2
      L3=LMAX-3
      M1=MMAX-1
      M2=MMAX-2
      IF (N1D.EQ.0) GO TO 260
C
C     COMPUTE THE 1-D INITIAL-DATA SURFACE
C
      CALL ONEDIM
      IF (IERR.NE.0) GO TO 10
C
C     COMPUTE THE INITIAL-DATA SURFACE MASS FLOW AND THRUST
C
260   IF (NPRINT.GT.0) GO TO 270
      NPRINT=-NPRINT
      GO TO 340
270   CALL MASFLO (0)
C
C     CALCULATE AND PRINT THE INITIAL-VALUE SURFACE
C
      DO 330 IU=1,2
      IF (IUO.EQ.1.AND.IU.EQ.2) GO TO 330
      IF (IUO.EQ.2.AND.IU.EQ.1) GO TO 330
      NLINE=0
      PRINT 660
      PRINT 810, TSTART,NSTART
      PRINT 820
      IF (IU.EQ.1) PRINT 830
      IF (IU.EQ.2) PRINT 840
      PRINT 670
      X=X1-DX
      DO 300 L=1,LMAX
      X=X+DX
      CALL MAP (0,L,1,AL,BE,DE,LD1,AL1,BE1,DE1)
      DYIO=DY/BE
      Y=YCB(L)-DYIO
      DO 300 M=1,MMAX
      Y=Y+DYIO
      VELMAG=SQRT(U(L,M,1)**2+V(L,M,1)**2)
      XMACH=VELMAG/SQRT(GAMMA*P(L,M,1)/RO(L,M,1))
      PRES=P(L,M,1)/PC
```

```
      RHO=RO(L,M,1)*G
      TEMP=P(L,M,1)/RHO/RGAS-TC
      XP=X
      YP=Y
      UP=U(L,M,1)
      VP=V(L,M,1)
      IF (IU.EQ.1) GO TO 280
      XP=XP*2.54
      YP=YP*2.54
      UP=UP*0.3048
      VP=VP*0.3048
      PRES=PRES*6.8948
      RHO=RHO*16.02
      VELMAG=VELMAG*0.3048
      TEMP=(TEMP+40.0)*5.0/9.0-40.0
280   NLINE=NLINE+1
      IF (NLINE.LT.55) GO TO 290
      PRINT 660
      PRINT 810, TSTART,NSTART
      PRINT 820
      IF (IU.EQ.1) PRINT 830
      IF (IU.EQ.2) PRINT 840
      PRINT 670
      NLINE=1
290   PRINT 850, L,M,XP,YP,UP,VP,PRES,RHO,VELMAG,XMACH,TEMP
300   CONTINUE
      IF (IU.EQ.2) GO TO 310
      PRINT 870, MASST,THRUST,MASSI,MASSE
      GO TO 320
310   MASST=MASST*0.4536
      MASSI=MASSI*0.4536
      MASSE=MASSE*0.4536
      THRUST=THRUST*4.4477
      PRINT 880, MASST,THRUST,MASSI,MASSE
320   IF (IUO.NE.3) GO TO 340
330   CONTINUE
340   IF (NPLOT.LE.0) GO TO 350
      CALL PLOT (TITLE,TSTART,NSTART)
      PRINT 1030, NSTART
350   IF (NMAX.EQ.0) GO TO 10
C
C     INITIALIZE THE TIME STEP INTEGRATION LOOP PARAMETERS
C
      N1=1 $ N3=2 $ DQM=0.0 $ NS=0 $ NCONV=0 $ NC=0 $ LDUM=1 $ NPC=0
      DXR=1.0/DX $ DYR=1.0/DY $ DXRS=DXR*DXR $ DYRS=DYR*DYR
      LD=81 $ MD=21 $ LMD=LD*MD
      IF (NASM.NE.0.AND.LT.NE.1) LDUM=LT-1
      NPD=0
      IF (JFLAG.EQ.0) GO TO 360
      UD(1)=U(LJET-1,MMAX,N1)
      VD(1)=V(LJET-1,MMAX,N1)
      PD(1)=P(LJET-1,MMAX,N1)
      ROD(1)=RO(LJET-1,MMAX,N1)
      UD(2)=UD(1)
      VD(2)=VD(1)
      PD(2)=PD(1)
      ROD(2)=ROD(1)
C
C     ENTER THE TIME STEP INTERGRATION LOOP
C
360   DO 580 N=1,NMAX
      NPD=NPD+1
      IF (NPD.NE.10) GO TO 370
      NP=N+NSTART
      PRINT 1040, NP
      NPD=0
370   CONTINUE
      LMD1=LMD*(N1-1)
      LMD3=LMD*(N3-1)
C
C     CALCULATE DELTA T
C
      DO 380 L=1,LMAX
      CALL MAP (0,L,MD,AL,BE,DE,LD1,AL1,BE1,DE1)
      DXDY=DXRS+BE*BE*DYRS
      DO 380 M=1,MMAX
      LMN1=L+LD*(M-1)+LMD1
      QS=U(LMN1)*U(LMN1)+V(LMN1)*V(LMN1)
      AS=GAMMA*P(LMN1)/RO(LMN1)
      UPA=SQRT(QS*DXDY)+SQRT(AS*DXDY)
      IF (L.EQ.1.AND.M.EQ.1) UPAM=UPA
      IF (UPA.GT.UPAM) UPAM=UPA
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 380   CONTINUE
       DT=FDT/UPAM
       T=T+DT
       IF (T.LE.TSTOP) GO TO 390
       T=T-DT
       DT=TSTOP-T
       T=TSTOP
C
C      DETERMINE IF THE EXIT FLOW IS SUBSONIC OR SUPERSONIC
C
 390   IVEL=0
       IF (QS.GE.AS) IVEL=1
C
C      CALCULATE THE NOZZLE WALL AND INTERIOR MESH POINTS
C
       IF (IAV.NE.0) CALL SHOCK (1)
       ICHAR=1
       IB=1
       CALL INTER
       CALL WALL
       IF (IERR.NE.0) GO TO 10
       IF (NGCB.EQ.0) GO TO 400
       IB=2
       CALL WALL
       IF (IERR.NE.0) GO TO 10
 400   ICHAR=2
       IB=1
       CALL INTER
       CALL WALL
       IF (IERR.NE.0) GO TO 10
       IF (NGCB.EQ.0) GO TO 410
       IB=2
       CALL WALL
       IF (IERR.NE.0) GO TO 10
C
C      EXTRAPOLATE THE EXIT MESH POINTS FOR SUPERSONIC FLOW
C
 410   DO 420 M=1,MMAX
       U(LMAX,M,N3)=U(L1,M,N3)+IEX*(U(L1,M,N3)-U(L2,M,N3))
       V(LMAX,M,N3)=V(L1,M,N3)+IEX*(V(L1,M,N3)-V(L2,M,N3))
       P(LMAX,M,N3)=P(L1,M,N3)+IEX*(P(L1,M,N3)-P(L2,M,N3))
       RO(LMAX,M,N3)=RO(L1,M,N3)+IEX*(RO(L1,M,N3)-RO(L2,M,N3))
       IF (P(LMAX,M,N3).GT.0.0.AND.RO(LMAX,M,N3).GT.0.0) GO TO 420
       P(LMAX,M,N3)=P(L1,M,N3)
       RO(LMAX,M,N3)=RO(L1,M,N3)
 420   CONTINUE
       V(LMAX,MMAX,N3)=-U(LMAX,MMAX,N3)*NXNY(LMAX)
       V(LMAX,1,N3)=-U(LMAX,1,N3)*NXNYCB(LMAX)
C
C      CALCULATE THE NOZZLE INLET MESH POINTS
C
       IF (ISUPER.EQ.0) CALL INLET
C
C      CALCULATE THE NOZZLE EXIT MESH POINTS FOR SUBSONIC FLOW
C
       IF (IVEL.EQ.0) CALL EXITT
       IF (N.LE.NST) CALL SHOCK (2)
C
C      DETERMINE THE MAXIMUM (DELTA U)/U
C
       IF (TCONV.LE.0.0) GO TO 440
       DQM=0.0
       DO 430 L=LDUM,LMAX
       DO 430 M=1,MMAX
       IF (U(L,M,N1).EQ.0.0) GO TO 430
       DQ=ABS((U(L,M,N3)-U(L,M,N1))/U(L,M,N1))
       IF (DQ.GT.DQM) DQM=DQ
 430   CONTINUE
 440   NC=NC+1
       NPC=NPC+1
       IF (DQM.GE.TCONV) GO TO 450
       NCONV=NCONV+1
       IF (NCONV.EQ.1) NCHECK=N-1
       IF (NCONV.GE.NCONVI) NC=NPRINT
 450   IF (N.EQ.NMAX) NC=NPRINT
       IF (N.GE.NCHECK+NCONVI) NCONV=0
       IF (T.EQ.TSTOP) NC=NPRINT
       IF (NC.EQ.NPRINT) GO TO 460
       IF (NPC.EQ.NPLOT) GO TO 550
       GO TO 570
C
C      COMPUTE THE SOLUTION SURFACE MASS FLOW AND THRUST
C
```

```
460   ICN=0
      IF (JFLAG.EQ.0) GO TO 470
      IF (LT.NE.LJET-1) GO TO 470
      UDUM=U(LT,MMAX,N3)
      RODUM=RO(LT,MMAX,N3)
      U(LT,MMAX,N3)=UD(3)
      RO(LT,MMAX,N3)=ROD(3)
      ICN=1
470   CALL MASFLO (1)
      IF (ICN.EQ.0) GO TO 480
      U(LT,MMAX,N3)=UDUM
      RO(LT,MMAX,N3)=RODUM
C
C     CALCULATE AND PRINT THE SOLUTION SURFACE
C
480   DO 540 IU=1,2
      IF (IUO.EQ.1.AND.IU.EQ.2) GO TO 540
      IF (IUO.EQ.2.AND.IU.EQ.1) GO TO 540
      NLINE=0
      PRINT 660
      TIME=T/LC
      DTIME=DT/LC
      NP=N+NSTART
      PRINT 860, NP,TIME,DTIME
      PRINT 820
      IF (IU.EQ.1) PRINT 830
      IF (IU.EQ.2) PRINT 840
      PRINT 670
      X=XI-DX
      DO 510 L=1,LMAX
      X=X+DX
      CALL MAP (0,L,1,AL,BE,DE,LD1,AL1,BE1,DE1)
      DYIO=DY/BE
      Y=YCB(L)-DYIO
      DO 510 M=1,MMAX
      Y=Y+DYIO
      VELMAG=SQRT(U(L,M,N3)**2+V(L,M,N3)**2)
      XMACH=VELMAG/SQRT(GAMMA*P(L,M,N3)/RO(L,M,N3))
      PRES=P(L,M,N3)/PC
      RHO=RO(L,M,N3)*G
      TEMP=P(L,M,N3)/RHO/RGAS-TC
      XP=X
      YP=Y
      UP=U(L,M,N3)
      VP=V(L,M,N3)
      IF (IU.EQ.1) GO TO 490
      XP=XP*2.54
      YP=YP*2.54
      UP=UP*0.3048
      VP=VP*0.3048
      PRES=PRES*6.8948
      RHO=RHO*16.02
      VELMAG=VELMAG*0.3048
      TEMP=(TEMP+40.0)*5.0/9.0-40.0
490   NLINE=NLINE+1
      IF (NLINE.LT.55) GO TO 500
      PRINT 660
      PRINT 860, NP,TIME,DTIME
      PRINT 820
      IF (IU.EQ.1) PRINT 830
      IF (IU.EQ.2) PRINT 840
      PRINT 670
      NLINE=1
500   PRINT 850, L,M,XP,YP,UP,VP,PRES,RHO,VELMAG,XMACH,TEMP
510   CONTINUE
      IF (IU.EQ.2) GO TO 520
      PRINT 870, MASST,THRUST,MASSI,MASSE
      GO TO 530
520   MASST=MASST*0.4536
      MASSI=MASSI*0.4536
      MASSE=MASSE*0.4536
      THRUST=THRUST*4.4477
      PRINT 880, MASST,THRUST,MASSI,MASSE
530   IF (IUO.NE.3) GO TO 550
540   CONTINUE
550   IF (NPLOT.LT.0) GO TO 560
      TIME=T/LC $ NP=N+NSTART
      CALL PLOT (TITLE,TIME,NP)
      PRINT 1030, NP
C
C     CHECK FOR CONVERGENCE OF THE STEADY STATE SOLUTION
C
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560   IF (DQM.LT.TCONV) GO TO 590
      IF (T.EQ.TSTOP) GO TO 590
      IF (N.EQ.NMAX) GO TO 590
      IF (NC.EQ.NPRINT) NC=0
      IF (NPC.EQ.NPLOT) NPC=0
570   CONTINUE
      NNN=N1
      N1=N3
      N3=NNN
580   CONTINUE
C
C     PUNCH A SIVS NAMELIST FOR RESTART
C
590   IF (NPLOT.GE.0) CALL ADV (10)
      IF (IPUNCH.EQ.0) GO TO 10
      DO 600 L=1,LMAX
      DO 600 M=1,MMAX
      P(L,M,N3)=P(L,M,N3)/PC
      RO(L,M,N3)=RO(L,M,N3)*G
600   CONTINUE
      PUNCH 930, NP,TIME
      DO 610 M=1,MMAX
      PUNCH 940, M
      PUNCH 950, (U(L,M,N3),L=1,LMAX)
610   CONTINUE
      DO 620 M=1,MMAX
      PUNCH 960, M
      PUNCH 950, (V(L,M,N3),L=1,LMAX)
620   CONTINUE
      DO 630 M=1,MMAX
      PUNCH 970, M
      PUNCH 980, (P(L,M,N3),L=1,LMAX)
630   CONTINUE
      DO 640 M=1,MMAX
      PUNCH 990, M
      PUNCH 1000, (RO(L,M,N3),L=1,LMAX)
640   CONTINUE
      PUNCH 1010
      NCARDS=(LMAX/7+2)*MMAX*4+2
      PRINT 1020, NCARDS
      GO TO 10
C
C     FORMAT STATEMENTS
C
650   FORMAT (8A10)
660   FORMAT (1H1)
670   FORMAT (1H )
680   FORMAT (1H0)
690   FORMAT (1H0,15X,100HNAP, A COMPUTER PROGRAM FOR THE COMPUTATION OF
     1 TWO-DIMENSIONAL, TIME-DEPENDENT, INVISCID NOZZLE FLOW,//,37X,59HB
     2Y MICHAEL C. CLINE, T-3 - LOS ALAMOS SCIENTIFIC LABORATORY)
700   FORMAT (1H0,10X,18HPROGRAM ABSTRACT -,//,26X,86HTHE EQUATIONS OF M
     1OTION FOR TWO-DIMENSIONAL, TIME DEPENDENT, INVISCID FLOW IN A NOZZ
     2LE,/,21X,93HARE SOLVED USING THE SECOND-ORDER, MACCORMACK, FINITE-
     3DIFFERENCE SCHEME. THE FLUID IS ASSUMED,/,21X,95HTO BE A PERFECT G
     4AS. ALL BOUNDARY CONDITIONS ARE COMPUTED USING A SECOND-ORDER, REF
     5ERENCE PLANE,/,21X,91HCHARACTERISTIC SCHEME. THE STEADY STATE SOLU
     6TION IS OBTAINED AS THE ASYMPTOTIC SOLUTION FOR,/,21X,91HLARGE TIM
     7E. THE NOZZLES MAY BE EITHER CONVERGING, CONVERGING-DIVERGING, OR
     8PLUG GEOMETRIES.)
710   FORMAT (1H0,10X,11HJOB TITLE -//21X,8A10)
720   FORMAT (1H0,10X,20HCONTROL PARAMETERS -)
730   FORMAT (1H0,20X,5HLMAX=,I2,2X,5HMMAX=,I2,3X,5HNMAX=,I4,2X,7HNPRINT
     1=,I4,2X,6HTCONV=,F6.3,3X,4HFDT=,F4.2,2X,6HNSTAG=,I1,5X,5HNASM=,I1,
     24X,6HIUNIT=,I1,/,21X,4HIUI=,I1,4X,4HIUO=,I1,5X,4HIEX=,I1,6X,7HNCON
     3VI=,I2,4X,6HTSTOP=,F7.5,2X,4HN1D=,I2,4X,6HNPLOT=,I4,2X,7HIPUNCH=,I
     41,2X,7HISUPER=,I1,/,21X,4HIAV=,I1,4X,4HCAV=,F4.1,2X,4HXMU=,F4.2,3X
     5,4HXLA=,F4.2,5X,5HRKMU=,F5.2,5X,4HCTA=,F4.2,2X,4HLSS=,I2,6X,4HSMP=
     6,F4.2,2X,4HNST=,I4)
740   FORMAT (1H0,10X,13HFLUID MODEL -,//,21X,36HTHE RATIO OF SPECIFIC H
     1EATS, GAMMA =,F6.4,26H AND THE GAS CONSTANT, R =,F9.4,15H (FT-LBF/
     2LBM-R))
750   FORMAT (1H0,10X,13HFLUID MODEL -,//,21X,36HTHE RATIO OF SPECIFIC H
     1EATS, GAMMA =,F6.4,26H AND THE GAS CONSTANT, R =,F9.4,9H (J/KG-K))
760   FORMAT (1H0,10X,21HBOUNDARY CONDITIONS -,//,21X,3HPT=,F9.4,7H (PSI
     1A),5X,3HTT=,F9.4,4H (F),5X,6HTHETA=,F9.4,6H (DEG),5X,3HPE=,F9.4,7H
     2 (PSIA))
770   FORMAT (1H0,10X,21HBOUNDARY CONDITIONS -,//,21X,3HPT=,F9.4,6H (KPA
     1),5X,3HTT=,F9.4,4H (C),5X,6HTHETA=,F9.4,6H (DEG),5X,3HPE=,F9.4,6H
     2(KPA))
780   FORMAT (1H0,10X,15HFLOW GEOMETRY -)
790   FORMAT (1H0,20X,47HTWO-DIMENSIONAL, PLANAR FLOW HAS BEEN SPECIFIED
     1)
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800   FORMAT (1H0,20X,36HAXISYMMETRIC FLOW HAS BEEN SPECIFIED)
810   FORMAT (1H ,30HINITIAL-DATA SURFACE - TIME = ,F10.8,8H SECONDS,4H
     1(N=,I4,1H))
820   FORMAT (1H0,11X,1HL,4X,1HM,9X,1HX,10X,1HY,10X,1HU,11X,1HV,12X,1HP,
     111X,3HRHO,9X,1HQ,11X,4HMACH,8X,1HT)
830   FORMAT (1H ,25X,4H(IN),7X,4H(IN),6X,5H(FPS),7X,5H(FPS),7X,6H(PSIA)
     1,6X,9H(LBM/FT3),4X,5H(FPS),10X,2HNO,8X,3H(F))
840   FORMAT (1H ,25X,4H(CM),7X,4H(CM),6X,5H(MPS),7X,5H(MPS),7X,6H (KPA)
     1,7X,7H(KG/M3),5X,5H(MPS),10X,2HNO,8X,3H(C))
850   FORMAT (1H ,7X,2I5,4F12.4,F13.4,F12.6,3F12.4)
860   FORMAT (1H ,20HSOLUTION SURFACE NO.,I5,3H - ,7HTIME = ,F10.8,20H S
     1ECONDS (DELTA T = ,F10.8,1H))
870   FORMAT (1H0,10X,5HMASS=,F9.4,10H (LBM/SEC),5X,7HTHRUST=,F11.4,6H (
     1LBF),5X,6HMASSI=,F9.4,5X,6HMASSE=,F9.4)
880   FORMAT (1H0,10X,5HMASS=,F9.4,9H (KG/SEC),5X,7HTHRUST=,F11.4,10H (N
     1EWTONS),5X,6HMASSI=,F9.4,5X,6HMASSE=,F9.4)
890   FORMAT (1H0,10X,21HBOUNDARY CONDITIONS -,//,22X,1HM,11X,8HPT(PSIA)
     1,10X,5HTT(F),10X,10HTHETA(DEG),10X,3HPE=,F7.3,7H (PSIA),/)
900   FORMAT (1H0,10X,21HBOUNDARY CONDITIONS -,//,22X,1HM,11X,7HPT(KPA),
     112X,5HTT(C),10X,10HTHETA(DEG),10X,3HPE=,F7.3,6H (KPA),/)
910   FORMAT (1H ,20X,I2,10X,F7.2,10X,F7.2,10X,F7.2)
920   FORMAT (1H0,78H***** THE RADIUS OF THE CENTERBODY IS LARGER THAN T
     1HE NOZZLE WALL RADIUS *****)
930   FORMAT (1X,18HSIVS NID=0,NSTART=,I4,8H,TSTART=,F14.10,1H,)
940   FORMAT (1X,4HU(1,,I2,5H,1) =)
950   FORMAT (1X,7(F10.3,1H,))
960   FORMAT (1X,4HV(1,,I2,5H,1) =)
970   FORMAT (1X,4HP(1,,I2,5H,1) =)
980   FORMAT (1X,7(F10.4,1H,))
990   FORMAT (1X,5HRO(1,,I2,5H,1) =)
1000  FORMAT (1X,7(F10.6,1H,))
1010  FORMAT (1X,1H$)
1020  FORMAT (1H0,27H***** EXPECT APPROXIMATELY ,I4,20H PUNCHED CARDS **
     1***)
1030  FORMAT (1H0,31H***** EXPECT FILM OUTPUT FOR N=,I4,6H *****)
1040  FORMAT (1H ,2HN=,I4)
      END
```

```
      SUBROUTINE GEOM
C
C     *****************************************************************
C
C     THIS SUBROUTINE CALCULATES THE NOZZLE RADIUS AND OUTER NORMAL
C
C     *****************************************************************
C
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
      GO TO (10,30,120,170), NGEOM
C
C     CONSTANT AREA DUCT CASE
C
10    PRINT 230
      IF (IUI.EQ.1) PRINT 250, XI,RI,XE
      IF (IUI.EQ.2) PRINT 260, XI,RI,XE
      LT=LMAX
      DX=(XE-XI)/(LMAX-1)
      XT=XE
      RT=RI
      RE=RI
      DO 20 L=1,LMAX
      YW(L)=RI
      NXNY(L)=0.0
20    CONTINUE
      IF (JFLAG.EQ.0) GO TO 210
C
      XWL=XI+(LJET-2)*DX
      IF (IUI.EQ.1) PRINT 370, XWL,LJET,LMAX
      IF (IUI.EQ.2) PRINT 380, XWL,LJET,LMAX
      GO TO 210
C
C     CIRCULAR-ARC, CONICAL NOZZLE CASE
C
30    PRINT 230
      IF (RCI.EQ.0.0.OR.RCT.EQ.0.0) GO TO 200
      ANI=ANGI*3.141593/180.0
      ANE=ANGE*3.141593/180.0
      XTAN=XI+RCI*SIN(ANI)
      RTAN=RI+RCI*(COS(ANI)-1.0)
      RT1=RT-RCT*(COS(ANI)-1.0)
      XT1=XTAN+(RTAN-RT1)/TAN(ANI)
      IF (XT1.GE.XTAN) GO TO 40
      XT1=XTAN
      RT1=RTAN
40    XT=XT1+RCT*SIN(ANI)
      XT2=XT+RCT*SIN(ANE)
      RT2=RT+RCT*(1.0-COS(ANE))
      RE=RT2+(XE-XT2)*TAN(ANE)
      LT=1
      DX=(XE-XI)/(LMAX-1)
      IF (IUI.EQ.1) PRINT 270, XI,RI,RT,XE,RCI,RCT,ANGI,ANGE,XT,RE
      IF (IUI.EQ.2) PRINT 280, XI,RI,RT,XE,RCI,RCT,ANGI,ANGE,XT,RE
      DO 110 L=1,LMAX
      X=XI+(L-1)*DX
      IF (X.GE.XI.AND.X.LE.XTAN) GO TO 50
      IF (X.GT.XTAN.AND.X.LE.XT1) GO TO 60
      IF (X.GT.XT1.AND.X.LE.XT) GO TO 70
      IF (X.GT.XT.AND.X.LE.XT2) GO TO 80
      IF (X.GT.XT2.AND.X.LE.XE) GO TO 90
C
50    YW(L)=RI+RCI*(COS(ASIN((X-XI)/RCI))-1.0)
      NXNY(L)=(X-XI)/(YW(L)-RI+RCI)
      GO TO 100
C
60    YW(L)=RT1+(XT1-X)*TAN(ANI)
      NXNY(L)=TAN(ANI)
      GO TO 100
```

```
C
70    YW(L)=RT+RCT*(1.0-COS(ASIN((XT-X)/RCT)))
      NXNY(L)=(XT-X)/(RCT+RT-YW(L))
      GO TO 100
C
80    YW(L)=RT+RCT*(1.0-COS(ASIN((X-XT)/RCT)))
      NXNY(L)=(XT-X)/(RCT+RT-YW(L))
      GO TO 100
C
90    YW(L)=RT2+(X-XT2)*TAN(ANE)
      NXNY(L)=-TAN(ANE)
C
100   IF (L.EQ.1) GO TO 110
      IF (YW(L).LT.YW(LT)) LT=L
110   CONTINUE
      IF (JFLAG.EQ.0) GO TO 210
C
      XWL=XI+(LJET-2)*DX
      IF (IUI.EQ.1) PRINT 370, XWL,LJET,LMAX
      IF (IUI.EQ.2) PRINT 380, XWL,LJET,LMAX
      GO TO 210
C
C     GENERAL NOZZLE CASE - INPUT WALL COORDINATES ONLY
C
120   PRINT 240
      PRINT 230
      XI=XWI(1)
      XE=XWI(NWPTS)
      DX=(XE-XI)/(LMAX-1)
      XW(1)=XI
      XW(LMAX)=XE
      YW(1)=YWI(1)
      YW(LMAX)=YWI(NWPTS)
      RI=YW(1)
      RE=YW(LMAX)
      LT=1
      DO 130 L=2,NWPTS
      IF (YWI(L).LE.YWI(LT)) LT=L
130   CONTINUE
      XT=XWI(LT)
      RT=YWI(LT)
      IF (IUI.EQ.1) PRINT 290, XT,RT,IINT,IDIF
      IF (IUI.EQ.2) PRINT 300, XT,RT,IINT,IDIF
      LT=1
      L1=LMAX-1
      IP=1
      DO 140 L=2,L1
      XW(L)=XI+DX*(L-1)
      CALL MTLUP (XW(L),YW(L),IINT,NWPTS,NWPTS,1,IP,XWI,YWI)
      IF (L.EQ.1) GO TO 140
      IF (YW(L).LE.YW(LT)) LT=L
140   CONTINUE
      LDUM=NWPTS
      IF (LMAX.GT.NWPTS) LDUM=LMAX
      DO 160 L=1,LDUM
      IF (L.GT.LMAX) GO TO 150
      SLOPE=DIF(L,IDIF,LMAX,XW,YW)
      NXNY(L)=-SLOPE
150   IF (L.LE.NWPTS.AND.L.LE.LMAX) PRINT 330, L,XWI(L),YWI(L),XW(L),YW(
     1L),SLOPE
      IF (L.GT.NWPTS.AND.L.LE.LMAX) PRINT 340, L,XW(L),YW(L),SLOPE
      IF (L.LE.NWPTS.AND.L.GT.LMAX) PRINT 350, L,XWI(L),YWI(L)
160   CONTINUE
      IF (JFLAG.EQ.0) GO TO 210
C
      IF (IUI.EQ.1) PRINT 370, XW(LJET-1),LJET,LMAX
      IF (IUI.EQ.2) PRINT 380, XW(LJET-1),LJET,LMAX
      GO TO 210
C
C     GENERAL NOZZLE CASE - INPUT WALL COORDINATES AND SLOPES
C
170   PRINT 240
      PRINT 230
      DX=(XE-XI)/(LMAX-1)
      RI=YW(1)
      RE=YW(LMAX)
      LT=1
      DO 180 L=2,LMAX
      IF (YW(L).LE.YW(LT)) LT=L
180   CONTINUE
      XT=XI+(LT-1)*DX
      RT=YW(LT)
      IF (IUI.EQ.1) PRINT 310, XT,RT
```

```
      IF (IUI.EQ.2) PRINT 320, XT,RT
      DO 190 L=1,LMAX
      XW(L)=XI+DX*(L-1)
      SLOPE=-NXNY(L)
      PRINT 360, L,XW(L),YW(L),SLOPE
190   CONTINUE
      IF (JFLAG.EQ.0) GO TO 210
C
      IF (IUI.EQ.1) PRINT 370, XW(LJET-1),LJET,LMAX
      IF (IUI.EQ.2) PRINT 380, XW(LJET-1),LJET,LMAX
      GO TO 210
C
200   PRINT 390
      IERR=1
      RETURN
C
210   IF (IUI.EQ.1) RETURN
      DO 220 L=1,LMAX
      YW(L)=YW(L)/2.54
220   CONTINUE
      XT=XT/2.54
      RT=RT/2.54
      IF (NGCB.NE.0) RETURN
      XI=XI/2.54 $ XE=XE/2.54
      DX=DX/2.54
      RETURN
C
C     FORMAT STATEMENTS
C
230   FORMAT (1H0,10X,17HNOZZLE GEOMETRY -)
240   FORMAT (1H1)
250   FORMAT (1H0,20X,46HA CONSTANT AREA DUCT HAS BEEN SPECIFIED BY XI=,
     1F8.4,10H (IN), RI=,F8.4,14H (IN), AND XE=,F8.4,5H (IN))
260   FORMAT (1H0,20X,46HA CONSTANT AREA DUCT HAS BEEN SPECIFIED BY XI=,
     1F8.4,10H (CM), RI=,F8.4,14H (CM), AND XE=,F8.4,5H (CM))
270   FORMAT (1H0,20X,56HA CIRCULAR-ARC, CONICAL NOZZLE HAS BEEN SPECIFI
     1ED BY XI=,F8.4,10H (IN), RI=,F8.4,6H (IN),,/,21X,3HRT=,F8.4,10H (I
     2N), XE=,F8.4,11H (IN), RCI=,F8.4,11H (IN), RCT=,F8.4,12H (IN), ANG
     3I=,F6.2,7H (DEG),,/,21X,9HAND ANGE=,F6.2,35H (DEG). THE COMPUTED V
     4ALUES ARE XT=,F8.4,13H (IN) AND RE=,F8.4,6H (IN).)
280   FORMAT (1H0,20X,56HA CIRCULAR-ARC, CONICAL NOZZLE HAS BEEN SPECIFI
     1ED BY XI=,F8.4,10H (CM), RI=,F8.4,6H (CM),,/,21X,3HRT=,F8.4,10H (C
     2M), XE=,F8.4,11H (CM), RCI=,F8.4,11H (CM), RCT=,F8.4,12H (CM), ANG
     3I=,F6.2,7H (DEG),,/,21X,9HAND ANGE=,F6.2,35H (DEG). THE COMPUTED V
     4ALUES ARE XT=,F8.4,13H (CM) AND RE=,F8.4,6H (CM).)
290   FORMAT (1H0,20X,68HA GENERAL NOZZLE HAS BEEN SPECIFIED BY THE FOLL
     1OWING PARAMETERS, XT=,F8.4,10H (IN), RT=,F8.4,6H (IN),,/,21X,5HIIN
     2T=,I1,7H, IDIF=,I1,1H,,//,22X,1HL,10X,7HXWI(IN),10X,7HYWI(IN),11X,
     36HXW(IN),11X,6HYW(IN),12X,5HSLOPE,/)
300   FORMAT (1H0,20X,68HA GENERAL NOZZLE HAS BEEN SPECIFIED BY THE FOLL
     1OWING PARAMETERS, XT=,F8.4,10H (CM), RT=,F8.4,6H (CM),,/,21X,5HIIN
     2T=,I1,7H, IDIF=,I1,1H,,//,22X,1HL,10X,7HXWI(CM),10X,7HYWI(CM),11X,
     36HXW(CM),11X,6HYW(CM),12X,5HSLOPE,/)
310   FORMAT (1H0,20X,68HA GENERAL NOZZLE HAS BEEN SPECIFIED BY THE FOLL
     1OWING PARAMETERS, XT=,F8.4,10H (IN), RT=,F8.4,6H (IN),,//,22X,1HL,
     211X,6HXW(IN),11X,6HYW(IN),12X,5HSLOPE,/)
320   FORMAT (1H0,20X,68HA GENERAL NOZZLE HAS BEEN SPECIFIED BY THE FOLL
     1OWING PARAMETERS, XT=,F8.4,10H (CM), RT=,F8.4,6H (CM),,//,22X,1HL,
     211X,6HXW(CM),11X,6HYW(CM),12X,5HSLOPE,/)
330   FORMAT (1H ,20X,I2,7X,F10.4,7X,F10.4,7X,F10.4,7X,F10.4,7X,F10.4)
340   FORMAT (1H ,20X,I2,41X,F10.4,7X,F10.4,7X,F10.4)
350   FORMAT (1H ,20X,I2,7X,F10.4,7X,F10.4)
360   FORMAT (1H ,20X,I2,7X,F10.4,7X,F10.4,7X,F10.4)
370   FORMAT (1H0,20X,69HAN EXHAUST JET CALCULATION HAS BEEN REQUESTED.
     1 THE NOZZLE ENDS AT X=,F8.4,11H (IN).  THE,/,21X,14HMESH POINTS L=
     2,I3,6H TO L=,I3,58H ARE AN INITIAL APPROXIMATION TO THE EXHAUST JE
     3T BOUNDARY.)
380   FORMAT (1H0,20X,69HAN EXHAUST JET CALCULATION HAS BEEN REQUESTED.
     1 THE NOZZLE ENDS AT X=,F8.4,11H (CM).  THE,/,21X,14HMESH POINTS L=
     2,I3,6H TO L=,I3,58H ARE AN INITIAL APPROXIMATION TO THE EXHAUST JE
     3T BOUNDARY.)
390   FORMAT (1H0,44H***** RCI OR RCT WAS SPECIFIED AS ZERO *****)
      END
```

```
      SUBROUTINE GEOMCB
C
C     ******************************************************************
C
C     THIS SUBROUTINE CALCULATES THE CENTERBODY RADIUS AND SLOPE
C
C     ******************************************************************
C
      DIMENSION YW(81), NXNY(81)
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
C
      GO TO (10,30,120,160), NGCB
C
C     CYLINDRICAL CENTERBODY CASE
C
10    IF (IUI.EQ.1) PRINT 230, XICB,RICB,XECB
      IF (IUI.EQ.2) PRINT 240, XICB,RICB,XECB
      LECB=LMAX
      DO 20 L=1,LMAX
      YCB(L)=RICB
      NXNYCB(L)=0.0
20    CONTINUE
      GO TO 200
C
C     CIRCULAR-ARC, CONICAL CENTERBODY CASE
C
30    XI=XICB
      RI=RICB
      RT=RTCB
      XE=XECB
      RCI=RCICB
      RCT=RCTCB
      ANGI=ANGICB
      ANGE=ANGECB
      RI=2.0*RT-RI
      IF (RCI.EQ.0.0.OR.RCT.EQ.0.0) GO TO 190
      ANI=ANGI*3.141593/180.0
      ANE=ANGE*3.141593/180.0
      XTAN=XI+RCI*SIN(ANI)
      RTAN=RI+RCI*(COS(ANI)-1.0)
      RT1=RT-RCT*(COS(ANI)-1.0)
      XT1=XTAN+(RTAN-RT1)/TAN(ANI)
      IF (XT1.GE.XTAN) GO TO 40
      XT1=XTAN
      RT1=RTAN
40    XT=XT1+RCT*SIN(ANI)
      XTCB=XT
      XT2=XT+RCT*SIN(ANE)
      RT2=RT+RCT*(1.0-COS(ANE))
      RE=RT2+(XE-XT2)*TAN(ANE)
      RECB=RE
      RI=2.0*RT-RI
      RE=2.0*RT-RE
      IF (IUI.EQ.1) PRINT 250, XI,RI,RT,XE,RCI,RCT,ANGI,ANGE,XT,RE
      IF (IUI.EQ.2) PRINT 260, XI,RI,RT,XE,RCI,RCT,ANGI,ANGE,XT,RE
      RI=2.0*RT-RI
      RE=2.0*RT-RE
      DO 110 L=1,LMAX
      X=XI+(L-1)*DX
      IF (X.GE.XI.AND.X.LE.XTAN) GO TO 50
      IF (X.GT.XTAN.AND.X.LE.XT1) GO TO 60
      IF (X.GT.XT1.AND.X.LE.XT) GO TO 70
      IF (X.GT.XT.AND.X.LE.XT2) GO TO 80
      IF (X.GT.XT2.AND.X.LE.XE) GO TO 90
C
50    YW(L)=RI+RCI*(COS(ASIN((X-XI)/RCI))-1.0)
      NXNY(L)=(X-XI)/(YW(L)-RI+RCI)
      GO TO 100
C
```

```
60    YW(L)=RT1+(XT1-X)*TAN(ANI)
      NXNY(L)=TAN(ANI)
      GO TO 100
C
70    YW(L)=RT+RCT*(1.0-COS(ASIN((XT-X)/RCT)))
      NXNY(L)=(XT-X)/(RCT+RT-YW(L))
      GO TO 100
C
80    YW(L)=RT+RCT*(1.0-COS(ASIN((X-XT)/RCT)))
      NXNY(L)=(XT-X)/(RCT+RT-YW(L))
      GO TO 100
C
90    YW(L)=RT2+(X-XT2)*TAN(ANE)
      NXNY(L)=-TAN(ANE)
C
100   YCB(L)=2.0*RTCB-YW(L)
      NXNYCB(L)=-NXNY(L)
      IF (YCB(L).GE.0.0) GO TO 110
      YCB(L)=0.0
      NXNYCB(L)=0.0
110   CONTINUE
      GO TO 200
C
C     GENERAL CENTERBODY CASE - INPUT CENTERBODY COORDINATES ONLY
C
120   PRINT 220
      IF (IUI.EQ.1) PRINT 270, IINTCB,IDIFCB
      IF (IUI.EQ.2) PRINT 280, IINTCB,IDIFCB
      L1=LMAX-1
      IP=1
      DO 130 L=1,LMAX
      XCB(L)=XICB+DX*(L-1)
      CALL MTLUP (XCB(L),YCB(L),IINTCB,NCBPTS,NCBPTS,1,IP,XCBI,YCBI)
130   CONTINUE
      LDUM=NCBPTS
      IF (LMAX.GT.NCBPTS) LDUM=LMAX
      DO 150 L=1,LDUM
      IF (L.GT.LMAX) GO TO 140
      SLOPE=DIF(L,IDIFCB,LMAX,XCB,YCB)
      NXNYCB(L)=-SLOPE
      IF (YCB(L).GE.0.0) GO TO 140
      YCB(L)=0.0
      NXNYCB(L)=0.0
      SLOPE=-NXNYCB(L)
140   IF (L.LE.NCBPTS.AND.L.LE.LMAX) PRINT 310, L,XCBI(L),YCBI(L),XCB(L)
     1,YCB(L),SLOPE
      IF (L.GT.NCBPTS.AND.L.LE.LMAX) PRINT 320, L,XCB(L),YCB(L),SLOPE
      IF (L.LE.NCBPTS.AND.L.GT.LMAX) PRINT 330, L,XCBI(L),YCBI(L)
150   CONTINUE
      GO TO 200
C
C     GENERAL CENTERBODY CASE - INPUT CENTERBODY COORDINATES AND SLOPES
C
160   PRINT 220
      IF (IUI.EQ.1) PRINT 290
      IF (IUI.EQ.2) PRINT 300
      DO 180 L=1,LMAX
      XCB(L)=XICB+DX*(L-1)
      IF (YCB(L).GE.0.0) GO TO 170
      YCB(L)=0.0
      NXNYCB(L)=0.0
170   SLOPE=-NXNYCB(L)
      PRINT 340, L,XCB(L),YCB(L),SLOPE
180   CONTINUE
      GO TO 200
C
190   PRINT 350
      IERR=1
      RETURN
C
200   IF (IUI.EQ.1) RETURN
      DO 210 L=1,LMAX
      YCB(L)=YCB(L)/2.54
210   CONTINUE
      XICB=XICB/2.54
      XECB=XECB/2.54
      DX=DX/2.54
      RETURN
C
C     FORMAT STATEMENTS
C
```

```
220   FORMAT (1H1)
230   FORMAT (1H0,20X,52HA CYLINDRICAL CENTERBODY HAS BEEN SPECIFIED BY
     1XICB=,F8.4,12H (IN), RICB=,F8.4,16H (IN), AND XECB=,F8.4,5H (IN))
240   FORMAT (1H0,20X,52HA CYLINDRICAL CENTERBODY HAS BEEN SPECIFIED BY
     1XICB=,F8.4,12H (CM), RICB=,F8.4,16H (CM), AND XECB=,F8.4,5H (CM))
250   FORMAT (1H0,20X,62HA CIRCULAR-ARC, CONICAL CENTERBODY HAS BEEN SPE
     1CIFIED BY XICB=,F8.4,5H (IN),7H, RICB=,F8.4,6H (IN),,/,21X,5HRTCB=
     2,F8.4,7H (IN), ,5HXECB=,F8.4,5H (IN),8H, RCICB=,F8.4,5H (IN),8H, R
     3CTCB=,F8.4,5H (IN),9H, ANGICB=,F6.2,7H (DEG),,/,21X,11HAND ANGECB=
     4,F6.2,8H (DEG). ,29HTHE COMPUTED VALUES ARE XTCB=,F8.4,5H (IN),10H
     5 AND RECB=,F8.4,6H (IN).)
260   FORMAT (1H0,20X,62HA CIRCULAR-ARC, CONICAL CENTERBODY HAS BEEN SPE
     1CIFIED BY XICB=,F8.4,5H (CM),7H, RICB=,F8.4,6H (CM),,/,21X,5HRTCB=
     2,F8.4,7H (CM), ,5HXECB=,F8.4,5H (CM),8H, RCICB=,F8.4,5H (CM),8H, R
     3CTCB=,F8.4,5H (CM),9H, ANGICB=,F6.2,7H (DEG),,/,21X,11HAND ANGECB=
     4,F6.2,8H (DEG). ,29HTHE COMPUTED VALUES ARE XTCB=,F8.4,5H (CM),10H
     5 AND RECB=,F8.4,6H (CM).)
270   FORMAT (1H0,20X,76HA GENERAL CENTERBODY HAS BEEN SPECIFIED BY THE
     1FOLLOWING PARAMETERS, IINTCB=,I1,9H, IDIFCB=,I1,1H.,//,22X,1HL,10X
     2,8HXCBI(IN),10X,8HYCBI(IN),9X,7HXCB(IN),10X,7HYCB(IN),11X,5HSLOPE,
     3/)
280   FORMAT (1H0,20X,76HA GENERAL CENTERBODY HAS BEEN SPECIFIED BY THE
     1FOLLOWING PARAMETERS, IINTCB=,I1,9H, IDIFCB=,I1,1H.,//,22X,1HL,10X
     2,8HXCBI(CM),10X,8HYCBI(CM),9X,7HXCB(CM),10X,7HYCB(CM),11X,5HSLOPE,
     3/)
290   FORMAT (1H0,20X,68HA GENERAL CENTERBODY HAS BEEN SPECIFIED BY THE
     1FOLLOWING PARAMETERS,,//,22X,1HL,11X,7HXCB(IN),10X,7HYCB(IN),11X,5
     2HSLOPE,/)
300   FORMAT (1H0,20X,68HA GENERAL CENTERBODY HAS BEEN SPECIFIED BY THE
     1FOLLOWING PARAMETERS,,//,22X,1HL,11X,7HXCB(CM),10X,7HYCB(CM),11X,5
     2HSLOPE,/)
310   FORMAT (1H ,20X,I2,7X,F10.4,7X,F10.4,7X,F10.4,7X,F10.4,7X,F10.4)
320   FORMAT (1H ,20X,I2,41X,F10.4,7X,F10.4,7X,F10.4)
330   FORMAT (1H ,20X,I2,7X,F10.4,7X,F10.4)
340   FORMAT (1H ,20X,I2,7X,F10.4,7X,F10.4,7X,F10.4)
350   FORMAT (1H0,48H***** RCICB OR RCTCB WAS SPECIFIED AS ZERO *****)
      END
```

```
      SUBROUTINE MTLUP(X,Y,M,N,MAX,NTAB,I,VARI,VARD)
C
C     ****************************************************************
C
C     THIS SUBROUTINE IS CALLED BY SUBROUTINE GEOM TO INTERPOLATE FOR
C     EQUALLY SPACED NOZZLE WALL COORDINATES FOR THE TABULAR INPUT
C     CASE. SUBROUTINE MTLUP WAS TAKEN FROM THE NASA-LANGLEY PROGRAM
C     LIBRARY. THE DATE OF THIS VERSION IS 09-12-69.
C
C     ****************************************************************
C
C        MODIFICATION OF LIBRARY INTERPOLATION SUBROUTINE  FTLUP
C       MULTIPLE TABLE LOOK-UP ON ONE INDEPENDENT VARIABLE TABLE
C      USES AN EXTERNAL INTERVAL POINTER (I) TO START SEARCH
C       I LESS THAN 0 WILL CHECK MONOTONICITY
C
      DIMENSION VARI(1), VARD(MAX,1), Y(1), V(3), YY(2)
      LOGICAL EX
C
      IF (M.EQ.0) GO TO 170
      IF (N.LE.1) GO TO 170
      EX=.F.
      IF (I.GE.0) GO TO 60
      IF (N.LT.2) GO TO 60
C
C     MONOTONICITY CHECK
C
      IF (VARI(2)-VARI(1)) 20,20,40
C
C     ERROR IN MONOTONICITY
C
10    K=LOCF(VARI(1))
      PRINT 190, J,K,(VARI(J),J=1,N)
      STOP
C
C     MONOTONIC DECREASING
C
20    DO 30 J=2,N
      IF (VARI(J)-VARI(J-1)) 30,10,10
30    CONTINUE
      GO TO 60
C
C     MONOTONIC INCREASING
C
40    DO 50 J=2,N
      IF (VARI(J)-VARI(J-1)) 10,10,50
50    CONTINUE
C
C     INTERPOLATION
C
60    IF (I.LE.0) I=1
      IF (I.GE.N) I=N-1
C
C     LOCATE I INTERVAL (X(I).LE.X.LT.X(I+1))
C
      IF ((VARI(I)-X)*(VARI(I+1)-X)) 100,100,70
C
C     IN GIVES DIRECTION FOR SEARCH OF INTERVALS
C
70    IN=SIGN(1.0,(VARI(I+1)-VARI(I))*(X-VARI(I)))
C
C     IF X OUTSIDE ENDPOINTS, EXTRAPOLATE FROM END INTERVAL
C
80    IF ((I+IN).LE.0) GO TO 90
      IF ((I+IN).GE.N) GO TO 90
      I=I+IN
      IF ((VARI(I)-X)*(VARI(I+1)-X)) 100,100,80
C
C     EXTRAPOLATION
C
90    EX=.T.
100   IF (M.EQ.2) GO TO 120
C
C     FIRST ORDER
C
      DO 110 NT=1,NTAB
110   Y(NT)=(VARD(I,NT)*(VARI(I+1)-X)-VARD(I+1,NT)*(VARI(I)-X))/(VARI(I+
     11)-VARI(I))
      IF (EX) I=I+IN
      RETURN
C
C     SECOND ORDER
C
```

```
120   IF (N.EQ.2) GO TO 10
      IF (I.EQ.(N-1)) GO TO 140
      IF (I.EQ.1) GO TO 130
C
C     PICK THIRD POINT
C
      SK=VARI(I+1)-VARI(I)
      IF ((SK*(X-VARI(I-1))).LT.(SK*(VARI(I+2)-X))) GO TO 140
130   L=I
      GO TO 150
140   L=I-1
150   V(1)=VARI(L)-X
      V(2)=VARI(L+1)-X
      V(3)=VARI(L+2)-X
      DO 160 NT=1,NTAB
      YY(1)=(VARD(L,NT)*V(2)-VARD(L+1,NT)*V(1))/(VARI(L+1)-VARI(L))
      YY(2)=(VARD(L+1,NT)*V(3)-VARD(L+2,NT)*V(2))/(VARI(L+2)-VARI(L+1))
160   Y(NT)=(YY(1)*V(3)-YY(2)*V(1))/(VARI(L+2)-VARI(L))
      IF (EX) I=I+IN
      RETURN
C
C     ZERO ORDER
C
170   DO 180 NT=1,NTAB
180   Y(NT)=VARD(1,NT)
      RETURN
C
190   FORMAT (1H1,49H TABLE BELOW OUT OF ORDER FOR MTLUP AT POSITION  ,I
     15,/31H X TABLE IS STORED IN LOCATION ,O6,//(8G15.8))
      END
```

```
      FUNCTION DIF (L,M,NP,VARI,VARD)
C
C     ****************************************************************
C
C     THIS FUNCTION IS CALLED BY SUBROUTINE GEOM TO CALCULATE THE
C     NOZZLE WALL SLOPE FOR THE TABULAR INPUT CASE. FUNCTION DIF WAS
C     TAKEN FROM THE NASA-LANGLEY PROGRAM LIBRARY. THE DATE OF THIS
C     VERSION IS 08-01-68.
C
C     ****************************************************************
C
C        THIS FUNCTION SUBPROGRAM FINDS THE DERIVATIVE AT A GIVEN POINT,
C        L, FOR THE DESIRED X AND Y IN A GIVEN TABLE.  THE N-POINT
C        LAGRANGIAN FORMULA IS USED WHERE N IS ODD.
C
C        L = INTEGER, THE POINT OF X AND Y AT WHICH DERIVATIVE IS FOUND
C        M = INTEGER, 1-5, TO DETERMINE THE POINT FORMULA, N.  N=2*M+1
C        NP= INTEGER, THE NUMBER OF POINTS IN TABLE OF VARIABLES
C        VARI = ARRAY OF INDEPENDENT VARIABLE, X.  VARI(NP)
C        VARD = ARRAY OF DEPENDENT VARIABLE, Y.    VARD(NP)
C
      DIMENSION VARI(NP), VARD(NP), X(11), Y(11)
      DIF=17770000000000000000B
      IF (M.LT.1) RETURN
      N=2*M+1
      IF (M.GT.5.OR.N.GT.NP) RETURN
      M1=M+1
      M2=NP-M+1
      K=L
      IF (L.LE.M1.OR.N.EQ.NP) GO TO 10
      K=M1
      IF (L.LT.M2) GO TO 10
      K=L-(NP-N)
10    MX=L-K
      DO 20 J=1,N
      MJ=MX+J
      X(J)=VARI(MJ)
20    Y(J)=VARD(MJ)
      A=1.
      B=0.
      C=0.
      DO 40 J=1,N
      IF (J.EQ.K) GO TO 40
      P=1.
      DO 30 I=1,N
      IF (I.EQ.J) GO TO 30
      P=P*(X(J)-X(I))
30    CONTINUE
      T=X(K)-X(J)
      B=B+Y(J)/(P*T)
      A=A*T
      C=C+1./T
40    CONTINUE
      DIF=A*B+Y(K)*C
      RETURN
      END
```

```
      SUBROUTINE ONEDIM
C
C     ******************************************************************
C
C     THIS SUBROUTINE CALCULATES THE 1-D INITIAL-DATA SURFACE
C
C     ******************************************************************
C
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
      MN3=0.01
      IF (N1D.EQ.-1.OR.N1D.GT.2) MN3=2.0
      GRGAS=1.0/(RGAS*G)
      NXCK=0
      ACOEF=2.0/(GAMMA+1.0)
      BCOEF=(GAMMA-1.0)/(GAMMA+1.0)
      CCOEF=(GAMMA+1.0)/2.0/(GAMMA-1.0)
      IF (N1D.LT.0) GO TO 20
C
C     OVERALL LOOP
C
      IF (NGCB.NE.0) GO TO 10
      RSTAR=RT
      RSTARS=RT*RT
      GO TO 20
10    RSTAR=YW(LT)-YCB(LT)
      RSTARS=YW(LT)**2-YCB(LT)**2
20    DO 130 L=1,LMAX
      IF (L.EQ.1.AND.N1D.EQ.-1) GO TO 130
      IF (L.EQ.1.AND.N1D.GT.2) GO TO 130
      X=XI+DX*(L-1)
      IF (N1D.LT.0) GO TO 50
      IF (NGCB.NE.0) GO TO 30
      IF (X.LT.XT) GO TO 50
      IF (X.GT.XT) GO TO 40
      MN3=1.0
      GO TO 100
30    IF (L.LT.LT) GO TO 50
      IF (L.GT.LT) GO TO 40
      MN3=1.0
      GO TO 100
40    IF (NXCK.EQ.1) GO TO 50
      IF (N1D.EQ.1.OR.N1D.EQ.3) MN3=1.1
      IF (N1D.EQ.2.OR.N1D.EQ.4) MN3=0.9
      NXCK=1
50    IF (NDIM.EQ.1) GO TO 60
      RAD=YW(L)-YCB(L)
      ARATIO=RAD/RSTAR
      GO TO 70
60    RADS=YW(L)**2-YCB(L)**2
      ARATIO=RADS/RSTARS
C
C     NEWTON-RAPHSON ITERATION LOOP
C
70    DO 90 ITER=1,20
      ABM=ACOEF+BCOEF*MN3*MN3
      ABMC=ABM**CCOEF
      FM=ABMC/MN3-ARATIO
      FPM=ABMC*(2.0*BCOEF*CCOEF/ABM-1.0/(MN3*MN3))
      OMN3=MN3
      MN3=OMN3-FM/FPM
      IF (MN3.GT.1.0.AND.OMN3.LT.1.0) MN3=0.99
      IF (MN3.LT.1.0.AND.OMN3.GT.1.0) MN3=1.01
      IF (MN3.GE.0.0) GO TO 80
      MN3=-MN3
      GO TO 90
80    IF (ABS(MN3-OMN3)/OMN3.LE.0.0005) GO TO 100
90    CONTINUE
      PRINT 140, L
```

```
C
C     FILL IN 2-D ARRAYS LOOP
C
100   DEM=1.0+GAM2*MN3*MN3
      DEMP=DEM**GAM1
      DNXNY=(NXNY(L)-NXNYCB(L))/M1
      DO 120 M=1,MMAX
      P(L,M,1)=PT(M)/DEMP
      TEMP=TT(M)/DEM
      RO(L,M,1)=P(L,M,1)*GRGAS/TEMP
      Q=MN3*SQRT(GAMMA*P(L,M,1)/RO(L,M,1))
      DN=NXNYCB(L)+DNXNY*(M-1)
      DNS=DN*DN
      IF (DNS.EQ.0.0) GO TO 110
      SIGN=1.0
      IF (DN.GT.0.0) SIGN=-1.0
      U(L,M,1)=Q/SQRT(1.0+DNS)
      V(L,M,1)=SIGN*Q/SQRT(1.0+1.0/DNS)
      GO TO 120
110   U(L,M,1)=Q
      V(L,M,1)=0.0
120   CONTINUE
130   CONTINUE
      RETURN
C
140   FORMAT (1H0,10X,93H***** THE 1-D SOLUTION FOR THE INITIAL-DATA SUR
     1FACE FAILED TO CONVERGE IN 20 ITERATIONS AT L=,I2,6H *****)
      END
```

```
      SUBROUTINE MAP(IP,L,M,AL,BE,DE,LD1,AL1,BE1,DE1)
C
C     ****************************************************************
C
C     THIS SUBROUTINE CALCULATES THE MAPPING FUNCTIONS
C
C     ****************************************************************
C
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
      BE=1.0/(YW(L)-YCB(L))
      IF (IP.EQ.0) RETURN
      Y=(M-1)*DY
      AL=BE*(NXNYCB(L)+Y*(NXNY(L)-NXNYCB(L)))
      DE=-BE*Y*XWI(L)
      IF (IP.EQ.1) RETURN
      BE1=1.0/(YW(LD1)-YCB(LD1))
      AL1=BE1*(NXNYCB(LD1)+Y*(NXNY(LD1)-NXNYCB(LD1)))
      DE1=-BE1*Y*XWI(LD1)
      RETURN
      END
```

```
      SUBROUTINE MASFLO(ISURF)
C
C     ****************************************************************
C
C     THIS SUBROUTINE CALCULATES THE INITIAL-DATA OR SOLUTION SURFACE
C     MASS FLOW AND THRUST
C
C     ****************************************************************
C
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
      LC2=LC*LC
      LDUM=LMAX-1
      IF (LT.EQ.LMAX) LT=LMAX-1
      IF (JFLAG.NE.0) LDUM=LJET-1
      IF (ISURF.EQ.1.OR.N1D.EQ.0) GO TO 30
C
C     CALCULATE THE MASS FLOW AND THRUST FOR THE 1-D INITIAL-DATA
C     SURFACE
C
      IF (NDIM.EQ.1) GO TO 10
      AREAI=(YW(1)-YCB(1))/LC2
      AREAT=(YW(LT)-YCB(LT))/LC2
      AREAE=(YW(LDUM)-YCB(LDUM))/LC2
      GO TO 20
10    AREAI=3.141593*(YW(1)**2-YCB(1)**2)/LC2
      AREAT=3.141593*(YW(LT)**2-YCB(LT)**2)/LC2
      AREAE=3.141593*(YW(LDUM)**2-YCB(LDUM)**2)/LC2
20    VMI=SQRT(U(1,1,1)**2+V(1,1,1)**2)
      VMT=SQRT(U(LT,1,1)**2+V(LT,1,1)**2)
      VME=SQRT(U(LDUM,1,1)**2+V(LDUM,1,1)**2)
      MASSI=RO(1,1,1)*VMI*AREAI*G
      MASST=RO(LT,1,1)*VMT*AREAT*G
      MASSE=RO(LDUM,1,1)*VME*AREAE*G
      THRUST=RO(LDUM,1,1)*U(LDUM,1,1)**2*AREAE
      RETURN
C
C     CALCULATE THE MASS FLOW AND THRUST FOR THE 2-D INITIAL-DATA
C     AND SOLUTION SURFACES
C
30    MASSI=0.0
      MASST=0.0
      MASSE=0.0
      THRUST=0.0
      DYI=DY*(YW(1)-YCB(1))
      DYT=DY*(YW(LT)-YCB(LT))
      DYE=DY*(YW(LDUM)-YCB(LDUM))
      ND=1
      IF (ISURF.EQ.1) ND=N3
      DO 60 M=1,M1
      RADI=(M-1)*DYI+YCB(1)
      RADT=(M-1)*DYT+YCB(LT)
      RADE=(M-1)*DYE+YCB(LDUM)
      IF (NDIM.EQ.1) GO TO 40
      AREAI=DYI/LC2
      AREAT=DYT/LC2
      AREAE=DYE/LC2
      GO TO 50
40    AREAI=3.141593*((RADI+DYI)**2-RADI**2)/LC2
      AREAT=3.141593*((RADT+DYT)**2-RADT**2)/LC2
      AREAE=3.141593*((RADE+DYE)**2-RADE**2)/LC2
50    ROUI=(RO(1,M,ND)*U(1,M,ND)+RO(1,M+1,ND)*U(1,M+1,ND))*0.5
      ROUT=(RO(LT,M,ND)*U(LT,M,ND)+RO(LT,M+1,ND)*U(LT,M+1,ND))*0.5
      ROUE=(RO(LDUM,M,ND)*U(LDUM,M,ND)+RO(LDUM,M+1,ND)*U(LDUM,M+1,ND))*0
     1.5
      ROU2E=(RO(LDUM,M,ND)*U(LDUM,M,ND)**2+RO(LDUM,M+1,ND)*U(LDUM,M+1,ND
```

```
     1)**2)*0,5
      MASSI=MASSI+ROUI*AREAI*G
      MASST=MASST+ROUT*AREAT*G
      MASSE=MASSE+ROUE*AREAE*G
      THRUST=THRUST+ROU2E*AREAE
60    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PLOT(TITLE,T,NP)
C
C     ****************************************************************
C
C     THIS SUBROUTINE PLOTS THE VELOCITY VECTORS AND DEPENDENT VARIABLE
C     CONTOUR PLOTS
C
C     ****************************************************************
C
      DIMENSION CQ(81,21), CON(9), XCO(4), YCO(4), TITLE(8)
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
C     GENERATE THE VELOCITY VECTOR PLOT
C
      ND=N3
      IF (N.EQ.0) ND=1
      CALL GETQ (4LKJBN,JNM)
      XL=XI
      XR=XE
      YT=YW(1)
      YB=YCB(1)
      DO 10 L=2,LMAX
      YT=AMAX1(YT,YW(L))
      YB=AMIN1(YB,YCB(L))
   10 CONTINUE
      VV=0.9*DX
      FIYB=916.0
      XD=(XR-XL)/(YT-YB)
      FIR=(1022.0-1022.0/FLOAT(L1)-1.0)/900.0
      IF(XD.LE.FIR) GO TO 20
      FIXL=0.0
      FIXR=1022.0-1022.0/FLOAT(L1)-1.0
      FIYT=916.0-FIXR/XD
      GO TO 30
   20 FIXL=511.0-450.0*XD
      FIXR=511.0+450.0*XD
      FIYT=16.0
   30 XCONV=(FIXR-FIXL)/(XR-XL)
      YCONV=(FIYT-FIYB)/(YT-YB)
      VMAX=0.0
      DO 40 L=1,LMAX
      DO 40 M=1,MMAX
      VMAX=AMAX1(VMAX,ABS(U(L,M,ND)),ABS(V(L,M,ND)))
   40 CONTINUE
      IF (VMAX.LT.1.0E-10) GO TO 60
      DROU=VV/VMAX
      CALL ADV (1)
      DO 50 L=1,LMAX
      IX1=FIXL+(FLOAT(L-1)*DX)*XCONV
      DY=(YW(L)-YCB(L))/FLOAT(MMAX-1)
      DO 50 M=1,MMAX
      IY1=FIYB+(YCB(L)+FLOAT(M-1)*DY-YB)*YCONV
      IX2=FIXL+(FLOAT(L-1)*DX+U(L,M,ND)*DROU)*XCONV
      IY2=FIYB+(YCB(L)+FLOAT(M-1)*DY-YB+V(L,M,ND)*DROU)*YCONV
      CALL DRV (IX1,IY1,IX2,IY2)
      CALL PLT (IX1,IY1,16)
   50 CONTINUE
      CALL LINCNT (59)
      WRITE (7,410)
      WRITE (7,350)JNM,TITLE,NP,T
C
C     GENERATE THE CONTOUR PLOTS
C
   60 I=0
   70 I=I+1
      GO TO (80,100,120,140,340), I
   80 DO 90 L=1,LMAX
      DO 90 M=1,MMAX
      CQ(L,M)=RO(L,M,ND)*G
```

```
90    CONTINUE
      GO TO 160
100   DO 110 L=1,LMAX
      DO 110 M=1,MMAX
      CQ(L,M)=P(L,M,ND)/PC
110   CONTINUE
      GO TO 160
120   DO 130 L=1,LMAX
      DO 130 M=1,MMAX
      CQ(L,M)=P(L,M,ND)/RO(L,M,ND)/RGAS/G-TC
130   CONTINUE
      GO TO 160
140   DO 150 L=1,LMAX
      DO 150 M=1,MMAX
      CQ(L,M)=SQRT((U(L,M,ND)**2+V(L,M,ND)**2)/(GAMMA*P(L,M,ND)/RO(L,M,N
     1D)))
150   CONTINUE
160   QMN=1.0E06
      QMX=-QMN
      DO 170 L=1,LMAX
      DO 170 M=1,MMAX
      QMN=AMIN1(CQ(L,M),QMN)
      QMX=AMAX1(CQ(L,M),QMX)
170   CONTINUE
      XX=QMX-QMN
      DQ=0.1*XX
      DO 180 K=1,9
      CON(K)=QMN+(FLOAT(K))*DQ
180   CONTINUE
      K=9
      CALL ADV (1)
      CALL LINCNT (59)
      GO TO (190,200,210,220), I
190   WRITE (7,360)
      GO TO 230
200   WRITE (7,370)
      GO TO 230
210   WRITE (7,380)
      GO TO 230
220   WRITE (7,390)
230   WRITE (7,400)QMN,QMX,CON(1),CON(K),DQ
      WRITE (7,350)JNM,TITLE,NP,T
      DO 320 L=2,LMAX
      DY=(YW(L-1)-YCB(L-1))/FLOAT(MMAX-1)
      DY1=(YW(L)-YCB(L))/FLOAT(MMAX-1)
      DO 320 M=2,MMAX
      NN=0
      DO 320 KK=1,K
      K1=K2=K3=K4=0
      IF (CQ(L-1,M-1).LE.CON(KK)) K1=1
      IF (CQ(L,M-1).LE.CON(KK)) K2=1
      IF (CQ(L-1,M).LE.CON(KK)) K3=1
      IF (CQ(L,M).LE.CON(KK)) K4=1
      IF (K1*K2*K3*K4.NE.0) GO TO 320
      IF (K1+K2+K3+K4.EQ.0) GO TO 320
      IF (NN.NE.0) GO TO 240
      NN=1
      XCO(1)=XI+FLOAT(L-2)*DX
      XCO(2)=XCO(1)+DX
      XCO(3)=XCO(1)
      XCO(4)=XCO(2)
      YCO(1)=YCB(L-1)+FLOAT(M-2)*DY
      YCO(2)=YCB(L)+FLOAT(M-2)*DY1
      YCO(3)=YCB(L-1)+FLOAT(M-1)*DY
      YCO(4)=YCB(L)+FLOAT(M-1)*DY1
240   LL=0
      IF (K1+K3.NE.1) GO TO 250
      IC1=1 $ IC2=3 $ LP1=L-1 $ MP1=M-1 $ LP2=L-1 $ MP2=M
      ASSIGN 250 TO KR1
      GO TO 280
250   IF (K1+K2.NE.1) GO TO 260
      IC1=1 $ IC2=2 $ LP1=L-1 $ MP1=M-1 $ LP2=L $ MP2=M-1
      ASSIGN 260 TO KR1
      GO TO 280
260   IF (K2+K4.NE.1) GO TO 270
      IC1=2 $ IC2=4 $ LP1=L $ MP1=M-1 $ LP2=L $ MP2=M
      ASSIGN 270 TO KR1
      GO TO 280
270   IF (K3+K4.NE.1) GO TO 320
      IC1=3 $ IC2=4 $ LP1=L-1 $ MP1=M $ LP2=L $ MP2=M
      ASSIGN 320 TO KR1
```

```
280   LL=LL+1
      XX=(CON(KK)-CQ(LP1,MP1))/(CQ(LP2,MP2)-CQ(LP1,MP1))
      IF (LL.EQ.2) GO TO 290
      IX1=FIXL+(XCO(IC1)+XX*(XCO(IC2)-XCO(IC1))-XL)*XCONV
      IY1=FIYB+(YCO(IC1)+XX*(YCO(IC2)-YCO(IC1))-YB)*YCONV
      GO TO KR1,(250,260,270,320)
290   IX2=FIXL+(XCO(IC1)+XX*(XCO(IC2)-XCO(IC1))-XL)*XCONV
      IY2=FIYB+(YCO(IC1)+XX*(YCO(IC2)-YCO(IC1))-YB)*YCONV
      CALL DRV (IX1,IY1,IX2,IY2)
      IF (KK.NE.1) GO TO 300
      CALL PLT (IX1,IY1,35)
300   IF (KK.NE.K) GO TO 310
      CALL PLT (IX1,IY1,24)
310   LL=0
      IF (LP2.NE.L) GO TO 320
      IF (MP2.NE.M-1) GO TO 320
      GO TO 260
320   CONTINUE
C
C     DRAW THE GEOMETRY CONTOURS
C
      DO 330 L=2,LMAX
      IX1=FIXL+(FLOAT(L-2)*DX)*XCONV
      IX2=FIXL+(FLOAT(L-1)*DX)*XCONV
      IY1=FIYB+(YCB(L-1)-YB)*YCONV
      IY2=FIYB+(YCB(L)-YB)*YCONV
      IY3=FIYB+(YW(L-1)-YB)*YCONV
      IY4=FIYB+(YW(L)-YB)*YCONV
      CALL DRV (IX1,IY1,IX2,IY2)
      CALL DRV (IX1,IY3,IX2,IY4)
330   CONTINUE
      GO TO 70
340   CONTINUE
      DY=1.0/FLOAT(MMAX-1)
      CALL ADV (1)
      RETURN
C
350   FORMAT (1H ,A10,4X,8A10,2X,2HN=,I4,2X,2HT=,1PE10.4,4H SEC)
360   FORMAT (1H ,7HDENSITY)
370   FORMAT (1H ,8HPRESSURE)
380   FORMAT (1H ,11HTEMPERATURE)
390   FORMAT (1H ,11HMACH NUMBER)
400   FORMAT (1H ,10HLOW VALUE=,1PE11.4,2X,11HHIGH VALUE=,E11.4,2X,12HLO
     1W CONTOUR=,E11.4,2X,13HHIGH CONTOUR=,E11.4,2X,14HDELTA CONTOUR=,E1
     21.4)
410   FORMAT (1H ,16HVELOCITY VECTORS)
      END
```

```
      SUBROUTINE SHOCK(IPASS)
C
C     ****************************************************************
C
C     THIS SUBROUTINE CALCULATES THE LOCAL ARTIFICAL VISCOSITY TERMS
C     FOR SHOCK COMPUTATIONS
C
C     ****************************************************************
C
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUD,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
      GO TO (10,160), IPASS
C
C     CALCULATE LOCAL ARTIFICAL VISCOSITY FOR SHOCK COMPUTATIONS
C
10    IF (N.NE.1) GO TO 30
      NC=0
      RG=RGAS*G
      CTA1=1.0-CTA
      DO 20 L=1,LMAX
      DO 20 M=1,MMAX
      QUT(L,M)=0.0
      QVT(L,M)=0.0
      QPT(L,M)=0.0
20    CONTINUE
30    RDUM=CAV*DT*DX*DY*2.0
      NC=NC+1
      NLINE=0
      IF (NC.NE.NPRINT) GO TO 40
      PRINT 200
      PRINT 190, N,PC
C
40    DO 150 L=LSS,L1
      DO 150 M=1,MMAX
      LMD2=LD*(M-1)+LMD1
      LMMD2=LD*(M-2)+LMD1
      LMPD2=LD*M+LMD1
      LMN1=L+LMD2
      LP=L+1+LMD2
      LM=L-1+LMD2
      MP=L+LMPD2
      MM=L+LMMD2
      LPMP=L+1+LMPD2
      LPMM=L+1+LMMD2
      LMMP=L-1+LMPD2
      LMMM=L-1+LMMD2
      CALL MAP (1,L,M,AL,BE,DE,LD1,AL1,BE1,DE1)
C
C     CHECK TO SEE IF THE DIVERGENCE OF THE VELOCITY IS NEGATIVE
C
      UX=0.5*(U(LP)-U(LM))*DXR
      IF (M.EQ.1) GO TO 50
      IF (M.EQ.MMAX) GO TO 60
      UY=0.5*(U(MP)-U(MM))*DYR
      VY=0.5*(V(MP)-V(MM))*DYR
      GO TO 70
50    UY=(U(MP)-U(LMN1))*DYR
      VY=(V(MP)-V(LMN1))*DYR
      GO TO 70
60    UY=(U(LMN1)-U(MM))*DYR
      VY=(V(LMN1)-V(MM))*DYR
70    DIV=UX+AL*UY+BE*VY
      IF (DIV.LT.0.0) GO TO 80
      QUT(L,M)=0.0
      QVT(L,M)=0.0
      QPT(L,M)=0.0
      GO TO 150
```

```
80    OQUT=QUT(L,M)
      OQVT=QVT(L,M)
      OQPT=QPT(L,M)
      UX1=(U(LMN1)-U(LM))*DXR
      UX2=(U(LP)-U(LMN1))*DXR
      VX1=(V(LMN1)-V(LM))*DXR
      VX2=(V(LP)-V(LMN1))*DXR
      TM=P(LM)/(RO(LM)*RG)
      T=P(LMN1)/(RO(LMN1)*RG)
      TP=P(LP)/(RO(LP)*RG)
      TX1=(T-TM)*DXR
      TX2=(TP-T)*DXR
      LDUM=L-1
      CALL MAP (1,LDUM,M,ALM,BEM,DE,LD1,AL1,BE1,DE1)
      LDUM=L+1
      CALL MAP (1,LDUM,M,ALP,BEP,DE,LD1,AL1,BE1,DE1)
      BE1=0.5*(BEM+BE)
      BE2=0.5*(BEP+BE)
      AL1=0.5*(ALM+AL)
      AL2=0.5*(ALP+AL)
      IF (M.EQ.1) GO TO 90
      IF (M.EQ.MMAX) GO TO 100
C
C     CALCULATE THE INTERIOR POINT QUANTITIES
C
      UY1=0.25*(U(MP)+U(LMMP)-U(MM)-U(LMMM))*DYR
      UY2=0.25*(U(MP)+U(LPMP)-U(MM)-U(LPMM))*DYR
      VY1=0.25*(V(MP)+V(LMMP)-V(MM)-V(LMMM))*DYR
      VY2=0.25*(V(MP)+V(LPMP)-V(MM)-V(LPMM))*DYR
      UX3=0.25*(U(LP)+U(LPMM)-U(LM)-U(LMMM))*DXR
      UX4=0.25*(U(LP)+U(LPMP)-U(LM)-U(LMMP))*DXR
      VX3=0.25*(V(LP)+V(LPMM)-V(LM)-V(LMMM))*DXR
      VX4=0.25*(V(LP)+V(LPMP)-V(LM)-V(LMMP))*DXR
      VY3=(V(LMN1)-V(MM))*DYR
      VY4=(V(MP)-V(LMN1))*DYR
      UY3=(U(LMN1)-U(MM))*DYR
      UY4=(U(MP)-U(LMN1))*DYR
      TMY=P(MM)/(RO(MM)*RG)
      TPY=P(MP)/(RO(MP)*RG)
      TMM=P(LMMM)/(RO(LMMM)*RG)
      TMP=P(LMMP)/(RO(LMMP)*RG)
      TPM=P(LPMM)/(RO(LPMM)*RG)
      TPP=P(LPMP)/(RO(LPMP)*RG)
      TY1=0.25*(TPY+TMP-TMY-TMM)*DYR
      TY2=0.25*(TPP+TPY-TPM-TMY)*DYR
      TX3=0.25*(TP+TPM-TM-TMM)*DXR
      TX4=0.25*(TPP+TP-TMP-TM)*DXR
      TY3=(T-TMY)*DYR
      TY4=(TPY-T)*DYR
      MDUM=M-1
      CALL MAP (1,L,MDUM,ALMY,BEMY,DE,LD1,AL1,BE1,DE1)
      MDUM=M+1
      CALL MAP (1,L,MDUM,ALPY,BEPY,DE,LD1,AL1,BE1,DE1)
      BE3=0.5*(BEMY+BE)
      BE4=0.5*(BEPY+BE)
      AL3=0.5*(ALMY+AL)
      AL4=0.5*(ALPY+AL)
      GO TO 110
C
C     CALCULATE THE CENTERLINE POINT QUANTITIES
C
90    UY1=0.5*(U(MP)+U(LMMP)-U(LMN1)-U(LM))*DYR
      VY1=0.5*(V(MP)+V(LMMP)-V(LMN1)-V(LM))*DYR
      UY2=0.5*(U(LPMP)+U(MP)-U(LP)-U(LMN1))*DYR
      VY2=0.5*(V(LPMP)+V(MP)-V(LP)-V(LMN1))*DYR
      UX3=VX3=UX4=VX4=0.0
      THEW=ATAN(-NXNYCB(L))
      THE=ATAN(V(MP)/U(MP))
      VMAG=SQRT(U(MP)*U(MP)+V(MP)*V(MP))
      RTHE=2.0*THEW-THE
      UR=VMAG*COS(RTHE)
      VR=VMAG*SIN(RTHE)
      UY3=(U(LMN1)-UR)*DYR
      VY3=(V(LMN1)-VR)*DYR
      UY4=(U(MP)-U(LMN1))*DYR
      VY4=(V(MP)-V(LMN1))*DYR
      TPY=P(MP)/(RO(MP)*RG)
      TMP=P(LMMP)/(RO(LMMP)*RG)
      TPP=P(LPMP)/(RO(LPMP)*RG)
      TY1=0.0
      TY2=0.0
      TX4=0.25*(TPP+TP-TMP-TM)*DXR
```

```
      TX3=TX4
      TY4=(TPY-T)*DYR
      TY3=-TY4
      BE3=BE
      AL3=AL
      MDUM=M+1
      CALL MAP (1,L,MDUM,AL4,BE4,DE,LD1,AL1,BE1,DE1)
      GO TO 110
C
C     CALCULATE THE WALL POINT QUANTITIES
C
100   UY1=0.5*(U(LMN1)+U(LM)-U(MM)-U(LMMM))*DYR
      VY1=0.5*(V(LMN1)+V(LM)-V(MM)-V(LMMM))*DYR
      UY2=0.5*(U(LP)+U(LMN1)-U(LPMM)-U(MM))*DYR
      VY2=0.5*(V(LP)+V(LMN1)-V(LPMM)-V(MM))*DYR
      UX3=VX3=UX4=VX4=0.0
      UY3=(U(LMN1)-U(MM))*DYR
      VY3=(V(LMN1)-V(MM))*DYR
      THEW=ATAN(-NXNY(L))
      THE=ATAN(V(MM)/U(MM))
      VMAG=SQRT(U(MM)*U(MM)+V(MM)*V(MM))
      RTHE=2.0*THEW-THE
      UR=VMAG*COS(RTHE)
      VR=VMAG*SIN(RTHE)
      UY4=(UR-U(LMN1))*DYR
      VY4=(VR-V(LMN1))*DYR
      TPM=P(LPMM)/(RO(LPMM)*RG)
      TMY=P(MM)/(RO(MM)*RG)
      TMM=P(LMMM)/(RO(LMMM)*RG)
      TY1=0.0
      TY2=0.0
      TX3=0.25*(TP+TPM-TM-TMM)*DXR
      TX4=TX3
      TY3=(T-TMY)*DYR
      TY4=-TY3
      MDUM=M-1
      CALL MAP (1,L,MDUM,AL3,BE3,DE,LD1,AL1,BE1,DE1)
      BE4=BE
      AL4=AL
C
110   UXY1=UX1+AL1*UY1
      UXY2=UX2+AL2*UY2
      UXY3=UX3+AL3*UY3
      UXY4=UX4+AL4*UY4
      UXY12=0.5*(UX1+UX2+AL3*UY3+AL4*UY4)
      VXY1=VX1+AL1*VY1
      VXY2=VX2+AL2*VY2
      VXY3=VX3+AL3*VY3
      VXY4=VX4+AL4*VY4
      VXY12=0.5*(VX1+VX2+AL3*VY3+AL4*VY4)
      BUY1=BE1*UY1
      BUY2=BE2*UY2
      BUY3=BE3*UY3
      BUY4=BE4*UY4
      BUY34=0.5*(BUY3+BUY4)
      BVY1=BE1*VY1
      BVY2=BE2*VY2
      BVY3=BE3*VY3
      BVY4=BE4*VY4
      BVY34=0.5*(BVY3+BVY4)
      TXY1=TX1+AL1*TY1
      TXY2=TX2+AL2*TY2
      TXY3=TX3+AL3*TY3
      TXY4=TX4+AL4*TY4
C
C     CALCULATE THE ARTIFICAL VISCOSITY COEFFICIENTS
C
      DIV=UXY12+BVY34
      VID=VXY12+BUY34
      RLA=XLA*RDUM*ABS(DIV)/BE
      RMU=XMU*RDUM*ABS(VID)/BE
      RK=RMU*GAM1*RG/RKMU
      RLP2M=RLA+2.0*RMU
      RLA2=2.0*RLA
      RMU2=2.0*RMU
      RLPM=RLA+RMU
      UVTA=0.0
      VVTA=0.0
      PVTA=0.0
      PCTA=0.0
      IF (NDIM.EQ.0) GO TO 130
```

```
C
C     CALCULATE THE AXISYMMETRIC TERMS
C
      IF (M.EQ.1.AND.YCB(L).EQ.0.0) GO TO 120
      Y=FLOAT(M-1)*DY/BE+YCB(L)
      VB=V(LMN1)
      UVTA=(RLPM*VXY12+RMU*BUY34)/Y
      VVTA=RLP2M*(BVY34-VB/Y)/Y
      PVTA=(RLP2M*VB*VB/Y+RLA2*VB*(BVY34+UXY12))/Y
      PCTA=RK*0.5*(BE4*TY4+BE3*TY3)/Y
      DUVTA=RLPM*BE*(VXY4-VXY3)*DYR+RMU*BE*(BUY4-BUY3)*DYR
      DVVTA=RLP2M*0.5*BE*(BVY4-BVY3)*DYR
      DPVTA=(RLP2M+RLA2)*BVY34*BVY34+RLA2*BVY34*UXY12
      DPCTA=RK*BE*(BE4*TY4-BE3*TY3)*DYR
      IF (ABS(UVTA).GT.ABS(DUVTA)) UVTA=DUVTA
      IF (ABS(VVTA).GT.ABS(DVVTA)) VVTA=DVVTA
      IF (ABS(PVTA).GT.ABS(DPVTA)) PVTA=DPVTA
      IF (ABS(PCTA).GT.ABS(DPCTA)) PCTA=DPCTA
      GO TO 130
  120 UVTA=RLPM*BE*(VXY4-VXY3)*DYR+RMU*BE*(BUY4-BUY3)*DYR
      VVTA=RLP2M*0.5*BE*(BVY4-BVY3)*DYR
      PVTA=(RLP2M+RLA2)*BVY34*BVY34+RLA2*BVY34*UXY12
      PCTA=RK*BE*(BE4*TY4-BE3*TY3)*DYR
C
C     CALCULATE THE ARTIFICAL VISCOSITY TERMS
C
  130 QUT(L,M)=(RLP2M*(UXY2-UXY1)+RLA*(BVY2-BVY1))*DXR+AL*(RLP2M*(UXY4-U
     1XY3)+RLA*(BVY4-BVY3))*DYR+RMU*BE*(VXY4+BUY4-VXY3-BUY3)*DYR+UVTA
      QVT(L,M)=RMU*(VXY2+BUY2-VXY1-BUY1)*DXR+RMU*AL*(VXY4+BUY4-VXY3-BUY3
     1)*DYR+BE*(RLA*(UXY4-UXY3)+RLP2M*(BVY4-BVY3))*DYR+VVTA
      QPT(L,M)=RO(LMN1)*(GAMMA-1.0)*(RLP2M*(UXY12*UXY12+BVY34*BVY34)+RMU
     1*(VXY12*VXY12+BUY34*BUY34)+RLA2*UXY12*BVY34+RMU2*BUY34*VXY12+RK*((
     2TXY2-TXY1)*DXR+AL*(TXY4-TXY3)*DYR+BE*(BE4*TY4-BE3*TY3)*DYR)+PVTA+P
     3CTA)
      QUT(L,M)=CTA*QUT(L,M)+CTA1*OQUT
      QVT(L,M)=CTA*QVT(L,M)+CTA1*OQVT
      QPT(L,M)=CTA*QPT(L,M)+CTA1*OQPT
C
C     PRINT THE ARTIFICAL VISCOSITY TERMS
C
      IF (NC.NE.NPRINT) GO TO 150
      NLINE=NLINE+1
      IF (NLINE.LT.55) GO TO 140
      PRINT 200
      PRINT 190, N,PC
      NLINE=1
  140 QPT(L,M)=QPT(L,M)/PC
      PRINT 180, L,M,QUT(L,M),QVT(L,M),QPT(L,M)
      QPT(L,M)=QPT(L,M)*PC
  150 CONTINUE
      IF (NC.EQ.NPRINT) NC=0
      RETURN
C
C     SMOOTH THE FLOW VARIABLES IF REQUESTED
C
  160 IF (SMP.LT.0.0.OR.SMP.GE.1.0) RETURN
      SMP4=.25*(1.0-SMP)
      DO 170 L=2,L1
      U(L,MMAX,N3)=SMP4*(U(L-1,MMAX,N3)+U(L+1,MMAX,N3)+2.0*U(L,M1,N3))+S
     1MP*U(L,MMAX,N3)
      V(L,MMAX,N3)=-U(L,MMAX,N3)*NXNY(L)
      P(L,MMAX,N3)=SMP4*(P(L-1,MMAX,N3)+P(L+1,MMAX,N3)+2.0*P(L,M1,N3))+S
     1MP*P(L,MMAX,N3)
      RO(L,MMAX,N3)=SMP4*(RO(L-1,MMAX,N3)+RO(L+1,MMAX,N3)+2.0*RO(L,M1,N3
     1))+SMP*RO(L,MMAX,N3)
      U(L,1,N3)=SMP4*(U(L-1,1,N3)+U(L+1,1,N3)+2.0*U(L,2,N3))+SMP*U(L,1,N
     13)
      V(L,1,N3)=-U(L,1,N3)*NXNYCB(L)
      P(L,1,N3)=SMP4*(P(L-1,1,N3)+P(L+1,1,N3)+2.0*P(L,2,N3))+SMP*P(L,1,N
     13)
      RO(L,1,N3)=SMP4*(RO(L-1,1,N3)+RO(L+1,1,N3)+2.0*RO(L,2,N3))+SMP*RO(
     1L,1,N3)
      DO 170 M=2,M1
      LMD2=LD*(M-1)+LMD3
      LMMD2=LD*(M-2)+LMD3
      LMPD2=LD*M+LMD3
      LMN3=L+LMD2
      LP=L+1+LMD2
      LM=L-1+LMD2
      MP=L+LMPD2
      MM=L+LMMD2
      U(LMN3)=SMP4*(U(LM)+U(LP)+U(MM)+U(MP))+SMP*U(LMN3)
      V(LMN3)=SMP4*(V(LM)+V(LP)+V(MM)+V(MP))+SMP*V(LMN3)
```

```
      P(LMN3)=SMP4*(P(LM)+P(LP)+P(MM)+P(MP))+SMP*P(LMN3)
      RO(LMN3)=SMP4*(RO(LM)+RO(LP)+RO(MM)+RO(MP))+SMP*RO(LMN3)
170   CONTINUE
      RETURN
C
180   FORMAT (1H ,5X,2I5,3F14.4)
190   FORMAT (1H ,63HLOCAL ARTIFICAL VISCOSITY PARAMETERS FOR SHOCK CALC
     1ULATIONS, N=I4,5H (PC=,F5.1,1H),//,10X,1HL,4X,1HM,10X,3HQUT,11X,3H
     2QVT,11X,3HQPT,/)
200   FORMAT (1H1)
      END
```

```
      SUBROUTINE INTER
C
C     ****************************************************************
C
C     THIS SUBROUTINE CALCULATES THE INTERIOR MESH POINTS
C
C     ****************************************************************
C
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
      ATERM=0.0
      IF (ICHAR.EQ.2) GO TO 40
C
C     COMPUTE THE TENTATIVE SOLUTION AT T+DT
C
      MDUM=1
      IF (NGCB.NE.0) MDUM=2
      DO 30 L=2,LMAX
      DO 30 M=MDUM,M1
      CALL MAP (1,L,M,AL,BE,DE,LD1,AL1,BE1,DE1)
      LMD2=LD*(M-1)
      LMN1=L+LMD2+LMD1
      LMN3=L+LMD2+LMD3
      L1MN1=L-1+LMD2+LMD1
      LM1N1=L+LD*(M-2)+LMD1
      UB=U(LMN1)
      VB=V(LMN1)
      PB=P(LMN1)
      ROB=RO(LMN1)
      ASB=GAMMA*PB/ROB
      IF (M.NE.1) GO TO 10
      DUDX=(UB-U(L1MN1))*DXR
      DPDX=(PB-P(L1MN1))*DXR
      DRODX=(ROB-RO(L1MN1))*DXR
      DVDY=(4.0*V(L,2,N1)-V(L,3,N1))*0.5*DYR
      V(LMN3)=0.0
C
      URHS=-UB*DUDX-DPDX/ROB
      RORHS=-UB*DRODX-ROB*DUDX-(1+NDIM)*ROB*BE*DVDY
      PRHS=-UB*DPDX+ASB*(RORHS+UB*DRODX)
      GO TO 20
10    IF (NDIM.EQ.1) ATERM=ROB*VB/((M-1)*DY/BE+YCB(L))
      UVB=UB*AL+VB*BE+DE
      DUDX=(UB-U(L1MN1))*DXR
      DVDX=(VB-V(L1MN1))*DXR
      DPDX=(PB-P(L1MN1))*DXR
      DRODX=(ROB-RO(L1MN1))*DXR
      DUDY=(UB-U(LM1N1))*DYR
      DVDY=(VB-V(LM1N1))*DYR
      DPDY=(PB-P(LM1N1))*DYR
      DRODY=(ROB-RO(LM1N1))*DYR
C
      URHS=-UB*DUDX-UVB*DUDY-(DPDX+AL*DPDY)/ROB
      VRHS=-UB*DVDX-UVB*DVDY-BE*DPDY/ROB
      RORHS=-UB*DRODX-UVB*DRODY-ROB*(DUDX+AL*DUDY+BE*DVDY)-ATERM
      PRHS=-UB*DPDX-UVB*DPDY+ASB*(RORHS+UB*DRODX+UVB*DRODY)
      V(LMN3)=V(LMN1)+VRHS*DT
20    U(LMN3)=U(LMN1)+URHS*DT
      P(LMN3)=P(LMN1)+PRHS*DT
      RO(LMN3)=RO(LMN1)+RORHS*DT
      IF (P(LMN3).LE.0.0) P(LMN3)=PLOW*PC
      IF (RO(LMN3).LE.0.0) RO(LMN3)=ROLOW/G
30    CONTINUE
      RETURN
C
C     COMPUTE THE FINAL SOLUTION AT T+DT
C
```

```
40    MDUM=1
      IF (NGCB.NE.0) MDUM=2
      DO 70 L=2,L1
      DO 70 M=MDUM,M1
      CALL MAP (1,L,M,AL,BE,DE,LD1,AL1,BE1,DE1)
      LMD2=LD*(M-1)
      LMN1=L+LMD2+LMD1
      LMN3=L+LMD2+LMD3
      L1MN3=L+1+LMD2+LMD3
      LM1N3=L+LD*M+LMD3
      UB=U(LMN3)
      VB=V(LMN3)
      PB=P(LMN3)
      ROB=RO(LMN3)
      ASB=GAMMA*PB/ROB
      IF (M.NE.1) GO TO 50
      DUDX=(U(L1MN3)-UB)*DXR
      DPDX=(P(L1MN3)-PB)*DXR
      DRODX=(RO(L1MN3)-ROB)*DXR
      DVDY=(4.0*V(L,2,N3)-V(L,3,N3))*0.5*DYR
      V(LMN3)=0.0
C
      URHS=-UB*DUDX-DPDX/ROB
      RORHS=-UB*DRODX-ROB*DUDX-(1+NDIM)*ROB*BE*DVDY
      PRHS=-UB*DPDX+ASB*(RORHS+UB*DRODX)
      GO TO 60
50    IF (NDIM.EQ.1) ATERM=ROB*VB/((M-1)*DY/BE+YCB(L))
      UVB=UB*AL+VB*BE+DE
      DUDX=(U(L1MN3)-UB)*DXR
      DVDX=(V(L1MN3)-VB)*DXR
      DPDX=(P(L1MN3)-PB)*DXR
      DRODX=(RO(L1MN3)-ROB)*DXR
      DUDY=(U(LM1N3)-UB)*DYR
      DVDY=(V(LM1N3)-VB)*DYR
      DPDY=(P(LM1N3)-PB)*DYR
      DRODY=(RO(LM1N3)-ROB)*DYR
C
      URHS=-UB*DUDX-UVB*DUDY-(DPDX+AL*DPDY)/ROB
      VRHS=-UB*DVDX-UVB*DVDY-BE*DPDY/ROB
      RORHS=-UB*DRODX-UVB*DRODY-ROB*(DUDX+AL*DUDY+BE*DVDY)-ATERM
      PRHS=-UB*DPDX-UVB*DPDY+ASB*(RORHS+UB*DRODX+UVB*DRODY)
      V(LMN3)=(V(LMN1)+V(LMN3)+VRHS*DT)*0.5
60    U(LMN3)=(U(LMN1)+U(LMN3)+URHS*DT)*0.5
      P(LMN3)=(P(LMN1)+P(LMN3)+PRHS*DT)*0.5
      RO(LMN3)=(RO(LMN1)+RO(LMN3)+RORHS*DT)*0.5
      IF (P(LMN3).LE.0.0) P(LMN3)=PLOW*PC
      IF (RO(LMN3).LE.0.0) RO(LMN3)=ROLOW/G
      IF (IAV.EQ.0) GO TO 70
C
C     ADD THE ARTIFICAL VISCOSITY FOR SHOCK CALCULATIONS
C
      U(LMN3)=U(LMN3)+QUT(L,M)
      V(LMN3)=V(LMN3)+QVT(L,M)
      IF (M.EQ.1) V(LMN3)=0.0
      P(LMN3)=P(LMN3)+QPT(L,M)
70    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE WALL
C
C     ****************************************************************
C
C     THIS SUBROUTINE CALCULATES THE BOUNDARY MESH POINTS AT THE NOZZLE
C     WALL, EXHAUST JET BOUNDARY, AND CENTERBODY
C
C     ****************************************************************
C
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(8
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
      IF (N.EQ.1) DELY=0.005
      XWID=0.0
      IF (IB.EQ.1) GO TO 10
      Y1=0.0 $ Y3=0.0 $ MDUM=1 $ MDUM1=2 $ SIGN=-1.0
      GO TO 20
10    Y1=1.0 $ Y3=1.0 $ MDUM=MMAX $ MDUM1=M1 $ SIGN=1.0
20    ATERM2=0.0
      ATERM3=0.0
      LDUM=LMAX
      IF (ICHAR.EQ.2) LDUM=L1
      LMDM=LD*(MDUM-1)
      LMDM1=LD*(MDUM1-1)
      DYS=SIGN*DYR
      DO 350 L=2,LDUM
      LMN1=L+LMDM+LMD1
      LMN3=L+LMDM+LMD3
      LM1N1=L+LMDM1+LMD1
      L1MN1=L-1+LMDM+LMD1
      L1MN3=L+1+LMDM+LMD3
      L1M1N1=L-1+LMDM1+LMD1
      IF (JFLAG.EQ.0) GO TO 50
      IF (IB.EQ.2) GO TO 50
C
      XWID=XWI(L)
      IF (ICHAR.EQ.1) GO TO 30
C
C     USE THE DUMMY ARRAYS TO MANIPULATE THE ONE-SIDED SOLUTIONS
C
      IF (L.NE.LJET-2) GO TO 30
      U(L1MN3)=UD(3)
      V(L1MN3)=VD(3)
      P(L1MN3)=PD(3)
      RO(L1MN3)=ROD(3)
      GO TO 50
30    IF (L.NE.LJET-1) GO TO 40
      IF (ICHAR.EQ.1) UOLD=U(LMN1)
      U(LMN1)=UD(1)
      V(LMN1)=VD(1)
      P(LMN1)=PD(1)
      RO(LMN1)=ROD(1)
      GO TO 50
40    IF (L.NE.LJET) GO TO 50
      U(L1MN1)=UD(2)
      V(L1MN1)=VD(2)
      P(L1MN1)=PD(2)
      RO(L1MN1)=ROD(2)
C
50    U1=U(LMN1)
      V1=V(LMN1)
      P1=P(LMN1)
      RO1=RO(LMN1)
      U2=U1
      V2=V1
      A1=SQRT(GAMMA*P1/RO1)
      A2=A1
      IF (ICHAR.EQ.2) GO TO 60
```

```
      U3=U1
      V3=V1
      P3=P1
      RO3=RO1
      A3=A1
      GO TO 70
60    U3=U(LMN3)
      V3=V(LMN3)
      P3=P(LMN3)
      RO3=RO(LMN3)
      A3=SQRT(GAMMA*P3/RO3)
C
C     CALCULATE THE PROPERTY INTERPOLATING POLYNOMIAL COEFFICIENTS
C
70    BU=(U1-U(LM1N1))*DYS
      BV=(V1-V(LM1N1))*DYS
      BP=(P1-P(LM1N1))*DYS
      BRO=(RO1-RO(LM1N1))*DYS
      CU=U1-BU*Y3
      CV=V1-BV*Y3
      CP=P1-BP*Y3
      CRO=RO1-BRO*Y3
C
C     CALCULATE THE CROSS DERIVATIVE INTERPOLATING POLYNOMIAL
C     COEFFICIENTS
C
      DU=(U1-U(L1MN1))*DXR
      DV=(V1-V(L1MN1))*DXR
      DP=(P1-P(L1MN1))*DXR
      DRO=(RO1-RO(L1MN1))*DXR
      DU1=(U(LM1N1)-U(L1M1N1))*DXR
      DV1=(V(LM1N1)-V(L1M1N1))*DXR
      DP1=(P(LM1N1)-P(L1M1N1))*DXR
      DRO1=(RO(LM1N1)-RO(L1M1N1))*DXR
      BDU=(DU-DU1)*DYS
      BDV=(DV-DV1)*DYS
      BDP=(DP-DP1)*DYS
      BDRO=(DRO-DRO1)*DYS
      CDU=DU-BDU*Y3
      CDV=DV-BDV*Y3
      CDP=DP-BDP*Y3
      CDRO=DRO-BDRO*Y3
C
C     CALCULATE Y2
C
      CALL MAP (1,L,MDUM,AL,BE,DE,LD1,AL1,BE1,DE1)
      ALS=SQRT(AL*AL+BE*BE)
      UV3=U3*AL+V3*BE+DE
      AL2=AL
      DO 90 ILL=1,3
      UV2=U2*AL2+V2*BE+DE
      Y2=Y3-(UV2+SIGN*ALS*A2+UV3+SIGN*ALS*A3)*DT*0.5
C
C     INTERPOLATE FOR THE PROPERTIES
C
      U2=BU*Y2+CU
      V2=BV*Y2+CV
      P2=BP*Y2+CP
      RO2=BRO*Y2+CRO
      AL2=Y2*AL
      AD=GAMMA*P2/RO2
      IF (AD.GT.0.0) GO TO 80
      PRINT 360, N,L,MDUM
      IERR=1
      RETURN
80    A2=SQRT(AD)
90    CONTINUE
C
C     INTERPOLATE FOR THE CROSS DERIVATIVES
C
      DU1=DU
      DV1=DV
      DP1=DP
      DRO1=DRO
      DU2=BDU*Y2+CDU
      DV2=BDV*Y2+CDV
      DP2=BDP*Y2+CDP
      DRO2=BDRO*Y2+CDRO
C
C     CALCULATE THE PSI TERMS
C
      IF (NDIM.EQ.0) GO TO 110
```

```
      IF (IB.EQ.2) GO TO 100
      ATERM2=RO2*V2/(YCB(L)+Y2/BE)
      GO TO 110
100   ATERM2=RO2*V2/(YCB(L)+Y2/BE)
      IF (IAV.EQ.0) GO TO 110
      ATDS=RO2*V(L,2,N1)*DYR*BE
      IF (ABS(ATERM2).GT.ABS(ATDS)) ATERM2=ATDS
C
110   PSI21=-U1*DU1-DP1/RO1
      PSI31=-U1*DV1
      PSI41=-U1*DP1+A1*A1*U1*DRO1
      PSI12=-U2*DRO2-RO2*DU2-ATERM2
      PSI22=-U2*DU2-DP2/RO2
      PSI32=-U2*DV2
      PSI42=-U2*DP2+A2*A2*U2*DRO2
      IF (ICHAR.EQ.1) GO TO 150
C
C     CALCULATE THE CROSS DERIVATIVES AT THE SOLUTION POINT
C
      IF (JFLAG.EQ.0) GO TO 120
      IF (IB.EQ.2) GO TO 120
      IF (L.EQ.2) GO TO 120
      IF (L.NE.LJET-1) GO TO 120
      IF (ILJET.EQ.2) GO TO 120
      GO TO 130
120   DU3=(U(L1MN3)-U3)*DXR
      DV3=(V(L1MN3)-V3)*DXR
      DP3=(P(L1MN3)-P3)*DXR
      DRO3=(RO(L1MN3)-RO3)*DXR
      GO TO 140
130   DU3=(U3-U(L-1,MDUM,N3))*DXR
      DV3=(V3-V(L-1,MDUM,N3))*DXR
      DP3=(P3-P(L-1,MDUM,N3))*DXR
      DRO3=(RO3-RO(L-1,MDUM,N3))*DXR
C
C     ENTER THE EXHAUST JET ITERATION LOOP
C
140   IF (JFLAG.EQ.0) GO TO 150
      IF (IB.EQ.2) GO TO 150
      IF (L.LT.LJET) GO TO 150
      YWI(L)=YW(L)
      UDUM=U(LMN3)
      VDUM=V(LMN3)
      PDUM=P(LMN3)
      RODUM=RO(LMN3)
150   DO 290 NJ=1,10
      IF (ICHAR.EQ.1) GO TO 250
      IF (JFLAG.EQ.0) GO TO 210
      IF (IB.EQ.2) GO TO 210
      IF (L.LT.LJET) GO TO 210
      IF (NJ.EQ.1) GO TO 200
      IF (NJ.GT.2) GO TO 180
160   YWOLD=YW(L)
      POLD=P(LMN3)
      IF (P(LMN3).LT.PE) GO TO 170
      YW(L)=YW(L)+DELY
      GO TO 190
170   YW(L)=YW(L)-DELY
      GO TO 190
180   IF (P(LMN3).EQ.POLD) GO TO 160
      DYDP=(YW(L)-YWOLD)/(P(LMN3)-POLD)
      YWNEW=YW(L)+DYDP*(PE-P(LMN3))
      YWOLD=YW(L)
      POLD=P(LMN3)
      YW(L)=YWNEW
190   IF (YW(L).LT.(0.98*YWOLD)) YW(L)=0.98*YWOLD
      IF (YW(L).GT.(1.02*YWOLD)) YW(L)=1.02*YWOLD
200   NXNY(L)=-(YW(L)-YW(L-1))*DXR
      XWI(L)=(YW(L)-YWI(L))/DT
      XWID=XWI(L)
      CALL MAP (1,L,MMAX,AL,BE,DE,LD1,AL1,BE1,DE1)
      ALS=SQRT(AL*AL+BE*BE)
      U(LMN3)=UDUM
      V(LMN3)=VDUM
      P(LMN3)=PDUM
      RO(LMN3)=RODUM
C
C     CALCULATE THE PSI TERMS AT THE SOLUTION POINT
C
210   IF (NDIM.EQ.0) GO TO 240
      IF (IB.EQ.2) GO TO 220
      ATERM3=RO3*V3/(YCB(L)+1.0/BE)
      GO TO 240
```

```
220   IF (YCB(L).EQ.0.0) GO TO 230
      ATERM3=RO3*V3/YCB(L)
      IF (IAV.EQ.0) GO TO 240
      ATDS=RO3*V(L,2,N3)*DYR*BE
      IF (ABS(ATERM3).GT.ABS(ATDS)) ATERM3=ATDS
      GO TO 240
230   ATERM3=RO3*V(L,2,N3)*DYR*BE
C
240   PSI13=-U3*DRO3-RO3*DU3-ATERM3
      PSI23=-U3*DU3-DP3/RO3
      PSI33=-U3*DV3
      PSI43=-U3*DP3+A3*A3*U3*DRO3
C
C     CALCULATE THE COMPATIBILITY EQUATION COEFFICIENTS
C
250   ABR=NXNY(L)
      IF (IB.EQ.2) ABR=NXNYCB(L)
      ALB=0.5*(AL2+AL)/ALS
      BEB=BE/ALS
      A1B=(A1+A3)*0.5
      A2B=(A2+A3)*0.5
      RO1B=(RO1+RO3)*0.5
      RO2B=(RO2+RO3)*0.5
      IF (ICHAR.EQ.1) GO TO 260
      PSI21B=(PSI21+PSI23)*0.5
      PSI31B=(PSI31+PSI33)*0.5
      PSI41B=(PSI41+PSI43)*0.5
      PSI12B=(PSI12+PSI13)*0.5
      PSI22B=(PSI22+PSI23)*0.5
      PSI32B=(PSI32+PSI33)*0.5
      PSI42B=(PSI42+PSI43)*0.5
      GO TO 270
260   PSI21B=PSI21
      PSI31B=PSI31
      PSI41B=PSI41
      PSI12B=PSI12
      PSI22B=PSI22
      PSI32B=PSI32
      PSI42B=PSI42
C
C     SOLVE THE COMPATIBILITY EQUATIONS FOR U,V,P, AND RO
C
270   U(LMN3)=(U(LMN1)-ABR*(V(LMN1)-XWID)+(PSI21B-ABR*PSI31B)*DT)/(1.0+A
     1BR*ABR)
      V(LMN3)=-U(LMN3)*ABR+XWID
      P(LMN3)=P2-SIGN*RO2B*A2B*(ALB*(U(LMN3)-U2)+BEB*(V(LMN3)-V2))+(PSI4
     12B+A2B*A2B*PSI12B+SIGN*RO2B*A2B*(ALB*PSI22B+BEB*PSI32B))*DT
      IF (P(LMN3).LE.0.0) P(LMN3)=PLOW*PC
      RO(LMN3)=RO(LMN1)+(P(LMN3)-P(LMN1)-PSI41B*DT)/(A1B*A1B)
      IF (RO(LMN3).LE.0.0) RO(LMN3)=ROLOW/G
      IF (IAV.EQ.0) GO TO 280
C
C     ADD THE ARTIFICAL VISCOSITY FOR SHOCK CALCULATIONS
C
      IF (ICHAR.EQ.1) GO TO 280
      U(LMN3)=U(LMN3)+(QUT(L,MDUM)-ABR*QVT(L,MDUM))/(1.0+ABR*ABR)
      V(LMN3)=-U(LMN3)*ABR
      P(LMN3)=P(LMN3)+QPT(L,MDUM)
C
280   IF (JFLAG.EQ.0) GO TO 350
      IF (IB.EQ.2) GO TO 350
      IF (L.LT.LJET-1) GO TO 350
      IF (L.EQ.LJET-1) GO TO 300
      IF (ICHAR.EQ.1) GO TO 350
      DELP=ABS((P(LMN3)-PE)/PE)
      IF (DELP.LE.0.001) GO TO 350
290   CONTINUE
      GO TO 350
C
C     SOLVE THE COMPATIBILITY EQUATIONS FOR THE DOWNSTREAM SIDE OF THE
C     NOZZLE WALL EXIT POINT
C
300   UD(3)=U(LMN3)
      VD(3)=V(LMN3)
      PD(3)=P(LMN3)
      ROD(3)=RO(LMN3)
      PD(4)=PE
      XM1=SQRT((UD(3)*UD(3)+VD(3)*VD(3))/(GAMMA*PD(3)/ROD(3)))
      DUMD=1.0+GAM2*XM1*XM1
      TD=PD(3)/ROD(3)/RGAS/G
      TTD=TD*DUMD
      IF (PE.GT.PD(3).AND.XM1.GE.1.0) GO TO 310
      TTD=TD*DUMD
```

```
      IF (PE.GT.PD(3).AND.XM1.GE.1.0) GO TO 310
      PTD=PD(3)*DUMD**GAM1
      ROD(4)=ROD(3)*(PE/PD(3))**(1.0/GAMMA)
      GO TO 320
310   PRD=PE/PD(3)
      GAMD=(GAMMA+1.0)/(GAMMA-1.0)
      ROD(4)=ROD(3)*(GAMD*PRD+1.0)/(PRD+GAMD)
320   TE=PE/ROD(4)/RGAS/G
      XMACH=SQRT((TTD/TE-1.0)/GAM2)
      SS=SQRT(GAMMA*PE/ROD(4))
      VMAG=XMACH*SS
      UD(4)=VMAG/SQRT(1.0+NXNY(LJET)*NXNY(LJET))
      VD(4)=-UD(4)*NXNY(LJET)
C
C     AVERAGE THE 1-SIDED MACH NOS FOR THE INTERIOR POINT CALCULATIONS
C
      XM2=SQRT((UD(4)*UD(4)+VD(4)*VD(4))/(GAMMA*PD(4)/ROD(4)))
      IF (XM1.GE.1.0) GO TO 350
      XMB=(XM1+XM2)/2.0
      IF (XMB.GE.1.0) GO TO 330
      DPL=1.0
      DPR=1.0
      GO TO 340
330   DPL=XM2-1.0
      DPR=1.0-XM1
      XMB=1.0
340   DPLR=DPR+DPL
      DUM=1.0+GAM2*XMB*XMB
      TEMP=TTD/DUM
      P(LMN3)=PTD/DUM**GAM1
      RO(LMN3)=P(LMN3)/(RGAS*TEMP*G)
      QA=SQRT(2.0*GAM1*(RGAS*TTD*G-P(LMN3)/RO(LMN3)))
      DNXNY=(DPR*NXNY(LJET)+DPL*NXNY(L))/DPLR
      U(LMN3)=QA/SQRT(1.0+DNXNY*DNXNY)
      V(LMN3)=-U(LMN3)*DNXNY
      IF (ICHAR.EQ.1) GO TO 350
      UD(1)=UD(3)
      VD(1)=VD(3)
      PD(1)=PD(3)
      ROD(1)=ROD(3)
      UD(2)=UD(4)
      VD(2)=VD(4)
      PD(2)=PD(4)
      ROD(2)=ROD(4)
350   CONTINUE
      IF (JFLAG.EQ.0) RETURN
      IF (IB.EQ.2) RETURN
      IF (ICHAR.EQ.1) RETURN
      U(LJET-1,MMAX,N1)=UOLD
      YWI(LMAX)=YW(LMAX)
      YW(LMAX)=2.0*YW(L1)-YW(L2)
      NXNY(LMAX)=-(YW(LMAX)-YW(L1))*DXR
      XWI(LMAX)=(YW(LMAX)-YWI(LMAX))/DT
      DELY=ABS(YW(LJET)-YWI(LJET))
      IF (DELY.EQ.0.0) DELY=0.0001
      RETURN
C
360   FORMAT (1H0,61H***** A NEGATIVE SQUARE ROOT OCCURED IN SUBROUTINE
     1WALL AT N=,I4,4H, L=,I2,8H, AND M=,I2,6H *****)
      END
```

```
      SUBROUTINE INLET
C
C     ******************************************************************
C
C     THIS SUBROUTINE CALCULATES THE BOUNDARY MESH POINTS AT THE NOZZLE
C     INLET FOR SUBSONIC FLOW
C
C     ******************************************************************
C
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1STAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
      GRGB=GAMMA*RGAS*G
      X3=XI
      ATERM2=0.0
      ATERM3=0.0
      DO 180 ICHAR=1,2
      DO 180 M=1,MMAX
      LMN1=1+LD*(M-1)+LMD1
      LMN3=1+LD*(M-1)+LMD3
      L1MN1=2+LD*(M-1)+LMD1
      L1M1N1=2+LD*(M-2)+LMD1
      LM1N1=1+LD*(M-2)+LMD1
      LM1N3=1+LD*M+LMD3
      CALL MAP (2,1,M,AL,BE,DE,2,AL1,BE1,DE1)
      U2=U(LMN1)
      A2=SQRT(GAMMA*P(LMN1)/RO(LMN1))
      IF (ICHAR.EQ.2) GO TO 10
      U(LMN3)=U2
      V(LMN3)=V(LMN1)
      A3=A2
C
C     CALCULATE THE PROPERTY INTERPOLATING POLYNOMIAL COEFFICIENTS
C
10    BU=(U(L1MN1)-U(LMN1))*DXR
      BV=(V(L1MN1)-V(LMN1))*DXR
      BP=(P(L1MN1)-P(LMN1))*DXR
      BRO=(RO(L1MN1)-RO(LMN1))*DXR
      BYCB=(YCB(2)-YCB(1))*DXR
      BAL=(AL1-AL)*DXR
      BBE=(BE1-BE)*DXR
      CU=U(1,M,N1)-BU*X3
      CV=V(1,M,N1)-BV*X3
      CP=P(1,M,N1)-BP*X3
      CRO=RO(1,M,N1)-BRO*X3
      CYCB=YCB(1)-BYCB*X3
      CAL=AL-BAL*X3
      CBE=BE-BBE*X3
C
C     CALCULATE THE CROSS DERIVATIVE INTERPOLATING POLYNOMIAL
C     COEFFICIENTS
C
      IF (M.EQ.1) GO TO 20
      DU=(U(L1MN1)-U(L1M1N1))*DYR
      DV=(V(L1MN1)-V(L1M1N1))*DYR
      DP=(P(L1MN1)-P(L1M1N1))*DYR
      DRO=(RO(L1MN1)-RO(L1M1N1))*DYR
      DU1=(U(LMN1)-U(LM1N1))*DYR
      DV1=(V(LMN1)-V(LM1N1))*DYR
      DP1=(P(LMN1)-P(LM1N1))*DYR
      DRO1=(RO(LMN1)-RO(LM1N1))*DYR
      GO TO 40
20    IF (NGCB.NE.0) GO TO 30
      DU=0.0
      DV=V(2,2,N1)*DYR
      DP=0.0
      DRO=0.0
      DU1=0.0
      DV1=V(1,2,N1)*DYR
```

```
      DP1=0.0
      DRO1=0.0
      GO TO 40
30    DU=(U(2,2,N1)-U(2,1,N1))*DYR
      DV=(V(2,2,N1)-V(2,1,N1))*DYR
      DP=(P(2,2,N1)-P(2,1,N1))*DYR
      DRO=(RO(2,2,N1)-RO(2,1,N1))*DYR
      DU1=(U(1,2,N1)-U(1,1,N1))*DYR
      DV1=(V(1,2,N1)-V(1,1,N1))*DYR
      DP1=(P(1,2,N1)-P(1,1,N1))*DYR
      DRO1=(RO(1,2,N1)-RO(1,1,N1))*DYR
40    BDU=(DU-DU1)*DXR
      BDV=(DV-DV1)*DXR
      BDP=(DP-DP1)*DXR
      BDRO=(DRO-DRO1)*DXR
      CDU=DU1-BDU*X3
      CDV=DV1-BDV*X3
      CDP=DP1-BDP*X3
      CDRO=DRO1-BDRO*X3
C
C     CALCULATE X2
C
      IF (ICHAR.EQ.2) A3=SQRT(GAMMA*P(LMN3)/RO(LMN3))
      DO 50 IL=1,2
      X2=X3-(U(1,M,N3)-A3+U2-A2)*0.5*DT
C
C     INTERPOLATE FOR THE PROPERTIES
C
      U2=BU*X2+CU
      P2=BP*X2+CP
      RO2=BRO*X2+CRO
      A2=SQRT(GAMMA*P2/RO2)
50    CONTINUE
      V2=BV*X2+CV
      YCB2=BYCB*X2+CYCB
      AL2=BAL*X2+CAL
      BE2=BBE*X2+CBE
      UV2=U2*AL2+V2*BE2
C
C     INTERPOLATE FOR THE CROSS DERIVATIVES
C
      DU2=BDU*X2+CDU
      DV2=BDV*X2+CDV
      DP2=BDP*X2+CDP
      DRO2=BDRO*X2+CDRO
C
C     CALCULATE THE PSI TERMS
C
      IF (NDIM.EQ.0) GO TO 70
      IF (M.EQ.1.AND.NGCB.EQ.0) GO TO 60
      ATERM2=RO2*V2/(DY*(M-1)/BE2+YCB2)
      GO TO 70
60    ATERM2=RO2*BE2*DV2
70    PSI12=-UV2*DRO2-RO2*AL2*DU2-RO2*BE2*DV2-ATERM2
      PSI22=-UV2*DU2-AL2*DP2/RO2
      PSI42=-UV2*DP2+A2*A2*UV2*DRO2
      IF (ICHAR.EQ.1) GO TO 130
C
C     CALCULATE THE CROSS DERIVATIVES AT THE SOLUTION POINT
C
      IF (M.EQ.1.AND.NGCB.EQ.0) GO TO 80
      IF (M.EQ.MMAX) GO TO 90
      DU3=(U(LM1N3)-U(LMN3))*DYR
      DV3=(V(LM1N3)-V(LMN3))*DYR
      DP3=(P(LM1N3)-P(LMN3))*DYR
      DRO3=(RO(LM1N3)-RO(LMN3))*DYR
      GO TO 100
80    DU3=0.0
      DV3=V(1,2,N3)*DYR
      DP3=0.0
      DRO3=0.0
      GO TO 100
90    DU3=(U(1,MMAX,N3)-U(1,M1,N3))*DYR
      DV3=(V(1,MMAX,N3)-V(1,M1,N3))*DYR
      DP3=(P(1,MMAX,N3)-P(1,M1,N3))*DYR
      DRO3=(RO(1,MMAX,N3)-RO(1,M1,N3))*DYR
C
C     CALCULATE THE PSI TERMS AT THE SOLUTION POINT
C
100   IF (NDIM.EQ.0) GO TO 120
      IF (M.EQ.1.AND.NGCB.EQ.0) GO TO 110
      ATERM3=RO(LMN3)*V(LMN3)/(DY*(M-1)/BE+YCB(1))
      GO TO 120
```

```
110   ATERM3=RO(LMN3)*BE*DV3
120   UV3=U(LMN3)*AL+V(LMN3)*BE
      PSI13=-UV3*DRO3-RO(LMN3)*AL*DU3-RO(LMN3)*BE*DV3-ATERM3
      PSI23=-UV3*DU3-AL*DP3/RO(LMN3)
      PSI43=-UV3*DP3+A3*A3*UV3*DRO3
      GO TO 140
130   PSI23=PSI22
      PSI43=PSI42
      PSI13=PSI12
C
C     SOLVE THE COMPATIBILITY EQUATIONS FOR U,V,P, AND RO
C
140   MN3=SQRT(U(LMN3)*U(LMN3)+V(LMN3)*V(LMN3))/A3
      T2=P2/(RO2*RGAS*G)
      PSI1B=(PSI12+PSI13)*0.5
      PSI2B=(PSI22+PSI23)*0.5
      PSI4B=(PSI42+PSI43)*0.5
      GPSI1B=GAMMA*PSI1B
      TTHETA=TAN(THETA(M))
      UCORR=0.5+0.5/SQRT(1.0+TTHETA*TTHETA)
C
      DO 160 ITER=1,20
      DEM=(1.0+GAM2*MN3*MN3)
      P(LMN3)=PT(M)/(DEM**GAM1)
      T3=TT(M)/DEM
      PB=(P2+P(LMN3))*0.5
      RTB=RGAS*(T2+T3)*0.5*G
      U(LMN3)=U2+DT*PSI2B+(P(LMN3)-P2-(PSI4B+RTB*GPSI1B)*DT)*SQRT(RTB/GA
     1MMA)/PB
      U(LMN3)=U(LMN3)*UCORR
      V(LMN3)=-U(LMN3)*TTHETA
      OMN3=MN3
      MN3=SQRT((U(LMN3)*U(LMN3)+V(LMN3)*V(LMN3))/(T3*GRGB))
      IF (OMN3.NE.0.0) GO TO 150
      IF (ABS(MN3-OMN3).LE.0.0001) GO TO 170
      GO TO 160
150   IF (ABS((MN3-OMN3)/OMN3).LE.0.001) GO TO 170
160   CONTINUE
C
      PRINT 190, M,N
170   RO(LMN3)=P(LMN3)/(RGAS*T3*G)
180   CONTINUE
      RETURN
C
190   FORMAT (1H0,58H***** THE SOLUTION FOR NOZZLE ENTRANCE BOUNDARY POI
     1NT ( 1,,I2,1H,,I4,43H) FAILED TO CONVERGE IN 20 ITERATIONS *****)
      END
```

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      SUBROUTINE EXITT
C
C     ******************************************************************
C
C     THIS SUBROUTINE CALCULATES THE BOUNDARY MESH POINTS AT THE NOZZLE
C     EXIT FOR SUBSONIC FLOW
C
C     ******************************************************************
C
      COMMON /AV/ IAV,CAV,NST,SMP,LSS,CTA,XMU,XLA,RKMU,QUT(81,21),QVT(81
     1,21),QPT(81,21)
      COMMON /ONESID/ UD(4),VD(4),PD(4),ROD(4)
      COMMON /SOLUTN/ U(81,21,2),V(81,21,2),P(81,21,2),RO(81,21,2)
      COMMON /CNTRLC/ LMAX,MMAX,NMAX,NPRINT,TCONV,FDT,GAMMA,RGAS,GAM1,GA
     1M2,L1,L2,L3,M1,M2,DX,DY,DT,N,N1,N3,NASM,IVEL,ICHAR,N1D,LJET,JFLAG,
     2IERR,IUI,IUO,DXR,DYR,LD,MD,LMD1,LMD3,IB,RSTAR,RSTARS,NPLOT,G,PC,TC
     3,LC,PLOW,ROLOW
      COMMON /GEMTRYC/ NGEOM,XI,RI,XT,RT,XE,RE,RCI,RCT,ANGI,ANGE,XW(81),
     1YW(81),XWI(81),YWI(81),NXNY(81),NWPTS,IINT,IDIF,LT,NDIM
      COMMON /GCB/ NGCB,XICB,RICB,XTCB,RTCB,XECB,RECB,RCICB,RCTCB,ANGICB
     1,ANGECB,XCB(81),YCB(81),XCBI(81),YCBI(81),NXNYCB(81),NCBPTS,IINTCB
     2,IDIFCB,LECB
      COMMON /BCC/ PT(21),TT(21),THETA(21),PE,MASSE,MASSI,MASST,THRUST,N
     1NSTAG
      REAL MN3,NXNY,MASSI,MASST,NXNYCB,MASSE
C
      X3=XE
      ATERM1=0.0
      ATERM2=0.0
      ATERM3=0.0
      DO 160 ICHAR=1,2
      DO 160 M=1,MMAX
      CALL MAP (2,LMAX,M,AL,BE,DE,L1,AL1,BE1,DE1)
      U1=U(LMAX,M,N1)
      U2=U1
      A1=SQRT(GAMMA*P(LMAX,M,N1)/RO(LMAX,M,N1))
      A2=A1
      IF (ICHAR.EQ.2) GO TO 10
      U(LMAX,M,N3)=U1
      A3=A1
C
C     CALCULATE THE PROPERTY INTERPOLATING POLYNOMIAL COEFFICIENTS
C
10    BU=(U(LMAX,M,N1)-U(L1,M,N1))*DXR
      BV=(V(LMAX,M,N1)-V(L1,M,N1))*DXR
      BP=(P(LMAX,M,N1)-P(L1,M,N1))*DXR
      BRO=(RO(LMAX,M,N1)-RO(L1,M,N1))*DXR
      BYCB=(YCB(LMAX)-YCB(L1))*DXR
      BAL=(AL-AL1)*DXR
      BBE=(BE-BE1)*DXR
      BDE=(DE-DE1)*DXR
      CU=U(LMAX,M,N1)-BU*X3
      CV=V(LMAX,M,N1)-BV*X3
      CP=P(LMAX,M,N1)-BP*X3
      CRO=RO(LMAX,M,N1)-BRO*X3
      CYCB=YCB(LMAX)-BYCB*X3
      CAL=AL-BAL*X3
      CBE=BE-BBE*X3
      CDE=DE-BDE*X3
C
C     CALCULATE THE CROSS DERIVATIVE INTERPOLATING POLYNOMIAL
C     COEFFICIENTS
C
      IF (M.EQ.1) GO TO 20
      DU=(U(LMAX,M,N1)-U(LMAX,M-1,N1))*DYR
      DV=(V(LMAX,M,N1)-V(LMAX,M-1,N1))*DYR
      DP=(P(LMAX,M,N1)-P(LMAX,M-1,N1))*DYR
      DRO=(RO(LMAX,M,N1)-RO(LMAX,M-1,N1))*DYR
      DU1=(U(L1,M,N1)-U(L1,M-1,N1))*DYR
      DV1=(V(L1,M,N1)-V(L1,M-1,N1))*DYR
      DP1=(P(L1,M,N1)-P(L1,M-1,N1))*DYR
      DRO1=(RO(L1,M,N1)-RO(L1,M-1,N1))*DYR
      GO TO 40
20    IF (LECB.EQ.LMAX) GO TO 30
      DU=0.0
      DV=V(LMAX,2,N1)*DYR
      DP=0.0
      DRO=0.0
      DU1=0.0
      DV1=V(L1,2,N1)*DYR
      DP1=0.0
      DRO1=0.0
      GO TO 40
```

```
30    DU=(U(LMAX,2,N1)-U(LMAX,1,N1))*DYR
      DV=(V(LMAX,2,N1)-V(LMAX,1,N1))*DYR
      DP=(P(LMAX,2,N1)-P(LMAX,1,N1))*DYR
      DRO=(RO(LMAX,2,N1)-RO(LMAX,1,N1))*DYR
      DU1=(U(L1,2,N1)-U(L1,1,N1))*DYR
      DV1=(V(L1,2,N1)-V(L1,1,N1))*DYR
      DP1=(P(L1,2,N1)-P(L1,1,N1))*DYR
      DRO1=(RO(L1,2,N1)-RO(L1,1,N1))*DYR
40    BDU=(DU-DU1)*DXR
      BDV=(DV-DV1)*DXR
      BDP=(DP-DP1)*DXR
      BDRO=(DRO-DRO1)*DXR
      CDU=DU-BDU*X3
      CDV=DV-BDV*X3
      CDP=DP-BDP*X3
      CDRO=DRO-BDRO*X3
C
C     CALCULATE X1 AND X2
C
      IF (ICHAR.EQ.2) A3=SQRT(GAMMA*P(LMAX,M,N3)/RO(LMAX,M,N3))
      DO 50 IL=1,2
      X1=X3-(U(LMAX,M,N3)+U1)*0.5*DT
      X2=X3-(U(LMAX,M,N3)+A3+U2+A2)*0.5*DT
C
C     INTERPOLATE FOR THE PROPERTIES
C
      U1=BU*X1+CU
      U2=BU*X2+CU
      P2=BP*X2+CP
      RO2=BRO*X2+CRO
      A2=SQRT(GAMMA*P2/RO2)
50    CONTINUE
      V1=BV*X1+CV
      P1=BP*X1+CP
      RO1=BRO*X1+CRO
      YCB1=BYCB*X1+CYCB
      AL1=BAL*X1+CAL
      BE1=BBE*X1+CBE
      DE1=BDE*X1+CDE
      UV1=U1*AL1+V1*BE1+DE1
      A1=SQRT(GAMMA*P1/RO1)
      V2=BV*X2+CV
      YCB2=BYCB*X2+CYCB
      AL2=BAL*X2+CAL
      BE2=BBE*X2+CBE
      DE2=BDE*X2+CDE
      UV2=U2*AL2+V2*BE2+DE2
C
C     INTERPOLATE FOR THE CROSS DERIVATIVES
C
      DV1=BDV*X1+CDV
      DP1=BDP*X1+CDP
      DRO1=BDRO*X1+CDRO
      DU2=BDU*X2+CDU
      DV2=BDV*X2+CDV
      DP2=BDP*X2+CDP
      DRO2=BDRO*X2+CDRO
C
C     CALCULATE THE PSI TERMS
C
      IF (NDIM.EQ.0) GO TO 70
      IF (M.EQ.1.AND.LECB.NE.LMAX) GO TO 60
      ATERM1=RO1*V1/(DY*(M-1)/BE1+YCB1)
      ATERM2=RO2*V2/(DY*(M-1)/BE2+YCB2)
      GO TO 70
60    ATERM1=RO1*BE1*DV1
      ATERM2=RO2*BE2*DV2
70    PSI31=-UV1*DV1-BE1*DP1/RO1
      PSI41=-UV1*DP1+A1*A1*UV1*DRO1
      PSI12=-UV2*DRO2-RO2*AL2*DU2-RO2*BE2*DV2-ATERM2
      PSI22=-UV2*DU2-AL2*DP2/RO2
      PSI42=-UV2*DP2+A2*A2*UV2*DRO2
      IF (ICHAR.EQ.1) GO TO 130
C
C     CALCULATE THE CROSS DERIVATIVES AT THE SOLUTION POINT
C
      IF (M.EQ.1.AND.LECB.NE.LMAX) GO TO 80
      IF (M.EQ.MMAX) GO TO 90
      DU3=(U(LMAX,M+1,N3)-U(LMAX,M,N3))*DYR
      DV3=(V(LMAX,M+1,N3)-V(LMAX,M,N3))*DYR
      DP3=(P(LMAX,M+1,N3)-P(LMAX,M,N3))*DYR
      DRO3=(RO(LMAX,M+1,N3)-RO(LMAX,M,N3))*DYR
      GO TO 100
```

```
80    DU3=0.0
      DV3=V(LMAX,2,N3)*DYR
      DP3=0.0
      DRO3=0.0
      GO TO 100
90    DU3=(U(LMAX,MMAX,N3)-U(LMAX,M1,N3))*DYR
      DV3=(V(LMAX,MMAX,N3)-V(LMAX,M1,N3))*DYR
      DP3=(P(LMAX,MMAX,N3)-P(LMAX,M1,N3))*DYR
      DRO3=(RO(LMAX,MMAX,N3)-RO(LMAX,M1,N3))*DYR
C
C     CALCULATE THE PSI TERMS AT THE SOLUTION POINT
C
100   IF (NDIM.EQ.0) GO TO 120
      IF (M.EQ.1.AND.LECB.NE.LMAX) GO TO 110
      ATERM3=RO(LMAX,M,N3)*V(LMAX,M,N3)/(DY*(M-1)/BE+YCB(LMAX))
      GO TO 120
110   ATERM3=RO(LMAX,1,N3)*BE*DV3
120   UV3=U(LMAX,M,N3)*AL+V(LMAX,M,N3)*BE+DE
      PSI13=-UV3*DRO3-RO(LMAX,M,N3)*(AL*DU3+BE*DV3)-ATERM3
      PSI23=-UV3*DU3-AL*DP3/RO(LMAX,M,N3)
      PSI33=-UV3*DV3-BE*DP3/RO(LMAX,M,N3)
      PSI43=-UV3*DP3+A3*A3*UV3*DRO3
C
C     CALCULATE THE COMPATIBILITY EQUATION COEFFICIENTS
C
130   IF (ICHAR.EQ.1) GO TO 140
      PSI31B=(PSI31+PSI33)*0.5
      PSI41B=(PSI41+PSI43)*0.5
      PSI12B=(PSI12+PSI13)*0.5
      PSI22B=(PSI22+PSI23)*0.5
      PSI42B=(PSI42+PSI43)*0.5
      GO TO 150
140   PSI31B=PSI31
      PSI41B=PSI41
      PSI12B=PSI12
      PSI22B=PSI22
      PSI42B=PSI42
C
C     SOLVE THE COMPATIBILITY EQUATIONS FOR U,V,P, AND RO
C
150   P(LMAX,M,N3)=PE
      RO(LMAX,M,N3)=RO1+2.0*(P(LMAX,M,N3)-P1-DT*PSI41B)/(A3*A3+A1*A1)
      U(LMAX,M,N3)=U2+((PSI42B+(RO2+RO(LMAX,M,N3))*(A2+A3)*PSI22B/4.0+(A
     1 2*A2+A3*A3)*PSI12B/2.0)*DT-(P(LMAX,M,N3)-P2))/(RO2+RO(LMAX,M,N3))/
     2(A2+A3)*4.0
      V(LMAX,M,N3)=V1+DT*PSI31B
160   CONTINUE
      V(LMAX,MMAX,N3)=-U(LMAX,MMAX,N3)*NXNY(LMAX)
      V(LMAX,1,N3)=-U(LMAX,1,N3)*NXNYCB(LMAX)
      IF (JFLAG.EQ.0) RETURN
      V(LMAX,MMAX,N3)=V(LMAX,MMAX,N3)+XWI(LMAX)
      RETURN
      END
```